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# Ranking Ranking Rules

**Abstract:** Transitivity is a fundamental requirement for consistency. Legal systems, especially when composed over time and by different agencies, may encounter non-transitive cycles, in which by one rule the law prefers one outcome *a* over another outcome *b*, by another rule *b* trumps some third result *c*, but a third rule ranks *c* higher than *a*. This paper discusses a new solution to such cycles in which the relevant rules of preferences are ranked and then applied until a transitive order of the options is obtained. The paper provides a formal generalization of this solution, and demonstrates its possible implementation to some legal issues. It is also shown that this solution can be traced to the Rabbinic literature, starting with the Mishnah and the Talmud (1st–5th c CE).

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## 1 Introduction

Legal systems are created over time and by different agencies, and it is therefore hardly surprising that occasionally inconsistencies are discovered and need to be resolved. Most systems have some built-in resolution mechanisms. For example, legislative agencies are ranked, as are courts. Such mechanisms are suitable for solving conflicts where two norms contradict each other. However, there is another type of inconsistency, where even though each pair of two laws is consistent, larger sets of laws are not. Such is the case of non-transitive cycles, where by one law one outcome *a* is ranked above *b*, by another law *b* is ranked higher than *c*, but a third law ranks *c* higher than *a*. This paper discusses a possible solution for circumstances in which a set of legal rules results in such a cycle. We show how it can be used to solve contemporary problems, and describe how it can be detected to the legal system of the Talmud.

There are several theoretical objections to violations of transitivity. The first relates to the requirement that judges use logical reasoning in their decisions (see Levi, 1949; Dworkin, 1986). If three rules create a cycle as above and a dispute between $a$ and $c$ needs to be resolved, then the logical conclusion that ranks $a$ higher than $c$ (since one law ranks $a$ above $b$, and another law ranks $b$ higher than $c$) clashes with the direct ruling concerning these two options, leaving the judge with contradicting instructions. Transitivity is also a key axiom in choice theory (Harsanyi, 1955; Sen, 1986; Kreps, 1988). The best support for transitivity in this context comes from the requirement that choice should be consistent (see, e.g. Sugden, 1985; Rubinstein, 2006). Let $X$ be a set of alternatives, and let $C$ be a choice function defined on all nonempty subsets of $X$. Suppose that for three alternatives $a$, $b$, and $c$, the choice function $C$ satisfies $C(a,b) = a$, $C(b,c) = c$, but $C(a,c) = c$. What should $C(a,b,c)$ be? If the outcome of $C$ is a singleton (say $a$), then there is another outcome that is even better. And if the outcome of $C(a,b,c)$ is not a singleton, then one of these outcomes is dominated by the other.

Sometimes, an implied assumption of the modern discussion of non-transitive cycles is that the rules or preferences that lead to this outcome enjoy a similar normative status. Consequently, an arbitrary agenda setting is inevitable to solve the cycle (for example, Levmore, 2005); and specifically in the context of collegial courts (Kornhauser, 1992; List and Pettit, 2002). The solution we offer points to the fact that some external set of norms may well provide a ranking of the ranking rules. This ranking, together with the logic of transitivity, may provide a solution to cycles. As we show in Section 3, this latter solution can be traced to medieval Rabbinic thought and is new to the literature concerning choice.

The basic structure of our model is this. There are $n \geq 3$ outcomes, and rules that rank possible pairs of these outcomes. We assume that these rules can be ranked and applied sequentially according to this ranking. A rule will be observed only if it does not violate conclusions from earlier rules. Under some assumptions that we discuss below, all outcomes will eventually be ranked by a transitive order.

To illustrate our idea, consider the following example. There are three outcomes, $a$, $b$, and $c$, and three rules. According to rule 1, $a$ prevails over $b$. According to rule 2, $b$ prevails over $c$, while according to rule 3, $c$ prevails over $a$.[1] Suppose that the rules are ranked 2, 1, 3. Applying rule 2 implies that $b$ precedes $c$. Applying rule 1 yields $a$ precedes $b$, hence the ranking of the outcomes

1 In the next section, we discuss more general forms of rules.

is *abc*. However, if the rules are ranked 2, 3, 1, then we get that $b$ precedes $c$, $c$ precedes $a$, and the outcomes are ranked *bca*. As indicated, it is a standard case that two rules contradict, as, for instance, when rule 1 ranks $a$ higher than $b$ while rule 2 ranks $b$ above $a$, and the conflict is resolved by ranking these ranking rules. In this paper we point at the need to rank the ranking rules even when they do not result in contradicting rankings. Specifically, a certain rule may be declared "inapplicable" even though it does not contradict (or directly infringe) a higher-ranked rule, on the basis of the ideal of avoiding cycling in the legal system. Moreover, whereas in the typical case of contradicting rules the ranking set by the top rule yields the outcome, this is not necessarily the result here, as illustrated by the above examples. As we explain in the next section, this is where our solution differs from standard lexicographic orders, and is thus not a special case of Tversky (1972) or Manzini and Mariotti (2007, 2010).

The paper proceeds as follows. We present our procedure in the next section. Section 3 tracks this solution to early Rabbinic Law, and show how the Talmud and its commentators deal with a case in which a choice function cannot be supported by a transitive relation and yet needs to be extended from pairs to larger sets. Section 4 illustrates possible implementations of this procedure, by addressing modern legal doctrines in the fields of property law, anti-discrimination law, and torts. The Appendix provides an exact algorithm for implementing this solution concept for the case where each rule applies to two alternatives only.

## 2 Ranking rules

As indicated, we suggest that a solution to non-transitive cycles created by a given set of ranking rules can be based on a ranking of the ranking rules. The ranking determines the sequence in which each of the ranking rules is to be applied, and in conjunction with the logic of transitivity may solve non-transitive cycles. The ranking should be based on some normative theory, and thus requires additional information to that provided by the ranking rules themselves. In the following sections, we discuss possible sources for such ranking. Our aim in this section is to present the algorithm for implementing the solution that we offer, given that the set of ranking rules is ranked according to such a normative theory.

Let $X$ be a finite set of options. A ranking rule is of the form $\{B\} \succ \{C\}$, where $B$ and $C$ are disjoint subsets of $X$. According to this rule, all elements of $B$ are ranked higher than all elements of $C$, but the rule says nothing about the relative ranking of elements in $B$ or $C$. Also, as we do not require that $B \cup C = X$, the ranking rule says nothing about the relationship between elements of $B$ and $C$ and options that are not in these two subsets.

Many different rankings are possible, some of which may contradict each other. We suggest to solve such conflicts by ranking the rules themselves. We start by referring to several examples, before formulating the general algorithm.

**Example 1:** $X = \{a, b, c\}$. There are three rules: $a \succ^1 b$, $b \succ^2 c$, and $c \succ^3 a$. That is, rule 1 ranks $a$ above $b$, rule 2 ranks $b$ above $c$, and rule 3 ranks $c$ above $a$ (this is the situation discussed in Sections 3 and 4 below). There are six possible rankings of these rules, each leading to a different ranking of the elements of the set $X$. For example, the ranking 2-3-1 leads to $b \succ c$ (rule 2) and $c \succ a$ (rule 3). From these two we deduce, using transitivity, that $b \succ a$, and rule 1 is never used. The three options are ranked $b \succ c \succ a$.

Of course, other rankings of the rules lead to different rankings of the options themselves. For example, ranking the rules 1-3-2 yields $a \succ b$, $c \succ a$, hence by transitivity $c \succ b$ and rule 2 is ignored. The three options are thus ranked $c \succ a \succ b$. □

Note that being top in the rule that is ranked highest does not imply top ranking in the set $X$, because each of the rules may apply only to a subset of $X$. Suppose that in the above example the rules are ranked 1-3-2. As explained before, this ranking of rules implies the order $c \succ a \succ b$. Although the first rule to be applied is rule 1, according to which $a$ is best in $\{a, b\}$, at the end $a$ comes second to $c$ because of the second-to-be-used rule implies $c \succ a$.

Sometimes only part of the implications of a rule can be used.

**Example 2:** $X = \{a, b, c, d\}$. There are two rules: $a \succ^1 b$ and $\{b, d\} \succ^2 \{a, c\}$. The second rule implies $b \succ^2 a$, $b \succ^2 c$, $d \succ^2 a$, and $d \succ^2 c$. But if we rank the first rule before the second then we have to ignore $b \succ^2 a$. We thus obtain $a \succ b$ (rule 1), $d \succ a$ (rule 2), and $b \succ c$ (rule 2). Using transitivity we obtain the complete order $d \succ a \succ b \succ c$. □

Not always do we have enough information how to rank all options. Consider the following example.

**Example 3:** $X = \{a, b, c, d\}$. There are two rules: $\{a, b\} \succ^1 c$ and $\{c, d\} \succ^2 a$. These two rules imply four pairwise comparisons: $a \succ^1 c$, $b \succ^1 c$, $c \succ^2 a$, and $d \succ^2 a$. Ranking rule 1 before rule 2 yields $a \succ c$, $b \succ c$, and $d \succ a$ (observe that $c \succ^2 a$ is rejected). By transitivity we also obtain $d \succ c$.

Although we know that all options are ranked higher than $c$, we do not have enough information to compare the pairs $\{a, b\}$ and $\{b, d\}$. It is possible that $d$ is ranked highest, and then we can have either $d \succ a \succ b$ or $d \succ b \succ a$, or $b$ is ranked highest, in which case $b \succ d \succ a$. □

Sometimes, given higher ranked rules, the added information contradicts itself and must therefore be ignored. This is the case in the next example.

**Example 4:** $X = \{a, b, c, d\}$. There are three rules: $a \succ^1 b$, $c \succ^2 d$, and $\{b, d\} \succ^3 \{a, c\}$. The third rule implies, as before, $b \succ^3 a$, $b \succ^3 c$, $d \succ^3 a$, and $d \succ^3 c$. But if we rank the rules 1-2-3, then we have to ignore $b \succ^3 a$ and $d \succ^3 c$. We are thus left with $b \succ^3 c$ and $d \succ^3 a$.

Given the ranking 1-2-3, if we use the third rule to obtain $b \succ c$, then we must deduce, using transitivity that $a \succ b \succ c \succ d$. However, if we use the third rule to obtain that $d \succ a$, then transitivity implies $c \succ d \succ a \succ b$. The two possible extensions contradict each other. As we don't have a reason to prefer the rule $b \succ c$ to $d \succ a$, we will ignore both. All we can therefore say in this case is that $a \succ b$ and $c \succ d$. □

Even when some information that a rule provides must be ignored, part of the information should be usded. Consider the following case.

**Example 5:** $X = \{a, b, c, d, e\}$. There are three rules: $a \succ^1 b$, $c \succ^2 d$, and $\{b, d, e\} \succ^3 \{a, c\}$, and the rules are ranked 1-2-3. Similar to the previous example, we cannot use either $b \succ^3 c$ or $d \succ^3 a$. But this time rule 3 is not entirely useless as nothing prevents us from using the information $e \succ^3 a$ and $e \succ^3 c$. By rules 1 and 2 and by transitivity, we thus obtain $e \succ a \succ b$ and $e \succ c \succ d$. □

In general, when it is the turn of rule $i$ to be used, we will first eliminate all its pairwise comparisons that are already contradicted by previous rules, or by transitive conclusions obtained from these rules. We then include all the pairwise comparisons obtained from rule $i$, provided they do not lead to contradictions with other rule $i$ comparisons or with their conclusions. In the Appendix we offer a complete algorithm for our method for the case where all rules are of the form $a \succ b$ and all pairs (there are $\frac{1}{2}n(n-1)$ of them) are thus compared. We show that in this case our algorithm yields a complete and transitive ranking of the $n$ options.

Our method should not be confused with lexicographic preferences. Tversky (1972) suggested the following ranking rule: There are $k$ (ordered) criteria. Option $a$ is ranked higher than $b$ if the first criterion in which $a$ and $b$ are not indifferent ranks $a$ above $b$. This idea was expanded by Manzini and Mariotti (2007, 2010) to include sequential (or inductive) elimination, where at step $i$ a relation $P_i$ is applied to the set $M_{i-1}$ of all options that survived the previous step, and only options that are not dominated in $P_i$ by any other option in $M_{i-1}$ survive to the next stage (this new set is labeled $M_i$).

Example 1 illustrates why the procedure we suggest is different from the above elimination mechanisms. In this example, $X = \{a, b, c\}$, $a \succ^1 b$, $b \succ^2 c$, $c \succ^3 a$, and the rules are ranked 1-2-3. Our procedure leads to $a \succ b$, then $b \succ c$, and by transitivity $a \succ c$. The ranking of the three alternatives is thus $a \succ b \succ c$. On the other hand, the lexicographic elimination procedure will apply first the rule $a \succ^1 b$ to the set $\{a, b, c\}$ and option $b$ is thus eliminated. Next we apply the rule $b \succ^2 c$ to the surviving set $\{a, c\}$ and obtain that none of these options is eliminated by this rule. Finally, we apply the rule $c \succ^3 a$ to the remaining set $\{a, c\}$ and obtain that $a$ is eliminated. The induced order is thus $c \succ a \succ b$.

More generally, rather than eliminating options, our solution eliminates rankings that contradict those yielded by the logic of transitivity. The ranking of the ranking rules required by our solution is thus not lexicographic but one that only determines the priority of each of the ranking rules in applying the logic of transitivity. In this sense, our solution requires "weaker" normative theory than that of the elimination procedure.

We conclude this section with a brief discussion of some other methods that were offered to analyze cycles. In certain contexts preserving a non-transitive cycle can be justified. A possible justification is the view that the relevant values (or priority rules) are incommensurable, thus suggesting that value-pluralism should prevail over the intuitive case for transitivity (Pildes and Anderson, 1990; Sunstein, 1994; Luban, 1996). Along this line, the Talmud sometimes intentionally avoids resolving non-transitive cycles. It may do so either because the cycle does not create any operational inconsistency or in order to force conflicting parties into a compromise (see Naeh and Segal, 2009). However, in many contexts cycles cannot be sustained and must be resolved.

The extensive discussion on transitivity in the legal literature that applies social choice theory follows, to a large extent, the implications of Arrow's Impossibility Theorem. It examines devices for lawmaking institutions and election rules that diminish the possibility of cyclical outcomes (see, for example, Easterbrook (1982), Easterbrook and Fischel (1983), Kornhauser and Sager (1986), Hayden (1999), and Hayden and Bodie (2009)). As mentioned above, the focus of this paper is different, as it discusses possible solutions for circumstances in which an *existing* set of legal rules results in a cycle.

A related, but different discussion than the one presented here was offered in the context of collegial courts (e.g. Kornhauser, 1992; List and Pettit, 2002; Chapman, 2002). It deals with the question how judges' positions should be aggregated in instances in which the outcome of the case is determined by the decision in two (or more) different legal disputes. For instance, in a typical contract law case, the proposition whether the defendant has breached the contract is a compound of the atomic decisions whether there was a contract at all, and whether there was an act constituting breach of such contract. Assume that two out of the three judges decide in favor of the defendant, but one of them decides so since she finds there was no contract (but if there were one, this judge believes it was breached), while the other judge is in the opposite position in both issues (there was a contract but it was not breached). The minority judge rules that there was a contract and it was breached. Aggregating these atomic positions separately results in ruling in favor of the plaintiff. This problem is often resolved by employing a decision-rule according to which the outcome is based on aggregating the judges' compound, rather than atomic decisions. In contrast, we analyze a different case. We refer to instances in which there is no dispute regarding each of the atomic positions, that is, regarding the ranking of each of the given pairs of possible outcomes. The problem we address stems from the fact that combining each of these rankings results in a cycle. In other words, we analyze a problem where a choice function defined over two-item sets cannot extend to include three-item sets.

## 3 Ranking rules in the Talmud

In this section we show how the Talmud and its commentators deal with a specific set of inconsistent laws that result in a cycle. A few remarks are needed here on the nature of the rabbinic materials. Essentially, Talmudic[2] Law encompasses all aspects of (Jewish) life, legal as well as ritual. The Rabbis treat all of them with the same seriousness and meticulousness. Moreover, they use the same analytical tools in their discussions of the different materials, be it criminal law, civil law, temple rituals, or other religious commandments, public or individual. Another important aspect of these materials is their interpretive nature. Typically, the Talmudic passage is built through interpretations and counter-interpretations of previous sources. In this article, we give special attention to the interpretive principles and processes, explicit or implicit, of

**2** Talmud, Babylonian (3rd–5th c CE) and Palestinian (3rd–4th c CE), is the collection of rabbinic materials of various genres – rulings, discussions, interpretations, stories, morals, and other literary forms. The Talmud is arranged as comprehensive interpretation to the Mishnah, the basic and authoritative collection of old rabbinic oral instructions edited circ. 220 CE. The Mishnah is composed of 60 tractates ordered in six "orders" (sedarim). Each single paragraph of the Mishnah is also called a Mishnah.

the texts. The example discussed below may seem contrived. Indeed it is. The way of the Talmud is to create such examples in order to isolate or examine the fine details of an idea or a rule (see Moscovitz, 2002).

The example we analyze deals with the precedences of private offerings in the Jerusalem temple.[3] The question is what is the order according to which an accumulation of different types of offering should be sacrificed?

Two sets of laws were created in two different early rabbinic compositions, a fact that may account for the apparent incongruence between them. Usually, the rabbis solve such conflicts by restricting the application of each ruling to a specific situation. In the case discussed below this solution is natural since the sources themselves offer such a distinction. However, in the Talmudic discussion the distinction was blurred and the Talmud was thus faced with a non-transitive cycle. The need to solve it eventually led medieval commentators to develop strategies for untangling such cycles. This section analyzes the hermeneutical development of the text, the cycle, and its possible solutions.

Ranking all possible types of private offerings, the Mishnah (1933:Zev.10:4) is using two rules:

**Rule 1:** Type of animal. Cattle (ox or lamb) precede birds.
**Rule 2:** Ritual function. Within each type of animal, offerings are ranked by their ritual importance. Sin-offering[4] and burnt-offering[5] precede peace-offering,[6] which in turn precedes tithe.[7]

The Mishnah is using these rules lexicographically, first by the type of the animal (Rule 1) and then by its ritual function (Rule 2). According to this procedure, the tithe, being from the cattle, precedes bird sin-offering even though ritually the sin-offering is several degrees higher than the tithe.

**3** The discussion in the Talmud is purely theoretical as by the time the issue was discussed the Jerusalem temple was destroyed for more than 200 years. This does not prevent the rabbis from analyzing the question as they maintained that the study of the old laws is culturally and spiritually important, regardless of their actual relevance.

**4** A sacrifice made for the atonement of an unintentional sin or ritual fault. As the Hebrew name *ḥattat* means also “purification” (see Milgrom, 1991-2001:758-759), sin-offering is obligatory in specific rituals even when no sin is committed, like after childbirth or after recovery from leprosy.

**5** An animal sacrifice which is entirely consumed by fire on the altar. Burnt-offering may be brought either as public or individual sacrifice. In the latter case, it can be obligatory, for certain rituals, or voluntary, as an act of piety.

**6** A voluntary individual offering, usually brought as thanks-giving offering or for the purpose of a feast. Except for some inner parts of the animal, the flesh of the peace-offering is eaten by the priests, the donor, and his family.

**7** A tenth of the newly born cattle. Animal tithe is given each year to the priests, who has to sacrifice and ritually eat it.

There may be another relevant criterion, about which the Mishnah is silent. In general, the gathering of a group of various offerings may occur in two different circumstances:

(i) Several offerings accumulated at random, as one person or more brought them simultaneously, for different ritual purposes.

(ii) A group of offerings is brought as one sacrificial unit for a specific ritual. For example, the pair of a bird sin-offering and a lamb burnt-offering, brought by a mother after giving birth.[8]

The Mishnah clearly intends to impose its ruling in the first situation – it ranks some offerings that can never form any of the known groups of the second – and it does not address the issue of sacrificial units that are composed of more than one animal (situation (ii) above). Such units are the interest of the Sifra,[9] another early rabbinic text from the same period. Restricting itself to sin and burnt offerings only, the Sifra says:

**Rule 3:** Sin-offerings precede burnt-offerings. "Be it a bird sin-offering with a bird burnt-offering, be it a bird sin-offering with a cattle burnt-offering or a cattle sin-offering with a cattle burnt-offering – sin-offerings always precede burnt-offering – sin-offerings always precede burnt-offerings *that are brought with them.*"[10]

The Sifra derives this ruling from Scripture.[11] Its formulation explicitly rules that when the choice is between burnt-offering and sin-offering, the only criterion to be used is the ritual function and not the type of the sacrificed animal. This seems to contradict Rule 1 of the Mishnah, where the type of animal is the first criterion to apply. However, one should notice the words "that are brought with them" at the end of Rule 3 of the Sifra, which imply that this rule must apply only to situation (ii) above. The Mishnah and the Sifra may therefore be reconciled if one assumes that the Mishnah deals only with cases of situation (i). Consequently, the Mishnah and the Sifra form together one system, in which the

**8** Lev. 12:6. Another example is that of a nazirite, who is required to bring, when accomplishing his vow, a group of three animals: a sin-offering, a burnt offering, and a peace-offering (Num. 6:16–17). A very common group is the pair of birds – a sin-offering and a burnt-offering – brought by the poor person who cannot afford the regular cattle offering (e.g. Lev. 5:7, 12:8, Luke 2:24).

**9** The old (3rd c CE) rabbinic commentary to Leviticus, attributed to the school of R. Akiva, the 2nd c CE mishnaic scholar.

**10** Sifra (1988, 18:5–6:189). Italics added.

**11** Through an exegesis to Lev. 5:8: "And [the priest] shall offer that which is for the sin-offering first."

three rules are applied in the following way: If two or more offerings accumulate at random, then they are sacrificed according to Rules 1 and 2. But if a set of offerings is brought as one sacrificial unit, then Rule 3 applies. It is only in the latter case that a bird sin-offering precedes a cattle burnt-offering.[12]

Yet this does not supply us with instructions for all possible events. What if situations (i) and (ii) occur simultaneously? Consider, for example, the following case. A person arrives at the temple with a tithe ($a$); at the same time a woman after giving birth brings a sacrificial pair of animals: a bird for sin-offering ($b$) and a lamb for burnt-offering ($c$). The internal order of the group $\{b, c\}$ is governed by Rule 3: $b \succ c$. But which should be brought first to the altar, the tithe $a$ or the woman's offerings $b$ and $c$? On the one hand, the woman's pair contains $c$, a burnt-offering (the lamb), and according to Rule 2, $c \succ a$. On the other hand, the actual choice is between $a$, the tithe, and $b$, the bird, and according to Rule 1, $a \succ b$. Apparently, this is a simple ranking problem of two items, where one of the items is composed of two animals, and the above two rankings are the only ways in which it can be answered. This is most probably the original form of the question asked by the Talmud (Zev. 90b): "A bird sin-offering and an animal burnt-offering and tithe – which of these come first?" meaning, "[a pair of] {a bird sin-offering and an animal burnt-offering} and tithe – which of these come first?" The Talmud offers two answers:

1. Here [= in Babylon] they held that an animal offering is superior [hence the tithe comes first];
2. In the West [= Palestine] they say: The superiority of the animal burnt-offering over tithe enters in the bird sin-offering and elevates it over the tithe [hence the bird sin-offering comes first].

The Babylonian solution[13] considers the two actual possible offerings – bird and tithe – and according to Rule 1 ranks the animal (tithe) before the bird.[14] The order is therefore $a \succ \{b, c\}$, that is, $\{tithe\}$ and then {bird, lamb}, where the pair {bird, lamb} is ordered "bird before lamb." The Palestinian solution on the other

12 Actually, the only example for such a combination is the set of offerings brought by a mother after giving birth (Lev. 12:6). See Talmod Bavli (1961:Zev.90a, s.v. ha'of). For Rashi, see footnote 2(')@ below.

13 A similar solution is offered in the Talmud Yerushalmi (1982-1994:Hor.3:7, 48a) to a similar question, based on a different set of rules and sacrifices.

14 The Babylonian's wording alludes to the formula used by the Mishnah to postulate the precedence of tithe to birds: "The tithe precedes the bird-offerings, since it is an animal-offering" (1933:Zev.10:3).

hand gives priority to the pair $\{b, c\}$ (that is, {bird, lamb}) because it includes the element $c$ (lamb) which is stronger than $a$ (tithe). The order is therefore {bird, lamb} and then tithe, whereas before, the lamb follows the bird.

This was probably the Talmud's original form. However, in the present form of the Talmud the wording of the explanatory paragraph that follows the question gives it a completely different meaning:

> A bird sin-offering, an animal burnt-offering, and tithe – which of these come first? Shall the bird come first? There is the tithe that must precede it! Shall tithe come first? There is the animal burnt-offering that must precede it! Shall the animal burnt-offering come first? There is the bird sin-offering that must precede it![15]

Undoubtedly, the explanatory paragraph ("Shall the bird…") does not present the question as choice between two (composite) elements, but as a non-transitive cycle of three independent items. According to this paragraph, the fact that the bird and the lamb are brought by one person as one sacrificial unit $\{b, c\}$ is insignificant and the two offerings are considered two separate entities. This implies a new understanding of Rule 3. As formulated above it only applies to closed sacrificial units of burnt and sin offering (situation (ii) above). The explanatory paragraph, however, employs Rule 3 as a general rule applicable to any combination of these offerings. (Such a shift in the meaning of Rule 3 may also be detected elsewhere in the Talmud, see, e.g. (1961:Hor.13a).) The question is now transformed from a simple inquiry of which of two items should come first (where one of the items by itself is a pair), a question that has two simple and intuitive answers, into a contradiction presented by a non-transitive cycle.

While the meaning of the question is transformed by this latter addition, the answers remain unchanged. This inevitably causes a severe incoherence in the text of the Talmud. The Babylonian and the Palestinian approaches that answered the original simple question no longer fit its new form. This is most evident in the Babylonian answer "an animal offering is superior," which is helpful when the choice is between a bird and an animal, but it cannot choose between the two animals, the tithe and the burnt-offering. The Palestinian solution too is not clear: why should one pair together the bird and the burnt-offering and not any of the other two possible pairs? Any two items paired together will be able to overcome the third.

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**15** Observe that the language of the Talmud is very clear – the question relates to a choice function, which is to select one out of a set of options. The question is not what is the right order at which the animals will be sacrificed, although eventually this question too will be answered.

The reader of the Talmud, who tries to understand the text "as is," is now encountered with a serious problem: how is one to interpret the answers as solutions to the question and its new form? We show here how this is done by Rashi,[16] who silently introduced two novel ideas in order to adapt the original answers to the new non-transitive puzzle. We focus on Rashi's explanation to the Babylonian answer, which implies an innovative (though quite intuitive) solution to the inconsistent choice rules. It is this idea that we formalize and apply to current legal issues in the subsequent section.

Simply put, Rashi's interpretation of the Babylonian solution is that the rules themselves should be ranked. Implicit in the Talmudic text is the assertion that Rule 3, being derived from scripture, should be the first to be applied. The statement "Here they held that an animal offering is superior" should be understood to say "Rule 1 precedes Rule 2." (As mentioned above, this is also the way the Mishna is ranking these two rules.)

The ranking of the rules implies a complete ranking of the three offerings. Rule 3 ranks the bird ($b$) before the lamb ($c$). Rule 1 then ranks the tithe ($a$) before the bird, and using transitivity, we now obtain $a \succ b \succ c$ (i.e. the ranking tithe, bird, lamb). Observe that in this analysis Rule 2 is not used, because by the time its turn arrives, we've already used the other rules to obtain a complete and transitive order of the three options.

This case is relatively simple as it is using only Rules 3 and 1. Nevertheless, one can imagine situations that are more complex where Rule 2 too will be used. Consider the case of four offerings: the above three ($a$: tithe, $b$: bird, and $c$: lamb) and $d$: an ox peace-offering, $d$. By Rule 3, the bird sin-offering precedes the lamb burnt-offering ($b \succ c$); by Rule 1 (which precedes Rule 2), the tithe and the peace-offering precede the bird sin-offering ($a \succ b$ and $d \succ b$); and then, by Rule 2, the peace-offering precedes the tithe ($d \succ a$). Therefore, the order of sacrifice is peace-offering, tithe, bird sin-offering, burnt-offering, $d \succ a \succ b \succ c$. In Section 2 we have showed how this idea can be generalized to sets with more than three items.

As happens in many cases, a hermeneutical crisis may bring about fresh and innovative ideas. In our case, the shift in the meaning of Rule 3 (from closed sacrificial pairs to any coincidental pair) caused a profound change in the conceptual structure of the passage, forcing the original answer to respond to a new question. An interpretive innovation is necessary to bridge the gap and to

**16** R. Shlomo b. Yitz ḥ ak (northern France, d. 1105), preeminent commentator to most parts of the Bible and the Babylonian Talmud. In this article, reference is made to Rashi's commentary printed on the inner margins of the traditional editions of the Babylonian Talmud.

adapt the answer to its new role. From this hermeneutic process, the original simple and intuitive solution came out as a novel strategy for breaking non-transitive cycles.[17]

# 4 Application to current legal dilemmas

In what follows, we illustrate possible implementations of our analysis to actual legal issues. We discuss three examples: conflict of title between an original owner, an intermediate purchaser, and a good-faith purchaser for value; the conflict between the "disparate-treatment" and "disparate-impact" doctrines of anti-discrimination laws; and the right of beneficiaries to recover from tortfeasors.

## 4.1 Conflict of title

Conflict of title rights in an asset between a first-in-time claimant (the "original owner") and a good-faith purchaser for value provides an example to the possibility of non-transitive cycles. These conflicts arise when parties not in contractual privity assert simultaneous claims of rights over the same asset whose concurrent discharge is legally impossible, and the law is called upon to resolve the conflict.

The original owner, $a$, and the good-faith purchaser, $c$, are linked through the activities of an intermediate seller, $b$, who transacted with each of them. The intermediate seller may be a wrongdoer, whose title is considered void, such as a thief or a trustee who acted in breach of his duty; or one who contracted with the original owner but whose title is voidable, since, for instance, he breached the contract or did not disclose a material fact.

A cycle might happen when according to one legal rule the good-faith purchaser $c$ prevails over the original owner $a$, whereas the rules that apply to each of the pairs $\{a, b\}$ and $\{b, c\}$ induce the opposite ranking. Consider the following scenario. Assume that the intermediate seller has the power to nullify his contract with the good-faith purchaser and to claim back the title, due to certain deficiencies in the contract, for instance mutual mistake. At the

17 Although this is not the topic of our paper, we should mention here that Rashi's interpretation of the Palestinian solution essentially claims that the Independence of Irrelevant Axiom (IIA) must be violated. According to this axiom, the ranking of two options does not depend on the existence of other alternatives. See Naeh and Segal (2009) for a further discussion of the Talmudic analysis of this idea.

same time, assume that the original owner too has a good claim against the intermediate seller, for instance due to material breach of the contract between them. The relations between each of the two contracting pairs, *a* and *b*, and *b* and *c*, are governed by the relevant contract law rules, and raise no difficulty. For instance:

**Rule 1:** In case of material breach, the aggrieved party may terminate the contract and recover the property transferred.

**Rule 2:** In case of mutual mistake regarding a substantial fact, each party may terminate the contract and recover the property transferred.

According to Rule 1, if the original owner employs her power to nullify the contract with the intermediate seller, she recovers the title, such that $a \succ b$.[18] And under Rule 2, if the intermediate seller employs his power to terminate his contract with the good-faith purchaser, he recovers the title, and thus $b \succ c$. But assume that the conflict is between *a* and *c*, as the original owner claims that the good-faith purchaser should transfer her the good. What is the result in the case of such a conflict between *a* and *c*?

According to the good-faith purchaser for value rule, if *c* acted in good faith and for value, and the transaction took place before *a* employed her power to terminate her contract with *b* and recover the title, *c* gains the title.[19] Recall that *b* has the power to terminate his contract with *c*, but we assume that *b* did not employ this power at the time of the conflict between *a* and *c*. The dilemma in such cases is whether the original owner can “subrogate” the intermediate seller in implementing the power to nullify the contract between the intermediate seller and the good-faith purchaser. According to some views the answer is no, such that the original owner cannot “trace” the power to nullify a voidable contract (a “limited tracing” rule). Thus, assume that:

**Rule 3:** A good-faith purchaser for value gains title, even if the seller’s title is voidable, and even if the good-faith purchaser’s title too is voidable.

In other words, as long as *b* did not exercise his power to nullify the voidable contract with *c*, *a* does not have the power to do so, and *c* prevails. If this is the case, we have a cycle: $a \succ b$ (Rule 1); $b \succ c$ (Rule 2); but $c \succ a$ (Rule 3).

**18** In this context, $\succeq$ means “has a priority regarding the title of the good.”

**19** In most legal systems, a good-faith purchaser for value may gain title even though the intermediate seller’s title is voidable (e.g. Section 2–403(1) of the UCC, “[a] Person with voidable title has power to transfer a good title to a good faith purchaser for value”). In some legal systems a good-faith purchaser for value may gain title even if the intermediate seller’s title is void (under the “market-overt” rule) (e.g. Medina (2003)).

If this cycle is not resolved, the ultimate result depends on the decision which conflict to resolve first. If the court starts with resolving the dispute between *a* and *b*, *a* would employ her power to terminate the contract and recover her title, but would then lose to *c*, as *c* purchased his title before *b*'s title was terminated. In contrast, if the court starts with resolving the conflict between *b* and *c*, such that *b* nullifies the contract, *a* would ultimately get the title. Note that even if one of the contestants is not party to the specific legal dispute, for instance when the intermediate seller *b* is missing, the court may well have to consider the cycle. The fact that *b* is missing does not necessarily mean that Rule 3 is the only one relevant to resolving the dispute. Otherwise, the outcome will not be determined according to some consistent internal logic of the law or relevant policy considerations, but rather on the arbitrary decision which dispute was resolved first.

Following our suggested procedure, the required normative decision is a ranking of the three rules. This ranking determines the sequence of applying the rules and thus, in the current context, which rule would not apply. If Rule 3 is ranked first, we get that $c \succ a$, and the remaining dispute is between *b* an *c*, to which Rule 2 applies and *b* prevails. Indeed, according to Rule 1 $a \succ b$, but this would result in a non-transitive cycle and Rule 1 should not apply here. Once again, observe that the winner of the top rule is not necessarily the overall winner.

## 4.2 Disparate-treatment and disparate-impact

Following the circumstances discussed in the Supreme Court's decision in Ricci v. Destefano (2009), suppose that a city is required to determine who of its firefighters are qualified for promotion to the rank of lieutenant or captain. It is known that applying a certain type of (facially neutral) examination would result in white candidates outperforming minority candidates, such that although minority candidates compose of more than one half of all candidates, almost none of them would be qualified for promotion. An alternative test – for instance, one that gives higher weight to oral components – is expected to result in a more balanced outcome. However, suppose that the statistical correlation between candidates' results in the alternative test and their success in fulfilling the role of lieutenant or captain is somewhat lower than that of the former test. What test should the city choose? And what if the former test was already administered, such that certain persons invested considerable amounts of efforts and money in preparation for the exam and were even told that they successfully passed it?

Title VII of the Civil Rights Act of 1964 contains two central anti-discrimination doctrines – the so-called "disparate-treatment" and "disparate-impact" prohibitions. The former norm proscribes an employer the use of an employment practice that adversely affects employees because of, among other things, their race (Title VII, §2000e-2(a)(1)). The latter prohibition addresses unintentional discrimination. It proscribes exercising even a facially neutral employment practice which results in an imbalanced effect on specific groups.[20] An employment practice which results in an imbalanced representation is unlawful not only when it cannot be shown that this practice is "job related for the position in question and consistent with business necessity," but also when the practice meets this requirement but there exists an alternative employment practice that would not cause the disparate impact (Title VII, §2000e-2(k)).

These two doctrines may conflict in determining the legitimacy of making race-conscious decisions when choosing between alternative employment practices (for instance, Primus (2003) and Kelman (2001)). On the one hand, the disparate-impact prohibition requires the employer to account for the racial impact of each of the relevant alternative employment practices in choosing which one of them to adopt. On the other hand, preferring one practice over another on the basis of its expected racial impact might be considered as a decision made "because of race," in violation of the disparate-treatment prohibition.[21]

Following the Supreme Court's decision in Ricci v. Destefano (2009), an account for the racial impact of a practice is legitimate only when the employer realizes (or has a reasonable basis to expect ex ante) that a specific employment practice has (or might have) a substantially unequal racial impact.[22] In such a case, the employer should first verify that the practice under consideration meets the threshold of relevancy (that is, that the practice is "job related for the position in question and consistent with business necessity"). Then the employer should inquire whether

**20** According to the standard set by the Equal Employment Opportunity Commission (EEOC), a test that is implemented to select candidates is considered imbalanced when it results in a selection rate of a specific race (that is, the ratio of those qualified out of all candidates of this race) which is less than 80% of the selection rate of the race with the highest selection rate. 29 CFR § 1607.4(D) (2008); Watson v. Fort Worth Bank & Trust, 487 U.S. 977, 995–96, n. 3.

**21** According to the prevailing (although not self-evident) approach, it is illegitimate to grant preferential treatment to an individual or to a group because of the race of such individual or group even if such treatment is aimed to insure a fair representation of this race.

**22** In Ricci v. Destefano, 557 U.S. 557, 585 (2009), the Supreme Court held that "once . . . employers have made clear their selection criteria, they may not then invalidate the test results. Doing so, absent a strong basis in evidence of an impermissible disparate impact, amounts to the sort of racial preference that Congress has disclaimed, and is antithetical to the notion of a workplace where individuals are guaranteed equal opportunity regardless of race."

there are other possible practices that are similarly job related (for instance, in terms of the predictive power of a qualification test regarding an employee's performance) and can be expected to have a more equal impact. In short:

**Rule 1:** An employer is required to discard an imbalanced promotion test in favor of an alternative, which is as job related as the former or at least close to it, and whose result is more racially balanced. However,

**Rule 2:** An employer is not allowed to discard a suggested test on the basis of its racially imbalanced impact in favor of an alternative test, which is not as job related as the former or at least sufficiently close to it.

These two rules seem to provide a coherent solution. However, determining the legal outcome in a specific case is more complicated once employees' reliance interest is taken into account. Rules 1 and 2 deal with instances in which discarding a specific promotion test does not infringe legitimate expectations of the employees, either since the test was not yet implemented or the change of the test will affect only future decisions. But what if discarding the promotion test would infringe the employees' legitimate expectations and reliance interests? In circumstances in which an employment practice (such as a promotion test) has already been applied, the expectation and reliance interests of the employees who are eligible to some benefit according to this practice impose a constraint on the employer's power to discard it.[23] This constraint is not an absolute one. Assume that it can be infringed only when discarding the original practice is required to avoid a sufficiently high disparate impact. Thus:

**Rule 3:** An employer is prohibited from discarding the results of an imbalanced promotion test that was already applied in favor of a similarly imbalanced test.

Note that addressing each pair of rules independently does not result in any incoherence. Rules 1 and 2 are consistent, as Rule 1 ranks two options that are similarly job related while Rule 2 ranks options that are not. Similarly, Rules 2 and 3 are coherent, as Rule 2 (but not 3) deals with the legitimacy of discarding a test on the basis of its racially imbalanced impact. For the same reason, Rules 1 and 3 too do not contradict each other. However, when all three Rules apply, the outcome may well be a non-transitive cycle.

To analyze the outcome of these three rules, consider the following three alternative employment practices:

**23** See Ricci v. Destefano, 129 S. Ct. 2658, 2676 (2009), in which the Court held that "[e] xaminations like those administered by the City create legitimate expectations on the part of those who took the tests," and invalidated the City's decision to discard these tests.

*a* A promotion test which is highly job relevant, but has an imbalanced impact.
*b* A promotion test which is only moderately job relevant, but its impact is more racially balanced.
*c* A promotion test which is moderately job relevant, its impact is imbalanced, but it was already relied upon by certain employees.

Suppose a court is asked to compare test *a* with *b*. Both are yet to be given, hence Rule 3 is irrelevant here. As the job relevancy of *b* is only moderate (compared to the high job relevancy of *a*), Rule 2 dictates that *a* cannot be replaced with *b* (see Table 1).

**Table 1:** The three promotion tests.

| | Rule 1 | Rule 2 | Rule 3 |
|---|---|---|---|
| | **Balanced impact** | **Job relevancy** | **Reliance interest** |
| *a* | Low | High | No |
| *b* | High | Moderate | No |
| *c* | Low | Moderate | Yes |

Comparing tests *b* and *c*, courts will observe that both are only moderately job-relevant, but *b* is racially balanced while *c* is not. Rule 1 implies that *b* is better than *c* while Rule 3 does not apply (as *b* is racially balanced, but *c* is not). Test *b* is thus better than *c*.

Consider now the ranking of tests *a* and *c*. As both are racially imbalanced, Rules 1 and 2 do not apply. Unlike test *a*, test *c* was already conducted, thus Rule 3 implies that *c* is better than *a*.

But how should courts rank the triple *a,b,c* when all three of them are possible? As in the preceding illustration, here too it may well be the case that in a specific dispute the court would have to rank only a pair of tests. However, it may well be wrong to resolve the dispute by referring exclusively to the rule that ranks the specific two tests that are in dispute, disregarding the two other rules, as it would deprive employers and employees information about the proper ranking of all three tests.

Following the solution discussed in Section 2, the cycle can be avoided by ranking the three rules. Assume, for instance, that it can be established that Rule 3 takes precedence over the two others and should therefore be the first one to be applied. As stated above, this rule applies only to the pair $\{a,c\}$. According to this rule, an employer is not allowed to discard *c* in favor of *a*, therefore $c \succ a$. Next, we should rank Rules 1 and 2. Suppose that Rule 1 is normatively superior to Rule 2 (for instance, since Rule 1 is explicit in Title VII, whereas Rule 2 results from an interpretation of this provision). This rule applies to the pair $\{b,c\}$ only, and according to it, $b \succ c$.

These two stages yield that $c \succ a$ and $b \succ c$. Transitivity then implies $b \succ a$. It is true that had the employer implemented the promotional test $a$, it would not have been legitimate, due to Rule 2, to discard $a$ in favor of $b$. However, given the assumed precedence of Rule 3, when $c$ is the already selected and relied upon test, and the precedence of Rule 1 over Rule 2, the ranking of $a$ and $b$ by Rule 2 should be ignored in order to avoid intransitivity.[24]

The essential decision is therefore that of ranking the relevant rules. Seemingly, a rule set forth by Congress (Rules 1 and 2 in our case) should be applied before a judge-made rule (such as Rule 3 here). However, substantive reasons may well be relevant too, and these may lead to alternative rankings of the rules. For instance, requiring an individual to bear the burden of a loss – other than a mere lost benefit – to prevent imbalanced outcome may be considered as a form of taking, in violation of the Fifth Amendment,[25] such that Rule 3 takes precedence over the two other rules.

Setting the ranking of the rules will often be based on a normative theory which is outside the rules themselves and the ranking of the options that they yield. It is beyond the scope of this paper to resolve the dilemma what is the correct priority among the above three rules, or what is the general theory that determines the ranking of rules. Our point is that ranking the rules will enable courts to avoid cycles. Indeed, ranking the rules is actually the central role of courts in this regard.

## 4.3 Beneficiaries' right to recovery from Tortfeasors

The legal-system's commitment to avoid cycles can serve as a resourceful interpretation tool. In this subsection we comment on such possible uses of the solution concept that we offer here.

Assume that a judge should resolve a dispute between outcomes $a$ and $b$, and that the relevant rule (Rule 1) prioritizes $a$ over $b$. In principle, the opposite

24 Note that in our example although $c$ wins in the first comparison (where Rule 3 is applied for the pair {a,c}), $c$ is not the best policy overall as it is defeated by $b$.

25 See, e.g., Local 28 of the Sheet Metal Workers, Int'l Ass'n v. EEOC, 478 U.S. 421, 479 (1986) (upholding an affirmative action plan in part because it "did not require any member of the union to be laid off, and did not discriminate against existing union members"); Wygant v. Jackson Bd. of Educ., 476 U.S. 267, 283 (1986) (emphasizing the importance of "not disrupting the settled and legitimate expectations of innocent parties"). See also Zamir (2012), Vars (1996:79), and Ayres and Vars (1998:1617).

ranking of these two outcomes can be justified by invoking some third outcome $c$, and applying the governing norms regarding the trio, $a$, $b$, and $c$. If some other rule (Rule 2) determines that $b \succ c$, and a third one (Rule 3) results in $c \succ a$, the judge may apply the above solution concept of ranking the ranking rules, and if Rule 3 prevails, the result would be $b \succ a$, notwithstanding Rule 1. This procedure can be justified as long as the outcome $c$, which is irrelevant in resolving the current dispute, may plausibly generate a cycle. In other words, in prioritizing two given outcomes an additional outcome can be invoked as long as it is likely that a ranking of all three outcomes will be required. Not so if it can be expected that in any actual dispute only two outcomes are to be ranked.

A related misuse of the solution offered here is where a cycle is a finite one. This, for instance, may occur when, regarding a given case, $a$ has a cause of action against $b$, and $b$ has a cause of action against $c$, but $c$ has a cause of action against $a$. As long as this "cycle" stops at one place, the problem of non-transitivity does not materialize, and all the relevant rules can apply. To demonstrate this case, consider the following example, which deals with instances in which an injured person, $a$, is entitled to be compensated (or reimbursed for his costs) from both the injurer, $c$, and another party, the beneficiary, $b$. The beneficiary may be obliged to cover the injured's costs either due to contractual obligation (as in the case of personal accident insurance) or according to a statue.

Consider in this respect the Medical Care Recovery Act (MCRA), which provides that when the government treats or pays for the treatment of a military member, retiree, or dependent, it may recover its expenses from any third party legally liable for the injury or disease. The MCRA creates an independent cause of action for the United States. The prevailing view is that procedural defenses that may prevent the injured person from recovering, such as a failure to properly file a complaint on the third party, do not prevent the United States from pursuing its own action to recover the value of medical treatment provided to the injured person.[26] For example, injured persons do not have the power to release or contract away the right of the United States to recover the reasonable

26 For instance, the right of the United States to pursue an independent action to recover from tortfeasor for reasonable care and treatment is not subject to any state statute of limitations (sometime 2 years), but only to the 3 years limitations set at the MCRA (U.S. v. Gera 409 F.2d 117 (3d Cir. 1969)). The government is also released from state doctrine of inter-spousal immunity which may prevent the injured from recovering from the injurer (e.g., United States v. Moore, 469 F.2d 788, 790 (3d Cir. 1972)).

cost of medical care furnished, and thus the injured person may be bound by a compromise agreement with the injured but the government isn't (e.g. U.S. v. Greene 266 F.Supp 976 (N.D.Ill. 1967); Holbrook v. Andersen Corp. 996 F.2d 1339 (3d.Cir. 1993). As a result, the injured person, $a$, is entitled to compensation from the government, $b$, for certain expenses, such that $a \succ b$ (Rule 1). Similarly, the government is entitled to recovery from the injurer, $c$, such that $b \succ c$ (Rule 2). However, $c$ is entitled to recover from $a$ (based, for instance, on their prior settlement agreement), such that $c \succ a$ (Rule 3). Nevertheless, a non-transitive cycle does not occur, since $a$ is not entitled to recover again from $b$. Therefore, even if Rule 3 is of a higher normative ranking than the other rules, there is no basis for overriding Rules 1 or 2.

We thus suggest that the transitivity argument should be used only when all elements (three or more) of a group should be ranked. If each pair of claims can be ranked independently of the other outcomes and there is no natural reason to rank all claims simultaneously, one cannot justifiably refer to the requirement of transitivity to override a certain legal rule.

# 5 Concluding remarks

The idea that rules should be ordered is both natural and normatively appealing. In the Condorcet's voting paradox, where society needs to rank $a$, $b$, and $c$, person 1 preferences are $a \succ_1 b \succ_1 c$, person 2 preferences are $b \succ_2 c \succ_2 a$ while $c \succ_3 a \succ_3 b$, majority rule dictates $a \succ b$, $b \succ c$, while $c \succ a$. Unless we have some information about the three individuals we will have to rank them equally, and a non-transitive cycle emerges. However, unlike voters, rules can be ranked. Many judicial decisions involve conflicting past rulings or legislations and therefore require courts and legislators to rank them, e.g. the superiority of the Constitution over other laws or the superiority of the precedents of the Supreme Court over those of other courts. The ranking of the ranking rules is the key by which non-transitivity can be resolved.

Of course, the rankings of the rules too may be subject to nontransitive cycles, yet this is less likely to happen. To understand why, consider the following scenario for a cycle. Suppose that according to a certain rule, alternative $a$ supersedes alternative $b$. Similarly, another rule implies that $b$ supersedes $c$ and a third rule dictates that $c$ supersedes $a$. The wider is the scope of the ranked items $a$, $b$, and $c$, the harder it is to find reasons that apply to only two of them and not to the third alternative as well. Of course, if a rule is relevant for the comparisons of all three pairs then there cannot be a cycle.

Finally, the fact that the scope of rules that rank specific laws is typically wider than the scope of specific laws establishes our claim.[27]

# Appendix: ranking rules

This appendix explains how to create a complete ranking from ordered pairwise comparisons.

Let $N = \{1, \ldots, n\}$ and let $c$ be a choice function such that for all $i \neq j$, $c(i,j)$ exists and is a singleton. Let $R$ be a linear ranking of the $n(n-1)/2$ pairs $\{i,j\}$, $i \neq j$, that is,

$$\{i_1, j_1\} R \{i_2, j_2\} R \ldots R \{i_{n(n-1)/2},\ j_{n(n-1)/2}\}$$

and for no $k > 1$, $\{i_k, j_k\} R \{i_{k-1}, j_{k-1}\}$. Let $\varnothing \neq T \subseteq N$ and construct inductively partial linear orders $\succ_0^T, \ldots, \succ_{n(n-1)/2}^T$ on $T$ as follows:

1. $\succ_0^T = \varnothing$ (that is, no elements of $T$ are compared by $\succ_0^T$). Set $k = 1$ and move to the next step.
2. If $k = \frac{n(n-1)}{2} + 1$ set $\succ^T = \succ_{n(n-1)/2}^T$ and the construction is complete. Otherwise, move to the next step.
3. If $\{i_k, j_k\} \nsubseteq T$, set $\succ_k^T = \succ_{k-1}^T$, increase the value of $k$ by 1, and move to step 2. Otherwise, move to the next step.
4. If $i_k$ and $j_k$ are comparable by $\succ_{k-1}^T$, set $\succ_k^T = \succ_{k-1}^T$,[28] increase the value of $k$ by 1, and move to step 2. Otherwise, move to the next step.
5. Define $\gtrdot_k^T$ as follows:
   - For all $\{i,j\} \neq \{i_k, j_k\}$, $i \gtrdot_k^T j$ if $i \succ_{k-1}^T j$.
   - If $c(i_k, j_k) = i_k$, then $i_k \gtrdot_k^T j_k$. If $c(i_k, j_k) = j_k$, then $j_k \gtrdot_k^T i_k$.

   And let $\succ_k^T$ be the transitive closure of $\gtrdot_k^T$. Increase $k$ by 1 and move to step 2.

By construction, the ranking $\succ^T$ is transitive. It is also complete. Let $i, j \in T$ and let $k$ such that $\{i_k, j_k\} = \{i, j\}$. Either $i_k$ and $j_k$ were comparable by $\succ_{k-1}^T$, and then they are also comparable by $\succ_k^T$, or $\succ_k^T$ is constructed to compare them. As $\succ_1^T \subseteq \ldots \subseteq \succ_{n(n-1)/2}^T = \succ^T$, the ranking $\succ^T$ is complete.

**27** Kalai et al. (2002) introduced the concept of *rationalization by multiple rationales*, where the set of strict preferences $\succ_1, \ldots, \succ_k$ over a set $S$ thus rationalize a choice function if for every $T \subseteq S$ there is $\succ_i$ such that $c(T)$ is $\succ_i$-maximal in $T$. The Babylonian solution does that and goes a step further by ranking the partial preferences $\{\succ_i\}$. The rules are applied one at a time until a complete and transitive order is created. Naturally, it may happen that not all rules are used.
**28** Even if $c(i_k, j_k)$ disagrees with $\succ_{k-1}^T$. In such a case $c$ is overruled by $\succ_{k-1}^T$.

This method may not work if we drop the requirement that for no $k > 1$, $\{i_k, j_k\} R \{i_{k-1}, j_{k-1}\}$. Let $N = \{a_1, \ldots, a_4\}$, and suppose that $c(a_1, a_2) = a_1$, $c(a_2, a_3) = a_2$, $c(a_3, a_4) = a_3$, and $c(a_1, a_4) = a_4$. Suppose further that the top four pairs are $\{a_1, a_2\} R \{a_3, a_4\} R \{a_2, a_3\} R \{a_1, a_4\}$ but also $\{a_1, a_4\} R \{a_2, a_3\}$. The two orders $a_1 \succ a_2 \succ a_3 \succ a_4$ and $a_3 \succ a_4 \succ a_1 \succ a_2$ are consistent with the above algorithm.

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