# Gravity, Productivity and the Pattern of Production and Trade*

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January 6, 2009

**Abstract**

The aggregated incidence of bilateral trade costs is derived from the gravity model. Incidence is equivalent to a TFP penalty. Sectoral and national differences in TFP have sharp implications for the equilibrium pattern of production and trade in a specific factors model of production. Unskilled labor is intersectorally mobile. Skilled labor acquires sector specific skills. Productivity shocks cause incidence shock that induce ex post inefficient allocation of skilled labor. Below (above) average TFP sectors produce less and have below (above) average skill premia. Ex ante efficient allocation is lower in sectors with riskier TFP incidence, despite risk neutrality.

JEL Classification: F10, D24

*I am grateful to Christian Broda for helpful comments on an earlier version at the NBER ITI Summer Institute, 2007. I thank participants in seminars at Drexel University, Brandeis University and Oxford University for comments on an earlier versions.

Distribution frictions intuitively drag down productivity, with implications for trade patterns. Total Factor Productivity (TFP) is modeled in this paper as the product of the sectoral incidence of trade frictions and Hicks neutral sectoral productivity frictions. Operational measures of sectoral incidence based on the structural gravity model aggregate the enormous real world complexity of bilateral trade costs. Sharp implications of TFP differences for the global equilibrium pattern of production and trade are drawn using the specific factors model of production.

Simple aggregate incidence measures are available under the assumption of trade separability (Anderson and van Wincoop, 2004): the distribution of goods across bilateral pairs is separable from the allocation of resources and expenditures across product lines within countries. It is as if each producing sector in each economy traded with a single world market. Specializing the trade allocation model to a CES structure results in the economic theory of gravity (Anderson, 1979; Anderson and van Wincoop, 2003, 2004). Outward and inward multilateral resistance give respectively the supply side and demand side incidence of trade frictions in conditional general equilibrium for each country and sector. It is not necessary to solve the full general equilibrium to solve for incidence.

The allocation of expenditure and resources across sectors within each economy is determined by a vector of world prices margined up by its national sellers' incidence. Global equilibrium requires that world prices clear world markets. Specializing the resource allocation model to the specific factors model permits sharp predictions. Unskilled labor is intersectorally mobile. The other factor is potentially mobile prior to production as well, but it must take on sector specific attributes to be deployed in production. Call this factor skilled labor to fix ideas. Sector specific skills are acquired, then productivity shocks are realized, prices are realized and the ex post efficient allocation of unskilled labor occurs. Neither factor of production is internationally mobile.

The global general equilibrium pattern of production in this model, call it specific gravity, is explained by sector specific factor endowments (a supply shifter), taste parameters (a demand shifter) and the equilibrium incidence of TFP. The monopolistic competition variant of the model endogenizes the taste shifter. Looking across countries, the (multi-) factoral terms of trade and hence wages and real incomes are negatively related to the incidence of TFP. Looking across sectors within a country, the sector specific skill premium is reduced by high incidence of productivity frictions.

The efficient ex ante allocation of skilled labor is characterized. Higher sectoral variance lowers ex ante efficient sectoral investment despite risk neutrality. Looking across countries, higher variance of the incidence of productivity shocks lowers ex post production efficiency. It is plausible that the national variance of the incidence rises with the mean, implying that ex post inefficiency is larger for economies with higher average trade costs.

The same qualitative properties obtain when the model is extended to include intermediate goods combined with vertical disintegration (outsourcing) using the model of selection into trade of Helpman, Melitz and Rubinstein (2007).

The closest related model is that of Eaton and Kortum (2002). They embed gravity in a Ricardian model of trade featuring productivity differences resulting from draws from nationally differing Frechet distributions. In equilibrium the model is observationally equivalent to the one good/many varieties gravity model (see Anderson and van Wincoop, 2004). Costinot and Komunjer (2007) extend the Eaton-Kortum framework to a multi-good setting. The specific gravity model nests the Costinot-Komunjer model as a special case when the efficient allocation of skilled labor is made after the realization of productivity draws.

The more general case has two advantages in descriptive power. It allows a role for relative factor endowment differences, and it allows a role for income distribution. Ex post specificity combines with productivity shocks to generate the well documented phenomenon of sectorally heterogeneous returns to otherwise identical skilled labor, positively correlated with export intensity. Another advantage (not exploited here) is that the model links easily to the interest group political economy model of trade policy that endogenizes part of trade frictions.

A less closely related recent literature that seeks to explain the pattern of production by international differences in endowments and technology lacks an appropriate general treatment of trade costs. Davis and Weinstein (2001) use the multi-cone Heckscher-Ohlin continuum of products model, but effectively assume that all the incidence of trade costs is on the demand side. Romalis (2004) considers the role of uniform trade costs in resource allocation using the multi-cone Heckscher-Ohlin continuum model, but in a North-South model with M identical countries in each half of the world. Trade costs disappear from his empirical work via a substitution that is valid only using the high degree of uniformity of the model. Trefler's HOV model (1995) allows for technology differences and home bias in preferences, but the home bias is not connected with gravity. The powerful empirical regularities of gravity estimation (Anderson and van Wincoop, 2004) suggests that the incidence of trade costs should have equal importance to endowments and technology in determining production and trade patterns.

Section 1 sets the stage by describing how the supply side incidence of trade frictions can be treated as equivalent to sectoral productivity penalties in the standard abstract model of production and trade. Section 2 provides the solution to incidence and aggregation of high dimensional productivity frictions in production and distribution. Section 3 sets out the specific factors model of production in a special case. The world equilibrium reduced form pattern of production and trade that results is set out and characterized in Section 4. Section 5 analyzes efficient ex ante allocation of specific factors facing random productivity draws in the world economy. Section 6 extends the discussion to treat intermediate products trade and the implications of selection into exporting. Section 7 concludes. The Appendix develops the endogenous determination of varieties in monopolistic competition, and fills out the connection of the model with the Costinot-Komunjer model. It also reviews selection into exporting.

# 1 Trade and Productivity Frictions

Each country produces and distributes goods to its trading partners. Productivity is reduced from the frontier by a Hicks neutral friction $a_k^j \geq 1$ for product $k$ in country $j$. Thus at the factor prices relevant for product $k$ in country $j$, $a_k^j - 1$ more factors are used than needed with the ideal practice. Distribution to destination $h$ requires additional factors to be used, in the proportion $T_k^{jh} - 1$ to their use in production: the metaphor of iceberg-melting distribution costs.

The cost at destination $h$ is given by $p_k^{jh} = a_k^j T_k^{jh} \widetilde{p}_k^j$, where $\widetilde{p}_k^j$ is the unit cost of production using ideal practice, or the 'efficiency unit cost'. Since the $a$'s and $T$'s enter the model multiplicatively, they combine in a productivity friction measure $t_k^{jh}$ that represents both trade frictions and frictions in the assimilation of technology. This useful simplification is exploited everywhere in what follows.[1]

[1] The decomposition of $t$ into $T$'s and $a$'s is always available, but mostly a distraction here.

With trade frictions, a key issue is incidence.[2] Looking across bilateral trade, buyers get some of the benefit of sellers' distribution productivity, the remainder being a benefit to the sellers.[3] Incidence analysis breaks the markup $t_k^{ij} = p_k^{ij}/\widetilde{p}_k^{ij}$ into its incidence on buyers and sellers. Let $p_k$ denote the hypothetical equilibrium price that would obtain in the absence of frictions to productivity and trade. Then $p_k^{jh} = p_k^j P_k^h$ and $\widetilde{p}_k^j \Pi_k^j = p_k^j$ provides a useful decomposition of $t_k^{jh} = P_k^h \Pi_k^j$.

Incidence is more complex than the preceding partial equilibrium analysis suggests with many countries. Country $i$ ships to many destinations at once, and competes in those destinations with products from many other origins. Section 2 shows that, using the gravity model, it is possible to appropriately aggregate the incidence of these frictions on the supply side into an index $\Pi_k^j$ for each product category $k$ in each country $j$. With efficiency unit production cost $\widetilde{p}_k^j$ in country $j$, it is as if there was an average ('world') destination price for goods $k$ delivered from $j$, $p_k^j = \widetilde{p}_k^j \Pi_k^j$. Similarly, on the demand side it is as if a single composite good $k$ shipped from a world market at markup $P_k^h$. Anderson and van Wincoop (2004) call $\Pi$'s and $P$'s outward and inward multilateral resistance, respectively.

The metaphor of iceberg melting trade costs extends to productivity frictions that 'melt' resources before the shipments begin their journey to market. Both types of friction operate in combination with given world prices to generate the allocation of production. A similar treatment works to generate the allocation of expenditure in the aggregate based on demand side incidence and world prices. The upper level general equilibrium requires solving for the world prices that clear global markets given the supply and demand incidences. The full general equilibrium requires that the allocations of resources and expenditure at the upper level for given incidences be consistent with the allocations that generate those same incidences in the lower level.

[2] The incidence of productivity frictions is in principle divided too, an important issue in general equilibrium comparative statics. For the cross sectional analysis of production and trade patterns, differences in productivity frictions have their incidence entirely on the supply side, as shown below.

[3] From the point of view of the world, the benefit to the buyers is relevant too, so incidence is not an issue.

## 2 Aggregate Incidence

The incidence of productivity in distribution and production is determined in general equilibrium. Insight and the prospect of operationality through aggregation are available with the specializing assumption of *trade separability* — the composition of expenditure or production within a product group is independent of prices outside the product group.

On the supply side, separability is imposed by the assumption the goods from $j$ in class $k$ shipped to each destination are perfect substitutes in supply.[4] On the demand side, separability is imposed by assuming that expenditure on goods class $k$ forms a separable group containing shipments from all origins. Goods are differentiated by place of origin, an assumption that has a deeper rationale in monopolistic competition, as developed in the Appendix. This setup enables two stage budgeting analysis. Specialization to CES structure for the separable groups yields yields procedures for calculating multilateral resistance indexes that do not require solving the full general equilibrium.

Subsection 2.1 derive a solution to the incidence and aggregation problems in a model of the upper level allocation of expenditure and production that encompasses a wide class of standard general equilibrium trade models. Subsection 2.2 imposes the CES structure that yields incidence measures as multilateral resistances using the structural gravity model.

### 2.1 General Equilibrium Allocation

Within class $k$ at each destination $h$ the buyers face prices $p_k^{jh} = \widetilde{p}_k^j t_k^{jh}, t_k^{jh} \geq 1$. The $t$'s will usually be called trade frictions for simplicity.

Buyers are consumers only for now; intermediate input purchases are treated in Section 6. Consumer expenditure is based on identical homothetic preferences that are separable with respect to the partition between goods classes. Exact price aggregators $P_k^h$ are defined in this case that index the prices of varieties from all origins to destination $h$ in class $k$. The domestic price vector for goods *classes* at location $h$ is given by $q^h = \{P_k^h\}, \forall k$. The expenditure function is given by $e(q^h)u^h$, where $u^h$ is the real income of the representative consumer in location $h$ and $e(q^h)$ is the true cost of living index at location $h$. The aggregate quantity of $k$ demanded from all sources

[4] This can be generalized to assuming that the output of $k$ from $j$ is a joint output of destination specific varieties characterized by constant elasticity of transformation, assumed in the text to be infinite.

by $h$ is $e_k(q^h)u^h$, by Shephard's Lemma. The quantity demanded of good $k$ from origin $j$ in destination $h$ is given by $e_k(q^h)u^h\partial P_k^h/\partial p_k^{jh}$, using Shephard's Lemma again.

Sellers produce quantities of goods in each class at each origin. GDP in origin $j$ is given by the GDP function $g^j(\widetilde{p}^j,\cdot)$, where $\widetilde{p}^j$ is the vector of $\widetilde{p}_k^j$'s and the $\cdot$ stands for a $j$-specific vector of technology parameters and factor endowments. The GDP function embeds perfectly competitive behavior for now; the same qualitative properties obtain for the Dixit-Stiglitz monopolistic competitors analyzed in the Appendix. By Hotelling's Lemma the supply of good $k$ is given by $g_k^j$ and its GDP share in origin $j$ by $s_k^j = g_k^j\widetilde{p}_k^j/g^j$. The supply function $g_k^j$ exists for a class of degree one homogeneous technologies for which the number of primary factors weakly exceeds the number of goods produced. The specific factors model developed in Section 3 is but one example.

The budget constraint for each economy is expressed assuming for simplicity no foreign owned factors or international transfers, and no tariffs;[5] hence the equilibrium real income is given by

$$u^h = g^h(\widetilde{p}^h,\cdot)/e(q^h).$$

Market clearance in the world economy requires that for each good $k$ from source $j$ the quantity produced is equal to the quantity demanded. Using the supply and demand structure above and the budget constraints, the market clearance condition is expressed as

$$g_k^j(\widetilde{p}^j,\cdot) = \sum_h e_k(q^h)\frac{g^h(\widetilde{p}^h,\cdot)}{e(q^h)}\frac{\partial P_k^h}{\partial p_k^{jh}}, \forall k, j. \tag{1}$$

With $N$ countries and $M$ goods classes, there are $MN$ $\widetilde{p}$'s (efficiency unit costs) to be determined by the $MN$ equations, obtaining the user prices from

[5]Tariffs impose an additional markup factor over the origin price, the difference being that the revenue is collected by the imposing government instead of the original shipper. When there are final good tariffs, the expression on the right below is multiplied by the foreign exchange multiplier. The foreign exchange multiplier under homothetic preferences is equal to $1/(1-\mu\tau^a)$ where $\tau^a \in [0,1)$ is the trade weighted average final goods tariff on the domestic price base and $\mu \in (0,1)$ is the share of total expenditure falling on tariff-ridden final goods. With intermediate goods tariffs, $g$ in the budget expression is augmented by the tariff revenue from the intermediate goods.

the $\widetilde{p}$'s margined up by the $t$'s and the price aggregator definitions. Due to homogeneity, only $MN-1$ relative prices can be determined.[6]

Taking advantage of trade separability, and identical homothetic preferences, the sellers incidence can be derived by exploiting the equivalence from their point of view to selling to a single world market. Let 'world prices' be given by $p_k^j = \widetilde{p}_k^j \Pi_k^j$ where $\Pi_k^j$ is the sellers' incidence. Evaluated at world prices, the sales share of good $k$ from origin $j$ is given by

$$Y_k^j / \sum_j Y_k^j = \frac{g_k^j(\widetilde{p}^j, v^j)\widetilde{p}_k^j \Pi_k^j}{\sum_j g_k^j(\widetilde{p}^j, v^j)\widetilde{p}_k^j \Pi_k^j}.$$

On the other side of the market, the buyers expenditure share on goods in class $k$ from origin $j$ when prices are equal to world prices is given by $P_{kj} p_k^j / P_k$, where $P_k = P_k(p_k^1, ..., p_k^N)$, the common 'true cost of living' index for goods in class $k$ and $P_{kj} \equiv \partial P_k / \partial p_k^j$. Equilibrium of the hypothetical unified world market requires that sellers' shares equal buyers' shares. The sellers' incidences can be solved at given equilibrium factory gate prices $\{\widetilde{p}_k^j\}$ (obtained from the solution to (1)) from the system

$$\frac{g_k^j(\widetilde{p}^j, v^j)\widetilde{p}_k^j \Pi_k^j}{\sum_j g_k^j(\widetilde{p}^j, v^j)\widetilde{p}_k^j \Pi_k^j} = \frac{\widetilde{p}_k^j \Pi_k^j P_{kj}(\{\widetilde{p}_k^j \Pi_k^j\})}{P_k(\{\widetilde{p}_k^j \Pi_k^j\})}. \tag{2}$$

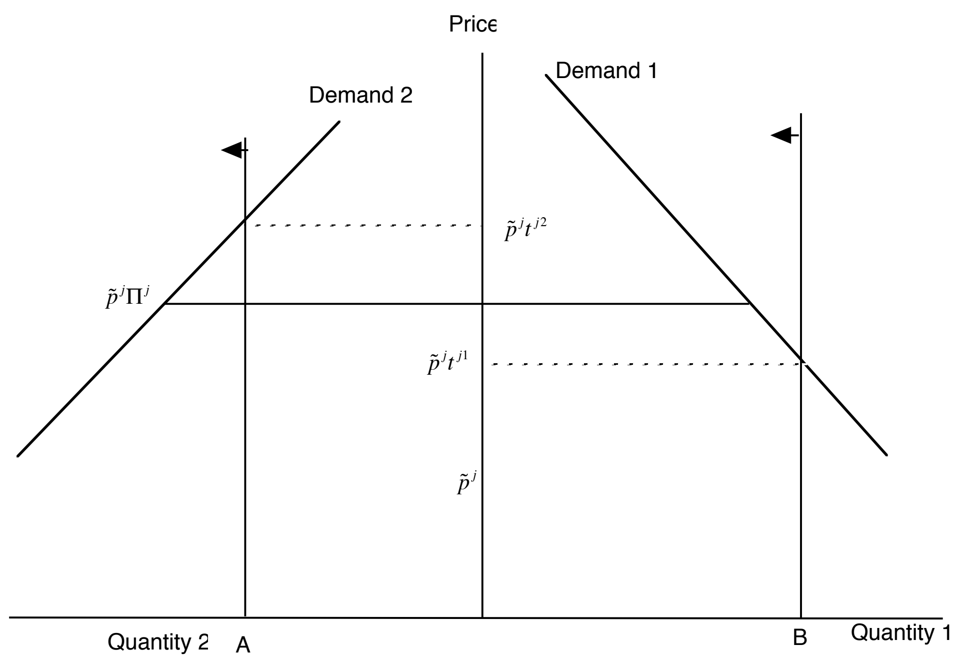
The $\Pi$'s are determined up to a normalization (since (2) sums to one. The natural normalization is $\sum_j g_k^j(\widetilde{p}^j, v^j)\widetilde{p}_k^j \Pi_k^j = 1$.[7]

Figure 1 illustrates the determination of $\Pi^j$ in this hypothetical equilibrium for the case of two markets in partial equilibrium, suppressing the goods class index $k$ for clarity. Market 1 to the right may be thought of as the home market with market 2 to the left being the export market. Distribution costs are lower in the home market than in the export market. The given equilibrium factory gate price $\widetilde{p}^j$ is preserved by maintaining the total quantity shipped while replacing the nonuniform trade costs with the uniform trade cost $\Pi^j$.

[6] Due to the assumptions, a unique equilibrium always exists.

[7] The solution exists and is unique. To see this, normalize by imposing $\sum_j g_k^j(\widetilde{p}^j, v^j)\widetilde{p}_k^j \Pi_k^j = 1$. Then the adding up condition implies that $P_k = 1$. Divide (2) through by $\widetilde{p}_k^j \Pi_k^j$. The left hand side is an endowment vector, given the $\widetilde{p}$'s. The right hand side is a vector of compensated demand functions with a unique inverse subject to the normalization.

Figure 1. Quantity-Preserving Aggregation




Total shipments AB are preserved by moving the goalposts left such that a uniform markup is applied to each shipment.

The bilateral buyers incidences can be calculated from $t_k^{jh}/\Pi_k^j$, given the sellers' incidences solved from (2). The average buyers' incidence is given by

$$P_k^h = P_k(p_k^1 t_k^{1h}/\Pi_k^1, ..., p_k^N t_k^{Nh}/\Pi_k^N). \tag{3}$$

From the point of view of buyers benefit it is as if all purchases were made from a single world market at a uniform markup $P_k^h$ over world prices:

$$P_k^h = \frac{P_k(p_k^1 t_k^{1h}/\Pi_k^1, ..., p_k^N t_k^{Nh}/\Pi_k^N)}{P_k(p_k^1, ..., p_k^N)}.$$

Numerical methods can compute incidence in particular cases (selecting functional forms and parameters for the underlying expenditure and production functions).

A particularly useful special case is when the sectoral demand structure is CES. In that case (2) becomes

$$\frac{g_k^j(\widetilde{p}^j, v^j)\widetilde{p}_k^j \Pi_k^j}{\sum_j g_k^j(\widetilde{p}^j, v^j)\widetilde{p}_k^j \Pi_k^j} = (\beta_k^j \widetilde{p}_k^j \Pi_k^j)^{1-\sigma_k},$$

where $\sigma_k$ is the elasticity of substitution parameter and $\beta_k^j > 0$ is a distribution parameter. The right hand side is the hypothetical world expenditure share under the adding up condition $\sum_j (\beta_k^j \widetilde{p}_k^j \Pi_k^j)^{1-\sigma_k} = 1$. The next section develops the CES case further, exploiting its properties to derive an operational solution for the $\Pi$'s and $P$'s from sectoral gravity estimation that does not require the observation (calculation) of the $\widetilde{p}$'s.

## 2.2 Incidence and Multilateral Resistance

Impose CES preferences on the sub-expenditure functions. Then $P_k^h$ for goods class $k$ in location $h$ is defined by

$$P_k^h \equiv \sum_j [(\beta_k^j \widetilde{p}_k^j t_k^{jh})^{1-\sigma_k}]^{1/(1-\sigma_k)},$$

where $(\beta_k^j)^{1-\sigma_k}$ is a quality parameter for goods from $j$ in class $k$.[8]

[8]In monopolistic competition models, $(\beta_k^j)^{1-\sigma_k}$ is endogenous, equal to the proportion of all varieties of class $k$ that are produced by $j$. See the Appendix for a full treatment.

The expenditure share for class $k$ in $h$, by Shephard's Lemma, is given by

$$\frac{\partial P_k^h p_k^{jh}}{\partial p_k^{jh} P_k^h} = \left\{\frac{\beta_k^j \widetilde{p}_k^j t_k^{jh}}{P_k^h}\right\}^{1-\sigma_k}.$$

The share of expenditure on $k$ from all origins at destination $h$ is given by

$$\theta_k^h = e_k(q^h)\frac{P_k^h}{e(q^h)},$$

where $e_k(q^h) = \partial e / \partial P_k^h$.

The value of shipments at *delivered* prices from origin $h$ in product class $k$ is $Y_k^h$. At efficiency production prices, the supply is $g_k^j \widetilde{p}_k^j$, with $Y_k^j = g_k^j \widetilde{p}_k^j \Pi_k^j$ margined up to reflect the cost of delivery. The expenditure in destination $h$ on product class $k$ is $E_k^h = \theta_k^h g^h$, using the balanced budget constraint.

Market clearance requires:

$$Y_k^j = \sum_h \left\{\frac{\beta_k^j \widetilde{p}_k^j t_k^{jh}}{P_k^h}\right\}^{1-\sigma_k} E_k^h. \tag{4}$$

Now solve (4) for the quality adjusted efficiency unit costs $\{\beta_k^j \widetilde{p}_k^j\}$:

$$(\beta_k^j \widetilde{p}_k^j)^{1-\sigma_k} = \frac{Y_k^j}{\sum_h (t_k^{jh}/P_k^h)^{1-\sigma_k} E_k^h}. \tag{5}$$

Based on the denominator in (5), define

$$(\Pi_k^j)^{1-\sigma_k} \equiv \sum_h \left\{\frac{t_k^{jh}}{P_k^h}\right\}^{1-\sigma_k} \frac{E_k^h}{\sum_h E_k^h}.$$

Divide numerator and denominator of the right hand side of (5) by total shipments of $k$ and use the definition of $\Pi$, yielding:

$$(\beta_k^j \widetilde{p}_k^j \Pi_k^j)^{1-\sigma_k} = Y_k^j / \sum_j Y_k^j. \tag{6}$$

The right hand side is the global expenditure share for class $k$ goods from country $j$. The left hand side is a 'global behavioral expenditure share', understanding that the CES price index is equal to one due to the normalization implied by summing (6):

$$\sum_j (\beta_k^j \widetilde{p}_k^j \Pi_k^j)^{1-\sigma_k} = 1. \tag{7}$$

The incidence of trade costs to sellers being given by the Π's, the incidence of bilateral trade costs on the buyers' side of the market is given by $t_k^{jh}/\Pi_k^j$, taking away the sellers' incidence. The average incidence of all bilateral costs to $h$ from the various origins $j$ is given by the buyers' price index $P_k^h$.

$P_k^h$ as an index of buyers' incidence is obtained by substituting for quality adjusted efficiency unit costs from (5) in the definition of the true cost of living index, using the definition of the Π's:

$$(P_k^h)^{1-\sigma_k} = \sum_j \left\{\frac{t_k^{jh}}{\Pi_k^j}\right\}^{1-\sigma_k} \frac{Y_k^j}{\sum_j Y_k^j}. \tag{8}$$

Collect this with the definition of the Π's:

$$(\Pi_k^j)^{1-\sigma_k} = \sum_h \left\{\frac{t_k^{jh}}{P_k^h}\right\}^{1-\sigma_k} \frac{E_k^h}{\sum_h E_k^h}. \tag{9}$$

These two sets of equations jointly determine the inward multilateral resistances, the $P$'s and the outward multilateral resistances, the Π's, given the expenditure and supply shares and the bilateral trade costs, subject to the normalization (7). A normalization of the Π's is needed to determine the $P$'s and Π's because (8)-(9) determine them only up to a scalar.[9]

Assurance that the normalized Π's and $P$'s calculated by this procedure do indeed give the sellers' and buyers' incidences follows from a property noted Anderson and van Wincoop (2004). Replace all the bilateral trade frictions with the hypothetical frictions $\widetilde{t}_k^{jh} = \Pi_k^j P_k^h$. The budget constraint (8) and market clearance (9) equations continue to hold at the same prices, even though individual bilateral trade volumes change. For each sale of $k$ from each country $j$ it as if a single shipment was made to the 'world market' at the average incidence $\Pi_k^j$. Similarly for each purchase of $k$ by country $h$ it is as if all purchases were made in a single 'world market' at the average incidence $P_k^h$.

Bilateral trade flows are given by the gravity equation

$$X_k^{jh} = \left\{\frac{t_k^{jh}}{\Pi_k^j P_k^h}\right\}^{1-\sigma_k} \frac{Y_k^j E_k^h}{\sum_j Y_k^j}. \tag{10}$$

[9] If $\{P_k^0, \Pi_k^0\}$ is a solution to (8)-(9), then so is $\{\lambda P_k^0, \Pi_k^0/\lambda\}$ for any positive scalar $\lambda$; where $P_k$ denotes the vector of $P$'s and the superscript 0 denotes a particular value of this vector, and similarly for $\Pi_k$.

This follows from the CES expenditure setup using (6) to substitute for $(\beta_k^j \widetilde{p}_k^j)^{1-\sigma_k}$. The interpretation of (10) reveals that trade frictions modify the frictionless flow $Y_k^j E_k^h / \sum_j Y_k^j$ by a power transform of the relative incidence of trade costs. The relative incidence ratio can be interpreted from either the demand side or the supply side viewpoint. From the demand side viewpoint, $\Pi_k^j$ being the incidence absorbed by supplier $j$, $t_k^{jh}/\Pi_k^j$ is the demand incidence absorbed by buyer $h$ on goods shipped from $j$. $P_k^h$ is the average buyer incidence in $h$ across all suppliers $j$, hence $t_k^{jh}/\Pi_k^j P_k^h$ is the relative demand side incidence of trade costs from $j$ to $h$ in goods class $k$. From the supply side viewpoint, $P_k^h$ being the incidence absorbed by buyer $h$, $t_k^{jh}/P_k^h$ is the supply incidence absorbed by seller $j$ on goods shipped to $h$. $\Pi_k^j$ is the average seller incidence in $j$ across all buyers $h$, hence $t_k^{jh}/\Pi_k^j P_k^h$ is the relative supply side incidence of trade costs from $j$ to $h$ in goods class $k$.

The relationship between the incidence of trade frictions and productivity frictions in the cross section is clarified by analyzing the special limiting case of frictionless trade, where $t_k^{jh} = a_k^j, \forall k, j$. The solution to (8)-(9) under the convenient normalization $P_k^1 = 1$[10] is $\Pi_k^j = a_k^j, \forall k, j$, and $P_k^h = 1, \forall k, h$. All the incidence of productivity is borne on the supply side. The reason is that in conditional general equilibrium the expenditure $E_k^j$ on good $k$ from source $j$ is given. With a fall in $a_k^j$, market clearance is achieved with a rise in the efficiency unit cost, so that all the benefit accrues to suppliers of $k$ from $j$. The further implication is that sectoral TFP is decomposable into a Hicks neutral production component and an equilibrium distribution incidence component.[11]

Estimates of gravity models are based on a stochastic version of (10). The inferred $T$'s are constructed based on regressions of bilateral trade flows on country fixed effects (for imports and exports separately) and a list of trade cost proxies. The exporter fixed effects control for the Hicks neutral parameters $\{a_k^j\}$, among other things.

The multilateral resistances are computed using the constructed $t$'s in (8)-(9) along with the normalization equation (7). In practice, for analyzing conditional equilibrium, another normalization will often be convenient due to missing information on the $\beta$'s and $\widetilde{p}$'s. For example, set one of the $P$'s

[10]For allocations within sectors, only the relative multilateral resistances are relevant for allocation, so allocation is invariant to the normalization.

[11]It is important to keep in mind that the comparative static incidence of a productivity improvement is still shared between buyer and seller; this decomposition applies in the cross section.

equal to 1. In any case, relative 'world' prices $\{\widetilde{p}_k^j \Pi_k^j\}$ are what controls allocation in conditional general equilibrium, so scaling factors for the $\widetilde{p}$'s and $\Pi$'s cancel out.

Separate information is required on the $a$'s to compute $\Pi$'s that include all productivity effects. In the absence of such informaiton, multilateral resistances is an index of trade frictions only, based on the gravity results.[12]

The link of the conditional general equilibrium in (8)-(9) to full general equilibrium uses the unified world market metaphor and $Y_k^j = g_k^j \widetilde{p}_k^j \Pi_k^j$. 'The full general equilibrium obtains when the world production shares that arise from solving (1) are consistent with the world production shares used to solve for the $\Pi$'s in (8)-(9) subject to (7) using the equilibrium $\widetilde{p}$'s.

# 3 The Specific Factors Model

Unskilled labor is intersectorally mobile but in fixed supply to the economy. Skilled workers are in fixed total supply to the economy prior to their acquisition of sector specific skills, after which they are in fixed supply to each sector. In Section 5, the allocation of skilled workers to sectors is chosen efficiently to equalize expected returns prior to the realization of productivity shocks. Ex post, the skill premia differ by sector and the allocation of skilled workers is inefficient. Both factors are internationally immobile. To ease notational clutter, the country superscript is suppressed temporarily.

Supply (understood for now as deliveries to final demand) in product class $k$ is given by

$$X_k = f^k(L_k, K_k)/a_k, \forall k \tag{11}$$

where the country index superscript $j$ is omitted for notational ease. $f^k$ is a concave homogeneous of degree one production function. Labor $L_k$ is mobile across sectors while $K_k$ is the specific skill endowment.

$\widetilde{p} = \{p_k/\Pi_k\}$, the vector of efficiency unit costs of production. The factory gate price (unit cost) is given by $c_k = a_k \widetilde{p}_k$. Gross domestic product $\sum_k \widetilde{p}_k X_k$

[12] Anderson and Yotov (2008) show that multilateral resistances are large for 2 digit industries in Canadian provinces, and vary quite a lot over commodities and over time. Outward multilateral resistance is on average around 5 times the size of inward multilateral resistance, and is negatively related to market share. Theoretical foundations for these regularities are provided for special cases. The results point to the importance of differences in outward multilateral resistance in determining the allocation of production between sectors and across countries and suggest the need for wider estimates of multilateral resistances.

is given by the maximum value GDP function $g(\widetilde{p}, L, \{K_k\})$. Hotelling's Lemma implies that the supply of $k$ is given by $g_{c_k} = X_k = g_{\widetilde{p}_k}/a_k$ and its value at factory gate prices by $g_{c_k} c_k = g_{\widetilde{p}_k} \widetilde{p}_k$. The equilibrium wage $w$ is given by $g_L$. Efficient allocation requires the value of marginal product conditions

$$w = \widetilde{p}_k f^k_{L_k}, \forall k$$

along with labor market clearance

$$\sum_k L_k = L.$$

Several important modeling purposes are served by more restrictive production functions. Assuming identical production functions (up to a productivity parameter $a$) ensures that the fully efficient equilibrium will be Ricardian (because the equilibrium relative factor intensities will be identical), and thus the model will nest the Eaton-Kortum and Costinot-Komunjer models. Imposing Cobb-Douglas structure results in a closed form solution for $g$ with very convenient properties. First, aggregate factor shares are stable, consistent with nearly constant empirical shares across periods of time when the composition of GDP has altered tremendously. Second, Stolper-Samuelson forces are shut down: the average skill premium is independent of international forces in the model. This is analytically convenient for thinking about a world in which skill premia seem to be rising simultaneously in both rich and poor countries.

Let $K = \sum_k K_k$ and let $f^k = L_k^\alpha K_k^{1-\alpha}$ where $\alpha$ is the parametric share parameter for labor. Then

$$g = L^\alpha K^{1-\alpha} G \tag{12}$$

where $G$ is given by

$$G = [\sum_k \lambda_k (\widetilde{p}_k)^{1/(1-\alpha)}]^{1-\alpha}, \tag{13}$$

and $\lambda_k = K_k/K$, the proportionate allocation of specific capital to sector $k$.[13] GDP is the product of real activity in production and distribution $R = L^\alpha K^{1-\alpha}$ and the real activity deflator $G$. $G$ is convex and homogeneous of degree one in the $\widetilde{p}$'s.

[13] Solve the labor market clearance condition for the equilibrium wage, then use the Cobb-Douglas property $wL/\alpha = g$.

The GDP function has a constant elasticity of transformation (CET). The elasticity of transformation is equal to $\alpha/(1-\alpha)$, the ratio of labor's share to capital's share.[14] The supply share for any good $k$ is given by:

$$s_k = \widetilde{p}_k X_k/g = \frac{\lambda_k \widetilde{p}_k^{\alpha/(1-\alpha)}}{\sum_k \lambda_k \widetilde{p}_k^{\alpha/(1-\alpha)}}. \tag{14}$$

# 4 World Trade Equilibrium

This section derives a world trade equilibrium 'reduced form' to characterize production and trade patterns. Across goods classes, production is allocated with the Cobb-Douglas specific factors model of Section 3. The model takes as given the incidence of trade costs solved from the allocation of given expenditure and supply across trading partners in Section 2. Sharp results are obtained by eventually restricting the elasticity of substitution parameter used in Section 2 to be the same across goods classes.

Market clearance with balanced trade implies

$$\Pi_k^j s_k^j g^j - \sum_h \theta_k^h \left\{ \frac{\beta_{kj} \widetilde{p}_k^j t_k^{jh}}{P_k^h} \right\}^{1-\sigma_k} g^h = 0, \tag{15}$$

$\forall k, j$. This system of equations determines the set of efficiency unit costs, $\widetilde{p}_k^j$, one for each $k$ and $j$. The system is homogeneous of degree zero in the unit costs (understanding that the $P$'s are homogeneous of degree one in the unit costs, being CES price indexes), hence relative unit costs only are determined. The task is to characterize the equilibrium.

## 4.1 Equilibrium Prices

Define $\omega^h \equiv g^h / \sum_h g^h$. Divide through in (15) by $\sum_h g^h$. Recognizing that $\theta_k^h g^h = E_k^h$ and using (9), (15) can be expressed as: $\Pi_k^j s_k^j \omega^j = (\Pi_k^j)^{1-\sigma_k} (\beta_k^j)^{1-\sigma_k} \Theta_k$, where $\Theta_k \equiv \sum_h \theta_k^h \omega^h$. Use (14) to substitute for $s_k^j$ in the market clearance equations, and solve for the equilibrium unit costs as:

$$\widetilde{p}_k^j = \left\{ \frac{D_k^j}{\omega^j \lambda_k^j (\Pi_k^j)^{\sigma_k}} \right\}^{\frac{1-\alpha}{\alpha+\sigma_k(1-\alpha)}} (G^j)^{1/(\alpha+\sigma_k(1-\alpha))} \tag{16}$$

[14]The CET form is commonly used in applied general equilibrium modeling. The microfoundations provided here may prove useful in this context.

where

$$D_k^j \equiv \beta_{kj}^{1-\sigma_k}\Theta_k.$$

On the right hand side of (16) the demand shifter $D_k^j$ is the product of a $k$ specific component $\Theta_k$ reflecting tastes in the global economy for good $k$ and a $(j,k)$ specific 'quality' parameter $\beta_{kj}^{1-\sigma_k}$ reflecting tastes within goods class $k$ for varieties from origin $j$. As for the $\theta$'s, any homothetic form yields qualitatively similar results with shares that depend on the $P$'s. In the Cobb-Douglas case, $\Theta_k$ is a parameter, hence so is $D_k^j$.

(16) in the empirically relevant case $\sigma_k > 1$ reveals that unit costs are increasing in the demand side drivers $D_k^j$. On the supply side, bigger country size $\omega^j$ and bigger sectoral allocations of specific factors $\lambda_k^j$ both reduce unit costs. The higher the incidence of trade costs in varieties from $j$ in class $k$, $\Pi_k^j$, the lower is the unit cost. The GDP deflator $G^j$ in the general equilibrium is the (multi-) factoral terms of trade. A rise in the factoral terms of trade raises unit cost, all else equal.

The factoral terms of trade $G^j$ can be solved for in global general equilibrium in terms of the $\lambda$'s, the $\Pi$'s and the $D$'s. The equilibrium GDP shares may be expressed as 'reduced form' equations in the international equilibrium using (16). Let $\eta_k = \alpha + \sigma_k(1-\alpha)$. Then:

$$s_k^j = \lambda_k^j \left( \frac{D_k^j}{\lambda_k^j (\Pi_k^j)^{-\sigma_k} \omega^j} \right)^{1/\eta_k} G_j^{1-\sigma_k}. \tag{17}$$

Use the adding up condition on the shares (17). Next, define the parametric 'real potential GDP' $R^j \equiv (L^j)^\alpha (K^j)^{1-\alpha}$, and note that $\omega^j = R^j G^j / \sum_j R^j G^j$. The adding up condition $\sum_j \omega^j = 1$ implies a natural normalization for the factoral terms of trade: $\sum_j R^j G^j = 1$, hence $\omega^j = R^j G^j$. Then the normalized factoral terms of trade are solved from

$$1 = \sum_k (\lambda_k^j)^{1-1/\eta_k} (\Pi_k^j)^{-\sigma_k/\eta_k} (D_k^j)^{1/\eta_k} (R^j)^{-1/\eta_k} G_j^{-\sigma_k/\eta_k}, \forall j.$$

A closed form solution exists when $\sigma_k = \sigma, \forall k$. The adding up condition on the shares (17) yields

$$G^j = (\Lambda^j / R^j)^{1/\sigma}, \tag{18}$$

where

$$\Lambda^j \equiv \Big\{ \sum_k \lambda_k^j \Big\{ \frac{D_k^j}{(\Pi_k^j)^\sigma \lambda_k^j} \Big\}^{1/\eta} \Big\}^\eta. \tag{19}$$

(18) implies, intuitively, that $G^j$ is homogeneous of degree minus one in the incidence of trade frictions. (18) also implies that bigger countries in real terms have lower factoral terms of trade in the cross section. Since $g^j = R^j G^j$ and the size elasticity of the terms of trade exceeds $-1$, this intuitive effect does not lower nominal GDP. (Size is not immiserizing in the cross section.)

$\Lambda$ is an efficiency measure of the match of sector specific factor allocations to the pattern of demand discounted by the incidence of trade costs. Consider as a benchmark the efficient allocation of skills to the sectors where it becomes specific. It is readily shown that $D_k^j/(\Pi_k^j)^\sigma \lambda_k^{j*} = \bar{c}^j, \forall k, j$.[15] Then $\Lambda^j = \bar{c}^j$ in (18), and any allocation less efficient will have $\Lambda^j < \bar{c}^j$.

(18) implies that the factoral terms of trade is *in*creasing in the relative efficiency of allocation of specific factors. This positive net effect decomposes into a direct effect, raising the value of GDP at given prices, and an indirect effect, lowering national $\widetilde{p}$'s due to, in effect, a country size effect on the terms of trade. The net effect is positive. The model thus yields a tightly specified way in which global economy shocks (showing up in $\Pi$'s, the incidence of productivity frictions) interact with the differential flexibility of national factor markets to differentially impact the factoral terms of trade.

The model has very strong implications for the cross section pattern of individual factor returns. The unskilled wage (using $w = g_L$) is given by

$$w^j = \alpha \left\{ \frac{K^j}{L^j} \right\}^{1-\alpha} \left\{ \frac{\Lambda^j}{R^j} \right\}^{1/[1+(\sigma-1)\eta]}, \forall j.$$

The national average return to skills is $\bar{r}^j = g_K^j$. Based on the preceding discussion,

**Proposition 1 (a)** *Wages are increasing in the human capital to labor ratio. The average skill returns are decreasing in the human capital to labor ratio. Both factor incomes are increasing in the relative efficiency of sector specific allocations, and decreasing in country size. (b) The average skill premium* $\bar{r}^j/w^j - 1 = [(1-\alpha)/(\alpha)](K^j/L^j)^{\alpha-1} - 1$ *is independent of international forces.*

Real national income (the product of average real income and the population) is given by $R^j G^j / \bar{P}^j$, the product of the real GDP $R^j$ and the factoral terms of trade $G^j/\bar{P}^j$. Here, $\bar{P}^j$ is the true cost of living index for country $j$, a homogeneous of degree one concave function of the vector of inward

[15] See Section 5 for details.

multilateral resistances $P^j$. Using (18), real income is given by

$$\frac{(\Lambda^j)^{1/[1+(\sigma-1)\eta]}(R^j)^{(\sigma-1)\eta/[1+(\sigma-1)\eta]}}{\bar{P}^j}$$

*Real national income is decreasing in the average national incidence of both inward and outward multilateral resistance,* by previous discussion.

The preceding algebra does not require that none of the bilateral trade flows be zero. The last section of the paper amplifies this claim in the presence of traded inputs and selection into trade. For present purposes, this property means that the distinction made in much earlier literature between traded and nontraded goods plays no central role; substitution on the extensive margin between traded and nontraded goods occurs in the background.

Proposition 1 (b) is a useful neutrality property of the model with respect to income distribution. But the distribution of sector specific factor incomes is powerfully affected by international forces. Sector specific factor returns are given by $r_k^j = g^j_{\lambda_k^j}/K^j$. Use the properties of the special Cobb-Douglas GDP function to yield

$$r_k^j = \bar{r}^j \frac{s_k^j}{\lambda_k^j}.$$

The properties of the national average returns to skill, $\bar{r}^j$, are given above. The sector specific part of the preceding expression will be developed following the analysis of equilibrium production shares.

## 4.2 Equilibrium Production and Trade Patterns

In the case of $\sigma_k = \sigma$ the reduced form production share equations simplify to

$$s_k^j = \frac{(\lambda_k^j)^{1-1/\eta}(\Pi_k^j)^{-\sigma/\eta}(D_k^j)^{1/\eta}}{\sum_k(\lambda_k^j)^{1-1/\eta}(\Pi_k^j)^{-\sigma/\eta}(D_k^j)^{1/\eta}}. \tag{20}$$

As compared to (17), (20) eliminates the effect of country size on the equilibrium pattern of production. Based on (20):

**Proposition 2** *In the special case of equal elasticities of substitution in expenditure (with $\sigma > 1$) and uniform Cobb-Douglas production functions, the equilibrium production share is*

1. *increasing in the capital allocation share $\lambda_k^j$;*

2. *increasing in the demand 'parameter'* $D_k^j$*;*

3. *increasing in the dispersion of* $D_k^j/\lambda_k^j(\Pi_k^j)^\sigma$ *and*

4. *decreasing in the incidence of trade costs* $\Pi_k^j$.

Proposition 2.3 follows because the denominator in (20) is concave in $D/\lambda\Pi^\sigma$ for $\eta{\geq}1$. The economic intuition is that the mismatch of the sectoral allocation of capital with the pattern of demand lowers GDP, hence raises the share of sector $k$ in the total (given the value of the denominator).

Sector specific factor returns can now be characterized drawing on Proposition 2. Using (20) and $r_k^j = \bar{r}^j s_k^j/\lambda_k^j$ yields

$$\frac{r_k^j}{\bar{r}^j} = \frac{[D_k^j/\lambda_k^j(\Pi_k^j)^\sigma]^{1/\eta}}{\sum_k \lambda_k^j[D_k^j/\lambda_k^j(\Pi_k^j)^\sigma]^{1/\eta}}. \tag{21}$$

Then:

**Proposition 1 (c)-(d)** *Sector specific factor returns are increasing in the national labor to human capital endowment ratio, decreasing in the sector specific allocation, decreasing in the sectoral incidence of trade costs and increasing in the sectoral demand parameter. (d) The distribution of skill premia is more dispersed the more inefficient is the sectoral allocation of human capital.*

(21) summarizes the properties of the inequality of specific factor returns in global equilibrium. Technology shocks affect the Π's primarily (exclusively under Cobb-Douglas upper level preferences so that $D_k^j$ is parametric.) Then for given allocations of skills, more dispersion of the incidence of productivity induces more ex post inequality.

The reduced form unit cost equations simplify when $\sigma_k = \sigma$. Using (19) in (18), substituting into (16) and simplifying yields

$$\widetilde{p}_k^j = \frac{(D_k^j/\lambda_k^j(\Pi_k^j)^\sigma)^{(1-\alpha)/\eta}}{[\sum_k \lambda_k^j(D_k^j/\lambda_k^j(\Pi_k^j)^\sigma)^{1/\eta}]^{\alpha/\sigma}}(R^j)^{-(1-\alpha+\alpha/\sigma)/\eta}. \tag{22}$$

Compared to (16), the special case (22) implies that larger countries have uniformly lower unit production costs.

The implications of (22) for equilibrium 'competitiveness' are very intuitive and sharp:

**Proposition 3** *In the special case model, all else equal:*

1. *larger specific endowments lower costs;*
2. *larger world demand for a good raises its cost;*
3. *higher quality costs more;*
4. *higher incidence of trade costs lowers unit costs;*
5. *bigger countries have lower costs.*
6. *higher dispersion of $D_k^j/\lambda_k^j(\Pi_k^j)^\sigma$ raises unit costs.*

That higher quality costs more is less obvious than it might seem. The CES model of preferences implies that some of each variety will be demanded, so it is not true that lower quality must have a lower price to be purchased by anyone.[16] Proposition 3.3 states that in general equilibrium, higher quality goods have higher unit costs, all else equal. Proposition 3.6, like Proposition 2.3, reflects the concavity of the deflator in (20) and (22) in $D/\lambda\Pi^\sigma$.

The model yields strong restrictions on the equilibrium pattern of trade. Production valued at world prices is given by $\Pi_k^j s_k^j g^j$. Own demand is given by (10). The ratio of gross exports (at 'world' prices) to GDP in the special case of equal elasticities of substitution is given by

$$\Pi_j^k s_k^j \left(1 - \frac{E_k^j}{Y_k}\left\{\frac{t_k^{jj}}{\Pi_k^j P_k^j}\right\}^{1-\sigma}\right).$$

Imposing Cobb-Douglas upper level preferences this reduces to

$$\Pi_k^j s_k^j \left(1 - G^j R^j \left\{\frac{t_k^{jj}}{\Pi_k^j P_k^j}\right\}^{1-\sigma}\right). \tag{23}$$

Here the identical expenditure shares assumption is used to replace $E_k^j/Y_k$ with $g^j/\sum_j g^j$ and the normalization of the $G$'s is applied to simplify to (23). Divide (23) by $\Pi_k^j$ to obtain exports to GDP at origin prices. The implications are that:

**Proposition 4** *in the special case model the ratio of gross exports to GDP is*

[16] The interpretation of $\beta_{kj}^{1-\sigma_k}$ as a quality parameter is natural from examining the sub-utility function that lies behind the CES expenditure function: starting from equal consumption of each variety, the consumer's willingness to pay is higher the larger is $\beta_{kj}^{1-\sigma_k}$.

1. *increasing in* $s_k^j$*, which moves according to Proposition 1;*

2. *decreasing in country size* $R^j$*,*

3. *decreasing in the factoral terms of trade* $g^j$*, and*

4. *decreasing in Constructed Home Bias* $\left\{\frac{t_k^{jj}}{\Pi_k^j P_k^j}\right\}^{1-\sigma}$.

Each item in the proposition is intuitive. The term Constructed Home Bias is coined by Anderson and Yotov (2008) and used to summarize the implications of gravity for the prominent empirical regularity called home bias.

Proposition 4 combined with Proposition 1 (c) implies that *export intensity is positively correlated with sectoral earnings premia*, a well documented empirical regularity in rich and poor countries alike. For sharper results in a specific factors continuum model, see Anderson (2008).

# 5 Equilibrium Specific Factor Allocation

In a plausible long run equilibrium, the specific factor allocations are determined by optimizing behavior. Investments in sectors become specific once made, but are allocated from a given stock $K$ so as to equalized anticipated returns. It is useful to consider the fully efficient equilibrium before proceeding to the more realistic equilibrium where investments are ex ante efficient but ex post inefficient due to the realizations of the productivity draws.

## 5.1 Fully Efficient Equilibrium

An instructive benchmark is the special case model when the specific factors are fully efficiently allocated. This arises if the specific factor becomes mobile; or equivalently, if the incidence of trade and productivity frictions is perfectly anticipated by agents selecting the specific factor investments. The identical Cobb-Douglas production function structure assumed here makes the production set effectively Ricardian when capital allocation adjusts efficiently.[17] Due to the love of variety structure of preferences, prices adjust

[17] The Ricardian production set is the outer envelope of specific factor production sets for fixed sectoral allocations.

in equilibrium to support diversification, avoiding the corner solutions that otherwise arise with Ricardian production.

The long run general equilibrium GDP shares reduce to[18]

$$s_k^j = \lambda_k^j = \frac{D_k^j(\Pi_k^j)^{-\sigma}}{\sum_k D_k^j(\Pi_k^j)^{-\sigma}}. \tag{24}$$

Supply adjusts to meet demand in absence of trade and productivity frictions. Trade and productivity frictions captured by the Π's redistribute sales through a CES structure, but the mechanism of an essentially demand driven equilibrium pattern of production remains.

Paralleling this feature, with efficient allocation the unit costs of (22) become invariant to $k$: $\widetilde{p}_k^j = \bar{p}^j, \forall k$, where $\bar{p}^j = (\Lambda^j)^{(1-\alpha-\alpha/\sigma)/\eta}(R^j)^{-(1-\alpha+\alpha)/\eta}$ and $\Lambda^j = \sum_k D_k^j(\Pi_k^j)^{-\sigma}$. The equilibrium national shares of world sales in each sector $k$ are given by $Y_k^j/Y_k = (\bar{p}^j)^{1-\sigma}(\beta_k^j\Pi_k^j)^{1-\sigma}$.

The specific gravity model takes the $\beta$'s as given, while the Eaton-Kortum model endogenizes them. The connection between the two models is seen as follows. Using the preceding equation to obtain $Y_k^j/Y_k(\Pi_k^j)^{1-\sigma}$ and substituting back into the gravity equation (10), the bilateral trade flows are given by

$$X_k^{jh} = (\beta_k^j)^{1-\sigma_k} E_k^h (\bar{p}^j)^{1-\sigma_k} (t_k^{jh}/P_k^h)^{1-\sigma_k}.$$

For any sector, the Eaton-Kortum assumptions result in equilibrium $\beta$'s such that the right hand side above is replaced with a gravity expression equivalent to (10), only with $1-\sigma$ replaced by $-\nu$ where $\nu$ is the dispersion parameter of the Frechet distribution. (See Eaton and Kortum, equation (11).) Demand side forces in the Eaton-Kortum model disappear into a constant term that cancels in equilibrium trade shares. Substitution is all on the extensive margin. In contrast, the Armington structure forces diversified production in each country by assuming that goods are differentiated by place of origin. Substitution is on the intensive margin. The distribution of the productivity penalties, the $a$'s, is unrestricted.

The Costinot-Komunjer extension of Eaton-Kortum to combine deterministic sector/country productivity with variety specific productivity draws from Frechet distributions results in the assignment of proportions of varieties

[18]Using (20) for $s_k^j$, $s_k^j/\lambda_k^j = 1$ can be solved for $\lambda_k^j = D_k^j(\Pi_k^j)^{-\sigma}/\sum_k[\lambda_k^j(D_k^j/\lambda_k^j(\Pi_k^j)^{\sigma})^{1/\eta}]^{\eta}$. $\sum_k \lambda_k^j = 1$ implies that $\sum_k[\lambda_k^j(D_k^j/\lambda_k^j(\Pi_k^j)^{\sigma})^{1/\eta}]^{\eta} = \sum_k D_k^j(\Pi_k^j)^{-\sigma}$. This yields the solution in the text.

within sectors as in Eaton-Kortum along with the assignment of sectoral allocations. The Appendix expands on the connection between the generalized Ricardian and specific factors models by analyzing monopolistic competition equilibrium when skill allocation is subsequent to productivity realizations.

The difference between the specific gravity and Eaton-Kortum/Costinot-Komunjer models is the specificity of skilled labor. Frechet distributions of productivity draws do not yield closed form predictions when factors are specific. Nevertheless, as preceding sections show, useful predictions about the pattern of production and trade can be made taking the $\lambda$'s and $\beta$'s as given.

## 5.2 Ex Ante Efficient Equilibrium

Suppose that the productivity of distribution and production is random. (Evidence from gravity estimation indicates that bilateral trade costs $T_k^{jh}$ are remarkably stable over time, while in contrast the sectoral productivity penalties $a_k^j$ and the multilateral resistances $\Pi_k^j$ appear to have significant randomness.) Investments in sectors must be made prior to the realization of the random variables, at which time the realized sector specific returns differ. Ex ante efficient equilibrium is characterized by equal expected rates of return. The Diamond stock market reaches such an equilibrium with risk neutral agents.[19]

The ratio of the realized rate of return in sector $k$ to the average realized rate of return is given by

$$\frac{r_k^j}{\sum_k \lambda_k^j r_k^j} = \frac{s_k^j}{\lambda_k^j}.$$

Substituting on the right hand side from the equilibrium share equation (20) and taking expectations, the allocation that equalizes expected returns satisfies

$$1 = E\left(\frac{(D_k^j/(\Pi_k^j)^\sigma \lambda_k^{j*})^{1/\eta}}{\sum_k \lambda_k^{j*}(D_k^j/(\Pi_k^j)^\sigma \lambda_k^{j*})^{1/\eta}}\right), \forall k. \tag{25}$$

[19] Anderson and Riley (1976) point out that the Diamond stock market decentralizes equilibrium in a trading economy under uncertain prices or technology shocks. Helpman and Razin (1978) develop the implications of international trade in securities when there is aggregate risk. Both papers develop the important resource allocation implications of risk aversion.

Here $E$ denotes the expectation operator. Note that since the right hand side is concave in $\Pi$, *riskier sectors receive less investment*, by Jensen's Inequality. This occurs despite risk neutrality.

An empirically tractable form of the share equation emerges from considerations of ex ante efficiency in monopolistic competition equilibrium. Realized $D$'s differ from ex post efficient equilibrium (including rational expectations) $D$'s by a white noise error term. The Appendix shows that realized shares are given by

$$s_k^j = \frac{\lambda_k^j \left( f_k f_j \epsilon_k^j (\Pi_k^j)^{-\sigma} \right)^{1/\eta}}{\sum_k \lambda_k^j \left( f_k f_j \epsilon_k^j (\Pi_k^j)^{-\sigma} \right)^{1/\eta}}, \tag{26}$$

where the $f$'s are fixed effects and the $\epsilon$'s are realizations of a unit mean random error that is orthogonal to the other terms. The orthogonality property is due to the assumption of ex ante efficient allocation.

Now consider the implications of randomness for the efficiency of allocation given ex ante efficiency.Taking expectations of (19), the concavity of $\Lambda$ in $\Pi$ guarantees that riskier incidence lowers efficiency for a given allocation. Moving to a risk-reducing allocation helps to offset this but cannot fully do so. Moreover, variance plausibly rises with the mean, in which case higher average incidence of trade and productivity frictions imposes an added burden through greater expected ex post inefficiency of allocation. In empirical exercises the $\lambda^*$'s can be calculated and compared to actual $\lambda$'s to decompose the inefficiency due to randomness into its avoidable and unavoidable components.

The full rational expectations equilibrium of the model requires that the expectations of $\Pi$'s be equal to the expectations of the realized $\Pi$'s obtained from (8)-(9) subject to (7).

# 6 Intermediate Inputs

Intermediate products trade comprises a large and growing share of world trade. A simple extension of the specific factors model of production encompasses intermediate products trade.

Vertical disintegration is apparent — an increasing share of components are imported, meaning some formerly potential trade becomes active. In the multi-country context, similar shifts in the qualitative pattern of trade arise as more of the potential bilateral trade links are activated by the choice of firms to initiate trade. The action on the extensive margin of trade introduced here also applies to final goods trade.

## 6.1 Specific Factors Production with Intermediates

Intermediate products enter for simplicity as just a single intermediate product, potentially produced as a variety at each location.[20] The CES aggregate of the varieties is an input into production of all final goods and the intermediate good at each location. To ease notation, suppress country indexes. The production function for product $k$ in the Cobb-Douglas case is given by

$$f_k = L_k^{\alpha} K_k^{1-\alpha-\nu} M_k^{\nu} / \bar{a}_k \Pi_k$$

where $M_k$ is the quantity of the CES aggregate intermediate input used in sector $k$ and sector $m$ is the intermediate goods production sector. Let $P_m$ denote the price of the intermediate input used by the home country, a CES aggregate of the intermediate products purchased from all trading origins. Cost minimization combines with the labor market clearance condition to yield the GDP function $g(\widetilde{p}, P_m, L, K, \{\lambda_k\})$ with a closed form given by

$$\{L^{\alpha} K^{1-\alpha} [(\sum_{k=1}^{n} \lambda_k \widetilde{p}_k^{1/(1-\alpha-\nu)})^{1-\alpha-\nu} P_m^{-\nu}]\}^{1/(1-\nu)} c. \tag{27}$$

Here, $c$ is a constant term combining the parameters, while $\widetilde{p}_k = p_k / \bar{a}_k \Pi_k$, the 'efficiency unit cost' in sector $k$. For some sector $k = m$, $\widetilde{p}_m$ is the efficiency unit price of the intermediate product from sector $m$ produced in the home country.

$P_m^j$ is a CES price aggregate for country $j$ of the elements of the vector $\{\widetilde{p}_m^i t_m^{ij}\}$. All the earlier procedures for multilateral resistance apply. Higher buyers' incidence of trade costs in intermediate inputs lowers GDP while lower sellers' incidence of intermediate products raises GDP.

Due to the separability of the GDP function, the reduced form production shares are independent of the incidence of trade costs on intermediate inputs $P_m$. This separability implies that all the production, trade and income distribution pattern results of Section 4 apply in the presence of intermediate goods.

[20] The methods used here readily scale up to any number of intermediate product classes.

## 6.2 The Extensive Margin, Productivity and Trade Patterns

The production function for each industry $k$ is comprised of the production functions of those firms that earn non- negative profits. The firms choose to enter production, commit a skilled labor force and then receive a Hicks-neutral productivity draw from a probability distribution. Those firms unlucky enough to receive draws too low to allow breaking even exit from production. The average productivity in industry $k$, $1/\bar{a}_k$, is determined by the cutoff productivity of the marginal firm in combination with the parameters of the productivity draw distribution. Average productivity is for present purposes taken as given.

Profits are earned by inframarginal firms, and form part of the rents earned by the sector specific factors.[21] The average productivity is associated with an average price, a constant markup over the the average unit cost of extant firms. See Melitz (2003) for details. The Melitz model differs in having only one factor of production, but the essentials remain the same, illustrated in the Appendix development of the monopolistic competition model. This setup allows aggregation of the heterogeneous firm model into a representative firm model easily linked to the general equilibrium production theory of preceding sections.

The second key contribution of Melitz is to introduce a second cutoff due to fixed costs of exporting. Expanding the iceberg metaphor, part of the iceberg shears off and is lost as it leaves the home glacier, the remainder melting as it travels to its destination. There are two consequences for the allocation of trade and a further consequence for the allocation of resources. As for trade, some (many in practice) trade links are shut down completely because no firm exports, and secondly, firm selection contributes to trade volume in active links. As for resource allocation, firms choosing to export must hire additional unskilled workers to meet the fixed cost.[22] The resulting rise in the wage raises the cost of production for all firms. Now the conditions of trade have an effect on average productivity: lowering the variable cost of trade induces more firms to incur the fixed cost of trade and raises the

[21] The division of rents between 'owners' and skilled workers is irrelevant to present purposes.

[22] (If skilled workers are not completely firm specific, firms can also hire skilled workers within their sector with some loss of skills. The implications of this Darwinian force in the sectoral skilled labor market is developed further in Anderson (2008).

average productivity of all surviving firms.

Helpman, Melitz and Rubinstein (2007) develop the implications of fixed costs of export for bilateral trade and the gravity model. For present purposes, note that the effect of action on the extensive margin is isolated in the multilateral resistance terms, $\Pi_k^j$ for outputs and $P_m^j$ for inputs in country $j$ and sector $k$. The Appendix develops the implications of their model for multilateral resistance.

# 7 Conclusion

This paper provides a platform for consistent aggregation of the fine structure of trade costs into productivity measures that are suitable for exploring implications of productivity differences across goods, countries and time. The implications of productivity differences at a point in time for the pattern of production and trade are explored in detail for the special case of the specific factors model.

The paper points to future empirical work. A first step is to estimate multilateral resistance indexes for an appropriately disaggregated set of goods for a set countries and years and combine them with sectoral productivity measures. These can be used in a number of settings, but the specific factors model gives TFP particularly strong implications for the pattern of production and trade. How well does specific factors do relative to the Eaton-Kortum model, or to the HOV model?

The paper also points to future theoretical refinement. The model has implicit in it a link between trade frictions and income distribution, a link that appears worth exploring in light of concerns about globalization causing inequality. The extreme simplicity of the model buys strong results, while hinting that the results hold in less restrictive cases. How robust is the model?

Finally, the analysis reveals important channels through which technology shocks in production and in distribution in one country are transmitted to productivity in all trading partners. The specific factors structure suggests gradual adjustment to long run equilibrium. Future research might profitably explore these channels for their implications about inference of productivity and about the international transmission of shocks.

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# 9 Appendix

## 9.1 Monopolistic Competition

The special form of monopolistic competition and trade that is the focus of most of the literature has essentially no effect on the equilibrium of the model for given allocations of the specific factor. Endogenizing the allocation of the specific factor has the additional important effect of endogenizing the expenditure share parameters.

The CES preferences in each sector now contain a very large number of potential brands produced by firms in each country. Each firm is a monopolistic competitor, marking up price over cost by a constant proportion $\sigma/(\sigma-1)$. The GDP shares have exactly the same form as in the text because differing elasticities act on the model exactly like differing technology frictions and become part of the $\Pi$'s while common elasticities cancel out.

The development of a brand takes $F$ units of skilled labor. The allocation of skilled labor is subject to the constraint $K^j = F\sum_k n_k^j + K_k^j$ where $n_k^j$ is the number of firms in sector $k$ and country $j$. The allocation share of skilled labor net of development requirements is given for each sector $k$ by $\lambda_k^j = K_k^j/(K^j - F\sum_k n_k^j)$.

The number of brands is determined in fully efficient equilibrium by the zero profit condition $r^j(Fn_k^j + K_k^j) + wL_k^j = p_k^j y_k^j$. Using the marginal revenue product functions for skilled and unskilled labor for the Cobb-Douglas production function in the zero profit condition and simplifying yields

$$n_k^j = \frac{K_k^j}{F}\left(\frac{\sigma}{\sigma-1} - 1\right) = \lambda_k^j \frac{K^j - F\sum_k n_k^j}{F(\sigma-1)}.$$

Sum and solve for $\sum_k n_k^j$, then substitute back into the right hand expression to yield

$$n_k^j = \lambda_k^j \frac{K^j}{F(\sigma-1)}\left(1 - \frac{1}{(\sigma-1)(1+F)}\right). \tag{28}$$

The supply of labor net of development requirements is in equilibrium given by

$$K^j\left(1 - \frac{1}{\sigma-1} + \frac{1}{(\sigma-1)^2(1+F)}\right)$$

Thus the GDP function remains exactly the same as in the text, with the understanding that $K^j$ is replaced by the expression above for net skilled labor and $\lambda$'s are defined as shares of net skilled labor.

Now consider the implications for the demand side of the model. For each sector $k$, the demand 'parameter' is

$$D_k^j = \theta_k n_k^j / \sum_j n_k^j \tag{29}$$

based on the Dixit-Stiglitz structure.

The efficient allocation in a frictionless world is $\lambda_k^j = \theta_k, \forall j, k$. This follows from solving (24) with the Π's equal to one.

For a world with trade frictions the equilibrium allocation is solved from $\lambda_k^j = s_k^j$, using (20) for $s_k^j$ and then replacing $D_k^j$ with a function of $\lambda$'s by using (28) in (29).

$$D_k^j = \theta_k \frac{\lambda_k^j}{\bar{\lambda}_k} \frac{K^j}{\sum_j K^j}, \tag{30}$$

where $\bar{\lambda}_k \equiv \sum_j \lambda_k^j K^j / \sum_j K^j$. Substituting in (20) and simplifying using $s_k^j = \lambda_k^j$ yields, for goods that are produced,

$$1 = \frac{\theta_k / \bar{\lambda}_k (\Pi_k^j)^\sigma}{\sum_k \lambda_k^j \theta_k / \bar{\lambda}_k (\Pi_k^j)^\sigma}. \tag{31}$$

(31) only holds (goods are produced) for goods with the same Π's. The Π's being endogenous, describing the equilibrium is difficult.

Eaton and Kortum resolve this difficulty by imposing a Frechet distribution on the $a$'s that differs nationally by a location parameter but has a common dispersion parameter. Eaton and Kortum predict the proportion of varieties that will be produced and exported in equilibrium by each country to each partner as a gravity equation. (See their equation (11).) Costinot and Komunjer extend the Eaton-Kortum approach by adding a deterministic country/sector specific component to productivity. Now the gravity model describes bilateral trade patterns in any sector while the country/sector productivity component shifts the country/sector production shares. Thus this generalized Ricardian approach is nested in the specific gravity approach when the specific factor is allocated after the productivity draws.

Admitting productivity shocks that are not revealed prior to the allocation of skilled labor, the efficient allocation is solved from using (28) in (29) and then substituting the result into (25). The pattern of production and trade predictions of the model remain those of the text for given $\lambda$'s and $D$'s, while the explantion of the $\lambda$'s and $D$'s is deeply implicit. An empirically tractable form of the share equation nevertheless emerges from considerations of ex ante efficiency. Realized $D$'s differ from ex post efficient equilibrium (including rational expectations) $D$'s by a white noise error term. Substituting the right hand side of (30) augmented by white noise into (20) yields

$$s_k^j = \frac{\lambda_k^j \left( f_k f_j \epsilon_k^j (\Pi_k^j)^{-\sigma} \right)^{1/\eta}}{\sum_k \lambda_k^j \left( f_k f_j \epsilon_k^j (\Pi_k^j)^{-\sigma} \right)^{1/\eta}}, \tag{32}$$

where the $f$'s are fixed effects and the $\epsilon$'s are realizations of a unit mean random error that is orthogonal to the other terms. The orthogonality property is due to the assumption of ex ante efficient allocation.

## 9.2 Selection to Trade

Helpman, Melitz and Rubinstein (2007) derive the gravity model with selection. The exposition below reviews their model, and reformulates it to highlight the role of multilateral resistance in both intensive and extensive margins. It eases notational clutter to suppress the separate accounting for each goods class $k$, and to move the location indexes to the subscript position.

The model of the preceding subsection applies to determine the number of firms that enter, taken here as given along with the other variables of conditional general equilibrium.

The cost of a firm to serve its own market (assuming that $t_{ii} = 1$ for simplicity) is given by $\widetilde{p}_i$ times $a_i$, the inverse of the firm's productivity draw. The aggregate expenditure at destination $j$ is $E_j$ and the CES expenditure system allocates expenditure across origins. Sales by $i$ to country $j \neq i$ are profitable only if $a_i \leq a_{ij}$ where $a_{ij}$ is defined by the zero profit condition:

$$\sigma^{-\sigma}(\sigma - 1)^{\sigma - 1} \left( \frac{t_{ij}\widetilde{p}_i a_{ij}}{P_k^j} \right)^{1-\sigma} E_j = f_{ij}.$$

Here, $f_{ij}$ denotes the fixed bilateral export cost. Extending the iceberg metaphor, $f$ is measured in uinits of the good, as if a chunk sheared off and was lost as the berg separated from the mother glacier. Note that the markup cancels in the numerator and denominator of the demand function facing the firm.

Define the selection variable $V_{ij}(a_{ij})$ where

$$V_{ij} = \int_{a_L}^{a_{ij}} a^{1-\sigma_k} dF(a)$$

for $a_{ij} \geq a_L$ while

$$V_{ij} = 0$$

otherwise. Here, $F$ is the cumulative density function. The value of shipments to all destinations from location $i$ is denoted $Y_i$.

Now derive the gravity model. The bilateral import value of shipments is given by

$$X_{ij} = \left(\frac{\widetilde{p}_i t_{ij}}{P_j}\right)^{1-\sigma} E_j n_i V_{ij}.$$

The total value of shipments is

$$Y_i = \sum_j X_{ij} = \widetilde{p}_i^{1-\sigma} n_i \sum_j (\frac{t_{ij}}{P_j})^{1-\sigma} V_{ij} E_j.$$

First, solve market clearance for $\widetilde{p}_i^{1-\sigma}$:

$$\widetilde{p}_i^{1-\sigma} = \frac{y_i/Y}{\Pi_i^{1-\sigma}}. \tag{33}$$

Here, $y_i$ denotes the shipments of the average firm in country $i$, $Y_i/n_i$ and $Y = \sum_i Y_i = \sum_j E_j$, while

$$\Pi_i^{1-\sigma} \equiv \sum_j (\frac{t_{ij}}{P_j})^{1-\sigma} V_{ij} E_j / Y \tag{34}$$

Substitution yields the bilateral flows as:

$$X_{ij} = (\frac{t_{ij}}{P_j \Pi_i})^{1-\sigma} V_{ij} Y_i E_j / Y,$$

where

$$P_j^{1-\sigma} = \sum_i (\frac{t_{ij}}{\Pi_i})^{1-\sigma} V_{ij} Y_i / Y. \tag{35}$$

The normalization condition for the $\Pi$'s follows from manipulating (33) and summing:

$$\sum_i n_i (\Pi_i \widetilde{p}_i)^{1-\sigma} = 1. \tag{36}$$

The selection equation can be restated to highlight the role of multilateral resistance. Selection is controlled by:

$$\sigma^{-\sigma}(\sigma - 1)^{\sigma-1}(\frac{a_{ij} t_{ij}}{P_j \Pi_i})^{1-\sigma} E_j y_i / Y = f_{ij}. \tag{37}$$

There are three implications. First, notice that the gravity model with selection combines the effects of trade costs on the intensive margin with their effects on the extensive margin acting through $V_{ij}$. Higher fixed costs reduce volume while larger markets draw more entrants. Second, $\sigma$ plays a role in selection. Incorporating variation across goods class, lower elasticity (higher markup) goods classes will have more firms selected into exporting, all else equal. Third, most importantly, the multilateral resistance variables incorporate both the productivity penalty imposed by the incidence of trade costs and the productivity gain garnered by the incidence of selection into trade.

The formal model is completed by specifying a distribution function for $G$. With the Pareto distribution used by Helpman, Melitz and Rubinstein, let the Pareto parameter be $\kappa$. Then

$$V_{ij} = \frac{\kappa a_L^{\kappa-\sigma+1}}{(\kappa-\sigma+1)(a_H - a_L)} W_{ij}$$

$$W_{ij} = max[(a_{ij}/a_L)^{\kappa-\sigma+1} - 1, 0].$$

Helpman, Melitz and Rubinstein estimate selection with a Probit regression, then use these estimates to control for selection in the second stage gravity model regression with positive trade flows.