LONG TERM DEPENDENCE IN STOCK RETURNS

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Keywords: Stock returns, long memory, fractal dynamics, spectral regression

JEL: G12, C14

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Abstract

We test for long term dependence in U.S. stock returns, analyzing composite and sectoral stock indices and firms' returns series to evaluate aggregation effects. Fractal dynamics are not detected in stock indices, but are present in some firms' returns series.

1. Introduction

Long memory, or long term dependence, describes the correlation structure of a series at long lags. If a series exhibits long memory (or the "biased random walk"), there is persistent temporal dependence even between distant observations. Such series are characterized by distinct but nonperiodic cyclical patterns. Mandelbrot (1977) characterizes long memory processes as having "fractal dimensions." The presence of long memory dynamics in asset prices would provide evidence against the weak form of market efficiency as it implies nonlinear dependence in the first moment of the distribution and hence a potentially predictable component in the series dynamics. It would also raise issues regarding linear modeling, forecasting, statistical testing of pricing models based on standard statistical methods, and theoretical and econometric modeling of asset pricing.

The most widely used tests for fractal dynamics are the rescaled-range (R/S) analysis introduced by Hurst (1951) and later refined by Mandelbrot (1972, 1975) and

Mandelbrot and Wallis (1969), the modified R/S analysis introduced by Lo (1991), and the spectral regression method suggested by Geweke and Porter-Hudak (1983). Long memory analysis has been conducted for stock returns series (Greene and Fielitz (1977), Aydogan and Booth (1988), Lo (1991), Cheung, Lai, and Lai (1993), Cheung and Lai (1995), Chow, Denning, Ferris, and Noronha (1995)) with most evidence suggesting the absence of fractal structure in stock returns. All these studies have used returns series on stock indices, whose construction entails a great deal of aggregation. If fractal structure does exist in individual stock returns series, its presence may be masked in aggregate returns series. This paper considers that possibility by employing the spectral regression method to test for long memory in a variety of aggregate and sectoral stock indices and stock returns series for individual companies.

The plan of this paper is as follows. Section 2 presents the technical details of the fractional integration test. Empirical results are discussed in Section 3. Finally, in Section 4 we summarize our results.

2. The Spectral Regression Test for Long Memory

A flexible and parsimonious way to model both the short term and long term behavior of a time series is by means of an autoregressive fractionally integrated moving average (ARFIMA) model. A time series $\mathbf{y} = \{y_1, \ldots, y_T\}$ with mean δ follows an autoregressive fractionally integrated moving average process of order (p,d,q), denoted by $\text{ARFIMA}(p,d,q)$, if

$$\Phi(L)(1-L)^d\left(y_t-\delta\right)=\Theta(L)\,\varepsilon_t, \qquad \varepsilon_t \sim \text{iid}(0,\ \sigma_\varepsilon^2) \tag{1}$$

where L is the backward-shift operator, $\Phi(L) = 1 - \phi_1 L - ... - \phi_p L^p$, $\Theta(L) = 1 + \vartheta_1 L + ... + \vartheta_q L^q$, and $(1-L)^d$ is the fractional differencing operator defined by

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d) L^k}{\Gamma(-d)\Gamma(k+1)} \qquad (2)$$

with $\Gamma(\cdot)$ denoting the gamma, or generalized factorial, function. The arbitrary restriction of d to integer values gives rise to the standard autoregressive integrated moving average (ARIMA) model, rendering the ARIMA model a special case of the ARFIMA model. The stochastic process $\mathbf{y}$ is both stationary and invertible if all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle and $|d| < 0.5$. The process is nonstationary for $d \geq 0.5$, as it possesses infinite variance (cf. Granger and Joyeux (1980)). Assuming that $d \in (-0.5, 0.5)$ and $d \neq 0$, Hosking (1981) showed that the correlation function, $\rho(\cdot)$, of an ARFIMA process is proportional to j^{2d-1} as $j \to \infty$. Consequently, the autocorrelations of the ARFIMA process decay hyperbolically to zero as $j \to \infty$, which is contrary to the faster, geometric decay of a stationary ARMA process. For $d \in (0, 0.5)$, $\sum_{j=-n}^{n} |\rho(j)|$ diverges as $n \to \infty$, and the ARFIMA process is said to exhibit long memory, or long-range positive dependence. The process exhibits intermediate memory, or long-range negative dependence, for $d \in (-0.5, 0)$.[1] The process possesses only short memory for $d = 0$ (corresponding to the standard ARMA model). For $d \in [0.5, 1)$ the process is mean reverting as there is no long run impact of an innovation to future values of the process.

Geweke and Porter-Hudak (1983) suggested a semi-parametric procedure to obtain an estimate of the fractional differencing parameter d based on the slope of the spectral density function around the angular frequency $\xi = 0$. More specifically, let $I(\xi)$ be the periodogram of $\mathbf{y}$ at frequency ξ defined by

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[1] Other authors refer to a process as a long memory process if $d \neq 0$.

$$I(\xi) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} e^{it\xi}(y_t - \bar{y})\right|^2. \tag{3}$$

Then the spectral regression is defined by

$$\ln\{I(\xi_\lambda)\} = \beta_0 + \beta_1 \ln\left\{\sin^2\left(\frac{\xi_\lambda}{2}\right)\right\} + \eta_\lambda, \qquad \lambda = 1,\ldots,\nu \tag{4}$$

where $\xi_\lambda = \frac{2\pi\lambda}{T}(\lambda = 0,\ldots,T-1)$ denotes the harmonic ordinates of the sample, T is the number of observations, and $\nu = g(T) << T$ is the number of harmonic ordinates included in the spectral regression.

Assuming that $\lim_{T\to\infty} g(T) = \infty$, $\lim_{T\to\infty}\left\{g(T)/T\right\} = 0$, and $\lim_{T\to\infty} \ln(T)^2/g(T) = 0$, the negative of the OLS estimate of the slope coefficient in (4) provides an estimate of d. Geweke and Porter-Hudak (1983) prove consistency and asymptotic normality for $d < 0$, while Robinson (1990) proves consistency for $d \in (0, 0.5)$. Hassler (1993a,b) proves consistency and asymptotic normality in the case of Gaussian ARMA innovations in (1). The spectral regression estimator is not $T^{1/2}$ consistent as it will converge at a slower rate. The theoretical asymptotic variance of the spectral regression error term is known to be $\pi^2/6$.

3. Data and Empirical Estimates

The series studied include three aggregate stock indices: two at daily frequencies and one at a monthly frequency. We also consider seven sectoral monthly stock indices, and daily prices for the thirty companies included in the Dow Jones Industrials index. Further details of the data set (constructed from CRSP Daily Stock Master and

CITIBASE databases) appear in the tables below. All subsequent analysis is done on the first-differenced log series (returns series).

Tables 1 and 2 report the empirical estimates for the fractional differencing parameter d as well as the test results regarding its statistical significance based on the spectral regression test. The number of low-frequency periodogram ordinates used in the spectral regression must be chosen judiciously. Improper inclusion of medium- or high-frequency periodogram ordinates will contaminate the estimate of d; at the same time too small a regression sample will lead to imprecise estimates. We report d estimates for $\nu = T^{0.50}, T^{0.55}$, and $T^{0.60}$ in order to evaluate the sensitivity of our results to the choice of ν. To test the statistical significance of the d estimates, two-sided ($d = 0$ versus $d \neq 0$) as well as one-sided ($d = 0$ versus $d < 0$ and $d = 0$ versus $d > 0$) tests are performed. The known theoretical variance of the spectral regression error $\pi^2/6$ is imposed in the construction of the $t-$statistic for d.

As Table 1 indicates, there does not appear to be any consistent, convincing evidence supporting the long memory (biased random walk) hypothesis for the returns series of any of the aggregate or sectoral stock indices. When we consider the returns series of the Dow Jones Industrials companies in Table 2, there is scattered evidence of fractal structure in some of the series. Strong evidence of long memory is only found for Boeing and Eastman Kodak, while weaker evidence is found for Merck, Sears, and Woolworth. These returns series exhibit long memory features. In the time domain, long memory implies that the series eventually exhibit strong positive dependence between distant observations while, in the frequency domain, the spectral density becomes unbounded as the frequency approaches zero. For International Paper and Texaco, there is clear evidence of intermediate memory; weaker evidence exists for Allied Signal. In the time domain, these returns series exhibit long-range negative dependence while, in the frequency domain, their spectral density approaches zero as the frequency approaches zero. For the remaining 22 stock returns series there is no evidence of fractal structure. Based upon this evidence, fractal dynamics does not appear to be a universal feature of stock returns at either the aggregate or disaggregate level. Limited evidence of fractal structure for some individual companies' stock returns is established, with the nature of fractional dynamics being dissimilar across these series.

4. Conclusions

We applied the spectral regression method to test for fractal structure in aggregate stock returns, sectoral stock returns, and stock returns for the companies included in the Dow Jones Industrials index. No evidence of fractal structure is found in the stock indices. Some evidence of long memory is found for five company returns series while intermediate memory appears to characterize the returns series for three other companies. There is no obvious characteristic linking firms in these two groupings. These results highlight the similarities and differences in fractal structure across different companies' series, implying that fractal structure (where it exists) may be masked in stock indices due to aggregation. However, the overall findings from both aggregate and disaggregate data do not offer convincing evidence against the martingale model.

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Table 1: Empirical Estimates for the Fractional Differencing Parameter d for Stock Index Returns

Returns Series	Number of Observations or Sample Period	$d(0.50)$	$d(0.55)$	$d(0.60)$
S&P 500 Index (D)	8180	-0.032	-0.018	-0.012
		(-0.443)	(-0.322)	(-0.270)
Nasdaq Index (D)	5571	0.057	0.076	0.107
		(0.704)	(1.182)	(2.113)**,##
Dow Jones Industrial Average (M)	47:01-95:11	-0.250	-0.109	-0.057
		(-1.552)!!	(-0.832)	(-0.521)
S&P Composite (M)	47:01-95:10	-0.288	-0.071	0.010
		(-1.789)*,!!	(-0.542)	(-0.091)
S&P Capital Spending (M)	47:01-95:10	-0.330	-0.068	-0.062
		(-2.055)**,!!	(-0.519)	(-0.573)
S&P Consumer Goods (M)	47:01-95:10	-0.212	0.014	0.014
		(-1.321)!	(0.108)	(0.132)
S&P Financials (M)	70:01-95:10	-0.061	-0.152	-0.178
		(-0.303)	(-0.923)	(-1.296)!
S&P Industrials (M)	47:01-95:10	-0.277	-0.073	0.019
		(-1.719)*,!!	(-0.553)	(0.176)
S&P Transportation (M)	70:01-95:10	-0.275	-0.204	-0.195
		(-1.365)!	(-1.234)	(-1.422)!
S&P Utilities (M)	47:01-95:10	-0.248	-0.201	-0.082
		(-1.545)!	(-1.524)!	(-0.755)

Notes: The last day of observation for the S&P 500 and Nasdaq index returns series is 12/30/94. $d(0.50)$, $d(0.55)$, and $d(0.60)$ give the d estimates corresponding to the spectral regression of sample size $v = T^{0.50}$, $v = T^{0.55}$, and $v = T^{0.60}$, respectively. The $t-$statistics are given in parentheses and are constructed imposing the known theoretical error variance of $\pi^2/6$. The superscripts ***, **, * indicate statistical significance for the null hypothesis $d = 0$ against the alternative $d \neq 0$ at the 1, 5, and 10 per cent levels, respectively. The superscripts ###, ##, # indicate statistical significance for the null hypothesis $d = 0$ against the one-sided alternative $d > 0$ at the 1, 5, and 10 per cent levels, respectively. The superscripts !!!, !!, ! indicate statistical significance for the null hypothesis $d = 0$ against the one-sided alternative $d < 0$ at the 1, 5, and 10 per cent levels, respectively. D (M) stands for daily (monthly) frequency.

Table 2: Empirical Estimates for the Fractional Differencing Parameter d for Daily Stock Returns of the Dow Jones Industrials Firms

Stock Returns Series	Number of Observations	$d(0.50)$	$d(0.55)$	$d(0.60)$
ATT	8178	0.018 (0.251)	-0.013 (-0.231)	-0.008 (-0.179)
Allied Signal	8177	-0.125 (-1.709)*,!!	-0.125 (-2.187)**,!!	-0.023 (-0.520)
Alcoa	8179	-0.033 (-0.460)	-0.078 (-1.367)!	-0.087 (-1.940)*,!!
American Express	4453	0.061 (0.697)	0.071 (1.031)	0.002 (0.039)
Bethlehem Steel	8179	-0.073 (-0.999)	-0.052 (-0.905)	-0.037 (-0.834)
Boeing	8179	0.165 (2.249)**,##	0.121 (2.119)**,##	0.078 (1.735)*,##
Caterpillar	8179	0.005 (0.070)	-0.027 (-0.474)	-0.064 (-1.442)#
Chevron	8179	-0.085 (-1.156)	-0.046 (-0.812)	-0.056 (-1.264)
Coca Cola	8179	0.070 (0.953)	0.052 (0.913)	0.050 (1.125)
Disney	8179	0.013 (0.189)	0.036 (0.643)	0.009 (0.218)
DuPont	8178	0.059 (0.809)	0.013 (0.237)	0.013 (0.298)
Eastman Kodak	8179	0.133 (1.816)*,##	0.090 (1.572)##	0.102 (2.276)**,##
Exxon	8179	0.065 (0.889)	-0.068 (-1.184)	-0.070 (-1.568)!
General Electric	8179	-0.012 (-0.173)	0.009 (0.170)	0.020 (0.465)
General Motors	8178	0.020 (0.279)	0.043 (0.758)	0.063 (1.419)#
Goodyear	8179	0.058 (0.799)	0.007 (0.133)	-0.006 (-0.154)
IBM	8176	0.007 (0.102)	-0.037 (-0.646)	0.007 (0.168)
International Paper	8179	-0.094 (-1.285)!	-0.146 (-2.555)**,!!!	-0.097 (-2.166)**,!!
McDonalds	7169	-0.081 (-1.069)	-0.022 (-0.370)	0.050 (1.081)
Merck	8179	0.153 (2.090)**,##	0.097 (1.702)*,##	0.033 (0.745)
Minn. Mining & Mfg.	8179	-0.061 (-0.834)	-0.063 (-1.096)	-0.052 (-1.154)
Morgan JP	6508	-0.049 (-0.626)	-0.003 (-0.055)	-0.024 (-0.496)
Phillip Morris	8178	-0.076 (-1.034)	-0.025 (-0.443)	-0.019 (-0.427)
Procter & Gamble	8179	-0.031 (-0.422)	-0.053 (-0.927)	-0.028 (-0.643)

Sears	8179	0.040 (0.551)	0.092 (1.615)##	0.111 (2.476)**,###
Texaco	8178	-0.136 (-1.858)*,!!	-0.098 (-1.718)*,!!	-0.078 (-1.748)*,!!
United Carbide	8179	0.080 (1.091)	0.004 (0.071)	0.014 (0.321)
United Technology	8177	0.001 (0.024)	0.024 (0.424)	-0.007 (-0.163)
Westinghouse	8179	0.055 (0.756)	0.107 (1.861)*,##	0.067 (1.508)#
Woolworth	8179	0.097 (1.322)	0.091 (1.585)#	0.095 (2.127)**,##

Notes: The last day of observation for the Dow Jones Industrials companies' returns series is 12/30/94. All series are of daily frequency. See notes in Table 1 for explanation of table.