International Trade and Income Inequality*

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Abstract

We propose a simple theory that shows a mechanism through which international trade entails wage and job polarization. In the basic model, we consider two symmetric countries in which individuals with different abilities work either as knowledge workers, who develop differentiated products, or as production workers, who engage in actual production processes. In equilibrium, *ex ante* symmetric firms attract knowledge workers with different abilities, which create firm heterogeneity in product quality. Market integration disproportionately benefits firms that produce high-quality products. This winner-take-all trend of product markets causes war for talents, which exacerbates income inequality within the countries. Indeed, we show that as a result of trade liberalization, the real wages of highest and lowest income earners increase, while those of middle-income class decline. We also extend the basic model to those with asymmetric countries and show among others that international trade worsens income inequality within the countries, particularly so in the smaller country.

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1 Introduction

It has been widely recognized that income inequality has increased significantly in many countries, especially in the developed countries. In Britain, for example, "chief executives can expect to receive average compensation in excess of £4.5m ($ 6.9m) this year. Pay at the top grew by over 300% between 1998 and 2010. At the same time, the median British worker's real wage has been pretty stagnant. These trends mean the ratio of executive to average pay at FTSE 100 firms jumped from 47 to 120 times in 12 years." (*The Economist*, January 14th-20th, 2012, p. 11) It has also been well documented that job polarization has occurred in many developed countries, including the United States and some European countries, such that the shares of employment in high-skilled occupations and low-skilled occupations grow while that of middle-wage occupation declines; polarization of earnings, as well as the employment polarization, has also been observed (Autor *et al.* 2008 and Acemoglu and Autor 2011).

This study proposes a simple theory to show that growing international trade in goods can cause job polarization; the proposed model can explain in particular that the real incomes for the top income earners and the lowest income earners rise while those for the middle-income class decline. A growing share of the middle-income class serves as an engine of economic growth. The middle-income class also plays an important role in political stability (Acemoglu and Robinson, 2006). So a decline in the middle-income class may have serious economic and political consequences.

Technological changes and globalization are often argued as the causes of the job polarization and the expansion of income inequality. Machines have replaced middle-skilled workers who engage in routine tasks. Offshoring routine tasks to low-wage countries reduces demands for middle-skilled workers in high-wage developed countries. Offshoring is unambiguously an important aspect of globalization. But we argue here that a greater opportunity of international trade in goods alone can cause job polarization and expand income inequality among different skill groups of workers.

Thanks to the ICT (information and communication technology) revolution, it has become much easier for consumers to access detailed product information and to make comparisons between similar products. As a result even a small difference in product quality can lead to a large differential in firms' profitability within industries: firms that sell high-quality products command disproportionally high market shares. But this winner-take-all trend of product markets causes war for talents since what determines the product quality is the talents of knowledge workers, such as managers and R&D workers hired by firms. Consequently, workers who work as knowledge workers in winning firms earn disproportionally high income as a rent for their talents. A greater opportunity of international trade amplifies this effect. A decrease in trade costs (including marketing and other costs in foreign countries), again caused mostly by the ICT revolution, has increased the volume of world trade. The proliferation of international trade naturally widens profit differentials among firms within industries and hence widens income inequality among different skill groups.

To show this phenomenon, we build a two-country variant of Lucas's (1978) model in which *ex ante* symmetric firms in a representative differentiated-good (manufacturing) sector hire knowledge workers with different abilities, thereby producing products with different qualities. Firms also hire production workers to produce their products. We assume that workers are homogeneous in their productivities when hired as production workers despite the difference in their abilities. In equilibrium, knowledge workers are sorted into different firms according to their abilities; firms that hire a group of highly-talented knowledge workers produce high-quality products, while those hire mediocre knowledge workers produce low-quality products. The wage gap may arise between knowledge workers and production workers even within a firm; the wage gap within a firm is particularly serious in profitable firms that produce high-quality products.

International trade affects firms' profitability differently across firms. Top-tier firms that produce high-quality products are the winners of globalization; getting access to additional markets gives them large benefits. Medium-tier and lowest-tier firms, on the other hand, lose from globalization. They suffer from foreign top-tier firms' penetration into their own market. Although the medium-tier firms may sell their products to foreign markets as well as their own market, additional export profits after the subtraction of fixed costs of export are not enough to offset the loss that they incur in their domestic market. Consequently, top income earners who are best talented and work in the top-tier exporting firms benefit from opening to trade; they are the winners of globalization. On the contrary, workers in the middle-income class, who work as knowledge workers in the middle-tier firms, are likely to suffer from opening to trade. Their incomes fall because the profits for the firms in which they are working fall after trade liberalization. Some workers with intermediate abilities may also drop from the pool of knowledge workers to the pool of production workers, thereby receiving lower wages, as fierce competition among firms in the globalized world pushes the weakest firms out of the market. Although the real wages for the middle-income class may still rise thanks to the increased varieties of products in their consumption after trade liberalization, we show that under a relatively mild condition their real wages unambiguously decline by opening to trade. Indeed, workers in the middle-income class are the only losers from trade liberalization. The real wages for the least talented workers, who work as production workers, increase when the country opens to trade, thanks to the increased varieties in consumption.

After presenting our main results of the paper in the basic model with two symmetric countries, we also extend the model in section 5 to the case where countries are asymmetric in their population size and ability distribution. We find through numerical simulations that international trade exacerbates income inequality within the countries. Income inequality in the trade equilibrium is greater in the smaller country in the case of population asymmetry, while it is greater in the talent-abundant country in the case where the ability distributions are different between the countries. In both cases, income inequality worsens as openness to trade increases.

We are certainly not the first to theoretically predict that international trade widens wage gap across different income groups. Blanchard and Willmann (2016), Costinot and Vogel (2010), Helpman *et al.* (2010a,b), Helpman *et al.* (2016), Manasse and Turrini (2001), Sampson (2014), and Yeaple (2005) among others show in their respective models that international trade in goods widens wage gap within countries.[1]

Among these studies, Manasse and Turrini (2001) and Yeaple (2005) are the closest to our paper.

Manasse and Turrini (2001) employ the same basic model structure as ours; *ex ante* symmetric firms produce products of different qualities because they are run by entrepreneurs with different skills. They show among others that skill earnings in non-exporting firms are reduced relative to those in exporting firms as a result of further trade integration. In their analysis, however, the mass of entrepreneurs (i.e., workers with skills), which is equal to the mass of firms in the differentiated good industry by construction, is fixed and it is not affected by opening to trade. As a consequence, they cannot analyze how trade liberalization affects individual worker's occupational choice in the event of an induced change in trade volume at the extensive margin. This channel is important when we assess the impact of trade liberalization on the wage distribution within countries, because the winner-take-all trend in product markets is reinforced by globalization, thereby reducing the number of firms in each industry and reducing the jobs as knowledge workers in the middle-income class. In addition, their model does not show that international trade adversely affects the middle-income class.

Yeaple (2005) derives similar predictions to ours in a similar model environment. In his model, firms choose both their individual production technologies and types of workers. A distinguishing feature of his model is the complementarity between the technology and skills of labor; high-productivity technology is matched with high-skilled workers. He shows among others that a reduction in trade costs *may* decrease the real wage of moderately skilled workers. Beside the fact that the endogenously-determined average talent of knowledge workers is the only source of firm heterogeneity in our model, our model is different from his in the important aspect that we separate labor into two endogenously allocated categories: knowledge workers and production workers. This distinction is a key to our analysis of the impact of trade on income redistribution. First, we can discuss differential effects of

[1] See Grossman (2013) for a thought-provoking survey on the impact of international trade on labor markets.

trade on workers within firms, which is an important wage gap besides the wage gap within sectors and within occupations. Second, we can show that under the mild condition (on the elasticity of substitution) the real incomes *unambiguously* fall as a result of globalization for some workers in the middle-income class because they drop out of the pool of knowledge workers and work as production workers after trade is liberalized. Third and more broadly, separating knowledge workers from production workers is important in understanding the effect of globalization on the labor market. Knowledge can be embedded into products so that it is duplicated unboundedly with the help of capital and production workers to possibly earn a fortune in the global market. Globalization does not necessarily increase the demands for knowledge workers. (Indeed, our model predicts that demands for them decrease as a result of globalization.) It only increases the demands for talent. Knowledge created by a limited number of knowledge workers is embedded in the products and travels over the world.

Monte (2011) and Egger and Kreickemeier (2012) also extend Lucas's (1978) model to examine the impact of international trade on income distribution. Since their interest is slightly different from ours, they do not show that trade reduces the real wage of the middle-income class.

We believe that our model is the simplest one to show the adverse effect of trade on the middle-income class, capturing the important aspect of globalization: the winner-take-all market and war for talents.

2 The Model

We consider a model of two countries (countries 1 and 2), one good (a differentiated good with many varieties), and one production factor (labor). The differentiated good consists of a continuum of varieties, each of which (denoted by $\omega \in \Omega$) is produced by a firm under the monopolistic competition. Product quality, which is represented by $\alpha(\omega)$, may vary across varieties. Following Manasse and Turrini (2001) and Caliendo and Rossi-Hansberg (2012), we represent a representative consumer's preferences by the utility function:

$$u = \left[\int_{\Omega} \alpha(\omega)^{\frac{1}{\sigma}} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $x(\omega)$ denotes the consumption level of a variety ω and $\sigma > 1$ denotes the elasticity of substitution. The higher the $\alpha(\omega)$, the higher the utility a consumer derives from the consumption of variety ω.

In each country $i = 1, 2$, there is a continuum of workers with the mass L_i; each worker provides 1 unit of labor. Labor is the only production factor in this economy. But a worker is employed either as a knowledge worker to develop a product or as a production worker to produce the good. We choose labor provided by production workers as the numeraire. Workers are heterogeneous in their abilities, which only matter when they are hired as knowledge workers. Thus, they are heterogeneous as knowledge workers, but homogenous as production workers. In the basic model, ability is measured by $a \in \mathbb{R}_+$, the distribution of which in country i is represented by the cumulative distribution function G_i with the probability density function g_i; the mass of workers with their abilities less than or equal to a is, therefore, given by $L_i G_i(a)$. In the basic model, we assume that countries 1 and 2 are symmetric: that is, $L_1 = L_2 = L$ and $G_1(a) = G_2(a) = G(a)$ for all $a \in \mathbb{R}_+$.

The good market is under the monopolistic competition with free entry and exit. To enter the market, firms need to develop the product by hiring l knowledge workers, which serves as entry costs. The average ability of these workers determines the quality of the product; we simply assume that the quality of the product $\alpha(\omega)$ is equal to the average ability of the knowledge workers employed in the firm. Production itself requires only production workers; 1 unit of labor produces 1 unit of the good.

There is no friction in the labor markets both for knowledge workers and production workers, nor does there exist any information asymmetry between workers and firms on individual workers' abilities. Since the average ability of knowledge workers determines the product quality, firms in the differentiated good sector compete for talent. They post wages for knowledge workers, and workers apply for those positions; we assume that each firm offers a wage that equally applies to every knowledge worker within the firm regardless of the workers' abilities.[2] Then, each firm chooses l workers from those who have applied for the firm's position.

In equilibrium, matching between firms and knowledge workers must be stable. As a consequence of the competition among the firms, the entire operating profits for each firm are given as a rent to the knowledge workers. Therefore, given that the average ability of knowledge workers determines the product quality and that they receive the same wage within the firms, they have a strong incentive to be matched with other knowledge workers with abilities that are greater than or equal to their own abilities. As a result, knowledge workers are sorted into the firms according to their abilities. Since the most talented workers are in limited supply, the firms post different wages for knowledge workers, attracting workers with different abilities. Workers are sorted according to their abilities such that workers with highest abilities are hired by the firms that post highest wages. Then, it follows from the assumption of a continuum of workers that all knowledge workers in a firm will have a common ability. Letting $w(\omega)$ denote the knowledge workers' wage (or rent) in the firm that produces the variety ω and $\tilde{\pi}(\omega)$ denote the firm's operating profits, we therefore have

$$w(\omega)l = \tilde{\pi}(\omega). \tag{2}$$

Firms produce varieties of different qualities in equilibrium. The exogenously-given ability distribution determines the firm distribution with respect to their product quality, since workers are sorted according to their abilities and the average ability of knowledge workers determines the product quality. The distribution of workers with respect to their abilities is characterized by a density function of $Lg(a)$. Since all knowledge workers in a firm will have a common ability, the density of firms that produce varieties of quality α, which is denoted

[2] We will show that in equilibrium, the abilities of knowledge workers are the same within the firms. If we assume alternatively that a firm posts a wage profile that possibly assigns a higher wage to a worker with a higher ability, we would have another equilibrium in which a firm attracts workers with different abilities. But such equilibrium would disappear if (even a slight) complementarity between knowledge workers' abilities within the firms is introduced. Since our aim is to construct a simple and yet plausible model to show that the middle-income class suffers from globalization, we make the assumption that each firm pays the same wage to all knowledge workers and examine the properties of the equilibrium in which knowledge workers are sorted into the firms according to their abilities.

by $f(\alpha)$, is given by

$$f(\alpha) = \frac{Lg(\alpha)}{l}. \tag{3}$$

Workers who are not hired as knowledge workers will work as production workers. In equilibrium, there will be a cutoff ability α^* such that all workers with $a \geq \alpha^*$ work as knowledge workers, while all workers with $a < \alpha^*$ work as production workers. Once α^* is given, together with (3), the quality distribution of operating firms is completely determined.

3 Autarkic Equilibrium

This section derives the autarkic equilibrium and shows that knowledge workers receive higher wages than production workers and that knowledge workers' wages increase proportionately with their abilities. Thanks to the symmetry assumed in the basic model, we need only consider a representative country to derive the autarkic equilibrium.

First, we use a consumer's (or worker's) demands derived from (1) to obtain a firm's production level and profits. Let I denote the aggregate income of a country. Since the wage rate of production workers is normalized to 1, each firm optimally selects the price $p(\alpha) = \sigma/(\sigma - 1)$, the constant mark-up price over the marginal cost of 1, regardless of its product quality α. Consequently, the firm that produces a variety of quality α sells

$$x(\alpha) = \frac{\alpha p(\alpha)^{-\sigma}}{\int_{\alpha^*}^{\infty} \alpha' p(\alpha')^{1-\sigma} f(\alpha') d\alpha'} I = \frac{\alpha}{\int_{\alpha^*}^{\infty} \alpha' f(\alpha') d\alpha'} \frac{(\sigma - 1)I}{\sigma} \tag{4}$$

units of the good. The firm's production level is higher, the higher is the quality of its product and the smaller is the quality index, $\int_{\alpha^*}^{\infty} \alpha' f(\alpha') d\alpha'$. The operating profits for the firm that produces a product with quality α are given by

$$\tilde{\pi}(\alpha) = \frac{\alpha I}{\sigma \int_{\alpha^*}^{\infty} \alpha' f(\alpha') d\alpha'}. \tag{5}$$

Henceforth, we identify a firm by the quality of its product rather than $\omega \in \Omega$, since all firms that produce the good of the same quality have common characteristics.

Letting $w(\alpha)$ denote the wage for a knowledge worker with ability $a = \alpha$, who is hired by the firm that produces the product of quality α (with a slight abuse of notation), we can write the profits for the firm as

$$\pi(\alpha) = \tilde{\pi}(\alpha) - w(\alpha)l.$$

If $\pi(\alpha)$ is strictly positive for some firm with α, an entrant would post a slightly higher wage than $w(\alpha)$ and get all the knowledge workers from that firm and profitably operate. Therefore, $\pi(\alpha) = 0$ in equilibrium, so that the knowledge workers' wage schedule is given by $w(\alpha) = \tilde{\pi}(\alpha)/l$ as (2) indicates.

The equilibrium is characterized by the two conditions: the free-entry (FE) condition and the labor-market clearing (LM) condition. The free-entry condition expresses that the operating profits for the cutoff firms with α^* are just large enough for them to pay the wage of 1 to each knowledge worker.[3] The knowledge workers in the cutoff firms earn the wage of 1, i.e., $w(\alpha^*) = 1$, since if $w(\alpha^*) > 1$ profitable entry by a firm that attracts knowledge workers with an ability slightly lower than α^* at the wage $[w(\alpha^*) + 1]/2$, for example, would arise. Thus, the free-entry condition can be written as

$$\frac{\alpha^* I}{\sigma \int_{\alpha^*}^{\infty} \alpha f(\alpha) d\alpha} = l. \tag{6}$$

The labor-market clearing condition, on the other hand, expresses that total labor demands, the sum of demands for knowledge workers and those for production workers, must equal the labor supply L. Total demands for knowledge workers are given by $l \int_{\alpha^*}^{\infty} f(\alpha) d\alpha$. Total demands for production workers equal $(\sigma - 1)I/\sigma$ as we can easily obtain from (4). Thus, the labor-market clearing condition can be written as

$$l \int_{\alpha^*}^{\infty} f(\alpha) d\alpha + \frac{\sigma - 1}{\sigma} I = L. \tag{7}$$

Figure 1 depicts the relationships between α^* and I that express the free-entry and labor-market clearing conditions. The free-entry condition is expressed by a negatively-sloped schedule FE, since the left-hand side of (6) increases with both α^* and I. The labor-market clearing condition, on the other hand, is expressed by a positively-sloped schedule LM, since

[3]Note that α^* denotes both threshold ability for knowledge workers and the quality of the cutoff firms' products since the cutoff firms that produce products of quality α^* hire knowledge workers with ability α^*.

the left-hand side of (7) decreases with α^* but increases with I. The intersection of these two schedules gives us the autarkic equilibrium values of α^* and I, which we call α^*_A and I_A.

Once the equilibrium threshold α^*_A is determined, the equilibrium wage schedule is readily obtained. As Figure 2 shows, wages are flat at 1 for all workers with their abilities smaller than α^*_A. Their wages are 1 because they work as production workers. Those who have abilities greater than α^*_A, on the other hand, work as knowledge workers. Their wages are the rents for their abilities and are greater than 1 except for the knowledge workers whose ability levels are exactly equal to α^*_A. As we can see from (5), the ratio of wages for knowledge workers with any two different levels of abilities is equal to the ratio of their abilities itself:

$$\frac{w_A(\alpha)}{w_A(\alpha^*_A)} = \frac{\tilde{\pi}(\alpha)}{\tilde{\pi}(\alpha^*_A)} = \frac{\alpha}{\alpha^*_A},$$

where we can compare the wage for a knowledge worker with ability $a = \alpha$ with that for a knowledge worker with ability $a = \alpha^*_A$. It follows from $w_A(\alpha^*_A) = 1$ that we can write the wage of a worker as a function of her ability:

$$w_A(a) = \begin{cases} 1 & \text{if} \quad 0 \leq a < \alpha^*_A \\ a/\alpha^*_A & \text{if} \quad a \geq \alpha^*_A. \end{cases}$$

We can view this result from the perspective of wage gaps within firms to obtain the following proposition.

Proposition 1 *The wage gap between knowledge workers and production workers within a firm is large, the higher is the quality of its product and hence larger is the firm.*

The equilibrium utility for a worker, which can also be considered as her real wage, can be readily derived. It is easy to infer from (4) that the worker with ability a consumes $\frac{\alpha}{\int_{\alpha^*_A}^{\infty} \alpha' f(\alpha') d\alpha'} \frac{(\sigma-1) w_A(a)}{\sigma}$ units of each variety of quality $\alpha \geq \alpha^*_A$. Then it follows from (1) that the worker's indirect utility in the autarkic equilibrium is given by

$$u_A(a) = \frac{(\sigma - 1) w_A(a)}{\sigma \left[\int_{\alpha^*_A}^{\infty} \alpha f(\alpha) d\alpha \right]^{\frac{1}{1-\sigma}}}. \tag{8}$$

Not surprisingly, the equilibrium utility for an individual depends on both her wage and the quality index of the market.

4 Trade Equilibrium

Let us turn to the analysis of the impact of international trade between the two symmetric countries on the firms' activities and the wage schedule in the individual countries. We suppose that firms incur f_X units of labor as the fixed cost of exporting. Exporting firms also incur iceberg trade cost such that they need to ship $\tau(>1)$ units of the good to supply 1 unit in the foreign market.

We consider a realistic case in which only a fraction of the firms export their products. Let α^X denote the threshold quality such that the products are exported (as well as supplied domestically) if and only if their individual qualities are higher than or equal to α^X. It is easy to see that the operating profits from the domestic sales and foreign sales are given by

$$\tilde{\pi}_d(\alpha) = \frac{\alpha I}{\sigma\left[\int_{\alpha^*}^{\infty}\alpha' f(\alpha')d\alpha' + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha' f(\alpha')d\alpha'\right]}, \tag{9}$$

$$\tilde{\pi}_X(\alpha) = \frac{\alpha\tau^{1-\sigma} I}{\sigma\left[\int_{\alpha^*}^{\infty}\alpha' f(\alpha')d\alpha' + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha' f(\alpha')d\alpha'\right]}, \tag{10}$$

respectively. Then, the profits for the firm that produces a product of quality α are equal to

$$\pi(\alpha) = \begin{cases} \tilde{\pi}_d(\alpha) - w(\alpha)l & \text{if } \alpha^* \le \alpha < \alpha^X \\ \tilde{\pi}_d(\alpha) + \tilde{\pi}_X(\alpha) - w(\alpha)l - f_X & \text{if } \alpha \ge \alpha^X. \end{cases} \tag{11}$$

Since the quality indices take the same value between the two countries due to the symmetry, it is easy to compare the operating profits from exporting and those from domestic sale. In particular, we compare the operating export profits for the export-cutoff firm with the product quality α^X, i.e., $\tilde{\pi}_X(\alpha^X) = f_X$, and the profits for the entry-cutoff firm with α^*, i.e., $\tilde{\pi}_d(\alpha^*)$. Then, it follows from (9) and (10) that the relationship between α^X and α^* can be expressed by the function:

$$\alpha^X(\alpha^*) = \frac{\tau^{\sigma-1} f_X}{l}\alpha^*. \tag{12}$$

We assume that $\tau^{\sigma-1} f_X > l$ so that only a fraction of the firms export their products.

The free-entry condition and the labor-market clearing condition can be written as

$$\frac{\alpha^* I}{\sigma\left[\int_{\alpha^*}^{\infty}\alpha f(\alpha)d\alpha+\tau^{1-\sigma}\int_{\alpha^X(\alpha^*)}^{\infty}\alpha f(\alpha)d\alpha\right]}=l, \tag{13}$$

$$l\int_{\alpha^*}^{\infty}f(\alpha)d\alpha+f_X\int_{\alpha^X(\alpha^*)}^{\infty}f(\alpha)d\alpha+\frac{\sigma-1}{\sigma}I=L, \tag{14}$$

respectively.

By comparing the free-entry condition (13) with the autarkic counterpart (6), we find that the FE schedule shifts up as Figure 3 indicates. International trade intensifies the domestic competition. In order for the same threshold producer to be break-even, the total income must increase. Similarly, by comparing (14) with (7), we see that the LM schedule shifts down. The labor market becomes tighter because trade creates additional demands for labor that is used as a fixed input for exporting. Thus, the income must decrease so that these increased demands are offset by the decreased demands for production workers. As Figure 3 indicates, trade-equilibrium income denoted by I_T may be greater or smaller than I_A, i.e., the impact of trade on the total income is ambiguous. However, trade will unambiguously raise the threshold quality α_T^* (Melitz, 2003). International trade intensifies competition in individual domestic markets, which lowers the profitability of firms that only serve their individual domestic markets. In addition, the labor market becomes tighter due to the demands for labor for exporting. These two effects work as factors to increase the bar to enter the industry.

Proposition 2 *International trade raises the entry-threshold quality of the differentiated good, i.e.,* $\alpha_A^* < \alpha_T^*$*, and hence decreases the mass of firms and raises the average quality of the good. Moreover, a decrease in the mass of firms implies that opening to trade induces some knowledge workers (with the lowest abilities among them) to work as production workers.*

The main focus of the paper is to examine the impact of trade on the wage schedule. It follows from (11) and $\tilde{\pi}_X(\alpha)/\tilde{\pi}_X(\alpha^X) = \alpha/\alpha^X$, together with $\tilde{\pi}_X(\alpha^X) = f_X$, that the equilibrium wage schedule is described by a piecewise linear function:

$$w_T(a) = \begin{cases} 1 & \text{for} \quad a \in [0, \alpha_T^*) \\ \frac{a}{\alpha_T^*} & \text{for} \quad a \in [\alpha_T^*, \alpha^X) \\ \frac{a}{\alpha_T^*} + \frac{(a-\alpha^X) f_X}{l\alpha^X} & \text{for} \quad a \in [\alpha^X, \infty). \end{cases}$$

Figure 4 shows how the wage schedule changes as a result of opening to trade. The wage schedule in trade equilibrium shows that there are more production workers, who earn the wage of 1, than those before opening to trade since trade decreases the mass of firms and hence the mass of knowledge workers. That is, workers whose ability a is between α_A^* and α_T^* used to work as knowledge workers but now work as production workers after opening to trade. In trade equilibrium, workers whose ability is between α_T^* and α^X work as knowledge workers in the firms that serve only their own domestic market. Their wages are lower in trade equilibrium than in autarky because their firms suffer from increased competition in their domestic market. Workers whose abilities are greater than α^X are the knowledge workers who work in firms that export their products. Profits for a firm that barely meets the criterion for exporting are smaller than those in autarky. This is because profits from exporting after paying the fixed costs of exporting are not sufficient to offset the losses in the domestic sales caused by the import penetration. Wages for knowledge workers who work in such firms also decline as a result of opening to trade. However, the wages for knowledge workers whose abilities are sufficiently high rise by opening to trade, since trade disproportionately increases the profits for the firms that hire such knowledge workers. Trade increases wages (measured by labor provided by the production workers) only for those who are highly talented. Extraordinary talent pays disproportionately in the world of globalization. Such talents are embodied in products and travel over the world.

Proposition 3 *International trade raises the income (measured by labor provided by the production workers) only for those who are most talented.*

Interpreting this result differently, we obtain the following corollary about the wage gap within firms.

Corollary 1 *As a result of trade liberalization, the wage gap between knowledge workers and production workers within the top-tier firms expands while this wage gap shrinks within medium-tier and lowest-tier firms, whose operating profits drop as a consequence.*

What is the impact of trade on real wages? Similarly to (8), the worker's indirect utility in the trade equilibrium can be written as

$$u_T(a) = \frac{(\sigma-1)w_T(a)}{\sigma\left[\int_{\alpha_T^*}^{\infty}\alpha f(\alpha)d\alpha + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha f(\alpha)d\alpha\right]^{\frac{1}{1-\sigma}}}. \tag{15}$$

It is easy to see that those who work as production workers both before and after opening to trade benefit from trade. Their real wages increase because the quality index rises due to the intensified competition in the product market while the (nominal) wages are unaffected. It is even more obvious that most talented workers whose (nominal) incomes increase after opening to trade also benefit from trade.

What about the impact on the middle-income class? To derive a clear-cut answer to this question, we assume now that the ability is distributed according to the Pareto distribution with its cumulative distribution function $G(a) = 1-(a_0/a)^k$, where $a_0 > 0$ and $k > 1$. Then, it can be shown that under the mild condition that $\sigma > 2$, international trade decreases the real wages for the middle-income class as Figure 5 shows.[4] The proof of the following proposition requires some calculations and thus relegated to the Appendix.

Proposition 4 *Suppose that there are two symmetric countries in which workers' ability distribution follows a Pareto distribution. Then, the lowest income earners who work as production workers as well as the highest income earners who work as knowledge workers are better off by international trade. Those who belong to the middle-income class, however, experience a decrease in real wages by opening to trade: there is a range of workers' ability such that the real wages of workers whose abilities are within this range fall as a result of*

[4] Many authors have estimated the elasticity of substitution and have reported the estimated values that exceed 2 in most cases; the estimate of the elasticity by Bernard, *et al.* (2003), for example, is 3.79. Broda and Weinstein (2006) estimate the elasticities of substitution for different industries at different levels of aggregation in different periods of time. Table IV of their paper shows that the simple average of the elasticities of substitution is 6.6 (with an outlier dropped) for five-digit (SITC) industries while it is 4.0 for three-digit (SITC) industries in the period of 1990-2001.

opening to trade if and only if $\sigma > 2$ holds. All knowledge workers who work in the firms that only serve their individual domestic markets in trade equilibrium belong to such middle-income class.

An individual that belongs to the middle-income class (e.g., knowledge workers with $a \in [\alpha_T^*, \alpha^X]$) benefits from trade as a consumer due to an increase in the quality index, but loses as a residual claimer of the firm since an increase in the quality index leads to a decrease in the firm's operating profits. The latter effects outweighs the former if and only if $\sigma > 2$, i.e., the elasticity of substitution is large enough that the latter competition-enhancing effect is dominant.

Having derived the impacts of trade on individual workers' real wages, we now turn to the impact on overall social welfare of each country. We use two measures of social welfare to evaluate the effect of international trade: the utilitarian social welfare and the Lorenz domination. If the ability is distributed according to a Pareto distribution, we have unambiguous results regarding the impact of trade on social welfare in both measures.

The following proposition, the proof of which is relegated to the Appendix, shows that trade unambiguously increases a simple aggregation of individuals' utilities.

Proposition 5 *Suppose that there are two symmetric countries in which workers' ability distribution follows a Pareto distribution. Then, international trade unambiguously improves utilitarian social welfare for individual countries.*

The other measure of social welfare is the Lorenz domination, which is a measure to evaluate the equality of income distribution. We define the Lorenz function by

$$\mathcal{L}(a) = \frac{\int_0^a w(a')dG(a')}{\int_0^\infty w(a')dG(a')},$$

the fraction of total income earned by those who have the ability a or less. We say that the income distribution characterized by the Lorenz function $\mathcal{L}_\mathcal{A}$ is *Lorenz-dominated* by the one characterized by $\mathcal{L}_\mathcal{B}$, if $\mathcal{L}_A(a) \leq \mathcal{L}_B(a)$ for any a with strict inequality for some a.

Proposition 6 *Suppose that there are two symmetric countries in which workers' ability distribution follows a Pareto distribution. Then, the income distribution under international trade is Lorenz-dominated by the one in autarky.*

The Appendix shows the proof of Proposition 6.

Propositions 5 and 6 give us clear and important message to us about the impact of international trade on income distribution. International trade benefits a country as a whole, but increases income inequality within the country. This message is reminiscent of the well-known results from the traditional, neoclassical trade theory: international trade benefits all trading countries but create winners and losers within the countries (the Stolper-Samuelson theorem). But there is an important difference. In traditional trade models, trade increases the reward to the factor whose reward has been suppressed due to the abundance in its supply, while it decreases the reward to the factor that had enjoyed a relatively high reward due to its scarceness. In contrast, our model predicts widening the income gap such that the reward to the scarce talent that had already claimed a disproportionately large proportion of the aggregate income disproportionately increases as a result of opening to trade.

5 Asymmetric Countries

In this section, we extend the basic model to the one in which the countries are asymmetric in population or ability distribution. We will show numerically that our main result that only the middle-income class suffers from trade liberalization remains valid also in the case of asymmetric countries. We show among others that income inequality worsens in both countries as openness to trade increases, but it does so particularly in the smaller country.

We examine the two cases (i) that country 1 is larger than country 2, i.e., $L_1 > L_2$, and (ii) that country 1 has relatively more workers of high ability than country 2. The second case is characterized by the first-order stochastic dominance of the Pareto distributions for workers' ability. Specifically, we assume that $k_1 < k_2$ so that $G_1(a) < G_2(a)$ for any $a > a_0$, where $G_i(a) = 1 - (a_0/a)^{k_i}$. When there is sufficiently large asymmetry between the two countries, there is a greater chance that all firms in one country, e.g., the smaller country, export their products. We allow this possibility, i.e., $\alpha_i^* = \alpha_i^X$, for $i = 1, 2$, in this section.

Before we examine these two cases separately, we shall derive the equilibrium conditions, noting that the firm distribution in country i is given by the density function similar to the one in (3):

$$f_i(\alpha) = \frac{L_i g_i(\alpha)}{l}. \tag{16}$$

We choose labor provided by the production workers in country 2 as a numeraire, so the wage rate for the production workers in country 2 equals 1. We let w denote the wage rate for the production workers in country 1.

To derive the equilibrium conditions, we first note that the total income for each country equals the total revenue of all the domestic firms. We can write the total income for country 1, for example, as

$$I_1 = \frac{w^{1-\sigma}\int_{\alpha_1^*}^{\infty}\alpha f_1(\alpha)d\alpha}{w^{1-\sigma}\int_{\alpha_1^*}^{\infty}\alpha f_1(\alpha)d\alpha + \tau^{1-\sigma}\int_{\alpha_2^X}^{\infty}\alpha f_2(\alpha)d\alpha} I_1 + \frac{w^{1-\sigma}\tau^{1-\sigma}\int_{\alpha_1^X}^{\infty}\alpha f_1(\alpha)d\alpha}{\int_{\alpha_2^*}^{\infty}\alpha f_2(\alpha)d\alpha + w^{1-\sigma}\tau^{1-\sigma}\int_{\alpha_1^X}^{\infty}\alpha f_1(\alpha)d\alpha} I_2.$$

Using (16), we can rewrite this equality as

$$I_1 = \frac{w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha}{w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha + \tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha} I_1 + \frac{w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha}{L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha + w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha} I_2.$$

Similarly, we have

$$I_2 = \frac{L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha}{L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha + w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha} I_2 + \frac{\tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha}{w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha + \tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha} I_1,$$

for country 2. Although they look different, it is easy to show that the last two equations are equivalent and can be written as

$$\frac{w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha}{L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha + w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha} I_2 = \frac{\tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha}{w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha + \tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha} I_1. \tag{17}$$

The left-hand side of this equation represents the total value of country 1's exports while the right-hand side represents country 2's counterpart. Thus, this equation shows that the trade should balance between the two countries, which is the first equilibrium condition of this extended model.

The second set of equilibrium conditions are the free-entry conditions for countries 1 and 2. Using (16), we can write the conditions as

$$\frac{w^{1-\sigma}\alpha_1^* I_1}{\sigma\left[w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha+\tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha\right]} = w, \tag{18}$$

$$\frac{\alpha_2^* I_2}{\sigma\left[L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha+w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha\right]} = 1. \tag{19}$$

Whereas these conditions determine the entry cutoffs for the two countries, the next two conditions determine the cutoff qualities for exporting firms:

$$\frac{lw^{1-\sigma}\tau^{1-\sigma}\alpha_1^X I_2}{\sigma\left[L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha+w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha\right]} = wf_X, \tag{20}$$

$$\frac{l\tau^{1-\sigma}\alpha_2^X I_1}{\sigma\left[w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha+\tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha\right]} = f_X, \tag{21}$$

for countries 1 and 2, respectively.[5]

The last two conditions are the labor-market clearing conditions:

$$\begin{aligned} L_1\int_{\alpha_1^*}^{\infty}g_1(\alpha)d\alpha &+ \frac{f_X L_1}{l}\int_{\alpha_1^X}^{\infty}g_1(\alpha)d\alpha+\frac{w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha}{w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha+\tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha}\frac{\sigma-1}{\sigma}\frac{I_1}{w} \\ &+ \frac{w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha}{L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha+w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha}\frac{\sigma-1}{\sigma}\frac{I_2}{w}=L_1, \end{aligned} \tag{22}$$

$$\begin{aligned} L_2\int_{\alpha_2^*}^{\infty}g_2(\alpha)d\alpha &+ \frac{f_X L_2}{l}\int_{\alpha_2^X}^{\infty}g_2(\alpha)d\alpha+\frac{L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha}{L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha+w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha}\frac{\sigma-1}{\sigma}I_2 \\ &+ \frac{\tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha}{w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha+\tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha}\frac{\sigma-1}{\sigma}I_1=L_2, \end{aligned} \tag{23}$$

for countries 1 and 2, respectively.

We have 7 equations, (17)–(23), and 7 unknowns (α_1^*, α_1^X, α_2^*, α_2^X, w, I_1, I_2). Naturally, it is rather difficult to analytically solve this simultaneous equation system. Thus, we conduct

[5] If α_2^X is so small that $\alpha_2^X=\alpha_2^*$ holds, for example, (19) and (21) are replaced by

$$\frac{l\alpha_2^* I_2}{\sigma\left[L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha+w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha\right]}+\frac{l\tau^{1-\sigma}\alpha_2^X I_1}{\sigma\left[w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha+\tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha\right]}=l+f_X$$

and $\alpha_2^X=\alpha_2^*$.

some numerical simulations to derive the effects of international trade on income inequality when the two countries are asymmetric. We select $\sigma = 4$, $l = 1$, $f_X = 1$, and $a_0 = 1$ for concreteness when we show our results graphically. The properties of the equilibrium, however, are robust to changes in parameter values.

What remains to be of our greatest interest is the impact of international trade on the real wage schedules. Similarly to (15) in the case of symmetric countries, we obtain the real wage schedules for countries 1 and 2 for this general model of asymmetric countries:

$$u_1(a) = \frac{(\sigma-1)w_1(a)}{l^{\frac{1}{\sigma-1}}\sigma\left[w^{1-\sigma}L_1\int_{\alpha_1^*}^{\infty}\alpha g_1(\alpha)d\alpha + \tau^{1-\sigma}L_2\int_{\alpha_2^X}^{\infty}\alpha g_2(\alpha)d\alpha\right]^{\frac{1}{1-\sigma}}}, \tag{24}$$

and

$$u_2(a) = \frac{(\sigma-1)w_2(a)}{l^{\frac{1}{\sigma-1}}\sigma\left[L_2\int_{\alpha_2^*}^{\infty}\alpha g_2(\alpha)d\alpha + w^{1-\sigma}\tau^{1-\sigma}L_1\int_{\alpha_1^X}^{\infty}\alpha g_1(\alpha)d\alpha\right]^{\frac{1}{1-\sigma}}}, \tag{25}$$

respectively.

5.1 Different Population

We first consider the case in which the two countries are different only in the population size. We assume that country 1 is larger than country 2: $L_1 = 100$ and $L_2 = 50$. The ability distributions are the same between the two countries: $k_1 = k_2 = 2$.

Figure 6 illustrates the simulation result on the impact of international trade on real wages, measured by the indirect utility given by (8) in the case of autarky and (24) and (25) in the case of the trade equilibrium when the openness to trade, defined by $\phi = \tau^{1-\sigma}$, equals 0.58 (i.e., $\tau \approx 1.2$). The first observation we want to emphasize is that in both countries, only the middle-income class suffers from opening to trade. That is, the main result obtained in the case of symmetric countries remains valid also in this case of asymmetric population size.

As for the comparison between the two countries, we first note that in autarky, the real wage is higher in country 1 than in country 2 for any ability a. This is because country 1 as the larger country host more firms, each of which produces a variety that is slightly different from others, so that the price index is smaller (or equivalently, the quality index is greater) in country 1. This advantage of living in the larger country remains even after opening to trade. We observe (but do not show here) that the wage rate is greater in the larger country 1 than in country 2, i.e., $w > 1$, which is known as the home market effect (Krugman, 1980, 1991). Together with the observation that the price index is smaller in the larger country 1 than in country 2, the real wage is still higher in country 1 than in country 2 in the trade equilibrium for any ability level. In addition, in all simulations with different parameter values, the results indicate that I_1/w and I_2 take the same values, respectively, throughout a change in $\phi(= \tau^{1-\sigma})$ from 0 to 1. Together with the observation that $w > 1$, this means that international trade creates the (nominal) income gap between the two countries, favoring the larger country.

We also infer from Figure 6 that international trade worsens income inequality, as observed in the symmetric-country case, and it does so more severely for the smaller country. Our model shares with Melitz's (2003) the important property that the entry threshold α_i^* increases while the export threshold α_i^X decreases as the openness to trade ϕ increases even when the countries are asymmetric. Compared with the case of autarky, in particular, the entry threshold in the trade equilibrium is higher and the (nominal) wage schedule shifts as indicated in Figure 4 in both countries. Together with the observation that the values of I_1/w and I_2 do not change by opening to trade, respectively, this means that the same logic as used in the proof of Proposition 6 (in the Appendix) applies here to conclude that the income distribution under international trade is again Lorenz-dominated by the one in autarky even in this case. This logic can also be applied to the difference between the countries in assessing the effect of opening to trade. As Figure 6 indicates, $\alpha_1^* < \alpha_2^*$ and $\alpha_1^X > \alpha_2^X$ in the trade equilibrium. The larger country 1 has a larger market than country 2 not just because of the larger population but also because of the greater equilibrium wage rate. Consequently, country 1 can accommodate more lower-quality firms (both domestic and foreign) than country 2, i.e., $\alpha_1^* < \alpha_2^*$ and $\alpha_2^X < \alpha_1^X$, which in turn means that the proportions of both lowest and highest income groups are larger in country 2 than in country 1. So we infer that the smaller country 2's income inequality worsens compared with country 1's in the sense of the Lorenz domination. This observation is confirmed by Figure 7, which illustrates the vertical difference between the Lorenz curve in the trade equilibrium and that in autarky, i.e., the income share in the trade equilibrium minus that in autarky, for countries 1 and 2.

Finally, we find that the income inequality monotonically worsens in both countries as the openness to trade increases, as expected from the observation that α_i^* increases while α_i^X decreases with ϕ. Figure 8 shows that the Gini coefficients increase in both countries as ϕ, the openness to trade, increases.[6] Moreover, the smaller country 2 always experiences greater income inequality than the larger country 1. International trade entails job polarization, which exacerbates income inequality. This effect is stronger, the smaller is the country.

5.2 Different Ability Distributions

We turn to the case where the two countries are different only in the ability distribution. We report the findings here when $k_1 = 3$ and $k_2 = 4$: country 1 has relatively more workers with high abilities than country 2. We assume that the two countries have the same population: $L_1 = L_2 = 100$.

Similarly to the case of population asymmetry, only the middle-income class suffers from trade in both countries, as Figure 9 shows. We also observe that the real wage for production workers is greater in the talent-abundant country 1 than in country 2 regardless of whether the two countries engage in trade. This is because country 1 has more firms that produce goods of high quality than country 2 so that demands for production workers are higher. When we compare knowledge workers with the same ability, however, the real wages for them are higher in the talent-scarce country 2 reflecting the scarcity of talent; the (nominal) wage for production workers is lower in country 2 than in country 1 so that the firms in country 2 earn more operating profits than their counterparts in country 1. We emphasize here that the fact that knowledge workers with the same ability earns less in country 1 than in country 2 does not mean that country 2 is wealthier than country 1. Indeed, as Figure 10 indicates, the (nominal) wage is higher in the talent-abundant country 1 than in country 2 at

[6] All firms in country 2 export their products, i.e., $a_2^* = a_2^X$, when the openness to trade is greater than $\phi = 0.76$ (i.e., $\tau \approx 1.10$). We observe that country 2's threshold ability $a_2^*(= a_2^X)$ and hence its Gini coefficient remain the same, respectively, beyond that critical level of openness.

any percentile of individuals, reflecting the difference in the ability distributions; individuals at the 90th percentile, for example, have the ability $a = 2.15$ and earns $w = 1.24$ in country 1 while their counterparts have the ability $a = 1.77$ and earns $w = 1.14$ in country 2.

Again, international trade exacerbates income inequality within the countries; Figure 11 indicates that in both countries, the income distribution under the trade equilibrium when $\phi = 0.58$ is Lorenz-dominated by that in autarky. Indeed, as Figure 12 shows, income inequality, measured by Gini coefficient, worsens in both countries as the openness to trade increases.[7] Figure 12 also indicates that talent-abundant country 1 always experiences greater income inequality, as a consequence of a thicker talented population that earn disproportionately high incomes.

6 Conclusion

In order to examine the impact of international trade on income inequality across workers with different abilities, we have built a two-country trade model in which the average ability of knowledge workers determines the quality of the product that the firm produces. Knowledge workers are sorted into firms according to their abilities, which entails firm heterogeneity in product quality. International trade benefits the firms that produce the high-quality products, while trade decreases the profits for those that barely export their products and those that serve only their individual domestic markets. Consequently, income inequality within knowledge workers, who earn higher income than production workers, expands. International trade increases the real wages for top income earners and lowest income earners, who benefit from a resulting fall of the price index, while decreases those for the middle-income class.

We have deliberately designed the model as simple as possible in order to highlight what we believe are the important factors in explaining why globalization entails job and wage polarization especially in developed countries: the winner-take-all market and war for talent. For that purpose, we have made some simplifying assumptions such that firms are

[7] We observe $a_2^* = a_2^X$ when $\phi \geq 0.98$ (i.e., τ is approximately less than 1.01). In such cases, the entry threshold and hence the Gini coefficient are the same, respectively, over different values of ϕ.

required to hire only a fixed number of knowledge workers and productivities as production workers are the same between workers with different abilities. Some of our results would surely be modified if we relax these assumptions. Our basic message, however, would remain valid. Globalization has created the opportunity for firms to reach people all over the world. Top-tier firms are the main beneficiaries of the globalization since the resulting increases in sale and profits are large. But other firms are likely to lose because of foreign top-tier firms' penetration to their own markets. These differential impacts of globalization on firms directly entail differential impacts on wages of knowledge workers as residual claimers. Most talented workers who work in top-tier exporting firms benefit from globalization, earning higher real wages than before. Those who are mediocre knowledge workers are likely to lose; their firms may suffer from increased competition in their domestic markets, or they may even drop out of a knowledge-worker pool.

Appendix

Proof of Proposition 4. First, we prove that international trade makes the production workers better off by showing that the product quality index increases (or equivalently, the price index drops) by trade. Then, we show that trade also makes top income earners better off. Finally, we derive the condition under which there exist workers with intermediate abilities such that their real wages fall as a result of opening to trade.

To calculate the utility in autarky, expressed by (8), and its counterpart in the trade equilibrium, we first derive the product quality indices as

$$\int_{\alpha_A^*}^{\infty} \alpha f(\alpha) d\alpha = \frac{k a_0^k L}{l} \int_{\alpha_A^*}^{\infty} \alpha^{-k} d\alpha = \frac{k a_0^k L}{(k-1) l \alpha_A^{*k-1}} \tag{26}$$

in autarky, and

$$\int_{\alpha_T^*}^{\infty} \alpha f(\alpha) d\alpha + \tau^{1-\sigma} \int_{\frac{\tau^{\sigma-1} f_X}{l} \alpha_T^*}^{\infty} \alpha f(\alpha) d\alpha = \frac{k a_0^k L}{(k-1) l \alpha_T^{*k-1}} \left[1 + \tau^{k(1-\sigma)} \left(\frac{l}{f_X} \right)^{k-1} \right] \tag{27}$$

in the trade equilibrium.

Now, we solve the autarkic FE and LM equations, (6) and (7), for α_A^* and I_A. Substituting the expression of (26) into (6), we obtain the aggregate income in autarky as

$$I_A = \frac{\sigma l}{\alpha_A^*} \times \frac{k a_0^k L}{(k-1) l \alpha_A^{*k-1}} = \frac{\sigma k L}{k-1} \left(\frac{a_0}{\alpha_A^*} \right)^k. \tag{28}$$

Then, we substitute (28) into (7) to obtain

$$\begin{aligned} l \times \frac{L}{l} \left(\frac{a_0}{\alpha_A^*} \right)^k + \left(\frac{\sigma - 1}{\sigma} \right) \frac{\sigma k L}{k-1} \left(\frac{a_0}{\alpha_A^*} \right)^k &= L \\ L \left(\frac{a_0}{\alpha_A^*} \right)^k \left(\frac{\sigma k - 1}{k-1} \right) &= L, \end{aligned}$$

which gives us

$$\alpha_A^* = a_0 \left(\frac{\sigma k - 1}{k-1} \right)^{\frac{1}{k}}. \tag{29}$$

Note that $\alpha_A^* > a_0$ holds because $\sigma > 1$. We also obtain I_A by substituting (29) back to (28):

$$I_A = \frac{\sigma k L}{k-1} \times \frac{k-1}{\sigma k - 1} = \frac{\sigma k L}{\sigma k - 1}. \tag{30}$$

Let us turn to the trade equilibrium. Substituting the expression of (27) into the FE condition (13), we obtain the aggregate income in the trade equilibrium as

$$I_T = \frac{\sigma k L}{k-1}\left(\frac{a_0}{\alpha_T^*}\right)^k \left[1+\tau^{k(1-\sigma)}\left(\frac{l}{f_X}\right)^{k-1}\right]. \tag{31}$$

Then, we substitute (31) into the LM condition (14) to obtain

$$\begin{aligned} l \times \frac{L}{l}\left(\frac{a_0}{\alpha_T^*}\right)^k + f_X \times \frac{L}{l}\left(\frac{\alpha_0}{\alpha_T^*}\right)^k \left(\frac{l}{\tau^{\sigma-1} f_X}\right)^k + \left(\frac{\sigma-1}{\sigma}\right)\frac{\sigma k L}{k-1}\left(\frac{a_0}{\alpha_T^*}\right)^k \left[1+\tau^{k(1-\sigma)}\left(\frac{l}{f_X}\right)^{k-1}\right] &= L \\ L\left(\frac{a_0}{\alpha_T^*}\right)^k \left[1+\tau^{k(1-\sigma)}\left(\frac{l}{f_X}\right)^{k-1}\right]\left(\frac{\sigma k-1}{k-1}\right) &= L. \end{aligned}$$

Thus, we have

$$\alpha_T^* = a_0\left[1+\tau^{k(1-\sigma)}\left(\frac{l}{f_X}\right)^{k-1}\right]^{\frac{1}{k}}\left(\frac{\sigma k-1}{k-1}\right)^{\frac{1}{k}}. \tag{32}$$

Substituting (32) back to (31), we obtain

$$\begin{aligned} I_T &= \frac{\sigma k L}{k-1} \times \frac{k-1}{\sigma k-1} \times \left[1+\tau^{k(1-\sigma)}\left(\frac{l}{f_X}\right)^{k-1}\right]^{-1} \times \left[1+\tau^{k(1-\sigma)}\left(\frac{l}{f_X}\right)^{k-1}\right] \\ &= \frac{\sigma k L}{\sigma k-1}. \end{aligned} \tag{33}$$

It is immediate from (30) and (33) that $I_A = I_T$.

Since the production worker's wage is normalized to 1, it is obvious from (8) and (15) that the production workers are better off by opening to trade if and only if the quality index increases by trade, i.e.,

$$\int_{\alpha_T^*}^{\infty} \alpha f(\alpha) d\alpha + \tau^{1-\sigma}\int_{\frac{\tau^{\sigma-1} f_X}{l}\alpha_T^*}^{\infty} \alpha f(\alpha) d\alpha > \int_{\alpha_A^*}^{\infty} \alpha f(\alpha) d\alpha. \tag{34}$$

Now, it follows from (26) and (29) that

$$\int_{\alpha_A^*}^{\infty} \alpha f(\alpha) d\alpha = \frac{k a_0 L}{(k-1)l}\left(\frac{k-1}{\sigma k-1}\right)^{\frac{k-1}{k}}. \tag{35}$$

Similarly, we obtain from (27) and (32) that

$$\begin{aligned} &\int_{\alpha_T^*}^{\infty} \alpha f(\alpha) d\alpha + \tau^{1-\sigma}\int_{\frac{\tau^{\sigma-1} f_X}{l}\alpha_T^*}^{\infty} \alpha f(\alpha) d\alpha \\ &= \frac{k a_0 L}{(k-1)l}\left(\frac{k-1}{\sigma k-1}\right)^{\frac{k-1}{k}}\left[1+\tau^{k(1-\sigma)}\left(\frac{l}{f_X}\right)^{k-1}\right]^{\frac{1}{k}}. \end{aligned} \tag{36}$$

The direct comparison between (35) and (36) reveals that (34) holds, and hence the production workers are better off by international trade.

Next, we show that the highest income earners are better off by opening to trade. As Figure 5 suggests, we need only show that $u'_T(a) > u'_A(a)$ for $a > \alpha^X$. Since $w_A(a) = \tilde{\pi}(a)/l$ for $a \geq \alpha^*_A$, where $\tilde{\pi}$ is defined by (5), and $w_T(a) = [\tilde{\pi}_d(a) + \tilde{\pi}_X(a) - f_X]/l$ for $a \geq \alpha^X$, we obtain from (5), (8), (9), (10), and (15) that

$$u_A(a) = \frac{(\sigma-1)aI_A}{l\sigma^2\left[\int_{\alpha^*_A}^{\infty}\alpha f(\alpha)d\alpha\right]^{\frac{\sigma-2}{\sigma-1}}} \text{ for } a \geq \alpha^*_A, \tag{37}$$

$$\begin{aligned} u_T(a) &= \frac{\sigma-1}{l\sigma\left[\int_{\alpha^*_T}^{\infty}\alpha f(\alpha)d\alpha + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha f(\alpha)d\alpha\right]^{\frac{1}{1-\sigma}}} \\ &\quad \times \left\{\frac{a(1+\tau^{1-\sigma})I_T}{\sigma\left[\int_{\alpha^*_T}^{\infty}\alpha f(\alpha)d\alpha + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha f(\alpha)d\alpha\right]} - f_X\right\} \text{ for } a \geq \alpha^X. \end{aligned} \tag{38}$$

Note that in both autarky and trade, a knowledge worker's utility is higher, the higher is the product quality index, if and only if $1 < \sigma < 2$. The higher the quality index, a knowledge worker is better off as a consumer, but worse off as a residual claimer of a firm that competes with other firms. The former effect outweighs the latter if and only if $\sigma < 2$, i.e., the elasticity of substitution is small enough that the negative impact of an increase in the quality index through a drop of the firm's operating profits is sufficiently small.

Now, it follows from (37) and (38) that we have $u'_T(a) > u'_A(a)$ if and only if

$$\left[\frac{\int_{\alpha^*_T}^{\infty}\alpha f(\alpha)d\alpha + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha f(\alpha)d\alpha}{\int_{\alpha^*_A}^{\infty}\alpha f(\alpha)d\alpha}\right]^{\frac{\sigma-2}{\sigma-1}} < 1 + \tau^{1-\sigma},$$

where we have used $I_A = I_T$. The expression in the square brackets on the left-hand side is greater than 1 as we have shown above. Thus, this inequality is satisfied if $\sigma \leq 2$. To see if this inequality is also satisfied even if $\sigma > 2$, we rewrite this inequality using(12), (26), and (27) as

$$\left[\left(\frac{\alpha^*_A}{\alpha^*_T}\right)^{k-1} + \left(\frac{\alpha^*_A}{\alpha^X}\right)^{k-1}\tau^{1-\sigma}\right]^{\frac{\sigma-2}{\sigma-1}} < 1 + \tau^{1-\sigma}. \tag{39}$$

Now, it follows from $\alpha^*_A < \alpha^*_T < \alpha^X$ and $k > 1$ that

$$\left(\frac{\alpha^*_A}{\alpha^*_T}\right)^{k-1} + \left(\frac{\alpha^*_A}{\alpha^X}\right)^{k-1}\tau^{1-\sigma} < 1 + \tau^{1-\sigma}.$$

Since $0 < (\sigma-2)/(\sigma-1) < 1$ when $\sigma > 2$ and $(\alpha_A^*/\alpha_T^*)^{k-1} + (\alpha_A^*/\alpha^X)^{k-1}\tau^{1-\sigma} > 1$ (which is equivalent to (34)), we have

$$\left[\left(\frac{\alpha_A^*}{\alpha_T^*}\right)^{k-1} + \left(\frac{\alpha_A^*}{\alpha^X}\right)^{k-1}\tau^{1-\sigma}\right]^{\frac{\sigma-2}{\sigma-1}} < \left(\frac{\alpha_A^*}{\alpha_T^*}\right)^{k-1} + \left(\frac{\alpha_A^*}{\alpha^X}\right)^{k-1}\tau^{1-\sigma} < 1+\tau^{1-\sigma},$$

so that we have shown that (39) holds, and hence that the higher income earners benefit from trade regardless of the value of $\sigma \in (1,\infty)$.

Finally, we derive the condition under which the middle-income earners are worse off by trade. In the trade equilibrium, the utility of a knowledge worker with $a \in (\alpha_T^*, \alpha^X)$ can be written as

$$u_T(a) = \frac{(\sigma-1)aI_T}{l\sigma^2\left[\int_{\alpha_T^*}^{\infty}\alpha f(\alpha)d\alpha + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha f(\alpha)d\alpha\right]^{\frac{\sigma-2}{\sigma-1}}}.$$

Using $I_A = I_T$, we compare this utility with $u_A(a)$, shown in (37), to find that $u_A(a) > u_T(a)$ for $a \in (\alpha_T^*, \alpha^X)$ if and only if

$$\left[\int_{\alpha_A^*}^{\infty}\alpha f(\alpha)d\alpha\right]^{\frac{\sigma-2}{\sigma-1}} < \left[\int_{\alpha_T^*}^{\infty}\alpha f(\alpha)d\alpha + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha f(\alpha)d\alpha\right]^{\frac{\sigma-2}{\sigma-1}}.$$

It follows from (34) that this inequality holds, and hence opening to trade makes the middle-income class worse off, if and only if $\sigma > 2$.

Proof of Proposition 5. It follows directly from (8) and (15) that each country's utilitarian social welfare can be written as

$$SW_A = \frac{(\sigma-1)I_A}{\sigma\left[\int_{\alpha_A^*}^{\infty}\alpha f(\alpha)d\alpha\right]^{\frac{1}{1-\sigma}}},$$

$$SW_T = \frac{(\sigma-1)I_T}{\sigma\left[\int_{\alpha_T^*}^{\infty}\alpha f(\alpha)d\alpha + \tau^{1-\sigma}\int_{\alpha^X}^{\infty}\alpha f(\alpha)d\alpha\right]^{\frac{1}{1-\sigma}}}.$$

Thus, trade improves utilitarian social welfare if and only if

$$\frac{I_A}{\left[\int_{\alpha_A^*}^{\infty}\alpha' f(\alpha')d\alpha'\right]^{\frac{1}{1-\sigma}}} < \frac{I_T}{\left[\int_{\alpha_T^*}^{\infty}\alpha' f(\alpha')d\alpha' + \tau^{1-\sigma}\int_{\frac{\tau^{\sigma-1}f_X}{l}\alpha_T^*}^{\infty}\alpha' f(\alpha')d\alpha'\right]^{\frac{1}{1-\sigma}}},$$

which is satisfied since $I_A = I_T$, as shown in the proof of Proposition 4, and the product quality index is greater in the trade equilibrium than in autarky, as shown in (34).

Proof of Proposition 6. Recall that the equilibrium wage schedules in autarky and in trade are given by

$$w_A(a) = \begin{cases} 1 & \text{for} \quad a \in [0, \alpha_A^*) \\ \frac{a}{\alpha_A^*} & \text{for} \quad \alpha \in [\alpha_A^*, \infty) \end{cases}$$

and

$$w_T(a) = \begin{cases} 1 & \text{for} \quad a \in [0, \alpha_T^*) \\ \frac{a}{\alpha_T^*} & \text{for} \quad a \in [\alpha_T^*, \alpha^X) \\ \frac{a}{\alpha_T^*} + \frac{(a-\alpha^X)f_X}{l\alpha^X} & \text{for} \quad a \in [\alpha^X, \infty), \end{cases}$$

respectively. It follows from $\alpha_A^* < \alpha_T^* < \alpha^X$ that there exists $\bar{\alpha}$ $(> \alpha^X)$ such that (i) $w_T(\bar{\alpha}) = w_A(\bar{\alpha})$; (ii) $w_T(\alpha) \leq w_A(\alpha)$ for all $\alpha \leq \bar{\alpha}$; and (iii) $w_T(\alpha) > w_A(\alpha)$ for all $\alpha > \bar{\alpha}$, as depicted in Figure 4. This implies that $\mathcal{L}_A(a) = \mathcal{L}_T(a)$ for all $a \leq \alpha_A^*$, and $\mathcal{L}_A(a) > \mathcal{L}_T(a)$ for all $a > \alpha_A^*$, since $w_A(a) = w_T(a) = 1$ for $a \leq \alpha_A^*$ and $\int_0^\infty w_A(a)dG(a) = \int_0^\infty w_T(a)dG(a)$, which is shown in the proof of Proposition 4 as $I_A = I_T$. Thus, the income distribution under international trade is Lorenz-dominated by the one in autarky.

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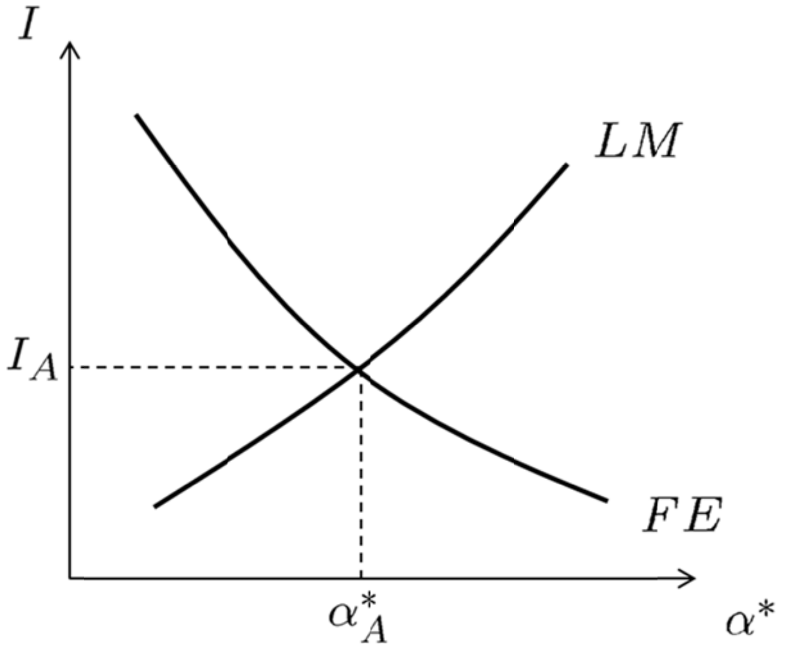


Figure 1. Autarkic equilibrium

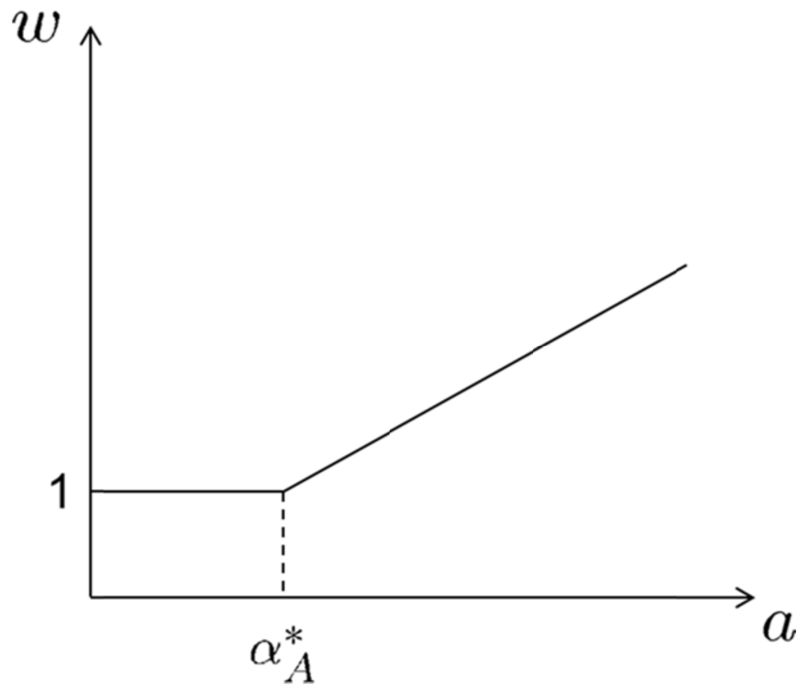


Figure 2. Autarkic equilibrium wage schedule

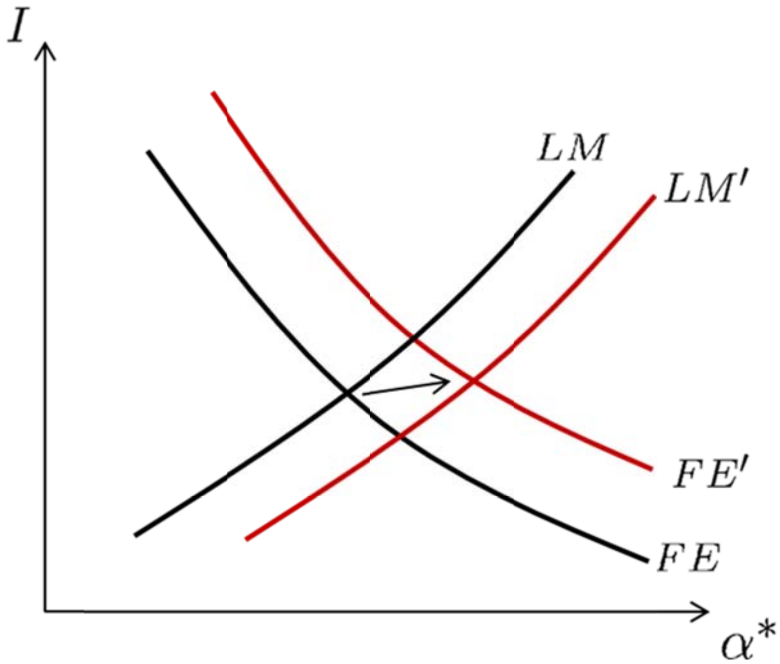


Figure 3. Trade equilibrium

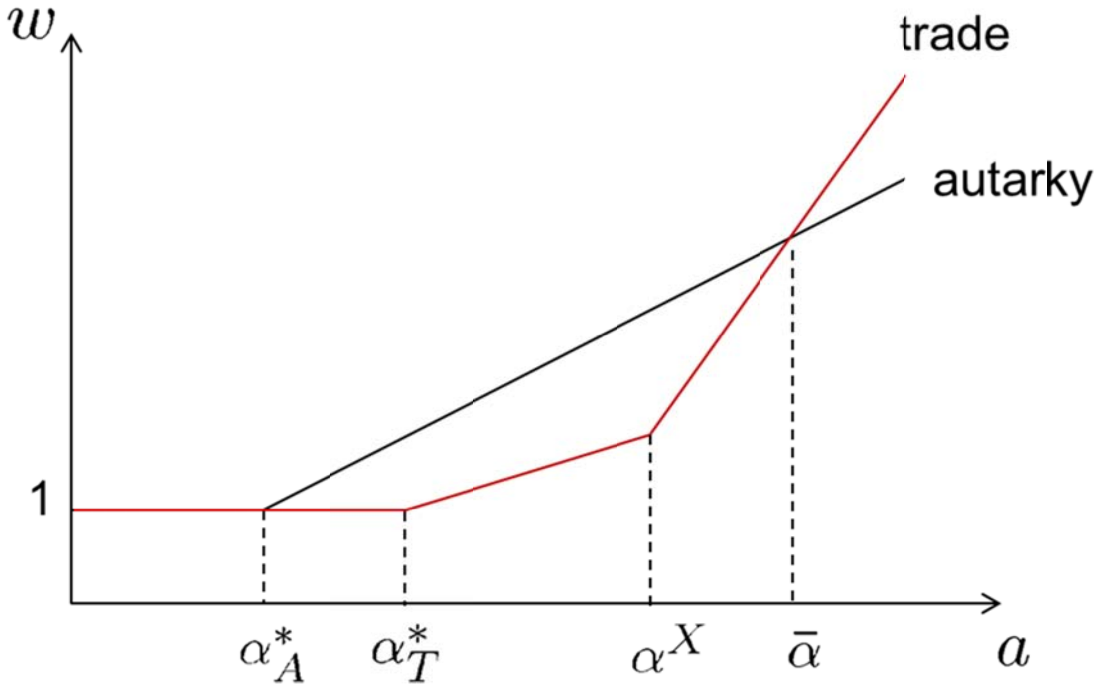


Figure 4. Impact of trade on nominal wages

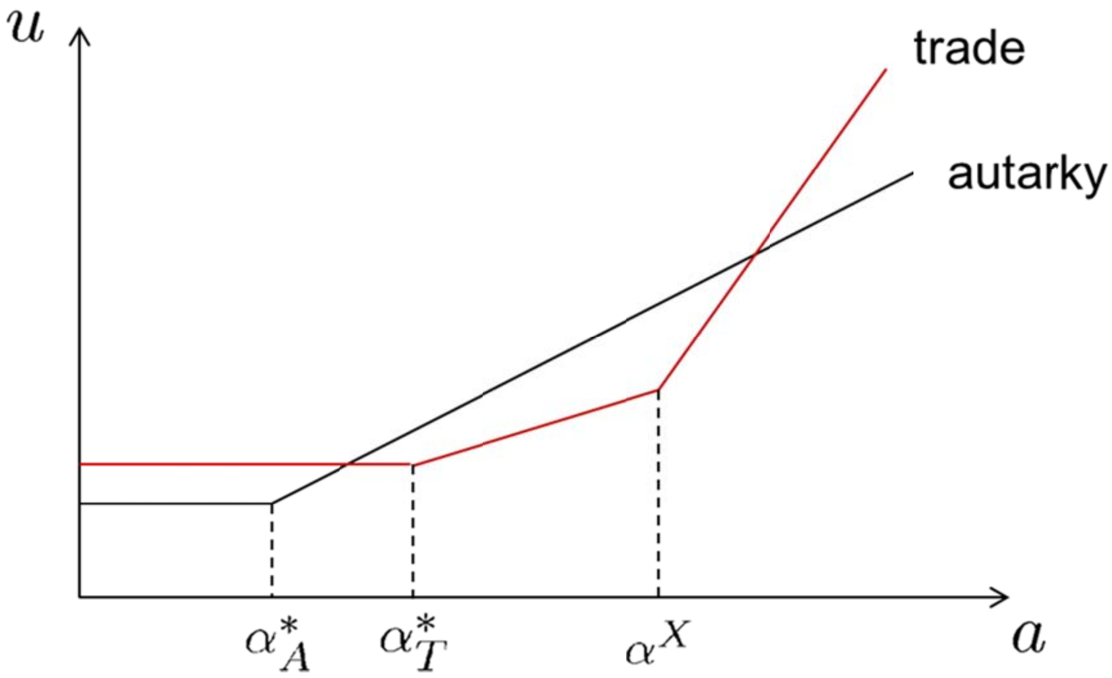


Figure 5. Impact of trade on real wages

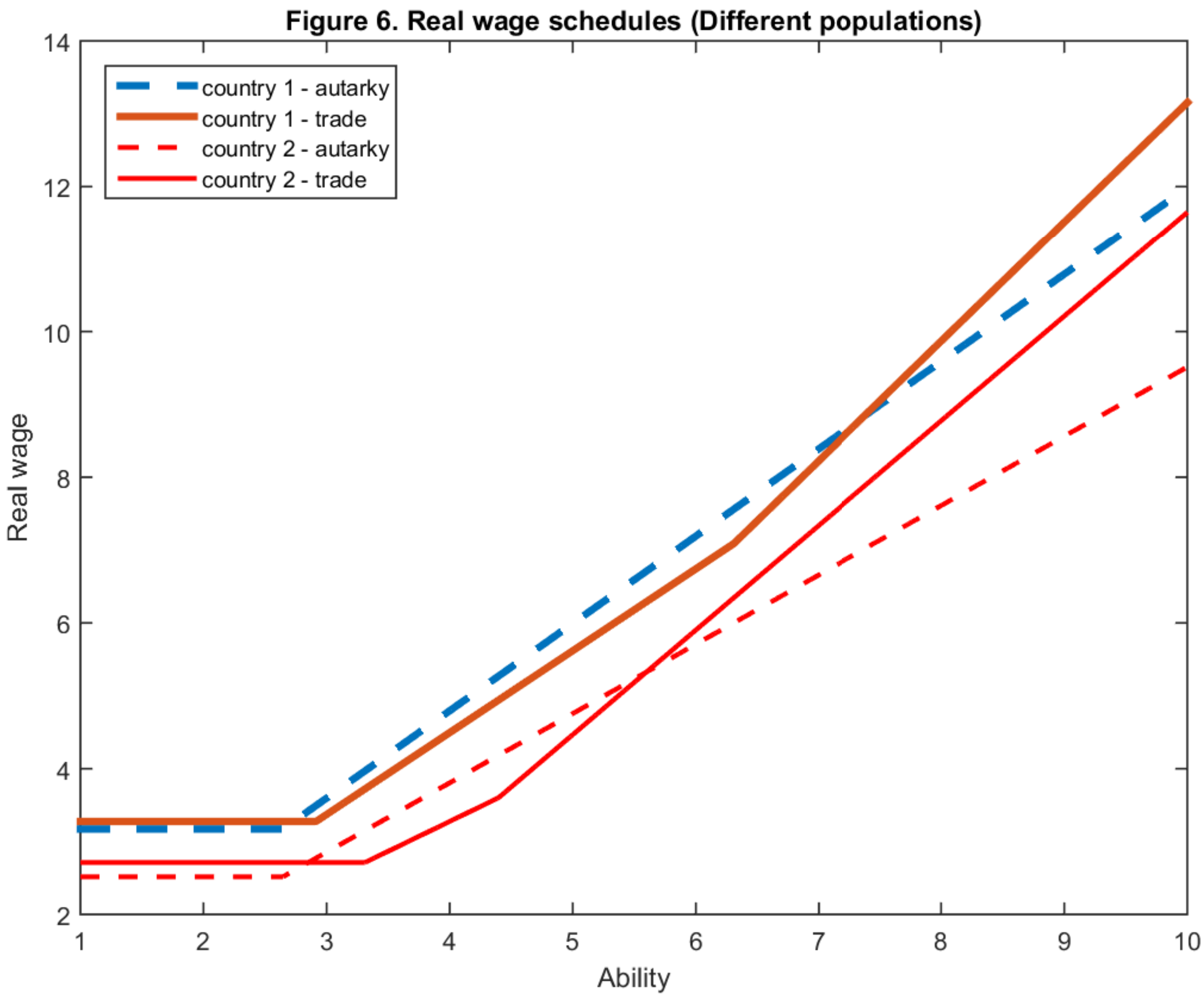
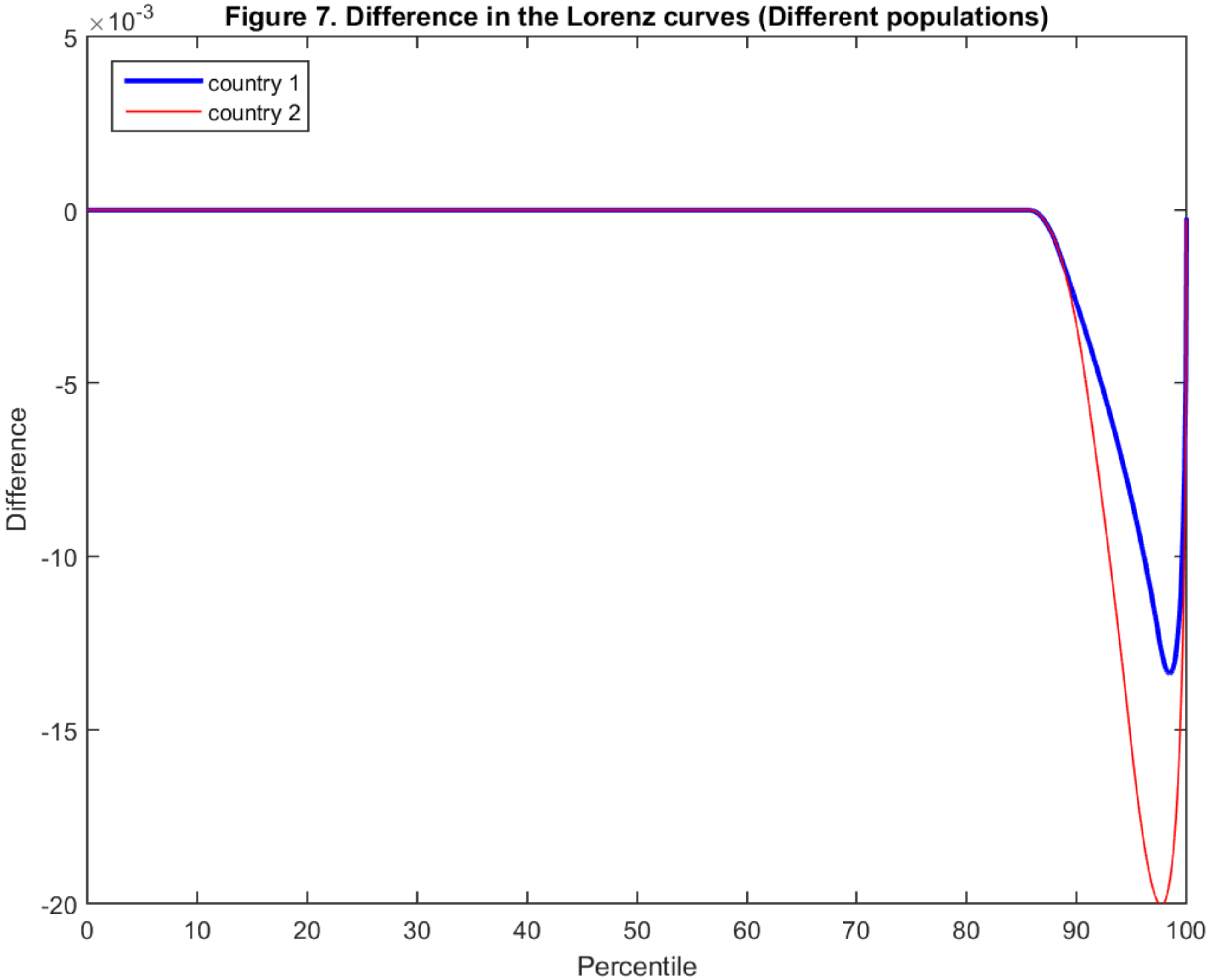


Figure 6. Real wage schedules (Different populations)
country 1 - autarky
country 1 - trade
country 2 - autarky
country 2 - trade
Real wage
Ability
14
12
10
8
6
4
2
1
2
3
4
5
6
7
8
9
10

Figure 7. Difference in the Lorenz curves (Different populations)



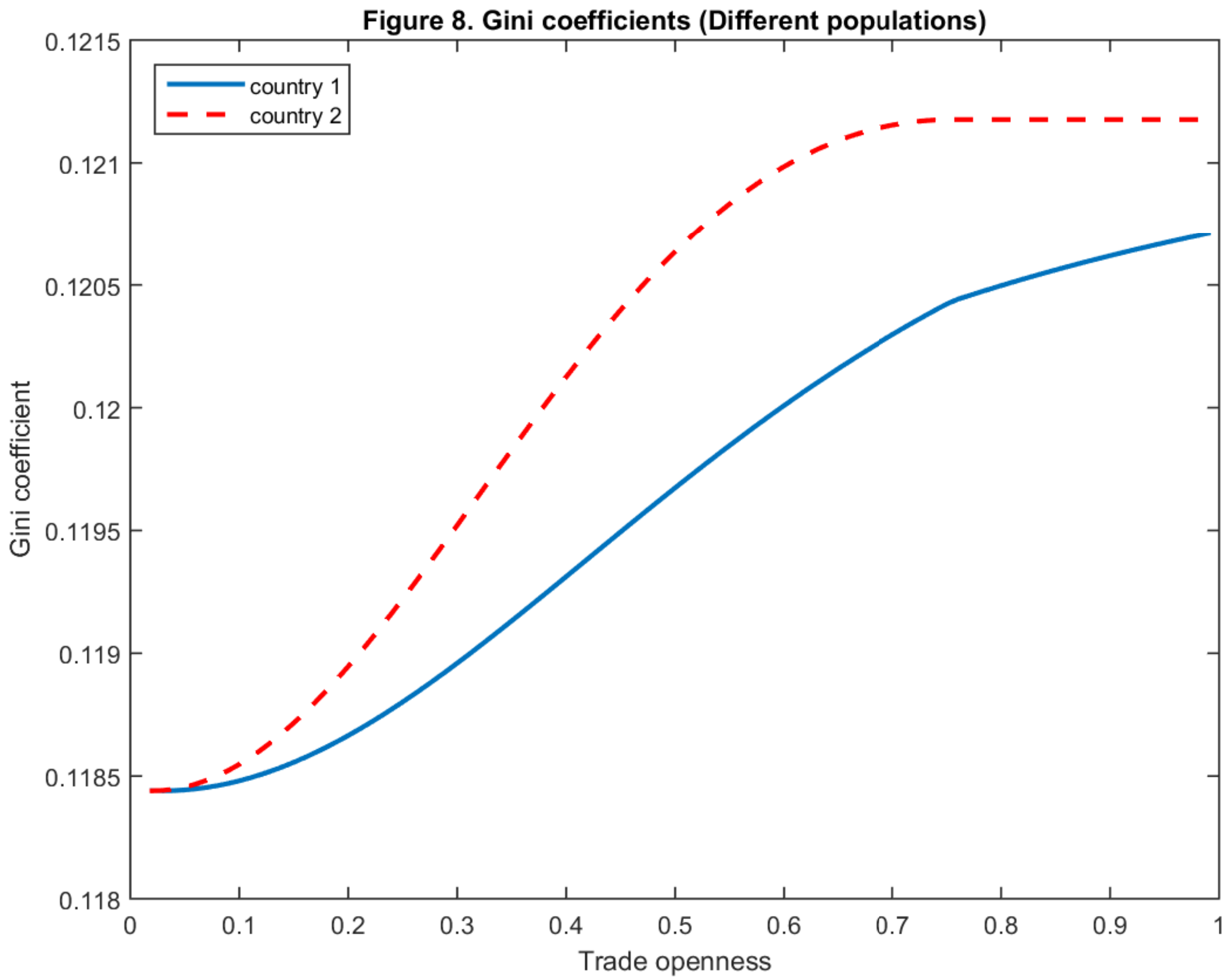


Figure 8. Gini coefficients (Different populations)
country 1
country 2
Gini coefficient
0.1215
0.121
0.1205
0.12
0.1195
0.119
0.1185
0.118
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Trade openness

Figure 9. Real wage schedules (Different abilities)

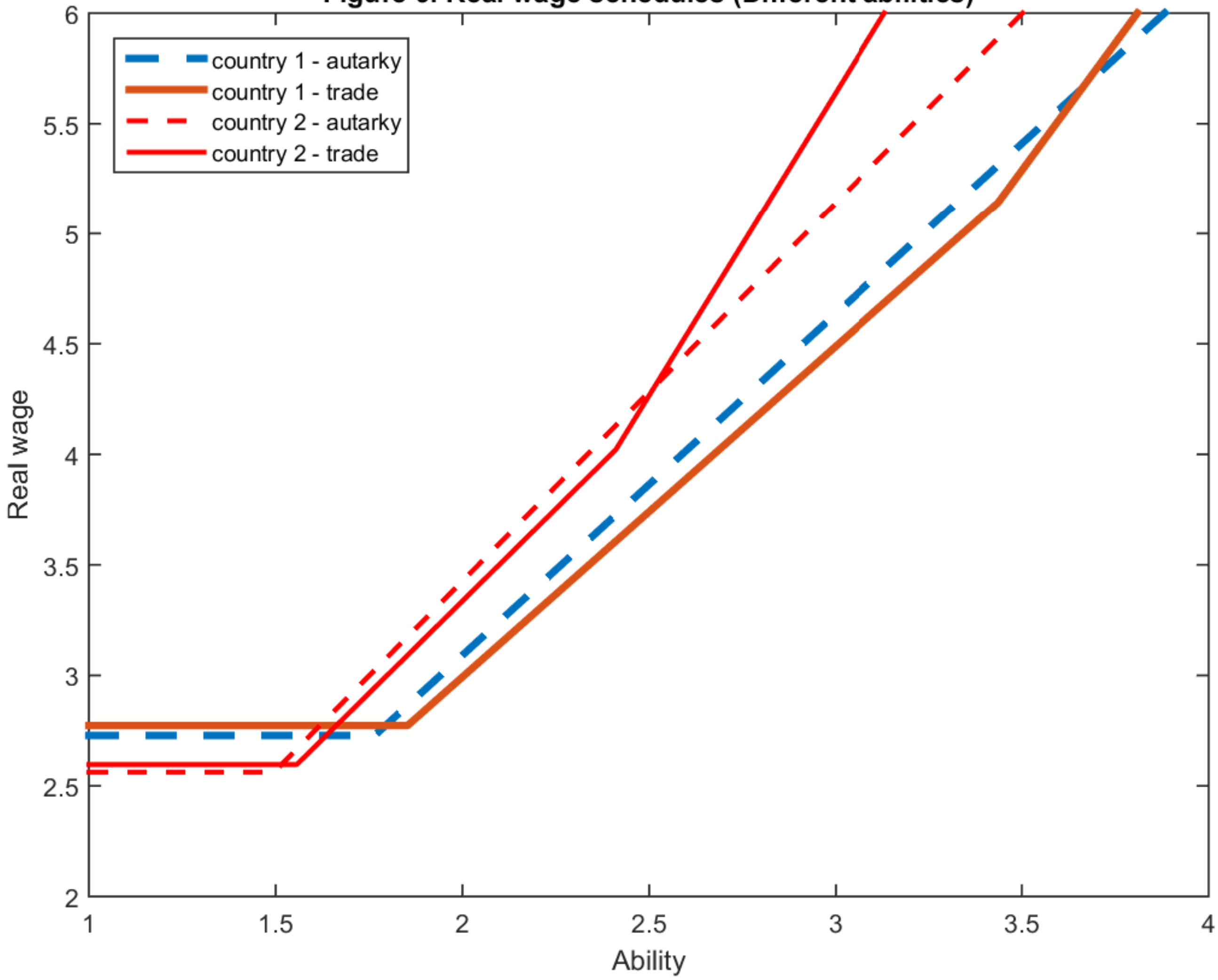


Figure 10. Wage over percentiles (Different abilities)

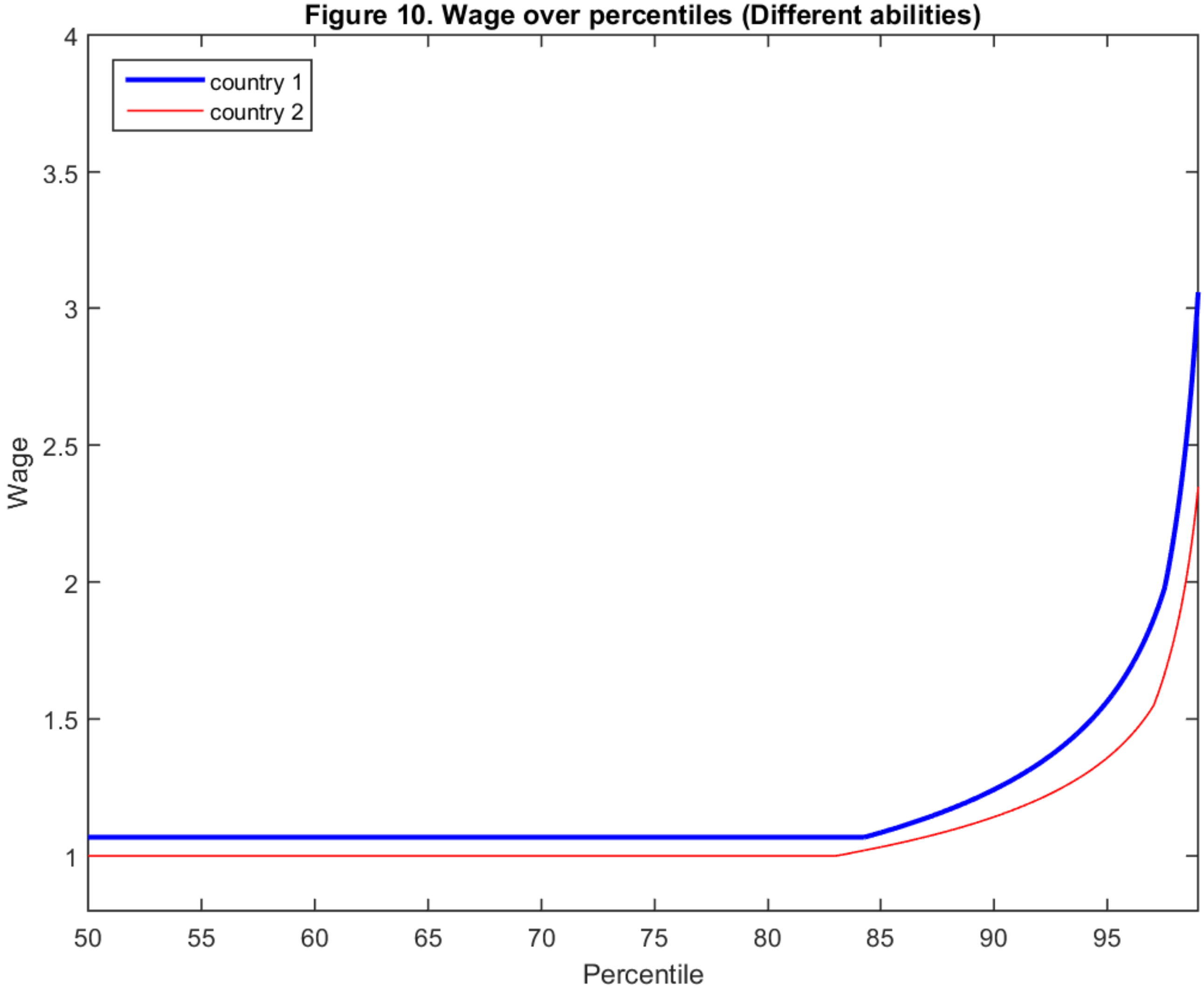


Figure 11. Difference the Lorenz curves (Different abilities)

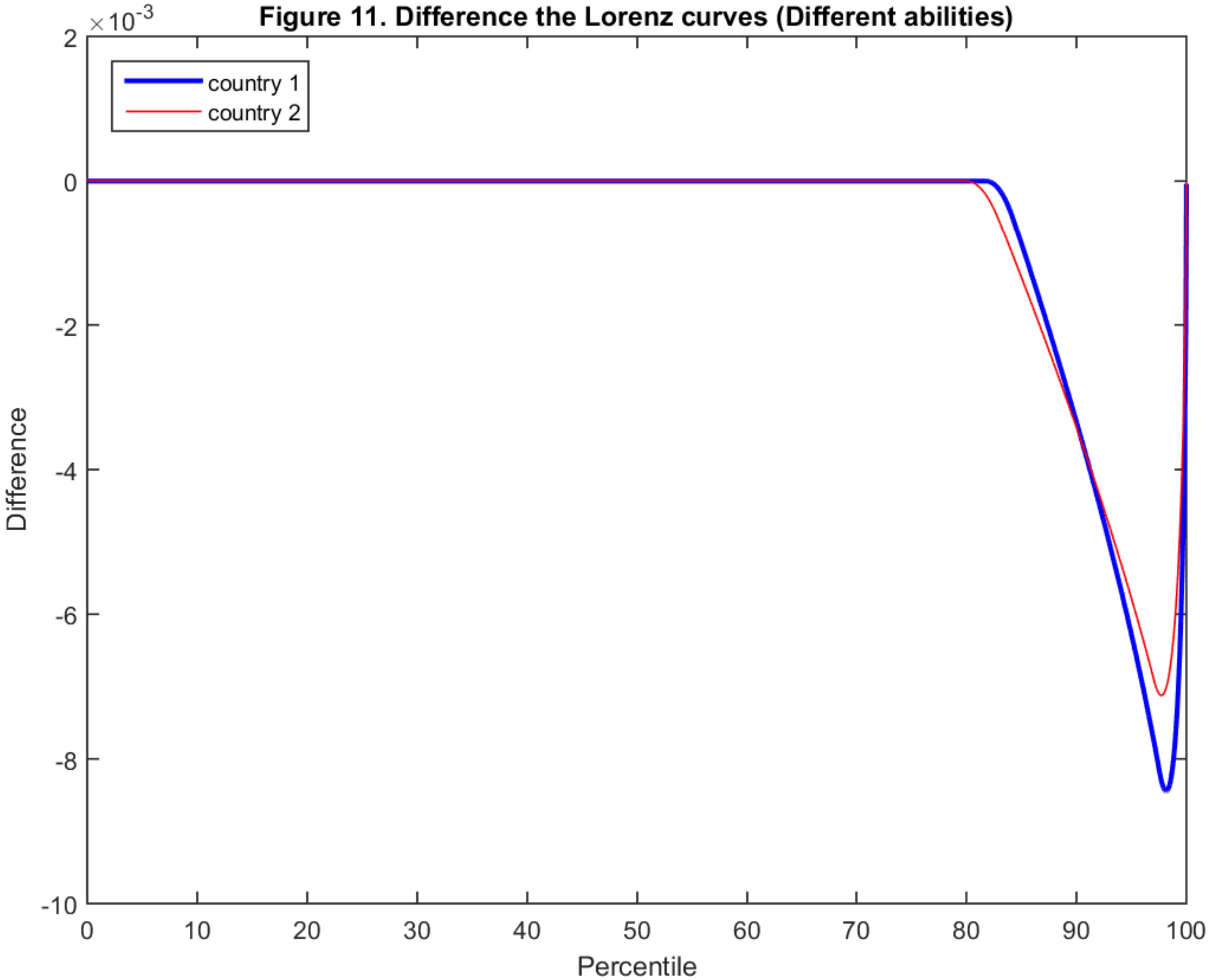


Figure 12. Gini coefficients (Different abilities)