Should Central Banks Target Investment Prices?*

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Abstract

Central banks nearly always state explicit or implicit inflation targets in terms of consumer price inflation. If there are nominal rigidities in the pricing of both consumption and investment goods and if the shocks to the two sectors are not identical, then monetary policy cannot stabilize inflation and output gaps in both sectors. Thus, the central bank faces a tradeoff between targeting consumption price inflation and investment price inflation. In this setting, ignoring investment prices typically leads to substantial welfare losses because the intertemporal elasticity of substitution in investment is much higher than in consumption. In our calibrated model, consumer price targeting leads to welfare losses that are at least three times the loss under optimal policy. A simple rule that puts equal weight on stabilizing consumption and investment price inflation comes close to replicating the optimal policy. Thus, GDP deflator targeting is not a good approximation to optimal policy. A shift in monetary policy to targeting a weighted average of consumer and investment price inflation may produce significant welfare gains, although this would constitute a major change in current central banking practice.

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1 Introduction

Since the 1990s, many central banks around the world have adopted inflation targeting as a means of conducting monetary policy and communicating policy commitments to the public. Over the years, most countries that have adopted an inflation targeting policy state their target in terms of an index of consumer price inflation.[1] In a recent blog post, Bernanke (2015) writes, "In practice, the FOMC has long been clear that its preferred measure of inflation is the rate of change in consumer prices, as reflected specifically in the deflator for personal consumption expenditures (PCE)." Later in the same post, he continues, "Frankly, I don't think there is much of a case for ... using the GDP deflator to measure inflation rather than using overall or core PCE inflation."

Bernanke summarizes the current state of central bank thinking about the appropriate price index to target. We believe this consensus should be examined using economic models. We present a model showing that targeting consumer prices alone leads to significantly larger welfare losses than targeting an index of consumer and investment-goods prices. We find that optimal policy in our model puts much *more* weight on investment price inflation than does the GDP deflator. In fact, even though investment is only about 20 percent of GDP, in our calibrated model the central bank should put approximately equal weight on stabilizing consumer and investment price inflation.

Why do we come to these conclusions? As we will show, targeting consumer prices alone would be innocuous if either investment goods prices were fully flexible, or if shocks to consumer and investment demand were symmetric.[2] The evidence does not appear to support either hypothesis. Our model has nominal rigidities for both consumption and investment goods and imperfectly correlated shocks to the two sectors, which reproduces key features of the data. Optimal policy in the model requires that the central bank should target

[1] The Federal Reserve targets inflation in the core PCE index, but also closely tracks CPI and PPI inflation. The primary objective of the European Central Bank is price stability and its Governing Council has announced that "price stability is defined as a year-on-year increase in the Harmonised Index of Consumer Prices (HICP) for the euro area of below 2%." For comprehensive overviews of the inflation indices targeted by monetary authorities around the world, see Svensson and Leiderman (1995), Bernanke et al. (1999), and Svensson (2010).

[2] To our knowledge, nearly all New Keynesian models with investment make one of these two assumptions, the exceptions being Basu et al. (2014) and Ikeda (2015). For example, models based on Smets and Wouters (2007) implicitly assume a flexible relative price for investment goods, since an investment-specific technology improvement in their framework immediately increases the number of investment goods that can be obtained by foregoing one consumption good.

investment prices. Furthermore, in the model omitting the investment price from the central bank's target leads to substantial welfare losses.

The intuition for the quantitative result stems from an important economic difference between consumption and investment goods: the intertemporal elasticity of substitution (IES) is likely to be much higher for investment than for consumption demand. The IES for consumption is relatively small – typical estimates put it around 0.5. Under a plausible set of assumptions, the IES in investment is nearly infinite.[3] Thus, small changes in the own real interest rate for investment due to expected changes in the price of investment goods have huge effects on investment demand, which is not the case for consumption. This difference in the IES leads to the asymmetry in optimal policy: in order to keep outcomes close to the social optimum, it is more important to avoid fluctuations in investment price inflation than in consumption price inflation.

Since our argument depends on the existence of nominal rigidities in the prices of investment goods, it is fair to ask whether the evidence supports this assumption. Available microeconomic evidence points to substantial price stickiness in several categories of investment goods: for example, Nakamura and Steinsson (2008) and Alvarez et al. (2006)). So does macroeconomic evidence. For example, Basu et al. (2013) find that consumption technology improvements lead to expansions in consumption, investment and hours, but investment technology improvements lead all three key variables to decline in the short run.[4] Basu et al. (2014) find that nominal rigidities in both the consumption sector and the investment sector are crucial for explaining the observed asymmetry of impulse responses to sector-specific technology shocks, as well as the evidence that the relative price of investment adjusts slowly to relative technology shocks. For our results to hold, in addition to nominal rigidities, we require that shocks to investment and consumption technology not be perfectly symmetric. *Prima facie* evidence to this effect is provided by the fact that the relative price of investment goods not only has a trend, but also fluctuates over time relative to this trend (in simple models, the relative price of investment reflects relative technology, at least over long periods of time). In addition, using Basu et al.'s (2013) series for sectoral technologies we find that investment-specific technology shocks are distinctly more volatile than consumption-specific technology shocks and the correlation between them is positive but far from one.

[3] See Barsky et al. (2007).

[4] The importance of investment technology shocks for aggregate fluctuations is stressed by Greenwood et al. (2000), Fisher (2006) and Justiniano et al. (2010).

The purpose of this paper is to investigate the optimal conduct of monetary policy in an economy characterized by shocks to both consumption and investment sectors when these shocks are not identical. To this end, we develop a micro-founded welfare criterion that allows normative analysis, and we discuss the nature of the trade-off that the central bank confronts. We characterize the properties of the optimal policy under commitment, and compare the welfare properties of alternative monetary policy rules.

Our framework is a two-sector, closed-economy model in which one sector produces non-durable consumption goods and the other produces investment goods. Prices in both markets are subject to the Calvo pricing friction. Labor and capital are immobile across sectors, and nominal wages are assumed to be flexible. Following the influential analysis of Rotemberg and Woodford (1997), we obtain a quadratic approximation to the social welfare function, and show that the deviation of welfare from its Pareto-optimal level depends not only on the variances of the consumption gap and CPI inflation, but also on the variances of the investment spending gap and investment price inflation. With two nominal frictions and only one instrument, the central bank is confronted with a nontrivial trade-off: indeed, it is generally impossible to stabilize inflation and the output gaps in both sectors simultaneously.[5]

We document that the optimal policy represents a compromise between the sectoral welfare losses. Most importantly, we find that the second-best policy places disproportionately high weight on the investment sector, notwithstanding its small relative size in the economy. Moreover, we find that monetary rules that ignore investment price inflation incur sizable welfare losses. In our calibrated model, a rule that targets only consumer price inflation leads to average welfare losses that are considerably larger than those obtained under an alternative simple rule that targets only investment price inflation.

Extending our study of simple rules, we study the behavior of a hybrid rule that responds only to the two sectoral inflation rates, and find that its performance is nearly optimal if the weights on the two inflation rates are chosen correctly. We find that such a rule places considerable weight on investment inflation, and thus its performance cannot be approximated by a Taylor rule that targets the GDP deflator. Finally, we document that our results are

[5]As an aside, we note that our model does not display what Blanchard and Galí (2007) term the "divine coincidence," even though we assume that both real and nominal wages are fully flexible. This result shows that the divine coincidence result obtains only due to the strong auxiliary assumption that all production functions in the economy are hit by the same technology shock. Since this assumption is clearly unrealistic and made only for modeling simplicity, "divine coincidence" is probably not important for the conduct of monetary policy.

robust to empirically plausible calibrations of the degree of price stickiness in the investment sector.

The literature on optimal monetary policy is vast, but most of its conclusions are drawn using models that abstract from capital accumulation. Important works include Erceg and Levin (2006), Huang and Liu (2005), Aoki (2001), and Benigno (2004). Erceg and Levin investigate the optimal monetary policy properties of a model that features non-durable and durable consumption goods. Consistent with our results, they find that the monetary authority should over-weight durable consumption goods prices in the price index that it targets.[6] Huang and Liu document the importance of targeting the producer price index (PPI) besides the CPI, in a model that features an input-output production structure. Aoki presents a model with a sticky-price sector and an otherwise identical flexible-price sector, and show that the optimal monetary policy is to target sticky-price inflation, rather than a broad inflation measure. This result is generalized by Benigno, who shows that in a two-country model an inflation targeting policy in which higher weight is given to the inflation in the region with higher degree of nominal rigidity is nearly optimal.[7] Mankiw and Reis (2003) take an analytical approach to characterizing the optimal price index that a central bank should target as a function of a number of sectoral characteristics; their main conclusion is that central banks should assign a high weight to the nominal wage rate.[8] In a recent related paper Ikeda (2015) shows that a trend towards a falling relative price of investment may increase the optimal inflation target up to around 2%. His work focuses on deriving the optimal inflation target rate on CPI inflation, in the context of an estimated medium-scale two-sector model with a downward trend in investment prices.

The paper is structured as follows. Section 2 outlines the dynamic general equilibrium model. Section 3 describes the solution method and parameter calibration. Section 4 discusses the second-order approximation to the welfare function. Section 5 examines characteristics of the optimal policy, and evaluates the performance of alternative rules. Section 6 presents conclusions and suggests directions for future research.

[6] See also the recent contribution by Barsky et al. (2016)

[7] The results of Aoki and Benigno thus generally support the Federal Reserve's procedure of targeting "core" rather than "headline" inflation.

[8] This particular conclusion depends on their belief that the average allocative wage in the US is very sticky in nominal terms. For evidence to the contrary, see Basu and House (2016).

2 The model

Our model consists of two sectors that produce non-durable consumption goods and investment goods. Labor-augmenting technology in both sectors has a trend, and both production functions are subject to stationary TFP fluctuations around that trend. Thus, we allow for the existence of a trend in the steady-state relative price of investment. The relative price of equipment investment has consistently declined over the past four decades. Both product markets exhibit monopolistic competition, and nominal prices do not change continuously. Each household has two types of workers that are permanently attached to their respective productive sectors. Households display separable preferences in the consumption good and in hours worked supplied to the two sectors. Labor markets are competitive. To finance consumption, households invest in riskless bonds and hold the sectoral capital stocks.

2.1 Firms

The model economy features two distinct sectors producing non-durable consumption goods (sector c) and investment goods (sector i). Each sector comprises a continuum of monopolistically competitive firms producing differentiated products. Let $Y_{j,t}$ denote sector-j output (real value added), for $j = \{c, i\}$:

$$Y_{j,t} = \left[\int_0^1 \left(Y_{j,t}(f) \right)^{\frac{1}{1+\theta_j}} df \right]^{1+\theta_j}$$

where $\theta_j > 0$ denotes the markup rate in the production composite of sector j. The aggregator chooses the bundle of goods that minimizes the cost of producing a given quantity of the sectoral output index $Y_{j,t}$, taking the price $P_{j,t}(f)$ of each good $Y_{j,t}(f)$ as given. The aggregator sells units of each sectoral output index at its unit cost $P_{j,t}$:

$$P_{j,t} = \left[\int_0^1 \left(P_{j,t}(f) \right)^{-\frac{1}{\theta_j}} df \right]^{-\theta_j}$$

It is natural to interpret $P_{j,t}$ as the price index for real value added in each sector. Thus, $P_{c,t}$ can be interpreted as the deflator for Personal Consumption Expenditures (PCE) or the Consumer Price Index (CPI).[9] $P_{i,t}$, on the other hand, denotes the price level in the

[9] In our model of a closed economy, the two indexes are identical up to a first-order approximation, which is the order to which we will analyze the model.

investment sector. One can also define an aggregate price index P_t as:

$$P_t = (P_{c,t})^{\phi} (P_{i,t})^{1-\phi}$$

where ϕ is the steady-state output share of consumption. The aggregate price index, P_t, is, to a first-order approximation, the GDP deflator of this model economy.

Households' demand for each good is given by:

$$Y^d_{j,t}(f) = \left[\frac{P_{j,t}(f)}{P_{j,t}}\right]^{-\frac{1+\theta_j}{\theta_j}} Y_{j,t} \tag{1}$$

Each differentiated good is produced by a single firm that hires capital services $K_{j,t}(f)$ and a labor index $L_{j,t}(f)$ defined below. All firms within each sector face the same Cobb-Douglas production function, with an identical level of technology $Z_{j,t}$ and labor-augmenting technical progress $\Gamma_{j,t}$:

$$Y_{j,t}(f) = Z_{j,t} (K_{j,t}(f))^{\alpha} (\Gamma_{j,t} L_{j,t}(f))^{1-\alpha}$$

We thus separate total factor productivity (TFP) into two components: Z, the part that is subject to shocks, and Γ, which grows steadily at a constant rate. Capital and labor are perfectly mobile across the firms within each sector, but cannot be relocated between sectors. Each firm chooses $K_{j,t}(f)$ and $L_{j,t}(f)$, taking as given the sectoral wage index $W_{j,t}$, and the sectoral rental price of capital $P_{c,t} r^k_{j,t}$, where $r^k_{j,t}$ is the sectoral real rental rate of capital in units of consumption goods. The conditional factor demand functions derived from the cost-minimization problem for labor and capital are, respectively:

$$W_{j,t} = MC_{j,t}(f)(1-\alpha)\frac{Y_{j,t}(f)}{L_{j,t}(f)}$$

$$P_{c,t} r^k_{j,t} = MC_{j,t}(f) (\alpha) \frac{Y_{j,t}(f)}{K_{j,t}(f)}$$

Since capital and labor can flow freely across firms *within* the same sector, and production functions feature constant returns to scale, all firms within each sector have identical nominal marginal costs per unit of output, which are given by:

$$MC_{j,t} = \frac{\tilde{\alpha}}{Z_{j,t}\Gamma_{j,t}} (P_{c,t} r^k_{j,t})^{\alpha} (W_{j,t})^{1-\alpha} \tag{2}$$

where $\tilde{\alpha} \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$.

We follow Calvo (1983) and assume that firms change their nominal prices only occasionally, and the probability that a firm changes its price is constant. Once a price is set, the firm must supply its differentiated product to meet market demand at the posted price. We follow Yun (1996) in assuming that the new price set in a generic period t is indexed to trend inflation.[10] Hence, even if the firm is not allowed to reoptimize its price, the preset price grows at the rate of trend inflation.

In Appendix A we show that, under full indexation to trend inflation, the steady state coincides with the flexible-price steady state: each firm sets its price as a constant markup over marginal cost. This assumption guarantees that the steady state is not distorted by trend inflation. Also, in the steady state the monopolistic markup is completely offset by the subsidy τ_j. This assumption guarantees that the steady state of the model is not distorted by imperfect competition. Furthermore, full indexation to trend inflation guarantees that, in each sector, the log-linearized Phillips Curve is identical to the one obtained under zero steady-state inflation.[11]

2.2 Households

We assume that there is a continuum of households indexed on the unit interval, and that each household supplies homogeneous labor services. Within every household, a fixed number of ν_c members work exclusively in the consumption sector, while the remaining ν_i members work exclusively in the investment sector. Each member of a given household $h \in [0,1]$ who works in sector $j = \{c, i\}$ has the same wage rate $W_{j,t}(h)$ and supplies the same number of hours $N_{j,t}(h)$. Households have no market power in the labor market, and the aggregation in each sectoral labor market is given by:

$$L_{j,t} = \nu_j \int_0^1 N_{j,t}(h)dh$$

In other words, a representative labor aggregator combines individual labor hours into a sectoral labor index $L_{j,t}$ using the same proportions that firms would choose. As a result of competitive labor markets, the sectoral wage index is the same across households within sector; that is, $W_{j,t}(h) = W_{j,t}$.

[10]Trend inflation in our model is induced by the presence of the trend component of TFP, Γ.

[11]For an analysis of the implications of trend inflation for New Keynesian models, see Ascari (2004).

In each period, the household purchases $Y_{c,t}$ (or, equivalently, C_t) units of consumption goods at price $P_{c,t}$, and $Y_{i,t}$ (or I_t) units of investment goods at price $P_{i,t}$. Investment contributes to the formation of new capital stock in either consumption or investment sector; that is, $I_t = I_{c,t} + I_{i,t}$. Thus, the households face the following intertemporal budget constraint:

$$\begin{aligned} P_{c,t}C_t + P_{i,t}I_t + \mathbb{E}_t D_{t,t+1}B_{t+1} \leq\ & W_{c,t}(h)N_{c,t}(h) + W_{i,t}(h)N_{i,t}(h) + \\ & + P_{c,t}r^k_{c,t}K_{c,t} + P_{c,t}r^k_{i,t}K_{i,t} + \Pi_t + B_t - T_t \end{aligned} \tag{3}$$

where B_{t+1} is a nominal state-contingent bond that represents a claim to one dollar in a particular event in period $t+1$, and this claim costs $D_{t,t+1}$ dollars in period t; $W_{j,t}(h)$ is sector-j nominal wage, $K_{j,t}$ is the beginning-of-period capital stock in sector j, Π_t is the profit share, and T_t is a lump-sum tax used by the government to finance subsidies to firms.

The capital stock in each sector evolves according to the following law of motion:

$$K_{j,t+1} = (1-\delta)K_{j,t} + \Psi\left(\frac{I_{j,t}}{K_{j,t}}\right)K_{j,t} \tag{4}$$

where the function $\Psi(\cdot)$ represents the adjustment cost in capital accumulation. We assume that $\Psi\left(\frac{I_j}{K_j}\right)$ satisfies $\Psi\left(\frac{I_j}{K_j}\right) = \frac{I_j}{K_j}$, $\Psi'\left(\frac{I_j}{K_j}\right) = 1$, and $\Psi''\left(\frac{I_j}{K_j}\right) = -\psi$ where $\psi > 0$, and $\frac{I_j}{K_j}$ is the share of investment to capital in sector j in steady state.

The household's expected lifetime utility is given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \mathbb{W}_{t+s}(h) \tag{5}$$

where the operator $\mathbb{E}_t$ here represents the conditional expectation over all states of nature, and the discount factor satisfies $0 < \beta < 1$. The period household utility function $\mathbb{W}_t(h)$ is additively separable with respect to the household's consumption C_t and the leisure of each household member:

$$\mathbb{W}_t = \mathbb{U}(C_t) - \mathbb{V}^c(N_{c,t}(h)) - \mathbb{V}^i(N_{i,t}(h)) \tag{6}$$

The subutility functions are defined as follows:

$$\mathbb{U}(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$$

$$\mathbb{V}^c(N_{c,t}) = \upsilon_c \frac{N_{c,t}(h)^{1+\eta}}{1+\eta}$$

$$\mathbb{V}^c(N_{i,t}) = \upsilon_i \frac{N_{i,t}(h)^{1+\eta}}{1+\eta}$$

where the parameters σ, υ_c, υ_i and η are all strictly positive.

Each household h maximizes (5) with respect to each of its components, subject to the budget constraint in (3) and the capital laws of motion in each sector in (4). The first order conditions for the utility-maximizing problem are given by

$$P_{c,t}\lambda_t = C_t^{-\sigma} \tag{7}$$

$$\upsilon_j N_{j,t}(h)^\eta = \frac{w_{j,t}}{C_t^\sigma} \tag{8}$$

$$1 = \beta \mathbb{E}_t \left[\frac{R_t^n}{\Pi_{c,t+1}} \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \right] \tag{9}$$

$$q_{i,t} = q_{j,t}^k \left[\Psi' \left(\frac{I_{j,t}}{K_{j,t}} \right) \right] \tag{10}$$

$$q_{j,t}^k = \beta \mathbb{E}_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left\{ q_{j,t+1}^k \left[1 - \delta + \Psi \left(\frac{I_{j,t+1}}{K_{j,t+1}} \right) - \Psi' \left(\frac{I_{j,t+1}}{K_{j,t+1}} \right) \left(\frac{I_{j,t+1}}{K_{j,t+1}} \right) \right] + r_{j,t+1}^k \right\} \tag{11}$$

where $w_{j,t} = \frac{W_{j,t}}{P_{c,t}}$ is the j-sector real wage, R_t^n is the time-t gross nominal interest rate, $\mathbb{E}_t \Pi_{c,t+1}$ represents the expected gross inflation rate in the consumption sector.

Denote by $q_{j,t}^k = \frac{\lambda_{j,t}^k}{\lambda_t P_{c,t}}$ the shadow value of j-sector capital stock in units of consumption goods. Then, Equations (10) and (11) become:

$$q_{i,t} = q_{j,t}^k \left[\Psi' \left(\frac{I_{j,t}}{K_{j,t}} \right) \right]$$

$$q_{j,t}^k = \beta \mathbb{E}_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left\{ q_{j,t+1}^k \left[1 - \delta + \Psi \left(\frac{I_{j,t+1}}{K_{j,t+1}} \right) - \Psi' \left(\frac{I_{j,t+1}}{K_{j,t+1}} \right) \left(\frac{I_{j,t+1}}{K_{j,t+1}} \right) \right] + r_{j,t+1}^k \right\} \tag{12}$$

where the last equation makes use of Equation (7) to eliminate the Lagrange multiplier λ_t.

The left-hand side of (12) represents the cost of acquiring a marginal unit of sector-j capital today; the right-hand side captures the benefit of holding one extra unit of sector-j capital which consists of the expected discounted future resale value and the expected rental value. The discount factor is the intertemporal marginal utility of consumption.

2.3 Fiscal and Monetary Policy

The government's budget is balanced in every period: lump-sum taxes equal output subsidies period by period. For simplicity, we assume that there are no government purchases.

In our analysis, the short-term nominal interest rate is used as the instrument of monetary policy, and we assume that the policymaker is able to commit to a time-invariant rule. We consider alternative specifications of the monetary policy rule. As discussed by Sveen (2014), all the rules that we will consider ensure model determinacy for plausible calibrations of the parameter that governs investment price stickiness.

2.4 Market Clearing

In equilibrium, the markets for bonds, consumption, investment, capital rentals and labor all clear. Bond market clearing implies that $B_t = 0$ for all t. Labor market clearing implies that $\int_0^1 L_{c,t}(f)df + \int_0^1 L_{i,t}(f)df = L_t$. Capital market clearing implies $\int_0^1 K_{c,t}(f)df + \int_0^1 K_{i,t}(f)df = K_t$. Goods market clearing in the two sectors implies that $Y_{c,t} = C_t$ and $Y_{i,t} = I_t$, where $I_t = I_{c,t} + I_{i,t}$. Consistent with the current procedure of the National Income and Product Accounts (NIPA), real GDP is defined as $Y_t = (C_t)^{\phi} (I_t)^{1-\phi}$, where ϕ is the expenditure share of consumption.

3 Solution and Calibration

Given fiscal and monetary policy, an equilibrium in this economy consists of prices and allocations such that (i) taking prices and real wages as given, each household's allocation solves its utility maximization problem; (ii) taking wages and all other firms' prices as given, each firm's factor demands and output price solve its profit maximization problem; (iii) the markets for bonds, labor, capital, and sectoral outputs all clear.

The trends in sectoral technologies render the model nonstationary. We focus on a stationary equilibrium with balanced growth in which output, consumption, investment, the capital stock, and the real wage all grow at constant rates, while hours worked remain constant. Furthermore, if trend growth in technology in the investment sector is faster than its consumption-sector counterpart, investment and capital will grow at a faster rate than consumption or GDP, and the relative price of investment will have a downward trend. The relative price of investment is defined in changes by:

$$q_{i,t} = q_{i,t-1} + \pi_{i,t} - \pi_{c,t} \tag{13}$$

Stationarity is obtained by appropriate variable transformations (see Appendix B). We study the dynamic properties of the model by taking a log-linear approximation of the equilibrium conditions around the balanced growth path (the resulting model log-linearized behavioral equations are listed in Table 1).

To close the model, we have to specify sectoral inflation dynamics. Sectoral Phillips curves are expressed by:

$$\pi_{c,t} = \tilde{\beta}\mathbb{E}_t\pi_{c,t+1} + \kappa_c v_{c,t} \tag{14}$$

$$\pi_{i,t} = \tilde{\beta}\mathbb{E}_t\pi_{i,t+1} + \kappa_i v_{i,t} \tag{15}$$

where the growth-adjusted discount factor $\tilde{\beta} = \beta(\gamma_c^{1-\alpha}\gamma_i^{\alpha})^{1-\sigma}$, $\kappa_j = \frac{(1-\xi_j)(1-\xi_j\tilde{\beta})}{\xi_j}$, and $v_{j,t}$ denotes sector-j real marginal costs. C-sector real marginal costs are expressed in consumption units, whereas i-sector real marginal costs are expressed in investment units.

As shown in Appendix C, under Woodford's (2003) definition of the natural rate,[12] sectoral real marginal costs can be rewritten as a function of consumption, investment, and relative price gaps:

$$v_{c,t} = \left[\frac{(1+\eta)}{1-\alpha} - (1-\sigma)\right]\tilde{c}_t$$

$$v_{i,t} = \left(\frac{\alpha+\eta}{1-\alpha}\right)\tilde{i}_t + \sigma\tilde{c}_t - \tilde{q}_{i,t}$$

where $\tilde{x}_t$ denotes the devation of variable x_t from its flexible-price counterpart, that is $x_t - x_t^*$. Then, the sectoral Phillips curves in (14) and (15) can be expressed as:

$$\pi_{c,t} = \tilde{\beta}\mathbb{E}_t\pi_{c,t+1} + \kappa_c\left[\left(\frac{\alpha+\eta}{1-\alpha} + \sigma\right)\tilde{c}_t\right] \tag{16}$$

$$\pi_{i,t} = \tilde{\beta}\mathbb{E}_t\pi_{i,t+1} + \kappa_i\left[\left(\frac{\alpha+\eta}{1-\alpha}\right)\tilde{i}_t + \sigma\tilde{c}_t - \tilde{q}_{i,t}\right] \tag{17}$$

Thus, as in the standard one-sector New Keynesian model, sectoral inflation dynamics respond to their respective measures of slack. In fact, the consumption-sector Phillips curve

[12] There exist two distinct definitions of the natural rate of output in New Keynesian models with endogenous capital accumulation. The difference between the two is discussed in Section 4.

Production functions	$y_{j,t} = z_{j,t} + \alpha k_{j,t} + (1-\alpha)h_{j,t}$
C-sector real marginal costs	$v_{c,t} = \alpha r^k_{c,t} + (1-\alpha)w_{c,t} - z_{c,t}$
I-sector real marginal costs	$v_{i,t} = \alpha r^k_{i,t} + (1-\alpha)w_{i,t} - z_{i,t} - q_{i,t}$
Factors demands	$w_{j,t} = r^k_{j,t} + k_{j,t} - h_{j,t}$
Labor-supply schedules	$h_{j,t} = (1/\eta)(w_{j,t} - \sigma c_t)$
Consumption Euler equation	$\mathbb{E}_t \Delta c_{t+1} = (1/\sigma)(R_t - \mathbb{E}_t \pi_{c,t+1})$
Shadow value of Capital	$q^k_{j,t} = q_{i,t} + \psi\left(\tilde{\delta}\gamma_i\right)(i_{j,t} - k_{j,t})$
Investment demands	$q^k_{j,t} = -\sigma \mathbb{E}_t \Delta c_{t+1} + \tilde{\beta}(1-\tilde{\delta})\mathbb{E}_t q^k_{j,t+1} +$ $\left(1 - \tilde{\beta}(1-\tilde{\delta})\right)\mathbb{E}_t r^k_{j,t+1} +$ $\tilde{\beta}\tilde{\delta}\psi\left(\tilde{\delta}\gamma_i\right)\mathbb{E}_t\left(i_{j,t+1} - k_{j,t+1}\right)$
Capital laws of motion	$k_{j,t+1} = (1-\tilde{\delta})k_{j,t} + \tilde{\delta} i_{j,t}$
Labor market clearing	$h_t = \phi h_{c,t} + (1-\phi)h_{i,t}$
Investment market clearing	$i_t = \phi i_{c,t} + (1-\phi)i_{i,t}$
Agg. resource constraint	$y_t = \phi c_t + (1-\phi)i_t$
	for $j = \{c, i\}$

Table 1: Log-linearized equilibrium conditions

Note that $c_t = y_{c,t}$ and $i_t = y_{i,t}$ also hold in equilibrium, as well as the definition of relative price, (13), and sectoral Phillips curves, (14) and (15). I-sector real marginal costs are expressed in investment units. Also, $\tilde{\delta} = 1 - \frac{1-\delta}{\gamma_i}$

is equivalent to the one resulting from the one-sector New Keynesian model. However, in addition to the investment gap, investment inflation dynamics depend upon the consumption gap as well as the deviation of the relative price of investment from its flexible-price counterpart. Intuitively, inefficiently high consumption induces a wealth effect that reduces hours worked in the investment sector; this leads to inefficiently high investment-sector wages and creates inflationary pressures. Also, when the relative price of investment is higher than its flexible-price level due to suboptimally high markups, investment-good producers adjust their prices downward to avoid incurring output losses.

The stationary components of sectoral technology follow a bivariate AR(1) process in logs:

$$\begin{pmatrix} z_{c,t} \\ z_{i,t} \end{pmatrix} = \begin{pmatrix} \rho_{z_c} & 0 \\ 0 & \rho_{z_c} \end{pmatrix} \begin{pmatrix} z_{c,t-1} \\ z_{i,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon^z_{c,t} \\ \varepsilon^z_{i,t} \end{pmatrix}$$

where the innovations are assumed to be i.i.d. and are allowed to have a non-zero contemporaneous correlation. Finally, the trend growth rates γ_c and γ_i are assumed to be constant. The model is calibrated at a quarterly frequency. We assume that the utility from consumption takes the logarithmic form ($\sigma = 1$). We set $\eta = 1/4$, corresponding to a Frisch elasticity of labor supply of 4. We set $\alpha = 1/3$, so that the labor share in each sector is 2/3. We set $\gamma_c = 1.004$ and $\gamma_i = 1.01$. The implied real per-capita consumption growth rate is $\gamma_c^{1-\alpha}\gamma_i^{\alpha} = 1.006$ or about 2.4% percent per year, which is close to the data. We calibrate δ so that the steady-state investment-to-capital ratio is about 0.15 per annum. In particular, the steady-state capital law of motion implies that

$$\tilde{\delta} \equiv \frac{\hat{I}}{\hat{K}} = 1 - \frac{1-\delta}{\gamma_i}$$

This relation implies that $\delta = 0.0279$ per quarter. We assume that $\beta = 0.995$, consistent with a steady-state real interest rate $r = \beta\left(\gamma_c^{1-\alpha}\gamma_i^{\alpha}\right)^{\sigma} = 1.0111$, or about 4.5% per year. Finally, we set the steady-state inflation rate for CPI, $\Pi_c = 1.005$, or 2% per annual, which implies that $\Pi_i = 1.0011$, or about 0.4% per annual. We set the markup rates such that $\mu_c = \mu_i = 1.11$, which are in line with micro studies (Basu and Fernald (1997)). As a benchmark, we set $\xi_c = \xi_i = 0.75$, implying that price contracts last on average for 4 quarters in each sector. The share of the investment sector in both output and employment $1 - \phi$ is set equal to 0.22, implying that the output share of consumption $\phi = 0.78$.[13] These reflect their empirical counterparts in the National Income and Product Accounts. Finally, we set $\psi = 2$, to reproduce the degree of investment volatility relative to output observed in the data.

To calibrate the variance of shocks, we use the use the consumption- and investment-sector series on total factor productivity estimated by Basu et al. (2013). The estimated persistence parameters for sectoral TFPs are $\rho_{z_c} = 0.98$ and $\rho_{z_i} = 0.95$. The estimated standard deviations of consumption and investment TFP innovations are $\sigma_{z_c} = 0.53\%$ and

[13]These determine the employment size parameters υ_c and υ_i in the subutility functions for leisure.

$\sigma_{z_i} = 1.33\%$. Importantly, our estimates indicate that $\sigma_{z_i,z_c} = 4.1342(10)^{-5}$, implying that sectoral TFP innovations are positively correlated, with a correlation coefficient of about 0.58.[14]

	Std. deviations				1st order autocorr.				Contemp. correlations			
	y	c/y	i/y	q_i/y	y	c	i	q_i	c,y	i,y	q_i,y	c,i
Model	1.00%	0.63	2.67	0.50	0.70	0.73	0.73	0.87	0.91	0.94	-0.09	0.71
Data	0.82%	0.45	2.97	0.85	0.89	0.82	0.89	0.75	0.82	0.95	-0.20	0.62

Table 2: Business cycle statistics: model vs data

All variables are in logarithms and have been detrended with the HP filter. The moments in this table are population moments computed from the solution of the model. We generate 500 simulations, each with the same number of observations available in the data (224), and report the average HP-filtered moments across these simulations. The monetary authority is assumed to follow a Taylor rule of the form $r_t^n = 1.5\pi_{c,t} + 0.5y$ where π_c is CPI inflation and y is the aggregate output gap.

Table 2 presents selected business cycle statistics for the benchmark economy:[15] the unconditional standard deviation of output is in line with its empirical counterpart; also, consistent with the data, consumption is about half as volatile as aggregate output whereas investment is about three times as volatile as output. The standard deviation of the relative price of investment implied by the model is somewhat smaller than the one observed in the data. The persistence generated by the model is high, but weaker than in the data for real output, consumption and investment, and stronger for the relative price of investment. In terms of comovements, the model performs remarkably well in capturing the contemporaneous correlations among the key variables. In line with the data, it captures the fact that both consumption and investment are highly correlated with output, and also captures the slight countercyclicality of the relative price of investment. Finally, the comovement between the two sectoral outputs is close to the positive value found in the data.

[14]Basu et al.'s (2013) TFPs are expressed in terms of log differences. We take the cumulative sum to obtain log levels of sectoral TFPs. To estimate the parameters of the bivariate technology shock process we use a seemingly unrelated regression (SUR) model. Also, we allow for time-varying linear trends, as a Bai-Perron test indicates that sectoral technologies contain at least one significant trend break in each sector. Finally, since Basu et al. (2013) provide us with annual data, we take the fourth root of annual persistence parameters and divide annual standard deviation parameters by four to obtain quarterly figures.

[15]In our model, "consumption" is naturally interpreted as comprising non-durable goods and services, whereas "investment" includes equipment, structures and residential investment.

4 The welfare function

We formally derive a second-order approximation to the social welfare function, and we compute its deviation from the welfare of the Pareto-optimal equilibrium, following Rotemberg and Woodford (1997). In our model, a Pareto-optimal equilibrium obtains when both consumption and investment sectors have fully flexible nominal prices. One should note that, however, in the context of sticky-price models with capital accumulation, there exist two distinct definitions of the natural rate of output.[16] One can define the natural rate of output based on the capital stocks implied by the flexible-price model (that is, $k^*_{j,t}$, for $j = \{c, i\}$), or the capital stocks that actually exists in the economy in that period (that is, the capital stock from the sticky-price model, $k_{j,t}$, for $j = \{c, i\}$).[17] In this paper, we adopt the latter definition of natural rate, proposed by Woodford (2003): thus, if the model economy operates under sticky-prices, then it is the period-t capital stock from that model that determines the current natural rate of output. We opt for this definition of natural rate of output, based on the *actual* capital stock, because it closely corresponds to what is generally thought of as *potential* output.

As derived in Appendix E, the unconditional welfare losses can be expressed as follows:

$$\mathbb{L} \simeq \frac{1}{2}\left[\frac{(1+\eta)}{1-\alpha} - (1-\sigma)\right] var\left(c - c^*\right) +$$

$$+\frac{1}{2}\left[\frac{1-\phi}{\phi}\left(\frac{1+\eta}{1-\alpha}\right)\right] var(i - i^*) + \frac{1}{2}\left(\frac{\theta_c}{1+\theta_c}\right)\left(\frac{1-\tilde{\beta}\xi_c}{1-\xi_c}\right)\frac{1}{\kappa_c} var(\pi_c) +$$

$$+\frac{1}{2}\frac{1-\phi}{\phi}\left(\frac{\theta_i}{1+\theta_i}\right)\left(\frac{1-\tilde{\beta}\xi_i}{1-\xi_i}\right)\frac{1}{\kappa_i} var(\pi_i) - \left[\frac{1-\phi}{\phi}(1+\eta)\right] cov(l^*_i, i - i^*) \tag{18}$$

In our two-sector model the variance of the consumption gap, $var\left(c - c^*\right)$, and the variance of the CPI inflation rate, $var(\pi_c)$, reduce household utility, as they do in the model without

[16]This difference arises only in models with endogenous capital accumulation. In such models, the natural rate of output in period-t depends upon the period-t capital stock, in addition to the model's exogenous disturbances

[17]The first concept has been advocated by Neiss and Nelson (2003), who construct their definition of the natural rate of output by assuming that the relevant capital stock is the one that which would have been in place had the economy always existed in a flexible-price world. To be precise, while the initial capital stock $k_{j,0}$, for $j = \{c, i\}$, is given, the capital stock that defines the natural rate of output in all subsequent periods is that from the flexible-price model, which is denoted by $\left\{k^*_{t+1}\right\}_{t=0}^{\infty}$.

endogenous capital accumulation.[18] In addition, given the presence of the investment sector, the variance of the investment gap, $var(i - i^*)$, also reduces household utility. Thus, an important aspect of the two-sector model with investment goods is that the *composition* of output, not just its aggregate value, has welfare implications. Besides, since investment goods prices are also sticky in nominal terms, the social welfare function also involves the variation in investment inflation, $var(\pi_i)$. As shown by Woodford (2003) and Erceg et al. (2000), inflation variation lowers welfare, since inflation in a given sector leads to price dispersion, which generates an inefficient allocation of production across firms.

Under the baseline calibration, the two sectors have the same markups and the same degree of price stickiness.[19] Thus, the welfare-function weight of CPI inflation relative to investment inflation is determined by the output share of consumption relative to investment (that is, about 4 to 1). Finally, a positive covariance between the investment gap and the natural rate of hours worked in the investment sector raises utility. It is difficult to place an intuitive interpretation on this term which, however, makes only a minor contribution to the welfare results reported below.[20]

5 Results

In our two-sector model monetary policy is confronted with a non-trivial trade-off if there are nominal rigidities in the pricing of both consumption and investment goods, and if the shocks to the two sectors are asymmetric. In this section we begin by formalizing this result for a simpler version of our model. To generalize, we then illustrate the stabilization problem faced by the central bank in response to a consumption-technology shock when both consumption and investment prices are sticky. Our graphical analysis serves two purposes. First, it highlights that in response to asymmetric sectoral shocks the attempt to stabilize fluctuations in one sector inevitably generates larger fluctuations in the other one. Hence, the second-best equilibrium – the optimal monetary policy that minimizes unconditional welfare losses under full commitment – represents a compromise between sectoral fluctuations. Second,

[18]See Edge (2003) for an excellent analysis of welfare functions in models with endogenous capital accumulation, under both Neiss and Nelson and Woodford's definition of natural rates.

[19]In each sector, the extent to which inflation variation affects household's utility is increasing in the degree of price stickiness and in the elasticity of substitution among goods. For given inflation volatility, higher price stickiness is associated with higher price dispersion and higher welfare costs. Also, when firms operate in highly competitive markets, price dispersion becomes particularly welfare diminishing, because even a relatively small dispersion of prices corresponds to considerable output dispersion.

[20]In addition, this term is nil whenever the investment gap is closed.

our impulse response analysis provides intuition for why optimal policy places relatively more weight on investment-sector stabilization. Later in this section, in fact, we examine the welfare properties of alternative monetary rules and show that, in general, welfare losses that arise from the investment sector are one order of magnitude larger than those stemming from the consumption sector. We finally show that the degree of rigidity of investment prices crucially affects the implications for optimal monetary policy.

5.1 The monetary policy trade-off

In general, when both consumption and investment prices are sticky the monetary authority cannot attain the flexible-price equilibrium. Given the complexity of our model, we are able to show this result analytically only for the special case in which there are no adjustment costs, and labor supply is infinitely elastic.

Proposition 1 *Assume that there are no sectoral adjustment costs ($\psi = 0$) and thus a unique capital stock for the economy with law of motion $k_{t+1} = (1-\tilde{\delta})k_t + \tilde{\delta}i_t$. Also, assume that the labor supply is perfectly elastic ($\eta = 0$). If prices in both sectors adjust at a finite speed ($\xi_c > 0$ and $\xi_i > 0$), then there exists no monetary policy that can attain the Pareto optimal allocation unless the two sectors are subject to identical disturbances ($z_{c,t} = z_{i,t}$ for each t).*

Proof. First, note that the relative price *gap* evolves according to:

$$\Delta\tilde{q}_{i,t} = \pi_{i,t} - \pi_{c,t} - \Delta q^*_{i,t} \tag{19}$$

which follows easily from equation (13). The assumptions of no adjustment costs and infinite Frisch elasticity guarantee that rental rates of capital and wages are equalized across sectors in each period, in units of consumption goods. As a result, in the flexible-price equilibrium relative price dynamics are solely governed by productivity differences (that is $q^*_{i,t} = z_{c,t} - z_{i,t}$). Thus, the flexible-price dynamics of the relative price gap evolves as:

$$\Delta q^*_{i,t} = \Delta z_{c,t} - \Delta z_{i,t} \tag{20}$$

Now suppose there was a monetary policy rule that would make the equilibrium allocation under sticky prices Pareto optimal. Then, in such an equilibrium, the gaps would be zero in every period, including the marginal cost gaps and the relative price gap (that is, $\tilde{q}_{i,t} = 0$ for all t). It follows from (14) and (15) that $\pi_{c,t} = \pi_{i,t} = 0$ for all t. In turn, this implies that $\Delta q_{i,t} = 0$ for all t, and (19) and (20) imply that $\pi_{i,t} - \pi_{c,t} = \Delta z_{c,t} - \Delta z_{i,t}$. This last equality contradicts the conclusion that $\pi_{i,t} = \pi_{c,t} = 0$ unless $\Delta z_{c,t} = \Delta z_{i,t}$ for all t. ■

To provide an intuition regarding the nature of the monetary policy trade-off, we begin by combining the log-linearized versions of the first-order conditions for bonds, (9), investment, (10), and capital, (11), together with the definition of the relative price of investment, (13). Using these equations, one can show that the Euler equation for sector-j Tobin's Q can be written as:

$$q_{j,t}^k - q_{i,t} = \tilde{\beta}\mathbb{E}_t\left(q_{j,t+1}^k - q_{i,t+1}\right) + \left(1 - \tilde{\beta}(1-\tilde{\delta})\right)\mathbb{E}_t(r_{j,t+1}^k - q_{i,t+1}) - (r_t^n - \mathbb{E}_t\pi_{i,t+1}) \quad (21)$$

where $r_t^n - \mathbb{E}_t\pi_{i,t+1}$ denotes the investment-real interest rate. By solving Equation (21) forward one obtains:

$$q_{j,t}^k - q_{i,t} = \sum_{s=0}^{\infty}\tilde{\beta}^s\mathbb{E}_t\left\{\left(1-\tilde{\beta}(1-\tilde{\delta})\right)\left(r_{j,t+s+1}^k - q_{i,t+s+1}\right) - \left(r_{t+s}^n - \pi_{i,t+s+1}\right)\right\} \quad (22)$$

The last equation shows that the current Tobin's Q of sector j (measured in units of investment goods) depends upon the discounted sum of the rental value of sector-j capital (in units of investment goods) as well as the discounted sum of the investment-real interest rate.[21] Moreover, for standard calibrations the term $1 - \tilde{\beta}(1-\tilde{\delta})$ takes values around 0.05. Thus, Tobin's Q is especially sensitive to the path of the investment-real interest rate: the investment-real interest rate channel dominates the response to capital returns. In turn, by the log-linearized FOC for investment, (10), the Tobin's Q (in investment good units) is a sufficient statistic for investment demand. As usual, agents' demand for investment depends upon the shadow value of capital, $q_{j,t}^k$, relative to its purchase price, $q_{i,t}$.[22]

Having understood the key forces driving investment demand, we move to analyzing the dynamic behavior of the model economy in response to disturbances that are not symmetric across sectors. Figure 1 reports the impulse response functions to a one-standard-deviation consumption-specific technology improvement. As shown by Basu et al. (2013), under flexible prices and logarithmic utility, consumption-specific technology shocks are neutral in the sense

[21] Note that the investment-real interest rate can also be expressed as the consumption real interest rate net of expected capital gains/losses, that is $(r_t^n - \mathbb{E}_t\pi_{c,t+1}) - \mathbb{E}_t\Delta q_{i,t+1}$.

[22] Here, both the shadow value of capital and the price of investment goods are expressed in consumption good units.

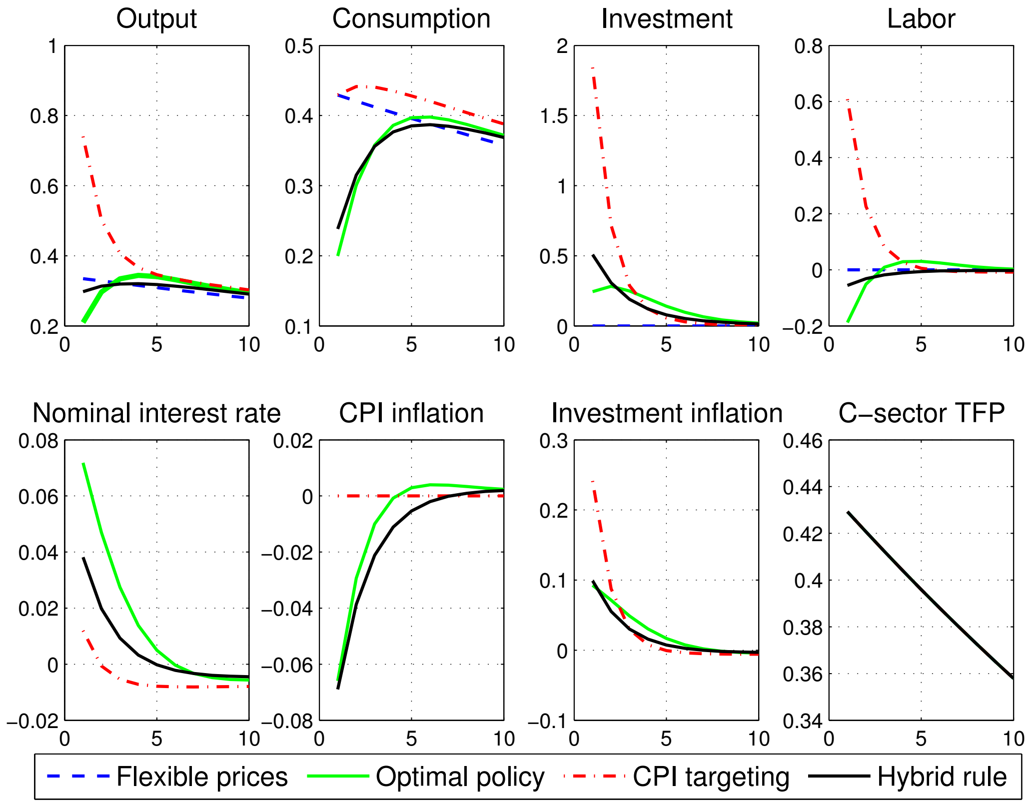


Figure 1: Impulse responses following a consumption-sector technology shock
Impulse responses are expressed as percent deviations from steady state.

that they have no effect other than increasing the quantity of consumption (and the relative price of investment) by the amount of the technology improvement. Thus, note that under flexible prices, both investment and hours worked remain at their steady-state values. This is the sense in which, under neoclassical conditions, consumption technology shocks cannot drive economic fluctuations, since they do not produce the comovements characteristic of business cycles. However, with sticky prices combined with the suboptimal but realistic policy of a Taylor rule that targets consumer prices, note that consumption technology shocks *do* produce positive comovements; the fluctuations in consumption, investment and labor input are qualitatively characteristic of business cycles, as is the ranking of volatility among investment, output and consumption.

The intuition behind the impulse responses is as follows. An improvement in consumption technology naturally increases the output of consumption goods. This raises the marginal product of capital in the consumption sector, which wishes to invest in capital goods. Under sticky investment goods prices, agents know that investment goods are temporarily cheap (the increase in demand will drive up their prices over time, but due to the Calvo friction, the nominal price of investment goods is basically unchanged on impact). Thus, there is a large spike in the demand for investment, which leads to a surge in investment inflation and a large increase in labor input.

When both consumption and investment prices are subject to nominal rigidities, an optimizing central bank faces a trade-off among competing ends. In response to a consumption-specific technology improvement, most firms in the consumption sector are unable to adjust prices downward to keep their markups constant. The result is that markups in the consumption sector rise and therefore consumption becomes inefficiently low, leading to a *negative* consumption gap and expected consumer price deflation. As the downward adjustment of consumption prices is imperfect, the relative price of investment is inefficiently low. This induces agents to demand too many investment goods, thereby generating a *positive* investment gap, and investment inflation. The monetary authority is unable to close both output gaps (or alternatively, to stabilize both inflation rates) because it can influence the economy only through a single instrument, the nominal interest rate. To stimulate consumption, the central bank must cut the nominal interest rate to offset the deflationary pressures in the consumption sector. However, this action would generate an even larger investment gap, by further reducing the investment real interest rate (see Equation (22)). On the other hand, to close the investment gap, the central bank should raise the nominal interest rate, but this policy would reduce consumption even further.

As potential output cannot be attained in both sectors, the optimal policy is a compromise between these two goals. In Figure 1, one can notice that the optimal policy succeeds in minimizing the investment gap by raising the nominal rate significantly. The cost of achieving a relatively small investment gap is to keep consumption below potential for five quarters, and aggregate output below potential for about two quarters.[23]

Through the same lens, we now analyze the dynamic response of the model economy when the central bank targets only CPI inflation, but chooses the size of the coefficient on the consumer price inflation rate optimally.[24] By solely responding to the consumption sector, the central bank cuts the nominal rate to respond to the c-sector deflationary outcome. Although this results in nearly optimal consumption dynamics, significantly negative investment real

[23]In fact, under optimal policy the consumption real interest rate is positive.

[24]Optimal coefficients of simple Taylor rules are chosen through a grid-searching algorithm.

rates further spur the demand for investment goods, resulting in an enormous investment gap as well as investment inflation volatility.

Finally, a hybrid rule which optimally targets both consumption and investment inflation manages to closely replicate the consumption and investment responses obtained under optimal policy. This suggests that targeting investment inflation generally has desirable stabilization properties. In the following section, we analyze this issue in greater depth.

5.2 Welfare implications of alternative rules

From both a normative and positive perspective, simple feedback interest rate rules are often considered effective ways to conduct and communicate monetary policy. The Taylor (1993) rule, under which the central bank sets the short-term nominal interest rate in response to fluctuations in inflation (and the output gap), is generally viewed as a simple but realistic description of monetary policy.[25] Our model suggests that the standard Taylor rule implemented with CPI/PCE inflation ignores some important variables in the welfare function, especially the investment inflation rate. Thus, we now investigate the welfare effects of various simple rules, and compare their performance to the optimal policy.

Table 3 displays the welfare losses under a set of interest rate rules. These allow the short-term rate to respond optimally to different combinations of CPI inflation, investment inflation, and sectoral output gaps. The results in Table 3 conform to expectations: a rule that strictly targets CPI inflation results in extremely large welfare losses. Specifically, under CPI-inflation targeting, the economy incurs welfare losses that are about three times larger than those of the optimal policy. Somewhat surprisingly, a rule that instead strictly targets investment inflation performs considerably better than the CPI targeting rule, notwithstanding the much smaller size of the investment sector. In other words, targeting investment inflation as opposed to CPI inflation has more desirable stabilizing properties. A formal intuition for this result is provided shortly.[26]

Importantly, a Taylor rule that optimally assigns about equal weight to CPI and investment inflation performs nearly as well as the optimal rule. As data on both CPI and

[25]Note that in his original paper, Taylor used inflation in the GDP deflator to implement his rule! While still far from optimal policy, we will show that targeting the GDP deflator performs significantly better in welfare terms than targeting PCE inflation.

[26]Note that adding the consumption (investment) gap to the CPI (investment) inflation rule as an additional targeting variable does not visibly affect welfare results. This is because the divine coincidence result holds for each sector in our model, although it does not hold in the aggregate.

investment inflation can be observed and communicated readily, we believe that this rule is a good candidate for realistic policy analysis.

Monetary rule target(s)	Welfare losses		Weights in opt. price index				
	Total	Rel. to OP	π_c	π_i	$\tilde{c}$	$\tilde{i}$	π_y
Optimal Policy	1.39	1					
Monetary rules that target sectoral variables							
CPI (π_c)	4.23	3.05	1.00				
π_c, $\tilde{c}$	4.22	3.04	0.40		0.60		
π_i	3.05	2.20		1.00			
π_i, $\tilde{i}$	3.05	2.05		0.40		0.60	
π_c, π_i	1.65	1.19	0.55	0.45			
Monetary rules that target aggregate variables							
GDP deflator (π_y)	2.08	1.50					1.00

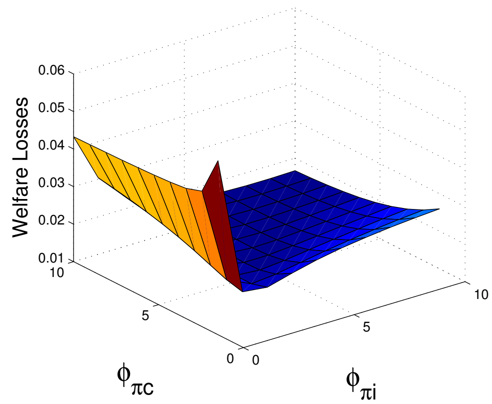
Table 3: Welfare costs of alternative policy rules

Total welfare losses are expressed as a percent of steady-state consumption (multiplied by 100). The third column reports welfare losses relative to those incurred under optimal policy.

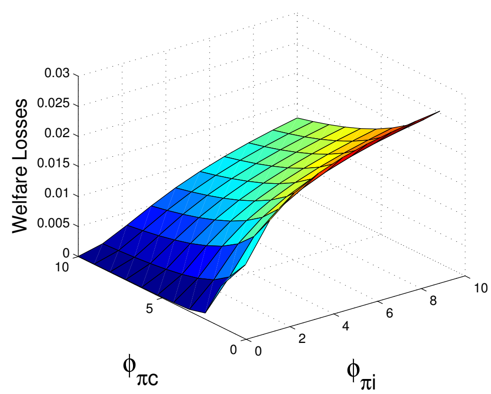
At this point one may wonder whether a rule that targets an "aggregate" price index, akin to the GDP deflator, might be a good approximation of the optimal rule. The bottom line of Table 3 shows that such rule leads to welfare losses of 2.08% of steady-state consumption, which is about 1.50 times the loss under optimal policy. Therefore, our results suggest that it is crucial to target investment-specific variables more than proportionally.

To provide further insight on the importance of targeting investment inflation relative to CPI inflation, Figure 2 displays the welfare losses incurred under alternative calibrations of the Taylor rule coefficients attached to the two inflation indexes.

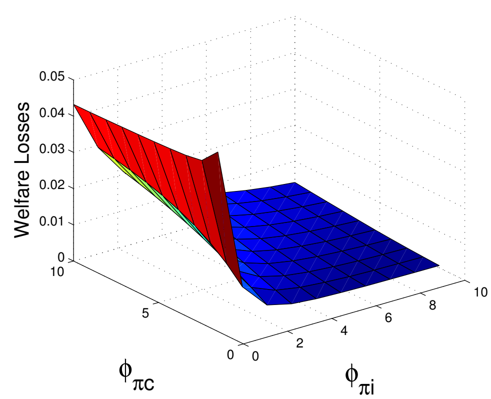
Figure 2(a) delivers a clear message: ignoring the investment inflation index typically creates significant welfare losses. On the other hand, when a standard Taylor rule accounts also for investment inflation, the welfare losses are quite close to those obtained under the optimal rule. Panels (b) and (c) of Figure 2 decompose the welfare losses into their sectoral components. A central bank that is solely concerned with CPI stabilization generates sizable welfare losses in the investment sector. When the monetary authority also targets investment inflation, the consumption sector becomes relatively more volatile. Overall, our model suggests that the second option is preferred from a welfare perspective: investment components of welfare are much larger than those arising from the consumption sector. As a result, their contribution is essential to account for the findings in Table 3.



(a) Overall welfare losses



(b) Consumption sector welfare losses



(c) Investment sector welfare losses

Figure 2: Welfare costs under alternative calibrations of the coefficient in the CPI-investment inflation hybrid rule

The rule that we consider in this experiment is $r_t^n = \phi_{\pi_c}\pi_{c,t} + \phi_{\pi_i}\pi_{i,t}$. Welfare losses are expressed as percent of steady-state consumption.

These results stem from the fact that the shadow value of capital is approximately unchanged in the wake of temporary disturbances. The illustration of this point follows the insights in Barsky et al. (2007), who first highlighted the dynamic implications of the near-constancy of the shadow value of long-lived goods. In our model, the shadow value of capital is defined by Equation (12). For simplicity, assume that there are no adjustment costs, so that Equation (12) becomes:[27]

$$q_{j,t}^{k} = \tilde{\beta}\gamma_i^{-1}\left[\mathbb{E}_t \sum_{i=1}^{\infty}\left(\frac{C_{t+i}}{C_{t+i-1}}\right)^{-\sigma}\left[\tilde{\beta}(1-\tilde{\delta})\right]^{i} MPK_{j,t+i+1}\right] \tag{23}$$

for $\{j = c, i\}$. As pointed out by Barsky et al. (2007), the shadow value of capital, $q_{j,t}^{k}$, will be largely unaffected by transitory shocks as long as the (growth-adjusted) depreciation rate of the capital stock, $\tilde{\delta}$, is sufficiently low. In fact, this would imply a high stock-flow ratio: even relatively high transitory changes in the production of investment goods would have little effect on the capital stock, and hence on its shadow value. Also, importantly, a low depreciation rate implies that the shadow value of capital in (23) is largely influenced by marginal products of capital in the distant future. And these are close to their steady-state values if shocks are temporary.

The near-constancy of the shadow value of long-lived capital goods implies that the demand for investment goods displays an almost infinite elasticity of intertemporal substitution: even a small rise in the price of the investment good today relative to tomorrow would cause people to delay their investment.[28] In contrast, (non-durable) consumption goods are subject to the consumption smoothing logic of the permanent income hypothesis. As a result, consumption is much less responsive to temporary price changes. This intrinsic difference has profound implications for monetary policy: as discussed above, a small intertemporal distortion in the price of investment goods generates high investment (and output) fluctuations, resulting in sizable welfare losses. For this reason, it becomes important to stabilize the investment sector. On the other hand, CPI targeting in a two-sector model of consumption and investment is generally incidental.

[27] Note that Equation (12) is also solved forward and we denote $MPK_{j,t} = r_{j,t}^{k}$. Equation (30) in Appendix B represents the scaled version of Equation (12)

[28] In fact, as stressed above, in their investment decision agents compare the marginal (or shadow) value of capital ($q_{j,t}^{k}$) to its marginal cost ($q_{i,t}$). Since the former is roughly constant, changes in the latter generate sizable fluctuations in investment demand.

5.3 The role of investment price rigidity

In this section we ask how sectoral differences in price stickiness affect our results. In particular, we analyze how the implications for monetary policy change as we change the degree of investment price rigidity. Figure 3 illustrates the outcome of this inquiry. A CPI inflation targeting rule displays very poor welfare properties as long as the average duration of investment price contracts is higher than two and a half quarters. With shorter durations (low values of ξ_i) investment price dispersion is lower, and ignoring investment inflation comes at a lower cost. As ξ_i rises, a CPI targeting rule becomes highly suboptimal because the investment components of welfare losses dominate those originating from the consumption sector. This feature of multi-sector models with heterogeneous price stickiness was first analyzed in Aoki (2001) (Benigno (2004)) who show that, in a two-sector (two-region) framework with heterogeneous degree of nominal rigidities, it is nearly optimal to implement an inflation targeting policy in which higher weight is given to the inflation of the stickier sector (region). These studies focus on the implications for monetary policy in models that features two sectors (regions) that are isomorphic to each other. That is, they are two otherwise identical final-good sectors producing non-durables that differ just with respect to their degree of nominal rigidity. Instead, in our framework the two sectors produce goods with intrinsically different characteristics.[29] Note that the hybrid rule performance is nearly optimal along the whole range of ξ_i. This is not surprising, since its coefficients are optimally set and it assigns different weights to the two inflation components as the nominal rigidity in the investment sector differs. Specifically, the coefficient on investment inflation rises as investment prices become more sticky.

In general, there is little microeconomic evidence on the frequency of price change of investment goods. However, available evidence supports high price stickiness in several categories of investment goods, both in the US and the Euro Area. According to Nakamura and Steinsson (2008), the median implied duration of prices of finished producer goods for the U.S. is 8.7 months and some categories exhibit an even higher duration: 26.5 months for machinery and equipment and 19.1 months for furniture and household durables. Similarly, Alvarez et al. (2006) summarize the vast evidence on micro price-setting recently obtained for Euro Area countries, and find that investment goods are the stickier components of producer

[29] In this respect, our results are closer to Erceg and Levin (2006).

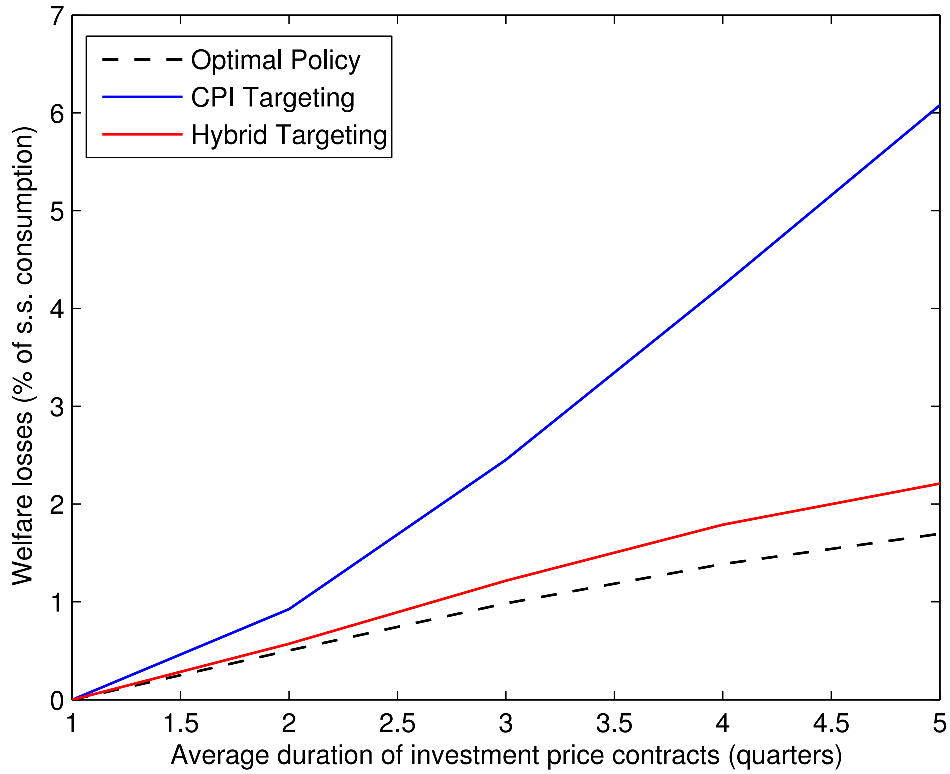


Figure 3: Welfare costs under alternative contract durations in the investment sector
The average contract duration in the consumption sector is set equal to 4 quarters. Welfare losses are expressed as percent of steady-state consumption, multiplied by 100.

prices: they report price stickiness of "capital goods" of 0.91 at monthly frequency, which would correspond to a quarterly frequency of as high as 0.75, or four quarters. In light of these studies, we find that investment goods price stickiness is plausibly at least as important as consumption goods price stickiness. This reinforces our message that central banks should, at a minimum, include investment goods prices in their target inflation indexes.

6 Conclusions

Using a model calibrated to reproduce some important empirical features, we asked whether the optimizing monetary authority should target both consumption and investment price inflation. We found that in general the answer is yes, it should. In and of itself, this result may not be surprising. A variety of previous work suggests that in general monetary policy should target all prices that are imperfectly flexible. If investment goods prices are sticky, then they should also feature in the inflation index that central banks target. The surprising element of our paper is the relative magnitudes of the costs of consumer versus investment price inflation. We find that welfare losses arising from markup variability in the investment sector are generally much larger than those that arise from similar variation in the consumption sector. This result stems from the fact that investment demand displays an almost infinite intertemporal elasticity of substitution, whereas intertemporal substitution of consumption over time is rather limited. Thus, predictable changes in investment goods prices lead to large and sub-optimal fluctuations in the quantity of investment, leading to large social losses.

We find that monetary rules that target solely CPI inflation generate substantial welfare losses. These are considerably reduced, however, if the central bank includes investment price inflation as part of its inflation target. In particular, we show that a Taylor rule that responds to both consumer and investment price inflation can nearly replicate the best feasible outcome, provided that investment sector stabilization is assigned a significantly higher weight than the share of investment in GDP. For this reason, a GDP deflator targeting rule is better but still unsatisfactory. Furthermore, we show that our results hold for any plausible degree of nominal rigidity in the investment sector. Our results contrast with standard central banking practice, as monetary policy in most countries currently targets only consumer price inflation.

A fully satisfactory treatment of optimal monetary policy should take into account all three issues that are known to influence welfare losses from price rigidity: differences in the average duration of prices across sectors; differences in the degree of strategic complementarity within and across sectors, as emphasized by Carvalho (2006); and differences in the durability of sectoral outputs, which is our contribution to this discussion. This should be done in a framework that distinguishes at least four categories of output: non-durable consumption goods; durable consumption goods; equipment investment output; and the output of residential and non-residential structures. Fortunately, Basu et al. (2013) provide measures of technology for all four categories of final demand, so our exercise could be extended to an elaborate model of this type. We have not done so in order to explain our argument in a simple framework that can be explained intuitively and analyzed analytically. We leave this more disaggregated treatment for future research.

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A Optimal price setting

Denote by $1 - \xi_j$ the probability that a firm in sector j can re-optimize price setting. A firm that can renew its price contract chooses $P_{j,t}(f)$ to maximize its expected discounted dividend flows given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \xi_j^s D_{t,t+s} \left[(1+\tau_j)\, P_{j,t}(f) Y_{j,t+s}^d(f) - V_{j,t+s}(f) \right]$$

where τ_j denotes a subsidy to sector-j output, $D_{t,t+s}$ is the period-t present value of a dollar in a future state in period $t+s$, and $V_{j,t+s}(f)$ is the total cost function; in maximizing its profits, each firm takes as given its demand schedule (1). The resulting optimal pricing rule is:

$$P_{j,t}(f) = \frac{\mu_j}{1+\tau_j} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \xi_j^s D_{t,t+s} v_{j,t+s} \left(\frac{P_{j,t+s}}{\Pi_j^s} \right)^{\frac{1+\theta_j}{\theta_j}} Y_{j,t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \xi_j^s D_{t,t+s} \left(\frac{P_{j,t+s}}{\Pi_j^s} \right)^{\frac{1}{\theta_j}} Y_{j,t+s}} \tag{24}$$

where $v_{j,t}$ denotes the real marginal cost function (in units of the respective sectoral price index) given by (2).

The steady state of (24), with constant *real* marginal costs, reads

$$\frac{P_{j,t}(f)}{P_{j,t}} = \frac{\mu_j}{1+\tau_j} v_j \tag{25}$$

It is important to note that, under full indexation to trend inflation, the steady state coincides with the flexible price steady state: each firm sets its price as a constant markup over marginal costs. This assumption guarantees that the steady state is not distorted by trend inflation. In the steady state, however, the markup is completely offset by the subsidy τ_j. This assumption guarantees that the steady state of the model is not distorted by imperfect competition.

The log-linearized counterpart of (24) – the log-linearized optimal price setting rule equation – is:

$$p_{j,t}^* - p_{j,t} = (1 - \xi_j \tilde{\beta}) \mathbb{E}_t \sum_{s=0}^{\infty} (\xi_j \tilde{\beta})^s \left[\pi_{j,t|t+s} + mc_{j,t+s} \right] \tag{26}$$

where mc_j denotes log-linearized sector-j *sector-j-real* marginal costs. One can show that the (sectoral) price level evolves as:

$$P_{j,t} = \left[\xi_j (\Pi_j P_{j,t-1})^{-\frac{1}{\theta_j}} + (1-\xi_j)(P^*_{j,t})^{-\frac{1}{\theta_j}}\right]^{-\theta_j} \tag{27}$$

where $1-\xi_j$ denotes the probability of adjusting the price in a given period, and θ_j represent the mark-up rate in sector j. One can rewrite (28) as:

$$1 = \xi_j \Pi_j^{-\frac{1}{\theta_j}} \Pi_{j,t}^{\frac{1}{\theta_j}} + (1-\xi_j)\left(\frac{P^*_{j,t}}{P_{j,t}}\right)^{-\frac{1}{\theta_j}} \tag{28}$$

Its log-linearized counterpart – the log-linearized general price level equation – is:

$$p^*_{j,t} - p_{j,t} = \frac{\xi_j}{1-\xi_j}\pi_{j,t} \tag{29}$$

Note that both (26) and (29) are independent from trend inflation *and* exactly coincide with their counterparts derived under zero steady-state inflation. As a result, putting them together, one obtains the usual (sectoral) New Keynesian Phillips curve(s).

B Stationary equilibrium and the deterministic steady state

To induce stationarity we transform variables as follows:[30]

$$\hat{C}_t = \hat{Y}_{c,t} = \frac{Y_{c,t}}{\Gamma^{1-\alpha}_{c,t}\Gamma^{\alpha}_{i,t}} \qquad \hat{I}_t = \hat{Y}_{i,t} = \frac{Y_{i,t}}{\Gamma_{i,t}} \qquad \hat{K}_{j,t+1} = \frac{K_{j,t+1}}{\Gamma_{i,t}} \qquad \hat{w}_{j,t} = \frac{W_{j,t}}{P_{c,t}\Gamma^{1-\alpha}_{c,t}\Gamma^{\alpha}_{i,t}}$$

$$\hat{q}_{i,t} = \frac{q_{i,t}}{\Gamma^{1-\alpha}_{c,t}\Gamma^{\alpha-1}_{i,t}} = \frac{P_{i,t}}{P_{c,t}}\left(\frac{\Gamma_{i,t}}{\Gamma_{c,t}}\right)^{1-\alpha} \qquad \frac{\hat{mc}_{i,t}}{q_{i,t}} = \frac{MC_{i,t}}{P_{i,t}} = \frac{MC_{i,t}}{P_{c,t}q_{i,t}} \qquad \hat{mc}_{c,t} = \frac{MC_{c,t}}{P_{c,t}} \qquad \hat{q}^k_{j,t} = q^k_{j,t}\left(\frac{\Gamma_{i,t}}{\Gamma_{c,t}}\right)^{1-\alpha}$$

$$\hat{r}^k_{j,t} = r^k_{j,t}\left(\frac{\Gamma_{i,t}}{\Gamma_{c,t}}\right)^{1-\alpha} \qquad \hat{Y}_t = \frac{Y_t}{\Gamma^{1-\alpha}_{c,t}\Gamma^{\alpha}_{i,t}}$$

The household's FOCs (8)-(11) are rendered stationary as follows:

$$\upsilon_j N_{j,t}(h)^{\eta} = \frac{\hat{w}_{j,t}}{\hat{C}^{\sigma}_t}$$

[30] Hatted variables are stationary

$$1 = \beta(\gamma_c^{1-\alpha}\gamma_i^{\alpha})^{-\sigma}\mathbb{E}_t\left[\frac{R_t^n}{\Pi_{c,t+1}}\frac{\hat{C}_{t+1}^{-\sigma}}{\hat{C}_t^{-\sigma}}\right]$$

$$\hat{q}_{i,t} = \hat{q}_{j,t}^k\left[\Psi'\left(\frac{\hat{I}_{j,t}\gamma_i}{\hat{K}_{j,t}}\right)\right]$$

$$\hat{q}_{j,t}^k = \tilde{\beta}\gamma_i^{-1}\mathbb{E}_t\left(\frac{\hat{C}_{t+1}}{\hat{C}_t}\right)^{-\sigma}\left\{\hat{q}_{j,t+1}^k\left[1-\delta+\Psi\left(\frac{\hat{I}_{j,t+1}\gamma_i}{\hat{K}_{j,t+1}}\right)-\Psi'\left(\frac{\hat{I}_{j,t+1}\gamma_i}{\hat{K}_{j,t+1}}\right)\left(\frac{\hat{I}_{j,t+1}\gamma_i}{\hat{K}_{j,t+1}}\right)\right]+\hat{r}_{j,t+1}^k\right\} \quad (30)$$

where γ_j denotes the growth rate of sector-j technology.

Sector-j production function reads:

$$\hat{Y}_{j,t} = Z_{j,t}\left(\gamma_i^{-1}\hat{K}_{j,t}\right)^{\alpha}(L_{j,t})^{1-\alpha}$$

Real marginal costs in the consumption sector:

$$\frac{MC_{c,t}}{P_{c,t}} = \frac{\tilde{\alpha}}{Z_{c,t}}(\hat{r}_{c,t}^k)^{\alpha}(\hat{w}_{c,t})^{1-\alpha}$$

Investment-real marginal costs in the investment sector:

$$\frac{MC_{i,t}}{P_{i,t}} = \frac{\tilde{\alpha}}{Z_{i,t}}(\hat{r}_{i,t}^k)^{\alpha}(\hat{w}_{i,t})^{1-\alpha}\frac{1}{\hat{q}_{i,t}}$$

Factor demands are given by:

$$\frac{\hat{w}_{j,t}}{\hat{r}_{j,t}^k} = \frac{1-\alpha}{\alpha}\frac{\gamma_i^{-1}\hat{K}_{j,t}}{L_{j,t}}$$

Capital laws of motion are denoted by:

$$\hat{K}_{j,t+1} = \frac{(1-\delta)}{\gamma_i}\hat{K}_{j,t} + \left(\Psi\left(\frac{\hat{I}_{j,t}\gamma_i}{\hat{K}_{j,t}}\right)\right)\hat{I}_{j,t}$$

The Euler equation in steady state imply:

$$R^n = \beta^{-1}(\gamma_c^{1-\alpha}\gamma_i^{\alpha})^{\sigma}\Pi_c$$

where Π_c denotes the steady-state CPI inflation rate. This has to satisfy:

$$\hat{q}_{i,t} = \frac{P_{i,t}}{P_{c,t}}\left(\frac{\Gamma_{i,t}}{\Gamma_{c,t}}\right)^{1-\alpha} \quad (31)$$

Divide both sides of (31) by its previous period counterpart:

$$1 = \frac{\Pi_{i,t}}{\Pi_{c,t}} \left(\frac{\gamma_i}{\gamma_c} \right)^{1-\alpha} \tag{32}$$

which states that the relative price of investment decline at a rate that is determined by the growth rate differential of sectoral technologies. Then, for given sectoral technology growth rates, we can calibrate one sectoral steady-state inflation rate, and the other would be uniquely determined, by (32).

The capital laws of motion in steady state imply:

$$\frac{\hat{I}_j}{\hat{K}_j} = 1 - \frac{1-\delta}{\gamma_i} \tag{33}$$

which is the investment-capital ratio in sector j. The log-linearized equilibrium conditions are reported in Table 1.

C The relationship between marginal cost gaps and output gaps

By using the log-linearized c-sector marginal costs, capital demand, labor supply, production function and resource constraint (see Table 1) we have:

$$v_{c,t} = \left[\frac{(1+\eta)}{1-\alpha} - (1-\sigma) \right] c_t - \alpha \frac{1+\eta}{1-\alpha} k_{c,t} - \frac{1+\eta}{1-\alpha} z_{c,t}$$

The last equation represents the real marginal costs in the consumption sector under sticky prices. The following equation defines the real marginal costs when prices are flexible, under the Woodford's definition of natural rates:[31]

$$v^*_{c,t} = \left[\frac{(1+\eta)}{1-\alpha} - (1-\sigma) \right] c^*_t - \alpha \frac{1+\eta}{1-\alpha} k_{c,t} - \frac{1+\eta}{1-\alpha} z_{c,t}$$

Recall that, in every time period, the variation in real marginal costs under flexible prices is nil. That is, prices are set as a constant markup over nominal marginal costs. Thus the

[31] As explained in Section 4, the Woodford's definition of natural rate implies that period-t capital stocks in the Pareto-optimal equilibrium coincide with period-t capital stocks in the sticky price equilibrium. Note that this is not true for $t+i$ if $i \neq 0$

marginal cost equals the marginal cost gap and is given by:

$$v_{c,t} - v^*_{c,t} = \left[\frac{(1+\eta)}{1-\alpha} - (1-\sigma)\right](c_t - c^*_t)$$

Similarly one can show that, under sticky prices, the real marginal costs in the investment sector, in units of investment goods, are:

$$v_{i,t} = \left[\frac{1+\eta}{1-\alpha} - 1\right] i_t - \alpha\frac{1+\eta}{1-\alpha}k_i + \sigma c_t - \frac{1+\eta}{1-\alpha}z_{i,t} - q_{i,t}$$

Under flexible prices, these correspond to

$$v^*_{i,t} = \left[\frac{1+\eta}{1-\alpha} - 1\right] i^*_t - \alpha\frac{1+\eta}{1-\alpha}k_i + \sigma c^*_t - \frac{1+\eta}{1-\alpha}z_{i,t} - q^*_{i,t}$$

The investment marginal costs gap is therefore:

$$v_{i,t} - v^*_{i,t} = \left[\frac{1+\eta}{1-\alpha} - 1\right](i_t - i^*_t) + \sigma(c_t - c^*_t) - (q_{i,t} - q^*_{i,t})$$

D Pareto optimum

Here we outline some flexible-price equilibrium relationships that will prove useful in deriving the second-order approximation to the social welfare function. By equating the marginal rate of substitution to the marginal product of labor in the consumption sector, under flexible prices, we obtain:

$$\sigma c^*_t + \eta h^*_{c,t} = c^*_t - h^*_{c,t}$$

Use the production function to substitute out for consumption-sector labor:

$$h^*_{c,t} = \frac{1}{1-\alpha}c^*_t - \frac{\alpha}{1-\alpha}k_{c,t} - \frac{1}{1-\alpha}z_{c,t}$$

$$\frac{1+\eta}{1-\alpha}(\alpha k_{c,t} + z_{c,t}) = \left[\frac{1+\eta}{1-\alpha} - (1-\sigma)\right]c^*_t \tag{34}$$

Note that, under Woodford's definition of natural rate, $k^*_{c,t} = k_{c,t}$ at the beginning of each time period.

The production function for investment goods, together with the market-clearing condition in the investment sector, imply:

$$z_{i,t} + \alpha k_{i,t} = i_t^* + (1-\alpha) h_{i,t}^* \tag{35}$$

E A second-order approximation to the welfare function

The approach adopted in this section largely follows Erceg et al. (2000) and Edge (2003).

To provide a normative assessment of alternative monetary policy choices, we measure social welfare as the unconditional expectation of average household lifetime utility:

$$\mathbb{E}_t \int_0^1 \left[\sum_{s=0}^{\infty} \tilde{\beta}^s \mathbb{W}_{t+s}(h) \right] dh$$

where $\tilde{\beta} = \beta(\gamma_c^{1-\alpha}\gamma_i^{\alpha})^{1-\sigma}$, and the term in large brackets is the discounted lifetime utility function of household h presented in the paper (Equations (5) and (6) after appropriate transformations). In this appendix, we follow the seminal analysis of Rotemberg and Woodford (1997) in deriving the second-order approximation to each component of the social welfare function and computing its deviation from the welfare of the Pareto-optimal equilibrium under flexible consumption and investment prices. We adopt Woodford's definition of natural rate of output: as explained in Section 4, when we enter period $t+1$ it is the capital stock that is *actually* present that determines how output in $t+1$ is defined. We will be more specific on the implications of this assumption below. The approach we take can be described as follows: we derive the second order approximation for the within-period welfare function in the sticky price model; we subtract its flexible-price counterpart; we take sum over t from 0 to ∞ and take the unconditional expectation.

It is useful to decompose household h's period utility function $\mathbb{W}_t(h)$ as follows:

$$\mathbb{W}_t = \mathbb{U}(\hat{C}_t) - \mathbb{V}^c(N_{c,t}(h)) - \mathbb{V}^i(N_{i,t}(h))$$

$$\mathbb{U}(\hat{C}_t) = \frac{\hat{C}_t^{1-\sigma}}{1-\sigma}$$

$$\mathbb{V}^c(N_{c,t}(h)) = \upsilon_c \frac{N_{c,t}(h)^{1+\eta}}{1+\eta}$$

$$\mathbb{V}^c(N_{i,t}(h)) = \upsilon_i \frac{N_{i,t}(h)^{1+\eta}}{1+\eta}$$

We use two approximations repeatedly. If A is a generic variable, the relationship between its arithmetic and logarithmic percentage change is

$$\frac{A-\bar{A}}{\bar{A}} = \frac{dA}{\bar{A}} \simeq a + \frac{1}{2}a^2, \; a \equiv \ln A - \ln \bar{A} \tag{36}$$

If $A = \left[\int_0^1 A(j)^{\phi} dj\right]^{\frac{1}{\phi}}$, the logarithmic approximation of A is

$$a \simeq E_j a(j) + \frac{1}{2}\phi\left(E_j a(j)^2 - (E_j a(j))^2\right) = E_j a(j) + \frac{1}{2}\phi var_j a(j) \tag{37}$$

We first consider the subutility functions that involve consumption terms, that is $\mathbb{U}(\hat{C}_t)$ and $\mathbb{V}^c(N_{c,t}(h))$; first, we approximate $\mathbb{U}(\hat{C}_t)$:[32]

$$\mathbb{U}(\hat{C}) \simeq \mathbb{U} + \mathbb{U}_{\hat{C}}\hat{C}\frac{d\hat{C}}{\hat{C}} + \frac{1}{2}\left(\mathbb{U}_{\hat{C}\hat{C}}\hat{C}^2\left(\frac{d\hat{C}}{\hat{C}}\right)^2\right)$$

Making use of Equation (36) we have:

$$\mathbb{U}(\hat{C}) \simeq \mathbb{U} + \mathbb{U}_{\hat{C}}\hat{C}\left(c + \frac{1}{2}c^2\right) + \frac{1}{2}\mathbb{U}_{\hat{C}\hat{C}}\hat{C}^2c^2 \tag{38}$$

Next we approximate $E_h\mathbb{V}^c(N_{c,t})$:

$$E_h\mathbb{V}^c(N_{c,t}(h)) \simeq \mathbb{V}^c + E_h\mathbb{V}^c_{N_c}N_c\frac{dN_c(h)}{N_c} + \frac{1}{2}\left(E_h\mathbb{V}^c_{N_cN_c}N_c^2\left(\frac{dN_c(h)}{\bar{N}_c}\right)^2\right)$$

Again, by (36) we can write:

$$E_h\mathbb{V}^c(N_{c,t}(h)) \simeq \mathbb{V}^c + \mathbb{V}^c_{N_c}N_c\left(E_hn_c(h) + \frac{1}{2}E_hn_c(h)^2\right) + \frac{1}{2}\left(\mathbb{V}^c_{N_cN_c}N_c^2E_hn_c(h)^2\right) \tag{39}$$

The aggregate supply of labor in the consumption sector is $L_c = \nu_c\int_0^1 N_c(h)dh$. Therefore,

$$l_c = \ln\nu_c + \ln\int_0^1 N_{c,t}(h)dh - \ln\bar{L}_c \simeq E_hn(h) + \frac{1}{2}var_hn_c(h) \tag{40}$$

[32]In this section we suppress the time subscript t for ease of notation.

where the constant terms are dropped in the last equation. The aggregate demand for labor by firms (in the consumption sector) is $L_c = \int_0^1 L_c(f)df = E_f L_c(f)$. Thus,

$$l_c = \ln E_f L_c(f) - \bar{L}_c \simeq E_f l_c(f) + \frac{1}{2} var_f l_c(f) \tag{41}$$

All firms in the consumption sector choose identical capital labor ratios $\left(\frac{\hat{K}_c(f)}{L_c(f)}\right)$ equal to the aggregate sectoral ratio $\frac{\hat{K}_c}{L_c}$ because they face the same factor prices, so

$$\hat{Y}_c(f) = Z_c \left(\frac{\hat{K}_c(f)}{L_c(f)}\right)^{\alpha} L_c(f) = Z_c \left(\frac{\hat{K}_c}{L_c}\right)^{\alpha} L_c(f)$$

The last equation implies:

$$y_c(f) = z_c + \alpha k_c - \alpha l_c + l_c(f)$$

which also implies:

$$E_f y_c(f) = z_c + \alpha k_c - \alpha l_c + E_f l_c(f) \tag{42}$$

and:

$$var_f y_c(f) = var_f l_c(f) \tag{43}$$

Using (42) and (43) into (41), and eliminating $E_f y_c(f)$ using (37) yields:

$$l_c \simeq \frac{1}{1-\alpha}(y_c - z_c) - \left(\frac{\alpha}{1-\alpha}\right) k_c + \frac{1}{2}\left(\frac{1}{1-\alpha}\right)\left(\frac{\theta_c}{1+\theta_c}\right) var_f y_c(f) \tag{44}$$

Solving (40) for $E_h n_c(h)$, eliminating l using (44), and noticing that, in absence of nominal wage rigidities, there is no variation in hours worked across *households* (i.e. $var_h n_c(h) = 0$), we have:

$$E_h n_c(h) \simeq \frac{1}{1-\alpha}(y_c - z_c - \alpha k_c) + \frac{1}{2}\left(\frac{1}{1-\alpha}\right)\left(\frac{\theta_c}{1+\theta_c}\right) var_f y_c(f) \tag{45}$$

Also note that $E_h n_c(h)^2 = var_h n_c(h) + [E_h n_c(h)]^2$. Thus, taking squares of both sides of (45):

$$E_h n_c(h)^2 \simeq \left(\frac{1}{1-\alpha}\right)^2 (y_c - z_c - \alpha k_c)^2 \tag{46}$$

Replacing (45) and (46) in (39) yields:

$$\begin{aligned}
E_h\mathbb{V}^c(N_{c,t}(h)) \;&\simeq\; \mathbb{V}^c+\mathbb{V}^c_{N_c}N_c\left[\frac{1}{1-\alpha}(y_c-z_c)-\left(\frac{\alpha}{1-\alpha}\right)k_c\right] \\
&+\frac{1}{2}\mathbb{V}^c_{N_c}N_c\left(\frac{1}{1-\alpha}\right)\left(\frac{\theta_c}{1+\theta_c}\right)var_f y_c(f) \qquad (47)\\
&+\frac{1}{2}\left[\mathbb{V}^c_{N_c}N_c+\mathbb{V}^c_{N_cN_c}N_c^2\right]\left[\left(\frac{1}{1-\alpha}\right)^2(y_c-z_c-\alpha k_c)^2\right]
\end{aligned}$$

Also, *in steady state*, if we equate the marginal rate of substitution to the marginal product of labor in sector j, we have:[33]

$$\frac{\mathbb{V}^j_{N_j}}{\mathbb{U}_{\hat{C}}}=(1-\alpha)\frac{\hat{Y}_j}{N_j}$$

When $j=c:$

$$\frac{\mathbb{V}^c_{N_c}N_c}{(1-\alpha)}=\mathbb{U}_{\hat{C}}\hat{C} \qquad (48)$$

When $j=i:$

$$\frac{\mathbb{V}^i_{N_i}N_i}{(1-\alpha)}=\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C} \qquad (49)$$

Also, since $\mathbb{V}^c(N_{c,t}(h))=\upsilon_c\frac{N_{c,t}(h)^{1+\eta}}{1+\eta}$, we have that $\mathbb{V}^c_{N_c}N_c=\upsilon_c {N_c}^{1+\eta}$ and $\mathbb{V}^c_{N_cN_c}N_c^2=\eta\mathbb{V}^c_{N_c}N_c$. Moreover, given that $\mathbb{U}(\hat{C}_t)=\frac{\hat{C}_t^{1-\sigma}}{1-\sigma}$, it also holds that $\mathbb{U}_{\hat{C}}\hat{C}=\hat{C}^{1-\sigma}$ and $\mathbb{U}_{\hat{C}\hat{C}}\hat{C}^2=-\sigma\mathbb{U}_{\hat{C}}\hat{C}$. Use (48) together with the above relationships into (47) and sum the resulting equation to (38). This leads to:[34]

$$\begin{aligned}
\mathbb{U}(\hat{C})-E_h\mathbb{V}^c(N_{c,t}(h)) \simeq\; &\mathbb{U}-\mathbb{V}^c+\mathbb{U}_{\hat{C}}\hat{C}\left(z_c+\alpha k_c\right)+\frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}\left(1-\sigma\right)c^2 \\
&-\frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}\left(\frac{\theta_c}{1+\theta_c}\right)var_f y_c(f) \qquad (50)\\
&-\frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}(1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(c-z_c-\alpha k_c)^2\right]
\end{aligned}$$

Let's now consider the component of the utility function related to investment. This coincides with the term that describes disutility from labor allocated in the investment

[33]Note that the markup and and subsidy exactly cancel each other out in steady state, in both sectors

[34]Here we also used the fact that $y_c=c$ which holds in our model since we abstract from government purchases.

sector. The derivation is similar to the one concerning $\mathbb{V}^c$. Thus:

$$
\begin{aligned}
E_h\mathbb{V}^i(N_i(h)) \simeq \mathbb{V}^i &+ \mathbb{V}^i_{N_i}N_i\left[\frac{1}{1-\alpha}(y_i-z_i)-\left(\frac{\alpha}{1-\alpha}\right)k_i\right] \\
&+\frac{1}{2}\mathbb{V}^i_{N_i}N_i\left(\frac{1}{1-\alpha}\right)\left(\frac{\theta_i}{1+\theta_i}\right)var_f y_i(f) \\
&+\frac{1}{2}\left[\mathbb{V}^i_{N_i}N_i+\mathbb{V}^i_{N_iN_i}N_i^2\right]\left[\left(\frac{1}{1-\alpha}\right)^2(y_i-z_i-\alpha k_i)^2\right]
\end{aligned}
\tag{51}
$$

By (49) and $i = y_i$ we can rewrite (51) as:

$$
\begin{aligned}
E_h\mathbb{V}^i(N_i(h)) \simeq \mathbb{V}^i &+ \frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\left[i-z_i-\alpha k_i\right]+\frac{1}{2}\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\left(\frac{\theta_i}{1+\theta_i}\right)var_f y_i(f) \\
&+\frac{1}{2}\left[\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\right](1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(i-z_i-\alpha k_i)^2\right]
\end{aligned}
\tag{52}
$$

To obtain the within-period welfare function under sticky prices, sum Eqs. (50) and (52):

$$
\begin{aligned}
\mathbb{W}_t \simeq\ & \mathbb{U}-\mathbb{V}^c+\mathbb{U}_{\hat{C}}\hat{C}\alpha k_c+\frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}\ \ (1-\sigma)\,c^2-\frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}\left(\frac{\theta_c}{1+\theta_c}\right)var_f y_c(f) \\
&-\frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}(1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(c-z_c-\alpha k_c)^2\right]-\mathbb{V}^i-\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\left[i-\alpha k_i\right] \\
&-\frac{1}{2}\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\left(\frac{\theta_i}{1+\theta_i}\right)var_f y_i(f)-\frac{1}{2}\left[\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\right](1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(i-z_i-\alpha k_i)^2\right]
\end{aligned}
\tag{53}
$$

Consider now the *first-order* terms in the first line of the above equation:

$$
\alpha k_{c,t}-\frac{1-\phi}{\phi}\left[i_t-\alpha k_{i.t}\right]
$$

Note that $i_t = \phi i_{c,t} + (1-\phi)i_{i,t}$ and for $j = \{c, i\}$: $k_{j,t+1} = (1-\tilde{\delta})k_{j,t} + \tilde{\delta} i_{j,t}$. Thus:

$$
\alpha k_{c,t}-\frac{(1-\phi)}{\tilde{\delta}}k_{c,t+1}+\frac{(1-\phi)(1-\tilde{\delta})}{\tilde{\delta}}k_{c,t}-\frac{(1-\phi)^2}{\phi\tilde{\delta}}k_{i,t+1}+\frac{(1-\phi)^2(1-\tilde{\delta})}{\phi\tilde{\delta}}k_{i,t}+\frac{(1-\phi)}{\phi}\alpha k_{i,t}
\tag{54}
$$

The net rental rate of capital in steady state (in units of investment goods) is:

$$
r_j^k = \frac{\gamma_i}{\tilde{\beta}}-(1-\delta)
\tag{55}
$$

By the law of motion in the investment sector, evaluated in steady state: $\frac{I_i}{K_i} = \frac{\hat{I}_i \gamma_i}{\hat{K}_i} = \gamma_i - (1-\delta)$. Also, by the investment-sector steady-state capital demand:

$$r_i^k = \alpha \frac{Y_i}{K_i} = \alpha \frac{[\gamma_i - (1-\delta)]\, I}{I_i} = \alpha \frac{[\gamma_i - (1-\delta)]}{1-\phi} \tag{56}$$

By equating (55) and (56):

$$\frac{1-\phi}{\tilde{\delta}} = \frac{\alpha\tilde{\beta}}{1-\tilde{\beta}(1-\tilde{\delta})} \tag{57}$$

Use (57) in (54):

$$\begin{aligned} &\alpha k_{c,t} - \frac{\alpha\tilde{\beta}}{1-\tilde{\beta}(1-\tilde{\delta})} k_{c,t+1} + \frac{\alpha\tilde{\beta}}{1-\tilde{\beta}(1-\tilde{\delta})}(1-\tilde{\delta})k_{c,t} \\ &+ \frac{(1-\phi)}{\phi}\left[-\frac{\alpha\tilde{\beta}}{1-\tilde{\beta}(1-\tilde{\delta})} k_{i,t+1} + \frac{\alpha\tilde{\beta}}{1-\tilde{\beta}(1-\tilde{\delta})}(1-\tilde{\delta})k_{i,t} + \alpha k_{i,t}\right] \\ &+ \left[\frac{\alpha}{1-\tilde{\beta}(1-\tilde{\delta})}\left(k_{c,t} - \tilde{\beta}k_{c,t+1}\right)\right] + \frac{(1-\phi)}{\phi}\left[\frac{\alpha}{1-\tilde{\beta}(1-\tilde{\delta})}\left(k_{i,t} - \tilde{\beta}k_{i,t+1}\right)\right] \end{aligned} \tag{58}$$

Now, recall that the second-order approximation of the *overall* utility function is given by the discounted sum of the second-order approximation to the within utility functions. We can therefore pull together all of the first-order terms that remain after simplification from each within period utility function to obtain:

$$\frac{\alpha}{1-\tilde{\beta}(1-\tilde{\delta})} E_0\left[k_{j,0} - \tilde{\beta}k_{j,1} + \tilde{\beta}(k_{j,1} - \tilde{\beta}k_{j,2}) + ... + \tilde{\beta}^t(k_{j,t} - \tilde{\beta}k_{j,t+1}) + \tilde{\beta}^{t+1}(k_{j,t+1} - \tilde{\beta}k_{j,t+2}) + ...\right]$$

which when we cancel terms from different periods is just equal to:

$$\frac{\alpha}{1-\tilde{\beta}(1-\tilde{\delta})} k_{j,0}$$

In other words, all linear terms in the square brackets in (58) disappear except for a term in $k_{j,0}$, for $j = \{c, i\}$, which denote the initial sectoral capital stocks and are assumed to be fixed and independent of policy.

Thus, the welfare function under sticky prices, (53), can be rewritten as:

$$
\begin{aligned}
\mathbb{W}_t \simeq \mathbb{U} - \mathbb{V}^c &+ \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}\left(1-\sigma\right)c^2 - \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}\left(\frac{\theta_c}{1+\theta_c}\right) var_f y_c(f) \\
&- \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}(1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(c - z_c - \alpha k_c)^2\right] - \mathbb{V}^i - \frac{1}{2}\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\left(\frac{\theta_i}{1+\theta_i}\right) var_f y_i(f) \\
&- \frac{1}{2}\left[\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\right](1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(i - z_i - \alpha k_i)^2\right]
\end{aligned} \tag{59}
$$

and, the corresponding flexible-price welfare function reads:

$$
\begin{aligned}
\mathbb{W}_t^* \simeq \mathbb{U}^* - \mathbb{V}^{c*} &+ \frac{1}{2}\mathbb{U}_{\hat{C}^*}\hat{C}^*\left(1-\sigma\right)c^{*2} - \frac{1}{2}\mathbb{U}_{\hat{C}^*}\hat{C}^*(1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(c^* - z_c - \alpha k_c)^2\right] \\
&- \mathbb{V}^{i*} - \frac{1}{2}\left[\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}^*}\hat{C}^*\right](1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(i^* - z_i - \alpha k_i)^2\right]
\end{aligned} \tag{60}
$$

A few things to note are as follows. First, the terms involving sectoral outputs dispersion are nil in the flexible-price equilibrium because all firms set the same price. Second, under the Woodford's definition of natural rate beginning of period sectoral capital stocks are identical to the beginning of period sectoral capital stocks under sticky prices. In other words, the assumption is that when we enter period $t+i$, it is the capital stock that is actually present, k_{t+i-1} that determines how in i^*_{t+i} is defined; hence, in $t+i$ the capital law of motion in the flexible price economy will read:

$$
k^*_{j,t+i} = (1-\tilde{\delta})k_{j,t+i-1} + \tilde{\delta} i^*_{j,t+i}
$$

Subtract the flexible price equilibrium, (59), to the sticky price one, (60):

$$
\begin{aligned}
\mathbb{W}_t - \mathbb{W}_t^* \simeq\ & \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}\left(1-\sigma\right)c^2 - \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}\left(\frac{\theta_c}{1+\theta_c}\right) var_f y_c(f) \\
&- \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}(1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(c - z_c - \alpha k_c)^2\right] \\
&+ \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}(1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(c^* - z_c - \alpha k_c)^2\right] - \frac{1}{2}\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\left(\frac{\theta_i}{1+\theta_i}\right) var_f y_i(f) \\
&- \frac{1}{2}\left[\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\right](1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(i - z_i - \alpha k_i)^2\right] \\
&+ \frac{1}{2}\left[\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\right](1+\eta)\left[\left(\frac{1}{1-\alpha}\right)(i^* - z_i - \alpha k_i)^2\right]
\end{aligned} \tag{61}
$$

Since the steady state of the sticky-price model is undistorted, the steady state terms cancel out. Also, terms involving only flexible-price variables or exogenous disturbances can be omitted since these are independent of policy (in fact these can be added and subtracted).

Equation (61) can be rewritten as:

$$
\begin{aligned}
\mathbb{W}_t - \mathbb{W}_t^* \simeq\ & \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}\,(1-\sigma)\,c^2 - \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}\left(\frac{\theta_c}{1+\theta_c}\right) var_f y_c(f) \\
& - \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}(1+\eta)\left[\left(\frac{1}{1-\alpha}\right)\left(c^2 - 2(z_c+\alpha k_c)c\right)\right] \\
& + \frac{1}{2}\mathbb{U}_{\hat{C}}\hat{C}(1+\eta)\left[\left(\frac{1}{1-\alpha}\right)\left(-2(z_c+\alpha k_c)c^*\right)\right] - \frac{1}{2}\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\left(\frac{\theta_i}{1+\theta_i}\right) var_f y_i(f) \\
& - \frac{1}{2}\left[\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\right](1+\eta)\left[\left(\frac{1}{1-\alpha}\right)\left(i^2 - 2(z_i+\alpha k_i)i\right)\right] \\
& + \frac{1}{2}\left[\frac{1-\phi}{\phi}\mathbb{U}_{\hat{C}}\hat{C}\right](1+\eta)\left[\left(\frac{1}{1-\alpha}\right)\left(-2(z_i+\alpha k_i)i^*\right)\right]
\end{aligned}
\tag{62}
$$

Note that terms involving differences between sticky-price and flexible-price *beginning-of-period* capital stocks cancel out, given that these two are the same.

By using Equations (34) and (35) to substitute out for the terms involving exogenous disturbances in the cross-product terms in (62) and divide both sides by $\mathbb{U}_{\hat{C}}\hat{C}$, one obtains:

$$
\begin{aligned}
\frac{\mathbb{W}_t - \mathbb{W}_t^*}{\mathbb{U}_{\hat{C}}\hat{C}} \simeq\ & -\frac{1}{2}\left[\frac{(1+\eta)}{1-\alpha} - (1-\sigma)\right](c-c^*)^2 \\
& - \frac{1}{2}\left[\frac{1-\phi}{\phi}\right]\left(\frac{1+\eta}{1-\alpha}\right)(i-i^*)^2 - \frac{1}{2}\left(\frac{\theta_c}{1+\theta_c}\right) var_f y_c(f) \\
& - \frac{1}{2}\frac{1-\phi}{\phi}\left(\frac{\theta_i}{1+\theta_i}\right) var_f y_i(f) + \left[\frac{1-\phi}{\phi}(1+\eta)\right](h_i^*)\,(i-i^*)
\end{aligned}
\tag{63}
$$

Also, Woodford (2003, Chapter 6) shows that the terms involving the variance of output dispersion are proportional to the variance of price dispersion, and in turn to the variance of inflation. That is, for $j = \{c, i\}$

$$
var_f y_j(f) = \left(\frac{1-\tilde{\beta}\xi_j}{1-\xi_j}\right)\frac{1}{\kappa_j} var(\pi_j)
$$

where $\kappa_j = \frac{(1-\tilde{\beta}\xi_j)(1-\xi_j)}{\xi_j}$. Also, when summing over infinite time the terms involving the squared gaps are nothing but the variance of the relative gaps. Therefore, the resulting welfare function is:

$$
\begin{aligned}
\frac{E_0 \int_0^1 \left[\sum_{t=0}^{\infty} \tilde{\beta}^t (\mathbb{W}_t(h) - \mathbb{W}_t^*(h))\right] dh}{\mathbb{U}_C C} \simeq & -\frac{1}{2}\left[\frac{(1+\eta)}{1-\alpha} - (1-\sigma)\right] var\left(c - c^*\right) \\
& - \frac{1}{2}\left[\frac{1-\phi}{\phi}\left(\frac{1+\eta}{1-\alpha}\right)\right] var(i - i^*) \\
& - \frac{1}{2}\left(\frac{\theta_c}{1+\theta_c}\right)\left(\frac{1-\tilde{\beta}\xi_c}{1-\xi_c}\right)\frac{1}{\kappa_c} var(\pi_c) \\
& - \frac{1}{2}\frac{1-\phi}{\phi}\left(\frac{\theta_i}{1+\theta_i}\right)\left(\frac{1-\tilde{\beta}\xi_i}{1-\xi_i}\right)\frac{1}{\kappa_i} var(\pi_i) \\
& + \left[\frac{1-\phi}{\phi}(1+\eta)\right] cov(h_i^*, i - i^*)
\end{aligned} \tag{64}
$$

which is Equation (18) in Section 4.