1. Classify the groups of order 105, up to isomorphism, and give a presentation of each group.

2. 
   a) Compute $\text{Ext}_i^K(\mathbb{Z}/6\mathbb{Z}, \mathbb{Z}/15\mathbb{Z} \oplus \mathbb{Z}/15\mathbb{Z})$ for all $i$.
   b) Compute $\text{Tor}_i^K(\mathbb{Z}/6\mathbb{Z}, \mathbb{Z}/15\mathbb{Z} \oplus \mathbb{Z}/15\mathbb{Z})$ for all $i$.

3. Let $W$ denote the unique irreducible two-dimensional complex representation of the symmetric group $S_3$. Determine the dimensions and multiplicities of the irreducible constituents of $\text{Ind}_{S_3}^K W$.

4. Suppose $R$ is a commutative local ring, and $M$ is a finitely generated $R$-module. Prove that $M$ is projective if and only if $M$ is free. Hint: you may use any form of Nakayama’s Lemma you like, provided you first state it correctly.

5. Let $f \in \mathbb{Z}[x]$ be an irreducible monic polynomial, let $K$ be a splitting field of $f$ and let $\alpha \in K$ be a root of $f$. Assume the Galois group $\text{Gal}(K/\mathbb{Q})$ is abelian.
   a) Prove that $K = \mathbb{Q}(\alpha)$.
   b) Assume there is a prime $p$ such that the image of $f$ in $\mathbb{F}_p[x]$ is irreducible. Determine the structure of $\text{Gal}(K/\mathbb{Q})$.

6. Let $K = \mathbb{C}(x)$ be the field of rational functions in one variable $x$. Fix an integer $n \geq 2$ and let $F \subset K$ be the field of rational functions fixed by the two automorphisms $\sigma : x \mapsto e^{2\pi i/n}x, \quad \tau : x \mapsto x^{-1}$.
   a) Determine the structure of the Galois group $\text{Gal}(K/F)$.
   b) Show that $F = \mathbb{C}(t)$, where $t = x^n + x^{-n}$, and determine the minimal polynomial of $x$ over $F$.

7. Show that every finite subgroup of $\text{GL}_2(\mathbb{Q})$ has order $2^a 3^b$ for some $a$ and $b$.

8. Let $F$ be a subfield of $\mathbb{C}$ that is finite and Galois over $\mathbb{Q}$. Suppose $\alpha \in F$ is an algebraic integer with the property that every Galois conjugate has complex absolute value 1. Prove that $\alpha$ is a root of unity. [Hint: show that the set of all such $\alpha$ in $F$ is finite.]