A NONPARAMETRIC INVESTIGATION OF THE 90-DAY T-BILL RATE

by

John T. Barkoulas

Visiting Assistant Professor of Economics
Department of Economics
Boston College
Chestnut Hill, MA 02167
617-552-3682

Christopher F. Baum

Associate Professor of Economics
Department of Economics
Boston College
Chestnut Hill, MA 02167
617-552-3673

Joseph Onochie

Assistant Professor of Finance
Box E-0621
Baruch College
The City University of New York
17 Lexington Avenue
New York, NY 10010
Tel: 212-802-6380
Fax: 212-802-6353

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Abstract

We employ a nonlinear, nonparametric method to model the stochastic behavior of changes in the 90-day U.S. T-bill rate. The estimation technique is locally weighted regression (LWR), a nearest-neighbor method, and the forecasting criteria are the root mean square error (RMSE) and mean absolute deviation (MAD) measures. We compare the forecasting performance of the nonparametric fit to the performance of two benchmark linear models: an autoregressive model and a random-walk-with-drift model. The nonparametric fit results in significant improvements in forecasting accuracy as compared to benchmark linear models both in-sample and out-of-sample, thus establishing the presence of substantial nonlinear mean predictability in the 90-day T-bill rate.

1. Introduction

Predicting the levels and direction of changes in interest rates is of interest to participants in financial markets. Owners and managers of fixed income portfolios will find accurate forecasts essential. Despite the sizable body of research focusing on the term structure of interest rates, models based on the analytics of this relationship–whether arbitrage-based (e.g. Merton (1973), Heath et al. (1992)) or of a general equilibrium nature (e.g. Cox et al. (1985a,b), Longstaff and Schwartz (1992))–have not proven to be reliable in the prediction of short-term interest rate movements. We are much better able to identify arbitrage opportunities at a point in time than we are able to forecast interest rate movements over a near-term horizon. Models of these dynamics, such as Cox et al. (1985b), generally rely upon restrictive assumptions of linearity on the dynamics of the underlying processes and stability of their conditional moments.

These assumptions would seem to be at odds with the data, as nonlinearities in many high-frequency asset returns have been well documented in the literature. To deal with these stylized facts, models of asset returns (such as Bollerslev's (1986) GARCH) assume linear conditional means in asset returns but nonlinearities in conditional variances or higher moments**.** But important questions are still unanswered. For example, are nonlinearities in conditional higher moments structural or are they misspecifications of models that serve merely as proxies for neglected nonlinearities in the conditional mean?

Analysis of nonlinearities in the dynamics of short-term interest rates is of great importance at both theoretical and empirical levels. From a theoretical perspective, it would challenge our understanding of the term structure, as short-term rates are the main input in models of the term structure of interest rates. Any nonlinearities in the dynamics of short-term rates will translate into nonlinearities in the term structure of interest rates and will imply that the levels relation between short and long rates is no longer linear.[1] Thus nonlinear interest rate dynamics have asset-pricing implications for the term structure of interest rates. From an empirical perspective, the presence of nonlinearities would form the basis for improved predictability of interest rates.

Recent empirical research documents nonlinear dynamics both in the mean and in the variance of interest rates. Hamilton (1988) applies a Markov switching model to U.S. short-term interest rate data and finds that this model fits the data better than a linear autoregressive model. Granger (1993) shows that the U.S. short-term interest rate depends in a nonlinear manner on the spread between long and short interest rates. Anderson (1994) provides additional evidence for the types of nonlinear effects reported in Granger. Kozicki (1994) finds asymmetry in the form of differing responses to positive and negative shocks. Naik and Lee (1993) and Das (1993) link the nonlinearities to changes in economic regimes and stochastic jumps, respectively. Finally, Pfann, Schotman, and Tscherning (1996) explore the scope of nonlinear dynamics in short-term interest rates and its implications for the term structure. Using SETAR models and accounting for heteroscedasticity, they find evidence for the presence of two regimes with distinct dynamics in the mean. Until interest rates reach double digits, they behave like a random walk. At higher levels, however, they show a mean-reverting tendency. This mounting evidence raises serious questions about the adequacy of a linear process to fit the short-term U.S. interest rate. These forms of nonlinearity, however, only represent a subset of the class of plausible nonlinear models.

To address these issues of model specification and predictability, this study models the dynamic behavior of a short-term interest rate, the 90-day U.S. Treasury bill (T-bill) rate, using a nonlinear, nonparametric estimation method. In contrast to parametric nonlinear approaches, our nonparametric approach does not impose any specific type of nonlinearity in the estimation process but, instead, lets the data determine a suitable regression function. Therefore, the nonparametric approach avoids the parametric-model selection problem and allows for a wider array of nonlinear behavior. Following Diebold and Nason (1990), we use the locally weighted regression method (henceforth LWR), a nonparametric estimation method, to model nonlinearities in mean returns of the 90-day U.S. T-bill rate. We measure the forecasting accurancy of our LWR model using both root mean square error (RMSE) and mean absolute deviation (MAD) criteria. We compare the forecasting performance of the nonparametric fit to the performance of two benchmark linear models: an autoregressive model and a random walk model with drift. We find that the LWR forecasts are superior to these linear forecasts both in-sample and out-of-sample. This evidence therefore establishes the presence of substantial nonlinear mean predictability in the 90-day T-bill rate as well as the effectiveness of the LWR method as a modeling strategy for short-term interest rates.

The plan of the paper is as follows. In section 2 we present the nonparametric method. Data, diagnostic tests, and empirical estimates are presented in section 3. Finally, we conclude in section 4 with a summary of our results and suggestions for future research.

2. The Locally Weighted Regression (LWR) Method

We attempt to uncover nonlinear relationships in the 90-day T-bill rate using the nonparametric locally weighted regression (LWR) method. LWR is a nearest-neighbor (NN) estimation technique, first introduced by Cleveland (1979) and further developed by Cleveland and Devlin (1988) and Cleveland, Devlin, and Grosse (1988). It is a way of estimating a regression surface through a multivariate smoothing procedure, fitting a function of independent variables locally and in a moving-average manner.

Suppose that the regression function is given by

$$y_t = g(x_t) + \varepsilon_t, \qquad t = 1, \ldots, n \tag{1}$$

where $x_t = (x_{1t}, \ldots, x_{pt})$ is a $1 \times p$ vector of (weakly) exogenous explanatory variables, $g()$ is a smooth function mapping $R^p \to R$, and ε_t is an independent and identically distributed disturbance with mean zero and variance σ^2.

LWR is a numerical algorithm that describes how $\hat{g}(x)$, the estimate of g at the specific value of x, is estimated. Let f be a smoothing constant such that $0 < f \leq 1$, and let $q_f = \text{int}(f\ n)$, where $\text{int}()$ extracts the integer part of its argument. The LWR uses the "window" of q_f observations nearest to x, where proximity is defined using the Euclidean distance. In LWR the conditional mean is determined from a weighted least squares regression of y on x for the relevant q_f observations. More specifically, given a point x called the current state, rank the x_t's by Euclidean distance from x. Let $\|\|$

measure Euclidean distance; then the Euclidean distance from x to its x_{q_f} nearest neighbors is

$$\mathrm{d}\left(x, x_{q_f}\right) = \left[\sum_{j=1}^{p}\left(x_{q_{fj}} - x_j^*\right)^2\right]^{1/2}. \tag{2}$$

Each of the q_f nearest neighbors is weighted by Euclidean distance from the current state and the rest of the observations are assigned a weight of zero. To set the observation weights, we use the tricube weighting function $w_{it} = \left(1 - u^3\right)^3$, where

$$u \equiv \frac{\left\|x_{it} - x_t\right\|}{\left\|x_q - x_t\right\|} \tag{3}$$

The value of the regression surface at x is then computed as

$$\hat{y} = \hat{g}(x) = x'\hat{\beta}, \tag{4}$$

where

$$\hat{\beta} = \operatorname{argmin}\left[\sum_{t=1}^{n} w_t\left(y_t - x_t'\beta\right)^2\right] \tag{5}$$

Stone (1977) formulated the problem of consistent estimation through regularity conditions on weights of the neighbors. Consistency of NN estimators (and therefore LWR) requires that the number of NNs used go to infinity with sample size, but at a slower rate, that is, as $n \to \infty, q \to \infty$, but

$q/n \to 0$. Consistency becomes a matter of imposing a selection rule on f. As f increases, the bias in $\hat{g}(x)$ tends to increase and the sampling variablility tends to decrease. In practice one needs to choose f to balance the trade-off between bias and variance.

The LWR estimator of $g()$ is linear in y:

$$\hat{g}(x) = \sum_{i=1}^{n} l_i(x) y_i \tag{6}$$

where the $l_i(x)$ depend on x_t, $t = 1,\ldots n$, w, d, and f, but not on the y_i. Therefore the statistical properties of the estimators can be derived with standard techniques. A difficulty arises since the projection matrix $(I - L)$ which generates LWR residuals is neither idempotent nor symmetric. Although the exact distribution of the error sum of squares is not χ^2 (as the eigenvalues of $(I - L)$ need not be all ones or zeros), it can be approximated by a constant multiplied by a χ^2 variable. The constant and degrees of freedom are chosen so that the first two moments of the approximating distribution match those of the distribution of the error sum of squares (Kendall and Stuart, 1977).

3. Empirical Estimates

A. Data and Preliminary Diagnostic Tests

Our data are quarterly observations for the 90-day U.S. T-bill rate, referred to as the T-bill rate hereafter. The sample period is 1957:1 to 1988:4 (training set) and observations from 1989:1 to 1993:4 (test set) are used for one-step ahead forecasts. The source is the International Monetary Fund's (IMF) International Financial Statistics (IFS) data base. We compare the LWR model to benchmark linear models, in particular, the linear autoregressive (AR) model and the random walk (RW) model with drift.

Figures 1 and 2 graph the T-bill rate and its first differences over the entire period 1957:1-1993:4. Table 1 presents selected summary statistics for the first-differenced T-bill rate over the training sample period. T-bill rate changes are symmetric but leptokurtic.

We first investigate the low-frequency properties of the T-Bill rate series. To do so, we apply the Phillips-Perron tests (PP) (Phillips (1987), Phillips and Perron (1988)) to both levels and first differences of the T-bill rate. Table 2 presents the PP test results. Inference is robust to the order of serial correlation allowed in the data. All PP tests fail to reject the unit root null hypothesis in the T-bill rate but strongly reject the unit root null in T-bill rate changes. Therefore, the T-bill rate series is an integrated process of order one and we subsequently apply our analysis to T-bill rate changes.

To obtain some preliminary evidence regarding the presence of nonlinearities, we perform the test proposed by Brock, Dechert, and Scheinckman (BDS, 1987) on T-bill rate changes and linearly filtered T-bill rate changes. The linear filter is an AR model with the order chosen by the Akaike information criterion (AIC). The BDS test tests the null hypothesis of independent and identical distribution (i.i.d.) in the data against an unspecified departure from i.i.d. A rejection of the i.i.d. null hypothesis in the BDS test is consistent with some type of dependence in the data, which could result from a linear stochastic system, a nonlinear stochastic system, or a nonlinear deterministic system. The BDS test statistic converges to a normal distribution with unit variance, i.e., $N(0,1)$. A brief description of the BDS test appears in the Appendix.

Table 3 reports the BDS test statistics for the T-bill rate changes and the residuals obtained from fitting an AR model to T-bill rate changes. The autoregression order chosen by the AIC criterion is 7 (the maximum autoregression order allowed is 12). We applied the BDS test to these two series for embedding dimensions of $m = 2,3,4$ and 5 and for each m, ε is set to 0.5 and 1.0 standard deviations (σ) of the data. The i.i.d. null hypothesis is rejected in all cases for T-bill rate changes. When the BDS test is applied to the AR-filtered series we still obtain several rejections of the i.i.d. null hypothesis, suggesting that linear dependence in the first moments does not fully account for rejection of i.i.d. in the changes of the T-bill rate series. Consequently, an AR model for T-bill rate changes may not adequately describe the stochastic properties of the series. Nonlinearities in the conditional mean of the T-bill rate changes appear to exist and capturing them is the focus of the next section.

B. LWR Estimation and Forecasting Performance

In this section we estimate nonparametric models for T-bill rate changes based on the LWR methodology and compare their forecasting performance to benchmark linear models. The observations for the period 1989:1-1993:4 are reserved for forecasting purposes (test set), the out-of-sample forecasting horizon is one-step ahead, and the criteria for forecasting performance are root mean squared error (RMSE) and mean absolute deviation (MAD). The linear models we consider are the AR model and a

RW model with drift. As mentioned earlier, the autoregressive order chosen on the basis of AIC is found to be 7 lags.

We estimated LWR models of various orders for the T-bill rate changes. Table 4 reports the in-sample RMSE and MAD performance of the LWR, AR, and RW models. We report LWR fits for window sizes between 30 per cent and 90 per cent of the size of the training set as smaller window sizes result in inadequate subsamples. We present the nonparametric fit for autoregressive orders two, four, and six. Inference is similar for alternative lag structures, suggesting robustness with respect to autoregression order.[2] Among the linear models, the AR model outperforms the RW model. The in-sample forecasting performance of the LWR model improves with higher lag structure and smaller window size, thus suggesting possible overfitting. In all but three cases, the nonlinear autoregressive fit significantly outperforms that of the linear models on both RMSE and MAD forecasting criteria.

The excellent in-sample fit of the LWR model could, however, be the result of overfitting the data, as the equivalent number of parameters estimated in the LWR model is greater than those of the linear models. Choosing a model on the basis of its in-sample performance is not a good modeling strategy, especially when judging the reliability of nonlinear models. It must be taken out of sample. If the LWR model maintains its superior performance over its linear counterparts without the benefit of hindsight, then we can acknowledge its suitability as a better modeling strategy for T-bill rate changes.

Table 5 reports the one-step ahead forecasting performance of the LWR and its linear counterparts, the AR and RW models. The twenty observations for the period 1989:1-1993:4 constitute our test set. We report forecasts based on the nonparametric autoregression of orders one through five. The AR outperforms the RW model on an out-of-sample basis as well. The nonlinear forecasting performance appears to deteriorate with increasing lag structures and window sizes. However, the nonparametric fit continues to outperform its linear counterparts in terms of both RMSE and MAD in the vast majority of cases. The maximum reductions in RMSE and MAD achieved by the LWR fit over the AR fit are 19.80% and 24.86%, respectively. The superior performance of the LWR model over the linear models appears to be robust with respect to both autoregression order and window size. This consistency enhances the view that the estimated nonlinearities are not a statistical artifact, but rather that they capture essential aspects of the data generating process.

4. Conclusions

We provide evidence that the nonparametric fit given by the LWR method results in improvements, compared to a linear autoregressive model and a random-walk-with-drift model, in forecasting 90-day T-bill rate changes. The in-sample and out-of-sample superior performance of the LWR methodology appears to be robust to autoregression order, window size, and forecasting measure. This evidence is much more encouraging than that found for exchange rates (Diebold and Nason (1990), Meese and Rose (1990)) and stock returns (LeBaron (1988), Hsieh (1991)).

Our results could be extended to multiple-step-ahead forecasting horizons. The empirical validity of the LWR methodology for other U.S. interest rate series as well as for international interest rate series should also be investigated. Finally, the relative forecasting performance of the LWR forecasts to other nonlinear (parametric as well as nonparametric) forecasts should be examined. These questions await future research.

APPENDIX

The BDS Test

The BDS test, proposed by Brock, Dechert, and Scheinkman (BDS, 1987), tests the null hypothesis of independent and identical distribution (i.i.d.) in the data against a general nonlinear alternative. BDS show that under the null hypothesis that the time series y_t is i.i.d. with a nondegenerate density G, $C_m(\varepsilon,T) \to C_1(\varepsilon,T)^m$ with probability one as $T \to \infty$ for any fixed m and ε, where m is the embedding dimension, ε is a distance value, and C_m is the correlation integral corresponding to embedding dimension m.[3] The correlation integral measures the number of vectors within an ε neighborhood of one another and is given by

$$C_m(\varepsilon,T) = \lim_{T \to \infty} \frac{1}{T^2} \times \#\left\{(j,k)\left\| y_j^m - y_k^m \right\| < \varepsilon\right\}, \quad m = 2,3,..., \tag{A1}$$

where $\| \|$ is some norm, T is the number of m histories, and # denotes the cardinality of the set. The sequence of m-histories of the series is defined as

$$y_j^m = \left(y_j,..., y_{j+m-1}\right) \tag{A2}$$

that is, the m-dimensional vectors obtained by putting m consecutive observations together.

BDS show that the test statistic $\sqrt{T}\, C_m(\varepsilon,T) - C_1(\varepsilon,T)^m$ has a normal limiting distribution with zero mean and variance V. The variance V can also be consistently estimated from the sample data as $V(m,\varepsilon,T)$.[4] Dividing the statistic by the estimate of the standard deviation gives

$$W_m(\varepsilon,T) = \frac{\sqrt{T}\left(C_m(\varepsilon,T) - C_1(\varepsilon,T)^m\right)}{V_m(\varepsilon,T)^{1/2}} \tag{A3}$$

which converges to a normal distribution with unit variance, i.e., $N(0,1)$.

Simulations presented by BDS show that this test has good power against simple nonlinear deterministic systems as well as nonlinear stochastic processes. Brock, Hsieh, and LeBaron (1991) and Hsieh and LeBaron (1988) also report Monte Carlo simulations showing that the asymptotic distribution is a good approximation to the finite sample distribution when there are more than 500 observations. They recommend using ε between one-half to two times the standard deviation of the raw data. The accuracy of the asymptotic distribution deteriorates for high embedding dimensions, particularly when m is 10 and above.

Endnotes

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[1] Sims (1984), Abel (1988), Hodrick (1987), Baldwin and Lyons (1988), and Nason (1988) have shown that economic theory does not rule out the possibility of nonlinear dependence in conditional means and higher-order conditional moments of asset returns.

[2] The only exception was nonlinear autoregression of order one for which the performance of the LWR fit was inferior to linear fits.

[3] Note that i.i.d. implies that $C_m(\varepsilon, T) = C_1(\varepsilon, T)^m$ but the converse is not true. Dechert (1988) offers several counter examples.

[4] See Brock, Dechert, and Scheinkman (1987) for definition of the variance V.

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Table 1: Summary Statistics of 90-day T-bill Rate Changes 1957:1-1988:4

Statistic	T-bill Rate Changes
Mean	0.0356
Standard Deviation	0.8983
Skewness	-0.0202
Excess Kurtosis	6.6496***
Minimum	-3.4100
Maximum	4.4700

The superscript *** indicates statistical significance at the 1 per cent level.

Table 2: Phillips-Perron Unit Root Tests on 90-day T-bill Rates, 1957:1-1988:4

		Test Statistic						
Series	Lags	$Z(a*)$	$Z(t_{a*})$	$Z(\phi_1)$	$Z(\tilde{a})$	$Z(t_{\tilde{a}})$	$Z(\phi_2)$	$Z(\phi_3)$
T-bill Rates	2	-6.62	-1.87	1.84	-12.74	-2.51	2.18	3.28
	4	-6.83	-1.89	1.89	-13.59	-2.59	2.32	3.49
	6	-7.33	-1.96	2.01	-14.93	-2.72	2.54	3.82
	8	-6.76	-1.88	1.87	-14.12	-2.64	2.41	3.62
T-bill Rate Changes								
	2	-94.99	-9.22	42.51	-95.02	-9.18	28.08	42.53
	4	-92.35	-9.20	42.28	-92.29	-9.16	27.90	42.29
	6	-95.10	-9.23	42.52	-94.96	-9.18	28.07	42.52
	8	-87.41	-9.18	41.99	-87.17	-9.13	27.67	41.99

The seven different statistics all test for a unit root in the univariate time-series representation for each of the series against a stationary or trend-stationary alternative. In constructing the test statistics the order of serial correlation allowed is 2, 4, 6, and 8. Inference is robust to the order of serial correlation. We used the lag window suggested by Newey and West (1987) to ensure positive semidefiniteness. For more details on the tests see Phillips (1987), and Phillips and Perron (1988). The critical values are as follows (Fuller (1976), and Dickey and Fuller (1979)) with rejections of the null hypothesis indicated by large absolute values of the statistics:

Critical Values	$Z(a*)$	$Z(t_{a*})$	$Z(\phi_1)$	$Z(\tilde{a})$	$Z(t_{\tilde{a}})$	$Z(\phi_2)$	$Z(\phi_3)$
10%	-11.3	-2.57	3.78	-18.3	-3.12	4.03	5.34
5%	-14.1	-2.86	4.59	-21.8	-3.41	4.68	6.25
2.5%	-16.9	-3.12	5.38	-25.1	-3.66	5.31	7.16
1%	-20.7	-3.43	6.43	-29.5	-3.96	6.09	8.27

Table 3: BDS Test Results for 90-day T-bill Rate Changes and AR-Filtered 90-day T-bill Rate Changes, 1957:1-1988:4.

		T-bill Rate Changes		AR-Filtered T-bill Rate Changes		5% Critical Values	
		ε/σ		ε/σ		ε/σ	
		0.5	1.0	0.5	1.0	0.5	1.0
m	2	6.72	5.44	1.68	0.77	-3.87	-2.58
						4.33	2.70
	3	6.65	5.58	2.73	3.05	-4.54	-2.55
						5.05	2.84
	4	8.99	5.45	3.75	3.83	-5.50	-2.63
						6.48	3.07
	5	15.28	6.36	5.16	4.29	-7.11	-2.63
						8.98	3.31

The $BDS(m, \varepsilon)$ tests for i.i.d. where m is the embedding dimension and ε is distance, set in terms of the standard deviation of the data σ to 0.5 and 1 standard deviations. Critical values for the BDS test are the 95 per cent quantiles reported by Brock et al. for 100 observations (1991, Tables C1, p. 232). The AR-filtered series are the residual series obtained from fitting an AR model of order 7 to first-differenced T-bill rates.

Table 4: In-sample Forecasting Performance from Alternative Models for Predicting 90-day T-bill Rate Changes, 1957:2-1988:4

Window Size	Lags=2	Lags=4	Lags=6
0.30	0.6612	0.3360	0.2035
	0.4616	0.2327	0.1327
0.40	0.6804	0.3936	0.2840
	0.4715	0.2792	0.1912
0.50	0.6915	0.4372	0.3513
	0.4761	0.3164	0.2412
0.60	0.7040	0.4736	0.3976
	0.4846	0.3471	0.2770
0.70	0.7313	0.5110	0.4382
	0.5017	0.3773	0.3072
0.80	0.7615	0.5512	0.4798
	0.5152	0.4055	0.3372
0.90	0.7820	0.6166	0.5437
	0.5228	0.4407	0.3767
AR(7)	0.7871		
	0.5003		
RW	0.9030		
	0.5827		

Window size refers to the percentage of the in-sample observations chosen as nearest neighbors in the locally weighted regression (LWR). The first entry of each cell is the root mean squared error (RMSE), while the second is the mean absolute deviation (MAD). The LWR model is a nonlinear autoregression of order 2, 4, and 6. AR(k) stands for a linear autoregression of order k, which is chosen on the basis of the AIC criterion. RW stands for random walk (with drift). RMSE and MAD obtained from the LWR method which are lower than those from the AR model are underlined.

Table 5: Out-of-sample RMSE and MAD from Alternative Models for Predicting 90-day T-bill Rate Changes, 1989:1-1993:4

Window Size	Lags=1	Lags=2	Lags=3	Lags=4	Lags=5
0.30	0.3595	0.3659	0.4583	0.5008	0.5092
	0.2831	0.2999	0.3716	0.4063	0.4160
0.40	0.3604	0.3731	0.4121	0.4510	0.4616
	0.2765	0.2931	0.3305	0.3689	0.3789
0.50	0.3699	0.3840	0.3837	0.4211	0.4258
	0.2789	0.2977	0.2984	0.3403	0.3503
0.60	0.3710	0.3882	0.3747	0.4173	0.4171
	0.2797	0.2934	0.2856	0.3335	0.3418
0.70	0.3725	0.3907	0.3769	0.4155	0.4154
	0.2809	0.2895	0.2826	0.3308	0.3374
0.80	0.3829	0.3928	0.3840	0.4194	0.4067
	0.2826	0.2869	0.2878	0.3292	0.3297
0.90	0.3978	0.4043	0.3872	0.4085	0.3987
	0.2902	0.2926	0.2887	0.3194	0.3093
AR(7)	0.4483				
	0.3680				
RW	0.4767				
	0.3680				

See notes in Table 4 for explanation of table.

Figure 1: 90-Day T-bill Rates, 1957:1-1993:4

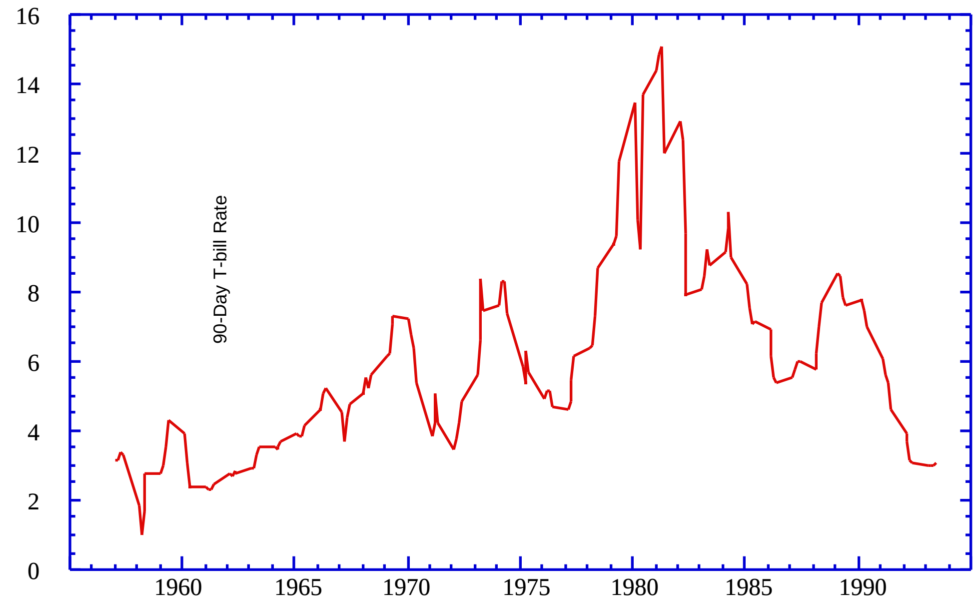

Figure 2: 90-Day T-bill Rate Changes, 1957:2-1993:4