Understanding Macroeconomic Interdependence:  
Do We Really Need to Shut Off the Current Account? 

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Abstract

This paper develops a small open economy, sticky-price model with a role for current account dynamics in the transmission of shocks. I solve the stationarity problem of incomplete markets, open economy models by adopting an overlapping-generations structure. I model nominal rigidity by assuming that firms face costs of output price inflation volatility. Markup dynamics affect labor demand and investment decisions. To illustrate the functioning of the model, I identify the home economy with Canada and analyze how a recession in the U.S. is transmitted to Canada under alternative inflation targeting rules. Stabilizing inflation (in consumer or producer prices) at a steady-state target in all periods results in a milder, but more persistent recession than a rule under which the interest rate reacts to inflation in a Taylor fashion. Markup dynamics and changes in asset holdings are central to this result.

Keywords: Current account; Markup; Model stationarity; Small open economy; Sticky prices  
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1. Introduction

The current account plays a crucial role in the transmission of shocks in Obstfeld and Rogoff’s (1995) model of macroeconomic interdependence. But the absence of a well defined, endogenously determined steady state makes the conclusions questionable from a theoretical and empirical perspective. The consumption differential between countries follows a random walk. So do an economy’s net foreign assets. Whatever level of asset holdings materializes in the period immediately following a shock becomes the new long-run position, until a new shock happens.

Determinacy of the steady state and stationarity fail because the average rate of growth of consumption in the model does not depend on average holdings of net foreign assets. Hence, setting consumption to be constant does not pin down steady-state asset holdings. This makes the choice of the economy’s initial position for the purpose of analyzing the consequences of a shock a matter of convenience. When the model is log-linearized, one is actually approximating its dynamics around a “moving steady state.” The results of comparative statics are thus questionable. The reliability of the log-linear approximation is low, especially for analyses with a longer time horizon than a short-run exercise, because variables wander away from the initial steady state. The long-run non-neutrality of money that characterizes the results can be attacked on empirical grounds. Finally, sensible stochastic analysis is impossible: As Schmitt-Grohé and Uribe (2003) point out, the unconditional variances of endogenous variables are infinite, even if exogenous shocks are bounded.

This paper develops a small open economy, sticky-price model that solves the indeterminacy/non-stationarity issue by changing the demographic structure relative to the familiar representative agent framework. The model follows Weil (1989a, b) in assuming that the world economy is populated by distinct infinitely lived households that come into being on different dates and are born owning no assets. This demographic structure, combined with the assumption that newly born agents have no financial wealth, generates an endogenously determined steady state to which the world economy returns over time following non-permanent shocks.1

Scholars of international macroeconomics had soon recognized the indeterminacy/non-stationarity problem of the Obstfeld-Rogoff (1995) model. Some decided to dismiss it.2 Others tried to finesse the current account issue in various ways. For example, Corsetti and Pesenti (2001a) build on insights in Cole and Obstfeld (1991) and develop a version of the Obstfeld-Rogoff model in which the intratemporal elasticity of substitution between domestic and foreign goods in consumption is equal to one. Under this assumption, the current account does not react to shocks if the initial net foreign asset position is zero, and thus it plays no role in the international business cycle. The dynamics of the terms of trade are the centerpiece in Corsetti and Pesenti’s model. Their assumption makes it possible to solve the model without resorting to log-linearization and yields interesting theoretical insights. Several papers subsequently adopted the approach, including Benigno, P. (2003) and Obstfeld and Rogoff (1998, 2000).

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1 In the model, households consume; hold bonds and shares in firms; and supply labor. Thus, I extend the Weil framework to allow for endogenous labor supply and differences in income across agents of different generations at each point in time.
2 See Lane (2001) for a survey of the literature.
Nevertheless, the Corsetti-Pesenti model shares the indeterminacy of the steady state with the original Obstfeld-Rogoff framework. There too, setting consumption to be constant does not pin down steady-state international asset holdings, for the same reason mentioned above. The choice of a zero-asset initial equilibrium, combined with the assumption on the elasticity of substitution between foreign and domestic goods, allows Corsetti and Pesenti to (de facto) shut off the current account channel. This makes stochastic analysis possible in a highly tractable framework, but at a cost in terms of realism. Any initial position that differs from the zero-asset equilibrium brings the non-stationarity back to the surface. Recent empirical work by Lane and Milesi-Ferretti (2001; 2002a, b) provides strong evidence of non-zero, long-run net foreign assets for a number of countries. Therefore, the assumption that fluctuations happen around a steady state with non-zero assets is reasonable. In addition, the trade literature abounds with estimates significantly above one for the elasticity in question (see Feenstra, 1994; Harrigan, 1993; Lai and Trefler, 2002; Shiells, Stern, and Deardorff, 1986).

An alternative way of dealing with the non-stationarity problem by de-emphasizing the role of net foreign asset dynamics in the transmission of shocks consists of assuming that financial markets are internationally complete. With complete markets, power utility, and unitary elasticity of substitution between domestic and foreign goods, the current account does not react to shocks in two-country models with zero initial net wealth that are popular in the literature. If the elasticity of substitution between domestic and foreign goods differs from one, the current account moves in response to output differences (even though perfect risk sharing ensures that the cross-country consumption differential is zero if purchasing power parity—PPP—holds). However, history independence of the equilibrium allocation ensures that net foreign assets are determined residually and their dynamics play no active role in shock transmission. Benigno, G. (1999), Benigno and Benigno (2001), and Gali and Monacelli (2002) are examples of papers that rely on the complete markets assumption. This too yields highly tractable models suitable for stochastic analysis at a cost in terms of realism. As pointed out in Obstfeld and Rogoff (2001), the complete markets assumption is at odds with empirical evidence.

The Corsetti-Pesenti and the complete markets assumptions have been dominant in the recent theoretical literature, especially in the construction of models for policy evaluation. But there are reasons to believe that these approaches risk missing important features of economic interdependence. The recent dynamics of the U.S. current account suggest that the latter plays an important role in generating interdependence between the United States and the rest of the world. Corsetti, Dedola, and Leduc (2002) and Duarte and Stockman (2001) argue that market incompleteness is a necessary ingredient of models that aim to explain important puzzles in international finance. When one wants to assume that markets are incomplete without setting the elasticity of substitution between domestic and foreign goods to one, it is necessary to deal with the indeterminacy/non-stationarity issue in other ways.

Building on an earlier version of this paper, Ghironi (2000) illustrates the functioning of Weil’s (1989a, b) demographic structure in a flexible-price, two-country model. Cavallo and Ghironi (2002) argue that a monetary version of the model provides a reasonable starting point for analyzing interdependence between the U.S. and the rest of the world and explaining recent dynamics of the dollar exchange rate and U.S. net foreign assets.  

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3 Schmitt-Grohé and Uribe (2003) survey alternative solutions to the non-stationarity issue that still rely on representative agent models while preserving a role for the current account. I discuss these approaches below. Devereux (2003) and Smets and Wouters (2002) use a similar approach to that in this paper.
For the sake of simplicity, the model in Ghironi (2000) and Cavallo and Ghironi (2002) assumes logarithmic preferences and does not include physical capital as a factor of production. Yet, assuming unitary elasticity of intertemporal substitution can prove too restrictive on quantitative grounds, as it implies a constant consumption-to-wealth ratio. Also, including investment in physical capital is consistent with the development of a model that attributes a role to net foreign asset dynamics.

This paper adopts a more general specification of utility, which generates a time-varying consumption-to-wealth ratio and allows me to prove the generality of the properties of the Weil setup with regard to steady-state determinacy and model stationarity. In addition, I assume that monopolistically competitive firms produce output using labor and physical capital.

Firms face quadratic costs of adjusting prices, as in Rotemberg (1982). This generates a markup that is endogenous to the conditions of the economy.\(^4\) Markup dynamics play an important role in business cycle fluctuations, consistent with Rotemberg and Woodford (1990, 1999). The dynamics of the real wage are not tied to those of the marginal product of labor. In addition, markup fluctuations affect the investment decisions of firms. Thus, markup dynamics matter for net foreign asset accumulation both through their impact on labor demand and consumption and through their effect on investment.

The second part of the paper contains an application of the model. There, I identify the home economy with Canada, which is small and open compared to the rest of the world. I approximate the latter with the United States. The small open economy assumption implies that foreign variables and world aggregates are exogenous from the perspective of the domestic economy.

I illustrate the functioning of the model by using the parameter estimates in Ghironi (1999) to calibrate it and analyze the transmission of a recession in the U.S. to Canada under alternative inflation targeting rules. I combine the theoretical model of the Canadian economy with a simple VAR that traces the co-movements of U.S. variables affecting Canada directly. The exercise illustrates the role of markup, relative price, and net foreign asset dynamics in the model. Stabilizing inflation (in consumer or producer prices) at a steady-state target in all periods results in a milder, but more persistent recession in Canada than a rule under which the interest rate reacts to inflation in a Taylor (1993) fashion.\(^5\)

The structure of the paper is as follows. Section 2 presents the model. Section 3 analyzes the determination of steady-state net foreign assets. Section 4 illustrates the example. Section 5 concludes.

2. The Model

The world consists of two countries, home and foreign. I denote variables referring to the foreign economy with an asterisk and world variables with a superscript \(W\). In each period \(t\), the world economy is populated by a continuum of distinct, infinitely lived households between 0 and \(N_t^W\). Each of these households consumes; supplies labor; and holds money balances, bonds, and shares in firms. Following Weil (1989a, b), I assume that households come into being on

\(^4\) Carré and Collard (2003), Hairault and Portier (1993), and Ireland (1997, 2001) adopt a similar assumption. Roberts (1995) shows that the Rotemberg specification yields aggregate dynamics that are similar to those of the Calvo (1983) model.

\(^5\) Ghironi (1998) evaluates the welfare performance of alternative monetary rules for Canada when the Canadian economy is subject to several sources of volatility.
different dates and are born owning no financial assets or cash balances. \( N_t \) —the number of households in the home economy—grows over time at the exogenous rate \( n \), so that \( N_{t+1} = (1 + n)N_t \). I normalize the size of a household to 1, so that the number of households alive at each point in time is also the economy’s population. Foreign population grows at the same rate as home. I assume that the ratio \( N_t / N_t^* \) is sufficiently small that home’s population is small relative to the rest-of-the-world’s. The world economy has existed since the infinite past. It is convenient to normalize world population at time 0 to the continuum between 0 and 1, so that \( N_0^W = 1 \).

At time 0, the number of households in the world economy equals the number of goods that are supplied. A continuum of goods \( i \in [0, 1] \) is produced in the world by monopolistically competitive, infinitely lived firms, each producing a single differentiated good. Over time, the number of households grows, but the commodity space remains unchanged. Thus, as time goes, the ownership of firms spreads over a larger number of households. Profits are distributed to consumers via dividends. The structure of the market for each good is given. The domestic economy produces goods in the interval \([0, a]\), which is also the size of the home population at time 0. The foreign economy produces goods in the range \((a, 1]\). The constant ratio \( N_t / N_t^* \) equals \( a/(1-a) \). Thus, the assumption that \( N_t / N_t^* \) is small is sufficient to ensure that home produces a small share of the goods available for consumption in each period.

For simplicity, and consistent with the evidence of home bias in equity markets, I assume that only home (foreign) households hold shares of home (foreign) firms. Similarly, only home (foreign) households hold home (foreign) currency. Nominal, uncontingent bonds are the only internationally traded assets. Each country issues bonds denominated in units of the country’s currency.

**2.a. Households**

Households have identical preferences over a consumption index \( C \), leisure \( LE \), and real money balances \( M/P \), where \( P \) is the price deflator. They are endowed with one unit of time in each period, part of which they supply as labor effort \( L \) in a competitive labor market (hence, \( LE = 1 - L \)). At time \( t_0 \), the representative home consumer \( j \) born in period \( \nu \in [-\infty, t_0] \) maximizes the intertemporal utility function:

\[
U_{t_0}^{\nu,j} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ C_t^{\nu,j} \left( 1 - L_t^{\nu,j} \right)^{-\rho} \right\}^{\frac{1}{1-\rho}} \left( 1 - \frac{1}{\sigma} \right) + \chi \left( M_t^{\nu,j} / P \right)^{\frac{1}{\sigma}} \left( 1 - \frac{1}{\sigma} \right),
\]

where \( 0 < \rho < 1 \) and \( \chi \) and \( \sigma \) are strictly positive.\(^6\)

The consumption index for the representative domestic consumer born in period \( \nu \) is:

\(^6\) Among others, Sbordone (2001) has demonstrated the importance of non-separable preferences over consumption and leisure for quantitative business cycle models. I restrict the intertemporal elasticity of substitution in utility from money holdings to equal the elasticity of substitution in utility from consumption and leisure as this makes it possible to aggregate the money demand equation across generations easily. This is not a strong restriction, since money demand does not play a central role in this paper.
\[ C^{\alpha}_{t} = \left( \int_{0}^{a} c^{\alpha, i}(i)^{\frac{\theta - 1}{\theta}} di + \int_{a}^{\infty} c^{\alpha, i}(i)^{\frac{\theta - 1}{\theta}} di \right)^{\frac{\theta}{\theta - 1}}, \] (2)

with \( \theta > 1 \). \( c^{\alpha, i}(i) \) (\( c^{\alpha, i}(i) \)) is consumption of good \( i \) produced in the home (foreign) country.

Since \( a \) is small, the share of domestic goods in the consumption basket is small. I assume that foreign agents have identical preferences for consumption.\(^7\)

The assumptions that the domestic population is small relative to the rest-of-the-world’s, that the number of goods produced in the home economy is small, and that the relative weight of foreign goods in the consumption basket is large—combined with that of free international borrowing and lending—are equivalent to the assumption that home is a small open economy, which has a negligible impact on the rest of the world.

The price deflator for nominal money balances (\( P \)) is the consumption-based money price index, or consumer price index (CPI). Letting \( p_{i} (i) \) (\( p_{i} (i) \)) be the home (foreign) currency price of good \( i \), CPIs at home and abroad are, respectively:

\[ P_{t} = \left( \int_{0}^{a} p_{i} (i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \quad P_{t}^{*} = \left( \int_{0}^{a} p_{i}^{*} (i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \] (3)

There are no impediments to trade. If firms have no incentive to price discriminate across markets, as I assume, the law of one price holds for each individual good. Letting \( \varepsilon \) denote the domestic currency price of one unit of the foreign currency, it is \( p_{i} (i) = \varepsilon p_{i}^{*} (i) \).\(^8\)

Using the law of one price, and recalling that the home economy produces goods in the range between 0 and \( a \), makes it possible to verify that \( P_{t} = \varepsilon P_{t}^{*} \). Consumption-based PPP holds because consumption baskets are identical across countries and there are no departures from the law of one price.\(^9\)

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\(^7\) I assume the same elasticity of substitution, \( \theta \), across all goods, domestic and foreign, in both countries. Corsetti and Pesenti (2001a) assume unitary elasticity of substitution between domestic and foreign sub-baskets to remove current account fluctuations from the model. The trade literature suggests that values between 3 and 6 are reasonable for the elasticity of substitution between imports and exports. This is the same range that is reasonable for the elasticity that determines the steady-state markup of prices over marginal cost in the numerical exercise below. Hence, I assume the same elasticity of substitution between all goods to simplify the analysis, given the intention to keep a role for net foreign asset dynamics. See also Tille (2001) for a discussion of the consequences of monetary shocks in the presence of differences in substitutability between goods inside each country and between the baskets of goods that each country produces.

\(^8\) The price index for each country solves the problem of minimizing total spending evaluated in units of the country’s currency subject to the constraint that the real consumption index be equal to 1. The assumption that consumers born at different points in time have the same preferences ensures that firms have no incentives to price discriminate across consumers of different ages.

\(^9\) Much literature following Obstfeld and Rogoff (1995) focused on PPP as the main weakness of the setup and extended it to allow for PPP deviations. See, for example, Benigno, G. (1999), Corsetti and Pesenti (2001b), and Devereux and Engel (1998). PPP holds in Corsetti and Pesenti (2001a). The main purpose of this paper is to illustrate the role of net foreign asset and markup dynamics in a model that can be related to that in Corsetti and Pesenti (2001a) while having more explicit quantitative features. Therefore, I retain PPP to keep the analysis relatively simple and the comparison transparent. (PPP is also a standard, albeit implicit assumption of small open economy, international RBC models applied to Canada, such as those in Schmitt-Grohé and Uribe, 2003.)
Households are subject to lump-sum taxes $T^{LS}$, payable in units of the composite consumption good. I let $V_t^i$ denote the date $t$ price of a claim to the representative domestic firm $i$'s entire future profits (starting on date $t + 1$) in units of home currency. $x_{t+1}^{v,j}$ is the share of the representative domestic firm $i$ owned by the representative domestic consumer $j$ born in period $u$ at the end of period $t$. $D_t^i$ denotes the nominal dividends the firm issues on date $t$.

The consumer enters period $t$ holding nominal bonds issued in the two countries, nominal money balances, and shares purchased during $t - 1$. He or she receives interest and dividends on the assets, earns capital gains or losses on shares, earns labor income, is taxed, and consumes. Savings are divided between increases in bond holdings and in the value of shares and money balances to be carried into the next period. Letting $A_{t+1}^{u,j}$ ($A_{t+1}^{u,j}$) denote holdings of domestic (foreign) bonds entering time $t + 1$, the period budget constraint in units of domestic currency is:

$$A_t^{u,j} - A_t^{u,j} + \epsilon_t^a \left( A_t^{u,j} - A_t^{u,j} \right) + \int_{0}^{a} \left( V_t^i x_{t+1}^{u,j} - V_t^{j,i} x_{t+1}^{u,j} \right) dt + M_t^{u,j} - M_{t-1}^{u,j} =$$

$$= i_t A_t^{u,j} + \epsilon_t^a, A_t^{u,j} + \int_{0}^{a} D_t x_{t+1}^{u,j} dt + \int_{0}^{a} \left( V_t^i - V_t^{j,i} \right) x_{t+1}^{u,j} dt + W_t L_t^{u,j} - P_t C_t^{u,j} - P_t T_t^{LS,v} .$$

$i_t$ ($i_t^*$) is the nominal interest rate on bonds denominated in home (foreign) currency between $t - 1$ and $t$. $W_t$ is the nominal wage paid for one unit of labor, taken as given by workers. $M_t^{u,j}$ denotes holdings of nominal money balances entering period $t$. Newly born individuals are not linked by altruism to individuals born in previous periods. Hence, individuals are born owning no financial wealth or cash balances (although they are born owning the present discounted value of their labor income): $A_{t+1}^{u,j} = A_{t+1}^{u,j} = x_{t+1}^{u,j} = M_{t-1}^{u,j} = 0$. This assumption is crucial to ensure that the model has an endogenously determined steady state, to which the economy returns following temporary shocks.

Dropping the $j$ superscript, because symmetric agents make identical equilibrium choices, optimal supply of labor is determined by the labor-leisure tradeoff equation:

$$L_t^{u} = 1 - (1 - \rho) C_t^{u} / (\rho W_t / P_t) .$$

At an optimum, the marginal cost of supplying labor equals the marginal utility of consumption generated by labor income.

Euler equations for holdings of domestic and foreign bonds yield uncovered interest parity (UIP):

$$1 + i_{t+1} = \left( 1 + i_{t+1}^* \right) e_{t+1} / e_t .$$

Letting $r_t^{u+1}$ ($r_t^{u+1}$) denote the home (foreign) consumption-based real interest rate between $t$ and $t + 1$, Fisher parity conditions imply $1 + i_{t+1} = \left( 1 + r_t^{u+1} \right) P_{t+1} / P_t$ , $1 + i_{t+1}^* = \left( 1 + r_t^{u+1}^* \right) P_{t+1}^* / P_t^*$ . Thus, UIP and PPP yield real interest rate equalization: $r_t^{u+1} = r_t^{u+1}$ . Because home is small compared to the rest of the world, $i_{t+1}$ and the world real interest rate $r_t^{u+1}$ are exogenous to the home economy.

The Euler equation for holdings of domestic bonds reduces to:
C_t^{\nu} = \frac{1}{\beta^\sigma (1 + r_{t+1})^\sigma} C_{t+1}^{\nu} \left( \frac{W_t / P_t}{P_{t+1}} \right)^{(1-\rho)(1-\sigma)} , \; \nu \leq t . \quad (7)

Unless \sigma = 1, in which case period utility is additively separable in consumption and leisure, consumption growth depends on real wage growth. Depending on whether \sigma is smaller or larger than 1, real wages that grow over time introduce an upward or downward tilt in the path of consumption, respectively.

Combining the first-order condition for the optimal choice of \( x_{t+1}^{\nu} \) with equation (7) shows that consumers are indifferent between bonds and shares as long as the gross rate of return on shares equals the gross real interest rate:

\[
1 + r_{t+1} = \frac{V_{t+1}^{i} + D_{t+1}^{i}}{V_{t}^{i}} \frac{P_{t}}{P_{t+1}} . \quad (8)
\]

Finally, demand for real balances is:

\[
M_t^{\nu} = \left( \frac{\lambda}{\rho} \right)^\sigma C_t^{\nu} \left( \frac{1 + i_{t+1}}{i_{t+1}} \right)^\sigma \left( \frac{1 - \rho}{\rho W_t / P_t} \right)^{(1-\rho)(1-\sigma)} . \quad (9)
\]

Real balances increase with consumption and decrease with the opportunity cost of holding money. The impact of a higher real wage depends on \sigma. If \sigma < 1, a higher real wage causes demand for real balances to decrease for any given level of consumption.10

2.b. Firms

2.b.1. Output Supply

Production requires labor and physical capital (K). Capital is a composite good, with the same composition and elasticity of substitution as the consumption bundle. Output supplied by the representative domestic firm \( i \) at time \( t \) is:

\[
Y_t^{Si} = Z_t K_t^{i} \left( E_t, L_t^{i} \right)^{(1-\gamma)} . \quad (10)
\]

It is not necessary to index output production and factor demands by a “date of birth” because all firms in the world economy have existed since the infinite past. \( Z_t \) is an economy-wide, exogenous productivity shock, held at an initial steady-state value of 1 in the numerical exercise below. \( E_t \) is exogenous, worldwide, labor-augmenting technological progress, such that \( E_t = (1 + g) E_{t-1} \) (\( g \) will be the steady-state rate of growth of aggregate per capita output). I assume \( 1 + r > (1 + n)(1 + g) \), where \( r \) is the steady-state world real interest rate, to ensure stability.

Rotemberg and Woodford (1993) argue that, when competition is not perfect, it is important to consider materials explicitly as a distinct input from capital. In their model, material inputs are a basket of all goods in the economy, with the same composition as the consumption bundle. I do not consider materials as a distinct input for two reasons. First is the desire to keep the model simple. Second, I assume below that purchases of goods are necessary to install new capital and make it operational and for marketing reasons when prices change. This provides a channel through which materials affect costs, and thus production, even if they do not enter the

10 As usual, first-order conditions and the period budget constraint must be combined with appropriate transversality conditions to ensure optimality.
production function directly. Another difference relative to Rotemberg and Woodford’s model is that I do not allow for the possibility of increasing returns and stick to a constant returns Cobb-Douglas technology for simplicity.

2.b.2. Output Demand and Price Stickiness

Output demand comes from several sources. Maximizing $C^\nu$ subject to a spending constraint yields the demands of goods produced in the two countries by the representative home consumer born in period $\nu$: $c^\nu_i(h) = (p_i(h)/P_t)^{-\theta} C^\nu_i$ and $c^\nu_i(f) = (p_i(f)/P_t)^{-\theta} C^\nu_i$, respectively. Identity of preferences implies analogous expressions for foreign consumers’ demands.

At time $t$, total demand for home good $i$ coming from domestic consumers is:

$$c_i(t) = a \left[ \frac{n}{1+n} c_i^{-1}(i) + \frac{n}{1+n} c^0_i(i) + \frac{n}{1+n} c^\nu_i(i) \right] = \left( \frac{p_i(i)}{P_t} \right)^{-\theta} \left[ a \left( \frac{n}{1+n} C_i^{-1} + \frac{n}{1+n} C^0_i + \frac{n}{1+n} C^\nu_i \right) \right]$$

where $C_i$ is aggregate per capita home consumption of the composite basket:

$$C_i = a \left[ \frac{n}{1+n} C_i^{-1} + \frac{n}{1+n} C^0_i + \frac{n}{1+n} C^\nu_i + n(1+n) C_i^1 + \frac{n}{1+n} C_i^2 + \frac{n}{1+n} C_i^3 + \frac{n}{1+n} C_i^4 \right]. (12)$$

Similarly, total demand for home good $i$ by foreign consumers is:

$$c^*_i(i) = \left( \frac{p_i(i)}{P_t} \right)^{-\theta} \left[ (1-a)(1+n) C^*_i \right]$$

where $C^*_i$ is foreign aggregate per capita consumption.

Capital accumulation obeys:

$$K_{i+1} - K_i = I_i - \delta K_i. (14)$$

$I_i$ is investment and $\delta$ is the rate of depreciation. Investment is a composite index of all goods produced in the world economy, defined as the consumption index.

Adjusting the capital stock is costly. I assume that the firm must purchase materials in the amount $CAC_i = \eta I_i^2 \left/ \left(2K_i \right) \right.$ to install new capital and make it operational. The quantity $CAC$, measured in units of the composite consumption good, represents the real cost of adjusting the firm’s capital stock. The cost is convex in the amount of investment. Faster changes in the

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11 Vintage $\nu = 0$ of home consumers, born at time 0, has size $a$. Home population in the next period is $a(1+n)$, of which $an$ individuals are new-born. In the following period, population contains $N_2 - N_1 = an(1+n)$ individuals born in that period. Continuing with this reasoning shows that generation $t$ consists of $an(1+n)^{t-1}$ households. Going back in time from $t = 0$, population at time $t = -1$ is $a/(1+n)$. Hence, generation $0$ consists of $an/(1+n)$ households. And so on. Vintage $-t$ consists of $an/(1+n)^{t+1}$ households.
capital stock are accompanied by more than proportional increases in installation costs. A larger amount of capital in place reduces adjustment costs because larger firms can absorb a given amount of new capital at a lower cost. In more traditional versions of this investment model, the adjustment cost is measured as a reduction in the firm’s output due to the investment activity. Here, it is natural to think of investment as causing costs due to the need of purchasing a set of goods that are required to make the installation possible and new capital operational.

Changing the output price is another source of costs. I follow Rotemberg (1982) and assume a quadratic cost of output price inflation volatility around a steady-state level \( \bar{\pi} \). The real cost for firm \( i \) is \( PAC_i^t = \phi \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 - \bar{\pi} \right)^2 K^t_i / 2 \). This cost is measured in units of the composite good. When the firm changes the price of its output, it must purchase a set of material goods (new catalogs, price tags, etc.). We can think of \( PAC_i^t \) as the amount of marketing materials that the firm must purchase when implementing a price change. Because the amount of these materials is likely to increase with the size of the firm, the cost of adjusting the price increases with capital, taken as a proxy for size. The cost is convex in inflation. Faster price movements are more costly. More marketing activity is likely to be required to preserve demand from falling too much as a consequence of a large price increase. Symmetrically, a large price cut gives the firm incentives to do more marketing as a way of letting a larger fraction of the public know about the lower price.

Home firms face an identical degree of nominal rigidity domestically and abroad. Combined with the assumption of identical elasticity of substitution across goods in the two economies and absence of market segmentation, this assumption implies that, even if firms were allowed to set prices in the currency of consumers, the law of one price would emerge as an equilibrium outcome (see Benigno, G., 1999). Hence, I do not consider local currency pricing as a source of deviations from the law of one price and PPP.12

Total demand of good \( i \) produced in the home country follows from adding the demands for that good originating in the two countries. It is:

\[
Y^t_{\hat{DW}} = \left( \frac{p_t(i)}{P_t} \right)^{\theta} \hat{Y}^t_{DW},
\]

where \( \hat{Y}^t_{DW} \) is aggregate (as opposed to aggregate per capita) world demand of the composite good, \( \hat{Y}^t_{DW} \equiv \hat{C}_i^w + \hat{I}_i^w + \hat{C}AC_i^w + \hat{PAC}_i^w \). (Where useful for clarity, I use a “hat” to denote aggregate levels of variables and differentiate the notation from that for aggregate per capita levels.)

2.b.3. Optimality Conditions

Equation (8) implies that, on date \( t_0 \),

\[
\frac{V^t_{t_0}}{P_{t_0}} = \frac{D^t_{t_0+1}}{P_{t_0+1}} + \frac{V^t_{t_0+1}}{P_{t_0+1}} + \frac{1}{1 + r_{t_0+1}}.
\]

Forward iteration in the absence of speculative bubbles yields:

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12 Benigno, G. (1999), Betts and Devereux (2000), Corsetti and Pesenti (2001b), and Devereux and Engel (1998), among others, propose models of interdependence that allow for local currency pricing. Engel and Rogers (1996) provide evidence of deviations from the law of one price between the U.S. and Canada, the economies on which I focus in the numerical exercise below. This notwithstanding, I limit myself to the simpler case to focus on other aspects of the model.
\[
V_{i_0}^t / P_{i_0} = \sum_{t'=0}^{\infty} R_{i_0,t'} \left( D_{i_0}^t / P_t \right),
\]  

where \( R_{i_0,t} \equiv 1 / \prod_{u=t_0}^{t-1} \left( 1 + r_u \right) \). Equation (17) states the familiar result that a firm’s market value on date \( t_0 \) is the present discounted value of the dividends the firm will issue starting on date \( t_0 + 1 \).

The real dividends paid by the representative domestic firm in period \( t \) are equal to revenues—\( \frac{p_t(i)}{P_t} Y_t^i \)—minus costs—\( \frac{W_t}{P_t} L_t + I_t^i + \frac{\eta (I_t^i)^2}{2 K_t^i} + \frac{\phi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 - \pi \right) K_t^i \) . Hence, the present discounted value of current and future real dividends at time \( t_0 \) is:

\[
\sum_{t=t_0}^{\infty} \left[ \frac{p_t(i)}{P_t} Y_t^i - \left( \frac{W_t}{P_t} L_t + I_t^i + \frac{\eta (I_t^i)^2}{2 K_t^i} + \frac{\phi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 - \pi \right) K_t^i \right) \right],
\]  

where \( R_{i_0,t_0} \) is interpreted as 1. The representative firm chooses the price of its product, labor, investment, and capital to maximize this expression subject to the constraints (10), (14), (15), and the market clearing condition \( Y_t^i = Y_t^{ST} = Y_t^{DR} \). The firm takes the wage, the aggregate price index, \( Z, E \), and world aggregates as given.

The first-order condition with respect to \( p_t(i) \) returns the pricing equation:

\[
p_t(i) = \Psi_t^i P_t \lambda_t^i.
\]  

The price of good \( i \) equals the product of the (nominal) shadow value of one extra unit of output—the (nominal) marginal cost, \( P_t \lambda_t^i \)—times a markup—\( \Psi_t^i \). Symmetric home firms make identical choices in equilibrium. Therefore, \( p_t(i) \) is also the home economy’s producer price index (PPI).

The markup \( \Psi_t^i \) depends on output demand as well as on the impact of today’s pricing decision on today’s and tomorrow’s cost of adjusting the output price:

\[
\Psi_t^i \equiv \theta Y_t^i \left( \theta - 1 \right) Y_t^i + \phi \left[ \frac{K_t^i}{p_t(i)} \left( \frac{p_t(i)}{p_{t-1}(i)} - 1 - \pi \right) + \frac{K_{t+1}^i}{p_t(i)} \left( \frac{p_{t+1}(i)}{p_t(i)} - 1 - \pi \right) \right]^{-1}.
\]  

If \( \phi = 0 \) (if prices are flexible), \( \Psi_t^i \) reduces to \( \theta / (\theta - 1) \), the familiar constant-elasticity markup.

Introducing price rigidity generates endogenous fluctuations of the markup in the presence of a constant elasticity specification of demand that is consistent with the assumptions of Rotemberg and Woodford’s (1993) basic model, where the markup is constant. Because \( \Psi_t^i \) depends on \( p_{t+1}(i) \), \( p_t(i) \), and \( p_{t-1}(i) \) (as well as on \( Y_t^i \)) if \( \phi \neq 0 \), equation (19) defines \( p_t(i) \) implicitly as the solution to a second-order, non-linear difference equation. It is possible to verify that, if it is optimal to have relatively higher (weighted) price inflation today than tomorrow, the firm will find it optimal to react to an increase in demand today by raising its

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13 I keep the superscript \( i \) for individual firm variables in this section because later I denote the aggregate per capita levels of some of these variables by dropping the superscript.
markup. If instead today’s optimal inflation is lower than tomorrow’s, an increase in demand will be accompanied by a decrease in the markup. The introduction of nominal price rigidity in a constant-elasticity framework that would otherwise be characterized by a constant markup generates predictions that resemble those of the implicit collusion model of Rotemberg and Woodford (1990).

If $\theta$ approaches infinite, firms have no monopoly power, and the markup reduces to the competitive level—1—regardless of the (finite) value of $\phi$. Under perfect competition, the presence of a cost of adjusting the price level is de facto irrelevant for the firm’s decisions. Some degree of monopoly power is necessary for nominal rigidity to matter. When the elasticity of substitution across goods is finite, $\Psi_i > 1$ as long as the real net revenue from output sale is larger than the real marginal cost of a price change, a condition that must be satisfied for the firm to be optimizing.

The interested reader can verify that, once the numerator and denominator of the markup expression in equation (20) are written in terms of detrended, aggregate per capita quantities (which converge to well defined steady-state levels—see below), log-linearizing equation (20) around the steady state with inflation $\pi$ yields a New Keynesian Phillips curve of familiar form that relates the current markup to current and future PPI inflation (see equation (44) below). In a closed economy, PPI and CPI coincide in equilibrium, and the New Keynesian Phillips curve can be rewritten in terms of current marginal cost and current and future inflation using the fact that the markup is the reciprocal of marginal cost. In an open economy, the New Keynesian Phillips curve can be rewritten in terms of PPI inflation, marginal cost, and the relative price of the representative domestic good, $RP_i = p_i(i)/P_t$, which can be taken as a measure of the economy’s terms of trade. Note that monopoly power of home firm $i$ over its product implies that, even if the home economy is a small open economy, the terms of trade are endogenous to domestic economic developments.

The first-order condition for the optimal choice of $L^i_t$ yields:

$$\lambda^i_t (1 - \gamma) Y^i_t / L^i_t = W^i_t / P_t.$$  (21)

At an optimum, the real wage index must equal the shadow value of the extra output produced by an additional unit of labor.

In a model in which firms have no market power and they take the price as given, labor demand is determined by the familiar equality between the real wage index—$W^i_t / P_i$—and the marginal product of labor in units of the composite good—$(p^i_i(i)/P_t)(1 - \gamma)(Y^i_t / L^i_t)$. Here, the combination of pricing and labor demand yields:

---

14 I am implicitly assuming that all variables in the definition of $\Psi^i_t$ have finite limit values as $\theta$ approaches infinite.


16 The terms of trade are actually given by $p^*_i (i)/\left(\varepsilon, p^*_i (f)\right)$, where $p^*_i (f)$ is the foreign PPI. Under the assumptions of this paper, the fraction of domestic goods in the world consumption bundle is negligible. Hence, $p^*_i (f)$ is only marginally different from $P^*_t$. Because of purchasing power parity, $p^*_i (i)/\left(\varepsilon, P^*_t\right) = p^*_i (i)/P_t$. See also Benigno and Benigno (2001).
\[ \frac{W_i}{P_i} = \frac{p_i(i)}{P_i \Psi_i} (1-\gamma) \frac{Y_i^L}{L_i}, \tag{22} \]

which reduces to the familiar condition that determines labor demand in a competitive framework when \( \theta \) approaches infinite. The presence of monopoly power introduces a wedge between the real wage index and the marginal product of labor. Because \( \Psi_i > 1 \) when \( \theta \) is finite, monopoly power causes firms to raise the marginal product of labor above the real wage, \( i.e., \) to demand less labor than they would under perfect competition, as in Rotemberg and Woodford (1993). The wedge between the real wage index and the marginal product of labor reflects also the presence of costs of adjusting the price level. A finite value of \( \theta \) causes price stickiness to have a direct effect on labor demand, which would disappear if firms had no monopoly power.

The cyclical behavior of the markup and its impact on labor demand are consistent with Rotemberg and Woodford’s (1990, 1999) result that markup variations play an important role in business cycle fluctuations. This is an argument that has become central in the recent New Keynesian literature (Goodfriend and King, 1997; Woodford, 2003). When the markup can move with the business cycle, the real wage is not tied to the marginal product of labor. As a consequence, the real wage can in principle adjust procyclically to shocks, letting the markup move in a countercyclical fashion. Changes in the markup transmit and propagate shocks by acting as changes in a tax on the demand for labor (and capital—see below).

The first-order condition for the optimal choice of \( I_i^t \) implies that firm \( i \)'s investment is positive if and only if the shadow value of one extra unit of capital in place at the end of period \( t \)—\( q_i^t \)—is larger than 1:

\[ I_i^t = K_i^t (q_i^t - 1)/\eta. \tag{23} \]

\[ q_i^t \] obeys the difference equation:

\[ q_i^t = \left( \frac{1}{1 + r_{i+1}} \right) \left[ q_{i+1}^t (1-\delta) + \frac{W_{i+1}^t}{P_{i+1}} \frac{\gamma}{1-\gamma} \frac{L_i^{t+1}}{K_i^{t+1}} + \eta \left( \frac{I_i^{t+1}}{K_i^{t+1}} \right)^2 - \phi \left( \frac{p_{i+1}(i)}{p_i(i)} - 1 - \pi \right)^2 \right]. \tag{24} \]

The shadow price of one unit of capital in place at the end of period \( t \) is the discounted sum of the shadow price of capital at time \( t + 1 \) net of depreciation, the shadow value of the incremental output generated by capital at \( t + 1 \), and the marginal contribution of capital in place at the end of period \( t \) to the costs of installing capital and changing the price of the firm’s output at time \( t + 1 \). Solving this equation in the absence of speculative bubbles yields:

\[ q_i^t = \sum_{s=t+1}^{\infty} R_{i,s} (1-\delta)^{-s-1} \left[ \frac{W_s}{P_s} \frac{\gamma}{1-\gamma} \frac{L_i^s}{K_i^s} + \eta \left( \frac{I_i^s}{K_i^s} \right)^2 - \phi \left( \frac{p_s(i)}{p_{i+1}(i)} - 1 - \pi \right)^2 \right]. \tag{25} \]

The shadow value of an additional unit of capital installed during period \( t \) equals the present discounted value of its marginal contributions to production and costs.

It is possible to show that an alternative expression for \( q \) is:

\[ q_i^t = \left[ \frac{V_i^t}{P_i} + \sum_{s=t+1}^{\infty} R_{i,s} \left( \frac{1}{\Psi_i^t} - 1 \right) \frac{p_s(i)}{P_s} Y_s^S \right] / K_i^{t+1}. \tag{26} \]

This is the result first obtained by Hayashi (1982). The ratio of real equity to capital—\( (V_i^t / P_i) / K_i^{t+1} \)—is the so-called average \( q — q^{AVG}_i \). Under perfect competition (when \( \theta \) is
infinite), the markup reduces to 1, and marginal and average $q$ coincide. When firms have monopoly power, the markup is higher than 1, and marginal $q$ is smaller than average $q$. The shadow value of an additional unit of capital installed at the end of period $t$ is smaller under monopolistic competition because a larger capital stock causes production to increase and the output price to decrease. This conflicts with a monopolist’s incentive to keep the price higher and supply less output than optimal in the absence of monopoly power. Markup fluctuations affect investment decisions by generating fluctuations in the discrepancy between average and marginal $q$.

2.c. Monetary Policy and Markup Dynamics

Monetary policy is conducted by setting the nominal interest rate $i_{t+1}$ according to interest setting rules discussed below. Different rules generate different CPI inflation volatility. Because firms react to CPI dynamics in their price setting (equation (19)), different CPI inflation volatility translates into different volatility in producer prices and the markup (equation (20)). The more volatile the latter, the more volatile labor demand and investment (equations (22), (23), and (26)). Labor market equilibrium and the labor-leisure tradeoff tie labor demand to consumption dynamics. Hence, volatile inflation ends up causing consumption to be more volatile via its impact on the markup. Put differently, alternative policy rules can produce different dynamics for the real economy by causing differences in the behavior of the relative price of the representative domestic good, $RP_t$. As observed in Goodfriend and King (1997), markup volatility could be removed completely by stabilizing producer price inflation at the steady-state level $\bar{\pi}$. Markup fluctuations—and changes in asset positions—are the main channel through which different monetary regimes have different implications for business cycles and welfare in a small open economy in which the domestic real interest rate is tied to the world rate, which is exogenous to the home economy.

3. Steady State Determinacy and Model Stationarity

The absence of a well-defined, endogenously determined steady state is one of the main problems in the benchmark model proposed by Obstfeld and Rogoff (1995, 1996 Ch. 10). The choice of the initial point for the purpose of performing comparative statics exercises is a matter of convenience. All shocks have permanent consequences via redistribution of wealth across countries regardless of their nature (real or nominal, temporary or permanent), a result that is debatable on empirical grounds. From a technical point of view, the models are log-linearized around an initial steady state to which the economy never returns. This raises suspicions on the reliability of the approximation.

De facto, indeterminacy of the steady state and non-stationarity preclude any sensible stochastic application of the framework, unless the current-account channel of international interdependence is de-emphasized. As mentioned in the Introduction, this can be accomplished either by assuming unitary intratemporal elasticity of substitution across domestic and foreign goods—along the lines of Corsetti and Pesenti (2001a)—or by assuming that financial markets

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17 The government budget constraint simply requires the government to rebate seignorage revenue to consumers via lump-sum transfers.

18 To be precise on this point, the ex ante home real interest rate is tied to the ex ante world real rate. Ex post real returns can differ in the period in which an unexpected shock happens. This provides an additional channel through which home monetary policy can affect the economy. See Section 4 on this.
are complete. Obstfeld and Rogoff (1998, 2000) rely on the Corsetti-Pesenti assumption when extending their model to the stochastic case. Benigno, G. (1999) and Galí and Monacelli (2002) resolve the stationarity issue by assuming that markets are complete. In all these models, net foreign asset dynamics play no role in shock transmission. The realism of the Corsetti-Pesenti hypothesis is an empirical issue. Estimates from the trade literature point to values above one for the elasticity in question. Complete markets appear at odds with the evidence. Regardless, the current account is an important channel of international interdependence for several economies.

Stationarity fails for an open economy whenever the equilibrium rate of aggregate per capita consumption growth is independent of the economy’s aggregate per capita net foreign assets. In that case, the requirement that consumption be constant in steady state does not determine a unique steady state for net foreign assets. The steady state of the model presented here is determined endogenously by the structural parameters and is stable, if appropriate conditions are satisfied. In the model, aggregate per capita consumption growth depends on aggregate per capita net foreign assets because of the discrepancy between the financial wealth of the newly born—zero—and the aggregate per capita financial wealth of those already alive. In this section, I focus on the determination of the constant steady-state level of home consumers’ detrended, aggregate per capita, real net asset holdings, $b_t ≡ B_t / E_t P_{t,1}$.

1

$B_{t+1} = A_{t+1} + \epsilon_t A_{t+1} + a V_t x_{t+1}$. $B_{t+1}$ denotes aggregate per capita household assets entering $t+1$.

The analysis clarifies how the demographic structure of the model and the assumption that newly born households have no financial wealth (so that Ricardian equivalence does not hold) play a crucial role for determinacy of the steady state.19

The derivation of a law of motion for consumers’ aggregate per capita assets that takes the optimal path of consumption into account is more complicated in this model—in which agents’ labor supply is governed by a labor-leisure tradeoff equation—than in Weil’s (1989a, b) or Obstfeld and Rogoff’s (1996 Ch. 3.7), where labor income is exogenous. If income is exogenous, one can assume that agents of different generations have identical income at each point in time.20 Under this hypothesis, aggregate per capita income at each point in time is equal to each individual household’s income. But assuming identical incomes for agents of different ages would be wrong here. Given that all agents face the same wage rate, the assumption would imply that agents of different generations are supplying the same amount of labor. By the labor-leisure tradeoff condition, this would require agents born at different dates to have identical consumption levels, which cannot be true, given that agents of different generations have accumulated different amounts of assets. The impossibility of constant labor income across generations complicates the solution of the model. This notwithstanding, the complications can be dealt with by making use of the consumption-Euler equation and the labor-leisure tradeoff condition.

The Euler equation for consumption implies that, in all periods after an initial shock,

$$C_s^\nu = \beta^{\sigma(1)} \left( R_{t,s} \right)^{-\sigma \left( \left( W_s / P_s \right) / \left( W_{t,s} / P_{t,s} \right) \right)^{1-p}} \sigma^{\nu(1)} C_t^\nu, \quad s > t. \quad (27)$$

Combining the Euler equation for consumption with the labor-leisure tradeoff yields an Euler equation for leisure:

19 I assume that the process for the nominal interest rate $i$ converges to a steady state and that shocks to productivity—$Z_t$—are distributed around a steady-state value of 1.

20 This is Weil’s (1989a, b) and Blanchard’s (1985) assumption, as well as Obstfeld and Rogoff’s (1996 Ch. 3.7).
\[ 1 - L^\nu_t = \beta^{-\sigma} (1 + r_{t+1})^{-\sigma} \left( \frac{W_t}{P_t} \frac{P_{t+1}}{W_{t+1}} \right)^{(1-\rho)(1-\sigma)^{-1}} \left( 1 - L^\nu_{t+1} \right). \] (28)

Hence,
\[ L^\nu_s = \beta^{\sigma(s-t)} \left( R_{t,s} \right)^{-\sigma} \left( \frac{W_t}{P_t} \frac{P_s}{W_s} \right)^{(1-\rho)(\sigma-1)^{-1}} \left( 1 - \beta^{\sigma(s-t)} \left( R_{t,s} \right)^{-\sigma} \left( \frac{W_t}{P_t} \frac{P_s}{W_s} \right)^{(1-\rho)(\sigma-1)^{-1}} \right), \quad s > t. \] (29)

In real terms, the (equilibrium) budget constraint for the representative home agent born in period \( \nu \) is: 21
\[ B^\nu_{t+1}/P_t = \left( 1 + r_t \right) B^\nu_t/\left( P_{t-1} + W_t L^\nu_t/P_t \right) - C^\nu_t \] (30)

Assuming that transversity and no-Ponzi-game conditions are satisfied, the intertemporal budget constraint is thus:
\[ \sum_{s=t}^{\infty} R_{t,s} C^\nu_s = \left( 1 + r_t \right) B^\nu_t/\left( P_{t-1} + \sum_{s=t}^{\infty} R_{t,s} W_s L^\nu_s/P_s \right) \] (31)

I now define:
\[ \Theta_t \equiv \sum_{s=t}^{\infty} \beta^{\sigma(s-t)} \left( R_{t,s} \right)^{-\sigma} \left( \frac{W_t}{P_t} \frac{P_s}{W_s} \right)^{(1-\rho)(\sigma-1)^{-1}}. \] (32)

Note that \( \Theta_t = 1/(1 - \beta) \) in the case of logarithmic utility \((\sigma = 1)\).

Substituting (27) and (29) into (31) and using (32) yields:
\[ C^\nu_t = \Theta_t^{-1} \left[ \left( 1 + r_t \right) B^\nu_t/\left( P_{t-1} + \sum_{s=t}^{\infty} R_{t,s} W_s/P_s \right) \right] - \left( 1 - L^\nu_t \right) W_t/P_t. \] (33)

Once the Euler equations for consumption and labor supply are taken into account, today’s consumption depends positively on today’s assets and on the present discounted value of the household’s lifetime endowment of time in terms of the real wage—human wealth. Consumption depends negatively on the real value of today’s leisure. \( \Theta_t^{-1} \) is the (time-varying) slope of the consumption function that relates consumption to asset holdings and human wealth. Slightly rearranging equation (33) gives:
\[ C^\nu_t + \left( 1 - L^\nu_t \right) W_t/P_t = \Theta_t^{-1} \left[ \left( 1 + r_t \right) B^\nu_t/\left( P_{t-1} + \sum_{s=t}^{\infty} R_{t,s} W_s/P_s \right) \right]. \] (34)

This equation shows that \( \Theta_t^{-1} \) can be interpreted as a generalized propensity to consume goods or leisure out of the expected path of the agent’s resources.

Substitution of (33) into (30) yields:
\[ B^\nu_{t+1}/P_t = \left( 1 + r_t \right) \left[ 1 - \Theta_t^{-1} \right] B^\nu_t/\left( P_{t-1} + W_t/P_t - \Theta_t^{-1} \sum_{s=t}^{\infty} R_{t,s} W_s/P_s \right). \] (35)

This equation expresses asset accumulation by the representative household born in generation \( \nu \) as a function of the time paths of the real wage and the real interest rate, which do not depend on the household’s date of birth. Equation (35) provides a generalization of the results in Weil (1989a, b) and Obstfeld and Rogoff (1996 Ch. 3.7)—as well as of Blanchard’s (1985) findings—to the case of endogenous labor income and time-varying consumption-to-wealth ratio.

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21 Equation (30) holds in all periods after an initial shock. At the time of the shock, no-arbitrage conditions that are used to obtain (30) may be violated ex post.
Applying the aggregation procedure in Section 2.b.2 to equation (35) (recalling that households are born holding no assets) and dividing both sides by trend productivity growth $E_t$, yields the following law of motion for detrended, aggregate per capita, real assets held by home consumers:

$$b_{t+1} = \frac{(1 + r_t)\{1 - \Theta_t^{-1}\}}{(1 + g)(1 + n)} b_t + \frac{w_t - \Theta_t^{-1} \sum_{s=t}^\infty R_{t,s}(1 + g)^{-s}w_s}{(1 + g)(1 + n)},$$

(36)

where $\Theta_t$ has been re-defined as $\Theta_t \equiv \sum_{s=t}^\infty \beta^{s-i}(R_{t,s})^{1-\sigma}(1 + g)^{(1-\rho)(1-\sigma)(s-i)}(w_s/w_s)$ and

$$w_t \equiv W_t/(E_tP_t).$$

If the (time-varying) slope coefficient is smaller than 1 and the forcing function (which depends on the path of interest rate and real wage) converges to a finite value, home consumers’ assets, $b_t$, converge to a steady-state level starting from any initial position.22

The steady state of the home economy is characterized by a constant detrended real wage, determined by labor market clearing, and by a constant real interest rate $r$, determined abroad. Assuming $[\beta(1 + r)]^\sigma (1 + g)^{(1-\rho)(1-\sigma)} < (1 + n)(1 + g)$, the steady-state level of detrended, aggregate per capita assets accumulated by home consumers as a function of the steady-state real wage and interest rate is:

$$\bar{b} = \frac{\beta^\sigma (1 + r)^\sigma (1 + g)^{(1-\rho)(1-\sigma)} - (1 + g)}{(r - g)[(1 + n)(1 + g) - \beta^\sigma (1 + r)^\sigma (1 + g)^{(1-\rho)(1-\sigma)}]}W.$$

(37)

Home consumers are net creditors in detrended, aggregate per capita terms if $[\beta(1 + r)]^\sigma (1 + g)^{(1-\rho)(1-\sigma)} > 1 + g$. They are net debtors if $[\beta(1 + r)]^\sigma (1 + g)^{(1-\rho)(1-\sigma)} < 1 + g$. To gain intuition on this result, consider the case in which $\sigma = 1$ and $g = 0$. Suppose also that the rest-of-the-world economy has already completed the transition to a steady-state position when the situation at home is taken into consideration, i.e., the world real interest rate is constant and equal to $r$ along the path to home’s steady state.23 To simplify the argument further, suppose that the real wage is already constant at its steady-state level. The law of motion of home consumers’ aggregate per capita assets reduces to:

$$\frac{B_{t+1}}{P_t} = [\beta(1 + r)/(1 + n)]B_t/P_{t-1} + \left\lfloor \frac{\beta(1 + r) - 1}{r(1 + n)} \right\rfloor \bar{W}.$$

(38)

If $\beta(1 + r)/(1 + n) < 1$, a steady-state level of real aggregate per capita assets exists and is stable. For this steady-state level to be positive, the intercept of the linear relation between $B_{t+1}/P_t$ and $B_t/P_{t-1}$ must be positive. Under the assumptions of the special case we are considering, $\beta(1 + r)$ is the slope of the time path of individual consumption. When $\beta(1 + r) > 1$, individual consumption is increasing over time. If income were exogenous (as in Blanchard, 1985; Weil, 1989a; b; and Obstfeld and Rogoff, 1996 Ch. 3.7) one could assume that agents of different generations have the same income at each point in time. Under the assumption of constant

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22 I assume that the conditions ensuring convergence are satisfied.

23 $r$ is determined by the structural characteristics of the foreign economy. The assumption that the latter is already in steady state, whereas home is not, is not innocuous in general. It can be made here because the disparity in the size of the economies ensures that changes in domestic variables over time have no impact on foreign ones. If the economies were of comparable size, it would be necessary to analyze the simultaneous convergence of the two economies to the steady state, because home variables would affect foreign ones.
individual labor income, for agents’ consumption to be increasing over time, it must be the case that households are accumulating financial assets. Hence, the steady state (existence and uniqueness of which is ensured by population growth and the assumption that newborn agents have no financial assets) must be characterized by positive aggregate per capita consumer assets, since no individual has negative asset holdings.

In a framework in which labor income is endogenous, individual labor income is not constant even when aggregate per capita income is, because agents of different generations supply different amounts of labor. When income is not constant, one can think of situations in which individual consumption increases over time while assets are being decumulated, for example, depending on the agent’s age. However, this is not the case in the steady state of the model. In fact, taking the Euler equation for labor supply into account removes the (direct) dependence of an agent’s accumulation of assets on the quantity of labor supplied (which depends on the individual’s date of birth) and shows that equilibrium asset accumulation is a function of the real wage alone (which does not depend on the individual’s age). When the economy is in steady state, individual asset accumulation follows:

\[
\frac{B_{i,t}}{P_i} = \beta (1 + r) \frac{B_{i,t-1}}{P_{i-1}} + \left[ \beta (1 + r) - 1 \right] r \bar{w},
\]

(39)

which shows that \( \beta (1 + r) > 1 \) is sufficient to ensure that the household’s assets are increasing over time regardless of the household’s date of birth. The intuition is clear if we look at the Euler equation for labor supply. The rate of change of an individual’s supply of labor between any two periods during which the economy is in steady state is:

\[
\frac{(\bar{T}_{i,t} - \bar{T}_{i,t-1})}{\bar{T}_{i,t}} = \left[ \beta (1 + r) - 1 \right] \left( 1 - \bar{T}_{i,t} \right) / \bar{T}_{i,t},
\]

(40)

which is negative if \( \beta (1 + r) > 1 \). Because labor income is declining over time in steady state, the household accumulates assets in order to sustain an increasing consumption. The individual consumption-tilt factor \( \beta (1 + r) - 1 \) determines whether or not the country’s consumers are creditors or debtors in steady state. If individual consumption is increasing over time, the consumers are net creditors in the long run. Else, they run a debt.

The result is robust to the adoption of a more general isoelastic utility function, in which \( \sigma \) is different from 1, and to the introduction of productivity growth. \( \frac{\beta^\sigma (1 + r)^\sigma (1 + g)^{(1-\rho)(1-\sigma)}}{(1 + g)} \) is the slope coefficient of a household’s consumption path under the assumption that the real wage and the real interest rate are constant. As in the simpler case, this expression determines also the tilt of labor supply. Again, a household’s consumption can increase over time in steady state only if the household is accumulating assets. \( \frac{\beta^\sigma (1 + r)^\sigma (1 + g)^{(1-\rho)(1-\sigma)}}{(1 + n)(1 + g)} \) is the slope coefficient of the law of motion for detrended, aggregate per capita, real asset holdings in this situation. If this coefficient is smaller than 1, i.e., if population growth is sufficiently fast, new households with no assets are entering the economy sufficiently quickly that detrended, aggregate per capita assets reach a stable steady state. This involves a positive level of asset holdings because \( \left[ \beta (1 + r) \right]^\sigma (1 + g)^{(1-\rho)(1-\sigma)} > 1 + g \) implies that there are no households with negative asset holdings. If it were \( \left[ \beta (1 + r) \right]^\sigma (1 + g)^{(1-\rho)(1-\sigma)} < 1 + g \), all households would be dissaving, and steady-state detrended, aggregate per capita assets would be negative. The existence/stability
condition \( \frac{\beta^\sigma (1 + r)^\sigma (1 + g)^{(1 + r)(1 - \sigma)}}{(1 + n)(1 + g)} < 1 \) determines the sign of the denominator of \( \bar{b} \), while the individual consumption-tilt factor determines the sign of the numerator.  

Given steady-state asset holdings, aggregate per capita consumption and labor supply can be obtained easily. Steady-state aggregate per capita labor supply is vertical in the \((L, \bar{w})\) space. In steady state, employment is determined by the amount of labor that is supplied, and the real wage adjusts to clear the market.  

Once the steady state is determined, the equations that govern the dynamics of detrended aggregate per capita variables can be log-linearized around it knowing that the transition dynamics following temporary shocks will bring the economy back to the original position over time provided proper stability conditions are satisfied. The model’s stationarity has several advantages. The arbitrariness of the starting point for the purpose of analyzing the consequences of a shock is removed. The reliability of the log-linear approximation is increased, with positive implications for the confidence in the results of the model’s dynamics. A stationary model facilitates econometric work. Finally, the presence of a steady state to which the economy returns over time makes it possible to perform simulations of the log-linear model in which “surprises” happen in all periods.

### 3.a. Antecedents and Alternative Approaches

I discussed two possible ways to circumvent the issue of indeterminacy of the steady state and non-stationarity, by assuming unitary elasticity of substitution between domestic and foreign goods in consumption and/or complete asset markets. These assumptions appear to dominate the recent literature, at least in what some call “new open economy macroeconomics.” As I mentioned, they amount to shutting off the current account channel of interdependence or relegating it to a secondary role in many scenarios. Here, I discuss other approaches to the problem that were proposed in the past and have reappeared only very recently in the literature on the international transmission of business cycles.

Buiter (1981) is the first antecedent to my approach. He shows that an overlapping-generations model with finite lifetimes can deliver a determinate, non-degenerate distribution of

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24 To determine whether the country as a whole is a debtor or a creditor, one needs to account for the fact that shares—which are assets from the consumers’ perspective—are a liability for firms. Using lower-case letters to denote detrended, aggregate per capita, real quantities in units of the consumption basket, the equity value of the home economy evolves according to:

\[ v_t = \frac{(1 + n)(1 + g)}{(1 + r_{t+1})} v_{t+1} + d_{t+1}(1 + r_{t+1}), \]

where \( v_t \equiv V_t / (P_tE_{t+1}) \), \( d_t \equiv D_t / (P_tE_t) \), and the reader should recall that nominal equity value and dividends without the superscript \( i \) are in aggregate per capita terms.

Letting \( a_{t+1} \) be the country’s detrended, aggregate per capita, real net foreign assets (aggregating consumers and firms) entering period \( t + 1 \), it is:

\[ (1 + n)(1 + g)a_{t+1} = (1 + r_t)a_t + y_t - c_t - inv_t - (\eta/2)inv_t^2/k_t - (\phi/2)[(p_t(i)/p_{t-1}(i)) - 1 - \pi_t^2]k_t, \]

where \( inv_t \equiv I_t/E_t \), \( k_t \equiv K_t/E_t \), \( y_t \) is detrended, aggregate per capita GDP in units of the consumption basket \( (y_t = RPZL^{\gamma}_{t-1}) \), and \( I_t, K_t, \) and \( L_t \) are in aggregate per capita terms.

25 Solutions for the steady-state levels of variables other than consumer asset holdings are available on request.

26 The log-linear model is presented in the appendix.
asset holdings across countries. Frenkel and Razin (1987) develop a model of macroeconomic interdependence that relies on Blanchard’s (1985) assumption that agents face a non-zero probability of dying at each point in time along with entry of new households with no assets in each period. When agents die, their assets are transferred to insurance companies that cover their outstanding debts. Frenkel and Razin use the Blanchard demographic structure “to conduct a meaningful analysis of budget deficits in the absence of distortions” (p. 311) rather than with the explicit purpose of generating stationary dynamics for their open economy model. (They use the departure from Ricardian equivalence implied by Blanchard’s structure to make the timing of taxes and spending matter.) However, it turns out that Blanchard’s structure works much in the same way as Weil’s in generating steady-state determinacy—the main difference being the inclusion of a probability of death. (Extending the Weil setup to Blanchard’s would be a matter of relatively straightforward algebra, but it would add little to the main points of this paper.)

Weil (1989b) uses a continuous-time version of the setup in this paper, with exogenous endowment income that is identical across generations, to generalize Buiter’s (1981) results. Finn (1990) combines an overlapping-generations framework in which agents live for two periods with the assumption of internationally complete asset markets. The assumption of overlapping generations pins down a unique steady state endogenously. If this assumption were replaced by that of a representative, infinitely-lived household, complete markets would still ensure stationary dynamics around an exogenously chosen initial position. Cardia (1991) uses the Blanchard framework in her small open economy model. Interestingly, the Blanchard-Weil specification is also an ingredient of the Quarterly Projection Model (QPM) and the Canadian Policy Analysis Model (CPAM) of the Bank of Canada. After the first draft of this paper was circulated, Smets and Wouters (2002) and Devereux (2003) put forth small open economy models that rely on the Blanchard and Weil specifications, respectively. A contribution of this paper in relation to this literature is also to clarify the role of the Weil assumption in ensuring steady-state determinacy and stationarity in a discrete-time, New Keynesian, monetary business cycle model in line with the recent open economy literature.

Other scholars have pursued different ways to generate determinacy and stationarity, which do not rely on changes in the dynamics of population. Correia, Neves, and Rebelo (1995) use a particular form of non-separability between consumption and labor effort in utility, first introduced by Greenwood, Hercowitz, and Huffman (1988), such that the marginal rate of substitution between consumption and labor effort is a function only of the latter. This implies that the household chooses effort independently of its consumption decisions. Using this assumption, Correia, Neves, and Rebelo develop a representative agent model of a small open economy in which a stable steady state exists for employment and the ratios capital/employment, consumption/capital, and net foreign assets/capital. The approach pursued here has the advantage of generating a steady state for variables that are directly relevant for normative analysis.

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27 See also Yaari (1965).
28 In Weil (1989b), it is exchange rate indeterminacy a la Kareken and Wallace (1981) that results in indeterminacy of the current account. Exchange rate indeterminacy can be resolved by designing monetary policy at home and abroad appropriately.
29 See Poloz, Rose, and Tetlow (1994) on QPM and Black and Rose (1997) on CPAM. References therein provide detailed information on the two models.
30 See also Devereux, Gregory, and Smith (1992) and Pierdzioch (2003).
Mendoza (1991) and Obstfeld (1990) obtain stationarity by assuming that the rate of time preference depends on consumption and, through this, on net foreign assets, an approach originally proposed by Uzawa (1968) and initially introduced in continuous-time, open economy models by Obstfeld (1981a, b). Smets and Wouters (1999) assume that agents derive utility from asset holdings. Heathcote and Perri (2002) and Neumeyer and Perri (1999) introduce a cost of bond holdings in the consumers’ period budget constraint, an approach originally followed by Turnovsky (1985). Kollmann (2002), Mendoza and Uribe (2000), Schmitt-Grohé and Uribe (2001), and Senhadji (2003) obtain stationarity of their small open economy models by assuming that the interest rate at which the home economy can borrow internationally is given by the world interest rate plus a premium that increases in the country’s stock of foreign debt. All these assumptions ensure that the equilibrium rate of consumption growth depends on asset holdings, so that setting consumption to be constant pins down a steady-state distribution of net foreign assets. They all yield tractable models, which avoid some of the complications of the framework developed here. Nonetheless, on one side, the gain in terms of tractability is limited: All models mentioned above are solved by numerical methods. On the other side, assumptions about the functional form of the discount factor, utility from asset holdings, cost of bond holdings, or the determination of the interest premium appear more ad hoc and harder to quantify on empirical grounds than assuming that a small number of households with no assets enter the economy in each period. In the Weil-world of this paper, each individual household in the economy behaves as the representative agent of the original Obstfeld-Rogoff setup. Aggregate per capita assets are stationary, individual household’s are not.

In a very recent paper, Schmitt-Grohé and Uribe (2003) compare five different versions of the small open economy model (Uzawa preferences, cost of portfolio adjustment, debt elastic premium, standard non-stationary setup, complete markets) and conclude that all models deliver similar dynamics at business cycle frequencies (though consumption is smoother under complete markets) when they are parameterized to match the behavior of the Canadian economy. This finding should not come as a complete surprise, at least as far as stationary, incomplete markets models are concerned. Different solutions to non-stationarity under incomplete markets should deliver similar results if they are parameterized to match a given economy. There is no presumption for it to be otherwise. The similarity of results across the stationary, incomplete markets results, the non-stationary model, and the complete markets world is more striking. Home and foreign goods are perfect substitutes in Schmitt-Grohé and Uribe (2003). This removes any role for terms of trade dynamics from their model. Ghironi (2000) shows that differences in results across stationary, incomplete markets economies, the non-stationary case,

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31 Corsetti, Dedola, and Leduc (2002), Hirose (2003), Kim and Kose (2003), McDonald and Guest (2001), Schmitt-Grohé (1998), and Uribe (1997) are more recent examples of the same approach.
32 Benigno, P. (2001) and Laxton and Pesenti (2003) make a similar assumption. The cost of adjusting financial positions is the approach that has been adopted in the IMF’s new Global Economy Model (GEM).
33 Searching for a determinate non-stochastic steady state is not the only approach to macroeconomic interdependence under incomplete markets. Ljungqvist and Sargent (2000) describe a number of models in which a stationary equilibrium is defined in terms of a stationary probability distribution of asset-holdings/state-of-nature pairs. They discuss conditions under which this distribution exists and is unique. Clarida (1990) and Devereux and Saito (1997) use a similar approach. However, this is different from the more traditional approach in international business cycle analysis, which typically starts from the linearization (or higher order approximation) of the model around a deterministic steady state.
and the complete markets world are sensitive to the degree of substitutability between home and foreign goods and to the persistence of exogenous shocks.35

4. A Recession in the World Economy

In Ghironi (1999), I discuss a plausible strategy to estimate the structural parameters of the model using quarterly data from Canada and the U.S. over the period 1980:1-1997:4. Most estimates are characterized by small standard errors and are in line with the findings of other studies.

In this section, I use parameter estimates from Ghironi (1999) to calibrate the model and I illustrate its functioning by analyzing the transmission of a recession in the U.S. (taken as a proxy of the rest-of-the-world economy) to Canada under alternative specifications for Canadian monetary policy.

Canada is a natural candidate for the exercise for two reasons: On one side, interdependence between Canada and the U.S. is an often-studied example of a small, open, developed economy interacting with a large partner on which the small economy is likely to have a small, if not negligible, impact. On the other side, the model of this paper shares features with the Quarterly Projection Model and the Canadian Policy Analysis Model of the Bank of Canada. Even if the Bank’s models are richer, and therefore more realistic, than that of this paper in several respects (for example, inclusion of non-traded goods and deviations from PPP), the fact that the Bank of Canada employs a similar overlapping-generations setup for its own policy evaluation exercises contributes to making Canada a natural experiment for this paper. The parameter values used in the exercise are displayed in Table 1.36

4.a. U.S. Dynamics

Shocks to U.S. variables cannot be taken in isolation. I have not modeled the structure of the U.S. economy as explicitly as Canada’s, but—at a minimum—one must recognize that four variables that appear in the equations for Canadian variables will be affected by shocks to the U.S. GDP or interest rate: Besides these, the U.S. CPI inflation rate and the real interest rate will change. One cannot analyze the consequences of a shock to U.S. output or the interest rate for Canada without explicitly accounting for the comovements in all relevant variables that are triggered by the initial disturbance.

35 Baxter and Crucini (1995) use a non-stationary, incomplete markets model in their comparison of international business cycles under complete versus incomplete markets. (See also Baxter and Farr, 2001.) They remove the effect of the unit root in net foreign assets on the results by filtering the series obtained from the simulation of the non-stationary, log-linear model prior to calculating second moments. Their argument is that this is legitimate because of the desire to focus on the business cycle properties of the model. Yet, the question of reliability of the initial log-linear approximation remains unresolved. Also, one could argue that log-linearization around the steady state should already be what delivers the focus on business cycle fluctuations.

36 I calculate \( g \) as the average quarterly rate of growth of aggregate per capita real Canadian GDP over the 1980-1997 period and \( r \)—the world real interest rate—by deflating a series of the U.S. Federal Funds Rate by U.S. CPI inflation. The condition \( 1 + r > (1 + n)(1 + g) \) must be satisfied for the model to be stable. The values of \( \rho \) and \( \theta \) are lower than estimated in Ghironi (1999), whereas the value of \( \eta \) is higher. Higher \( \rho \) results in Canadian agents working an unrealistically large fraction of time in steady state. I set \( \theta \) to obtain a steady-state markup (adjusted for taxation of firm revenues—which is omitted from the model) consistent with Schmitt-Grohé (1998) to facilitate comparison of results. The value of \( \theta \) in Table 1 is in the range of estimates from the trade literature. Doubling \( \theta \) leaves almost all qualitative results below unaffected. Lower \( \eta \) generates unrealistically volatile investment.
I impose a minimal amount of structure on the U.S. economy. I take the Federal Funds Rate to be the relevant nominal interest rate and the policy instrument of the Federal Reserve. Following Rotemberg and Woodford (1997), I assume that the Fed sets the nominal interest rate based on a reaction function that depends on past levels of the rate and on the current and past levels of CPI inflation and GDP. Using sans-serif fonts to denote percentage deviations from the steady state, it is:

\[
\tilde{i}_{t+1} = \sum_{k=1}^{n_i} \varphi_{1k} \tilde{i}_{t+1-k} + \sum_{k=0}^{n_y} \varphi_{2k} \tilde{\pi}_{t-k} + \sum_{k=0}^{n_y} \varphi_{3k} y_{t-k}^W + \xi_t^*,
\]

where a tilde denotes the percentage deviation of a gross rate from its steady-state level; from now on, \(\pi_t\) denotes the percentage deviation of the corresponding inflation rate from the steady state rather than the level of inflation, unless otherwise noted; and \(\xi_t^*\) is an i.i.d., zero-mean exogenous shock to U.S. monetary policy. Because Canada is small relative to the U.S., the Fed’s reaction function does not incorporate any Canadian variable. The negligible impact of Canadian GDP on world aggregates allows me to identify U.S. GDP with \(y^W\) in the model.

I model the U.S. economy as a recursive, structural VAR that includes equation (41) and equations for GDP and inflation. The state vector is \([\tilde{\pi}_t, y_t^W, i_t^*, y_{t+1}^W, \xi_t^*]\), and the causal ordering of variables is the order in which they are listed. I follow Rotemberg and Woodford (1997) in assuming that the interest rate affects output and inflation only with a lag, but I do not include future inflation and GDP in the time-\(t\) state vector, because I choose not to take future consumption and inflation levels as entirely predetermined.

I estimate the VAR with three lags using full information maximum likelihood. I use data between 1980:1 and 1997:4. The estimated coefficients for the three equations and the standard errors are in the columns of Table 2. Seasonal dummies were not significant, as well as further lags. The estimated coefficients for the Fed’s reaction function suggest behavior in line with a generalized Taylor rule, consistent with the findings of Rotemberg and Woodford (1997).

Figure 1 shows the responses of GDP (\(y^W\)), inflation (\(\text{cpi}^*\)), the Federal Funds Rate (\(i^*\)), and the world real interest rate (\(r\)) to a 1 percent decrease in U.S. GDP.\(^{37}\) \(^{38}\) The deviation of GDP from the steady state increases in the first two quarters. Inflation reacts with a lag, and subsequently drops. The Fed reacts immediately by lowering the Federal Funds Rate to sustain GDP. Since the nominal interest rate is lowered by more than the decrease in inflation, the world real interest rate (\(r\)) is below the steady state throughout the transition. Over time, all variables go back to the steady state.

4.b. Canada

The paths of U.S. variables generated by the shock constitute the paths of the world-economy variables following the initial impulse in my model of the Canadian economy. I include the estimated VAR equations in the system of equations that govern the dynamics of the world economy.

\(^{37}\) Because markets clear in the model, an exogenous decrease in U.S. GDP can be interpreted both as the consequence of a generalized decline in world demand for goods and as the outcome of a negative supply shock. I interpret the shock as an exogenous contraction in demand. The interpretation is consistent with the fact that U.S. inflation declines following the disturbance.

\(^{38}\) In the impulse responses, the level of the nominal interest rate at each date \(t\) is the value chosen by the monetary authority at \(t\) for the period between \(t\) and \(t+1\). The real interest rate at each date \(t\) is the \textit{ex ante} real interest rate between \(t\) and \(t+1\). I omit tildes from the figures.
economy following the initial shock, along with the model equations for Canada and the monetary rule followed by the Bank of Canada. I then solve the overall system using Uhlig’s (1999) implementation of the method of undetermined coefficients.

I consider three alternatives for Canadian monetary policy to illustrate the properties of the model: an inflation-targeting interest rate reaction function, strict CPI inflation targeting, and a policy that stabilizes the markup at its steady-state level in all periods.

4.b.1. Inflation-Targeting Taylor Rule

The Bank of Canada has an interval target for CPI inflation. In Black, Macklem, and Rose (1997), the Canadian Policy Analysis Model specifies policy in terms of a reaction for the slope of the term structure of interest rates (the short rate relative to the long rate) as a function of expected future CPI inflation. Extensions of the basic CPAM rule allow for a reaction of the slope of the term structure to measures of the output gap or to the deviation of the price level from a target.

In the spirit of Black, Macklem, and Rose (1997), I specify a benchmark inflation-targeting rule for the Bank of Canada of the following type:

\[ \tilde{I}_{t+1} = \alpha \tilde{z}_{t+1}, \quad \alpha > 1, \quad v = PPI, CPI, \quad s = 0,1. \]  

(42)

The central bank targets (current or expected) inflation, in the sense that it sets the interest rate in reaction to movements in the (PPI or CPI, current or expected) inflation rate. The reaction is more than proportional, i.e., the Taylor principle is satisfied. I interpret rule (42) as a flexible inflation targeting \((FIT)\) regime. When \(v = PPI\) and \(s = 0\), rule (42) is a special case of the floating exchange rate, inflation-targeting rules in Benigno and Benigno (2001).

Carlstrom and Fuerst (2000) demonstrate that current- or forward-looking interest rate rules of the type in (42) are prone to indeterminacy in a closed economy, sticky-price model with investment, no capital adjustment costs, and cash-in-advance (CIA) timing for money balances in the utility function (i.e., the money balances that enter the utility function are those at the beginning of the period). For this reason, they advocate backward-looking interest rate rules for the central bank. I adopt the more conventional cash-when-I-am-done (CWID) timing in this paper: Money balances in the utility function are those at the end of the period. The results in Carlstrom and Fuerst (2001) establish that the combination CWID timing plus forward-looking (current-looking) rule is equivalent to CIA timing plus current-looking (backward-looking) rule. Therefore, the case \(s = 1\) in (42) corresponds to a current-looking rule under Carlstrom and Fuerst’s CIA timing, and the case \(s = 0\) corresponds to a backward-looking rule under CIA timing. In the case \(s = 1\), Carlstrom and Fuerst’s (2000) results suggest that indeterminacy may be an issue. Numerical solution of the model for the parameterization in Table 1 and the values of \(s = 0\) I consider did not reveal existence of indeterminacy. Capital adjustment costs may be the reason for this difference in results. In any case, to minimize the risk of indeterminacy, most of the discussion below focuses on the safe case \(s = 0\).

Impulse responses to an exogenous drop in U.S. GDP under a \(FIT\) regime with \(v = PPI, s = 0,\) and \(\alpha = 1.1\) are in Figure 2. In this case, the central bank reacts to current PPI inflation “just enough” to ensure that the Taylor principle holds. As the figure shows, recession in the U.S. causes a drop in demand for Canadian goods and a prolonged recession in Canada, with lower

\[\text{This interpretation of inflation targeting differs from that of Svensson (2000, 2003), where inflation targeting (flexible or strict) is defined in relation to the targets that are featured in the central bank’s loss function.}\]
labor demand, investment, and consumption. In fact, Canada’s foreign debt worsens as agents borrow in an effort to sustain consumption. Lower demand for Canadian products causes PPI inflation (\( ppi \)) to fall, even if firms raise the markup component of prices (\( mkup \)) to sustain profits, and the terms of trade appreciate, consistent with a higher markup and a larger drop in Canadian than U.S. GDP. The central bank reacts to lower PPI inflation by lowering the interest rate by more than the decrease in the Federal Funds Rate. This generates expected future appreciation, and the Canadian dollar appreciates throughout the transition. Reaction of the interest rate to inflation in rule (42) results in familiar non-stationarity of nominal levels, so that the Canadian dollar appreciates permanently against the U.S. dollar in response to the recession (\( eps \)). Appreciation and lower U.S. inflation combine to deliver lower CPI inflation in Canada (\( cpi \)). This notwithstanding, a higher, countercyclical markup keeps dividends and share prices above the steady state.

To understand the dynamics of consumption, observe that consumption in each period depends on the generalized propensity to consume defined in Section 3, on the realized, gross real return on the consumers’ asset position entering that period, and on human wealth. Wage and \( ex \ ante \) real interest rates dynamics combine to deliver a decrease in the generalized propensity to consume.\(^{41} \) Human wealth (the present discounted value of the real wage stream from \( t \) on, denoted with \( h \)) rises above the steady state because the effect of a lower \( ex \ ante \) real interest rate prevails on that of a lower wage.

The home \( ex \ ante \) real interest rate is tied to the foreign one in all periods. In all periods after the initial shock, \( ex \ ante \) and \( ex post \) real interest rates coincide, as there is no further unexpected shock. However, \( ex post \) real returns can differ across countries in the period of the shock. The unexpected shock happens at time 0. Real interest rates are such that \( \tilde{r}_0 = \tilde{i}_0 - \tilde{\pi}_0^{\text{CPI}} \) and \( \tilde{r}_0^* = \tilde{i}_0^* - \tilde{\pi}_0^{\text{CPI}*} \), where \( \tilde{i}_0 \) and \( \tilde{r}_0 \) are the home nominal and real interest rates between periods – 1 and 0, respectively, and the same timing notation holds for foreign rates. From the perspective of period – 1 (\( ex \ ante \)), real interest rates at home and abroad are equal: \( \tilde{r}_0 = \tilde{r}_0^* = 0 \).

\( ex post \), the realized real returns at time 0 are such that \( \tilde{r}_0^{EP} = \tilde{r}_0^{EP*} = - (\tilde{\pi}_0^{\text{CPI}} - \tilde{\pi}_0^{\text{CPI}*}) \) = \( \tilde{\epsilon}_0 = - \epsilon_o \), where the superscript \( EP \) stands for \( ex post \). Since nominal interest rates between periods – 1 and 0 were set at time – 1, before any shock happened, they were set at the respective steady-state levels. It follows that \( ex post \) real returns at time 0 at home and abroad equal the negative of the respective CPI inflation rates, and PPP implies that the \( ex post \) real interest rate differential is equal to the negative of depreciation at time 0 (or to the negative of the deviation of the exchange rate from the steady state at time 0). In a nutshell, real interest rate equalization follows from UIP and PPP, but UIP can be violated \( ex post \) at the time of an unexpected shock. This \( ex post \) real interest rate differential disappears after period 0, since no new unexpected shock happens.

Albeit short-lived, the time-0 difference in \( ex post \) real returns across countries provides an additional channel through which home interest rate policy can affect the economy, as the initial movement of the home exchange rate is determined by interest rate setting from time 0 on at home and abroad via UIP. Since U.S. CPI inflation reacts to the shock with a lag, the U.S. real

\(^{40} \) In the figure, capital at time \( t \) is capital at the end of that period. The same is true of asset holdings.

\(^{41} \) An increase in \( TH \) corresponds to a lower value of \( \Theta_t^{-1} \) in equation (34).
interest rate at time 0 is the same *ex ante* and *ex post*. As home CPI inflation falls on impact (the home currency appreciates), the time-0 domestic, *ex post* real interest rate is above the steady state. Both \( \delta \) and \( \bar{\alpha} \) are negative for the parameter values in Table 1 (households’ debt more than offsets the value of equity in steady state). Hence, a higher than expected real interest rate has a negative effect on asset accumulation and consumption by increasing the burden of debt at time 0. Lower propensity to consume and larger interest burden of previously accumulated debt more than offset the increase in human wealth and deliver the initial drop in consumption in Figure 2, even if households borrow more to sustain their consumption levels.

In all periods after time 0, the *ex post* real interest rate coincides with the *ex ante* rate for that period, which is below the steady state, with a beneficial effect on asset accumulation and consumption. The latter climbs back toward the steady state as the propensity to consume strengthens and the households’ asset position improves. Asset holdings return to the steady state very slowly due to slow population growth. (All real variables eventually return to the steady state in Figure 2 and in the following figures.)

Figure 3 shows the impulse responses for the case in which \( v = \text{CPI}, s = 0, \) and \( \bar{\alpha} = 1.1 \). Relative to the previous scenario, the central bank is now targeting CPI inflation. Most results are broadly similar to those in Figure 2 on qualitative grounds. As expected, CPI inflation and the rate of depreciation respond by less to the shock when the central bank reacts to CPI rather than PPI inflation. Also consistent with the change in the policy rule, the deviation of PPI inflation from the steady state is somewhat more persistent than in Figure 2. This results in a smaller, smoother increase in the markup and in a smaller decrease in labor effort, wage, and GDP. As a consequence of the change in the wage profile, the propensity to consume does not decrease as much as in Figure 2. A smaller impact response of CPI inflation to the shock implies that the unexpected increase in the time-0 *ex post* real interest rate is smaller than under PPI targeting. Since the increase in human wealth is roughly the same, the smaller decrease in the propensity to consume and the smaller initial real interest rate shock are responsible for the increase in consumption on impact. This is quickly reversed as human wealth falls toward the steady state. Foreign debt increases by less, and consumer assets react positively to the shock. Combined with a lower *ex ante* and *ex post* real interest rate in all periods after time 0, the improvement in consumer asset positions drives consumption above the steady state approximately five years after the shock. The recession in Canada is less profound, though more persistent if the central bank reacts to CPI rather than PPI inflation.

Suppose now that the central bank reacts more aggressively to inflation, so that \( \bar{\alpha} = 1.5, \) as in the rule advocated as a benchmark for Federal Reserve policy by Taylor (1993). Figure 4 shows the impulse responses for the case \( v = \text{PPI} \) and \( s = 0 \). More aggressiveness in the reaction to inflation implies that consumption rises above the steady state in response to the shock and remains higher throughout the transition. More policy aggressiveness has a stabilizing impact through two channels: On one side, it implies a smaller increase in the markup, and thus a smaller drop in labor effort and wage—with a favorable effect on the propensity to consume. On the other side, it results in a smaller initial real interest rate shock, as CPI inflation falls by less than in figures 2 and 3. Indeed, a more aggressive reaction to inflation generates also an initial expansion in investment and capital as firms make less use of their monopoly power to increase the markup and substitute capital for labor to sustain production. This initial investment

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42 Reaction to CPI inflation does not lead to indeterminacy as in De Fiore and Liu (2002) because there is no home bias in consumption preferences in the model of this paper.
expansion is short-lived, though, and it is followed by a long-lasting contraction before the economy returns to the steady state.

Figure 5 illustrates the case $\alpha = 1.5, \nu = PPI$ and $s = 1$: This is a forward-looking, PPI inflation-targeting rule, with the Taylor (1993) reaction coefficient. Comparison of figures 4 and 5 shows that adoption of a forward-looking rule leaves qualitative dynamics roughly unaffected.

4.b.2. Strict CPI Inflation Targeting

To further illustrate the importance of inflation volatility and the markup channel in the model, I first consider an alternative monetary regime of strict inflation targeting (SIT). Under this regime, I assume that the Bank of Canada sets the Canadian nominal interest rate to keep CPI inflation at its steady-state level in all periods, including when an unexpected shock happens:

$$\pi^{CPI}_t = 0 \ \forall t \geq t_0.$$  

This policy goal could be accomplished by following rule (42) with $\nu = CPI$, $s = 0$, and a very high value of the reaction coefficient $\alpha$. But this would be impractical and quite risky: As observed by Svensson (2003), policy errors would be immensely costly under a super-aggressive interest rate rule.

There are several other interest rate rules that would implement the SIT regime in the model of this paper. For example, the Bank of Canada could exploit UIP and set the interest rate to react proportionally to Federal Reserve policy, expected exchange rate depreciation, and current CPI inflation:

$$\widetilde{I}_{t+1} = \widetilde{I}^*_{t+1} + \widetilde{\Theta}_{t+1} + \widetilde{\pi}^{CPI}_{t+1}.$$  

Combining (43) with UIP, $\widetilde{I}_{t+1} = \widetilde{I}^*_{t+1} + \widetilde{\Theta}_{t+1}$, yields $\pi^{CPI}_t = 0 \ \forall t \geq t_0$.  

Abstracting from the possibility of policy errors, all interest rate rules that implement the regime $\pi^{CPI}_t = 0 \ \forall t \geq t_0$ result in identical dynamics for the Canadian economy given determinacy of the equilibrium. Impulse responses to an exogenous drop in U.S. GDP under this regime are in Figure 6.

Because of PPP, CPI inflation stability requires the Bank of Canada to engineer a rate of depreciation such that $\widetilde{\epsilon} = -\pi^{CPI}_t$. Since the shock causes U.S. CPI inflation to decrease, the Canadian dollar must depreciate to keep Canadian CPI inflation constant. Combined with the fact that there is no ex post real interest rate shock at time 0 under the SIT regime, exchange rate depreciation dampens the extent of the recession relative to figures 2-5. In fact, Canadian PPI inflation now rises, while firms lower the markup component of prices to avoid depressing output demand by too much. The lower markup and ex ante real interest rate initially boost investment and capital accumulation by raising the value of installed capital, but equilibrium employment falls persistently. This is so for two reasons. On one side, lower world consumption demand tends to depress home labor demand. On the other side, the value of consumer assets increases persistently due to the initial, expansionary effect of a lower ex ante real interest rate on share prices and the lower real interest burden on debt. As a consequence, agents can sustain higher consumption while supplying less labor. This causes the real wage to rise, with a further

43 If it were useful to increase credibility, which is implicitly assumed to be perfect in this paper, the central bank could announce a version of (43) that allows for a more than proportional reaction to inflation a la Taylor. Combining the amended rule with UIP would still result in $\pi^{CPI}_t = 0 \ \forall t \geq t_0$ in equilibrium. It is also possible to obtain an interest setting rule that implements the SIT regime from the money demand equation in growth rates.
positive effect on asset holdings and a negative effect on labor demand. Higher human wealth, a muted decrease in the propensity to consume, and the wealth effect of asset accumulation under strict CPI inflation targeting allow Canadian households to increase their consumption on impact and keep it above the steady state through the transition. As under the inflation-targeting Taylor rule above, a deeper, more persistent GDP decline in Canada than in the U.S. is mirrored by a higher relative price of Canadian goods.

4.4.3. Markup Stability
Mimicking the flexible price equilibrium by keeping inflation at the steady state is the optimal monetary policy under commitment in many closed economy, sticky-price models with monopolistic competition. This is so because zero inflation implies that the markup is constant at its steady-state level, thus removing the distortion associated with sticky prices. Woodford (2003) reviews the argument in detail.

Benigno and Benigno (2003) demonstrate that the policy of mimicking the flexible price equilibrium is optimal for open economies only under very special assumptions, which are not satisfied in the model of this paper. Nevertheless, markup dynamics play such a key role in the model, and the policy of markup stability has received so much attention in the literature, that it is interesting to understand how such policy would influence model dynamics.

Log-linearizing the markup equation (20) after expressing numerator and denominator in terms of detrended, aggregate per capita variables yields:

$$\psi_t = - \phi \left(1 + \pi^*\right) \frac{k^*}{(\theta - 1)\bar{y}} \left[ \tilde{\pi}_{PPI} - \pi^* \right]$$

where $$\psi_t$$ is the percentage deviation of the markup from the steady state. From equation (44), we see that the markup is constant at its steady state level if and only if producer price inflation is, i.e., if and only if $$\tilde{\pi}_{PPI} = 0$$ for all $$t \geq t_0$$. This goal could be accomplished by following rule (42) with $$r = PPI$$, $$s = 0$$, and a very high value of the reaction coefficient $$\alpha$$, but this would be subject to the same objection as in the case of the SIT regime above. As before, an alternative interest setting rule that implements markup stability can be obtained by exploiting UIP. Suppose the Bank of Canada reacts proportionally to Federal Reserve policy, expected exchange rate depreciation, and current PPI inflation:

$$\tilde{I}_{t+1} = \tilde{I}^*_t + e_{t+1} + \tilde{\pi}_{PPI}$$

(45)

Combining (45) with UIP implies $$\tilde{\pi}_{PPI} = 0$$ for all $$t \geq t_0$$, and therefore $$\psi_t = 0$$ for all $$t \geq t_0$$.

Impulse responses under a regime of markup stability are in Figure 7. CPI inflation drops on impact as the Canadian dollar appreciates, but CPI dynamics stabilize quickly as the rate of depreciation adjusts to offset the movement of U.S. inflation in a similar fashion to the SIT regime. The dynamics of several key variables are qualitatively similar to those under strict CPI inflation targeting. Even if the markup does not fall, a lower ex ante real interest rate boosts the value of installed capital and initially expands investment and capital accumulation. Employment falls persistently due to lower output demand and the effect of consumer asset accumulation on

---

44 As for the SIT case, a more than proportional reaction to PPI inflation in (45) will still implement $$\psi_t = 0$$ for all $$t \geq t_0$$. Similarly, it is possible to obtain an alternative rule that implements this regime from the money demand equation in growth rates. Again, equilibrium determinacy ensures that dynamics are identical across rules that implement the markup stability regime.
labor supply. A higher real wage and a muted time-0 \textit{ex post} interest rate shock push consumption above the steady state, where it remains during the transition owing also to consumer asset accumulation.

Schmitt-Grohé (1998) documents that a negative innovation to U.S. gross national product causes Canadian output, employment, investment, exports, imports, and the terms of trade to fall. Figure 2 shows that the model of this paper can capture several qualitative features of the transmission of a worldwide recession to Canada under the assumption that the Bank of Canada is reacting moderately to PPI inflation. In this case, we observe a decline in GDP, consumption, investment, labor effort, and the wage, as well as increasing foreign debt, at least for the first few years after the shock. Some variables move in empirically counterfactual fashion—usually, we expect recessions to go along with stock market contraction rather than expansion. Similarly, the terms of trade improve rather than falling as Schmitt-Grohé’s evidence says they should. Rich as it is, the model clearly needs extension in several directions if it is to match more stylized facts. Allowing for local currency pricing appears a promising way to resolve the terms of trade puzzle that confronts this paper and the RBC models in Schmitt-Grohé’s exercise.

Comparison of figures 2-7 shows that strict CPI inflation targeting or the policy of producer inflation stability that mimics the flexible price equilibrium reduce the amplitude of fluctuations in the Canadian economy in the aftermath of a recession in the U.S. relative to an inflation-targeting Taylor rule. However, the recession in Canadian GDP is more persistent under strict CPI inflation targeting or markup stability than under the FIT rule. Inflation, markup, and asset accumulation dynamics play a crucial role in these results.

5. Conclusions

This paper has developed a model that makes it possible to analyze macroeconomic interdependence without de-emphasizing the current account channel. Absence of an endogenously determined steady state is a problem in several recent open economy models with internationally incomplete asset markets. The model presented here has the advantage that its steady state is entirely determined by the structural parameters—and, for some variables, by the steady-state levels of policy instruments—and is stable, if appropriate conditions are satisfied. Determinacy of the steady state is achieved by changing the demographic structure from the usual representative agent framework to an overlapping-generations structure a la Weil (1989a, b), in which new infinitely lived households enter the economy at each point in time and are born owning no financial assets. The existence of an endogenously determined steady state removes the arbitrariness of the choice of the initial point. It makes it possible to calculate reliable log-linear approximations. It guarantees that temporary shocks do not have permanent consequences. One can use the log-linear model to perform sensible stochastic simulations.

Costs of adjusting output prices generate an endogenously variable markup in the model. This unties the dynamics of the real wage and the marginal product of labor. The markup plays an important role in business cycle fluctuations, and the real wage can be procyclical, consistent with the empirical evidence.

Finally, the setup features investment and capital accumulation, adopting a standard \( q \) model of investment. Incorporating investment is particularly important in a model that attributes a role to the current account. Markup dynamics play a role in investment decisions.
I illustrated the functioning of the model by calibrating it to analyze the transmission of a recession in the U.S. to Canada under alternative monetary policy rules. Consistent with the goal of combining theoretical rigor with quantitative flexibility, I used a simple VAR to trace the comovements in U.S. variables. I combined the estimated VAR equations for the U.S. with the model equations for Canada to determine the response of the Canadian economy to the initial shock. I considered three policy rules: an inflation-targeting Taylor rule, strict CPI inflation targeting, and a rule that mimics the flexible price equilibrium by keeping PPI inflation constant. I found that stabilizing inflation (in consumer or producer prices) at a steady-state target in all periods results in a milder, but more persistent recession than a rule under which the interest rate reacts to inflation in a Taylor fashion. Markup dynamics and changes in asset holdings were central to this result.

The model of this paper provides an alternative to flexible-price, international RBC models such as those in Schmitt-Grohé (1998) and Schmitt-Grohé and Uribe (2003) for the analysis of business cycle fluctuations in small open economies. It constitutes an alternative to sticky-price, complete-markets models such as Galí and Monacelli’s (2002) for the study of monetary policy. Several papers in the recent literature on monetary policy in open economies obtain interesting analytical results at the cost of rather restrictive assumptions. The approach of this paper sacrifices some analytical power in favor of a more flexible, quantitative approach. Clearly, the model must be extended to allow for deviations from the law of one price and PPP if it is to match empirical features of interdependence between Canada and the U.S. closely. Steps in this direction have been undertaken at the Bank of Canada. I leave combining my work with recent developments in the literature on local currency pricing, habits, and variable factor utilization for future research.

Appendix. The Log-Linear Economy

Detrended, aggregate per capita, real net foreign assets entering period $t + 1$:

$$
a_{t+1} = \frac{1 + r}{(1 + n)(1 + g)} \left( i_t - \tilde{c}_{t+1} \right) + \frac{1 + r}{(1 + n)(1 + g)} a_t + \frac{\tilde{y}}{(1 + n)(1 + g)} y_t - \frac{\tilde{c}}{(1 + n)(1 + g)} c_t +$$

$$- \frac{\tilde{inv}}{(1 + n)(1 + g)} inv_t - \frac{\tilde{inv}^2}{2 (1 + n)(1 + g)} (2 inv_t - k_t).$$

(A.1)

The deviation of $a_t$ from the steady state is defined by $a_t \equiv (a_t - \tilde{a})/|\tilde{a}|$ to allow for the possibility of negative steady-state net foreign asset holdings.

Detrended, aggregate per capita real equity value of the home economy entering period $t + 1$:

$$v_t = -\tilde{i}_{t+1} + \tilde{c}_{t+1} + \frac{(1 + n)(1 + g)}{1 + r} v_{t+1} + \frac{\tilde{d}}{(1 + r)} d_{t+1}. \quad (A.2)$$

Detrended, aggregate per capita real dividends:

$$d_t = \frac{\tilde{y}}{d} y_t - \frac{\tilde{w} L}{d} (w_t + L_t) - \left( \frac{\tilde{inv}^2}{d} + \eta \frac{\tilde{inv}^2}{kd} \right) inv_t + \frac{\eta \tilde{inv}^2}{2 kd} k_t. \quad (A.3)$$

Detrended, aggregate per capita real consumer assets entering period $t$:
\[ b_t = \frac{a_t}{p} + \frac{v}{\bar{p}} v_{t-1}. \]

The deviation of \( b_t \) from the steady state is defined by \( b_t \equiv \left( b_t - \bar{b} \right) / \bar{b} \) to allow for the possibility of negative steady-state consumer asset holdings.

Let \( \hat{\Theta}_t \) denote the percentage deviation of \( \Theta_t \) (the reciprocal of the generalized propensity to consume defined in Section 3) from the steady state. Then:

\[
\hat{\Theta}_t = \frac{\beta^{\sigma}(1+r)^{\sigma(1+g)(1-\rho)(1-\sigma)}}{1+r} \left[ \hat{\Theta}_{t+1} - \left( 1 - \sigma \right) \left( \widetilde{i}_{t+1} - \pi_{t+1} \right) + (1-\rho)(1-\sigma)(w_{t+1} - w_t) \right].
\]  

Let \( h_t \) denote the present discounted value of the detrended real wage stream (human wealth): \( h_t = \sum_{t=1}^{\infty} R_{t-1} (1+g)^{t-1} w_t \). The percentage deviation from the steady state, \( h_t \), obeys:

\[
h_t = \frac{1+g}{1+r} \left( h_{t+1} - \tilde{i}_{t+1} + \pi_{t+1} \right) + \frac{r-g}{1+r} w_t.
\]  

Detrended, aggregate per capita consumption:

\[
c_t = -\hat{\Theta}_t + \frac{(r-g)\bar{p}}{(r-g)\bar{p} + W} \left( \tilde{i}_t - \pi_{t+1} \right) + \frac{(r-g)\bar{b}}{(r-g)\bar{b} + \bar{w}} b_t + \frac{\bar{w}}{(r-g)\bar{b} + \bar{w}} h_t.
\]  

Aggregate per capita labor supply:

\[
L_t = \frac{1-\rho}{\rho} \frac{\bar{\sigma}}{\bar{w}L} (w_t - c_t).
\]  

Growth rate, detrended, aggregate per capita, nominal money balances:

\[
\bar{g}_t = \pi_{t+1} + c_t - c_{t-1} - \frac{\sigma}{\bar{\nu}} \left( \tilde{i}_t - \tilde{i}_{t-1} \right) - (1-\rho)(1-\sigma)(w_t - w_{t-1}).
\]  

Detrended, aggregate per capita capital:

\[
k_{t+1} - k_t = \frac{(1+n)(1+g) - (1-\delta) \left( \text{inv}_t - k_t \right)}{(1+n)(1+g)}.
\]  

Detrended, aggregate per capita investment:

\[
\text{inv}_t - k_t = \frac{1+n}{\eta \left( 1+n \right) (1+g) - (1-\delta)} q_t.
\]  

Tobin’s \( q \):

\[
q_t = \frac{1-\delta}{1+r} q_{t+1} + \frac{\gamma}{1-\gamma \frac{\bar{w}L}{q} (1+r)} \left( w_{t+1} + L_{t+1} - k_{t+1} \right) + \frac{\eta \left( 1+n \right)(1+g) - (1-\delta)}{\eta \left( 1+n \right)(1+g) - (1-\delta)} \left( \text{inv}_{t+1} - k_{t+1} \right).
\]  

Aggregate per capita labor demand:

\[
L_t = \psi_t - y_t - w_t.
\]  

Markup:

\[
\psi_t = -\frac{\phi \left( 1+\pi \right) \bar{k}}{(\theta-1)\bar{p}} \left[ \pi_{t+1} \left( \text{PPI}_t \right) - \frac{1+n}{1+r} \pi_{t+1} \right].
\]
PPI inflation:
\[ \pi_{t}^{PPI} = \pi_{t}^{CPI} + \gamma_{t} - \gamma_{t-1} + w_{t} - w_{t-1} + \gamma \left[ L_{t} - L_{t-1} - (k_{t} - k_{t-1}) \right] - (Z_{t} - Z_{t-1}). \]  
(A.15)

Relative price:
\[ R_{t} = -\frac{\gamma_{t}}{\theta} k_{t} - \frac{1 - \gamma_{t}}{\theta} L_{t} + \frac{1}{\theta} w_{t} - \frac{1}{\theta} Z_{t}. \]  
(A.16)

Detrended, aggregate per capita GDP (in units of consumption):
\[ y_{t} = R_{t} + \gamma_{t} k_{t} + (1 - \gamma) L_{t} + Z_{t}. \]  
(A.17)

PPP:
\[ \bar{\pi}_{t}^{CPI} = \bar{\theta}_{t} + \bar{\pi}_{t}^{CPI*}. \]  
(A.18)

UIP:
\[ \bar{\pi}_{t+1} - \bar{\pi}_{t+1}^{*} = \bar{\epsilon}_{t+1}. \]  
(A.19)

Rate of depreciation:
\[ \bar{\epsilon}_{t} = \epsilon_{t} - \epsilon_{t-1}. \]  
(A.20)

Acknowledgments

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References


Table 1. Structural parameters, world steady-state, and steady-state interest rate *

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* $\bar{y}^W$ is average, detrended, aggregate per capita, real U.S. GDP over the period 1980:1-1997:4.

Table 2. The U.S. economy **

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** Standard errors in parenthesis.
Figure 1. The U.S. economy
Figure 2. Current-looking, PPI inflation-targeting, Taylor rule, $\alpha = 1.1$
Figure 3. Current-looking, CPI inflation-targeting, Taylor rule, $\alpha = 1.1$
Figure 4. Current-looking, PPI inflation-targeting, Taylor rule, $\alpha = 1.5$
Figure 5. Forward-looking, PPI inflation-targeting, Taylor rule, $\alpha = 1.5$
Figure 6. Strict inflation targeting
Figure 7. Markup stability