

QUALIFYING EXAM IN GEOMETRY AND TOPOLOGY, SUMMER 2015

Part I: Algebraic Topology

- (1) Consider an annulus. Identify the antipodal points on the outer circle. Also identify the points on the inner circle that are 120 degree (i.e. $2\pi/3$) apart. Compute the homology and cohomology (with integer coefficients) of this quotient space obtained from the operations above.
- (2) Let G be a finite graph
 - (a) Prove that $\pi_1(G)$ is a finitely generated free group
 - (b) Prove any finite index subgroup of a finitely generated free group is also a finitely generated free group.
 - (c) Prove that if F is a finitely generated free group and N is a nontrivial normal subgroup of infinite index, then N is not finitely generated.
 - (d) Show that a finitely generated free group has only a finite number of subgroups of a given finite index.
- (3) Let Σ_g be the closed orientable surface of genus g . Prove that there is an n -sheeted covering $M_g \rightarrow M_h$ if and only if $g = n(h - 1) + 1$.
- (4) Let A be the union of two once-linked circles in S^3 ; let B be the union of two unlinked circles in S^3 , as shown in the picture. Show that the cohomology groups of $S^3 - A$ and $S^3 - B$ are isomorphic, but the cohomology rings are not.

