Sticky-Price Models of the Business Cycle:
Specification and Stability

Peter N. Ireland*
Boston College and NBER

July 1999

Abstract

This paper focuses on the specification and stability of a dynamic, stochastic, general equilibrium model of the business cycle with sticky prices. Maximum likelihood estimates reveal that the data prefer a version of the model in which adjustment costs apply to the price level but not to the inflation rate. Formal hypothesis tests provide evidence of instability in the estimated parameters, concentrated in the Euler equation linking consumption growth to the interest rate.

*JEL: E31, E32, E52.

*Please address correspondence to: Peter N. Ireland, Boston College, Department of Economics, Carney Hall, 140 Commonwealth Avenue, Chestnut Hill, MA 02467-3806. Tel: (617) 552-3687. Fax: (617) 552-2308. E-mail: irelandp@bc.edu.
1. Introduction

Recent years have witnessed a heightened interest in the role played by nominal price rigidity in shaping key features of the American business cycle. A growing body of literature, surveyed by Nelson (1998a) and Taylor (1998), shows how sticky prices can be fruitfully incorporated into dynamic, stochastic, general equilibrium (DSGE) models of economic fluctuations.

These new sticky-price models of the business cycle, like the earlier models of Fischer (1977) and Taylor (1980), assume that private agents have rational expectations. These new models go beyond their predecessors, however, by providing explicit accounts of how the optimizing behavior of households and firms helps determine the time paths of aggregate variables such as output and inflation. Thus, as emphasized by Ireland (1997) and Rotemberg and Woodford (1997), these new models respond to the Lucas (1976) critique by identifying parameters describing private agents' tastes and technologies—parameters that ought to remain invariant to changes in the monetary policy regime.

Recent work with sticky-price models of the business cycle is criticized, however, by Estrella and Fuhrer (1999), who call attention to the fact that little evidence has been brought to bear in assessing whether or not these new models
actually live up to their promise of being truly structural. Thus, one purpose of this paper is to provide and examine such evidence. Accordingly, the paper develops and estimates a DSGE model with sticky prices. It then performs a series of formal, econometric hypothesis tests capable of determining whether the model’s estimated parameters have remained stable in the face of the important changes in monetary policy that are believed to have occurred over the past four decades and of pinpointing where the instability, if any, lies.

Recent work with sticky-price models of the business cycle is also criticized by Fuhrer and Moore (1995) and Nelson (1998b), who suggest that a full explanation of the US time series data may require a model in which the inflation rate, as well as the price level, responds sluggishly to the shocks that hit the economy. Thus, a second purpose of this paper is to reconsider the price adjustment mechanism in a DSGE framework. Accordingly, the business cycle model developed here generalizes those used previously by allowing for rigidity in both the price level and the inflation rate. The maximum likelihood procedure used to estimate the model lets the data decide which form of nominal rigidity is most important.

Thus, as its title suggests, this paper focuses on the specification and stability of a sticky-price model of the business cycle. Section 2, below, sets up the model. Section 3 describes the data, estimates, and tests. Section 4 concludes.
2. The Model

2.1. Overview

Here, the model developed in Ireland (1997) is extended in three ways. First, Ireland (1997) assumes that nominal goods prices move sluggishly because firms face a quadratic cost of price adjustment, as originally suggested by Rotemberg (1982). Here, a more general specification for the costs of price adjustment, proposed by Price (1992), Nelson (1998a), and Tinsley (1998), gives rise to sluggishness in the inflation rate as well as the price level. As noted above, this first extension allows the data to decide on the relative importance of sticky prices and sticky inflation in the US economy. Second, Ireland (1997) characterizes monetary policy as one that adjusts the growth rate of a broad monetary aggregate in response to the shocks that hit the economy. Here, policy is instead characterized as one that adjusts the short-term nominal interest rate in response to changes in output, inflation, and money growth. This second extension reflects growing recognition, following the influential work of Taylor (1993), that Federal Reserve policy is more accurately described by its effects on interest rates than by its effects on the monetary aggregates. Third, and finally, Kimball (1995), Kim (1996), and King and Watson (1996) find that adjustment costs for physical capital help DSGE models...
explain the behavior of interest rates; these adjustment costs, absent in Ireland (1997), are introduced here.

The model economy consists of a representative household, a representative finished goods-producing firm, a continuum of intermediate goods-producing firms indexed by \( i \in [0, 1] \), and a monetary authority. During each period \( t = 0, 1, 2, \ldots \), each intermediate goods-producing firm produces a distinct, perishable intermediate good. Hence, intermediate goods are also indexed by \( i \in [0, 1] \), where firm \( i \) produces good \( i \). The model features enough symmetry, however, to permit the analysis to focus on the activities of a representative intermediate goods-producing firm, identified by the generic index \( i \).

2.2. The Representative Household

The representative household carries \( M_{t-1} \) units of money, \( B_{t-1} \) bonds, and \( K_t \) units of capital into period \( t \). At the beginning of the period, the household receives a lump-sum nominal transfer \( T_t \) from the monetary authority. Next, the household’s bonds mature, providing \( B_{t-1} \) additional units of money. The household uses some of its money to purchase \( B_t \) new bonds at nominal cost \( B_t/R_t \), where \( R_t \) denotes the gross nominal interest rate between \( t \) and \( t + 1 \).

During period \( t \), the household supplies \( H_t(i) \) units of labor and \( K_t(i) \) units
of capital to each intermediate goods-producing firm $i \in [0, 1]$; its choices of $H_t(i)$ and $K_t(i)$ must satisfy

$$H_t = \int_0^1 H_t(i) di,$$

where $H_t$ denotes total hours worked, and

$$K_t = \int_0^1 K_t(i) di$$

for all $t = 0, 1, 2, \ldots$. Thus, the household receives nominal factor payments $W_t H_t + Q_t K_t$ during period $t$, where $W_t$ denotes the nominal wage and $Q_t$ denotes the nominal rental rate for capital. The household also receives nominal profits $D_t(i)$ from each intermediate goods-producing firm $i \in [0, 1]$, for a total of

$$D_t = \int_0^1 D_t(i) di$$

during each period $t = 0, 1, 2, \ldots$.

The household uses its funds to purchase the finished good from the representative finished goods-producing firm at the nominal price $P_t$; the household divides its purchase into an amount $C_t$ to be consumed and an amount $I_t$ to be invested. By investing $I_t$ units of the finished good during period $t$, the household
increases the capital stock \( K_{t+1} \) during period \( t + 1 \) according to

\[
K_{t+1} = (1 - \delta) K_t + I_t, \tag{1}
\]

where \( 1 > \delta > 0 \). The household must also pay a capital adjustment cost, measured in terms of the finished good and given by

\[
\frac{\phi_K}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 K_t.
\]

where \( \phi_K > 0 \). The household then carries \( M_t \) units of money, \( B_t \) bonds, and \( K_{t+1} \) units of capital into period \( t + 1 \); these quantities must satisfy the budget constraint

\[
\frac{M_{t-1} + B_{t-1} + T_t + W_t H_t + Q_t K_t + D_t}{P_t} \geq C_t + I_t + \frac{\phi_K}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 K_t + \frac{B_t/B_t + M_t}{P_t}.
\]

The household, therefore, chooses \( C_t, H_t, M_t, B_t, I_t \), and \( K_{t+1} \) for all \( t = 0, 1, 2, \ldots \) to maximize its expected utility, given by

\[
E \sum_{t=0}^{\infty} \beta^t \left[ a_t \left( \frac{\gamma}{(\gamma - 1)} \right) \ln \left( C_t^{(\gamma - 1)/\gamma} + b_t^{1/\gamma} (M_t/P_t)^{(\gamma - 1)/\gamma} \right) + \eta \ln (1 - H_t) \right], \tag{3}
\]
subject to the constraints imposed by (1) and (2) for all $t = 0, 1, 2, \ldots$. In (3), $1 > \beta > 0$, $\gamma > 0$, and $\eta > 0$. The preference shock $a_t$ follows the autoregressive process

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at},$$

where $1 > \rho_a > 0$ and $\varepsilon_{at}$ is a zero-mean, serially uncorrelated innovation. There are two possible interpretations of this aggregate disturbance. First, $a_t$ enters into the Euler equation linking consumption growth to the interest rate; thus, McCallum and Nelson (1997) show that $a_t$ resembles a shock to the IS curve in more traditional, Keynesian analyses. Second, Benhabib, Rogerson, and Wright (1991) and Bencivenga (1992) demonstrate that a preference shock like $a_t$ captures the effects of shocks to a household production technology that occur in more detailed models in which goods are produced both at home and in the marketplace.

The preference shock $b_t$ follows the autoregressive process

$$\ln(b_t) = (1 - \rho_b) \ln(b) + \rho_b \ln(b_{t-1}) + \varepsilon_{bt},$$

where $b > 0$, $1 > \rho_b > 0$, and $\varepsilon_{bt}$ is a zero-mean, serially uncorrelated innovation. Kim (1996) and Ireland (1997) show that the first-order conditions for the
The household’s problem can be combined to obtain the money demand equation

\[
\ln(\frac{M_t}{P_t}) \approx \ln(C_t) - \gamma \ln(r_t) + \ln(b_t),
\]

(4)

where \( r_t = R_t - 1 \) denotes the net nominal interest rate. Equation (4) reveals that \( \gamma \) measures the interest elasticity of money demand, while \( b_t \) acts as a serially correlated money demand shock.

2.3. The Representative Finished Goods-Producing Firm

The representative finished goods-producing firm uses \( Y_t(i) \) units of each intermediate good \( i \in [0, 1] \), purchased at the nominal price \( P_t(i) \), to produce \( Y_t \) units of the finished good according to the technology described by

\[
\left[ \int_0^1 Y_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \geq Y_t,
\]

(5)

where \( \theta > 1 \). Thus, during each period \( t = 0, 1, 2, \ldots \), the finished goods-producing firm chooses \( Y_t \) and \( Y_t(i) \) for all \( i \in [0, 1] \) to maximize its profits,

\[
P_t Y_t - \int_0^1 P_t(i) Y_t(i) di.
\]
subject to the constraint imposed by (5). The first-order conditions for this problem imply that

\[ Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t, \tag{6} \]

which reveals that \(-\theta\) measures the price elasticity of demand for intermediate good \(i\). Competition in the market for the finished good drives the representative firm’s profit to zero; along with (6), this zero-profit condition determines \(P_t\) as

\[ P_t = \left[ \frac{1}{\theta} \int_0^1 P_t(i)^{1-\theta} \, di \right]^{1/(1-\theta)} \]

for all \(t = 0, 1, 2, \ldots\).

2.4. The Representative Intermediate Goods-Producing Firm

The representative intermediate goods-producing firm hires \(H_t(i)\) units of labor and \(K_t(i)\) units of capital from the representative household during period \(t\) to produce \(Y_t(i)\) units of intermediate good \(i\) according to the technology described by

\[ K_t(i)^{\alpha} [z_t H_t(i)]^{1-\alpha} \geq Y_t(i), \tag{7} \]
where $1 > \alpha > 0$. In (7), the aggregate technology shock $z_t$ follows the autoregressive process

$$\ln(z_t) = (1 - \rho_z) \ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt},$$

where $z > 0$, $1 > \rho_z > 0$, and $\varepsilon_{zt}$ is a zero-mean, serially uncorrelated innovation.

Since the intermediate goods substitute imperfectly for one another in the production function (5) for the finished good, the representative intermediate goods-producing firm sells its output in a monopolistically competitive market. Thus, during period $t$, the intermediate goods-producing firm sets the price $P_t(i)$ for its output, subject to the requirement that it satisfy the representative finished goods-producing firm’s demand (6), taking $P_t$ and $Y_t$ as given.

In addition, the intermediate goods-producing firm faces costs of adjusting its nominal price, measured in terms of the finished good and given by

$$\frac{\phi_{p1}}{2} \left[ \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 Y_t + \frac{\phi_{p2}}{2} \left[ \frac{P_t(i)}{P_{t-1}(i)} - 1 \right]^2 Y_t,$$

(8)

where $\phi_{p1} > 0$, $\phi_{p2} > 0$, and $\pi > 1$ denotes the gross steady-state rate of inflation. The specification in (8) generalizes Rotemberg’s (1982) quadratic cost of price
adjustment as suggested by Price (1992) and Tinsley (1998), so that costs apply to changes in both the price level and the inflation rate. Nelson (1998a) shows that generalized adjustment costs of this form allow the model to reproduce the dynamics implied by Fuhrer and Moore’s (1995) model of inflation persistence; Brayton et al. (1997) describe how these generalized price adjustment costs are incorporated into the Federal Reserve’s large-scale, FRB/US econometric model. Generalized adjustment cost specifications like (8) are also used by Pesaran (1991) to model employment in the UK coal industry and Koiziki and Tinsley (1998) to model the demand for producers’ durable equipment in the US.

The costs of adjustment in (8) make the intermediate goods-producing firm’s problem dynamic: it chooses $H_t(i), K_t(i), Y_t(i)$, and $P_t(i)$ for all $t = 0, 1, 2, ...$ to maximize its total market value, equal to

$$E \sum_{t=0}^{\infty} \left( \beta^t \Lambda_t / P_t \right) D_t(i), \tag{9}$$

subject to the constraints imposed by (6) and (7) for all $t = 0, 1, 2, ...$, where

$$D_t(i) = P_t(i) Y_t(i) - W_t H_t(i) - Q_t K_t(i)$$

$$- \frac{\phi_{P1}}{2} \left[ \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 P_t Y_t - \frac{\phi_P}{2} \left[ \frac{P_t(i) / P_{t-1}(i)}{P_{t-1}(i) / P_{t-2}(i)} - 1 \right]^2 P_t Y_t$$
measures the firm’s nominal profits during period $t$. In (9), $\Lambda_t$ is the nonnegative Lagrange multiplier on the budget constraint (2) from the representative household’s problem, so that $\beta^t \Lambda_t / P_t$ measures the marginal utility value to the household of an additional dollar in profits received during period $t$.

2.5. The Monetary Authority

The monetary authority conducts monetary policy by adjusting the short-term nominal interest rate $R_t$ in response to changes in output $Y_t$, inflation $\pi_t = P_t / P_{t-1}$, and money growth $\mu_t = M_t / M_{t-1}$ according to policy rule

$$\ln (R_t / R) = \rho_Y \ln (Y_t / Y) + \rho_\pi \ln (\pi_t / \pi) + \rho_\mu \ln (\mu_t / \mu) + \varepsilon_{R_t},$$  \hspace{1cm} (10)$$

where $R$, $Y$, $\pi$, and $\mu$ are the steady-state values of $R_t$, $Y_t$, $\pi_t$, and $\mu_t$ and where $\varepsilon_{R_t}$ is zero-mean, serially uncorrelated innovation. The policy rule (10) resembles the one used by Taylor (1993) to describe Federal Reserve behavior from 1987 through 1992, but generalizes Taylor’s specification by allowing policy to respond to changes in money growth as well as output and inflation. Different monetary policy regimes correspond to different choices of the steady-state inflation rate $\pi$ and the response coefficients $\rho_Y$, $\rho_\pi$, and $\rho_\mu$. 

12
2.6. Symmetric Equilibrium

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that $P_t(i) = P_t$, $Y_t(i) = Y_t$, $H_t(i) = H_t$, $K_t(i) = K_t$, and $D_t(i) = D_t$ for all $i \in [0, 1]$ during each period $t = 0, 1, 2, \ldots$. In addition, the market-clearing conditions $M_t = M_{t-1} + T_t$ and $B_t = B_{t-1} = 0$ must hold for all $t = 0, 1, 2, \ldots$. These equilibrium conditions, together with the first-order conditions for the representative agents' problems, the laws of motion for the aggregate shocks, and the policy rule (10), form a system of difference equations describing the model's symmetric equilibrium. This system implies that in the absence of shocks, the economy converges to a steady state. The system can be log-linearized around its steady state; the methods of Blanchard and Kahn (1980) can then be applied to obtain a solution of the form

$$s_t = \Pi s_{t-1} + \Omega \varepsilon_t$$  \hspace{1cm} (11)

and

$$f_t = Us_t$$  \hspace{1cm} (12)

for all $t = 0, 1, 2, \ldots$. 

13
In (11) and (12), the vector $s_t$ keeps track of the model's state variables, which include the lagged values of real balances $m_{t-1} = M_{t-1}/P_{t-1}$ (because prices are sticky) and inflation $\pi_{t-1}$ (because inflation is sticky) as well as the current values of the capital stock $K_t$ and the shocks $a_t$, $b_t$, $z_t$, and $\varepsilon_{R_t}$. The vector $f_t$ keeps track of the model's flow variables, which include the interest rate $R_t$, output $Y_t$, consumption $C_t$, investment $I_t$, hours worked $H_t$, the real factor prices $w_t = W_t/P_t$ and $q_t = Q_t/P_t$, real profits $d_t = D_t/P_t$, money growth $\mu_t$, and the multiplier $\Lambda_t$ on (2) that enters into (9). The vector $\varepsilon_t$ contains the four innovations $\varepsilon_{at}$, $\varepsilon_{bt}$, $\varepsilon_{zt}$, and $\varepsilon_{Rt}$; for the purposes of estimation, $\varepsilon_t$ is assumed to be normally distributed with covariance matrix

$$E\varepsilon_t\varepsilon'_t = \begin{bmatrix} \sigma_a^2 & 0 & 0 & 0 \\ 0 & \sigma_b^2 & 0 & 0 \\ 0 & 0 & \sigma_z^2 & 0 \\ 0 & 0 & 0 & \sigma_R^2 \end{bmatrix}. $$

The elements of the matrices $\Pi$, $\Omega$, and $U$ all depend on the parameters describing private agents' tastes and technologies and the parameters of the policy rule (10).
3. Data, Estimates, and Tests

3.1. Data

Equations (11) and (12) take the form of a state-space econometric model, driven by the four innovations in $\varepsilon_t$. Thus, maximum likelihood estimates of the parameters in $\Pi$, $\Omega$, and $U$ can be obtained, as described by Hamilton (1994, Ch.13), using the Kalman filter together with data on four variables: output $Y_t$, real money balances $m_t$, inflation $\pi_t$, and the short-term nominal interest rate $R_t$.

Thus, in the data, output is measured by real GDP, while real balances are measured by dividing the M2 money stock by the GDP implicit price deflator. Inflation is measured by changes in the GDP deflator, and the interest rate is measured by the yield on three-month Treasury bills. All series, except for the interest rate, are seasonally adjusted; the series for output and real balances are expressed in per-capita terms by dividing by the civilian, noninstitutional population, age 16 and over.

The data are quarterly and run from 1959:1 through 1998:4. Since one objective of this paper is to test for the stability of the model’s structural parameters, however, the data are divided into two subsamples, the first covering the period from 1959:1 through 1979:2 and the second covering the period from
1979:3 through 1998:4. The breakpoint, of course, corresponds to the beginning of Paul Volker’s tenure as Chairman of the Federal Reserve System. A fundamental change in US monetary policy is widely-believed to have occurred at that time (see, for example, Clarida, Gali, and Gertler 1998), making a comparison of estimates obtained from the two subsamples ideal in assessing whether the DSGE model lives up to its promise of being truly structural.

Distinct upward trends appear in the series for output and real balances, reflecting the secular growth of the US economy. Ireland (1997) accounts for these trends in data through 1995 by including a deterministic term in the production function (7) that captures the effects of labor-augmenting technological progress. The model then implies that \( Y_t \) and \( m_t \) grow at the same rate \( g \) along a balanced growth path; \( g \) can be estimated together with the other parameters describing tastes and technologies. Two recent developments preclude the use of the same approach here. First, real M2 has grown at a much slower rate than output since 1990; Mehra (1997) shows that conventional M2 demand equations such as (4) must be modified to correct for this shift. Second, the Federal Reserve redefined its monetary aggregates in 1996, removing overnight repurchase agreements and Eurodollar deposits from M2; Orphanides and Porter (1998) find that this change introduced differential trends into real M2 and output even in pre-1990 data. To
accommodate these developments, output and real balances are expressed here as deviations from separate linear trends that are allowed to change across subsamples. Thus, the model is not required to explain the institutional changes, described more fully in Ireland (1994a, 1994b, 1995), that have generated long-run trends in the income velocity of money, nor is the model required to account for the productivity slowdown that lowers the annualized, trend rate of growth in real, per-capita output from 1.9 percent in the pre-1979 data to 1.6 percent in the post-1979 data.

Here, as in Ireland (1997), the data do not contain enough information to estimate all of the model’s parameters; some must be fixed prior to estimation. In particular, the parameters \( \eta, \delta, \alpha, \) and \( \phi_K \) are difficult to estimate without data on hours worked and investment. Thus, the weight \( \eta \) on leisure in the representative household’s utility function (3) is set equal to 1.5, implying that the household spends about one-third of its time working. The quarterly depreciation rate \( \delta \) is set equal to 0.025, and capital’s share in production \( \alpha \) is set equal to 0.36, values commonly used in the literature. The capital adjustment cost parameter \( \phi_K \) is set equal to 10; higher values for \( \phi_K \) led to unreasonably large estimates of \( \sigma_z \), the standard deviation of the innovation \( \varepsilon_{zt} \) to the technology shock \( z_t \). Finally, the model’s monopolistically competitive market structure works mainly to lower the
equilibrium level of output; the effects of changes in \( \theta \), measuring the degree of market power possessed by the representative intermediate goods-producing firm, are difficult to distinguish from the effects of changes in \( z \), measuring the average value of the technology shock. Thus, \( \theta \) is set equal to 6, implying an average markup of price over marginal cost equal to Rotemberg and Woodford’s (1992) benchmark of 20 percent.

3.2. Estimates

Table 1 displays maximum likelihood estimates of the model’s remaining 17 parameters, together with their standard errors, computed by taking the square roots of the diagonal elements of negative one times the inverted matrix of second derivatives of the maximized log likelihood function. Focusing first on the parameters of the policy rule (10), the estimates of \( \pi \) translate into annualized, steady-state inflation rates of 5.3 percent for the pre-1979 subsample and 3.5 percent for the post-1979 subsample. Related, the interest rate response to inflation, measured by \( \rho_\pi \), is larger when estimated with post-1979 data.

These estimates imply that Federal Reserve Chairmen Volker and Greenspan have been more aggressive than their predecessors in fighting inflation. Clarida, Gali, and Gertler (1998), who estimate policy rules resembling (10), reach a similar
conclusion; indeed, their estimates suggest that by responding only weakly to inflation in the pre-1979 period, the Fed failed to guarantee the uniqueness of the economy’s equilibrium. Unlike the rules estimated by Clarida, Gali, and Gertler (1998), however, (10) includes the rate of money growth in the list of variables with which the Fed is concerned. Here, the combined response of the interest rate to changes in inflation and money growth makes the model’s equilibrium unique, even under the pre-1979 estimates.

Turning next to the parameters describing the costs of price adjustment, the estimates of $\phi_{p1}$ are large, though imprecise, while the estimates of $\phi_{p2}$ are quite small. Thus, the data appear to prefer a version of the model in which costs of adjustment apply to the price level but not to the inflation rate.

Why is rigidity in inflation found to be unimportant here, contradicting the results in Fuhrer and Moore (1995) and Nelson (1998b)? Rotemberg (1997) suggests that the special features of the Fuhrer-Moore model are unessential once persistent shocks to preferences and technologies are allowed for. Here, in fact, the estimates of $\rho_a$ and $\rho_z$ imply that these shocks are very persistent. Thus, figure 1, which shows the impulse responses of inflation to one standard deviation innovations to $a_t$ and $z_t$, reveals that these shocks generate persistent movements in inflation even in the absence of large adjustment costs. In each case, infla-
tion remains away from its steady-state level more than ten years after the initial disturbance.

Figure 2 displays the impulse responses of the interest rate and output to a one standard deviation policy shock $\varepsilon_{Rt}$. This disturbance represents an exogenous tightening of monetary policy: the annualized interest rate rises on impact by 8 basis points using the pre-1979 estimates and by 12.5 basis points using the post-1979 estimates. Persistent movements in the variables on the right-hand side of (10) generate persistence in the interest rate response, even without the smoothing terms that Clarida, Gali, and Gertler (1998) include in their specification. Output falls sharply on impact and remains below its steady-state level more than three years after the shock.

But while the model allows monetary policy shocks to have large and persistent effects, the small estimates of $\sigma_R$ imply that in both subsamples, most of the observed variation in interest rates reflects the Fed’s deliberate response to other shocks that have hit the US economy. Thus, table 2, which decomposes forecast error variances in output into components attributable to each of the model’s four disturbances, indicates that the preference shock $a_t$ and the technology shock $z_t$ drive nearly all of the movements in output during both sample periods.
3.3. Tests

Andrews and Fair (1988) describe procedures that can be used to test for the stability of the model’s estimated parameters across the two subsamples. Let the vector \( \Theta_q^1 \) contain \( q \) parameters estimated with pre-1979 data, let the vector \( \Theta_q^2 \) contain the same \( q \) parameters estimated with post-1979 data, and let \( H_q^1 \) and \( H_q^2 \) denote the covariance matrices of \( \Theta_q^1 \) and \( \Theta_q^2 \). Andrews and Fair (1988) show that the Wald statistic

\[ W = (\Theta_q^1 - \Theta_q^2)’(H_q^1 + H_q^2)^{-1}(\Theta_q^1 - \Theta_q^2) \]

is asymptotically distributed as a chi-square random variable with \( q \) degrees of freedom under the null hypothesis of parameter stability: \( \Theta_q^1 = \Theta_q^2 \).

Table 3 shows that the Wald test rejects the null hypothesis that all of the model’s 17 estimated parameters are stable across the two subsamples. Estrella and Fuhrer (1999) also find evidence of parameter instability in a forward-looking monetary business cycle model, but their framework is not detailed enough to pinpoint the exact source of this instability. Thus, table 3 goes on to classify the 17 parameters estimated here into five groups and to test the parameters in each group for stability.
Surprisingly, perhaps, the test fails to reject the null hypothesis that the five parameters describing monetary policy, $\pi$, $\rho_Y$, $\rho_{\pi}$, $\rho_{\mu}$, and $\sigma_R$, have remained stable. The test also fails to reject the hypothesis that the price adjustment cost parameters $\phi_{p1}$ and $\phi_{p2}$ have remained stable, although this result may reflect the fact, noted above, that the estimates of $\phi_{p1}$ are imprecise. The three parameters describing technology shocks, $z$, $\rho_z$, and $\sigma_z$, appear stable as well. Instead, the null hypothesis of stability is rejected for the set of three parameters, $\beta$, $\rho_a$, and $\sigma_a$, describing preferences and the set of four parameters, $\gamma$, $b$, $\rho_b$, and $\sigma_b$, describing money demand.

That the parameters of the money demand equation (4) exhibit instability comes as no surprise; Goldfeld and Sichel (1990), among others, describe the chronic instability that has plagued US money demand specifications for most of the past 25 years. Nor does this instability in money demand necessarily present a problem for DSGE models of the business cycle; Rotemberg and Woodford (1997), for example, show how a sticky-price DSGE model can be constructed without any reference to the demand for, or supply of, money.

Instability in the preference parameters $\beta$, $\rho_a$, and $\sigma_a$ is more troubling, however. McCallum and Nelson (1997) show that these parameters govern the behavior of what corresponds to the model’s IS curve and, as emphasized by Estrella and
Fuhrer (1999), this curve will lie at the core of any monetary business cycle model. Here and in McCallum and Nelson (1997), the IS curve is simply a log-linearized version of the Euler equation linking the representative household's consumption growth to the interest rate. Thus, the instability in $\beta$, $\rho_a$, and $\sigma_a$ detected here may be related to Mankiw (1981) and Hansen and Singleton’s (1983) rejections of the restrictions implied by that Euler equation; more stable specifications might be found in the literature on asset prices and consumption that responds to those rejections. Alternatively, Benhabib, Rogerson, and Wright (1991) and Bencivenga (1992) show that the preference shock $a_t$ can be viewed as standing in for a disturbance to a home production technology, suggesting that the model’s stability might be improved through a more detailed description of the household sector.

4. Conclusion

As its title suggests, this paper focuses on the specification and stability of an estimated, sticky-price model of the business cycle. Regarding specification, the results indicate that the US data prefer a DSGE model in which adjustment costs apply to the price level but not to the inflation rate; here, persistent movements in inflation are attributed to persistence in exogenous shocks to preferences and technologies rather than to large costs of adjustment. Regarding stability, the
results provide evidence that the DSGE model fails to deliver on its promise of being fully structural. The parameter instability found here is concentrated in the Euler equation linking an optimizing household’s consumption growth to the rate of interest. Evidently, future work with sticky-price models of the business cycle must confront the fact that more detailed descriptions of the household sector are needed to explain the joint time series behavior of consumption (or output) and the interest rate.

5. References


Ireland, Peter N. "Endogenous Financial Innovation and the Demand for Money." *Journal of Money, Credit, and Banking* 27 (February 1995): 107-123.

Ireland, Peter N. "A Small, Structural, Quarterly Model for Monetary Policy Evaluation." *Carnegie-Rochester Conference Series on Public Policy* 47


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pre-1979 Estimate</th>
<th>Standard Error</th>
<th>Post-1979 Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9974</td>
<td>0.0012</td>
<td>0.9911</td>
<td>0.0013</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1908</td>
<td>0.0401</td>
<td>0.1184</td>
<td>0.0315</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.9399</td>
<td>0.0197</td>
<td>0.8920</td>
<td>0.0369</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.03057</td>
<td>0.00487</td>
<td>0.03512</td>
<td>0.00785</td>
</tr>
<tr>
<td>$b$</td>
<td>1.438</td>
<td>0.242</td>
<td>1.913</td>
<td>0.242</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.8919</td>
<td>0.0419</td>
<td>0.9680</td>
<td>0.0205</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.01736</td>
<td>0.00262</td>
<td>0.01092</td>
<td>0.00129</td>
</tr>
<tr>
<td>$z$</td>
<td>3999</td>
<td>210</td>
<td>4528</td>
<td>250</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.9203</td>
<td>0.0439</td>
<td>0.9564</td>
<td>0.0302</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.01586</td>
<td>0.00462</td>
<td>0.008186</td>
<td>0.002049</td>
</tr>
<tr>
<td>$\phi_{p1}$</td>
<td>72.01</td>
<td>34.56</td>
<td>77.10</td>
<td>84.62</td>
</tr>
<tr>
<td>$\phi_{p2}$</td>
<td>0.00004830</td>
<td>0.24697628</td>
<td>0.000003661</td>
<td>0.087183406</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.01290</td>
<td>0.00211</td>
<td>1.008681</td>
<td>0.002812</td>
</tr>
<tr>
<td>$\rho_Y$</td>
<td>0.04986</td>
<td>0.02555</td>
<td>0.08227</td>
<td>0.04726</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>0.8617</td>
<td>0.0984</td>
<td>0.9918</td>
<td>0.3650</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>0.7351</td>
<td>0.1928</td>
<td>0.5867</td>
<td>0.2697</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.007078</td>
<td>0.001531</td>
<td>0.005069</td>
<td>0.001342</td>
</tr>
</tbody>
</table>
Table 2. Forecast Error Variance Decompositions for Output

Pre-1979

<table>
<thead>
<tr>
<th>Quarters Ahead</th>
<th>Preference</th>
<th>Money Demand</th>
<th>Technology</th>
<th>Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45</td>
<td>23</td>
<td>6</td>
<td>26</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
<td>8</td>
<td>37</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>5</td>
<td>50</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>38</td>
<td>4</td>
<td>54</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>35</td>
<td>3</td>
<td>58</td>
<td>4</td>
</tr>
<tr>
<td>40</td>
<td>33</td>
<td>3</td>
<td>61</td>
<td>3</td>
</tr>
<tr>
<td>∞</td>
<td>33</td>
<td>3</td>
<td>61</td>
<td>3</td>
</tr>
</tbody>
</table>

Post-1979

<table>
<thead>
<tr>
<th>Quarters Ahead</th>
<th>Preference</th>
<th>Money Demand</th>
<th>Technology</th>
<th>Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>20</td>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
<td>10</td>
<td>18</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>58</td>
<td>6</td>
<td>31</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>52</td>
<td>5</td>
<td>39</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>44</td>
<td>4</td>
<td>49</td>
<td>3</td>
</tr>
<tr>
<td>40</td>
<td>37</td>
<td>4</td>
<td>57</td>
<td>3</td>
</tr>
<tr>
<td>∞</td>
<td>35</td>
<td>4</td>
<td>59</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: The table shows the percentage of each forecast error variance attributable to each of the model’s four shocks.
Table 3. Tests for Parameter Stability

<table>
<thead>
<tr>
<th>Category</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 Estimated Parameters</td>
<td></td>
<td>$W = 132.3078^{***}$</td>
</tr>
<tr>
<td>5 Policy Parameters: $\pi, \rho_Y, \rho_\pi, \rho_\mu, \sigma_R$</td>
<td></td>
<td>$W = 8.2066$</td>
</tr>
<tr>
<td>2 Price Adjustment Parameters: $\phi_{P1}, \phi_{P2}$</td>
<td></td>
<td>$W = 0.0032$</td>
</tr>
<tr>
<td>3 Technology Parameters: $z, \rho_z, \sigma_z$</td>
<td></td>
<td>$W = 4.6781$</td>
</tr>
<tr>
<td>3 Preference Parameters: $\beta, \rho_a, \sigma_a$</td>
<td></td>
<td>$W = 19.8826^{***}$</td>
</tr>
<tr>
<td>4 Money Demand Parameters: $\gamma, b, \rho_b, \sigma_b$</td>
<td></td>
<td>$W = 53.5484^{***}$</td>
</tr>
</tbody>
</table>

*Note:*** denotes significance at the 1% level.*
FIGURE 1

Response of Inflation to Preference Shock

Response of Inflation to Technology Shock