

**FRACTIONAL DYNAMICS IN A SYSTEM OF
LONG TERM INTERNATIONAL INTEREST RATES**

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Abstract

DeGennaro, Kunkel, and Lee (1994) studied the long run dynamics of a system of long term interest rates of five industrialized countries by means of sophisticated cointegration methods. They found little evidence in support of the cointegration hypothesis, thus concluding that a separate set of fundamentals drives the dynamics of each of the individual long term interest rate series. In this study, we extend the existing literature by exploring the possibility of very slow mean reverting dynamics (fractional cointegration) in the same system of five long term interest rates. We use the GPH test as our testing methodology for fractional integration and cointegration. Through rigorous investigation of the full system of the five long term interest rate series and its various subsystems, we provide evidence that the error correction term follows a fractionally integrated process with long memory, that is, it is mean reverting, though not covariance stationary. Despite significant persistence in the short run, a shock to the system of long term interest rates eventually dissipates so that an equilibrium relationship prevails in the long run.

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I. Introduction

DeGennaro, Kunkel, and Lee (1994), DKL hereafter, rigorously investigated the integration and cointegration properties of interest rates on long term government bonds issued by the U.S., Canada, Germany, U.K., and Japan. Using a number of unit root and cointegration tests, they established that the long term interest rates of these industrial countries are individually best characterized as integrated processes of order one but there is no equilibrium relationship binding them together in the long run. This absence of cointegration implies that each interest rate series follows its own set of fundamentals and that the development of error correction models is not warranted, as it is not likely to improve forecasting performance. This evidence is in contrast to that of Mougoue (1992) for a system of short term interest rates (who finds a cointegrating relationship) but is in agreement with that of Diebold et al. (1994) for a system of foreign exchange rates (no evidence of a cointegrating relationship).

This study extends the existing literature by allowing deviations from equilibrium in the system of long term international interest rates to follow a fractionally integrated process. The cointegration methods employed by DKL (Johansen, Stock-Watson) only allow for an integer order of integration in the equilibrium error process, which is a rather restrictive and ad hoc assumption. We suggest a generalized form of cointegration, known as fractional cointegration, as a characterization of the long run dynamics of the system of long term interest rates. Fractional cointegration analysis allows the integration order of the error correction

term to take any value on the real line, that is, to be fractionally integrated. By doing so, the knife-edged $I(1)$ and $I(0)$ distinction is avoided and a wider range of mean reverting behavior can be captured. More specifically, a fractionally integrated error correction term implies the existence of a long run equilibrium relationship, as it can be shown to be mean reverting, though not exactly $I(0)$. Despite its significant persistence in the short run, the effect of a shock to the system eventually dissipates, so that an equilibrium relationship among the system's variables prevails in the long run. Fractional cointegration, which uses the notion of fractional differencing suggested by Granger and Joyeux (1980) and Hosking (1981), was first proposed by Engle and Granger (1987). It has been applied by Cheung and Lai (1993) and Baillie and Bollerslev (1994) among others. The authors of the latter study found the system of seven daily foreign exchange rates to be fractionally cointegrated, contrary to the conclusions of the study by Diebold et al., which was based on standard cointegration methodology. In this study, we apply fractional cointegration analysis to the system of five long term interest rates investigated by DKL. We find that each individual interest rate series possesses a single unit root but the error correction term follows a fractionally differenced process. The system of long term interest rates is found to be cointegrated, exhibiting subtle mean reverting dynamics.

The plan of the paper is as follows. Section 2 presents the concepts of fractional integration and cointegration analysis and the fractional integration test employed. The data and empirical results are discussed in Section 3. Finally, in Section 4 we summarize our results and offer suggestions for future research.

2. *Econometric Methodology*

A series is said to be integrated of order d , denoted by $I(d)$, if it has a stationary, invertible autoregressive moving average (ARMA) representation after applying the differencing operator $(1-L)^d$. The series is said to be fractionally integrated if d is not an integer. A system of time series $y_t = \{y_{1t}, y_{2t}, \dots, y_{nt}\}$ is said to be cointegrated of order $I(d, b)$ if the linear combination $z_t = \alpha y_t$, called the error correction term, is $I(d - b)$ with $b > 0$. Under the general hypothesis of cointegration of order $I(d, b)$ with $b > 0$, Cheung and Lai (1993) showed that the least squares estimate of the cointegrating vector is consistent and converges at the rate $O(T^b)$ as opposed to the rate of $O(T)$ in standard cointegration analysis in which $d = b = 1$. The system of variables is said to be fractionally cointegrated if the error correction term z_t is fractionally integrated. The relevant case in this study is one in which the $I(1)$ hypothesis for the individual time series cannot be rejected, but the error correction term is found to be $I(1 - b)$ with $b > 0$ and taking a noninteger value. In this scenario, the error correction term is mean reverting, though not necessarily covariance stationary, as a shock to the system persists, but it eventually dies out. There is a binding long run equilibrium among the system variables even though adjustments to equilibrium may take a long time to complete.

Fractional cointegration analysis is implemented in two steps. In the first step, the error correction term is obtained through ordinary least squares estimation of the cointegrating regression. The error correction term is subjected to a unit root test in the second step to determine its order of integration. The unit root test employed allows for a fractional exponent in the differencing process of the error correction term.

Granger (1986) showed that the fractionally cointegrated system has an error representation of the form

$$\Psi(L)(1-L)^d y_t = -\gamma \left[1 - (1-L)^b \right] (1-L)^{d-b} z_t + c(L) \varepsilon_t \quad (1)$$

where $\Psi(L)$ is a matrix polynomial in the lag operator L with $\Psi(0)$ being the identity matrix, $c(L)$ is a finite lag polynomial, and ε_t is a white noise error term. If expanded in powers of L , the lag function $\left[1 - (1-L)^b \right]$ has no term in L^0 , so only lagged values of the error correction term z_t enter the right hand side of equation (1). This suggests that improvements in the forecasting accuracy of the system's variables can be attained over that of the benchmark ARMA models.

The hypothesis of fractional cointegration requires testing for fractional integration in the error correction term. The fractional testing methodology employed here is the semi-parametric test suggested by Geweke and Porter-Hudak (GPH, 1983). We now proceed to describe the properties of a fractionally integrated process and the GPH testing method.

A flexible and parsimonious way to model short term and long term behavior of time series is by means of an autoregressive fractionally integrated moving average (ARFIMA) model. A time series $\mathbf{y} = \{y_1, \dots, y_T\}$ follows an autoregressive fractionally integrated moving average process of order (p, d, q) with mean μ , denoted by ARFIMA(p, d, q), if

$$\Phi(L)(1-L)^d (y_t - \mu) = \Theta(L)u_t, \quad \varepsilon_t \sim \text{i.i.d.}(0, \sigma_\varepsilon^2) \quad (2)$$

where L is the backward-shift operator, $\Phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p$, $\Theta(L) = 1 + \vartheta_1 L + \dots + \vartheta_q L^q$, and $(1-L)^d$ is the fractional differencing operator defined by

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)} \quad (3)$$

with $\Gamma(\cdot)$ denoting the gamma, or generalized factorial, function. The parameter d is allowed to assume any real value. The arbitrary restriction of d to integer values gives rise to the standard autoregressive integrated moving average (ARIMA) model. The stochastic process \mathbf{y} is both stationary and invertible if all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle and $|d| < \frac{1}{2}$. The process is nonstationary for $d \geq \frac{1}{2}$, as it possesses infinite variance, i.e. see Granger and Joyeux (1980).

ARFIMA series exhibit both long term dependence and short memory. The effect of the differencing parameter d on observations widely separated in time decays hyperbolically as the lag increases, thus describing the high-order correlation structure of the series. At the same time, the temporal effects of the autoregressive and moving average parameters, $\Phi(L)$ and $\Theta(L)$, decay exponentially, describing the low-order correlation structure of the series. Assuming that $-\frac{1}{2} < d < \frac{1}{2}$ and $d \neq 0$, Hosking (1981) showed that the correlation function, $\rho(\cdot)$, of an ARFIMA process is proportional to j^{2d-1} as $j \rightarrow \infty$. Consequently, the autocorrelations of the ARFIMA process decay hyperbolically to zero as $j \rightarrow \infty$, which is contrary to the faster, geometric decay of a stationary ARMA process. For $0 < d < \frac{1}{2}$, $\sum_{k=-n}^n |\rho(k)|$ diverges as $n \rightarrow \infty$, and the ARFIMA process is said to exhibit long memory.¹ The process exhibits short memory for $d=0$ and intermediate memory for $-\frac{1}{2} < d < 0$. For $d < 1$ the series is mean-reverting in that the cumulative impulse response of future values of the series to a unit innovation at an infinite horizon, denoted by c_∞ , is zero even though it is not covariance stationary for $\frac{1}{2} < d < 1$. For $d=1$, c_∞ is finite and nonzero; and for $d > 1$, c_∞ is infinite.

The existence of a fractional order of integration can be determined by testing for the statistical significance of the sample differencing parameter d , which is also interpreted as the long memory parameter. To estimate d and perform hypothesis

testing, we employ the semi-parametric procedure suggested by Geweke and Porter-Hudak (1983). They obtain an estimate of d based on the slope of the spectral density function around the angular frequency $\xi = 0$. More specifically, let $I(\xi)$ be the periodogram of y at frequency ξ defined by

$$I(\xi) = \frac{1}{2\pi T} \left| \sum_{t=1}^T e^{it\xi} (y_t - \bar{y}) \right|^2. \quad (4)$$

Then the spectral regression is defined by

$$\ln\{I(\xi_\lambda)\} = \beta_0 + \beta_1 \ln\left\{\sin^2\left(\frac{\xi_\lambda}{2}\right)\right\} + \eta_\lambda, \quad \lambda = 1, \dots, \nu \quad (5)$$

where $\xi_\lambda = \frac{2\pi\lambda}{T}$ ($\lambda = 0, \dots, T-1$) denotes the Fourier frequencies of the sample, T is the number of observations, and $\nu = g(T) \ll T$ is the number of Fourier frequencies included in the spectral regression.

Assuming that $\lim_{T \rightarrow \infty} g(T) = \infty$, $\lim_{T \rightarrow \infty} \left\{g(T)/T\right\} = 0$, and $\lim_{T \rightarrow \infty} \ln(T)^2/g(T) = 0$, the negative of the OLS estimate of the slope coefficient in (5) provides an estimate of d . Geweke and Porter-Hudak (1983) prove consistency and asymptotic normality for $d < 0$, while Robinson (1990) proves consistency for $0 < d < 1/2$. Hassler (1993) proves consistency and asymptotic normality in the case of Gaussian ARMA innovations in (1). The spectral regression estimator is not $T^{1/2}$ consistent and will converge at a slower rate. The theoretical asymptotic variance of the spectral regression error term is known to be $\pi^2/6$.

To ensure that stationarity and invertibility are achieved, we apply the GPH test to the first differences of the series. The differencing parameter in the first-differenced data is denoted by \tilde{d} in which case the fractional differencing parameter for the level series is $d = 1 + \tilde{d}$.

3. *Data and Empirical Estimates*

A. *Data*

The data set is that employed by DKL, consisting of interest rates on long term government bonds issued by the central governments of the U.S., Canada, Germany, U.K., and Japan. They are of monthly frequency and span the period January 1967 to December 1990 for a total of 288 observations. They are obtained from the *International Financial Statistics* data tape of the International Monetary Fund.

B. *Fractional Integration and Cointegration Analysis*

Before we proceed with our analysis, we briefly describe the results obtained by DKL. By means of Dickey-Fuller (Dickey and Fuller (1981), Fuller (1976)), Phillips-Perron (Phillips (1987), Phillips and Perron (1988)), KPSS (Kwiatkowski, Phillips, Schmidt, and Shin (1992)) and Bayesian (Sims (1988)) unit root tests, DKL established that each interest rate series is $I(1)$. They then applied the Johansen (Johansen and Juselius (1990)) and Stock & Watson (1988) cointegration methods and established the absence of any common nonstationary components in the system of long term interest rates. We have reproduced the results obtained by DKL.

As the unit root tests employed by DKL allow for only integer orders of integration, the interest rate series are each first checked for a fractional exponent in the differencing process using the GPH test. The unit-root hypothesis is tested by determining whether the GPH estimate of \tilde{d} from the first-differenced interest rate

series is significantly different from zero. The number of low-frequency periodogram ordinates used in the spectral regression in (5) must be chosen judiciously. Improper inclusion of medium- or high-frequency periodogram ordinates will contaminate the estimate of \tilde{d} ; at the same time too small a regression sample will lead to imprecise estimates. For both the interest rate series as well as the error correction term below, we report \tilde{d} estimates for $\nu = T^{0.55}, T^{0.575}$, and $T^{0.60}$ to check the sensitivity of our results to the choice of ν . This choice is made in light of the recommended choice by Geweke and Porter-Hudak, based on forecasting experiments, and the test performance of simulation experiments conducted by Cheung and Lai (1993). Table 1 reports the empirical estimates for the fractional differencing parameter $\tilde{d} = 1 - d$ as well as the test results regarding its statistical significance based on the GPH test. To test the statistical significance of the \tilde{d} estimates, two-sided ($\tilde{d} = 0$ versus $\tilde{d} \neq 0$) as well as one-sided ($\tilde{d} = 0$ versus $\tilde{d} < 0$ and $\tilde{d} = 0$ versus $\tilde{d} > 0$) tests are performed. We have imposed the known theoretical variance of the spectral regression error $\pi^2/6$ in the construction of the t -statistic for \tilde{d} in order to increase estimation efficiency. The GPH test statistics cannot reject the unit-root null hypothesis for any interest rate series. Some evidence of long memory is obtained for the Japanese interest rate series but it is not robust across the various sample sizes of the spectral regression considered. Consistent with evidence obtained by DKL based on integer order unit root tests, the results obtained here support the hypothesis that each interest rate series possesses a single unit root, that is, it is $I(1)$.

We now turn to investigate the time dependencies in the error correction term of the system of long term interest rates. In the classical paradigm of cointegration, the system variables y_t are $I(1)$ and the error correction term $z_t = \alpha y_t$ is $I(0)$. This $I(1)$ versus $I(0)$ characterization of the low frequency properties of the error correction term is strict and ad hoc as the error correction term can be mean

reverting without being exactly $I(0)$. The error correction term could be $I(d)$ with $0 < d < 1$, in which case deviations from equilibrium are persistent but the cumulative impulse response of a shock to the system equals zero at an infinite horizon. In this case, the error correction term follows a fractionally integrated process and the system's variables form a fractionally cointegrated system. It must be noted that, by simulation methods, Diebold and Rudebusch (1991) found that standard unit root tests have very low power against fractional alternatives. Cheung and Lai (1993) found similar evidence when the unit root null hypothesis for the error correction term is tested against fractional alternatives.

The hypothesis of fractional cointegration requires testing for fractional integration in the error correction term. The GPH test can be used for that purpose but the critical values for the GPH test derived from the standard normal distribution cannot be used in testing for fractional cointegration. This is due to the fact that the error correction term is not actually observed but estimated by minimizing the residual variance of the cointegrating regression. The error correction term thus obtained tends to be biased toward stationarity, causing the null hypothesis of non-cointegration in the GPH test to be rejected more often than suggested by the nominal size of the GPH test. Since the critical values for the GPH test for cointegration are non-standard, we estimate them through Monte Carlo simulations. Table 2 reports the empirical size of the GPH test for cointegration corresponding to our sample size of 288 observations for various dimensions of the system and sample sizes of the spectral regression. As seen, the empirical distribution of the GPH test statistic for cointegration is negatively skewed, which is consistent with the argument above that critical values from the standard normal distribution would bias inference toward finding cointegration too often. It is also apparent that the empirical size depends upon the dimension of the system.

To ensure robustness of our evidence as well as to obtain insights into the long term dynamics of the system of interest rates, we test for fractional cointegration for the full system as well as for all possible subsystems. More specifically, we investigate the low frequency properties of the error correction term obtained from estimating all possible bivariate, trivariate, and four-variate subsystems of interest rates as well as the full system. For all cases considered, we run the cointegrating regression with all possible normalizations. However, we report results for those cases for which the GPH test statistics are statistically significant and for those normalizations in each subsystem which result in the highest adjusted coefficient of determination (\bar{R}^2) in the corresponding cointegrating regression.²

Table 3 reports the GPH cointegration test results. Concentrating first at the bivariate systems of interest rates, there is evidence of fractional cointegration only between the Canadian and U.S. interest rate series. This finding is hardly surprising given the similarities and interdependencies between the two economies. The error correction term corresponding to this subsystem is not covariance stationary as $0.5 < d < 1$ but it is mean reverting. For no other bivariate subsystem is the null hypothesis of no cointegration rejected.

Examining the trivariate systems of interest rates, we find evidence of fractional cointegration for the subsystems of interest rates of the following countries: (Canada, U.S., Germany), (Canada, U.S., U.K.), and (Canada, U.S., Japan). A common feature of these three fractionally cointegrated subsystems is the inclusion of the Canadian and U.S. interest rates in each of them. The same pattern is evident in two subsystems of four interest rates: (Canada, U.S., Germany, U.K.) and (Canada, U.S., Germany, Japan), for which evidence of a mean reverting, though not covariance stationary, error correction term is obtained.

The full system of long term interest rates is now investigated. As Table 3 reports, all estimates of the fractional differencing parameter for the error correction term lie between 0 and 1. Formal hypothesis testing indicates that $d < 1$ except for the case corresponding to a sample size of the spectral regression of $T^{0.55}$. In this case, the fractional differencing parameter is large but it is associated with a rather large standard error, and the null hypothesis of a unit root is not rejected. For the other two sizes of the spectral regression considered, the evidence supports the hypothesis that $d < 1$.

The overall evidence from the full system of interest rates and the various subsystems suggests that the error correction term follows a fractionally differenced process with long memory. The evidence indicates that there exists a binding long run equilibrium relationship among the long term interest rates of the five major industrial countries. Even though the individual interest rate series wander widely, deviations from the cointegrating relationship are mean-reverting indicating that a shock to the system will eventually die out, and an equilibrium relationship among the interest rates will prevail in the long run.

To obtain further insight into the long run behavior of the system of long term interest rates, Figures 1 through 8 graph the first 60 autocorrelation coefficients of the error correction terms obtained from the cointegrating regressions reported in Table 3. Concentrating first on the correlogram of the error correction term from the full system, the autocorrelation function appears to decay at a rather rapid rate. The shape of the autocorrelation function of the error correction term differs greatly from the shape of the autocorrelation function of a typical $I(1)$ series, which exhibits a very slow linear decay. For comparison purposes, we graph in Figure 9 the sample autocorrelation function of the interest rate series for the U.S., which was found to be $I(1)$. A visual inspection of the sample autocorrelation function clearly suggests differences in the dynamical behavior between the error correction term and an $I(1)$

process. It is immediately clear that the error correction term is a long memory process with long term cycles in its autocorrelation. However, standard cointegration tests mistakenly interpret the subtle mean reverting dynamics in the error correction term as an indication of $I(1)$ behavior. The evidence is similar for the error correction terms obtained from the fractionally cointegrated subsystems of long term interest rates.

Evidence of fractional cointegration in the system of long term interest rates raises the possibility of improved forecasting performance through the development of an appropriate error correction model. Given that the system of interest rates is $I(1,b)$, it has an error correction representation of the form

$$\Psi(L)(1-L)y_t = -\gamma[1-(1-L)^b](1-L)^{1-b}z_t + c(L)\varepsilon_t. \quad (6)$$

This representation involves only $I(0)$ terms since after expanding the lag function in square brackets in terms of powers of L , only lagged values of the error correction term z_t enter in the right hand side of equation (6). Adding the deviations from the cointegrating relationship in a VAR model of interest rates should improve predictive accuracy. Given that the impact of long memory is likely to be persistent, any improvement in forecasting accuracy should be apparent over long forecasting horizons. The development of such an error correction model is beyond the scope of this paper.

Finally, the possibility of the need to have a time series representation more general than an ARFIMA model to completely describe the fractional behavior of the error correction term is now investigated. As Figures 1 through 8 indicate, the autocorrelation coefficients for the error correction term do not exhibit a monotonic decline, but rather sinusoidal behavior. A more complete description of cyclical components in the autocorrelation function for the error correction term necessitates

the use a model that can allow for long memory harmonic behavior. One possible candidate is the Gegenbauer autoregressive moving average (GARMA) model considered by Gray, Zhang, and Woodward (1989), which makes use of the generating function of the Gegenbauer polynomials. The general form of the GARMA(p, u, λ, q) model, $\lambda \neq 0$, is given by

$$\Phi(L)(1-2uL+L^2)^\lambda z_t = \Theta(L)\varepsilon_t \quad (7)$$

which reduces to the ARFIMA model for $u = 1$ and $\lambda = d/2$. Provided that all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle, the process \mathbf{y} is a long memory process if $0 < \lambda < 1/4$ when $u = \pm 1$ or $0 < \lambda < 1/2$ when $|u| < 1$. The autocorrelation function, $\rho(\cdot)$, of the GARMA process for $0 < \lambda < 1/2$ and $|u| < 1$ is proportional to $j^{2\lambda-1} \sin(\pi\lambda - j\omega_0)$ as $j \rightarrow \infty$, where ω_0 is the Gegenbauer frequency.

The significance of the extension to a GARMA model is the inclusion of periodic or quasi-periodic data in the long memory model.³ A GARMA model for the error correction term might be able to account for the periodic component in its autocorrelation function. Given the dearth of literature considering parameter estimation of the GARMA model, this possibility is not pursued here.

4. Conclusions and Implications

We reexamined the long run dynamics of a system of long term interest rates of the U.S., Canada, Germany, the U.K., and Japan by allowing deviations from equilibrium to follow a fractionally integrated process (fractional cointegration). Contrary to previous evidence by DeGennaro et al. (1994), which was based on cointegration tests allowing for only integer orders of integration in the error

correction term, we find that the long term interest rates form a fractionally cointegrated system. Even though each interest rate series is best characterized as a unit root process, the system of interest rates appears to possess a common fractional, nonstationary component. Shocks to the system exhibit significant persistence in the short run but they eventually dissipate, so that an equilibrium relationship is obeyed in the long run. Close examination of the long run behavior of the various subsystems of interest rates identifies a strong comovement between the Canadian and U.S. interest rates, which appears to be robust with respect to the dimension of the system. This is hardly surprising considering the strong ties between the two economies. Our evidence is consistent with that of Baillie and Bollerslev (1994) that a fractional cointegrating relationship exists among daily exchange rates for seven major currencies.

Our findings have several implications concerning modeling and forecasting of long term interest rates. There appears to be a common set of fundamentals that binds the long term interest rates together in the long run. Two reasons can be hypothesized to account for the presence of long memory in the cointegrating relationship among the long term interest rates. One reason could be the presence of long memory in the common set of fundamentals. Another possible reason to account for the observed long term behavior is the periodic interventions of the monetary authorities to affect the level or direction of interest rates. A direct implication of the presence of cointegration is the possibility of improved forecasting, especially over longer forecasting horizons via the estimation of an appropriate error correction model. However, it must be noted that estimation of such an error correction model does not automatically imply improvements in near-term forecasting accuracy. The impact of long memory is, by definition, spread over a lengthy period, so that any improvement in forecasting accuracy may only be apparent in the very long run.

Future research should extend the framework of fractional cointegration to a system of international short term interest rates. Even though Mougoue (1992) found that such a system exhibits cointegration, utilizing a more appropriate VAR specification in the Johansen procedure might overturn his original conclusion. Also, the possibility that nonlinear cointegration might exist should also be addressed.

Notes

¹ Some authors refer to a process as a long memory process for all $d \neq 0$.

² A high \bar{R}^2 in the cointegrating regression is a desirable feature since Monte Carlo simulations have shown that the bias in estimating the cointegrating vector diminishes as \bar{R}^2 approaches unity (Banerjee, Dolado, Hendry, and Smith (1986)). The strongest GPH evidence corresponds to the cointegrating regressions with the normalization that results in the highest \bar{R}^2 . Full GPH test results for all subsystems with all possible normalizations are available upon request from the authors.

³ While the ARFIMA model has a peak in the spectrum at $f = 0$, the GARMA process can model long term periodic behavior for any frequency $0 \leq f \leq 0.5$.

References

- Baillie, R. T. and T. Bollerslev (1989), Common Stochastic Trends in a System of Exchange Rates, *Journal of Finance*, 44, 167-181.
- Baillie, R. T. and T. Bollerslev (1994), Cointegration, Fractional Cointegration and Exchange Rate Dynamics, *Journal of Finance*, 49, 737-745.
- Banerjee, A. J., J. Dolado, D. F. Hendry, and G. W. Smith (1986), Exploring equilibrium relationships in econometrics through static models: Some Monte Carlo evidence, *Oxford Bulletin of Economics and Statistics*, 48, 253-278.
- Cheung Y. and K. Lai (1993), A fractional cointegration analysis of purchasing power parity, *Journal of Business and Economic Statistics*, 11, 103-112.
- DeGennaro, R., R. Kunkel, and J. Lee (1994), Modeling international long-term interest rates, *Financial Review*, 29, 577-597.
- Dickey, D.A. and W.A. Fuller (1981), Likelihood ratio statistics for autoregressive time series with a unit root, *Econometrica*, 49, 1057-1072.
- Diebold, F. X., J. Gardeazabal and K. Yilmaz (1994), On Cointegration and Exchange Rate Dynamics, *Journal of Finance*, 49, 727-735.
- Diebold, F. X. and G. D. Rudebusch (1991), On the power of Dickey-Fuller tests against fractional alternatives, *Economics Letters*, 35, 155-160.
- Engle, R. and C.W.J. Granger (1987), Cointegration and Error-Correction: Representation, Estimation, and Testing, *Econometrica*, 55, 251-276.
- Fuller W. (1976), *Introduction to Statistical Time Series*, New York: John Wiley & Sons.
- Geweke J. and S. Porter-Hudak (1983), The Estimation and Application of Long Memory Time Series Models, *Journal of Time Series Analysis*, 4, 221-238.
- Granger, C. W. J. (1986), Developments in the study of cointegrated economic variables, *Oxford Bulletin of Economics and Statistics*, 48, 213-228.

- Granger, C. W. J. and R. Joyeux (1980), An introduction to long-memory time series models and fractional differencing, *Journal of Time Series Analysis*, 1, 15-39.
- Gray, Henry, N. Zhang and W. Woodward (1989), On generalized fractional processes, *Journal of Time Series Analysis* 10, 233-257.
- Hassler, U. (1993), Regression of spectral estimators with fractionally integrated time series, *Journal of Time Series Analysis*, 14, 369-380.
- Hosking, J. R. M. (1981), Fractional Differencing, *Biometrika*, 68, 165-176.
- Johansen, S. and K. Juselius (1990), Maximum Likelihood Estimation and Inference on Cointegration-With Application to the Demand for Money, *Oxford Bulletin of Economics and Statistics*, 52, 169-210.
- Kwiatkowski, D., Phillips P. C. B., Schmidt P., and Y. Shin (1992), Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?, *Journal of Econometrics*, 54, 159-178.
- Mougoue, Mbodja (1992), The term structure of interest rates as a cointegrated system: Empirical evidence from the Eurocurrency market, *Journal of Financial Research*, 15, 285-296.
- Phillips, P. C. B. (1987), Time Series Regression with a Unit Root, *Econometrica*, 55, 277-301.
- _____ and P. Perron (1988), Testing for a Unit Root in Time Series Regression, *Biometrika*, 75, 335-346.
- Robinson, P. (1990), Time series with strong dependence, *Advances in Econometrics*, 6th World Congress. Cambridge: Cambridge University Press.
- Sims, C. A. (1988), Bayesian Skepticism on Unit Root Econometrics, *Journal of Economic Dynamics and Control*, 12, 463-474.
- Stock J. H. and M. W. Watson (1988), Testing for common trends, *Journal of the American Statistical Association*, 83, 1097-1107.

Table 1: Empirical Estimates for the Fractional-Differencing Parameter \tilde{d}

Interest Rate Series	$\tilde{d}(0.55)$	$\tilde{d}(0.575)$	$\tilde{d}(0.60)$
U.S.	0.106 (0.626)	0.147 (0.939)	0.054 (0.382)
Canada	0.075 (0.443)	0.125 (0.801)	0.017 (0.121)
Germany	0.093 (0.551)	0.113 (0.723)	0.106 (0.746)
U.K.	-0.067 (-0.396)	-0.040 (-0.257)	-0.074 (-0.520)
Japan	0.336 (1.973) ^{**} ,##	0.190 (1.215)	0.131 (0.917)

Notes: $\tilde{d}(0.55)$, $\tilde{d}(0.575)$, and $\tilde{d}(0.60)$ give the \tilde{d} estimates corresponding to the GPH spectral regression of sample size $\nu = T^{0.55}$, $\nu = T^{0.575}$, and $\nu = T^{0.60}$, respectively. The t -statistics are given in parentheses and are constructed imposing the known theoretical error variance of $\pi^2/6$. The superscripts ***, **, * indicate statistical significance for the null hypothesis $\tilde{d} = 0 (d = 1)$ against the alternative $\tilde{d} \neq 0 (d \neq 1)$ at the 1, 5, and 10 per cent levels, respectively. The superscripts ###, ##, # indicate statistical significance for the null hypothesis $\tilde{d} = 0 (d = 1)$ against the one-sided alternative $\tilde{d} > 0 (d > 1)$ at the 1, 5, and 10 per cent levels, respectively.

Figure 1: Correlogram of the Error Correction Term for the System of Interest Rates for Canada and U.S.

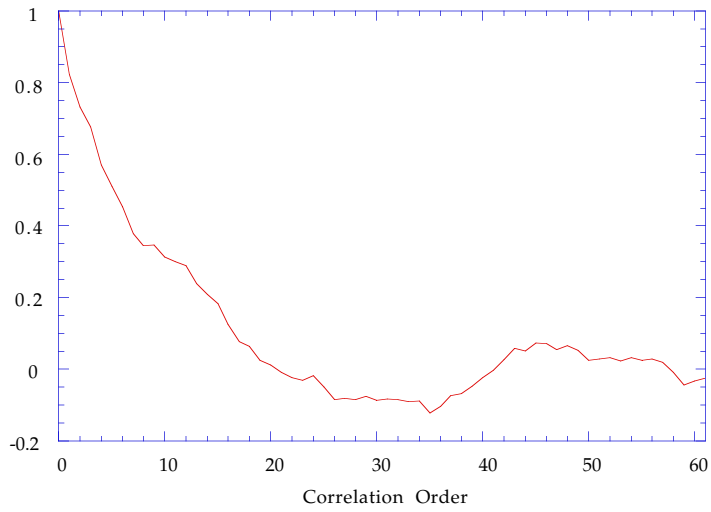


Figure 2: Correlogram of the Error Correction Term for the System of Interest Rates for Canada, U.S., and Germany

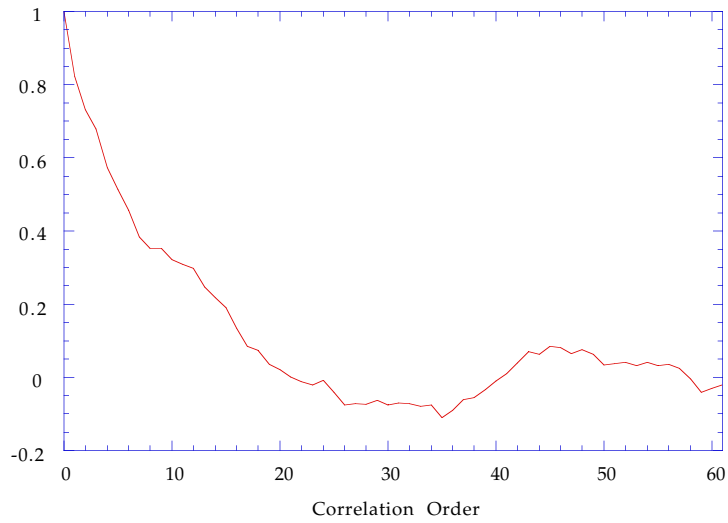


Figure 3: Correlogram of the Error Correction Term for the System of Interest Rates for Canada, U.S., and U.K.

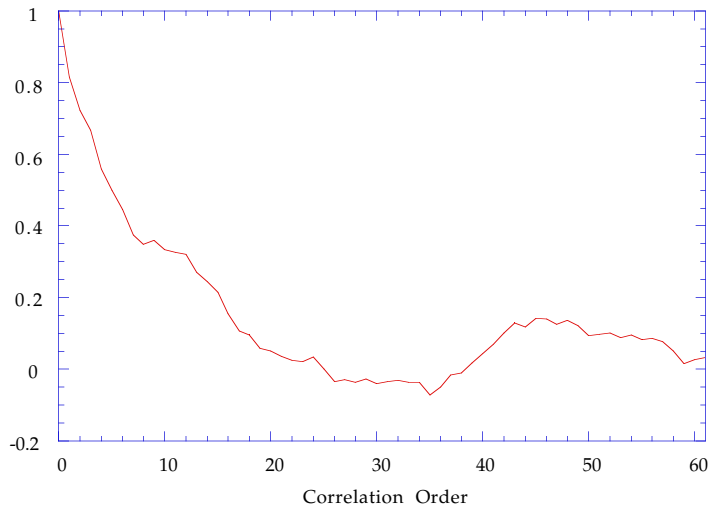


Figure 4: Correlogram of the Error Correction Term for the System of Interest Rates for Canada, U.S., and Japan

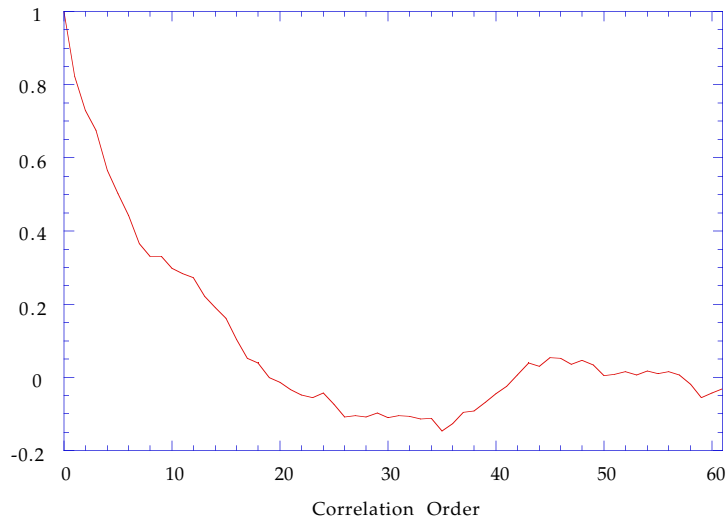


Figure 5: Correlogram of the Error Correction Term for the System of Interest Rates for Canada, U.S., Germany, and U.K.

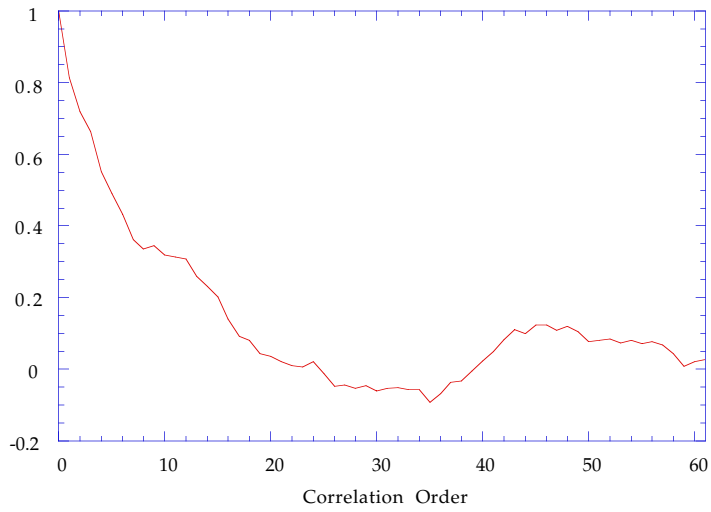


Figure 6: Correlogram of the Error Correction Term for the System of Interest Rates for Canada, U.S., Germany, and Japan

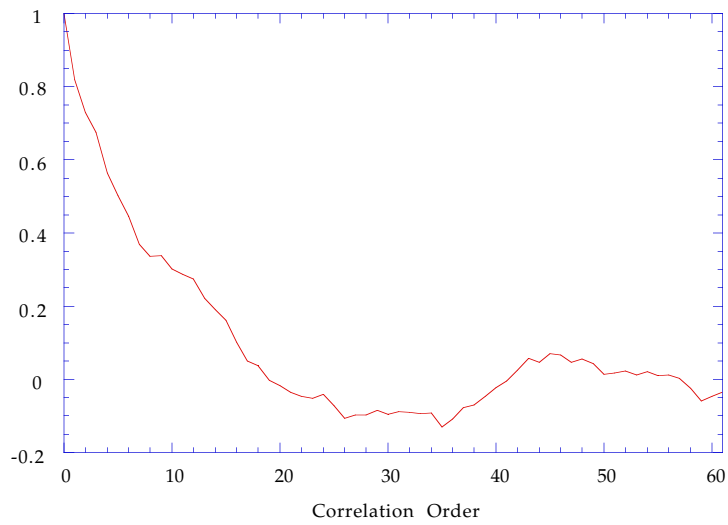


Figure 7: Correlogram of the Error Correction Term for the System of Interest Rates for Canada, U.S., U.K., and Japan

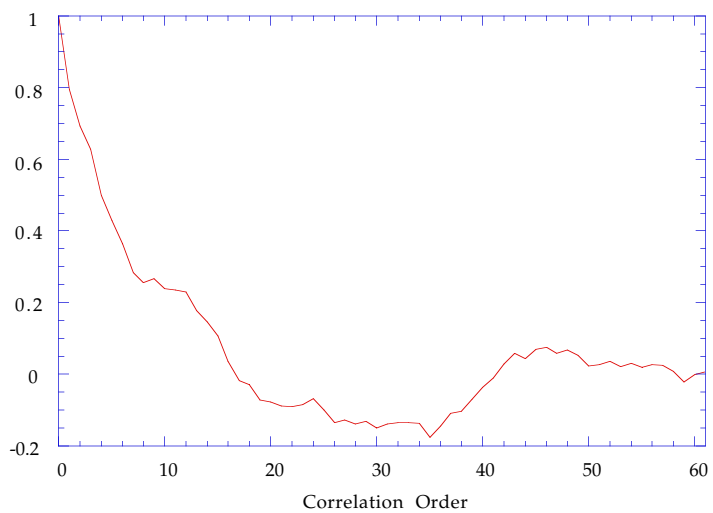


Figure 8: Correlogram of the Error Correction Term for the Full System of Interest Rates

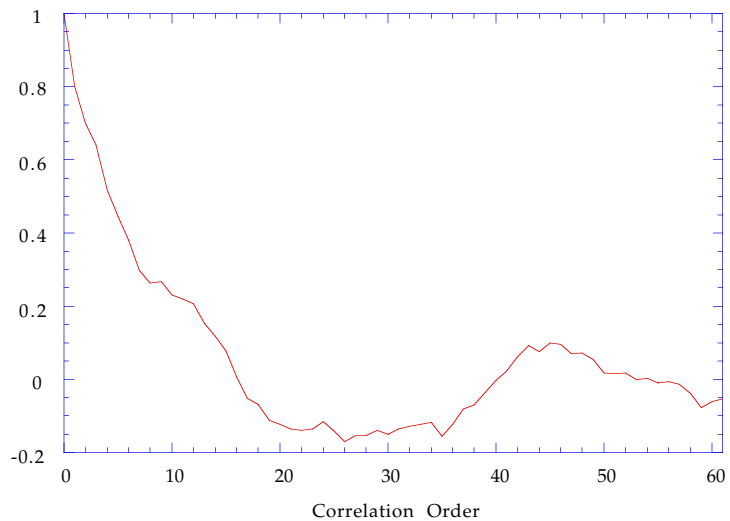


Figure 9: Correlogram of the U.S. Interest Rate Series

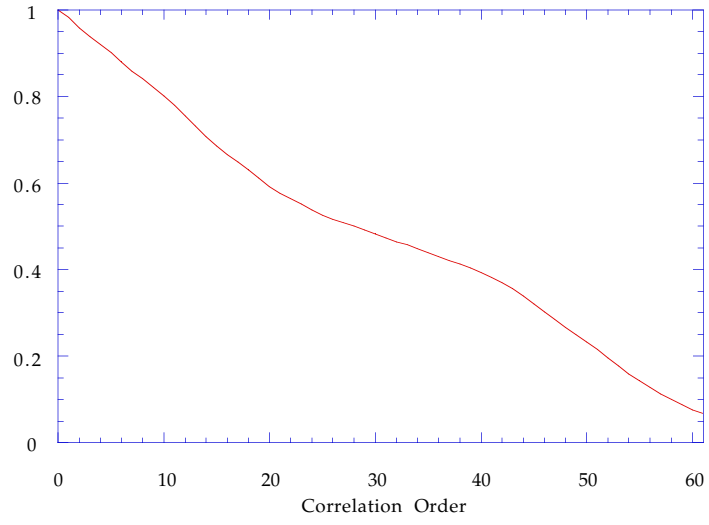


Table 2. Empirical Size for the GPH Test for Cointegration

Percentile	Dimension of the System											
	2			3			4			5		
	$\mu = 0.55$	$\mu = 0.575$	$\mu = 0.60$	$\mu = 0.55$	$\mu = 0.575$	$\mu = 0.60$	$\mu = 0.55$	$\mu = 0.575$	$\mu = 0.60$	$\mu = 0.55$	$\mu = 0.575$	$\mu = 0.60$
0.005	-3.190	-3.081	-3.152	-3.611	-3.560	-3.513	-4.064	-4.016	-4.003	-4.438	-4.425	-4.377
0.010	-2.840	-2.778	-2.845	-3.235	-3.194	-3.185	-3.670	-3.641	-3.624	-4.098	-4.048	-4.012
0.025	-2.350	-2.324	-2.346	-2.719	-2.703	-2.698	-3.129	-3.114	-3.100	-3.542	-3.504	-3.507
0.050	-1.956	-1.950	-1.955	-2.306	-2.304	-2.290	-2.705	-2.703	-2.691	-3.093	-3.064	-3.076
0.100	-1.520	-1.528	-1.525	-1.857	-1.862	-1.841	-2.260	-2.244	-2.237	-2.626	-2.595	-2.604
0.200	-1.034	-1.040	-1.034	-1.352	-1.347	-1.342	-1.722	-1.708	-1.709	-2.074	-2.043	-2.055
0.300	-0.692	-0.693	-0.692	-1.005	-0.999	-0.994	-1.353	-1.344	-1.346	-1.685	-1.662	-1.674
0.400	-0.415	-0.415	-0.412	-0.715	-0.706	-0.700	-1.046	-1.038	-1.040	-1.367	-1.352	-1.357
0.500	-0.160	-0.163	-0.152	-0.453	-0.445	-0.434	-0.766	-0.761	-0.764	-1.073	-1.067	-1.065
0.600	0.083	0.081	0.096	-0.190	-0.188	-0.171	-0.495	-0.484	-0.493	-0.798	-0.787	-0.782
0.700	0.339	0.343	0.361	0.075	0.086	0.092	-0.216	-0.208	-0.207	-0.506	-0.491	-0.486
0.800	0.637	0.646	0.664	0.383	0.394	0.406	0.104	0.110	0.121	-0.175	-0.166	-0.147
0.900	1.048	1.043	1.062	0.812	0.816	0.837	0.543	0.546	0.561	0.276	0.282	0.302
0.950	1.385	1.378	1.401	1.147	1.168	1.179	0.903	0.909	0.921	0.641	0.650	0.681
0.975	1.669	1.674	1.702	1.461	1.452	1.477	1.210	1.205	1.235	0.945	0.962	0.997
0.990	2.010	2.021	2.027	1.798	1.780	1.832	1.555	1.564	1.582	1.326	1.329	1.362
0.995	2.249	2.254	2.273	2.031	2.036	2.065	1.810	1.815	1.807	1.570	1.570	1.619
Mean	-0.208	-0.207	-0.198	-0.497	-0.491	-0.479	-0.820	-0.812	-0.808	-1.135	-1.118	-1.115
Skewness	-0.284	-0.268	-0.278	-0.295	-0.285	-0.272	-0.295	-0.292	-0.269	-0.297	-0.293	-0.259
Excess Kurtosis	0.383	0.348	0.329	0.400	0.335	0.342	0.316	0.308	0.278	0.293	0.277	0.237

Notes: The sample size for the GPH spectral regression is given by $\nu = T^\mu$, where T equals 288, the number of observations in our sample, and $\mu = 0.55, 0.575, \text{ and } 0.60$. Dimension of the system refers to the number of variables in the system. The empirical size is based on 50,000 replications, assuming that the true system consists of an appropriate number of non-cointegrated random-walk processes, where the number of variables corresponds to the dimension of the system.

Table 3: Empirical Estimates for the Cointegration Fractional-Differencing Parameter \tilde{d}

System of Interest Rate Series	Cointegrating Vector	\bar{R}^2	$\tilde{d}(0.55)$	$\tilde{d}(0.575)$	$\tilde{d}(0.60)$
Canada-U.S.	(1.000, 0.971, 1.341)	0.945	-0.316 (-1.856) [‡]	-0.384 (-2.447) ^{**} , ^{‡‡}	-0.412 (-2.878) ^{**} , ^{‡‡‡}
Canada-U.S.-Germany	(1.000, 0.968, 0.015, 1.250)	0.945	-0.329 (-1.934) [‡]	-0.396 (-2.523) [*] , ^{‡‡}	-0.425 (-2.973) ^{**} , ^{‡‡}
Canada-U.S.-U.K.	(1.000, 0.939, 0.062, 0.925)	0.948	-0.384 (-2.255) [‡]	-0.459 (-2.927) ^{**} , ^{‡‡}	-0.488 (-3.414) ^{**} , ^{‡‡‡}
Canada-U.S.-Japan	(1.000, 0.974, -0.026, 1.499)	0.945	-0.288 (-1.694)	-0.359 (-2.288) [‡]	-0.386 (-2.699) ^{**} , ^{‡‡}
Canada-U.S.-Germany-U.K.	(1.000, 0.941, -0.030, 0.070)	0.948	-0.363 (-2.133)	-0.442 (-2.815) [*] , ^{‡‡}	-0.467 (-3.266) ^{**} , ^{‡‡}
Canada-U.S.-Germany-Japan	(1.000, 0.965, 0.073, 0.072, 1.349)	0.946	-0.296 (-1.740)	-0.364 (-2.320) [‡]	-0.398 (-2.783) [*] , ^{‡‡}
Canada-U.S.-U.K.-Japan	(1.000, 0.924, 0.121, -0.135, 1.357)	0.952	-0.310 (-1.821)	-0.398 (-2.535) [‡]	-0.415 (-2.902) [*] , ^{‡‡}
Canada-U.S.-Germany-U.K.-Japan	(1.000, 0.912, 0.088, 0.125, -0.195, 1.170)	0.953	-0.348 (-2.047)	-0.428 (-2.728) [‡]	-0.447 (-3.126) [*] , ^{‡‡}

Notes: The first column gives the countries for which the interest rate series enter the cointegrated system. The second column gives the cointegrating vector for the corresponding cointegrated system: The normalizing variable, that is, the regressand has a coefficient value of unity while the last term in the cointegrating vector gives the coefficient value for the constant term. \bar{R}^2 is the adjusted coefficient of determination in the corresponding cointegrating regression. $\tilde{d}(0.55)$, $\tilde{d}(0.575)$, and $\tilde{d}(0.60)$ give the \tilde{d} estimates corresponding to the GPH spectral regression of sample size $\nu = T^{0.55}$, $\nu = T^{0.575}$, and $\nu = T^{0.60}$, respectively, for the error correction term obtained from the corresponding cointegrating regression. The t -statistics are given in parentheses and are constructed imposing the known theoretical error variance of $\pi^2/6$. The superscripts ***, **, * indicate statistical significance for the null hypothesis $\tilde{d} = 0 (d = 1)$ against the alternative $\tilde{d} \neq 0 (d \neq 1)$ at the 1, 5, and 10 per cent levels, respectively. The superscripts ‡‡‡, ‡‡, ‡ indicate statistical significance for the null hypothesis $\tilde{d} = 0 (d = 1)$ against the one-sided alternative $\tilde{d} < 0 (d < 1)$ at the 1, 5, and 10 per cent levels, respectively. Critical values are based on the simulated values presented in Table 2.