A re-evaluation of empirical tests of the Fisher hypothesis

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Abstract

This paper shows that the recent literature that tests for a long-run Fisher relationship using cointegration analysis is seriously flawed. Cointegration analysis assumes that the variables in question are $I(1)$ or $I(d)$ with the same d. Using monthly post-war U.S. data from 1959-1997, we show that this is not the case for nominal interest rates and inflation. While we cannot reject the hypothesis that nominal interest rates have a unit root, we find that inflation is a long-memory process. A direct test for the equality of the fractional differencing parameter for both series decisively rejects the hypothesis that the series share the same order of integration.

Keywords: Fisher hypothesis, cointegration, long memory

JEL: E43, E44, C22, C52

1 Introduction

The existence of a one-to-one long-run equilibrium relationship between after-tax nominal interest rates and expected inflation, known as the Fisher hypothesis, has been tested extensively in the literature. Since most studies cannot reject the null hypothesis of a unit root in nominal interest rates and inflation, tests of the Fisher hypothesis have centered on testing for the existence of a cointegrating relationship between interest rates and inflation. For example, Mishkin (1992) and Evans and Lewis (1995) find cointegration between interest rates and inflation using single equation testing methods (such as Engle and Granger's (1987) least squares estimator), while Crowder and Hoffman (1996) obtain more robust findings supporting the Fisher equation by applying Johansen's (1988) maximum likelihood system estimation methodology.

We argue that it is inappropriate to use cointegration methods to test the Fisher relationship in postwar U.S. data. Using monthly series spanning the period from 1959:1 to 1997:8, we apply a thorough testing procedure to the individual data series to determine their trending and long-memory behavior, and conclude that nominal interest rates and inflation are characterized by different orders of integration. This implies that standard cointegration analysis, which assumes that all variables are integrated of order one (or are $I(1)$) cannot be applied to these series. The concept of fractional cointegration, which generalizes the methodology to allow for fractionally integrated series (or $I(d)$ with d different from unity) and equilibrium errors, is also inapplicable since it also assumes that the series share the same order of integration (i.e. the series being studied must be $I(d)$ with the same d).

We employ standard unit root tests, unit root tests that allow for structural breaks, and various methods that directly estimate the fractional differencing parameter for the individual series. While a number of previous studies have established that inflation rates follow mean-reverting fractionally integrated processes (see, for example, Hassler and Wolters (1995), Baum, Barkoulas and Caglayan (1999) and Crato and Rothman (1994)), the evidence on nominal yields is somewhat mixed. Several papers have called into question the common finding that nominal interest rates are integrated processes (i.e. Wu and Zhang (1996), Hauser and Kunst (1998)) and Dueker and Startz (1998)). However, since unit root tests have low power in rejecting the unit root hypothesis in favor of fractional alternatives, direct tests for the existence of fractional integration seem necessary. In addition, to directly address the issue of a common order of integration, we make use of tests that jointly estimate the fractional differencing parameter for both series, and test for their equality. The evidence from this battery of tests leads us to conclude that nominal interest rates contain unit roots (or are $I(1)$) whereas inflation series are long memory (or $I(d)$) stochastic processes, so that nominal interest rates and inflation do not share the same order of integration.

These findings cast serious doubt on the validity of recent tests of the Fisher hypothesis that have appeared in the literature. In addition, cointegration tests that incorrectly assume the underlying variables are integrated processes may be misleading. A recent study by Gonzalo and Lee (1998) compares the robustness of the Engle and Granger (1987) and Johansen's Likelihood Ratio cointegration tests with respect to varying characteristics of the underlying data generating processes. They find that, depending on the size of the fractional differencing parameter, incorrectly assuming that a process is $I(1)$ will lead to a false finding for cointegration.

The next section briefly reviews the current literature on empirical tests of the Fisher equation. Our empirical methodology and results are presented in Section 3, as well as comparisons of our findings to those appearing in the literature. Section 4 concludes and offers suggestions for future research.

2 The Fisher equation and cointegration

The Fisher hypothesis simply states that the nominal interest rate, i_t, is equal to the real interest rate, r_t, plus expected inflation, π_t^e, as shown in equation (1) below:

$$i_t = r_t + \pi_t^e, \tag{1}$$

Earlier tests of Fisher's theory often consisted of estimating a regression of the form:

$$i_t = \alpha + \beta\pi_t + \epsilon_t, \tag{2}$$

where α and β are parameters to be estimated, π_t is actual inflation, and ϵ_t is a composite error term under the assumption of rational expectations.[2] The null hypothesis to be tested is that movements in inflation are fully reflected in nominal interest rates or that $\beta = 1$.[3] In this case, the parameter α estimates the mean of the *ex post* real interest rate which is defined as $i_t - \pi_t$.[4] As is well known in the literature, testing whether $\beta = 1$ using conventional regression methods is valid only if nominal interest rates and inflation are stationary (or $I(0)$) variables since the distribution of the parameters is nonstandard otherwise. If i_t and π_t are characterized by unit roots (or are $I(1)$), the Fisher hypothesis may be tested by examining whether these two variables form a linear combination which is stationary, or whether ϵ_t is $I(0)$. Assuming the latter is true, equation (2) may be interpreted as a cointegrating regression, reflecting an equilibrium relationship between nominal interest rates and inflation, with the error term ϵ_t representing short-run deviations from a long-run equilibrium. Therefore, a finding of cointegration where the cointegrating vector is $(1, -1)$ (i.e. if the coefficient on inflation is indistinguishable from unity) is supportive of a "long-run" Fisher relationship and implies that the real interest rate, r_t, is stationary.

One recent study using this methodology to test the Fisher hypothesis is that of Crowder and Hoffman (1996). In this paper, the authors employ Johansen's (1988) fully-efficient maximum likelihood estimator to test for cointegration between three-month Treasury bill rates and quarterly inflation for the 1952:1 to 1991:4 period. Their estimates of the cointegrating vector indicate that after-tax nominal rates respond fully to movements in inflation, consistent with the Fisher hypothesis. Evans and Lewis (1995) and Mishkin (1992) also examined the long-run Fisher relationship in U.S. data with cointegration analysis, but using different estimation methods.[5] Using Engle and Granger's (1987) bivariate cointegration test, Mishkin's (1992) results do not support the Fisher equation, while Evans and Lewis (1995) report that after-tax nominal rates move one-for-one with

[2] If forecasts of inflation are rational, $\pi_t = \pi_t^e + e_t$, where the forecast error, e_t, is orthogonal to all variables in the information set at time t. Since ϵ_t is the sum of the regression error (which is orthogonal to π_t by construction) and the inflation forecast error e_t, the parameters of equation (2) can be consistently estimated using ordinary least squares.

[3] if nominal interest rates are measured on a before-tax basis, then the null hypothesis to be tested is that $\beta = 1/(1-\tau)$ where τ is the tax rate.

[4] Alternatively, one can examine the correlation between the nominal interest rate and inflation by regressing inflation on the nominal rate. This has been termed the "Fama specification" in reference to Fama (1975). The choice of normalization is of consequence; some authors have reached very different conclusions depending on their choice of normalization, as Crowder and Hoffman (1996, p.104) indicate.

[5] Crowder and Hoffman (1996) provide a detailed comparison of the three studies.

inflation only when regime shifts in the expected inflation process are taken into account. According to Crowder and Hoffman (1996), Monte Carlo analysis shows that ordinary least squares (OLS) estimates of the cointegrating relationship suffer from small sample bias when inflation is characterized as an integrated moving average process, as they find is the case in their data set. They conclude that the inability of previous studies to support the long-run Fisher hypothesis is due to the choice of estimator of the cointegrating relationship.

Standard cointegration analysis, as applied by the aforementioned studies, may only be used to test for the existence of a long-run Fisher relationship if nominal rates and inflation are both unit root processes. However, a number of recent studies have shown that inflation is best described as a long-memory or $I(d)$ process, where the order of integration, d, is not restricted to the values of 0 or 1.[6] Using monthly Consumer Price Index (CPI) inflation rates for five industrialized countries including the U.S., Hassler and Wolters (1995) find that, for most countries over their 1969-1992 sample period, inflation is a fractionally integrated (long-memory) process with the integration order, d, found to be close to the stationary value of 0.5, implying that shocks to the inflation process are persistent but that the process is mean-reverting. Baillie, Chung and Tieslau (1996) also find fractional integration in inflation rates using an extended model which allows for conditional heteroskedasticity in the error term. Using monthly inflation rates extending over the period from 1948 to 1990, they find that for nine of the ten countries considered, inflation is a stationary long-memory process with time-dependent heteroskedasticity. More recently, Baum, Barkoulas and Caglayan (1999) report similar findings of fractional integration using inflation rates that are computed from both CPIs and WPIs for a larger set of industrialized and developing countries.

Given the widespread evidence that inflation rates are not unit-root processes, the interpretation of previous studies' cointegration results becomes questionable. Standard unit root tests are known to have low power against fractional alternatives, as discussed by Diebold and Rudebusch (1991), Sowell (1990), Cochrane (1991), and Hassler and Wolters (1994). Given this weakness, it seems prudent to directly test for fractional integration in nominal interest rates as well before proceeding with tests of the Fisher equation. Recent evidence of differing orders of integration is provided by Phillips (1998), who estimates the integration parameter for nominal interest rates, inflation

[6] Details on the definition of long-memory processes and the relevant statistical tests are given in section 3.

and the ex post real interest rate for the period 1934-1997. He finds that the nominal rate is nonstationary ($d = 0.95$) while the inflation rate and the ex post real interest rate are fractionally integrated processes with $d = 0.53$, clearly rejecting unit root nonstationarity and short memory dependence for the entire period and for postwar subsamples.

Whether nominal interest rates and inflation are $I(d)$ or $I(1)$, the existence of a long-run equilibrium relationship between the two series is still possible if they share the same order of integration. Although it is not generally recognized, Engle and Granger's (1987) original development of the concept of cointegration included the possibility that the error term in the cointegrating regression might be fractionally integrated, rather than stationary. In our context, that would imply that deviations from the long-run relationship shared by nominal rates and inflation take a long time to dissipate and return the two series to their equilibrium relationship. If ϵ_t is a long-memory stationary process, then nominal rates and inflation are said to be fractionally cointegrated. However, fractional cointegration bears the same prerequisite as the cointegration of $I(1)$ series: each of the series must be integrated of the same order. In this study, we show that cointegration analysis should not be applied to tests of the Fisher hypothesis since nominal yields and inflation do not share the same order of integration. We find that inflation is fractionally integrated, while nominal interest rates are best characterized as unit-root processes. Therefore the concept that the two series share a stochastic trend which represents a long-run relationship is not applicable.

3 Empirical results

3.1 Data

We conduct our investigation using one- to nine-month yields on Treasury bills acquired from the CRSP Monthly Government Bond dataset's TB12 table. Inflation is calculated from the CPI-U measure (series PZUNEW in the DRI Basic Economics dataset) as 1200 $(\log P_{t+1} - \log P_t)$ for the one-month inflation rate (π_t^1), as 600 $(\log P_{t+2} - \log P_t)$ for the two-month inflation rate (π_t^2) , and so on. Monthly data for 1964:6 through 1997:6 are employed.

3.2 Time series characteristics of the data

We first apply conventional unit root tests to the nominal yields and inflation rate series in order to ascertain their degree of integration. We employ the efficient test for an autoregressive unit root proposed by Elliott, Rothenberg and Stock (ERS, 1996). Their test, referred to as the DFGLS test, is similar to the Augmented Dickey-Fuller t test as it applies GLS detrending before the (detrended) series is tested via the Dickey-Fuller regression. Compared with the ADF and Phillips-Perron tests, the DFGLS test is asymptotically "most powerful invariant" (Stock, 1994, p.2768), possessing the best overall performance in terms of small-sample size and power, with "substantially improved power when an unknown mean or trend is present." (ERS, p. 813) We also employ the Kwiatkowski et al. (KPSS, 1992) test, which has a null hypothesis of stationarity. The DFGLS test and KPSS test results, both allowing for trends in the data generating processes, are presented in Table 1. None of the nominal yields nor inflation rates reject the DFGLS null hypothesis of a unit root in these series at the five percent level of significance. The KPSS tests uniformly reject trend stationarity for all series tested at the one percent level of significance. The combination of this evidence strongly implies that both nominal rates and inflation rates are not stationary ($I(0)$) processes.

These conventional unit root tests have frequently been employed to establish the nonstationarity of time-series processes. One critique of that strategy has been levelled by Perron (1990, 1997) and Perron and Vogelsang (1992), who have demonstrated that shifts in the intercept and/or slope of the trend function of a stationary time series biases these standard unit-root tests toward nonrejection. Their work has been extended to handle a specification allowing for two shifts in the mean of the stochastic process by Clemente et al. (1998). These tests, allowing for either "additive outliers" (mean shifts) or "innovational outliers" (trend shifts), may give a more convincing indication of whether apparent unit-root behavior is truly indicative of those dynamics, or merely an artifact of structural breaks in the series (caused, for instance, by the oil price shocks of the 1970s, or changes in monetary operating procedures of the early 1980s). We applied the Clemente et al. tests for both additive outliers and innovational outliers for one-, two- and three-month tenors. [7] Both sets of tests detected two distinct breaks in the series: for yields, in 1978-79 and

[7] To save space, these test results are not presented here, but are available on request from the authors.

1984-85; for inflation, in 1972-73 and late 1981.[8] The additive outlier unit-root tests could never reject the unit-root null. The innovational outlier unit-root tests could not reject $I(1)$ for nominal yields, but rejected the unit-root null at the five percent level for each of the three inflation series. Thus, the inflation processes exhibit some characteristics that do not confirm unit-root behavior, but those findings are sensitive to the test employed. In summary, considering only the $I(0)/I(1)$ distinction of conventional unit-root tests, both nominal rates and inflation appear to resemble unit root processes.

3.3 Fractional integration tests

The common findings of unit-root behavior in the yield and inflation rate series may be an artifact of the testing methodology in a different sense. Conventional unit-root tests, whether their null is $I(1)$ or $I(0)$ behavior, fail to consider the possibility that the order of integration of these series may be fractional: $I(d)$, rather than integer, $I(1)$ versus $I(0)$. The mean-reverting properties of nominal yields and inflation rate series may not be detectable by standard integer-order unit-root tests, which as cited above are known to have low power against fractional alternatives. We employ a fractional integration framework to overcome this criticism and allow for fractional integration in the time-series processes.

We make use of two nonparametric tests for the order of fractional integration: Phillips' (1999a) modified log-periodogram regression estimator and Robinson's (1995) log-periodogram regression estimator. Phillips' estimator is a recently-proposed extension of the well-known Geweke/Porter-Hudak (GPH, 1983) test that addresses some of the weaknesses of the GPH test. Robinson's log-periodogram estimator may be applied to individual time series or, as in our setting, to a group of time series, allowing for comparison of their respective orders of integration. Before describing those tests, we discuss the characteristics of a fractionally integrated time series. The model of an autoregressive fractionally integrated moving average process of order (p, d, q), denoted by ARFIMA (p, d, q), with mean μ, may be written as

$$\Phi(L)(1-L)^d (y_t - \mu) = \Theta(L)u_t, \ u_t \sim i.i.d.(0, \sigma_u^2) \tag{3}$$

[8] Malliaropulos (2000) demonstrates that common structural breaks in the nominal rate and inflation series, if not taken into account, can lead to incorrect inference of a common stochastic trend among trend-stationary series.

where L is the backward-shift operator, $\Phi(L) = 1 - \phi_1 L - .. - \phi_p L^p$, $\Theta(L) = 1 + \vartheta_1 L + ... + \vartheta_q L^q$, and $(1-L)^d$ is the fractional differencing operator defined by

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)} \tag{4}$$

with $\Gamma\ (\cdot)$ denoting the gamma function. The parameter d is allowed to assume any real value. The arbitrary restriction of d to integer values gives rise to the standard autoregressive integrated moving average (ARIMA) model. The stochastic process y_t is both stationary and invertible if all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle and $|d| < 0.5$. The process is nonstationary for $d \geq 0.5$, as it possesses infinite variance, i.e. see Granger and Joyeux (1980).

Assuming that $d \in [0, 0.5)$, Hosking (1981) showed that the autocorrelation function, $\rho(\cdot)$, of an ARFIMA process is proportional to k^{2d-1} as $k \to \infty$. Consequently, the autocorrelations of the ARFIMA process decay hyperbolically to zero as $k \to \infty$ in contrast to the faster, geometric decay of a stationary ARMA process. For $d \in (0, 0.5)$, $\sum_{j=-n}^{n} |\rho(j)|$ diverges as $n \to \infty$, and the ARFIMA process is said to exhibit long memory, or long-range positive dependence. The process is said to exhibit intermediate memory (anti-persistence), or long-range negative dependence, for $d \in (-0.5, 0)$. The process exhibits short memory for $d = 0$, corresponding to stationary and invertible ARMA modeling. For $d \in [0.5,\ 1)$ the process is mean reverting, even though it is not covariance stationary, as there is no long-run impact of an innovation on future values of the process.

3.3.1 Phillips' Log-Periodogram Regression Estimator[9]

Geweke and Porter-Hudak (1983) proposed a semiparametric procedure to obtain an estimate of the memory parameter d from n observations on a fractionally integrated process X_t in a model of the form

$$(1-L)^d X_t = u_t, \tag{5}$$

where u_t is stationary with zero mean and continuous spectral density $f_u(\lambda) > 0$. The estimate $\hat{d}$ is obtained from the application of ordinary least squares to

$$\log\left(I_x\left(\lambda_s\right)\right) = \hat{c} - \hat{d} \log \left|1 - e^{i\lambda_s}\right|^2 + residual \tag{6}$$

[9]This section draws upon Phillips (1999a) in its exposition and notation.

computed over the fundamental frequencies $\left\{\lambda_s = \frac{2\pi s}{n}, s = 1, ..., m\right\}$ where m, the number of frequencies to be included in the regression, is chosen as a fraction of the sample size to prevent contamination of the estimate by "short-memory" (high-frequency) components. We define $\omega_x(\lambda_s) = \frac{1}{\sqrt{2\pi n}} \sum_{t=1}^{n} X_t e^{it\lambda_s}$ as the discrete Fourier transform (dft) of the time series X_t, $I_x(\lambda_s) = \omega_x(\lambda_s)\,\omega_x(\lambda_s)^*$ as the periodogram, and $x_s = \log\left|1 - e^{i\lambda_s}\right|$. Ordinary least squares on (6) yields

$$\hat{d} = 0.5 \frac{\sum_{s=1}^{m} x_s \log I_x(\lambda_s)}{\sum_{s=1}^{m} x_s^2}. \tag{7}$$

Geweke and Porter-Hudak (1983) prove consistency and asymptotic normality for $d < 0$, while Robinson (1995) proves consistency and asymptotic normality for $d \in (0, 0.5)$ in the case of Gaussian ARMA innovations in (5).

Phillips (1999a) points out that the prior literature on this semiparametric approach does not address the case of $d = 1$, or a unit root, in (5), despite the broad interest in determining whether a series exhibits unit-root behavior or long memory behavior. In recent work with Kim (Kim and Phillips, 1999), it has been shown that the $\hat{d}$ estimate of (7) is inconsistent when $d > 1$, with $\hat{d}$ exhibiting asymptotic bias toward unity.

Phillips (1999b) proposes a modification of the GPH estimator that circumvents these difficulties, based on an exact representation of the dft in the unit root case. This representation "...shows that the dft of a fractionally integrated process comprises two distinct components. The first of these is the dft of the innovations u_t, scaled by the transfer function of the differencing filter ... The second involves a weighted sinusoidal sum ...of the observations X_t." (1999b, p.5) Phillips shows that when $d = 1$, the latter sum is an expression depending only on the final sample observation. Thus, the modified estimator expresses the dft as $v_x(\lambda_s) = \omega_x(\lambda_s) + \frac{e^{i\lambda_s}}{1-e^{i\lambda_s}} \frac{X_n}{\sqrt{2\pi n}}$ with associated periodogram ordinates $I_v(\lambda_s) = v_x(\lambda_s)\,v_x(\lambda_s)^*$ (1999a, p.9). He notes that both $v_x(\lambda_s)$ and, thus, $I_v(\lambda_s)$ are observable functions of the data. The log-periodogram regression is now the regression of $\log I_v(\lambda_s)$ on $a_s = \log\left|1 - e^{i\lambda_s}\right|$. Defining $\bar{a} = m^{-1} \sum_{s=1}^{m} a_s$ and $x_s = a_s - \bar{a}$, the modified estimate of the long-memory parameter (MODLPR) becomes

$$\tilde{d} = 0.5 \frac{\sum_{s=1}^{m} x_s \log I_\nu(\lambda_s)}{\sum_{s=1}^{m} x_s^2}. \tag{8}$$

Phillips proves that, with appropriate assumptions on the distribution of u_t, the distribution of $\tilde{d}$

follows

$$\sqrt{m}\left(\tilde{d}-d\right) \stackrel{d}{\longrightarrow} N\left(0,\frac{\pi^2}{24}\right), \tag{9}$$

so that $\tilde{d}$ has the same limiting distribution at $d = 1$ as does the GPH estimator in the stationary case (and, as Kim and Phillips show, applies over the range $0 < d < 2$), so that $\tilde{d}$ is consistent for values of d around unity. A semiparametric test statistic for a unit root against a fractional alternative is then based upon the statistic (1999a, p.10):

$$z_d = \frac{\sqrt{m}\left(\tilde{d}-1\right)}{\pi/24} \tag{10}$$

with critical values from the standard normal distribution. This test, consistent against both $d < 1$ and $d > 1$ fractional alternatives, is that employed in our empirical analysis.[10]

3.3.2 Robinson's Log-Periodogram Regression Estimator[11]

Robinson (1995) proposes an alternative log-periodogram regression estimator which he claims provides "modestly superior asymptotic efficiency to $\bar{d}(0)$" ($\bar{d}(0)$ being the Geweke and Porter-Hudak estimator)(1995, p.1052). Robinson's formulation of the log-periodogram regression also allows for the formulation of a multivariate model, providing justification for tests that different time series share a common differencing parameter. Normality of the underlying time series is assumed, but Robinson claims that other conditions underlying his derivation are milder than those conjectured by GPH.

We present here Robinson's multivariate formulation, which applies to a single time series as well. Let X_t represent a $G-$dimensional vector with g^{th} element $X_{gt}, g = 1, ..., G$. Assume that X_t has a spectral density matrix $\int_{-\pi}^{\pi} e^{ij\lambda} f(\lambda)\, d\lambda$, with (g, h) element denoted as $f_{gh}(\lambda)$. The g^{th} diagonal element, $f_{gg}(\lambda)$, is the power spectral density of X_{gt}. For $0 < C_g < \infty$ and $-\frac{1}{2} < d_g < \frac{1}{2}$, assume that $f_{gg}(\lambda)\ \tilde{}\ C_g \lambda^{-2d_g}$ as $\lambda \to 0+$ for $g = 1, ..., G$. The periodogram of X_{gt} is then denoted as

$$I_g(\lambda) = (2\pi n)^{-1} \left| \sum_{t=1}^{n} X_{gt} e^{it\lambda} \right|^2, g = 1, ...G \tag{11}$$

[10] This estimation procedure is implemented in routine *modlpr*, written for *Stata* version 6 (Baum and Wiggins, 2000).

[11] This section draws upon Robinson (1995) in its exposition and notation.

Robinson's method also allows for averaging the periodogram over adjacent frequencies and omission of l initial frequencies from the regression. Use of those options does not qualitatively alter our findings. Without averaging, we may define $Y_{gk} = \log I_g(\lambda_k)$. The least squares estimates of $c = (c_1, ...c_G)'$ and $d = (d_1, ..., d_G)'$ are given by

$$\begin{bmatrix} \tilde{c} \\ \tilde{d} \end{bmatrix} = vec\left\{Y'Z(Z'Z)^{-1}\right\}, \tag{12}$$

where $Z = (Z_1, ...Z_m)'$, $Z_k = (1, -2\log\lambda_k)'$, $Y = (Y_1, ...Y_G)$, and $Y_g = (Y_{g,1}, ...Y_{g,m})'$ for m periodogram ordinates. Standard errors for $\tilde{d}_g$ and for a test of the restriction that two or more of the d_g are equal may be derived from the estimated covariance matrix of the least squares coefficients.[12]

3.4 Univariate Test Findings

For the nominal yields, the Phillips MODLPR tests, presented in Table 2, fail to reject the null hypothesis of $d = 1$ at the five percent level of significance for all but a few sample sizes (i.e. m) when applied to the levels of the series. The Robinson LPR tests, presented in Table 4, show that the null hypothesis of $d^* = 0$ cannot be rejected. Since the Robinson test is applied to the differences of the original series, this implies that a unit root in the original levels series cannot be rejected.

For the inflation rate series, the findings from fractional integration tests are distinctly different. As other authors have found (cf. Baillie et al. (1996), Hassler and Wolters (1995)) inflation rate series may well exhibit fractional orders of integration. Table 3 presents test results for the MODLPR test; evidence for $d < 1$ is quite widespread for the 1-5 month inflation rate series, with an order of integration for inflation in the 0.5–0.8 range.[13] The Robinson LPR test results, presented in Table 5, largely corroborate these findings for 1-4 month inflation rate series, with broadly similar d estimates and implied orders of integration of 0.5–0.8 for inflation proper. Both the Phillips tests and Robinson tests provide some evidence of $d > 1$ for 7-9 month inflation rate

[12] This estimation procedure is implemented (in more general form) as routine *roblpr*, written for *Stata* version 6 (Baum and Wiggins, 2000).

[13] Phillips suggests that if a deterministic trend is present in the series, it should be removed prior to testing. Significant linear trends were detected in the inflation series, and the test applied to the detrended series.

series. The inflation series appear to be characterized by fractional behavior, with an order of integration clearly distinguishable from both zero and unity, especially for short horizons.

3.5 Multivariate Test Findings

The Robinson LPR test may be applied to a vector of time series, yielding a vector of $\hat{d}$ estimates, which may be tested for their joint equality via a standard F statistic. For each maturity and sample size (choice of m) , we have jointly estimated the long memory parameters for (the differences of) nominal rates and inflation, as well as the test statistic for their equality. The results, presented in Table 6, are quite striking. 38 of the 45 F-tests for equality reject the null at the five percent level, with none of the maturities failing to reject more than twice over the five sample sizes tested. Thus, the findings appear quite robust: when both (the changes in) nominal rates and inflation for a given tenor are modelled as fractional processes, the data decisively reject the hypothesis that their orders of integration are equal. In most cases, the long-memory parameter for the change in the nominal rate cannot be distinguished from zero, supporting the earlier findings of a unit root in nominal yields. In contrast, the long-memory parameter for the change in inflation can generally be distinguished from both zero and its counterpart for the change in the nominal yield. The strength of this evidence should cast serious doubt on the use of any methodology relying upon a shared order of integration for these two processes.

3.5.1 Comparison with Wu and Zhang findings on nominal yields

Further evidence on the dissimilarity of nominal yields' and inflation rates' stochastic properties may be developed through application of the Robinson LPR test with equality constraints. Wu and Zhang (1996) point out that previous researchers' failure to reject a unit root for nominal yields may be an artifact of the low power of univariate unit root tests. They propose employing a multivariate, or pooled, test of the hypothesis that a unit root is jointly present in all series. A similar test in the context of fractional integration may be performed with the Robinson LPR test, applying the linear constraints of coefficient equality across series. We tested one- to three-month nominal yields and one- to three-month inflation rate series under the maintained hypothesis of a common d coefficient across tenors.[14] The results support the findings presented in Table 6. The common d coefficient estimate on 1-3 month yields ranges from -0.003 to -0.133, and is distinguishable from the null of unit root behavior for two of the powers employed at 5%. The equivalent coefficient for inflation ranges from -0.194 to -0.503, distinguishable from zero at any level of significance. When the six coefficients are estimated jointly, subject to the four constraints that the yields share d_i and the inflation series share d_π, the F-test for equality of d_i and d_π rejects its null at any level of significance. Thus, although we cannot decisively reject Wu and Zhang's conclusion that the nominal yields are stationary (rather than unit root) processes, we can clearly distinguish their order of integration from that of the inflation rate series. We concur with Wu and Zhang's argument that "recent efforts devoted to the cointegration analysis...on the long-run Fisher relationship might not be appropriate..." (1996, p.606) and cannot dispute their conclusion that "...shocks to interest rates are found to be fairly persistent." (1996, p. 619) However, for cointegration (or fractional cointegration) analysis to proceed, it is the degree of persistence that matters. We find clear evidence of disparities.

3.6 Comparison with Crowder and Hoffman findings

To evaluate the robustness of our results, we applied the same battery of tests to the 1952-1991 quarterly sample employed by Crowder and Hoffman (1996).[15] We used the GDP implicit deflator from DRI Basic Economics (GD) rather than the implicit deflator for personal consumption expenditures (GDC) (since the latter is no longer available prior to 1959) and the three-month Treasury bill rate, FYGM3. Although all of the tests employed produce sharper findings on the larger monthly data set, their findings for this quarterly sample are broadly similar. The DFGLS test cannot reject the presence of a unit root in either series, while the KPSS test can reject stationarity in both series, analogous to the findings in Table 1. The MODLPR tests cannot reject $d = 1$ at the ten percent level in either series (contrary to the stronger findings in Table 3 for inflation) but the ROBLPR tests reject $d = 1$ for two of the powers considered for the inflation rate series. The multivariate tests can reject equality of the two series' d coefficients for three of the five power levels considered at the five percent level. Therefore, although the ability to detect fractional inte-

[14]For brevity, the detailed results are not tabulated, but are available on request from the authors.

[15]These results are not presented here in the interests of brevity, but are available from the authors on request.

gration via nonparametric methods in a considerably smaller sample is hindered, our results on the Crowder and Hoffman sample are consistent with those from the later monthly sample employed in earlier sections. There is evidence of fractional integration, particularly in the inflation series, casting question on application of cointegration methodologies that rely upon $I(1)$ processes.

4 Conclusions

We find clear evidence that the research strategy pursued in several recent contributions to the Fisher equation literature is flawed. The application of cointegration techniques, of either the Engle-Granger or Johansen types, depends crucially on the series being considered exhibiting unit root behavior. Tests of the Fisher equation that presume that nominal interest rates and inflation measures share the same unitary order of integration are in conflict with a growing body of evidence that inflation rates, in the U.S. and in other industrialized countries, are fractional processes which exhibit long memory behavior. In this case, the application of standard cointegration techniques is inappropriate.

We have demonstrated, using a variety of tenors of short-term Treasury yields and several recently developed tests for fractional integration, that a Fisher equation constructed from those yields and appropriate measures of inflation is a spurious regression. In the sense of Engle and Granger, the two variables cannot be shown to share the same order of integration. This finding of long-memory behavior in the inflation series casts serious doubt on the findings of those studies that have assumed that both variables may be treated as unit root processes.

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Table 1: DFGLS and KPSS tests for trend stationarity of nominal yield and inflation rate series

	DFGLS	*lags*	*KPSS* *6 lags*	*12 lags*
i_1	-2.078	12	0.768	0.439
i_2	-2.161	14	0.763	0.435
i_3	-2.125	14	0.769	0.438
i_4	-2.058	16	0.781	0.444
i_5	-2.015	16	0.793	0.450
i_6	-1.804	15	0.802	0.455
i_7	-1.801	15	0.811	0.460
i_8	-1.938	11	0.815	0.462
i_9	-1.911	11	0.819	0.464
π_1	-2.350	16	0.526	0.324
π_2	-1.662	15	0.529	0.322
π_3	-2.009	16	0.532	0.322
π_4	-1.658	16	0.535	0.321
π_5	-1.955	16	0.541	0.321
π_6	-2.079	15	0.548	0.323
π_7	-1.881	15	0.556	0.325
π_8	-1.445	16	0.562	0.326
π_9	-2.259	16	0.567	0.327

Notes: the 10% (5%) critical values for the DFGLS test range from -2.538 to -2.599 (-2.820 to -2.887), respectively, depending on lag order.
Lags refers to the lag order used in the DFGLS test, chosen by the Ng-Perron (1995) criterion.
The 10% and 5% critical values for the KPSS test are 0.119 and 0.146, respectively.

Table 2: Modified Log-Periodogram Regression tests for stationarity of nominal yield series

Power	*0.50*	*0.55*	*0.60*	*0.65*	*0.70*
i_1	1.136	1.114	0.909	0.831	0.846
	(0.356)	(0.253)	(0.393)	(0.067)	(0.053)
i_2	1.176	1.153	0.951	0.846	0.932
	(0.232)	(0.222)	(0.644)	(0.095)	(0.391)
i_3	1.269	1.225	1.011	0.874	0.946
	(0.067)	(0.073)	(0.920)	(0.172)	(0.496)
i_4	1.363	1.281	1.029	0.877	0.963
	(0.014)	(0.026)	(0.783)	(0.185)	(0.638)
i_5	1.319	1.261	1.043	0.883	0.963
	(0.030)	(0.038)	(0.688)	(0.205)	(0.644)
i_6	1.294	1.270	1.009	0.862	0.961
	(0.046)	(0.032)	(0.931)	(0.135)	(0.623)
i_7	1.255	1.228	0.974	0.845	0.960
	(0.083)	(0.069)	(0.807)	(0.094)	(0.612)
i_8	1.224	1.203	0.957	0.841	0.973
	(0.129)	(0.106)	(0.685)	(0.085)	(0.738)
i_9	1.192	1.166	0.938	0.831	0.949
	(0.192)	(0.187)	(0.563)	(0.068)	(0.525)

Notes: the MODLPR test (Phillips, 1999a) is applied to the levels of the series after removal of a linear trend. Power indicates the sample used:
T^{power} ordinates are included, where $T = 397$.
The P-value for $H_0 : d = 1$ is given in parentheses.

TABLE 3: MODIFIED LOG-PERIODOGRAM REGRESSION TESTS FOR STATIONARITY OF INFLATION RATE SERIES

Power	*0.50*	*0.55*	*0.60*	*0.65*	*0.70*
π_1	0.923	0.835	0.753	0.518	0.541
	(0.602)	(0.190)	(0.021)	(0.000)	(0.000)
π_2	0.941	0.845	0.753	0.528	0.571
	(0.687)	(0.219)	(0.021)	(0.000)	(0.000)
π_3	0.976	0.863	0.755	0.545	0.628
	(0.868)	(0.275)	(0.022)	(0.000)	(0.000)
π_4	1.002	0.882	0.774	0.585	0.722
	(0.987)	(0.349)	(0.035)	(0.000)	(0.000)
π_5	0.993	0.892	0.808	0.652	0.844
	(0.961)	(0.390)	(0.073)	(0.000)	(0.050)
π_6	0.990	0.909	0.856	0.746	1.044
	(0.945)	(0.470)	(0.177)	(0.006)	(0.575)
π_7	1.010	0.938	0.918	0.876	1.242
	(0.944)	(0.624)	(0.443)	(0.180)	(0.002)
π_8	1.033	0.973	0.971	1.059	1.250
	(0.824)	(0.830)	(0.789)	(0.523)	(0.002)
π_9	1.057	1.014	1.016	1.199	1.211
	(0.698)	(0.910)	(0.884)	(0.032)	(0.008)

Notes: the MODLPR test (Phillips, 1999a) is applied to the levels of the series after removal of a linear trend. Power indicates the sample used: T^{power} ordinates are included, where $T = 397$.
The P-value for $H_0 : d = 1$ is given in parentheses.

Table 4: Robinson Log-Periodogram Regression tests for stationarity of changes in nominal yield series

Power	*0.60*	*0.70*	*0.80*
Δi_1	-0.074	-0.147	-0.045
	(0.530)	(0.000)	(0.430)
Δi_2	-0.016	-0.055	-0.069
	(0.900)	(0.520)	(0.250)
Δi_3	0.040	-0.008	-0.042
	(0.77)	(0.930)	(0.510)
Δi_4	0.125	0.008	-0.002
	(0.91)	(0.930)	(0.970)
Δi_5	-0.015	0.051	0.004
	(0.870)	(0.670)	(0.960)
Δi_6	-0.035	0.005	-0.017
	(0.700)	(0.950)	(0.770)
Δi_7	-0.055	-0.011	-0.008
	(0.530)	(0.880)	(0.880)
Δi_8	-0.061	-0.001	0.023
	(0.470)	(0.990)	(0.690)
Δi_9	-0.062	-0.021	-0.006
	(0.470)	(0.760)	(0.910)

Notes: the ROBLPR test (Robinson, 1995) is applied to the first differences of the series.
Power indicates the sample used: T^{power} ordinates are included, where $T = 396$.
The P-value for $H_0 : d^* = 0$ ($d = 1$) is given in parentheses.

Table 5: Robinson Log-Periodogram Regression tests for stationarity of changes in inflation rate series

Power	*0.60*	*0.70*	*0.80*
$\Delta\pi_1$	-0.317	-0.459	-0.482
	(0.018)	(0.000)	(0.000)
$\Delta\pi_2$	-0.304	-0.428	-0.366
	(0.020)	(0.000)	(0.000)
$\Delta\pi_3$	-0.283	-0.362	-0.102
	(0.030)	(0.000)	(0.210)
$\Delta\pi_4$	-0.254	-0.265	0.345
	(0.048)	(0.014)	(0.001)
$\Delta\pi_5$	-0.215	-0.110	0.220
	(0.096)	(0.330)	(0.015)
$\Delta\pi_6$	-0.166	0.169	0.206
	(0.200)	(0.220)	(0.027)
$\Delta\pi_7$	-0.103	0.356	0.405
	(0.400)	(0.008)	(0.000)
$\Delta\pi_8$	-0.024	0.349	0.386
	(0.850)	(0.004)	(0.000)
$\Delta\pi_9$	0.081	0.379	0.278
	(0.510)	(0.004)	(0.002)

Notes: the ROBLPR test (Robinson, 1995) is applied to the first differences of the series.
Power indicates the sample used: T^{power} ordinates are included, where $T = 396$.
The P-value for $H_0 : d^* = 0$ ($d = 1$) is given in parentheses.

Table 6: Robinson Multivariate Log-Periodogram Regression for nominal yields and inflation rates

Power	*0.65*		*0.70*		*0.75*		*0.80*		*0.85*	
	i	π	i	π	i	π	i	π	i	π
1 mo	-0.16	-0.52	-0.15	-0.46	-0.12	-0.51	-0.05	-0.48	-0.03	-0.52
	(0.13)	(0.00)	(0.11)	(0.00)	(0.13)	(0.00)	(0.49)	(0.00)	(0.57)	(0.00)
F	6.29	(0.01)	6.02	(0.02)	13.73	(0.00)	21.99	(0.00)	39.05	(0.00)
2 mo	-0.14	-0.51	-0.05	-0.43	-0.07	-0.45	-0.07	-0.37	0.01	-0.27
	(0.20)	(0.00)	(0.56)	(0.00)	(0.35)	(0.00)	(0.31)	(0.00)	(0.89)	(0.00)
F	6.03	(0.02)	7.80	(0.01)	11.89	(0.00)	9.76	(0.00)	12.14	(0.00)
3 mo	-0.10	-0.48	-0.01	-0.36	-0.06	-0.32	-0.04	-0.10	0.01	0.21
	(0.36)	(0.00)	(0.93)	(0.00)	(0.44)	(0.00)	(0.57)	(0.17)	(0.85)	(0.04)
F	5.81	(0.02)	6.43	(0.01)	4.79	(0.03)	0.34	(0.56)	4.17	(0.04)
4 mo	-0.11	-0.42	0.01	-0.26	-0.05	-0.09	-0.00	0.35	0.03	0.14
	(0.29)	(0.00)	(0.93)	(0.01)	(0.55)	(0.29)	(0.98)	(0.00)	(0.67)	(0.05)
F	4.86	(0.03)	4.07	(0.05)	0.11	(0.74)	7.71	(0.01)	1.27	(0.26)
5 mo	-0.04	-0.35	0.05	-0.11	-0.03	0.28	0.00	0.22	0.04	0.24
	(0.73)	(0.00)	(0.66)	(0.34)	(0.76)	(0.00)	(0.97)	(0.01)	(0.60)	(0.00)
F	2.93	(0.09)	0.97	(0.33)	4.58	(0.03)	3.40	(0.07)	4.45	(0.04)
6 mo	-0.12	-0.26	0.01	0.17	-0.05	0.29	-0.02	0.21	0.05	0.30
	(0.21)	(0.00)	(0.96)	(0.14)	(0.61)	(0.00)	(0.83)	(0.01)	(0.44)	(0.00)
F	1.01	(0.32)	1.06	(0.31)	6.49	(0.01)	4.19	(0.04)	6.77	(0.01)
7 mo	-0.13	-0.11	-0.01	0.36	-0.06	0.20	-0.01	0.41	0.07	0.25
	(0.16)	(0.22)	(0.92)	(0.00)	(0.50)	(0.02)	(0.92)	(0.00)	(0.29)	(0.00)
F	0.02	(0.89)	6.11	(0.02)	4.47	(0.04)	13.17	(0.00)	3.52	(0.06)
8 mo	-0.13	0.14	-0.00	0.35	-0.05	0.18	0.02	0.39	0.09	0.35
	(0.19)	(0.17)	(1.00)	(0.00)	(0.53)	(0.03)	(0.76)	(0.00)	(0.14)	(0.00)
F	3.72	(0.06)	6.59	(0.01)	4.23	(0.04)	12.02	(0.00)	8.18	(0.00)
9 mo	-0.14	0.39	-0.02	0.38	-0.07	0.31	-0.01	0.28	0.08	0.28
	(0.20)	(0.00)	(0.84)	(0.00)	(0.38)	(0.00)	(0.93)	(0.00)	(0.20)	(0.00)
F	12.09	(0.00)	7.64	(0.01)	10.23	(0.00)	8.04	(0.01)	4.90	(0.03)

Notes: the multivariate form of the ROBLPR test (Robinson, 1995) is applied to the first differences of the series.
Power indicates the sample used: T^{power} ordinates are included.
P-values for $H_0 : d^* = 0$ are given below the coefficient estimates.
F is a test of $H_0 : d_i^* = d_\pi^*$, and its P-value is given in parentheses.