Information Choice and Portfolio Bias in a Dynamic World

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Abstract

Contrary to standard theory, observed portfolios are concentrated in asset classes which comove strongly with the non-financial income of investors – e.g. home bias in equities. As an explanation, I propose a dynamic framework of endogenously generated information asymmetry, where rational agents optimally choose to focus their limited attention on (home) factors that drive both their non-financial income and some of the risky asset payoffs. In turn, the agents concentrate their portfolios in assets driven by those endogenously familiar factors. I build on the existing literature of endogenous information and portfolio choice by considering a dynamic framework. I show that introducing dynamics naturally leads to decreasing returns to information, in contrast to the increasing returns that typically arise in the standard static framework. This is an important distinction, as the decreasing returns are important for an information-based explanation of portfolio concentration that is consistent with the secular decline in the home bias in equities and salient empirical regularities in household-level portfolios.

JEL Codes: F3, G11, G15, D8, D83

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1 Introduction

Portfolio bias is one of the best established and longest standing puzzles in financial economics. It boils down to the fact that observed portfolios are not well diversified, seemingly leaving investors exposed to avoidable idiosyncratic risk. Making matters worse, portfolios are specifically concentrated in assets that are positively correlated with the investors’ non-financial income, and thus appear to provide little hedging benefits either. This is in sharp contrast to traditional economic theory, which suggests that agents should diversify holdings, and in particular favor assets which are negatively correlated with their non-financial income. These systematic biases are prevalent in both aggregate national data, e.g. the home equity bias, and in household-level portfolios, which are underdiversified in general, and also exhibit local and own company bias.¹

An intuitive and long standing potential explanation is information asymmetry among investors. If agents are differentially informed about different risk factors, in equilibrium they would hold portfolios that differ from the usual benchmarks. A key advance in this literature are models of endogenous information choice, which have shown that general equilibrium forces could generate increasing returns to information, and thus lead to full specialization in learning.² In turn, this could act as a powerful amplification mechanism for any ex-ante information asymmetry between agents that could lead to strong asymmetry in equilibrium, even if prior differences are small. The key to the result is that investors profit most from information that the average market participant does not know, since it allows them to buy under-valued assets. The existing studies, however, utilize static frameworks where financial markets are only open once and asset payoffs are exogenous.

In this paper, I extend the literature by developing and analyzing a dynamic framework of endogenous information and portfolio choices. In this model asset markets are open every period, and hence asset payoffs are determined both by future dividends, given by an exogenous process, and by the future equilibrium price. This introduces an important endogenous component to asset payoffs, which turns out weakens the result of increasing returns to information. In this recursive dynamic framework, investors enjoy increasing returns to information only when information is relatively scarce, but are faced with decreasing returns when their ability to acquire information is larger. This has important implications about the resulting portfolio choice decisions and about the nature of equilibrium portfolio

bias. In particular, in this model diversification generally increases, rather than decreases, with investor sophistication, and diversified portfolios outperform concentrated portfolios.

The emergence of decreasing returns to information is the key differentiating feature of the dynamic framework. The basic intuition is that increasing returns obtain globally in environments where the endogenous information choice could potentially eliminate all uncertainty about the future payoffs, as for example in a static model where agents can acquire increasingly precise signals about the future dividends. In a dynamic equilibrium model, however, agents face a measure of ‘unlearnable uncertainty’ because asset returns are partially determined by the future equilibrium price. This price is an endogenous variable determined by next period’s actions and hence is measurable in next period’s aggregate information set. Crucially, in a recursive framework information sets become increasingly finer over time as agents pick up new pieces of information. Hence, the future equilibrium price is not measurable in the available information at time $t$, since in the future agents have access to pieces of information that are outside of their scope today. As a result, information acquisition ultimately exhausts its ability to reduce the uncertainty of future payoffs, and this introduces decreasing returns to information and interior solutions for information choice.

The decreasing returns to information have two important implications. First, in order to preserve information asymmetry and thus portfolio bias, it becomes important to consider endogenous reasons for valuing one type of information more than another in the broader context of the economic framework of the agent. Without such a force, since it is no longer optimal to fully specialize in learning, the endogenous information choice would not only no longer necessarily amplify ex-ante information differences, but could in fact fully undo them. This paper shows that labor immobility, and non-diversifiable non-financial income in general, is one such force. Households want to reduce uncertainty about their equilibrium consumption, and hence particularly value factors that drive both labor income and one or more asset payoffs. Since information is a non-rival good, any information about a common factor can be used equally well to learn about both future labor income and future dividends. This gives the households an endogenous reason to disproportionately value information about factors driving their labor income, and this generates information asymmetry in equilibrium even in the case of decreasing returns to information.\(^3\)

As agents tilt their information acquisition towards such common factors, they reduce the uncertainty of assets that comove with their labor income, and in equilibrium they find it optimal to tilt their portfolios towards those assets as well. This increases the agent’s

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\(^3\)Non-diversifiable labor income plays a similar role in swaying information choice in Nieuwerburgh and Veldkamp (2006). The key difference is that they analyze a static model that exhibits increasing returns to information acquisition, while I study a dynamic framework, and analyze the interaction between the resulting decreasing returns to information and labor income.
incentives to acquire information about the labor income factors further, which creates a feedback loop between information and portfolio choice. This loop is at the heart of the increasing returns to information in a standard framework, and would typically lead to corner solutions for information acquisition. In my dynamic model, however, information acquisition eventually exhausts its ability to reduce uncertainty, which diminishes the feedback effect and introduces decreasing returns to information. As a result, the optimal information choice is an interior solution that equalizes the marginal benefits of different types of information. As information is no longer a straightforward amplification mechanism of any ex-ante differences, it becomes crucial to consider the broader economic context of the agents and analyze other forces that affect the value of different kinds of information.

The second important implication of the decreasing returns to information is that information asymmetry and portfolio concentration have a non-monotonic relationship with an agent’s ability to acquire information. Agents with a low capacity for information find it optimal to specialize fully in information acquisition, and learn only about a single factor, while agents with a high capacity choose to spread their efforts across a variety of different factors. Thus, the resulting information asymmetry and the degree of portfolio concentration is increasing in the capacity for information when that capacity is low, and decreasing when it is high. Hence, this framework can generate portfolio bias in a way that is consistent with the empirical observation that an increase in the ability to acquire information is associated with an increase in portfolio diversification.

The key ingredients to the model are non-diversifiable non-financial income (incomplete-insurance), information processing constraints and unlearnable uncertainty. The agents see the current value of the persistent economic fundamentals and also observe both public and private signals about the value of fundamentals one period ahead. They make an information choice in choosing the precision of their private signals optimally, subject to an entropy constraint in the tradition of the Rational Inattention literature. If the households had infinite information capacity, they will be able to fully reveal the value of the future state. However, in this recursive information framework, tomorrow’s agents also have information on fundamentals two periods ahead. This makes next period’s asset prices a function of the innovation to fundamentals two periods ahead, which is a stochastic variable that today’s agents cannot learn, even if they had unlimited information capacity.

Thus, in a recursive dynamic model, agents are exposed to “unlearnable” uncertainty, or uncertainty that the household cannot reduce today even if it was able to acquire and process all information available to markets today. The existence of unlearnable uncertainty is what breaks the increasing returns to information. It weakens the feedback loop between information choice and portfolio choice, as further and further reductions in learnable uncertainty lead
to smaller and smaller proportional drops in the overall uncertainty faced by the agent. Importantly, the model shows that unlearnable uncertainty, and hence decreasing returns to information, are a fundamental feature of dynamic models as compared to static models.

I empirically evaluate several of the key differing implications of the dynamic framework, relative to the static one, and show that the data favors a dynamic model. I focus particularly on the international evidence of the home equity bias, as labor is particularly immobile between countries. First, I show that the home bias is negatively related to information capacity. Second, I show that controlling for information capacity, home bias is positively related to the size of labor income, which supports the idea that this is indeed an important force that affects information choice, and through it portfolio choice. Third, I exploit the cross-sectional variation in the foreign part of national portfolios, to show that not only are aggregate portfolios home biased, but the rest of the portfolio holdings are also biased in a way that supports the model. Overall, the paper shows through both theoretical and empirical results that dynamic considerations and unlearnable uncertainty play a key role in understanding empirical portfolio puzzles. Since this implies that full specialization in learning is typically no longer optimal, the results imply that portfolio choice cannot be studied in isolation – the rest of the economic environment is crucial in determining information choice, and as a result the optimal portfolio choice.

The idea that information asymmetry can generate concentrated portfolios is quite intuitive, and goes at least as far back as Merton (1987) who imposed the information structure exogenously. More recent work by Peress (2004), Van Nieuwerburgh and Veldkamp (2009, 2010) develops models where the agents make optimal information acquisition choices and the information structure arises endogenously. A key result is that increasing returns to information arise in equilibrium, which leads to full specialization in learning and thus the information choice can amplify arbitrarily small ex-ante information differences. This type of framework has become the benchmark for a number of subsequent papers, and has been very successful at explaining the overall level of home bias in national portfolios for example, but it is harder to square with some other facts such as the secular decline in the home bias. In this paper, I extend the literature by showing that dynamics, unlearnable uncertainty and non-diversifiable labor income produce an information-based theory that lines up well with all the facts.

The paper is also related to the open macroeconomics literature on the home bias, and specifically the strand of literature that considers the importance of labor income in the determination of international portfolios. Coeurdacier and Gourinchas (2011) and Heathcote and Perri (2007) develop two distinct frameworks where the joint determination of the equilibrium real exchange rate, labor income and asset returns generates a positive labor
income-hedging demand for the home equity asset. This paper shares the literature’s key insight that labor income plays an important role in the formation of home biased portfolios, but the mechanisms are fundamentally different. In my model, labor income does not provide a positive hedging demand, but rather is the reason that the agents decide to bias their information acquisition strategy towards the home asset. The two frameworks are complimentary and would amplify each other’s results if put together. An important difference, however, is that the current model can simultaneously explain the portfolio concentration in both international and domestic portfolios, while the hedging motive identified by the macro literature only operates in an international environment with volatile exchange rates.

As a result, however, such models imply that information asymmetry and portfolio bias are increasing in the ability to acquire information. This seemingly puts information-based explanations of portfolio bias at odds with the secular decline in the home bias over the past two decades, which coincides with the IT revolution, and with the micro-data regularity that sophisticated households tend to hold more diversified portfolios.4

2 The Static Model

The key feature of the dynamic framework is the emergence of decreasing returns to information, which come about as a result of the fact that unlearnable uncertainty arises endogenously in a dynamic model. In order to more transparently highlight the crucial role of unlearnable uncertainty, I start with analyzing a static version of the model where it is introduced exogenously. Moreover, this model is also directly comparable to the existing literature, and would facilitate comparisons later on.

For ease of exposition, I will present the model in an international framework, assuming that there are “home” and “foreign” agents, choosing between home and foreign assets. This allows for the unambiguous terminology “home” and “foreign” but all results carry over to a closed, multi-sector (region) economy where an agent has the option of investing in different sectors (regions) of the economy.

There is a continuum of agents of mass one that live in each country, earn their non-financial income there,5 and can trade a riskless bond and “home” and “foreign” risky securities. I will describe the problem of the home agent, with the understanding that the foreign agents are perfectly symmetric. The agents live for two periods – they make

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5In this paper I model the non-financial income as labor income, but in actuality this is the income stream coming from all other sources, but the publicly traded financial investments of an agent. Hence, this also includes the profits/dividends received from any privately-owned companies, private investments and etc.
information and portfolio choices in period $t$, and consume their resulting wealth in period $t+1$. Agents face the static problem of maximizing utility over period $t+1$ consumption, and do not value leisure. Therefore agents supply their whole labor endowment, normalized to 1, at the wage rate they face. Thus each agent faces the following period $t+1$ budget constraint,

$$c_{h,t+1} = \delta w_{h,t+1} + x_{ht}d_{h,t+1} + x_{ft}d_{f,t+1} + b_{ht}R,$$

where $w_{h,t+1}$ is the home wage rate, $d_{h,t+1}$ is the payoff (dividend) of the home asset, $d_{f,t+1}$ is the payoff (dividend) of the foreign asset, $R$ is the gross return on the riskless bond, and $x_{ht}$ and $x_{ft}$ are the quantities of the home and foreign asset the agent chose in period $t$. In this static version of the model asset markets are not open next period, thus the dividends of the risky assets are the entirety of their payoff. The first term in the above equation, $\delta w_{ht}$, is the non-financial (wage) income of the individual, and the rest is his financial based income. The parameter $\delta$ controls the relative weight of labor income in the agent’s total income. When $\delta = 0$ the agent has no labor income and period $t+1$ consumption relies entirely on financial income, and the importance of labor income increases as $\delta$ grows. The existence of labor income is central to the mechanism of the model, and the parameter $\delta$ will be useful for comparative static exercises later on.

As is standard, I model all exogenous payoff processes as Gaussian. The only departure from the standard framework is that each payoff is the sum of two separate components:

$$w_{h,t+1} = a_{h,t+1} + e_{w_{h},t+1}$$
$$d_{h,t+1} = a_{h,t+1} + e_{d_{h},t+1}$$
$$d_{f,t+1} = a_{f,t+1} + e_{d_{f},t+1}$$

where all variables are normally distributed with $[a_{h,t+1}, a_{f,t+1}]' \sim N(\mu_a, \Sigma_a)$ and $e_{j,t+1}$ are iid $N(0, \sigma_{e_j}^2)$, $j \in \{w_h, d_h, d_f\}$. In this static model it is not needed to further specify the time-series processes for the exogenous factors, hence I simply assume that $\mu_a$ and $\Sigma_a$ are constant and are not indexed by $t$.

The factor $a_{h,t}$ is the home fundamental that drives both labor income, $w_{h,t}$, and the dividend to the home risky asset $d_{h,t}$. This captures the idea that labor income and home equity are both affected by some of the same forces.\footnote{One can think of $a_{h,t}$ as home country/sector TFP and of $z_f$ as the foreign TFP.} However, the two are not perfectly correlated as the factors $e_{w_{h},t}$ and $e_{d_{h},t}$ are independent from each other, which ensures labor income is not traded and hence is not perfectly diversifiable. The imperfect relationship

\footnote{One can think of $a_{h,t}$ as home country/sector TFP and of $z_f$ as the foreign TFP.}
between home dividends and labor income is the minimum complexity required for the main results, but it is straightforward to generalize their actual structural relationship. For simplicity, the framework also assumes \( \Sigma_a \) is diagonal and hence there is no relationship between home and foreign payoffs.\(^7\)

Period \( t \) is split into two sub-periods. In the morning, the agents make their information acquisition choices, and in the afternoon they observe their idiosyncratic informative signals, update beliefs and participate in the financial market. In that latter part of the period, an agent chooses her portfolio allocations subject to her initial budget constraint and taking the equilibrium asset prices as given,

\[
W^{(i)}_{h,t} = p_{ht}x_{ht} + p_{ft}x_{ft} + b_{ht},
\]

where \( W^{(i)}_{h,t} \) is the initial wealth of agent \( i \) in the home country, \( p_{ht} \) is the price of the home asset and \( p_{ft} \) is the price of the foreign asset.

Prior to making the portfolio allocation choice the agents receive informative, but noisy, signals about \( \{a_{h,t+1}, a_{f,t+1}\} \) but not about any of the \( e_{j,t+1} \). This captures the idea that payoffs are subject to two types of uncertainty – uncertainty that an agent can reduce by acquiring and processing information available today (newspapers, Federal Reserve announcements, Analyst reports, etc.) and uncertainty that the agents cannot reduce with any of today’s information. I will refer to the first type as “learnable” uncertainty and to the other one as “unlearnable”. Without such a dual structure of uncertainty the model would imply that an agent with sufficiently large information processing capacity could forecast the future arbitrarily well. In the static model, the unlearnable uncertainty is a realistic, but exogenous assumption and is controlled by the choice of \( \sigma_{e_j}^2 \). As we will see, however, in the dynamic model unlearnable uncertainty arises naturally even when \( \sigma_{e_j}^2 = 0 \).

Agent \( i \) receives unbiased signals of the form:

\[
\eta_{ht}^{(i)} = a_{h,t+1} + \varepsilon_{\eta_{ht}}^{(i)}
\]

\[
\eta_{ft}^{(i)} = a_{f,t+1} + \varepsilon_{\eta_{ft}}^{(i)}
\]

where \( \varepsilon_{\eta_{ht}}^{(i)} \) and \( \varepsilon_{\eta_{ft}}^{(i)} \) are independent of all other random variables, and \( \varepsilon_{\eta_{ht}}^{(i)} \sim N(0, \sigma_{\eta_{ht}}^2) \), \( \varepsilon_{\eta_{ft}}^{(i)} \sim N(0, \sigma_{\eta_{ft}}^2) \).

Importantly, the precision of the signals is not exogenous, but is optimally chosen by the agents. This choice is made subject to the constraint that the total amount of information

\(^7\)This too can be generalized, as long we stay within a joint Gaussian framework.
carried in the chosen signals is limited by the following entropy reduction constraint:

$$H(a_{h,t+1}, a_{f,t+1} | I_t^p) - H(a_{h,t+1}, a_{f,t+1} | I_t^{(i)}) \leq \kappa,$$

where $H(X)$ is the entropy of random variable $X$ and $H(X|Y)$ is the entropy of $X$ conditional on knowing $Y$.\(^8\) Entropy is the standard measure of uncertainty in information theory, and consequently reductions in entropy measure information flow. Moreover, $I_t^p$ is the information set consisting of all public signals (in this case the equilibrium prices $p_{ht}$ and $p_{ft}$), and $I_t^{(i)}$ is the private information set of agent $i$, which combines the public signals with her two private signals. The expression $H(a_{h,t+1}, a_{f,t+1} | I_t^p) - H(a_{h,t+1}, a_{f,t+1} | I_t^{(i)})$, measures the amount of information about $\{a_{h,t+1}, a_{f,t+1}\}$ contained in the signals $\{\eta_{ht}, \eta_{ft}\}$, over and above the information encoded in the equilibrium prices. Thus, the above constraint states that agents can only choose signals that carry no more than a total of $\kappa$ bits of information. Intuitively, the agents can only acquire a limited amount of information (e.g. they only have so much time to analyze financial information, news and etc.) and will need to choose how to apportion their attention optimally between the home and foreign factors.

The independence of $\varepsilon_{ht}^{(i)}$ and $\varepsilon_{ft}^{(i)}$ implies that agents can only reduce the posterior variance of $a_{h,t}$ and $a_{f,t}$ but cannot choose a correlation structure for their posterior beliefs – the correlation structure is taken as given from their priors. In other words, this amounts to assuming that learning about $a_{h,t}$ and $a_{f,t}$ are independent and unrelated activities that the agents carry out separately from one another, which appears to be a natural assumption here given that the factors $a_{h,t}$ and $a_{h,t}$ themselves are assumed to be independent.\(^9\) After observing the described signals, the agents use Bayesian updating with the correct prior to form their posterior beliefs. Contrary to the standard approach in the literature, I assume the agents have identical priors over both the home and foreign factors. In this sense, there is no exogenously imposed information asymmetry and the rest of the paper focuses on showing that even without prior exogenous information advantages, the home agents’ utility maximizing decisions lead to ex-post information asymmetry.

Lastly, I assume that the agent’s have mean-variance utility, which is the standard choice in this literature, and thus they maximize

$$E(c_{h,t+1}^{(i)} | I_{ht}^{(i)}) - \frac{\gamma}{2} \text{Var}(c_{h,t+1}^{(i)} | I_{ht}^{(i)}),$$

where $\gamma$ is the absolute risk aversion coefficient (common across all agents) and $I_{ht}^{(i)}$ is home

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\(^8\)Entropy is defined as $H(X) = -E(\ln(f(x)))$, where $f(x)$ is the probability density function of $X$.

\(^9\)In any case, at the cost of some analytical tractability, this assumption can be relaxed along the lines of Mondria (2010), but is not important to the main point of the paper.
agent \(i\)'s information set. This is the standard utility function used in the literature on endogenous information choice and portfolio choice, due to its analytical tractability.\(^{10}\) In essence, this is an exponential (CARA) utility function with an added desire for early resolution of uncertainty.\(^{11}\)

### 2.1 Portfolio Choice

It is convenient to define the following notation for the posterior and prior variances of \(a_{ht}\) and \(a_{ft}\), \(\text{Var}_i^{(t)}(a_{ht}) = \hat{\sigma}_h^2\), \(\text{Var}_i^{(t)}(a_{ft}) = \hat{\sigma}_f^2\), and \(\text{Var}(a_{ht}) = \text{Var}(a_{ft}) = \sigma_a^2\), and for the posterior means \(E_i^{(t)}(a_{ht}) = \hat{\mu}_h\) and \(E_i^{(t)}(a_{ft}) = \hat{\mu}_f\). The information set of the agents consists of their private signals \(\eta^{(i)}\) and of the equilibrium prices \(p_{ht}\) and \(p_{ft}\), which are publicly observed informative signals. They also contain information about the fundamentals because they aggregate the information revealed by the agents’ portfolio choices. The agents use the standard Bayesian updating formulas to obtain the posterior distributions of the fundamentals conditional on the observed signals, which are Normal, and thus the optimal portfolio choice is

\[
x_{ht} = \frac{\hat{\mu}_h - p_{ht} R}{\gamma(\hat{\sigma}_h^2 + \sigma_{eh}^2)} - \frac{\delta \hat{\sigma}_h^2}{\hat{\sigma}_h^2 + \sigma_{eh}^2}
\]

\[
x_{ft} = \frac{\hat{\mu}_f - p_{ft} R}{\gamma(\hat{\sigma}_f^2 + \sigma_{ef}^2)}
\]

Two forces could potentially affect the agent’s desire to alter her portfolio holdings from being split equally between the two assets. She has an additional hedging motive to trade the home asset (the second term in the expression for \(x_{ht}\)) that does not factor into the demand for the foreign asset and the speculative portion of the portfolios themselves are affect by any information asymmetry, i.e. \(\sigma_h^2 \neq \sigma_f^2\).

In addition to the home and foreign informed traders, each risky asset is also traded by a measure of liquidity or “noise” traders, which trade for exogenous reasons outside of the model. The net noise trader demand for home assets is \(z_{ht} \sim N(0, \sigma_z^2)\) and similarly for foreign assets \(z_{ft} \sim N(0, \sigma_z^2)\). Market clearing requires that the sum of the informed agents trades and the noise traders equals the fixed supply of assets, which is \(\bar{z}_h\) and \(\bar{z}_f\) respectively, and thus

\(^{10}\)See Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010).

\(^{11}\)See Van Nieuwerburgh and Veldkamp (2010) for more details.
I conjecture and verify that the equilibrium prices are linear in the state variables and given by

\[ p_{ht} = \tilde{\lambda}_h + \lambda_{ah} a_{h,t+1} + \lambda_{zh} z_{ht} \]

\[ p_{ft} = \tilde{\lambda}_f + \lambda_{af} a_{f,t+1} + \lambda_{zf} z_{f,t} \]

where the coefficients \( \lambda \) can be solved for, and are functions of the structural parameters of the model and the information choices of the agents. Explicit formulas are given in the Appendix.

### 2.2 Information Choice

At the beginning of period \( t \) the agent chooses the precision of the signals she will receive before making her portfolio choice, \( \sigma^2_{\eta_h} \) and \( \sigma^2_{\eta_f} \). Given the assumptions that factors are Gaussian and independent, the information constraint of a home agent reduces to

\[ \kappa \geq \frac{1}{2} \left( \ln(\tilde{\sigma}^2_h) - \ln(\hat{\sigma}^2_h) \right) + \frac{1}{2} \left( \ln(\tilde{\sigma}^2_f) - \ln(\hat{\sigma}^2_f) \right) \]

where for ease of notation I have suppressed the index \( i \), since it will be the case that in equilibrium, all home agents make the same information choice and end up with the same \( \hat{\sigma}^2_h \). Moreover, the terms \( \hat{\sigma}^2_j \) is the variance of the home and foreign fundamental, respectively, conditional on the public information set, i.e.:

\[ \hat{\sigma}^2_j = \text{Var}(a_{j,t+1} | \mathcal{T}_t^p) \]

for \( j \in \{h, f\} \). Finally, it is useful to redefine the information problem of the agent in terms of choosing

\[ k_h = \frac{1}{2} \left( \ln(\tilde{\sigma}^2_h) - \ln(\hat{\sigma}^2_h) \right) \]
and
\[ k_f = \frac{1}{2} \left( \ln(\hat{\sigma}^2_f) - \ln(\hat{\sigma}^2_f) \right) \]

which are amounts of information capacity the agent allocates to home and foreign information respectively. The beginning of period \( t \) information choice problem is then

\[
\max_{k_h, k_f} U_t = E(c_{h,t+1}^{(i)}) - \gamma \frac{1}{2} \Var(c_{h,t+1}^{(i)})
\]

s.t.
\[
k_h + k_f \leq \kappa \\
k_h \geq 0, k_f \geq 0
\]

Note that the expectations are now not conditional on anything, but are taken in respect to the agent’s priors. At the time of information choice, asset markets are not open yet hence there is no public signal, and he has not received his private signals yet either.

The first main result of the paper is that as long as the agent has some measure of non-diversifiable labor income risk, i.e. \( \delta > 0 \) and \( \sigma^2_{ew} > 0 \), the optimal information acquisition is always biased towards domestic information. This is formalized in Proposition 1 below.

**Proposition 1.** If \( \delta > 0 \) and \( \sigma^2_{ew} > 0 \), the home agent optimally chooses \( k_h > k_f \), i.e. she chooses to acquire more information about the home fundamentals. Similarly, the foreign agent chooses to acquire more information about his own domestic fundamentals and picks \( k^*_f > k^*_h \).

**Proof.** Sketched in text, more details in the Appendix.

To build some intuition about this result, it useful to derive the objective function that is maximized at the time of the information choice. The agent is forward looking, and takes into account the form of the optimal portfolio choices \( x_{ht} \) and \( x_{ft} \) and how they are affected by the signals she receives. Substituting that in the \( t+1 \) budget constraint, plugging everything back in (1) and evaluating the expectations we arrive at

\[
U_t = \gamma W_t + \gamma \delta \mu_h + \frac{1}{2} \Var(\hat{\mu}_h - p_{ht}R) \exp(-k_h) \sigma^2_h + \sigma^2_{eh} + \frac{1}{2} \Var(\hat{\mu}_f - p_{ft}R) \exp(-k_f) \sigma^2_f + \sigma^2_{ef} + \gamma \delta E(\hat{\mu}_h - p_{ht}R) \exp(-k_h) \sigma^2_h + \sigma^2_{eh} + \gamma \delta E(\hat{\mu}_f - p_{ft}R) \exp(-k_f) \sigma^2_f + \sigma^2_{ef} - \gamma \delta \mu_h \exp(-k_h) \sigma^2_h + \sigma^2_{eh} + \gamma \delta \sigma^2_{ew} \sigma^2_{eh} + \gamma \delta \sigma^2_{ef} \sigma^2_{eh} + \frac{\gamma^2 \delta^2 \sigma^2_{eh} \sigma^2_{ef}}{2 \exp(-k_h) \sigma^2_h + \sigma^2_{eh} + \exp(-k_f) \sigma^2_f + \sigma^2_{ef}}
\]
Intuitively, information is valuable to the agent for two reasons. First, it decreases the uncertainty associated with asset payoffs and allows her to make better portfolio choices – this channel is active for both the home and the foreign asset. Second, more precise home information is also helpful in reducing the uncertainty associated with the labor income of the agent. This second type of benefit is not present with foreign information, but only with home information, reflecting the basic structural assumption that labor is immobile and more closely associated with local factors.

Agents want to learn about risk factors that are associated with assets that have high expected excess returns $E(\hat{\mu}_h - p_{ht}R)$, which in equilibrium are

$$E(\hat{\mu}_j - p_{jt}R) = \gamma(\hat{\sigma}_j^{(a)})^2 \left[ \bar{z}_j + \frac{\delta}{2} \hat{\sigma}_j^2 + \sigma_z^2 \right]$$

where I have defined the perceived variance of asset $j$’s payoffs for the average market participant as

$$(\hat{\sigma}_j^{(a)})^2 = \left( \frac{1}{2(\hat{\sigma}_{jt}^2 + \sigma_{e,j}^2)} + \frac{1}{2(\hat{\sigma}_{jt}^2 + \sigma_{e,j}^2)} \right)^{-1}$$

On the one hand, the equilibrium excess returns depend on the total supply of the asset – the more risky assets that the investors have to hold, the bigger the risk-premium they require. The total supply of risky assets is not just $\bar{z}_j$, but also the hedging portfolio of the home agent, in case of home assets, and the hedging portfolio of the foreign agent for the foreign asset. Both agents have an incentive to take a short position in their domestic asset to offset their labor income risk, and that short position is something the market as a whole has to bear as well. The risk-premium also depends on their risk aversion ($\gamma$), and also on the precision of beliefs of the average market participant. As $(\hat{\sigma}_j^{(a)})^2$ decreases, the market is better informed about the uncertainty encoded in the payoffs of the asset $j$, and requires a lower risk-premium for being exposed to it.

Agents also want to learn about more uncertain assets that have more volatile expected excess returns, as information about those assets reduces a bigger portion of the volatility faced by the agent. We can show that the ex-ante variance of the excess returns is given by

$$\text{Var}(\mu_j^{(i)} - p_{jt}R) = (\hat{\sigma}_h^{(a)})^2 \left( \Phi_h^{(a)} + (\hat{\sigma}_h^{(a)})^2(\gamma^2 \sigma_z^2 + \gamma Q_h \Phi_h^{(a)}) \right)$$

where $\Phi_h^{(a)}$ is the learnable portion of the residual uncertainty faced by the average market participant,

$$\Phi_h^{(a)} = \frac{\hat{\sigma}_{ht}^2}{2(\hat{\sigma}_{ht}^2 + \sigma_z^2)} + \frac{\hat{\sigma}_{ht}^2}{2(\hat{\sigma}_{ht}^2 + \sigma_z^2)}.$$
and $Q_h$ is the average market precision, weighted by the learnable portion of the residual uncertainty:

$$Q_h = \left(\frac{\hat{\sigma}_{ht}^2}{\gamma^2(\hat{\sigma}_{ht}^2 + \sigma_e^2)} \frac{1}{\sigma_{nh}^2} + \frac{\hat{\sigma}_{ht}^2}{\gamma^2(\hat{\sigma}_{ht}^2 + \sigma_e^2)} \frac{1}{\sigma_{nh}^2}\right)$$

This tells us that several important things. First, again we see that agents like to learn about assets that the average market participant knows less about, and hence $(\hat{\sigma}_j^{(a)})^2$ is higher. Second, the agent likes to learn about assets for which the market still faces a relatively high proportion of learnable uncertainty (high $\Phi_h^{(a)}$). And third, investors would like to invest in assets for which the average market participant has a low precision. The last result, while not immediately obvious from the equation above, obtains because the market average variance $(\hat{\sigma}_j^{(a)})^2$ itself is decreasing in $Q_h$, the average precision. Overall, we see that the model displays strategic substitutability in learning – the more the other investors learn about asset $j$, the lower $(\hat{\sigma}_j^{(a)})^2$, and the lower is the benefit of learning about that asset. Ideally, investors would like to learn about assets that the average market participant is less informed about.

I will focus on a symmetric world, where the unconditional means and volatilities of the home and foreign factors are the same, and the home and foreign agents face mirror image problems. In equilibrium, the home and foreign asset markets will be symmetric in the sense that the average market participant would be equally unsure about both: $(\hat{\sigma}_h^{(a)})^2 = (\hat{\sigma}_f^{(a)})^2$. Together with the rest of the symmetry assumptions, we have that the ex-ante volatilities and mean excess returns of the home and foreign assets are equivalent. Then, the marginal benefits of home and foreign information become,

$$\frac{\partial U_t}{\partial k_h} = (A + \delta B) \frac{\exp(-k_h)}{(\exp(-k_h)\hat{\sigma}_h^2 + \sigma_{e_{dh}}^2)^2} > 0$$

$$\frac{\partial U_t}{\partial k_f} = A \frac{\exp(-k_f)}{(\exp(-k_f)\hat{\sigma}_f^2 + \sigma_{e_{df}}^2)^2} > 0$$

where $\delta B \geq 0$, which shows that ceteris paribus, home information is more useful than foreign information, and that extra value of home information comes from the non-diversifiable labor income component ($\delta > 0$). This is at the heart of the result in Proposition 1.\textsuperscript{12}

In conclusion, even in a completely symmetric world, the model generates a home bias in information acquisition, which biases the optimal portfolios towards domestic assets. The existence of a non-diversifiable, forecastable component of non-financial income is key. If $\delta = 0$, then home and foreign risks are of equal importance to the agent and if $\sigma_{e_{dh}}^2 = 0$, then the learnable risk component $a_{h,t+1}$ in labor income can be perfectly hedged with the home

\textsuperscript{12}Details on the constants $A$ and $B$ are derived in the Appendix.
asset, and thus home and foreign risk again become of equally important.

### 2.3 Unlearnable Uncertainty and Specialization in Learning

As mentioned above, the result in Proposition 1 is enough to establish that there is home bias in information acquisition. This opens up an information channel that tilts portfolios towards domestic assets, which is a force that can help us explain both portfolio under-diversification and the general observation of home (or local) bias in portfolios. Still, how much portfolio bias this channel produces will depend on the strength of the generated information asymmetry, i.e. on the actual difference between the optimal $k_h$ and $k_f$.

Clearly, one can achieve the biggest such difference if it was optimal for agents to fully specialize in learning in the home asset, so that $k_h = \kappa > k_f = 0$. Indeed, this corner solution is the case that arises most often in the standard models, as they focus on the case with no unlearnable uncertainty, $\sigma_{eh}^2 = \sigma_{ef}^2 = 0$. However, as shown in Proposition 2 below, this model can also feature decreasing returns and interior solutions to information when unlearnable uncertainty is present, i.e. $\sigma_{ej}^2 > 0$.

**Proposition 2.** *The objective function of the information choice problem, is convex in home information when $\hat{\sigma}_h^2 \geq \sigma_e^2$ and concave otherwise. Similarly it is convex in foreign information when $\hat{\sigma}_f^2 \geq \sigma_e^2$ and concave otherwise.*

*Proof.* Sketched in text, more details in the Appendix. □

The basic force behind this result is that the marginal benefits of both home and foreign information are decreasing, as long as $\sigma_{ej}^2 > 0$, i.e. there is some measure of unlearnable uncertainty in the home or foreign asset respectively. To build some intuition, it is useful to realize that there is a feedback effect between learning and portfolio choice. The more the agent learns about a particular asset, the lower is the posterior variance of its returns, and thus the agent expects to hold more of it. The bigger the expected holdings, the more important determinants they are of his terminal wealth, and the bigger the incentive to learn about this asset. This feedback loop generates the increasing returns. But since expected holdings are a function of the total variance of the asset, $\hat{\sigma}_j^2 + \sigma_e^2$, as $\hat{\sigma}_h^2$ falls, any further falls result in smaller and smaller actual increases in expected portfolio holdings. As the feedback loop weakens, the model transitions into featuring decreasing returns to information.

To see this more clearly, take a second derivative of the objective function, and rewriting it in the more intuitive terms of the posterior variances $\hat{\sigma}_j^2$, we get

$$\frac{\partial^2 U_{l^-}}{(\partial k_j)^2} = A_h \frac{\hat{\sigma}_h^2(\hat{\sigma}_j^2 - \sigma_{ej}^2)}{(\hat{\sigma}_j^2 + \sigma_{ej}^2)^3} > 0$$
Thus the model is convex and displays increasing returns to information, when the remaining reducible uncertainty, $\hat{\sigma}_h^2$ in case of home information and $\hat{\sigma}_f^2$ in case of foreign information, is bigger than the corresponding unlearnable uncertainty. The intuition is that specialization in learning pays off when extra information can reduce a large proportion of the remaining uncertainty facing the agent. However, if the residual uncertainty faced by the agent is unlearnable, then he is better off to diversify his information and expand his information acquisition into the foreign factor as well.

It is then immediate, that the returns to information are always increasing when there is no unlearnable uncertainty, $\sigma_e^2 = 0$. This is the case the majority of existing models focus on, and in that case the optimal action is to always be at the corner where the agents exhaust their whole available information capacity on learning about the home asset. In this case, any increases in capacity would naturally only exacerbate information asymmetry and thus increase portfolio concentration and home bias. On the one hand, this makes information acquisition a powerful amplification mechanism that is helpful in generating empirically meaningful portfolio bias.

On the other hand, it might make it seem like information based models of portfolio concentration are at odds with the observation that more sophisticated households tend to hold more diversified portfolios and that the home bias has been steadily trending downward, just as information technology has greatly expanded everyone’s ability to acquire information. However, as we can also guess from Proposition 2, this is not the case in an information model with some unlearnable uncertainty. With $\sigma_e^2 > 0$ the returns to information start to decrease once one has acquired a lot of information about the same factor, and this has the natural consequence that eventually, information asymmetry trends downward as information capacity increases. This is quite intuitive, as one can imagine that heavily constrained people are likely to focus on learning the single most pertinent information, but eventually, as more and more information is accumulated, it value of specialization declines. These results are formalized in Proposition 3 below.

**Proposition 3.** Let $\delta, \gamma$ and $\sigma_{eh}^2 = \sigma_{ef}^2 = \sigma_e^2$ be given (symmetric assets), Then,

1. If $\sigma_e^2 = 0$, there is no unlearnable uncertainty and the model displays increasing returns to information globally. The optimal information choice is to allocation all attention to the domestic asset, $k_h = \kappa^* = \kappa$, and not acquire any information about the foreign asset $k_f = k_h^* = 0$.

2. If $\sigma_e^2 > 0$, then corner solutions are optimal for low values of $\kappa$, and high values of $\kappa$ are associated with interior solutions. In particular,
There exists a value $\bar{\kappa} > 0$ such that $k_h = k_f^* = \kappa$ and $k_f = k_h^* = 0$ when $\kappa \leq \bar{\kappa}$, but the agents acquire information about both home and foreign factors, $k_h > 0$, $k_f > 0$, $k_h^* > 0$, $k_f^* > 0$, when $\kappa > \bar{\kappa}$.

The home bias in information acquisition, i.e. $k_h - k_f > 0$ and $k_f^* - k_h^* > 0$, is increasing in $\kappa$ when $\kappa \leq \bar{\kappa}$, and decreasing otherwise.

The home bias in information acquisition is always positive, and persists in the limit, with $\lim_{\kappa \to \infty} k_h - k_f > 0$.

Proof. The proof is in the Appendix.

The first result follows directly from Proposition 2, which informs us that with $\sigma_e^2 = 0$ the problem is convex and hence we should look for the corner solution that implies home bias in information acquisition. In this type of a world, the increasing benefits of information specialization are never exhausted and agents always focus their entire information capacity on the home asset.

The second set of results informs us that if there is some unlearnable uncertainty, then agents fully specialize in learning when they have relatively little capacity, $\kappa \leq \bar{\kappa}$. In that case the constraint is strongly binding and agents find themselves in a situation in which even after exhausting all of their learning capacity on a single factor, there is still a large amount of learnable uncertainty left. It is only optimal to diversify learning once the agent has sufficient information capacity to reduce the learnable portion of uncertainty below the amount of unlearnable uncertainty he faces. But once the agent has enough capacity to find it optimal to diversify learning, any additional capacity is disproportionately allocated to the foreign information that he knows less about. Thus, in that part of the space information asymmetry is decreasing monotonically with the ability to acquire information.

As a result, the information force generating home bias in portfolios is also steadily weakening, and thus now we will see a gradual, downward relationship between information capacity and portfolio bias. Through the prism of this model, the empirical observations are hence a result of the fact that information capacity is sufficiently large to make full specialization in learning suboptimal, which is a reasonable assumption. Importantly, the information asymmetry never unravels completely and exists even in the limit as capacity for information goes to infinity. But it’s overall size and whether it is strong enough to generate sizable portfolio bias, even in the limit, would depend on the endogenous forces that differentiate home and foreign information, in this case labor income. This is a quantitative question I examine later.

Figure 1 plots the size of information asymmetry, $\Lambda = k_h - k_f$, and the ratio of posterior variances $\frac{\hat{\sigma}_f}{\hat{\sigma}_h}$ on $\kappa$. One can clearly see the two regions defined by Proposition 3. At first, the
agent devotes all attention to the home asset and $\Lambda$ is growing linearly with $\kappa$. After a certain point he starts paying attention to the foreign asset as well, and information asymmetry slowly starts to unravel but still eventually converges to a positive number, signifying a home bias in the limit.

![Figure 1: Information Asymmetry and Information Capacity](image)

### 2.3.1 Sizing up the Information Asymmetry

In a world where information specialization is not always optimal, it becomes important to consider how strong equilibrium information asymmetry can be once agents decide to diversify learning and what factors affect it. Figure 1 illustrates that the value of $\kappa$ is clearly important. At the two extremes of scarce and abundant information capacity information asymmetry is smallest, and is highest for moderate values of $\kappa$. Proposition 4 looks at the effect of the other structural components of the model – labor income, risk-aversion and the size of unlearnable uncertainty.

**Proposition 4.** Let the conditions of Proposition 1 be satisfied. Then, $\bar{\kappa}$ is an increasing function of $\gamma$, $\delta$ and $\alpha$, where $\alpha = \frac{\sigma^2}{\sigma^2_z}$. Moreover, $\Lambda$ - the overall amount of information asymmetry - is an increasing function of $\gamma$ and $\delta$.

**Proof.** The proof is in the Appendix.

Perhaps unsurprisingly, we see that information asymmetry is increasing with the importance of labor income in the agent’s terminal wealth ($\delta$). This is a direct result of the fact that labor income is the key mechanism that endogenously increases the marginal benefit
of home information. The more labor income risk the agent is exposed to, the bigger is his incentive to focus on reducing home uncertainty and thus acquire more home information. On the other hand, risk-aversion primarily acts as an amplification to the other forces that determine the relative value of information. Higher risk-aversion makes the agent dislike uncertainty more, and exacerbates their desire to reduce the most important sources of uncertainty first.

Figure 2 plots information asymmetry ($\Lambda$) and the ratio of posterior variances $\hat{\sigma}_f/\hat{\sigma}_h$, as a function of risk aversion in the left panel, and as a function of $\delta$ in the right panel. The effects of both parameters are similar. Increases are associated with larger values of the tipping point $\bar{\kappa}$, and larger information asymmetry for all values of capacity $\kappa$, including in the limit of infinite capacity.

The right panel of Figure 2 also hints at the fact that for high values of information capacity $\kappa > \bar{\kappa}$ information asymmetry could completely disappear, as $\delta \to 0$. The reason is that if an agent has sufficient capacity to be at an interior solution, then the optimal information choice sets the marginal benefit of home information equal to the marginal benefit of foreign information:

$$\frac{\partial U_t^-}{\partial k_h} = A \frac{\hat{\sigma}_h^2}{\hat{\sigma}_h^2 + \hat{\sigma}_e^2} = A \frac{\hat{\sigma}_f^2}{\hat{\sigma}_f^2 + \hat{\sigma}_e^2} = \frac{\partial U_t^-}{\partial k_f}$$

As we see from the equation above, this implies that the posterior variance of the home factor equals the posterior variance of the foreign factor:
\[ \dot{\sigma}_h^2 = \dot{\sigma}_f^2 \]

When the agents have sufficient information capacity, and no endogenous reason to value home information more than foreign information, then they perfectly diversify their learning to make the residual uncertainty about home and foreign factors equal. Clearly, in this situation the agents no more have any information-based incentive to bias their portfolios one way or the other. This showcases, that portfolio and information choice cannot be studied in isolation from the bigger economic environment of the agents. With unlearnable uncertainty, \( \sigma_e^2 > 0 \), full specialization in learning is no longer always optimal, and information choice would not necessarily amplify any arbitrarily small ex-ante differences.

In a world like this, it becomes important to consider what other endogenous considerations might make the agent value certain types of information more than others. In this paper I show that non-diversifiable labor risk, and the basic observation that labor is immobile, especially compared to financial assets, is a basic economic force that can generate home bias in information, and as a result provide an incentive to bias portfolios towards home assets. This is in line with the results in Massa and Simonov (2006), who find that the concentration in household-level portfolios is indeed information driven, and is also strongly associated with the labor income of investors. However, this is not to say that labor income is the only such force – for example housing is similarly immobile.

Lastly, the proposition shows that the scope for increasing returns increases, as we increase the ratio of learnable to unlearnable uncertainty. This makes sense, given that Proposition 2 shows that decreasing returns occur once the agents have reduced the learnable uncertainty below the unlearnable uncertainty. The more learnable uncertainty we have, the more information capacity it takes to reduce it. Importantly, however, this ratio of learnable to unlearnable uncertainty also has another effect – it reduces the amount of non-diversifiable labor income. In the limit of \( \sigma_e^2 = 0 \), the agent can fully eliminate all of his labor income exposure, and this creates a situation where the agents have no incentive to have any bias in their information acquisition. Thus, \( \alpha \) has a non-linear effect on the overall information asymmetry, where it might be increasing it for certain value of parameters, but could decrease it for others.

### 2.4 Home Bias in Investor’s Portfolios

So far we have focused on analyzing the home bias in information, but at the end of the day we are most interested in the resulting portfolios. In this section I will first discuss how to define and measure the home bias in portfolios, and how it relates to the optimal information
choice. Next, I will turn to a quantitative evaluation of whether the model can generate empirically relevant numbers.

First, I define two benchmark portfolios. One is the mean perfectly diversified market portfolio,\(^{13}\)

\[
\begin{align*}
x_h^{Market} &= \bar{z}_h \\
x_f^{Market} &= \bar{z}_f
\end{align*}
\]

This portfolio would prevail in a world with no learning \(\kappa = 0\) and no labor income considerations \(\delta = 0\). In that case, agents are identical across both countries and in equilibrium all hold the market portfolio, given by the per-capita supply of assets. The other benchmark is the mean equilibrium portfolio in a world with non-diversifiable labor income \(\delta = 0\), but still no learning. In that case,

\[
\begin{align*}
x_h^{NoInfo} &= \bar{z}_h - \frac{\delta}{2} \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2} \\
x_f^{NoInfo} &= \bar{z}_f + \frac{\delta}{2} \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}
\end{align*}
\]

Compared to the first portfolio, this one is tilted towards foreign assets due to the hedging motive of the labor income. To measure this divergence, or bias, away from the market portfolio, I will use the standard Equity Home Bias index (EHB). The EHB index is a commonly used measure of portfolio concentration in the literature on home equity bias. It is defined as,

\[
EHB = 1 - \frac{\text{Foreign Equity as a Share of the National Portfolio}}{\text{Foreign Equity as a Share of the World Market Portfolio}}
\]

and measures how much a country’s equity portfolio deviates from the global market portfolio.\(^{14}\) The index is positive when the analyzed portfolio differs from the market portfolio by holding a larger proportion of home assets (i.e. exhibits home bias), negative when it exhibits foreign bias, and is 0 when it is exactly equal to the market portfolio.

For simplicity, I normalize the supplies of the two assets to be equal across countries \(\bar{z}_h = \bar{z}_f\), and then we can compute the EHB of the no learning portfolio as:

\(^{13}\)The mean is over the realizations of the noise traders \(z_t\). We can view this as the stochastic steady-state portfolio.

\(^{14}\)The market portfolio is the equilibrium portfolio in standard CAPM theory (Sharpe (1964)) and is the most commonly used benchmark of the “fully diversified” portfolio.
Clearly, this portfolio exhibits foreign bias. Next, consider the mean aggregate home portfolio of the optimally informed agents, given an information capacity constraint $\kappa$:

$$x_h = E\left(\int (x_h^{(i)}) = \frac{(\hat{\sigma}_h^{(a)})^2}{\bar{\sigma}_h + \sigma_e^2} \left[\bar{\sigma}_h + \frac{\delta}{2} \frac{\hat{\sigma}_h^2}{\bar{\sigma}_h^2 + \sigma_e^2}\right] - \delta \frac{\hat{\sigma}_h^2}{\bar{\sigma}_h^2 + \sigma_e^2}\right)$$

$$x_f = E\left(\int (x_f^{(i)}) = \frac{(\hat{\sigma}_f^{(a)})^2}{\bar{\sigma}_f + \sigma_e^2} \left[\bar{\sigma}_f + \frac{\delta}{2} \frac{\hat{\sigma}_f^2}{\bar{\sigma}_f^2 + \sigma_e^2}\right] - \delta \frac{\hat{\sigma}_f^2}{\bar{\sigma}_f^2 + \sigma_e^2}\right)$$

Computing the EHB I get,

$$EHB^{OptInfo} = 1 - \frac{(\hat{\sigma}_h^{(a)})^2}{\bar{\sigma}_h^2 + \sigma_e^2} - \frac{(\hat{\sigma}_f^{(a)})^2}{\bar{\sigma}_f^2 + \sigma_e^2} \frac{\delta}{2} \frac{\sigma_a^2}{\bar{\sigma}_h^2 + \sigma_e^2}$$

The second component works through the informational channel. Since optimal information is always skewed towards the domestic asset, then $1 - \frac{(\hat{\sigma}_h^{(a)})^2}{\bar{\sigma}_h^2 + \sigma_e^2} > 0$ and this generates a home bias pressure in portfolios as well. The hedging motive still exists, and it still provides an incentive to short the home asset, but optimal information acquisition lowers the effect of the hedging asset as well. This operates through both $\frac{(\hat{\sigma}_h^{(a)})^2}{\bar{\sigma}_h^2 + \sigma_e^2} < 1$ and the fact that $\hat{\sigma}_h^2 < \sigma_e^2$.

While the resulting EHB could be both negative and positive, depending on whether the information channel or the hedging motive dominates, we can show that learning always tilts portfolios towards the domestic assets, and thus generates greater home bias. Formally, the home bias of the Optimal Information portfolio is greater than that of the No Information portfolio, although whether or not it is actually positive will depend on actual parameters.

### 3 The Dynamic Model

In this section I present the full, dynamic version of the model. It will show that unlearnable risk is a natural implication of any recursive economic setup, and exist even if $\sigma_e^2 = 0$. Moreover, we will see that all results on unlearnable risk from the static model carry over.

In each country, there are overlapping generations of agents that live for two periods, making information and portfolio choice decisions in the first period, and consuming in the second. The generation that gets born at time $t$ looks exactly like the agents studied in the static model – they are born with some initial wealth $W_t$, make information choice decisions in the first part of time period $t$, then observe signals and asset prices and make their portfolio
choice. At time \( t + 1 \), a new generation of agents is born in each country, who make their own information choices and then participate in the financial markets. The time \( t \) generation sells their portfolio holdings at the prevailing equilibrium prices at time \( t + 1 \), consumes their resulting wealth and exits.\(^{15}\)

The terminal wealth of the time \( t \) generation is thus

\[
c_{h,t+1} = \delta w_{h,t+1} + x_{ht}(p_{h,t+1} + d_{h,t+1}) + x_{ft}(p_{f,t+1} + d_{f,t+1}) + b_{ht} R
\]

where \( p_{j,t+1} \) is the time \( t + 1 \) equilibrium price of the \( j \)-th asset. The key difference between the dynamic model and the static model is that the payoff of the risky asset is no more purely exogenous, as it depends on the future prevailing equilibrium price. These time \( t + 1 \) prices depend on the time \( t + 1 \) information set of agents, and as we’ll see, in any recursive information setup those future information sets include variables that are outside of the scope of the learning ability of the current generation. Thus, the existence of financial markets in all periods naturally introduces a measure of unlearnable uncertainty, which introduces decreasing returns to information.

To showcase the endogenous unlearnable uncertainty I assume that the exogenous payoffs in each country are driven by a single factor, the fundamental \( a_{j,t} \). They are no longer the sum of two components, in this model, the agents will be able to learn about all payoff relevant factors, and thus there is no structurally assumed unlearnable uncertainty.

\[
w_{j,t+1} = d_{j,t+1} = a_{j,t+1}
\]

The exogenous factors follow a straightforward AR(1) process:

\[
a_{j,t+1} = \mu_j (1 - \rho_j) + \rho_j a_{j,t} + \varepsilon_{j,t+1}
\]

where all innovations are iid Normal \( \varepsilon_{j,t+1} \sim N(0, \sigma_j^2) \), and \( j \in \{h, f\} \).

Period \( t \) is again split into two sub-periods. In the morning, the young generation observes all variables up to \( t - 1 \) and make their information choices. In the afternoon, the time \( t \) shocks are realized and revealed to agents, the young generation makes their portfolio choices and the old generation sells their whole portfolio and consumes the resulting wealth. The young agents choose portfolios subject to the initial budget constraint

\[
W_{h,t}^{(i)} = p_{ht}x_{ht} + p_{ft}x_{ft} + b_{ht},
\]

\(^{15}\)This model extends the OLG framework of Bacchetta and van Wincoop (2006) to endogenous information choice.
where $W_{h,t}^{i}$ is the initial wealth of agent $i$ from the young generation in the home country. Since at each point in time only the young generation makes decisions, I have simplified notation by not indexing the two separate generations. Without risk of confusion, agent $i$ always indexes agents in the currently young generation.

Prior to making their portfolio allocation choice, each young agent $i$ receives unbiased, informative signals about the future fundamentals:

$$
\eta_{ht}^{(i)} = a_{h,t+1} + \varepsilon_{ht}^{(i)}
$$

$$
\eta_{ft}^{(i)} = a_{f,t+1} + \varepsilon_{ft}^{(i)}
$$

where $\varepsilon_{jt}^{(i)}$ are iid mean-zero normal variables, whose variance is chosen by agent $i$, subject to the familiar entropy constraint:

$$
H(a_{h,t+1}, a_{f,t+1}|I_t^p) - H(a_{h,t+1}, a_{f,t+1}|I_t^{(i)}) \leq \kappa.
$$

The information set of agent $i$, part of the young generation at time $t$, contains the whole history of fundamentals up to time $t$, the current equilibrium prices $p_{ht}$ and $p_{ft}$, and also the two private signals about future fundamentals:

$$
I_t^{(i)} = \{a_{h}^t, a_{f}^t, p_{ht}, p_{ft}, \eta_{ht}^{(i)}, \eta_{ft}^{(i)}\}
$$

The public information set contains the history of fundamentals plus the asset prices:

$$
I_t^{(p)} = \{a_{h}^t, a_{f}^t, p_{ht}, p_{ft}\}
$$

Lastly, it is also useful to define the *aggregate* information set at time $t$, which aggregates over the information sets of all agents $i$ at time $t$. This is the information set of a hypothetical social planner that is able to observe the information sets of all individual agents:

$$
I_t^{Agg} = \{a_{h}^t, a_{f}^t, p_{ht}, p_{ft}, a_{h,t+1}, a_{f,t+1}, z_{ht}, z_{ft}\}
$$

Note that this aggregate information set includes the actual realizations of the future fundamentals $a_{h,t+1}$ and $a_{f,t+1}$. This is because the idiosyncratic signals of the agents are unbiased, with iid noise, and hence aggregating over them perfectly reveals the future fundamentals. Moreover, notice that if an agent was able to aggregate all of the dispersed information about the fundamentals $a_{j,t+1}$, then he would be able to also invert the equilibrium prices, and uncover the measure of noise traders $z_{ht}$ and $z_{ft}$ as well. Thus, the aggregate information set contains both the future fundamentals, and the current measure of noise
traders, both of which are unknown to any single investor.

Notice that this is also the information set of an agent with unlimited capacity for information that is able to pick \( \sigma^2_{a} = \sigma^2_{d} = 0 \). In other words, it contains all the uncertainty that is learnable at time \( t \), and thus presents the scope of learning of time \( t \) agents. Crucially, the aggregate information set at time \( t + 1 \) contains the time \( t + 2 \) fundamentals, \( a_{h,t+2} \) and \( a_{f,t+2} \), that are not part of the aggregate information set at time \( t \). In particular, we have that the time \( t \) aggregate information set is a strict subset of the aggregate information set at time \( t + 1 \),

\[
\mathcal{I}^\mathrm{Agg}_{t} \subset \mathcal{I}^\mathrm{Agg}_{t+1} \quad \text{and} \quad \mathcal{I}^\mathrm{Agg}_{t+1} \setminus \mathcal{I}^\mathrm{Agg}_{t} \neq \emptyset
\]

Since, \( \mathcal{I}^\mathrm{Agg}_{t} \) is coarser than \( \mathcal{I}^\mathrm{Agg}_{t+1} \), this means that the future aggregate information set \( \mathcal{I}^\mathrm{Agg}_{t+1} \) contains uncertainty that is unlearnable for agents at time \( t \). Thus, a dynamic recursive information framework introduces endogenous unlearnable uncertainty, because aggregate information sets are nested.

3.1 Portfolio Choice

To form their portfolios the agents need to forecast future returns, which in this case are the sum of the future equilibrium price and the future dividend:

\[
y_{j,t+1} = p_{j,t+1} + d_{j,t+1}
\]

I solve the model by conjecturing, and verifying, that the equilibrium price are linear, time-invariant functions of the state variables in the current aggregate information set that forecast the future payoff relevant variables \( a_{j,t+1} \) and \( z_{j,t+1} \):

\[
p_{jt} = \bar{\lambda}_{j} + \lambda_{a}a_{j,t} + \lambda_{aj}a_{j,t+1} + \lambda_{zh}z_{ht}
\]

where \( z_{jt} \) is the measure of noise trader demand for asset \( j \) at time \( t \). I assume that the noise trader demands are iid Normal \( z_{jt} \sim N(0, \sigma^2_{z}) \). Thus, prices and overall returns Gaussian, and with the conditional expected payoff

\[
\hat{\mu}_{jt}^{(i)} = E(p_{j,t+1} + d_{j,t+1} | \mathcal{I}_{t}^{(i)}) = \bar{\lambda} + (1 + \bar{\lambda}_{a} + \lambda_{aj}\rho_{a})E(a_{j,t+1} | \mathcal{I}_{t}^{(i)})
\]

and conditional variance

\[
\text{Var}(p_{j,t+1} + d_{j,t+1} | \mathcal{I}_{t}^{(i)}) = (1 + \bar{\lambda}_{a} + \lambda_{aj}\rho_{a}) \text{Var}(a_{j,t+1} | \mathcal{I}_{t}^{(i)}) + \lambda_{aj}\sigma^2_{a} + \lambda_{zj}\sigma^2_{z}
\]

24
The conditional expectation and variance of the future fundamental \(a_{j,t+1}\) follows the standard formulas. The agents have two sources of signals, the idiosyncratic signal \(\eta_{jt}\), and the equilibrium price itself contains a signal \(\tilde{p}_{jt} = a_{j,t+1} + \frac{\lambda_{aj}}{\lambda_{zj}}z_{jt}\), which they combine with their prior knowledge of \(a_{jt}\) to compute:

\[
E(a_{j,t+1}|I_{ht}) = \hat{\sigma}_{jt}^2 \left( \frac{\rho_a a_{j,t}}{\sigma_a^2} + \frac{\lambda^2_{gh}}{\lambda^2_{zh} \sigma_a^2} \left( a_{j,t+1} + \frac{\lambda_{aj}}{\lambda_{zj}}z_{jt} \right) + \frac{1}{\sigma_{\eta_h}^2} \eta_{ht} \right)
\]

\[
\text{Var}(a_{j,t+1}|I_{ht}) = \hat{\sigma}_{jt}^2 = \left( \frac{1}{\sigma_a^2} + \frac{\lambda^2_{gh}}{\lambda^2_{zh} \sigma_a^2} + \frac{1}{\sigma_{\eta_h}^2} \right)^{-1}
\]

Thus, we again have the familiar mean-variance optimal portfolios:

\[
x^{(i)}_{ht} = \frac{\hat{\mu}^{(i)}_h - p_{ht}R}{\gamma((1 + \lambda_{ah} + \lambda_{ah} \rho_a) \hat{\sigma}_{ht}^2 + \lambda_{ah} \sigma_{ah}^2 + \lambda_{zh} \sigma_{z}^2)} - \delta \frac{(1 + \lambda_{ah} + \lambda_{ah} \rho_a) \hat{\sigma}_{ht}^2 + \lambda_{ah} \sigma_{ah}^2 + \lambda_{zh} \sigma_{z}^2}{(1 + \lambda_{ah} + \lambda_{ah} \rho_a) \hat{\sigma}_{ht}^2 + \lambda_{ah} \sigma_{ah}^2 + \lambda_{zh} \sigma_{z}^2}
\]

\[
x^{(i)}_{ft} = \frac{\hat{\mu}^{(i)}_f - p_{ft}R}{\gamma((1 + \lambda_{af} + \lambda_{af} \rho_a) \hat{\sigma}_{ft}^2 + \lambda_{af} \sigma_{af}^2 + \lambda_{zf} \sigma_{z}^2)} - \delta \frac{(1 + \lambda_{af} + \lambda_{af} \rho_a) \hat{\sigma}_{ft}^2 + \lambda_{af} \sigma_{af}^2 + \lambda_{zf} \sigma_{z}^2}{(1 + \lambda_{af} + \lambda_{af} \rho_a) \hat{\sigma}_{ft}^2 + \lambda_{af} \sigma_{af}^2 + \lambda_{zf} \sigma_{z}^2}
\]

For future reference, notice that the denominator in the above expressions, which is the posterior variance of returns, is not proportional to \(\hat{\sigma}_{jt}^2\), the posterior variance of the \(t+1\) fundamental factor that the agents are learning about. Returns also depend on the two-period ahead fundamental innovations, \(\varepsilon_{a_{j,t+2}}\) because those innovations would be in the aggregate information set at time \(t+1\) and future prices would depend on them. Moreover, returns also depend on the future noise traders, since they also affect the future equilibrium price \(p_{j,t+1}\). Both of those extra components represent unlearnable risk for the time \(t\) generation of young agents – this is uncertainty that they cannot reduce, even with unlimited information capacity.

In equilibrium, the sum of the young informed agents trades plus the noise trader demands \(z_{jt}\) must equal the supply of the risky assets, which is equalized to one. In practice, the old generation is selling the risky assets to the new generation plus the noise traders, but in equilibrium the old generation always carries the whole supply of assets \(\bar{z}_j\). Thus, market clearing requires:

\[
\bar{z}_h + z_{ht} = \frac{1}{2} \int_0^1 x^{(i)}_{ht} di + \frac{1}{2} \int_0^1 x^{*(i)}_{ht} di
\]

\[
\bar{z}_f + z_{ft} = \frac{1}{2} \int_0^1 x^{(i)}_{ft} di + \frac{1}{2} \int_0^1 x^{*(i)}_{ft} di
\]
Since the prices and the optimal portfolios are linear functions of the underlying state variables, at this point we can substitute in the guesses for the price function and match coefficients to find the equilibrium prices. This amounts to finding a fixed point of a system of equations detailed in the Appendix.

### 3.2 Information Choice

In the previous section we assumed a stationary equilibrium where asset prices are time-invariant functions of the state variables. As we saw in the static model, however, asset prices are in general a function of the posterior variance of a hypothetical, average market participant $\hat{\sigma}_{jt}^{(a)}$, which depends on the aggregate information choices of the agents. If that choice is time-varying, then we clearly cannot have a time-invariant equilibrium in asset prices either. The first result of this section is to establish that, indeed, the an invariant aggregate information choice is indeed an equilibrium.

**Proposition 5.** The optimal allocation of information is time-invariant, i.e. $k_{ht} = k_h$, $k_{ft} = k_f$, $k_{ht}^* = k_h^*$, $k_{ft}^* = k_f^*$ for all $t$.

**Proof.** Intuition is sketched out in the text, and details are in the Appendix.

This result tells us that at any time period $t$, the currently young generation allocates its finite information capacity in the same way that next period’s generation would do. Thus, the posterior variance of the average market participant is time-invariant. To gain intuition about the result, it’s useful to derive the objective function for the information choice of a home agent that was born at time $t$:

\[
U_t = \gamma W_t + \gamma \delta \rho_a a_{h,t} + \frac{1}{2} \sum_{j \in \{h, f\}} \frac{\text{Var}_t(\hat{\mu}_{jt}^{(i)} - p_{jt} R) + (E_t(\hat{\mu}_{jt}^{(i)} - p_{jt} R))^2}{(1 + \lambda_{ah} + \lambda_{ah} \rho_a) \exp(-k_h)\sigma_h^2 + \lambda_{ah} \sigma_{a_h}^2 + \lambda_{ah} \sigma_{a_h}^2} - \frac{\gamma \delta E_t(\hat{\mu}_{ht}^{(i)} - p_{ht} R)(1 + \lambda_{ah} + \lambda_{ah} \rho_a) \exp(-k_h)\sigma_h^2 + \frac{\gamma^2 \delta^2}{2} \sigma_{ewh}^2 \sigma_{eh}^2}{(1 + \lambda_{ah} + \lambda_{ah} \rho_a) \exp(-k_h)\sigma_h^2 + \lambda_{ah} \sigma_{a_h}^2 + \lambda_{ah} \sigma_{a_h}^2 + \lambda_{ah} \sigma_{a_h}^2}
\]

The structure of the objective function is the same as before. The agent values information about volatile assets that have high excess returns, and also information that helps reduce the uncertainty about his labor income. The key to the fact that time-invariant information choice is an equilibrium, is that given a constant information choice, the ex-ante expected excess return is constant through time as well. Plugging-in the expression for $\hat{\mu}_{jt}^{(i)}$ and the equilibrium price $p_{jt}$, we get...
\[ E(\hat{\mu}_j - p_{jt}R) = \gamma (\hat{\sigma}_{jt}^{(a)})^2 \left[ \bar{z}_j + \frac{\delta}{2} \hat{\sigma}_{jt}^2 \sigma^2 \right] \]

where \((\hat{\sigma}_{jt}^{(a)})^2\) is the perceived variance of asset \(j\)'s payoffs for the average market participant, defined as

\[
(\hat{\sigma}_{jt}^{(a)})^2 = \left( \frac{1}{2((1 + \lambda_{ah} + \lambda_{ah}\rho_a)\hat{\sigma}_{ht}^2 + \lambda_{ah}\sigma_{ah}^2 + \lambda_{zh}\sigma_{zh}^2)} + \frac{1}{2((1 + \lambda_{ah} + \lambda_{ah}\rho_a)\hat{\sigma}_{ht}^2 + \lambda_{ah}\sigma_{ah}^2 + \lambda_{zh}\sigma_{zh}^2)} \right)^{-1}
\]

Thus, the excess return again depends on the risk-adjusted supply, and a similar evaluation shows that the ex-ante volatility of the excess returns is also a function of the posterior variance faced by the average market participant. Now, assume that the aggregate information choice is time-invariant and thus \((\hat{\sigma}_{jt}^{(a)})^2 = (\hat{\sigma}_{jt}^{(a)})^2\), with all home agents choosing \(k_{ht} = k_h\) and the foreign agents \(k_{ht}^* = k_h^*\). Then, the optimal information choice of a single agent \(i\) is also time-invariant, \(k_{ht}^{(i)} = k_h\), and to find the equilibrium we look for an aggregate capacity choice \(k_h\), such that no individual agent has an incentive to deviate – \(k_{ht}^{(i)} = k_h\) for all \(i\).

For transparency, I again focus on a symmetric equilibrium, where it will again be the case that \((\hat{\sigma}_{ht}^{(a)})^2 = (\hat{\sigma}_{jt}^{(a)})^2\), and the marginal benefits of home and foreign information can be expressed as:

\[
\frac{\partial U_t^-}{\partial k_h} = (A + \delta B) \frac{\exp(-k_h)(1 + \lambda_{ah} + \lambda_{ah}\rho_a)\hat{\sigma}_{ht}^2}{(\exp(-k_h)(1 + \lambda_{ah} + \lambda_{ah}\rho_a)\hat{\sigma}_{ht}^2 + \lambda_{ah}\sigma_{ah}^2 + \lambda_{zh}\sigma_{zh}^2)^2} > 0
\]

\[
\frac{\partial U_t^-}{\partial k_f} = A \frac{\exp(-k_f)(1 + \lambda_{af} + \lambda_{af}\rho_a)\hat{\sigma}_{ft}^2}{(\exp(-k_f)(1 + \lambda_{af} + \lambda_{af}\rho_a)\hat{\sigma}_{ft}^2 + \lambda_{af}\sigma_{af}^2 + \lambda_{zf}\sigma_{zf}^2)^2} > 0
\]

where \(\delta B \geq 0, A \geq 0\), which again shows that ceteris paribus, home information is more useful than foreign information. Thus, perhaps unsurprisingly, we have a new version of Proposition 1, which shows that the optimal information acquisition strategy is again biased towards home information.

**Proposition 6.** If \(\delta > 0\) the home agent optimally chooses \(k_h > k_f\), i.e. she chooses to acquire more information about the home fundamentals. Similarly, the foreign agent chooses to acquire more information about his own domestic fundamentals and picks \(k_f^* > k_h^*\).

**Proof.** Sketched in text, more details in the Appendix.  

\(\square\)
The only difference is that this time we do not need to explicitly assume that labor income is not perfectly diversifiable, as this now emerges endogenously in the equilibrium of the model. Asset returns also depend on the future asset price, which depends on the future aggregate information set and future noise traders, which ensure that it won’t be perfectly correlated with labor income. Interestingly, this is the case even though labor income and the dividend process are in fact perfectly correlated.

Next, I turn to the question of convexity. Given that the objective function is of the same form as before, it is straightforward to show that again, the problem exhibits increasing returns for low values of capacity $\kappa$, but decreasing returns for high values. In particular, the relevant cutoff is again the point at which the reducible uncertainty, in this case \((1 + \lambda_{ah} + \lambda_{ah}\rho_a)\hat{\sigma}_{ht}^2\), becomes less than the unlearnable uncertainty, in this case \(\lambda_{ah}\sigma_{ah}^2 + \lambda_{zh}\sigma_z^2\).

**Proposition 7.** The objective function of the information choice problem, is convex in $k_j$ when the learnable portion of the uncertainty about asset $j$ is greater than the unlearnable uncertainty, i.e. \((1 + \lambda_{aj} + \lambda_{aj}\rho_a)\hat{\sigma}_j^2 \geq \lambda_{aj}\sigma_{aj}^2 + \lambda_{az}\sigma_z^2\) and concave otherwise.

*Proof.* Sketched in text, more details in the Appendix.

The intuition about the result is the same as before. There is a feedback loop between information choice and portfolio choice, where the more the agent learns about asset $j$, the more he expects to hold of that asset, and the more he values information about that asset. This loops generates increasing returns to information when the agent has acquired relatively little overall information. However, since the agent is facing both learnable and unlearnable uncertainty, the feedback loop weakens as the agent acquires more information, Eventually, the increasing returns are exhausted, and the agent faces decreasing returns to information.

**Proposition 8.** Let $\delta$, $\gamma$ and $\sigma_{ah}^2 = \sigma_{af}^2$ be given (symmetric assets), Then,

1. The optimal information acquisition choice is as characterized as follows

   (i) There exists a value $\bar{\kappa} > 0$ such that $k_h = k_f^* = \kappa$ and $k_f = k_h^* = 0$ when $\kappa \leq \bar{\kappa}$, but the agents acquire information about both home and foreign factors, $k_h > 0$, $k_f > 0$, when $\kappa > \bar{\kappa}$.

   (ii) The home bias in information acquisition, i.e. $k_h - k_f^* > 0$ and $k_f^* - k_h^* > 0$, is increasing in $\kappa$ when $\kappa \leq \bar{\kappa}$, and decreasing otherwise.

   (iii) The home bias in information acquisition is always positive, and persists in the limit, with $\lim_{\kappa \to \infty} k_h - k_f > 0$.

2. Where $\bar{\kappa}$ is an increasing function of $\gamma$, $\delta$ and $\alpha$, where $\alpha = \frac{\sigma_z^2}{\sigma_e^2}$. Moreover, $\Lambda$ - the overall amount of information asymmetry - is an increasing function of $\gamma$ and $\delta$.
Proof. The proof is in the Appendix.

So we see that the dynamic model always displays full information specialization for low values of $\kappa$, and information diversification for higher values. Thus, the model can potentially explain both the observed home bias in portfolios and the downward trend in the home bias over the last two decades. The next section analysis the effects on portfolio choice in more details.

3.3 Quantitative Evaluation

4 (Preliminary) Empirical Evidence

As we have seen, a model with some measure of unlearnable uncertainty has two major departures from a model with no unlearnable uncertainty. First, for high values of information capacity $\kappa > \bar{\kappa}$, a model with unlearnable uncertainty would imply that portfolio concentration will fall with information capacity, while the other framework implies that portfolio concentration is always rising in capacity. Second, with unlearnable uncertainty the information choice could affected by other endogenous reasons for valuing information, such as non-diversifiable labor income. In contrast, with no unlearnable uncertainty, information choice is always at the corner of focusing entirely on acquiring domestic information. In the particular case of labor income, a model with no unlearnable uncertainty would unambiguously imply that more labor income should lead to lower home bias, due to the hedging motive in portfolio choice. On the other hand, in the model with unlearnable uncertainty labor income also introduces an information based channel that generates home bias, thus the relationship is ambiguous.

In this section I exploit these differentiating features to empirically test the two frameworks. I do so by focusing on the empirical phenomenon of the home bias in national portfolios. To this end, I have collected aggregate data on 35 OECD countries for the time period between 2001 and 2008. I have data on portfolio positions, proxies for $\kappa$, labor income and financial wealth per capita, the Chinn-Ito index of financial openness and real PPP GDP per capita. All monetary series are in PPP terms and the data come from the IMF, the OECD and the World Bank and full details are given in the Appendix.\footnote{I am in the process of extending the data set with more recent observations. The initial data set was restricted by the past availability of data on IMF’s Coordinated Portfolio Investment Survey (CPIS), the highest quality international positions data.}

I start by examining the cross-section, and first time-average variables to come up with 35 cross-sectional observations. Then, I regress the degree of home bias (EHB) on proxies for
κ, the relative size of non-financial income, and other controls.

\[ E\beta B_i = \text{const} + \beta_k \kappa_i + \beta_\delta \delta_i + \beta' X_i + \epsilon_i \]

As a benchmark proxy for information capacity, κ, I use the number of Internet users per 100 people, which I obtain from the World Bank. The availability of the Internet is a natural proxy for the aggregate information processing capacity for two reasons. First, Internet technology greatly reduces the time associated with collecting information, and second, an Internet connection also presupposes the availability of a computer, which further enhances a person’s ability to sift through and analyze information. Moreover, for the time period at hand - 2001 through 2008 - the Internet was readily available to all 35 countries in the sample, but at the same time the countries exhibit significant variation in the extent to which the Internet had penetrated their populations.

I base my proxy for non-financial income on total labor compensation. What matters in the model, is not the total size of labor income but rather its size relative to capital income and I consider several different proxies that try to capture this relationship. First, I use the standard calculation of labor share of aggregate income:

\[ \delta_i = \frac{\text{Total Labor Compensation}_i}{\text{GDP}_i} \]

This first proxy is most relevant in a closed economy setup, where the total income of the representative agent is comprised by GDP. In an open economy setup, however, it is more appropriate to also factor in any income earned on foreign investments. To control for this, I compute a second version of Labor Share as follows:

\[ \text{Labor Share}_2 = \frac{\text{Total Labor Compensation}_i}{\text{Total Labor Compensation}_i + \text{Total Financial Assets}_i \times r_{US}^{3m}} \]

Here I express the labor share as the ratio of labor income and total income, where total income also includes the capital income flow from the Financial Assets (both domestic and foreign) of the representative agent. Unfortunately, I do not have data on the total capital income but only on the portfolio position, hence I proxy for the flow capital income by multiplying the portfolio position by the yield on the 3-months US Treasury. I have analyzed a number of other possible portfolio returns instead of the 3-months US Treasury and found that the actual return used has negligible effects on the results. The implicit assumption here is that the relevant return does not vary too much across countries.

Lastly, I also include Total Labor Income and Total Financial Assets as separate
regressors and put no restrictions on their coefficients. Since I do not have a direct observation of $\delta$, this third strategy aims to control for the relative size labor income without making explicit functional assumptions about the relationship between $\delta$ and the observable income proxies. For this specification, the regression is given by:

\[
\text{EHB}_i = \text{const} + \beta_k \kappa_i + \beta_l \ln(\text{LabIncome}_i) + \beta_f \ln(\text{FinWealth}_i) + \beta'X_i + \epsilon_i
\]

The vector of controls has two elements. First, I include the Chinn-Ito Index, which measures the degree of existing capital controls in a given country (Chinn and Ito (2006)), in an attempt to control for home bias which arises purely from institutional barriers to foreign investment. Second, I also include the initial period (year 2001) level of real PPP GDP per capita which controls for initial conditions, and especially ensures that the coefficient on Internet Users per 100 people does not simply pick up the fact that richer countries tend to be more diversified. \(^{17}\)

The coefficients of primary interest are $\beta_k$ and $\beta_\delta$. The first empirical restriction I test can be summarized by $H_0 : \beta_k > 0$ vs $H_a : \beta_k \leq 0$, and the second is given by $H_0 : \beta_\delta < 0$ vs $H_a : \beta_\delta \geq 0$. A rejection of the first null hypothesis would mean that the degree of home bias is decreasing in information processing capacity and a rejection of the second hypothesis would imply that home bias is increasing in the relative size of labor (non-financial) income. \(^{18}\)

Table 1 presents OLS estimates with the corresponding heteroskedasticity robust standard errors in parentheses. The first column shows results when I do not include any proxy for $\delta$ and only include the Chinn-Ito Index as an additional control. Columns (2)-(4) add, one at a time, the three different proxies for $\delta$. Lastly, columns (5)-(8) repeat the regressions in (1)-(4) by adding the initial period real GDP per capita as well.

The main result is that all estimate of $\beta_k$ are found to be negative and statistically significant at standard levels in 7 out of the 8 specifications. The only regression which shows up with an insignificant $\beta_k$ coefficient does not include a proxy for $\delta$ and hence is misspecified. The data is conclusive that information processing capacity is negatively related to the degree of home bias and this leads to a rejection of the first null hypothesis being tested. \(^{19}\)

\(^{17}\)The Appendix includes regressions which consider additional control variables, such as non-tradable consumption (Stockman and Tesar (1995),Pesenti and Van Wincoop (2002),Lewis (1999)), but I do not include them in the main body of the paper in the sake of parsimony of the empirical model and because the results remain unchanged.

\(^{18}\)The Appendix includes results from a regression where I include the square of $\kappa$ to capture potential non-linearities (which is, strictly speaking, the model’s prediction). This more flexible setup does find the non-monotonic relationship predicted by Proposition 3 - EHB is increasing in $\kappa$ for countries with low $\kappa$ and decreasing otherwise (only 5 countries are found to be in the increasing region).

\(^{19}\)Mondria and Wu (2010) use a slightly different proxy for $\kappa$, which is a measure of IT technology per
an information asymmetry model of the home bias that is based on increasing returns to learning has counter-factual implications. If increasing returns to learning generate home bias, then it must be the case that higher learning capacity leads to a higher degree of home bias but the data points to the opposite.

The empirical results are not quite as inconclusive in regards to the relationship between home bias and the relative size of labor income, but that was to be expected. This null hypothesis has lower power, since labor income has two opposite forces on home bias. In any case, I find a positive coefficient on labor income for all three proxies, and that coefficient is statistically significant in two out of the three regressions that include all controls. This suggests that indeed labor income imparts a strong positive force on the home bias in portfolios, that is in opposite to the standard hedging motive which puts negative pressure on the home bias. Thus, the data is again in support of a framework with unlearnable uncertainty, where the broader context of an agent’s economic environment, such as his labor income, have an important role on his information choice, and through that on his portfolio.

Interestingly, this is a phenomenon present in micro-level data as well. Using data on each individual investor’s labor income, financial income and portfolio composition, Massa and Simonov (2006) find that investors actively tilt their portfolios towards assets that are $1000 of economic activity rather than IT technology per capita, and find the opposite result that the Home Bias is positively related with their measure of $\kappa$. In the Appendix I reestimate my regressions using the Mondria and Wu (2010) measure of $\kappa$ and again find a statistically significant negative relationship. There I also discuss three possible reasons for the difference in the results: the different data sets, the extra controls I include, and differences in methodology.
positively correlated with their labor income process. They find that while the average correlation between the stock market as a whole and an individual’s labor income is roughly zero, the average correlation between an individual’s labor income and his portfolio proceeds is positive and statistically significant. Moreover, they also find evidence that investors act deliberately in skewing their portfolios towards assets that are positively correlated with their labor incomes. They show on average the stocks that investors buy increases the correlation of portfolio returns and non-financial income, while the stocks they sell decrease this correlation. And furthermore, they find evidence that the resulting concentrated portfolios are indeed driven by informational reasons.

Lastly, note that this paper considers only technological measures of information processing capacity. While in the 21st century information technology is certainly a very important, if not the chief, determinant of information processing ability, human capital is also likely to play an important role. Examining the empirical relationship between portfolio concentration and investors’ human capital is beyond the scope of this paper, but there already exists an empirical literature on this issue. The interested reader is directed to Goetzmann and Kumar (2008) who show that portfolio concentration is decreasing in investors’ educational level, investing experience and sophistication, and also to, Kimball and Shumway (2010) who document that the international home bias is decreasing in a number of different measures of investor sophistication.
References


A  Appendix A: Proofs

[In progress]