1. Classify the groups of order 24 having trivial center.

2. Let $V$ be a finite dimensional vector space over a field $F$ of characteristic $p$ and let $T : V \to V$ be a linear transformation such that $T^p = I$, the identity transformation of $V$.
   a) Determine the eigenvalues of $T$.
   b) Show that there is a basis of $V$ for which the matrix of $T$ is upper triangular,

3. Let $G$ be the group with presentation
   
   $$G = \langle x, y \mid x^4 = 1, x^2 = y^3 \rangle,$$


4. Let $\zeta = e^{\pi i/10}$. Find all the subfields $K \subset \mathbb{Q}(\zeta)$ such that $[K : \mathbb{Q}] = 2$ and express each of them in the form $K = \mathbb{Q}(\sqrt{d})$ for some $d \in \mathbb{Z}$.

5. Suppose $n$ and $m$ are positive integers, let $R = \mathbb{Z}[X]/(X^n)$ and let $M$ be an $R$-module, let $x$ be the image of $X$ in $R$, and let $(x^m)$ be the ideal in $R$ generated by $x^m$. Compute $\text{Tor}_i^R(M, (x^m))$ for all $i$.

6. Let $k$ be a field. Find the minimal primes and compute the Krull dimension of $R = k[x, y, z]/(xy, xz)$.

7. Let $R$ be an Artinian local ring. Prove that an $R$-module is flat if and only if it is free.

8. Suppose that $R$ is a Noetherian ring and $p \subset R$ is a prime ideal such that $R_p$ is an integral domain. Show that there is an $f \notin p$ such that $R_f$ is an integral domain. (Recall that $R_f = S^{-1}R$ where $S = \{1, f, f^2, f^3, \ldots \}$.)