Trade Integration and the Fragility of Trade Relationships: Theory and Empirics

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April 15, 2016

Abstract

We provide a theoretical framework to analyze the effects of economic integration agreements on the stability of product-level trade relationships and verify the predictions empirically. Specifically, we examine how economic integration agreements affect the value of trade at the start of a new trade relationship, the length of trade relationships, and how quickly trade grows within a relationship. Using annual trade data at the 5-digit SITC level for over 180 countries from 1962 to 2005, we find evidence of an interesting dichotomy which highlights the relevance of trade costs. While economic integration increases the length and growth of trade relationships that started prior to the agreement, it reduces the length and growth of those started after the agreement as well as their initial transaction value. Integration leads to additional entry.

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1 Introduction

Economic integration agreements (EIA) are more common now than they have ever been with more than a half of the entire value of international trade taking place between countries which are economically integrated to some extent. As illustrated by Figure 1[1] the volume of world trade taking place between countries with an agreement increased from roughly 20% of world trade in 1962 to some 55% by 2005. Over the same period of time the fraction of all country pairs with an agreement has increased from just over 1% to over 20%, while the number of trade relationships has increased from less than 15% to more than 40%. While much effort has been dedicated to investigating the effects of economic integration agreements on aggregate trade, their effects on disaggregated trade outcomes remain unexplored. The goal of our paper is to examine the effect of trade agreements on a disaggregated level by providing a theoretically guided empirical investigation.

In a seminal contribution Baier and Bergstrand (2007) argued that aggregate trade between two countries that sign a trade agreement doubles after ten years. Little is understood about how such an increase in aggregate trade occurs at a disaggregated level. One possibility is that all existing trade relationships between the two countries double in the ten years after signing an agreement. This simplistic possibility is almost surely wrong as it would entail that trade agreements only have an effect on the intensive margin. Since trade agreements reduce the costs of trading, they are bound to have an effect on the extensive margin ushering the creation of new trade relationships. Often this is one of the motivating factors behind signing a trade agreement, to open doors of new trading opportunities. Baier, Bergstrand, and Feng (2014) indeed show that trade agreements do have an effect on both the intensive and

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[1] This figure is similar to Figure 1 in Bergstrand, Egger, and Larch (2012) who investigate the determinants of the timing of preferential trade agreements using a duration framework and also plot the number of country pairs with an agreement. The two plots differ somewhat due to their inclusion of only PTAs, FTAs, and currency unions, and the fact that their plot is based only on agreements used in estimation. Our plot is based on all available data on agreements.
extensive margins, with the former effect dominating in the short run.

The fact that trade agreements have an effect on the extensive margin raises additional questions about their effects. In our analysis we differentiate between trade relationships that started before and after the agreement itself. We seek to understand whether post-agreement trade relationships are larger than pre-agreement ones at their starting point, whether they grow faster, and whether they are longer lived. One might expect pre- and post-agreement relationships to be different as it is likely that a good number, if not the majority, of post-agreement relationships will tend to be relationships which were not feasible prior to the agreement. Our effort aims to provide a more granular understanding of the intensive and extensive margin effects identified by Baier, Bergstrand, and Feng (2014).

We develop a dynamic model of international trade which allows us to track the
entire evolution of a trade relationship: the initial size, the duration, and the rate of
growth. We do so by combining Melitz (2003) with Klepper and Thompson (2006),
the former guiding the firm decision and the latter describing the evolution of the
entire trade relationship. As is common in most models of trade with heterogeneous
firms the decision of a firm to enter a market depends on its productivity and the
characteristics of the destination market: size, trade barriers, and competitive envi-
ronment. In our model, however, the decision to enter is only half the story. Upon
entry we require a firm to identify a possible buyer in the country of destination. If
successful, both parties establish a business relation. Buyers in the destination mar-
ket appear and disappear following a process that is independent of the selling firm.
This independence of the two processes makes it possible for selling firms to re-enter
a market and begin exporting again after some period of not having exported to a
particular destination. The possibility of re-entry of a once abandoned market is a
new result in this literature. Using this set-up we are able to track the evolution of
trade relationships, which are the aggregation of business relations across the same
country of origin and country of destination in a specific product category. Thus, a
trade relationship exists due to the activity of at least one exporting firm.

Our parsimonious model delivers a rich set of predictions about the dynamic
evolution of disaggregated trade. The first set of predictions pertains to the survival
and growth of trade relationships. Trade relationships can and do cease to exist, with
both the probability of ceasing and their growth rate decreasing the longer they are
active and the larger they are. The second set of predictions revolve around the effect
of economic integration agreements. Trade relationships which are active when an
agreement begins benefit from the agreement by becoming less likely to cease (longer
in duration) and by growing faster. Moreover, trade relationships which begin after
the agreement are more fragile: they are more likely to cease and grow less. Finally,
economic integration agreements result in an increase in entry and creation of new
trade relationships as more firms are able to export once an agreement reduces costs of trading.

In our empirical application we use data on disaggregated trade flows along with data on economic integration agreements. Our trade data are annual 5-digit SITC revision 1 imports between 1962 and 2005 for all countries available in the UN Comtrade database. Data on agreements come from the Database on Economic Integration Agreements constructed by Scott Baier and Jeffrey Bergstrand (2007). The database provides annual information on the existence of (various types of) economic integration agreements annually between pairs of countries. The object of our investigation is a trade relationship defined as a pair of countries exchanging a product, for example, Argentina exporting beef to the United States. We examine the effect of trade liberalization on the initial value of trade of new trade relationships, their duration, and their growth while they are active. Using these data we are able to empirically confirm the two sets of our theoretical predictions: longer lived and larger relationships grow less and are less likely to cease, while economic integration makes already active trade relationships more stable but those starting after the agreement less stable. We also show that agreements result in increased entry and more trade relationships between the countries that sign an agreement.

We make a contribution to an increasing literature that relates to export dynamics. Most of this literature concentrates on the expansion of the geographical coverage of trade as a firm continues to access more distant markets. Chaney (2014) provides theory and evidence on the expansion of trade networks and the dynamic evolution of trade frictions. Albornoz et al. (2012) and Defever, Heid, and Larch (2010) using a simpler model of market access, provide evidence that current export relationships influence the decision of where to export next. Complementary to those models, we provide the first theoretical model able to capture the dynamic evolution of existing trade relationships.
Our model is complementary to a recent set of papers that focus on the destination market, more so than on the firms in the country of origin. Bernard, Moxnes, and Ulltveit-Moe (2014) show that heterogeneity in the characteristics of buyers in the destination market matters for explaining trade relationships. Using highly disaggregated Norwegian data they find that the extensive margin of the number of buyers plays an important role in explaining the variation in exports at the aggregate level and at the firm level. Carballo, Ottaviano, and Volpe Martinacus (2013) use highly disaggregated data from Costa Rica, Ecuador, and Uruguay to show that while most firms serve only very few buyers abroad, the number of buyers and the skewness of sales across them increases with size and accessibility of destinations. Because we assume the process that generates buyers varies across destinations, our model can explain some of the results in these papers.

A large and still growing literature aims to examine the effects of economic integration agreements on trade. The majority of papers in this literature focus on aggregate effects of integration agreements.\(^2\) The most disaggregated approach using the gravity framework is offered by Anderson and Yotov (2011) who examine the effects of free trade agreements using 2-digit manufacturing goods data. Our contribution to the literature is to provide a comprehensive investigation of the effects of integration agreements on as detailed a level of analysis as possible.

We make a contribution to the duration of trade literature by providing the first theoretical model which makes predictions about the hazard of a relationship ceasing, thus explaining now standard results in the literature.\(^3\) Most similar to our work in terms of hazard effects is Kamuganga (2012) who shows that regional trade

\(^2\)See for example Baier and Bergstrand (2007) who estimate that free trade agreements, on average, double trade between member countries. While some studies, such as Carrère (2006) and Kohl (2012), allow for differences across individual arrangements, others estimate an average effect, based on a single dummy variable for all arrangements.

cooperation within Africa reduces the hazard of exports ceasing across all types of agreements. Our effort is broader in scope, analyzing data for all available countries and agreements, as well as a much longer time frame.

We also make a contribution to the literature examining the growth of trade at disaggregated levels. Araujo, Mion, and Ornelas (2011) use Belgian firm-level data to show that countries with weaker institutions experience faster growth of exports from a given exporting firm. Muûls (2014) also uses Belgian firm level data to examine the role of credit constraints on firm’s exports, including their growth. Besedeš, Kim, and Lugovskyy (2014) find that more credit constrained exporters have faster growing relationships, conditional on survival.

Unlike the issue of the hazard of trade ceasing and trade growth, the final element of our investigation, the value of trade at the start of a relationship, has rarely been studied before. Besedeš and Prusa (2006b) show that trade in differentiated goods typically starts with lower volumes relative to homogeneous goods. Besedeš (2008) showed that larger initial volumes are associated with longer lasting relationships and lower hazard rates, a result our theoretical model can now explain.

2 Conceptual Framework

The model presented in this section provides us with a framework to think about the dynamic behavior of trade relationships and guides us in the interpretation of empirical results discussed below. We start with a few definitions. There are two countries, origin $o$ and destination $d$. A business relation consists of a firm in country $o$ selling its product to a firm in country $d$. We refer to firms in the country of origin as sellers and to firms in the country of destination as buyers. A trade relationship is the collection of all business relations trading in the same product category between

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4 Most papers investigating the hazard of trade ceasing use the initial volume as an explanatory variable for the hazard reflecting a relationship’s initial conditions, but few papers focus on understanding the determinants of the initial volume.
the origin and the destination countries. Finally, a *trade spell* is a realization of a trade relationship or the period of time, in consecutive years, during which the trade relationship is active. Among other things, we are interested in characterizing trade spells.

At the beginning, a seller identifies potential buyers and bids for a business opportunity to sell its product. We follow Klepper and Thompson (2006) and assume potential buyers of a particular product in the destination country appear following a Poisson process with parameter $\lambda$. Once a seller successfully contracts with a buyer, the business relation is active only for an exogenously determined length of time, $z$, drawn from the exponential distribution $H(z) = 1 - e^{-z/\mu}$ with mean $\mu$. After period $z$, the buyer disappears.\footnote{The buyer disappears for at least two reasons. First, it could be that it went out of business following a random idiosyncratic shock. Second, it is possible the seller was replaced by a new firm selling the product to the buyer. This process of creative destruction is not modelled explicitly in this paper, but it can be rationalized along the lines of Klette and Kortum (2004). It is also possible to reconcile the process of arrival of new buyers with a model of advertising similar to the one developed in Arkolakis (2008).}

The probability that a seller will enter the destination market is $\theta$ and the size of the business relation is randomly drawn from a distribution $F(r)$ where $r$ is the revenue of the seller. While most of the results below are independent of the exact form of $\theta$ and $F(r)$, we borrow the characterization of these two quantities from Melitz (2003).

In Melitz (2003) firms are characterized only by their productivity levels, indexed by $\phi$. Firms in the origin country selling in the destination country incur per-unit trade costs $\tau > 1$ and must pay fixed exporting costs $f_x$ to set up operations in the destination country.\footnote{All firms that export will also produce in their domestic market. Because we are only looking at business relations across different countries, we focus on the fraction of profits and the probability of entry derived from exporting and thus, we only mention the fixed costs of exporting, $f_x$. Domestic firms that do not export will also have to pay a fixed cost, $f_e$ to set up operations, but we are not concerned with that set of domestic-only firms.} The probability of a firm entering the destination country depends on the productivity of the firm, the per-unit trade costs, and the set-up

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costs. Thus, we characterize the probability of entering the destination country as 
\[ \theta = \theta(\phi, \tau, f_x) \]. Only the most productive firms will enter the domestic market and 
among those, only the most productive firms will export, \[ \partial \theta / \partial \phi > 0 \].

Similarly, the size of each firm, characterized here by its revenue, is a function 
of the same three parameters presented above. That is \[ r = r(\phi, \tau, f_x) \]. It follows 
from the results in Meltiz (2003) that more productive firms are larger, \[ \partial r / \partial \phi > 0 \].
The distribution of firms’ sizes in the destination country is denoted by \( F(r) \) with 
expected value \( E[r] \) and variance \( var[r] \).

We could model trade liberalization events in two ways, either by reducing per-unit 
costs, \( \tau' < \tau \), or by reducing set-up costs, \( f_x' < f_x \). As \( \tau \) or \( f_x \) decrease the productivity 
cut-off value also decreases making it possible for marginally less productive firms to 
enter the destination market. That is:

\[
(1) \quad \frac{\partial \theta}{\partial \tau} < 0 \quad \text{and} \quad \frac{\partial \theta}{\partial f_x} < 0.
\]

Similarly, increasing \( \tau \) or \( f_x \) erodes the revenue and profit margins of the firms, 
resulting in smaller firms in equilibrium. Because only the best firms export, an 
increase in trade costs narrows the distribution of firms in the destination country. 
That is:

\[
(2) \quad \frac{\partial E[r]}{\partial \tau} < 0 \quad \text{and} \quad \frac{\partial E[r]}{\partial f_x} < 0,
\]

\[
(3) \quad \frac{\partial var[r]}{\partial \tau} < 0 \quad \text{and} \quad \frac{\partial var[r]}{\partial f_x} < 0.
\]
2.1 Characterizing trade spells

Define $v_k(t)$ as the probability that a trade spell has exactly $k$ business relations at time $t$. This probability is distributed according to:

$$v_k(t) = e^{-\theta \rho(t)} (\theta \rho(t))^k / k!$$

which is a Poisson distribution with parameter $\theta \rho(t) = \theta \lambda \mu \left(1 - e^{-t/\mu}\right)$. This is the probability that $k$ sellers draw costs lower than the cutoff cost $c < \hat{c}$ and that they had successfully bid for a business opportunity in the destination country. Notice that as time approaches infinity, $\rho(t)$ approaches $\lambda \mu$ and the stationary distribution is $v_k = e^{-\theta \lambda \mu} (\theta \lambda \mu)^k / k!$. In the long run, the probability that a trade spell has exactly $k$ business relations is a function of the probability of entry and parameters associated with the process that generates buyers in the destination market. Notice that any trade policy that affects the terms of trade will also affect $\theta$, and as a result the trade policy will affect the long term stationary distribution of trade relationships.

2.1.1 Size, Duration, and Survival

A trade spell starts when a business relation was not present in period $t$ and at least one exists in period $t + \Delta t$. Symmetrically, a trade spell ceases to exist when at least one business relation existed in period $t$ and no such relation exists in $t + \Delta t$. The duration of a trade spell, $s(t)$, is then defined as the length of time that has elapsed since it was last inactive. In our model, trade spells can appear, disappear, and reappear at various occasions. That is, there is re-entry resulting in multiple spells of the same trade relationship. The possibility of re-entry is a novel feature of our model and usually not found in previous attempts at modeling dynamic behavior of firms, such as Nguyen (2012).

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7The proofs and several other derivations are in Klepper and Thompson (2006). We also replicate them in the appendix for completeness.
The number of business relations in a trade spell is a function of the duration of the spell. Define \( w_k(s(t), t) \) as the probability that a spell with duration \( s \) at time \( t \) has exactly \( k \) active business relations. Then \( w_k(s(t), t) \) is distributed Poisson according to:

\[
(5) \quad w_k(s(t), t) = e^{-\theta \rho(s)} (\theta \rho(s))^k / k!
\]

with the mean given by \( \theta \rho(s) = \theta \lambda \mu \left( 1 - e^{-s/\mu} \right) \), which is increasing in duration of trade, \( s \). Economic integration, by increasing \( \theta \), should increase the number of business relations in any given trade relationship.

Denote by \( n(t) \) the number of business relations in a trade spell at time \( t \). The size of the trade spell is \( y = \sum_{0}^{n(t)} r \), where \( n(t) \) is a random number and each term in the sum is a random draw from \( F(r) \). It can be shown that the distribution of sizes of all active trade spells has mean

\[
(6) \quad E[y] = E[r] \theta \rho(s)
\]

and variance

\[
(7) \quad \text{var}[y] = E[r^2] \theta \rho(s).
\]

The quantities \( \theta, E[r] \) and \( \text{var}[r] \) increase when \( \tau \) decreases, thus:

**Result 1** *Holding everything else equal, trade spells in more open trade relationships (with a lower \( \tau \)) are necessarily larger and have a higher variance when compared to trade relationships with larger trade barriers (large \( \tau \)).*

Because we have assumed the distribution \( H(z) \) is exponential, the arrival of new buyers is independent of the duration of previous relations and \( n(t) \) is enough to explain the probability of exit. In other words, the more business relations in a trade
spell, the lower the chance of the spell ending in any finite time period.

**Result 2** For any $t, T \in (0, \infty)$, the probability of a trade spell ending by time $(t + T)$ is strictly decreasing in $n(t)$.

Moreover, both the duration and size of a spell are related to $n(t)$, but in different ways because size is drawn from a distribution that is independent of $n(t)$ and the process that generates buyers. Therefore, the probability of exit will decline with the size of the trade spell, holding duration constant, and will decline with duration, holding firm size constant.

**Result 3** For any $t, T \in (0, \infty)$, the probability of a trade spell stopping by time $(t + T)$ is decreasing in its size, $y(t)$, and age, $s(t)$.

### 2.1.2 Growth

The model allows us to describe the relationship between the growth rate of a trade spell and its size and duration. The growth rate of a trade spell is given by

\[
g_y(t, t + T; y, s) = \frac{E(y(t + T|s)) - y(t, s)}{y(t, s)} = \left(\frac{\theta \lambda \mu E(r)}{y} - 1\right) \left(1 - e^{-\frac{T}{\mu}}\right)
\]

which is a decreasing function of size $y$, but is independent of the duration conditional on $y$.

Conditioning on survival, however, increasing the size of the spell, $y$, decreases the average growth rate. Smaller trade spells have a greater probability of disappearing, which reduces the overall growth rate. Conditioning on survival increases the growth rates of younger spells more than older ones. Denote the mean growth rate of surviving trade spells as $g_y(t, t + T; y, s|n(t + T) > 0)$ and the growth rate of disappearing trade spells as $-1$. The probability of trade spells disappearing is given
by $\Pr\{n(t+T) = 0|y(t), s(t)\}$. Using this definition, we can write the growth rate as

$$g_y(t, t+T; y, s) = g_y(t, t+T; y, s|n(t+T) > 0)(1 - \Pr\{n(t+T) = 0|y(t), s(t)\})$$

$$+ \Pr\{n(t+T) = 0|y(t), s(t)\}(-1).$$

Now we can solve for the average growth rate, conditional on survival

$$g_y(t, t+T; y, s|n(t+T) > 0) = \frac{g_y(t, t+T; y, s) + \Pr\{n(t+T) = 0|y(t), s(t)\}}{1 - \Pr\{n(t+T) = 0|y(t), s(t)\}}.$$

Given that the probability of a trade spell disappearing is decreasing in the duration and size of the spell, we obtain the following result:

**Result 4** Conditional on survival, the growth rate of a trade spell is strictly decreasing in size conditional on duration and strictly decreasing in duration conditional on size.

### 2.2 Trade liberalization

There are two important results concerning the effects of trade liberalization on trade relationships: the fraction of firms exporting increases and the average size of the exporting firm increases. Characterizing the dynamic behavior of trade allows us to understand the effects of trade liberalization and to differentiate these effects depending on the timing of the trade liberalization event. Namely, we expect the effects of trade liberalization to affect existing trade spells differently than *new trade spells*, those formed after trade liberalization.

To fix ideas, Figure 2 provides a schematic illustration of the types of trade spells a pair of countries can have as they relate to a trade agreement, or more generally an economic integration agreement (EIA), they enter into. The advent of an EIA allows us to distinguish between three types of spells. There will be spells such as...
spell $A$, which begin and end before the agreement goes into effect. These spells are unaffected by the agreement. There are also spells such as spell $B$ which start before the agreement, but do not end until after the agreement goes into effect. These spells will be directly affected by the agreement. Finally, there are also spells, such as spell $C$, which start after the agreement has been established.

In our model, trade spells formed before the episode of trade liberalization, such as $B$, are different from those formed after trade liberalization, such as $C$, for two reasons. First, business relations already in place experience an increase in their individual size because, while holding their productivity constant, they incur lower trade costs. This will in turn increase their duration while boosting their growth rates, albeit temporarily. Second, new business relations include marginal firms that are able to export only because their effective costs have been reduced. In combination, standard trade models tell us that the average new business relation is larger due to trade liberalization, but separating the old business relations from the new, would show that new business relations are, on average, smaller than the old ones. We also know from our results above that new trade relations are shorter lived, simply because they have not been able to accumulate enough business relations.

The next result summarizes this intuition regarding the effects of trade liberalization on the size, duration, and growth of trade spells:

![Figure 2: Effects of trade liberalization on trade](image)
Result 5  *Trade spells that started before the episode of trade liberalization last longer and grow faster as a result of trade liberalization. Trade spells starting after the episode of trade liberalization may start larger or smaller, but exhibit lower growth, and have shorter duration than those that began before trade liberalization.*

The ambiguity in the effect of trade liberalization on initial values owes to the fact that a newly traded product in the wake of an agreement is potentially exported by two different types of exporters: highly productive firms who never exported before because they were unsuccessful in finding a buyer and marginally productive firms who begin to export only because the agreement reduced trade costs. If the former dominate, the initial volume will likely be larger, ceteris paribus, while if the latter dominate the initial volume will likely be smaller compared to a product which began to be exported before the agreement was signed.

Finally, the last result we investigate is the predicted increase in the number of exporting firms in the wake of an agreement. This is a consequence of a trade agreement reducing the exporting productivity cutoff $\theta$ allowing more firms to become exporters (see equation 1).

Result 6 *Trade agreements result in increased entry as more firms can export.*

3 Data

We have three sets of results the empirical verification of which is a function of available data. Results 1 and 2 are verifiable only with very detailed firm-level data, where one observes some form of a business relation. This could be a destination-product pair, or if taken very literally, every single business partner a firm obtains in a foreign market. While the former types of data exist, the detailed nature of the latter type are not yet readily available and we leave their empirical verification for future work.
Results 3 and 4 are the second set of results our model generates and pertain to spells of trade. Since spells of trade are some form of aggregation of the fundamental business relations our model is based on, data required to examine results 3 and 4 are more readily available. One could examine them using the aforementioned firm-level data or country-product- or country-industry-level data. These two results provide a theoretical underpinning for well established results in the duration of trade literature and we only briefly examine them below.

The third set of results are summarized by results 5 and 6. They pertain to the effect of trade liberalization on incumbent and newly started trade spells as well as the rate of entry of new spells. These two results are the primary focus of this paper. To investigate this set or results we must combine trade flow data at the firm or more aggregated levels along with data on various kinds of economic integration agreements. In order to cast as wide a net as possible in terms of various kinds of economic integration agreements, we chose to conduct our empirical investigation using as large as possible a data set with the richest coverage of both products, countries, and economic integration agreements.

We combine data from two sources. Trade flow data come from UN’s Comtrade Database. We use the longest possible panel available with trade recorded annually from 1962 until 2011 using the 5-digit SITC revision 1 classification. As Comtrade provides data on both imports and exports, we use data as reported by importers given their widely perceived greater accuracy. Since we use imports of all countries available through Comtrade, our analysis can be equivalently thought of as an analysis of imports or of exports. However, we shall simply use the term trade to avoid any confusion.

Data on economic integration agreements come from the Database on Economic Integration Agreements compiled by Scott Baier and Jeffrey Bergstrand (2007). It is available at [http://www.nd.edu/~jbergst](http://www.nd.edu/~jbergst).

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8At the 5-digit level, there are 944 product categories.
9Available at [http://www.nd.edu/~jbergst](http://www.nd.edu/~jbergst).
collects data on various economic integration agreements as entered into by 195 countries on an annual basis between 1950 and 2005. Our sample observations are defined by the temporal intersection of our two sources, from 1962 to 2005.

One advantage of using trade data at the SITC revision 1 level, reaching back to 1962, is the relative paucity of economic integration agreements at the beginning of the sample period. Thus, for the vast majority of EIAs that have been in existence in the post-World War II period, we observe their effect from the start of the agreement itself. This would not be the case if we used data at the 6-digit HS level as HS data are only available from 1989. Note from Figure 1 that in 1962, when our sample begins, only 1.1% of country pairs have an agreement in place. Thus, not taking into account the exact starting point of this small number of EIAs likely generates a small bias. By 1989, when the HS data become available, the fraction of country pairs with an agreement increases by an order of magnitude to 14.8%. By the end of our sample, around 21% of country pairs share an agreement.\footnote{The drop in the utilization rate in the early 1990s (1991 through 1994 to be precise) stems for the break up of the eastern block countries in Europe, Czechoslovakia, Soviet Union, and Yugoslavia. By 1995 the utilization rate returns to its pre-breakup levels.}

Since we examine the effect of economic integration agreements on trade relationships we define as a unit of observation a continuous trade spell involving two countries and a specific product. By this we mean consecutive years, beginning with the clearly observed starting point, during which a trade relationship is active. Consistent with our model, we differentiate between trade spells and trade relationships since a relationship denotes an exporter-importer-product triplet, while a spell indicates the consecutive years during which a relationship is active.

There are a total of 29,671,095 observations on (positive) trade flows between 1962 and 2005. Of these we have no information on economic integration agreements for 2,021,121 observations (about 7% of trade flow observations), which account for 1.7% of total observed trade in our sample. Most often this pertains to instances of
trade with very small economies, or countries which disappeared during the observed period as the database does not offer a historical perspective on agreements.\footnote{One could interpret these observations as no agreement existing, but that would be incorrect as one would have to make sure no agreement in fact was in place.}

<table>
<thead>
<tr>
<th>Type of agreement</th>
<th>Number of observations</th>
<th>Number of observations used in estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>16,990,281</td>
<td>15,237,989</td>
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<tr>
<td>Non-Reciprocal Preferential Trade Agreement</td>
<td>2,468,555</td>
<td>2,389,726</td>
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<tr>
<td>Preferential Trade Agreement</td>
<td>1,459,940</td>
<td>1,418,321</td>
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<td>Free Trade Agreement</td>
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<td>Economic Union</td>
<td>465,962</td>
<td>375,559</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>27,649,671</strong></td>
<td><strong>24,510,480</strong></td>
</tr>
</tbody>
</table>

Table 1: \textbf{Number of Observations by Agreement Type}

Of the remaining 27,649,671 observations, as documented in Table 1, some 61% involve pairs of countries which have no economic integration agreement in place at any point during our sample. These observations account for 41.5% of all observed trade. The remaining observations account for 56.7% of all observed trade and belong to the six types of agreements in the data: non-reciprocal preferential trade agreements (NR-PTA), (reciprocal) preferential trade agreements (PTA), free trade agreements (FTA), currency unions, common markets, and economic unions. FTAs are the most common accounting for 14% of observed disaggregated trade flows, followed by NR-PTAs with 9% and PTAs with 5% of observations. Deeper integration schemes are typically less frequent. Currency unions account for roughly 5% of the bilateral trade observations, while common markets account for 4% and economic unions for only 2%. For the purpose of understanding the effect of economic integration on the product-level patterns of trade, we do not distinguish between the different types of agreements, but rather focus on the sheer existence of an agreement of some sort. This simplifying assumption allows us to ignore issues arising from
countries upgrading or downgrading their agreements\textsuperscript{12}

We are primarily interested in the effects of economic integration agreements in a multicountry context. It follows from our theory that we need to include standard variables capturing country characteristics\textsuperscript{13}. We use the CEPII gravity data as the source for both the exporter’s and the importer’s GDP, distance, and existence of a common border and a common language\textsuperscript{14}.

In the forty four years in our data set relationships may be characterized, and frequently are, by multiple spells of service, a fact our model accounts for. There are a total of 3,109,559 trade relationships in our data with 7,191,964 observed active spells, or 2.3 per relationship. Some 45\% of all trade relationships have only one active spell, with 22\% having two active spells, and less than 7\% having six or more active spells. Table 2 shows that the vast majority of observed spells of trade are of very short duration, with slightly more than 55\% of all spells observed for just a single year and 90\% observed for seven or fewer years.

The second column of Table I shows the number of observations on each type of agreement in the dataset used in estimation. Our estimation sample is smaller by 3,139,494 observations, or some 10\%, due to two factors. The majority of these observations, 2,843,686 to be precise, are omitted since they belong to spells of trade that are left censored. For all spells which are active in the first year in which an importing country reports data, the actual start of the spell is not observed. For example, the first year in which the U.S. reports imports in our data set is 1962. Consequently, all spells involving the U.S. in 1962 are left censored, and we omit

\textsuperscript{12}The former is far more common than the latter. As an example, Germany and Austria signed a free trade agreement in 1973, upgraded it to a common market in 1994, and again to an economic union in 1999. To properly investigate the effects of specific types of agreements, we would need to control for such changes dynamically. We felt this worthy task is better left for a future paper. We refer the interested reader to Besedeš (forthcoming) who examines similar dynamic issues as they pertain to the integration of the European Union.

\textsuperscript{13}In our model we use Melitz (2003) to characterize individual firm behavior. It has been shown in Cheney (2008) that Melitz translates into a distorted gravity equation, so we need to account for those variables as well.

\textsuperscript{14}Available at http://www.cepii.fr/anglaisgraph/bdd/gravity.htm
all such observations from our analysis. The remaining omitted observations, almost 300,000, have missing gravity data and are not used.

### 4 Methodology

As our model shows, to properly identify the effects of EIAs, we need to differentiate between spells active when the agreement begins and spells which begin after the agreement. In order to properly capture all effects of economic integration agreements we use four variables. One variable, labeled ‘EIA in effect,’ identifies the years during which an agreement is in force, thus capturing the differential effect of the agreement itself. Since our model predicts that spells which start after the agreement are different from already active ones, we use a second dummy variable, ‘Spell starts after EIA,’ which identifies all spells which started after the agreement is put in force. The ‘EIA in effect’ and ‘Spell starts after EIA’ variables in conjunction identify the effect on spells which begin after the agreement is in effect. The first of our two specifications only includes these two variables. The second specification adds two more variable

<table>
<thead>
<tr>
<th>Spell length</th>
<th>Number of spells</th>
<th>Fraction of spells</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4,009,321</td>
<td>55.7%</td>
</tr>
<tr>
<td>2</td>
<td>1,109,540</td>
<td>15.4%</td>
</tr>
<tr>
<td>3</td>
<td>507,534</td>
<td>7.1%</td>
</tr>
<tr>
<td>4</td>
<td>294,258</td>
<td>4.1%</td>
</tr>
<tr>
<td>5</td>
<td>213,270</td>
<td>3.0%</td>
</tr>
<tr>
<td>6</td>
<td>174,633</td>
<td>2.4%</td>
</tr>
<tr>
<td>7</td>
<td>115,726</td>
<td>1.6%</td>
</tr>
<tr>
<td>8</td>
<td>99,488</td>
<td>1.4%</td>
</tr>
<tr>
<td>9</td>
<td>80,455</td>
<td>1.1%</td>
</tr>
<tr>
<td>10</td>
<td>80,313</td>
<td>1.1%</td>
</tr>
<tr>
<td>11-20</td>
<td>327,288</td>
<td>4.6%</td>
</tr>
<tr>
<td>21-30</td>
<td>82,061</td>
<td>1.1%</td>
</tr>
<tr>
<td>31-43</td>
<td>98,077</td>
<td>1.4%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>7,191,964</strong></td>
<td><strong>100.0%</strong></td>
</tr>
</tbody>
</table>

Table 2: Distribution of Spell Lengths
designed to capture any change in the effect of an agreement the longer the agreement itself is in effect. The third variable measures how long an agreement has been in place. This variable identifies at a micro level whether the effect of an agreement depends on how long it has been in place, as has been shown to be the case in aggregate measures by Baier and Bergstrand (2007) and Baier, Bergstrand, and Feng (2011). The fourth variable measures the effect of how long the agreement has been in place only on spells which start after the agreement, the post-agreement spells.

We are interested in the effect agreements have on three attributes of trade spells: the volume of trade in the first year, the growth of the volume of trade while the spell is active, and the conditional probability it will cease to be active or the hazard rate. We are also interested on the rate of entry of new spells. We examine the effect on initial volumes and the growth of trade within an active spells by estimating two separate OLS regressions:

\[
\ln(y(1)_{kodt}) = \alpha + \mathbf{EIA}_{odt}\beta + \gamma_{od} + \delta_{ot} + \zeta_{dt} + \eta_k + \epsilon_{kodt}
\]

and

\[
\ln(g_{kodt}) = \alpha + \mathbf{EIA}_{odt}\beta + \kappa \ln(s(t)_{kodt}) + \lambda \ln(y(t-1)_{kodt}) + \\
+ \gamma_{od} + \delta_{ot} + \zeta_{dt} + \eta_k + t_k + \xi_t + \lambda + \epsilon_{kodt}
\]

where \(\ln(y(1)_{kodt})\) is the logged volume of trade in the first year of spell \(k\) of a trade relationship between origin \(o\) and destination \(d\) in product \(i\) which occurs in calendar year \(t\), \(\ln(g_{kodt})\) is the log of the growth of trade from year \(t-1\) to \(t\) of product \(i\)'s spell \(k\) between \(o\) and \(d\), \(\mathbf{EIA}_{odt}\) is the vector of variables describing an agreement between origin \(o\) and destination \(d\) in year \(t\), \(\ln(s(t)_{kodt})\) is the log of the age of spell \(k\) in year \(t\), \(\ln(y(t-1)_{kodt})\) is the size of trade in the previous year of the spell, \(\gamma_{od}\) are origin-destination pair fixed effects, \(\delta_{ot}\) are origin-year fixed effects, \(\delta_{dt}\) are destination-year
fixed effects, $\zeta_k$ are spell fixed effects, $\iota_k$ are spell length fixed effects, $\xi_t$ are calendar year fixed effects, $\lambda$ are 3-digit SITC industry fixed effects, and $\epsilon_{kodv}$ and $\varepsilon_{ktodv}$ are the two error terms.

The third characteristic of spells we examine is the hazard of a spell of trade ceasing, $h_{kodt}^i$. The hazard is the probability of exports of product $i$ from country $o$ to country $d$ in spell $k$ ceasing at time $t + n$ conditional on it having survived until time $t$ (or in our notation age $s(t)$), $P(T_{kod}^i \leq t + n | T_{kod}^i \geq t)$, where $T_{kod}^i$ is a random variable measuring the survived duration of spell $kod$. We estimate the hazard of exports ceasing at time $n$ by estimating a discrete hazard using random effects probit specification

$$h_{kodt}^i = P(T_{kod}^i \leq t + n | T_{kod}^i \geq t) = \Phi \left( \text{EIA}_{odt} \beta + \text{X}_{od} \omega + \kappa \ln(s(t)_{kod}) + \lambda \ln(y(t-1)_{kod}) + \rho \ln \text{GDP}_o + \tau \ln \text{GDP}_d + \eta_k + \nu_{kod}^i \right)$$

(12)

where we use the same variables as in the above OLS specifications except that instead of origin-destination, origin-year, and destination-year fixed effects, we use the log of origin’s and destination’s GDP and a vector of of bilateral time-invariant gravity variables $\text{X}_{od}$ (distance, common border, and common language). Relationship-specific random effects are captured by $\nu_{kod}^i$.

The last object of our empirical investigation is the rate of entry of new spells of trade between two countries in a particular year. To investigate the effect of agreements on the rate of entry we estimate an OLS regression of the following specification:

$$\ln(\text{entry rate}_{odt}) = \alpha + \text{EIA}_{odt} \beta + \delta_{od} + \zeta_{dt} + \sigma_{odt}$$

(13)

where $\ln(\text{entry rate}_{odt})$ is the logged percentage of new spells of trade from country $o$ to country $d$ in year $t$, $\sigma_{odt}$ is the error term, and the remaining variables are defined
above. In this regression, rather than investigating characteristics of individual spells, we simply count the number of new spells in any given year and use that figure expressed as percentage of all spells in a given year as our dependent variable. We should point out that our results for entry rates have the highest chance of being affected by aggregation bias. Our model predicts there will be more entry at the firm level. Our data are, however, at the product level. As such, an existing trade spell may mask a lot of entry on the part of firms beginning to export a product which is already being exported. Our data preclude us from observing this entry. The only entry we can observe is a new spell of trade becoming active involving a product that no firm was previously exporting.

5 Results

We discuss our empirical results in the same order as they were derived in Section 2. Since our data are not sufficiently detailed to examine Results 1 and 2, we begin with Results 3 and 4. These two results have already been shown to hold in the literature, thus our discussion is purposefully brief and is included for completeness. We devote most of our discussion to Results 5 and 6, a set of results new to the literature.

5.1 Duration and Growth

Result 3 states that the probability of a trade relationship ceasing is decreasing in its size and age (or duration). A natural way to examine this result is to estimate a hazard model where the hazard of interest is that of a trade relationship ceasing. Result 4 states that the growth rate is decreasing in size conditional on duration and decreasing in duration conditional on size.

To estimate the hazard of trade ceasing we estimate the specification given by

\[ \text{Hazard} = \text{function of size, duration, and other covariates} \]

See Besedeš and Prusa (2006b), Nitsch (2009), and Besedeš, Lugovskyy, and Kim (2014) among others.
equation (12) and to estimate the growth rate we use the specification given by equation (11), in both cases without the economic integration variables. As implied by our model in both regressions we include duration or age of a spell and its current size measured as the value of trade. The latter is the biggest departure in our analysis from the extant duration literature. However, our theoretical model predicts that both growth and duration depend on the current size of the spell, not the initial one (see equation 9 in the case of growth). The standard size variable used in the literature is the initial value of trade. Our specification for the growth regression is similar to Muûls (2014), who also includes the volume of trade in period $t$ to explain the growth of firm-level trade from $t$ to $t + 1$. To estimate the hazard we include the standard gravity variables, GDP of both the importer and the exporter, distance between the two, as well as a dummy indicating the existence of a common border and a common language that the two countries share. In the growth regression, instead of using standard gravity variables, we use country-pair, origin-year, and destination-year fixed effects in order to fully control for unobserved multilateral resistance terms.

Results collected in Table 3 are consistent with the predictions of our model and are in line with the literature. Both the hazard and the growth rate are decreasing in duration, indicating that longer lived spells are less likely to cease and also grow less. Both are also decreasing in size, indicating that larger spells are less likely to cease and also grow less. Our results for growth are consistent with Muûls (2014) who examines firm-level growth and finds it to be decreasing in age as well as size.

5.2 Effects of Economic Integration Agreements

We examine the effect of a generic economic integration agreement on the initial size, growth, and duration of trade spells. To do so we add the four above-discussed EIA variables to the set of variables used in Table 3: a dummy indicating when the agreement is in effect (EIA in effect), a dummy identifying a spell of trade starting after
<table>
<thead>
<tr>
<th></th>
<th>Hazard</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(RE probit)</td>
<td>(OLS)</td>
</tr>
<tr>
<td>Duration (ln)</td>
<td>-0.433***</td>
<td>-0.055***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Size (ln)</td>
<td>-0.126***</td>
<td>-0.253***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Importer GDP (ln)</td>
<td>-0.013***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>Exporter GDP (ln)</td>
<td>-0.086***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>Distance (ln)</td>
<td>0.123***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>Contiguity</td>
<td>-0.110***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>Common language</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.328***</td>
<td>3.722***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>Observations</td>
<td>24,510,480</td>
<td>17,555,604</td>
</tr>
<tr>
<td>Relationships</td>
<td>3,109,593</td>
<td>1,871,657</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.</td>
<td>0.143</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.168***</td>
<td></td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses, with *, **, *** denoting significance at 10%, 5%, and 1%.

Table 3: Hazard and Growth Regressions

the agreement is in effect (Spell starts after EIA), and the two variables measuring how long the agreement has been in place, one for all spells and one only for post-agreement spells. Unlike in Table 3, we now estimate an OLS regression on initial volumes as well. We collect all results in Table 4. We estimate two separate specifications, one without and one with variables measuring how long the agreement has been in effect.

5.2.1 Initial volume of trade

Our first investigation pertains to the effect of economic integration agreements on the initial volume of trade. Since we are examining a single value at the starting point of a spell, our ability to identify different effects of economic integration agreements
<table>
<thead>
<tr>
<th></th>
<th>Initial Value (OLS)</th>
<th>Hazard (RE probit)</th>
<th>Growth (OLS)</th>
<th>Initial Value (OLS)</th>
<th>Hazard (RE probit)</th>
<th>Growth (OLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration (ln)</td>
<td>-0.423*** (0.001)</td>
<td>-0.060*** (0.001)</td>
<td>-0.426*** (0.001)</td>
<td>-0.060*** (0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Imports (ln)</td>
<td>-0.126*** (0.000)</td>
<td>-0.253*** (0.000)</td>
<td>-0.125*** (0.000)</td>
<td>-0.254*** (0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Importer GDP (ln)</td>
<td>-0.013*** (0.000)</td>
<td></td>
<td>-0.015*** (0.000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exporter GDP (ln)</td>
<td>-0.086*** (0.000)</td>
<td></td>
<td>-0.085*** (0.000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance (ln)</td>
<td>0.121*** (0.001)</td>
<td></td>
<td>0.119*** (0.000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contiguity</td>
<td>-0.112*** (0.002)</td>
<td></td>
<td>-0.114*** (0.002)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common language</td>
<td>-0.002 (0.001)</td>
<td></td>
<td>-0.006*** (0.001)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EIA in effect</td>
<td>-0.010** (0.004)</td>
<td>-0.224*** (0.002)</td>
<td>0.080*** (0.002)</td>
<td>-0.012*** (0.004)</td>
<td>-0.239*** (0.002)</td>
<td>-0.004 (0.002)</td>
</tr>
<tr>
<td>Spell started after EIA</td>
<td>0.245*** (0.002)</td>
<td>-0.074*** (0.002)</td>
<td>0.219*** (0.002)</td>
<td>0.219*** (0.003)</td>
<td>-0.065*** (0.003)</td>
<td></td>
</tr>
<tr>
<td>Duration of EIA (ln)</td>
<td>0.001*** (0.002)</td>
<td></td>
<td>0.003*** (0.000)</td>
<td>0.010*** (0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duration of EIA for post-agreement spells</td>
<td></td>
<td></td>
<td></td>
<td>0.095*** (0.001)</td>
<td>-0.005*** (0.001)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>9.786 (25.350)</td>
<td>1.341*** (0.006)</td>
<td>3.708*** (0.023)</td>
<td>9.780 (19.551)</td>
<td>1.365*** (0.006)</td>
<td>3.685*** (0.023)</td>
</tr>
<tr>
<td>Observations</td>
<td>7,353,211</td>
<td>24,510,177</td>
<td>17,555,604</td>
<td>7,353,211</td>
<td>24,510,177</td>
<td>17,555,604</td>
</tr>
<tr>
<td>Relationships</td>
<td>3,185,092</td>
<td>3,109,593</td>
<td>1,871,657</td>
<td>3,185,092</td>
<td>3,109,593</td>
<td>1,871,657</td>
</tr>
<tr>
<td>R²</td>
<td>0.304</td>
<td>0.143</td>
<td>0.304</td>
<td>0.143</td>
<td></td>
<td>0.143</td>
</tr>
<tr>
<td>ρ</td>
<td>0.164</td>
<td></td>
<td>0.166</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses for OLS regressions with *, **, *** denoting significance at 10%, 5%, and 1%.

Table 4: Effects of Economic Integration Agreements
is reduced. A spell either starts before or after the agreement. As a result, the effect of an agreement taking effect only applies to spells starting after the agreement. We thus have two variables identifying the effects of an agreement: a dummy variable identifying the years when the agreement is in effect (EIA in effect) and a variable reflecting how long the agreement has been in effect when a spell starts.

The first and fourth columns of Table 4 collect the results from estimating equation (10). Using only the dummy variable identifying when the agreement is in effect we find that initial volumes decrease with an agreement by 0.010 log points. We then add the variable measuring how long the agreement was in effect when the spell started. Doing so results in a fixed (with respect to time) effect of an agreement decreasing initial volumes by 0.012 log points, as well as a time-dependent effect which increases initial volumes by 0.001 log points for every year of the agreement being in force. Since both duration variables (of the spell and the agreement being in place) are measured in logs, while the initial value of trade increases after the agreement, it increases at such a low rate that it will not result in a post-agreement initial value being larger than the pre-agreement value in any reasonable time frame.\[16\]

\section*{5.2.2 Hazard of trade ceasing}

We estimate the hazard of trade ceasing by estimating equation (12) using random effects probit, which allows us to take into account unobserved heterogeneity. The use of a probit estimator necessitates that we specify how the hazard depends on the duration of a spell for which we use the logarithm of the current length of the spell (age) at every point in time (measured in years). To evaluate whether a variable has a significant effect on the hazard we first calculate the predicted hazard at the mean of every variable and then calculate the predicted hazard while changing the

\[\text{To be precise, the negative fixed effect of an agreement being in place and the positive temporal effect of how long it has been in place will offset each other after 162,754.8 years (the solution to the equation } -0.012 + 0.001 \ln(x) = 0).\]
value of the variable of interest. For example, to evaluate whether active spells of trade are affected by the onset of the agreement, we would calculate and plot the estimated hazard with the EIA in effect dummy first set to zero and then set to one, while keeping all other variables at their respective means. We plot both the estimated hazard along with the 99th percentile confidence interval, which is plotted with dotted lines.\textsuperscript{17} As long as confidence intervals do not overlap, the effect of the agreement is deemed to be statistically significant.\textsuperscript{18} In fact, in virtually every plot we examine below, we find that the differences are statistically significant. Such an approach to examining the effect of a covariate is necessary as the effect and the precision with which it is estimated depend on the standard errors of all estimated coefficients, all pairwise covariances, and the distributional specification of the probit model.

Results in columns (2) and (5) of Table 4 for gravity variables are in line with the literature and similar to those in Table 3 as is the case with the effect of the size of the spell and its duration. As predicted by our model, the effect of an agreement on already active spells is to reduce their likelihood of ceasing, thus making them longer. However, spells which begin after the agreement is in place are more likely to cease. This effect is slightly larger than the effect of an agreement being in place. Thus, the net effect on spells starting after the agreement, as we illustrate below, is to increase their hazard to a level higher than what it would be in the absence of an agreement. As indicated by results in column (5), the longer an agreement is in place the lower is the hazard of pre-agreement spells. Thus, the beneficial effect these spells receive from an agreement increases as they survive after the agreement. For post-agreement

\textsuperscript{17}We include confidence intervals for every plotted curve throughout the paper. The corresponding confidence interval is always represented with a dotted line and of the same color as the curve depicting the predicted hazard. In most instances the confidence interval is imperceptible given the high precision of our estimated coefficients and the large number of observations on which they are based.

\textsuperscript{18}See Sueyoshi (1995) for a longer discussion of how to evaluate whether the effect of a variable is significant when using a probit approach to estimate the hazard and Besed\v{s} and Prusa (2015) for an application in international trade.
spells the story is very different. For every year they survive their hazard increases by a net of 0.086 log points (the sum of the two length of EIA in place variables) in addition to the fixed effect of having started after the agreement of 0.017 log points.

To examine the magnitude and properly evaluate the statistical significance of the effects, we turn now to predicted hazards. Since our results and our model indicate that taking into account the timing of when an agreement takes effect and when a spell starts is important, we evaluate the effects of these variables under the following set of arbitrarily chosen characteristics. As our comparison benchmark we use the hazard in the absence of an agreement. We compare that hazard to the hazard profile for spells which are in their sixth year as the agreement comes into effect.\textsuperscript{19}

For spells which start after the agreement we assume that they start in the sixth year of the agreement. Given the small magnitude of the coefficient on the length of an agreement, changing in which year of the spell an agreement starts or in which year of the agreement a spell starts only has minimal effects on our plotted hazard profiles.

We note that when examining the effect of an EIA on either already active spells or spells which start after the EIA, we evaluate the effect for the remaining possible duration of a spell given the time span of our data. Thus, for those spells affected by an EIA in their sixth year, we examine the effect during the remaining 37 possible years assuming it would survive that long. For spells which begin after the EIA, we plot the estimated hazard for 43 years, the longest possible spell length we can observe. To summarize the effect of an EIA on already active spells, we calculate the difference in the hazard of spells affected by an EIA and those unaffected over the years 6 through 43, average the difference and divide it by the average hazard over years 6 through 43 for unaffected spells or the relevant comparison hazard.

We collect the plots in Figure\textsuperscript{3} where the top panel corresponds to column (2) of

\textsuperscript{19}Note that given the distribution of spell lengths (as tabulated in Table 2\textsuperscript{2}), a full 85\% of spells do not make it into year six, our chosen year to illustrate the effects of EIA. This should not be particularly troubling as year six was chosen purely for illustrative purposes. Moving the onset of the EIA to an earlier year of the spell would not drastically affect our conclusions.
Figure 3: Simulated Effects of EIAs on Hazard
Table 4 and the bottom panel to column (5). The decreasing nature of the hazard of trade ceasing makes it somewhat tricky to discuss the magnitude of the effect of an agreement. To illustrate, consider the top panel of Figure 3. The predicted hazard for post-agreement spells and those not affected by an agreement are very close to each other. The absolute difference between the two hazards is hence small, on average 0.41 percentage points. However, the relative difference between the two is 20.4%. This is a consequence of the hazard in later stages of spells being very low, on which case even small differences between two hazard profiles are large in a relative sense. Therefore, to provide a better sense of relative differences in various hazard profiles we plot, we summarize the relative differences in Table 5 where we provide average differences over the first five years that an agreement has an effect, followed by years 6 through 10, then 11 through 20, 21 through 30, and then all years. Given that 85.3% of all spells in our sample are observed for five years or less (see Table 2), we consider the effects during the first five years to be of most interest and most representative.

Focusing on the top panel of Figure 3 first, an agreement reduces the hazard of spells active when the agreement begins. Over the remaining course of an already active or pre-agreement spell, the hazard is 1.15 percentage points or 42.5% lower. Over the first five years of an agreement, the hazard is 3 percentage points or 35% lower. The relative effect increases as the spell survives, reaching a reduction of 45% between years 21 and 30 of a spell. Spells which begin after the agreement face a higher hazard than those in the absence of an agreement. In the first five years of an agreement post-agreement spells face a 3.1% higher hazard. This effect increases to 21.3% higher hazard for years 21 through 30 of the spells, which is largely a consequence of the hazard during those years being very low, as discussed above.

Turning to the bottom panel of Figure 3 adding the variable indicating how long the agreement has been in place reduces the positive effect of an agreement on active spells and increases the negative effect on spells which start after the agreement. The
agreement now reduces the hazard of pre-agreement spells by an average of 23.2% over the first five years, a full 12 percentage points smaller effect. Across the entire remaining length of the spell the hazard is now 31.5% lower. New spells which start after the agreement now face a higher hazard than spells outside an agreement. The absolute difference is 1.12 percentage points or a 51.0% on average higher hazard. During the first five years post-agreement spells face an 11.5% higher hazard, with the magnitude of the effect increasing progressively into the later stages of the spell.

<table>
<thead>
<tr>
<th>Spell years</th>
<th>No length of EIA variable</th>
<th>Length of EIA Variable Included</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-agreement spells</td>
<td>Post-agreement spells</td>
</tr>
<tr>
<td>1-5</td>
<td>-35.0%</td>
<td>3.1%</td>
</tr>
<tr>
<td>6-10</td>
<td>-38.8%</td>
<td>7.6%</td>
</tr>
<tr>
<td>11-20</td>
<td>-42.1%</td>
<td>13.8%</td>
</tr>
<tr>
<td>21-30</td>
<td>-45.0%</td>
<td>21.3%</td>
</tr>
<tr>
<td>All years</td>
<td>-42.5%</td>
<td>20.4%</td>
</tr>
</tbody>
</table>

Table 5: **Effect of Agreement on the Hazard of Trade Ceasing**

We present Figure 4 to help understand the impact of each of the four EIA related variables used in column 5 of Table 4 and summarize their magnitudes in Table 6. In each panel we plot the estimated hazard with each EIA-related variable set to zero and its appropriate agreement value (either one in the case of the two dummies or a count of the how long the agreement has been in place) in turn, while keeping the other three variables at zero. Thus, the relevant comparison is to the hazard in the absence an agreement. This illustrates the pure effect of each variable over the entire possible length of a spell. These plots allow us to clearly illustrate the effect of each variable and better understand how they combine to affect the hazard of trade ceasing.

The pure effect of an agreement is to reduce the hazard by an average of 25.9% over the course of an entire spell relative to the hazard in the absence of an agreement.
Figure 4: Pure Effects of EIAs on Hazard
The magnitude of this effect varies from a reduction of 16.1% over the first five years to a 27.9% reduction between years 21 through 30 of a spell. This is the effect most beneficial to pre-agreement spells. These spells also benefit the longer the agreement is in place with the hazard being an additional 1.3% lower over the first five years and 7% lower for years 21 through 30, averaging to a 5.9% lower hazard over the entire spell. While spells which begin after the agreement also benefit from these two effects, they also face some strong head winds from the other two variables. Spells which start after the agreement have on average a 4.1% higher hazard, the effect which ranges from 2.4% higher hazard over the first five years and 4.5% higher hazard for years 21 through 30. The much stronger headwinds come from the effect of how long the agreement has been in place. Over the first five years of an agreement, spells which started after the agreement face a 14.5% higher hazard. Between years 21 and 30 this effect increases to a 110.2% larger hazard, more than double the hazard faced in the absence of an agreement. Across all years of a spell, this effect averages to a 94% higher hazard.

<table>
<thead>
<tr>
<th>Spell years</th>
<th>EIA in effect</th>
<th>Spell started after EIA</th>
<th>Duration of EIA</th>
<th>Duration of EIA for post-agreement spells</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>-16.1%</td>
<td>2.4%</td>
<td>-1.3%</td>
<td>14.7%</td>
</tr>
<tr>
<td>6-10</td>
<td>-21.9%</td>
<td>3.4%</td>
<td>-3.4%</td>
<td>43.4%</td>
</tr>
<tr>
<td>11-20</td>
<td>-25.4%</td>
<td>4.0%</td>
<td>-5.3%</td>
<td>75.5%</td>
</tr>
<tr>
<td>21-30</td>
<td>-27.9%</td>
<td>4.5%</td>
<td>-7.0%</td>
<td>110.2%</td>
</tr>
<tr>
<td>All years</td>
<td>-25.9%</td>
<td>4.1%</td>
<td>-5.9%</td>
<td>94.0%</td>
</tr>
</tbody>
</table>

Table 6: **Pure Hazard Effects of Agreement-Related Variables**

An economic integration agreement has a dual effect on the hazard of trade ceasing, just as our model predicted. It reduces the hazard of already active spell, but increases it for any spell which starts after the agreement. To put it in different terms, economic integration seems to promote the stability of trade spells active when the agreement is signed and reduces the stability of those which commence in its wake.
5.2.3 Growth of trade

We now turn to examining the effect of economic integration agreements on the growth of trade embodied in active spells. In particular, we examine the growth of trade conditional on spell survival. To put it differently, we are not concerned with explaining the negative growth that occurs with the complete decrease in the volume once the spell ends. Our specification, given by equation (11), follows Besedeš, Kim, and Lugovskyy (2014) and includes spell number, calendar year, spell length, 3-digit SITC, origin-year, destination-year, and origin-destination fixed effects. Our results are collected in columns (3) and (6) of Table 4.

Similar to the results of Besedeš, Kim, and Lugovskyy (2012), we find that the rate of growth of trade within a spell decreases the longer the duration of the spell, just as our model predicts. Larger spells grow less. The effect of economic integration agreements is as our model predicts. Without accounting for how long an agreement has been in place, the effect on active spells is an increase of 8.0 percentage points in the growth rate, while the effect on spells which start after the agreement is a reduction in their growth rate of 7.5 percentage points. Thus, on net, spells which start after the agreement have a 0.5 percentage point higher growth rate than what would be the case in the absence of an agreement. Accounting for how long the agreement has been in place changes the two static effects. The agreement itself no longer has a statistically distinguishable from zero effect on the hazard. The positive effect of an agreement on pre-agreement spells is entirely captured by the temporal effect which indicates that for every year that the agreement is in effect, the growth rate increases by 1.0 percentage point. Post-agreement spells have a 6.5 percentage point lower growth rate. This negative effect of agreements on the growth of post-agreement spells is compounded by the negative temporal effect of 0.5 percentage points per year.

To illustrate these magnitudes we produce Figure 5 which is similar in nature
Figure 5: Simulated Effects of EIAs on Growth
We fit our model at the average of all our variables and examine the effect of EIA on the predicted growth rate. Note that the fitted growth rates differ considerably between the two specifications. The static specification provides a slower rate of decrease in the growth rate with duration, while the dynamic one which accounts for how long the agreement has been in place provides for a faster reduction in growth rates.

We can conclude that economic integration agreements have a positive effect on the growth of spells already active when an agreement starts and a smaller, though still positive, effect on spells which started after the agreement. Importantly, from the point of view of our theoretical predictions, spells started after the agreement grow less than those which started before the agreement.

5.2.4 Rate of entry of new spells

In addition to our model predicting that an agreement will increase the rate of entry, or re-entry, of (new) spells, these results are important from another point of view. The results we have presented so far indicate that economic integration agreements have a dual effect. While they make already active spells more stable and larger, their effect on newly created spells is the opposite. Spells created after an agreement are more likely to fail and their growth rate is marginally higher than the growth rate in the absence of an agreement. Such dual effects create a potential puzzle: if the effects at the disaggregated level indicate that old spells become more stable and new spells become less stable, what is driving the strong growth of trade found at the aggregate level? Baier and Bergstrand (2007) conclude that ten years after an agreement is signed the amount of trade between the two countries doubles. An additional factor contributing to this puzzle is the distribution of length of spells shown in Table 2 which indicates that the majority of spells are short lived. Thus, even the positively affected spells eventually end. They are replaced by more fragile spells which grow far
less. A possible explanation behind the observed aggregate growth is that agreements create new spells which account for much of the aggregate growth. This explanation is along the lines of Baier, Bergstrand, and Feng (2014) who show that in the short run the effect of agreements tends to be on the intensive margin, but with time much of the effect moves to the extensive margin. We now examine whether we can detect this effect in disaggregated data.

We estimate the specification given by equation (13) where we regress the percentage of new spells on agreement related variables as well as destination-year and origin-year fixed effects. Unlike our previous regressions where the basic unit of account was at the origin-destination-product level, our basic unit of account now is at the origin-destination level. Our results are collected in Table 7.

Using only the static effect of agreements indicates that the rate of entry decreases. However, the addition of the variable indicating how long an agreement has been in place changes results drastically. The static effect now indicates that an agreement increases the rate of entry by 30.4%, which is then slowly reduced by 0.04 log points for every year that an agreement is in place. This is a very slow rate of reduction with the rate of entry 43 years after an agreement has been in place still some 15.3% higher than would be the case in the absence of an agreement. Thus, taken together our results indicate that while new spells of trade which begin after an agreement are

### Table 7: Effect of Economic Integration Agreements on Entry

<table>
<thead>
<tr>
<th>EIA in effect</th>
<th>-0.056***</th>
<th>0.304***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>Duration of EIA (ln)</td>
<td>-0.040***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.144***</td>
<td>-1.477***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.040)</td>
</tr>
<tr>
<td>Observations</td>
<td>367,715</td>
<td>367,715</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.294</td>
<td>0.294</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses for OLS regressions with *, **, *** denoting significance at 10%, 5%, and 1%.
more likely to fail and grow less than those started before the agreement, but their sheer numbers play an important role in the aggregate growth of trade in the wake of an agreement.

6 Conclusion

In this paper we characterize the dynamic behavior of trade, on both the intensive and extensive margins, and analyze the effects of trade liberalization on trade dynamics. We start by building a theory model which characterizes the behavior of a trade relationship observed at the product level by starting from firm decisions. We characterize the decision of the firm using Melitz (2003) and aggregate to the trade relationship using Klepper and Thompson (2006). In our model, firms acquire new business relations and by accumulating new business relations, an exporter can grow its presence in the market. If an exporter loses all business relations the trade relationship will go dormant until a new business relation is acquired by an exporter firm, or seller.

Our model creates predictions about both duration and growth of trade of active spells of trade, an active instance of a trade relationship. Duration increases in size and age of a spell (and its converse, the hazard, is decreasing in both). The growth rate of a spell is decreasing in duration as well as its size. Both of these predictions are borne by our data. Moreover, our model is able to generate both exit of a once active trade relationship as well as its regeneration. This feature matches a fact present in international trade data that a number of trade relationships are present in multiple distinct instances.

Using the model we are able to predict the effects of economic integration agreements on trade. We examine three attributes of trade embodied in trade relationships defined as importer-exporter-product triplets: the initial value of trade, the growth
of trade within a spell, and the hazard of trade ceasing. Our model predicts that an economic integration agreement will reduce the likelihood of trade ceasing and will increase the growth of trade in an active spell. However, the effect will be reversed for spells started after the agreement, which start with somewhat smaller values, but are more likely to cease and grow less. In addition, our model predicts that there will be additional entry of new trade relationships or spells of trade after an agreement is signed. Using revision 1 SITC 5-digit level data in conjunction with Baier and Bergstrand (2007) data on economic integration agreements spanning the period between 1962 and 2005, we empirically confirm all theoretical predictions.

Our results are potentially puzzling. On the one hand, spells active when an agreement begins become longer lasting and larger, while those which start after the agreement are more likely to be shorter and to grow less. These two types of spells exert opposite forces on the aggregate level of trade, the former contributing to the growth of aggregate trade, while the latter potentially contributing to a decrease in aggregate trade. Our results also show that there is a large increase in entry and creation of new trade relationships or spells of trade after the agreement, thus offering a channel for the observed large increases in aggregate trade after an agreement – while new spells may be shorter and grow less, there is such a large number of them after the agreement that their sheer number increases aggregate trade. Thus, we can account for the Baier, Bergstrand, and Feng (2014) result that trade agreements initially increase trade along the intensive margin, followed by an increase on the extensive margin.
References


A Klepper and Thompson Framework

As we mentioned in the paper, most proofs follow directly from the results in Klepper and Thompson (2006). We present them here for completeness.

Preliminary results: We start by characterizing the process that generates buyers in the destination country, \( d \). Suppose \( N(t) \) buyers have been generated by time \( t \). New buyers disappear after some length of period distributed exponentially. So the probability of the \( i^{th} \) buyer still being active at time \( t \) is \( 1 - H(t - t_i) \). Because the arrival of new buyers is distributed according to a Poisson process, the probability that the \( i^{th} \) buyer is still alive at time \( t \) is given by

\[
(A.1) \quad \Pr(\text{buyer } i \text{ is active at } t) = \frac{\int_0^t 1 - H(v)dv}{t}
\]

It follows that, conditional on there being \( N(t) \) buyers, the number of buyers alive at time \( t \), apart from the first, \( n^*(t) \), is binomial:

\[
(A.2) \quad \Pr(n^*(t) = k|N(t)) = \binom{N(t)}{k} \left[ \frac{1}{t} \int_0^t (1 - H(v))dv \right]^k \left[ \frac{1}{t} \int_0^t H(v)dv \right]^{N(t) - k}
\]

Next, recall that \( N(t) \) is distributed Poisson with parameter \( \lambda t \) so the CDF is given by

\[
(A.3) \quad CDF = \sum_{N=k}^{\infty} \frac{(\lambda t)^N e^{-\lambda t}}{N!}
\]
Then the unconditional distribution is

\[ p_k(t) = \sum_{N=k}^{\infty} \frac{(\lambda t)^N}{N!} \binom{N}{k} \left[ \frac{1}{t} \int_0^t (1 - H(v))dv \right]^k \left[ \frac{1}{t} \int_0^t H(v)dv \right]^{N-k} \]  

(A.4)

\[ = \sum_{N=k}^{\infty} \frac{(\lambda t)^N}{N!} \frac{N!}{k!(N-k)!} \left[ \frac{1}{t} \int_0^t H(v)dv \right]^{N-k} \]

\[ = \frac{\lambda^k e^{-\lambda t}}{k!} \left[ \int_0^t (1 - H(v))dv \right]^k \sum_{N=k}^{\infty} \frac{\lambda^{N-k}}{(N-k)!} \left[ \int_0^t H(v)dv \right]^{N-k} \]

(A.5)

We can change variables, \( z = N - k \), to obtain

\[ p_k(t) = \frac{\lambda^k e^{-\lambda t}}{k!} \left[ \int_0^t (1 - H(v))dv \right]^k \sum_{z=0}^{\infty} \frac{\lambda^z}{z!} \left[ \int_0^t H(v)dv \right]^z \]

(A.6)

and using the series expansion \( e^x = \sum_{z=0}^{\infty} \frac{x^z}{z!} \) we can rewrite the expression above as

\[ p_k(t) = \frac{\lambda^k e^{-\lambda t}}{k!} \left[ \int_0^t (1 - H(v))dv \right]^k e^{\int_0^t H(v)dv} \]

\[ p_k(t) = \frac{1}{k!} \left[ \lambda \int_0^t (1 - H(v))dv \right]^k e^{-\lambda \int_0^t (1 - H(v))dv} \]

\[ p_k(t) = \frac{\rho(t)^k}{k!} e^{-\rho(t)} \]

(A.7)

where \( \rho(t) = \lambda \mu \left( 1 - e^{-t/\mu} \right) \). Finally the probability of the first buyer still being alive is \( 1 - H(t) \). With these results in hand, we can write the probability of exactly \( k \) buyers being active at time \( t \) as

\[ \Pi_k(t) = \begin{cases} H(t)p_k(\rho(t)) & k = 0 \\ (1 - H(t))p_{k-1}(\rho(t)) + H(t)p_k(\rho(t)) & k = 1, 2, 3, \ldots \end{cases} \]

(A.8)

where we have shown \( p_k(\rho(t)) \) is the probability of exactly \( k \) events from a Poisson distribution with mean \( \rho(t) = \lambda \int_0^t (1 - H(v))dv \). Because we have assumed \( H(z) \) is exponential with mean \( \mu \) we find \( \rho(t) = \lambda \mu \left( 1 - e^{-t/\mu} \right) \). As \( t \) approaches infinity, the
first market vanishes with probability 1, and the stationary distribution is Poisson with mean $\lambda \mu$.

The number of business relations in a trade spell, excluding the first buyer, is the sum of $n$ Bernoulli trials with probability of success $\theta$ where $n$ is distributed Poisson with mean $\rho(t)$. The distribution of this random sum is

\[
(A.9) \quad p_k(t) = \sum_{n=k}^{\infty} \binom{n}{k} \frac{e^{-\rho(t)}\rho(t)^n}{n!} \theta^k(1-\theta)^{n-k}
\]

which following the same steps as above we can write as

\[
(A.10) \quad p_k(t) = \frac{e^{-\theta \rho(t)}(\theta \rho(t))^n}{n!}
\]

Adding to this the probability $\theta(1-H(t))$ that the business relation with the first buyer is still active at time $t$, we find

\[
(A.11) \quad v_k(t) = \begin{cases} 
(\theta H(t) + (1-\theta))p_k(\theta \rho(t)) & k = 0 \\
\theta(1-H(t))p_{k-1}(\theta \rho(t)) + (\theta H(t) + (1-\theta))p_k(\theta \rho(t)) & k = 1, 2, 3, \ldots
\end{cases}
\]

As $t \to \infty$ the first buyer dies and the stationary distribution is Poisson with parameter $\theta \rho(t)$.

Because we defined the duration of a trade spell as the time that has elapsed since the trade spell became active again, and because buyers die independently of new arrivals, the duration of a trade relation is also independent of new arrivals. Then, the distribution for $w(s(t), t)$, is the same as $v_k(t)$ replacing $t$ by $s$ and ignoring the first buyer.

We are finally ready to proof Result 1. To to this, recall the size of a trade spell is given by $y(t) = \sum_0^{n(t)} r$, where $n(t)$ is a random number following the distribution $w(s(t), t)$ and $r$ is a random draw from the distribution $F(r)$. We can use the result
that the characteristic function of a sum of random variables is equivalent to the multiplication of their characteristic functions. The characteristic function for the unconditional distribution of trade spell sizes is obtained by taking the expectation over all \( n \)

\[
\phi_y(u; s) = E_n[\phi_r(u)^n|s]
\]

\[
= \sum_{k=0}^{\infty} w(s(t), t)\phi_r(u)^k
\]

\[
= \sum_{k=0}^{\infty} e^{-\theta \rho(s)}(\theta \rho(s))^n \frac{n!}{n!} \phi_r(u)^k
\]

\[
= e^{\theta \rho(s)(\phi_r(u)-1)}
\]

To find the expected value we calculate

\[
E[y] = \frac{\partial \phi_y(u; s)}{\partial u} |_{u=0} = \theta \rho(s) \frac{\partial \phi_r(u)}{\partial u} |_{u=0} = E[r] \theta \rho(s)
\]

and to find the variance we calculate

\[
E[y^2] = \frac{\partial^2 \phi_y(u; s)}{\partial u^2} |_{u=0}
\]

\[
= \theta \rho(s) \frac{\partial^2 \phi_r(u)}{\partial u^2} |_{u=0} + \left( \theta \rho(s) \frac{\partial \phi_r(u)}{\partial u} |_{u=0} \right)^2
\]

\[
= \theta \rho(s) \frac{\partial^2 \phi_r(u)}{\partial u^2} |_{u=0} + E[y]^2
\]

From here we find

\[
\text{var}[y] = \theta \rho(s) E[r^2]
\]

Result 1 follows directly from these outcomes.

To show Result 2, we first need a definition and a result. Let \( G_n(\tau|z_1, z_2, \ldots, z_n) \)
denote the distribution of the first passage time, \( \tau \), to a state of zero active business relations for a trade spell with \( n \) business relations of ages \( z_i \). Now add one business relation of age \( z_{n+1} \). By construction, the first passage distribution is given by

\[
G_{n+1}(\tau|z_1, z_2, \ldots, z_n, z_{n+1}) = \frac{H(z_{n+1} + \tau) - H(z_{n+1})}{1 - H(z_{n+1})} G_n(\tau|z_1, z_2, \ldots, z_n) < G_n(\tau|z_1, z_2, \ldots, z_n)
\]

Then, for **Result 3**, we recognize that \( n(t) \) is positively related to duration, \( s(t) \), according to Result 1. Since the size of a trade spell equals the product of \( n(t) \) and the average size of business relations in each trade spell, it is also positively related to \( n(t) \). Duration and size are related to \( n(t) \) in different ways, and thus both will be positively related to \( n(t) \) even conditional on the other. A more direct proof is provided by Klepper and Thompson (2006).

**Result 4** requires one more definition and a result. Let’s define \( G(z; s) \) as the distribution of ages of all the business relations in a trade relation of duration \( s \). In the case of \( H(z) \) exponential, the distribution \( G(z; s) \) is equal to

\[
G(z; s) = \frac{1 - e^{-z/\mu}}{1 - e^{-s/\mu}} \tag{A.16}
\]

which is the exponential \( H(z) \) with the support truncated at \( s \). This is the simplicity afforded by the exponential distribution. The future depends only in the current state of affairs.

With this result in hand, we proceed to calculate the growth rate of a trade spell. Consider a business relation of duration \( z \). Then, the probability that it vanishes in the subsequent period \( T \) is simply given by

\[
\frac{\int_z^{z+T} dH(z)}{1 - H(z)} = 1 - e^{-T/\mu}.
\]
Taking expectations over all possible ages, \( z \in [0, s] \), using the distribution \( G(z; s) \), we find

\[
(A.17) \quad E[\text{Number of lost business relations after interval } T|s] = n(s) \int_0^s \frac{e^{-z/\mu}(1 - e^{-s/\mu})}{\mu(1 - e^{-s/\mu})} dz
\]

where each lost relation has an expected size \( \bar{r}_n \) which is independent of \( n \).

Using the distribution for \( w(s(t), t) \), the expected number of new business relations appearing in the interval of length \( T \) is given by

\[
(A.18) \quad E[\text{Number of new business relations during interval } T|s] = \theta \rho(T)
\]

Each new relation has an expected size \( E[r] \).

We can define the growth rate as the difference between the new arrivals and the losses:

\[
(A.19) \quad g_y(t, t + T; y, s) = \frac{E(y(t + T|s)) - y(t, s)}{y(t, s)}
\]

\[
= \frac{E[r] \theta \rho(T)}{y(t, s)} - (1 - e^{-T/\mu})
\]

\[
= \left( \frac{E[r] \theta \mu}{y(t, s)} - 1 \right) (1 - e^{-T/\mu})
\]

Let’s denote the growth of trade spell that survive the interval time \( T \) as \( g_y(t, t + T; y, s | n(t + T) > 0) \) and the probability of dying as \( \Pr\{n(t + T) = 0|y(t), s(t)\} \). Then, it follows that

\[
(A.20) \quad g_y(t, t + T; y, s) = (1 - \Pr\{n(t + T) = 0|y(t), s(t)\}) g_y(t, t + T; y, s | n(t + T) > 0) + \Pr\{n(t + T) = 0|y(t), s(t)\} (-1)
\]
from where

\[(A.21) \quad g_y(t, t+T; y, s|n(t+T) > 0) = \frac{g_y(t, t+T; y, s) + \Pr\{n(t+T) = 0|y(t), s(t)\}}{1 - \Pr\{n(t+T) = 0|y(t), s(t)\}}\]

We showed that the probability of a trade spell ceasing to exist is decreasing in its size and age, thus mean firm growth decreases with size and age, as described in Result 4.