ANALYSIS QUALIFYING EXAM

JANUARY 19, 2012

REAL ANALYSIS

Answer all 4 questions. In your proofs, you may use any major theorem, except the fact you are trying to prove (or a variant of it). State clearly what theorems you use. Good luck.

Question 1
a) Let $f \in L^1(m)$ and $F(x) = \int_{-\infty}^{x} f(t)dt$. Then F is continuous.
b) Show for $a > 0$ $\int_{-\infty}^{\infty} e^{-x^2} \cos(ax)\ dx = \sqrt{\pi} e^{-a^2/4}$.

Question 2
State and prove the Hahn-Decomposition Theorem for signed measures.

Question 3
Let $1 \leq p < \infty$ and m be Lebesgue measure. Let M be a closed subspace of $L^p([0,1], m)$ such that M is contained in the space of continuous functions $C([0,1])$ (i.e. every element of M has a continuous function in its equivalence class, which is necessarily unique). Show that there exists a $C_p > 0$ such that for all $f \in M$

$$||f||_u \leq C_p.||f||_p$$

where $||.||_u$ is the sup norm on $C([0,1]) \subseteq L^p([0,1], m)$.

Question 4
a) Prove the uniqueness of a (left-invariant) Haar measure on a locally compact hausdorff topological group.
b) Prove that Haar measure for a compact group or abelian group is both left and right invariant.