ALGEBRA QUALIFYING EXAM SYLLABUS 2015

RECOMMENDED TEXTBOOKS

• Abstract Algebra by Dummit and Foote
• Introduction to Commutative Algebra by Atiyah and Macdonald
• Algebra by Lang.

TOPICS COVERED

I. Groups.
   • Group actions, orbit-stabilizer theorem, Sylow theorems, semi-direct products, Jordan-Holder theorem.
   • Examples: symmetric, alternating, dihedral groups, general and special linear groups.

II. Linear algebra.
   • Modules over a PID, elementary divisor theorem.
   • Invariant factors and similarity classes of matrices.
   • Jordan and rational canonical forms, Cayley-Hamilton theorem.

III. Fields.
   • Polynomial rings, Gauss lemma, Eisenstein criterion.
   • Finite fields: construction, classification, structure of the units.
   • Normal and separable extensions, Galois groups and the Galois correspondence.
   • Computing Galois groups of low degree extensions, cyclotomic fields.
   • Discriminants, symmetric polynomials, insolvability of the general quintic.
   • Transcendence degree.

IV. Rings and commutative algebra.
   • Noetherian and Artinian rings and modules.
   • Discrete valuation rings, local rings, localization, Nakayama’s lemma.
   • Primary decomposition.
   • Integral extensions. Going-up and going-down theorems.

V. Modules and homological algebra.
   • Tensor product of modules and algebras.
   • Exact sequences. Projective, injective, flat modules.
   • Complexes. Projective and injective resolutions, Ext and Tor.
   • Localization of modules.

VI. Algebraic Geometry.
   • Zariski topology, Spec of a commutative ring, algebraic sets in affine space.
   • Hilbert’s Nullstellensatz, Noether Normalization, Krull dimension.

VII. Algebraic Number Theory.
   • Algebraic integers, discriminants.
   • Prime factorization in Dedekind rings.