Expected inflation and other determinants of Treasury yields

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Abstract

Shocks to nominal bond yields can be decomposed into news about expected future inflation, news about expected future real short rates, and expected excess returns—all over the life of the bond. This paper estimates the magnitude of the first component for short and long maturity Treasury bonds. At a quarterly frequency, variances of news about expected inflation accounts for between 10 to 20 percent of variances of yield shocks. This result is robust statistically and stable across time. Standard dynamic models with long-run risk imply corresponding variance ratios close to one. Habit formation models fare somewhat better. The influences of shocks to real rates and expected excess returns cannot be disentangled reliably in the data owing to statistical uncertainty of the persistence of real-rate shocks.

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1 Introduction

A large and expanding literature explores the relation between nominal bond yields and inflation. Ang and Piazzesi (2003) make a particularly important contribution, introducing Gaussian macro-finance dynamic term structure models to determine the compensation investors require to face shocks to inflation and macroeconomic activity. The related literature has quickly branched out to include unspanned macro risks, non-Gaussian dynamics, and fundamental explanations for inflation risk premia that are grounded in investor preferences and New Keynesian macro models.

It is difficult to uncover from this literature any widely accepted conclusions about the joint dynamics of inflation and the nominal term structure. Ang, Bekaert, and Wei (2008) made the same point to motivate their own attempt to produce some basic facts. Research since Ang et al. has not exhibited any tendency to converge on their conclusions or any other set of core results. Thus it is not clear what branches of the macro-finance literature are likely to be fruitful and which should be abandoned.

This paper makes yet another effort to identify a robust empirical property that can be used to guide future research. How much inflation risk is embedded in nominal Treasury bonds? The term “inflation risk” is a little loose. The definition used here is derived from an accounting identity rather than from any assumptions about dynamics or risk premia. Inflation risk for a bond over a horizon such as a month or a quarter is the fraction of the variance of innovations in the bond’s yield that is attributable to news about expected average inflation over the life of the bond. This fraction can be computed for all models that specify the joint dynamics of inflation and bond yields. Therefore if we can agree on a range of estimates of inflation risk as defined here, we can evaluate existing models and construct new ones based on their ability to generate plausible values.

Using 45 years of quarterly inflation expectations from surveys, I conclude that shocks to expected inflation are a small part of shocks to the nominal bond yields. Approximately 10 to 20 percent of variances of quarterly shocks to Treasury bond yields are attributable to news about expected inflation over the life of the bond. This conclusion holds for a wide range of maturities, is robust statistically, and holds across subsamples. In particular, estimates of inflation risk for a sample that ends with Greenspan’s arrival at the Federal Reserve are similar to those for a sample that begins with his arrival.
This magnitude of inflation risk is strongly at odds with values implied by standard equilibrium models of inflation and bond yields. Long-run risk models imply variance ratios close to one at both short and long maturities. These models do not generate either volatile, persistent short-term real rates or volatile, persistent expected excess bond returns. Thus in these models, inflation shocks must dominate. Habit formation models perform better. For short maturities, implied variance ratios are a little more than a half. For long maturities, implied ratios are close to the estimates here. In these models, variations in surplus consumption can drive persistent fluctuations in both short-term real rates and risk premia.

The properties of the data that underlie the estimates are easy to summarize. Although expectations of future inflation are highly persistent, shocks are small. For example, estimates of the standard deviation of quarterly shocks to average expected inflation over a ten-year horizon are in the neighborhood of 20 basis points. Estimates of the standard deviation of quarterly shocks to the ten-year bond yield are around 50 basis points. Squaring and dividing produces a variance ratio estimate close to 15 percent.

Thus mechanically, innovations to expected short-term real rates and term premia are the primary drivers of yield shocks. There is insufficient information in the data to disentangle the relative contributions of these two components. Again, the relevant properties of the data are easy to summarize. Shocks to short-term real rates are large, and long-term nominal yields covary strongly with them. If short-term real rates are highly persistent, then the variation in long-term yields is explained by shocks to average expected future short-term real rates. If short-term real rates die out quickly, the variation is explained by term premia that positively covary with short-term real rates. Point estimates of the persistence are consistent with the latter version, but statistical uncertainty in these estimates cannot rule out the former version.

The next section describes how I measure the quantity of inflation risk and discusses in detail the survey data used to construct shocks to inflation expectations. Section 3 documents the low level of inflation risk in bond yields. Section 4 summarizes properties of inflation risk in various macro-finance equilibrium models. Section 5 attempts to determine the relative roles of news about expected future short rates and news about expected future returns. Section 6 considers (and rejects) some possible objections to the modeling framework, such as informationally-sticky survey forecasts.
2 Defining inflation risk

Inflation risk is not a clearly defined concept. There is no unique or best way to measure inflation risk in nominal bonds. The primary ambiguity is that shocks to yields cannot be divided cleanly between those that are associated with shocks to the path of expected inflation and those that are not.

Rather than adopt a specific model’s decomposition of shocks into inflation and non-inflation components, I measure inflation risk with an accounting approach that has its roots in the dividend/price decomposition of Campbell and Shiller (1988), as extended to returns by Campbell (1991). The measure is straightforward to estimate with available data. Any dynamic model of both inflation and bond yields—a class of that includes almost all dynamic macro models—implies a value of this measure of inflation risk. The main use of this measure is to shed light on the ability of these models to fit the joint dynamics of inflation and yields.

2.1 An accounting identity

I closely follow Campbell and Ammer (1993), who decompose unexpected bond returns into news about future real rates, news about future inflation, and news about future excess returns. The only mechanical difference is that I examine innovations in yields rather than innovations in returns. However, as Section 6.2 discusses, the conclusions I draw about the role of inflation contrast sharply with those of Campbell and Ammer.

Begin with some notation. All yields are continuously compounded and expressed per period. For example, with quarterly periods, a yield of 0.02 corresponds to eight percent per year.

\[ y_t^{(m)}: \text{Yield on a nominal zero-coupon bond maturing at } t + m. \]
\[ \pi_t: \text{Log change in the price level from } t - 1 \text{ to } t. \]
\[ r_t: \text{Ex-ante real rate, the yield on a one-period nominal bond less expected inflation, } r_t \equiv y_t^{(1)} - E_t(\pi_{t+1}). \]
For the moment we will not be precise about how these expectations are calculated. In no-arbitrage, complete-market models the ex-ante real rate differs from the yield on a one-period real bond owing to both a Jensen’s inequality term associated with price shocks and the compensation investors require to face uncertainty in next period’s price level.

The log return to holding an \( m \)-period nominal bond from \( t \) to \( t+1 \) in excess of the log return to a one-period nominal bond is

\[
ex_{t+1}^{(m)} = \left( my_t^{(m)} - (m - 1)y_t^{(m-1)} \right) - y_t^{(1)}. \tag{1}
\]

An accounting identity decomposes the \( m \)-maturity yield into future average inflation, ex-ante real rates, and these excess log returns. It is

\[
y_t^{(m)} = \frac{1}{m} \sum_{i=1}^{m} \pi_{t+i} + \frac{1}{m} \sum_{i=1}^{m} r_{t+i-1} + \frac{1}{m} \sum_{i=1}^{m} ex_{t+i}^{(m-i+1)}. \tag{2}
\]

The accounting identity formalizes observations such as for a given bond yield, higher inflation over the life of the bond must correspond to either lower ex-ante real rates or lower excess returns.

The time-\( t \) expectation of (2) decomposes the \( m \)-period yield into expectations of average inflation, average ex-ante real rates, and average excess returns over the life of the bond:

\[
y_t^{(m)} = \frac{1}{m} \sum_{i=1}^{m} E_t (\pi_{t+i}) + \frac{1}{m} \sum_{i=1}^{m} E_t (r_{t+i-1}) + \frac{1}{m} \sum_{i=1}^{m} E_t \left( ex_{t+i}^{(m-i+1)} \right). \tag{3}
\]

In (3), the third sum on the right is often described as the bond’s term premium. Again, we will not yet be precise about how the expectations in (3) are calculated. The accounting identity puts no structure on the term premium. In a frictionless no-arbitrage setting the term premium is determined by the risk premium investors require to hold the bond and a Jensen’s inequality component associated with the log transformation. In models with frictions the term premium may also include a safety or convenience component.

Using this accounting framework, express the innovation in the \( m \)-maturity yield from \( t-1 \) to \( t \) as the sum of news about expected average inflation, ex-ante real rates, and excess
returns. Denote the news by

$$\tilde{y}_t^{(m)} \equiv y_t^{(m)} - E_{t-1}y_t^{(m)},$$

$$\eta_{\pi,t}^{(m)} \equiv E_t \left( \frac{1}{m} \sum_{i=1}^{m} \pi_{t+i} \right) - E_{t-1} \left( \frac{1}{m} \sum_{i=1}^{m} \pi_{t+i} \right),$$

$$\eta_{r,t}^{(m)} \equiv E_t \left( \frac{1}{m} \sum_{i=1}^{m} r_{t+i-1} \right) - E_{t-1} \left( \frac{1}{m} \sum_{i=1}^{m} r_{t+i-1} \right),$$

$$\eta_{ex,t}^{(m)} \equiv E_t \left( \frac{1}{m} \sum_{i=1}^{m} e_{x_{t+i}}^{(m-i+1)} \right) - E_{t-1} \left( \frac{1}{m} \sum_{i=1}^{m} e_{x_{t+i}}^{(m-i+1)} \right).$$

(4)

A yield shock is then the sum of news, or

$$\tilde{y}_t^{(m)} = \eta_{\pi,t}^{(m)} + \eta_{r,t}^{(m)} + \eta_{ex,t}^{(m)}. \quad (5)$$

This is an accounting rather than a structural decomposition of yield shocks. No reasonable macro-finance model characterizes the three shocks on the right side of (5) as exogenous, fundamental shocks.

The payoff to this decomposition lies in the associated variance decomposition. The unconditional variance of yield innovations is the sum of the unconditional variances of the individual components on the right side of (5) and twice their unconditional covariances:

$$\text{Var} \left( \tilde{y}_t^{(m)} \right) = \text{Var} \left( \eta_{\pi,t}^{(m)} \right) + \text{Var} \left( \eta_{r,t}^{(m)} \right) + \text{Var} \left( \eta_{ex,t}^{(m)} \right)$$

$$+ 2 \text{Cov} \left( \eta_{\pi,t}^{(m)}, \eta_{r,t}^{(m)} \right) + 2 \text{Cov} \left( \eta_{\pi,t}^{(m)}, \eta_{ex,t}^{(m)} \right) + 2 \text{Cov} \left( \eta_{r,t}^{(m)}, \eta_{ex,t}^{(m)} \right). \quad (6)$$

Divide (6) by the variance on the left to express the fraction of yield variance explained, in an accounting sense, by news about inflation, real rates, and expected excess returns. The component at the center of this paper’s empirical analysis is a measure of inflation risk, the ratio of two variances:

$$\text{inflation risk} \equiv VR_{\pi}^{(m)} = \frac{\text{Var} \left( \eta_{\pi,t}^{(m)} \right)}{\text{Var} \left( \tilde{y}_t^{(m)} \right)}. \quad (7)$$
2.2 Measuring inflation expectations

The relative shares of the variance decomposition (6) depend on the way expectations are formed. For example, two investors that disagree about the persistence of inflation shocks will also disagree about the magnitude of inflation risk (7), even though (3) holds for both sets of beliefs. Thus there is no “true” value of (7). As economists, we are interested in the magnitude of inflation risk as perceived by sophisticated investors as well as implied by the forecasts of the most accurate econometric models.

Conveniently, these two types of inflation forecasts do not noticeably differ. Like many other researchers beginning with Pennacchi (1991), I use consensus inflation forecasts from surveys of market practitioners. These are close in spirit to the subjective expectations of a sophisticated investor, although no one agent may have these exact expectations. Moreover, substantial research concludes that econometric models are not more accurate than consensus survey forecasts. Ang, Bekaert, and Wei (2007) document that survey forecasts are more accurate than model-based forecasts constructed using the history of inflation and other non-survey information. In addition, they find no evidence that using realized inflation in addition to survey forecasts helps reduce survey-based forecast errors. Faust and Wright (2009) and Croushore (2010) draw the same conclusion. Chernov and Mueller (2012) cannot reject the hypothesis that the subjective probability distribution of future inflation, as inferred from surveys, equals the true probability distribution. In a comprehensive handbook chapter, Faust and Wright (2012) concur: “...purely judgmental forecasts of inflation are right at the frontier of our forecasting ability.”

Because of the difference between point-in-time price levels and average-over-time price levels, survey forecasts do not match up exactly with expectations in the accounting decomposition of Section 2.1. In the decomposition, expected inflation over the life of a bond is the expected log change in the price level from the date at which a bond’s yield is observed to the date on which the bond matures. (This change is divided by the bond’s maturity.) This is a log change in point-to-point prices. Forecasting in practice focuses on time-averaged inflation. For example, the Survey of Professional Forecasters tracks forecasts of the GDP Price Index in future calendar quarters. This price index is an average of prices in the quarter. The Blue Chip Survey has forecasts of the quarter-to-quarter percentage change in the CPI, where quarterly CPI is defined as the average of the three monthly CPI values in the quarter.
Figure 1 contrasts the theory and the practice. At date $T_1$ in 1997Q2, agents make predictions of the one-year bond yield at $T_2$, a date in 1997Q3 that is precisely three months after $T_1$. At $T_1$ agents also make predictions about average inflation during the life of the bond. The bond matures at $T_3$ in 1998Q3, precisely one year after $T_2$. Therefore agents at $T_1$ forecast the log change in prices from $T_2$ to $T_3$. This time span is indicated in the figure with a line from $T_2$ to $T_3$. At $T_2$, the bond yield is realized and agents have updated predictions about inflation during the life of the bond. The difference between the inflation predictions at $T_2$ and $T_1$ is the news about expected inflation, as defined in (4).

Researchers do not observe these $T_2$-to-$T_3$ inflation forecasts. In practice, surveys taken at dates $T_1$ and $T_2$ ask for predictions of quarter-to-quarter inflation for calendar quarters 1997Q4 through 1998Q3. Combining these calendar-quarter predictions creates forecasts of the log change from average prices during 1997Q3 to average prices during 1998Q3. The line in the figure labeled “Practice” connects the average of 1997Q3 to the average of 1998Q3.

Inflation news as defined by the theory will differ from inflation news as measured using surveys. The primary reason is that none of point-to-point change in prices has been realized at $T_2$, but part of the average-to-average change has been realized at $T_2$. Therefore investors at $T_2$ know more about the average-to-average measure of inflation than they do about the point-to-point measure.

A stark example is helpful. Imagine that expected inflation is always five percent per year. Variability in realized inflation is due entirely to permanent shocks to prices that are completely unforecastable. Then there is no news revealed between $T_1$ and $T_2$ about the expected log price change from $T_2$ to $T_3$. However, at date $T_2$ agents know the price-level shocks that have been realized since the beginning of 1997Q3. If these shocks are, on average, positive (negative), agents will predict an inflation rate from 1997Q3 to 1998Q3 that is greater (less) than five percent.

This effect of partially realized inflation raises the measured volatility of news about expected inflation relative to the true volatility. However, the effect is unlikely to be large even for short horizons. At long horizons it is even less important, since the distortion arises for only part of the first quarter of inflation news.
2.3 The survey data

Inflation forecasts are from two Blue Chip (BC) surveys and the Survey of Professional Forecasters (SPF). Respondents to the BC Economic Indicators Survey (EI) and the BC Financial Forecasts Survey (FF) predict CPI inflation. Respondents to the SPF forecast the GDP price index. The BC data are monthly beginning with March 1980 for BC-EI and July 1984 for BC-FF. The SPF data are quarterly beginning in 1968Q4. The data samples used here run through 2013. The BC consensus forecasts are means across respondents. The SPF consensus forecast is the mean across the respondents, dropping outliers.¹

Panel A of Table 1 illustrates the construction of inflation news using BC-FF survey responses. (The dates in this example correspond to those in Figure 1.) Consensus forecasts in the table are drawn from surveys published at the beginning of June and September 1997. The survey responses are gathered at the end of the May and August respectively, which are the dates reported in the table. The May-end survey has consensus forecasts of quarter-to-quarter inflation for five future quarters, from 1997Q3 through 1998Q3. The August-end survey has consensus forecasts for the five future quarters 1997Q4 through 1998Q4. The table reports consensus forecasts for the four future quarters that the surveys have in common.

These quarter-to-quarter forecasts are used to calculate, for each survey, the expected annualized log change in prices from 1997Q3 to each of the four quarters 1997Q4 through 1998Q3. The change from the May-end to August-end forecasts is defined as news about expected average inflation. In this example, forecasts of future average inflation drop by about 30 basis points at the one-quarter horizon. The magnitude of the decline shrinks to about 20 basis points at the four-quarter horizon.

Survey forecasts are limited in the cross section. The BC-FF surveys have forecasts of at most five future quarters. Therefore the longest-horizon quarterly news that can be constructed is for a four-quarter horizon, as in Table 1. BC-EI forecasts are available intermittently for horizons up to seven quarters ahead. The SPF has forecasts for only four future quarters, thus the longest-horizon quarterly news is for a three-quarter horizon.

Figure 2 displays monthly realizations of quarterly news about average expected inflation produced using the BC surveys.² The data are for the BC-FF survey, augmented by the

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¹I follow the procedure of Bansal and Shaliastovich (2013) to discard outliers from the SPF. Data limitations prevent me from dropping BC outliers.

²The displayed realizations contain overlapping information because of the monthly frequency and quar-
BC-EI survey for news realizations that cannot be produced with the BC-FF survey. It is worth emphasizing that these realizations, which are available for horizons up to six quarters, are model-free. A glance at the figure reveals two unsurprising patterns: news is positively correlated across forecast horizons and the magnitude of news declines with the forecast horizon.

Quarterly realizations of quarterly news about average expected inflation from the SPF are displayed in Figure 3. As in Figure 2, these realizations are model-free. The long time series reveals that the volatility of news about expected inflation was substantially larger prior to the appointment of Greenspan in 1987Q3 than after.

The mean of squared news shocks in any panel of these figures is a sample estimate of the numerator of the inflation risk measure (7) for a bond maturity equal to the panel’s horizon. We now turn to calculating shocks to short-maturity yields in order to calculate the denominator of the measure.

### 2.4 Yield data

The one-quarter yield is from the Federal Reserve Board’s H15 release. Yields on zero-coupon bonds with maturities from two to six quarters are produced by Anh Le as described in Le and Singleton (2013). I use both month-end yields and mid-month yields, depending on whether the yields are to be matched with BC forecasts or SPF forecasts. All yields are continuously compounded and expressed at an annual rate.

### 2.5 Estimating yield innovations

Shocks to bond yields as defined by (4) are realizations less one-period-ahead predictions. Survey forecasts of Treasury yields are available for a variety of maturities. Like inflation forecasts, they are forecasts of average values within a quarter. Unlike inflation forecasts, they are not useful here. Although the inflation forecasts are forecasts of averages over time, the forecasts themselves are made at specific dates. In the example of Figure 1, the two dates

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3 Thanks very much to Anh for sharing the data.

4 For comparison with the quarterly data of the SPF, yields are observed on the 15th of the second month in the quarter. If the 15th is not a trading day, yields are observed on the last trading day prior to the 15th.
are \( T_1 \) and \( T_2 \). Therefore the yield shocks must also be measured as date-to-date innovations. Otherwise the left and right sides of (5) are not aligned.

In the absence of survey forecasts, I construct innovations using off-the-shelf forecasting techniques. A long literature dating back to at least Campbell and Shiller (1991) concludes that the shape of the term structure helps predict future short-maturity yields. I form forecasts of future short-maturity yields by projecting them on the short end of the term structure using

\[
y_{t+1}^{(m)} - y_t^{(m)} = b_{0,m} + b_{1,m}' \begin{pmatrix} y_t^{(1)} & y_t^{(4)} & y_t^{(6)} \end{pmatrix}' + \tilde{y}_{t+1}^{(m)}.
\]

In (8), both time and maturities are measured in quarters. The parameters \( b_{0,m} \) and \( b_{1,m} \) are a scalar and a length-three vector respectively. In words, quarterly changes in bond yields are predicted using yields on one-quarter, four-quarter, and six-quarter bonds. The residuals are the innovations.

One way to implement (8), although not the most efficient, is with ordinary least-squares (OLS). Panel B of Table 2 illustrates the construction of short-maturity yield innovations. The first line reports end-May yields for bonds with maturities between one and four quarters. Equation (8) is estimated with OLS over the sample for which BC surveys are available. In-sample fitted one-quarter-ahead forecasts as of the end of May 1997 are reported in the second line of Panel B. Realized end-August yields are also reported, and the corresponding fitted innovations in yields.

Panel B reports that short-maturity yields were somewhat higher in August than anticipated in May, with yield innovations ranging from 3 to 30 basis points. Equation (5) expresses these innovations as the sum of news about expected future inflation, expected future short-term real rates, and expected excess returns. Panel A shows that the news about expected future inflation is around negative 25 basis points at these maturities. Therefore expected future real rates and/or expected excess returns must have increased unexpectedly. Section 5 contains a model that attempts to disentangle these possibilities.

Figure 4 displays monthly realizations of quarterly innovations in yields. The bond maturities correspond to the news about average inflation from BC surveys in Figure 2. Figure 5 displays quarterly realizations of quarterly innovations in yields. These correspond to news about average inflation from the SPF surveys in Figure 3.

A glance at the scales of the vertical axes of these four figures reveals the main result of this paper. The typical magnitude of yield innovations is more than twice as large as the
typical magnitude of corresponding news about average expected inflation. An immediate
implication is that the measure of inflation risk — the ratio of the two variances — is less than
a quarter. The next section provides more precise evidence, extends the analysis to longer
maturities, and evaluates statistical significance. But nothing that follows is surprising, given
the evidence of Figures 2 through 5.

3 Measuring inflation risk

This section estimates the inflation risk measure (7) at both short and long horizons. Survey
data allow the model-free construction of news about average expected inflation over short
horizons. Thus the short-horizon estimates are little more than variance calculations using
the realizations plotted in Figures 2 through 5. These are discussed in Section 3.1. Longer-
horizon forecasts require a dynamic model of inflation expectations. A simple model drawn
from the literature on inflation expectations is used in Section 3.2 to estimate inflation risk
at multi-year horizons.

3.1 Short-horizon forecasts

A few maintained assumptions allow us to estimate standard errors for inflation risk. For
short-horizon forecasts drawn from Figures 2 through 5, we use four. First, assume consensus
forecasts from surveys are valid expectations of point-to-point quarterly inflation. This is a
convenient approximation to reality. Section 2.2 explains the imperfect mapping from survey
forecasts to point-to-point inflation. The term “valid” is purposely imprecise. It is reasonable
to think of consensus forecasts as expectations of researchers examining historical data.
Researchers know they cannot improve systematically on the consensus forecasts. However,
consensus means are sample means rather than common forecasts of investors. Sections 3.2
and 5.1 relax this assumption by allowing for measurement error, such as sampling error, in
consensus forecasts.

Second, assume bond yields are measured without error. Again, this is a convenient
approximation. All but the one-quarter yield is interpolated from yields on coupon bonds.
Bekaert, Hodrick, and Marshall (1997) estimate that, for maturities around one year, stan-
dard deviations of interpolation measurement error are in the range of eight to nine basis
points. Since the focus here is on quarterly shocks to yields, the relative contribution of such measurement error is relatively small. For parsimony, it is ignored entirely in this exercise. Measurement error in yields is introduced in Sections 3.2 and 5.1.

Third, innovations to yields are assumed to satisfy the forecasting equation (8). Fourth, the innovation for maturity $m$ is assumed to be homoskedastic and jointly normally distributed with contemporaneous news about expected inflation over the horizon $m$. Overwhelming evidence, including the patterns of shocks in Figures 2 through 5, documents stochastic volatility of both yields and inflation.\(^5\) Volatilities of both inflation news and yield shocks are higher in the first part of the sample than they are in the second. There appear to be two volatility regimes, split roughly at the time of Greenspan’s arrival at the Fed in the third quarter of 1987. Split-sample results that assume homoskedasticity will be more convincing than full-sample results that make the same assumption.

Table 2 reports results of maximum likelihood (ML) estimation of this simple model for the Blue Chip surveys of CPI inflation. Only a subset of the Blue Chip data are used. Although the data are monthly, estimation uses only surveys from the end of the second month of each quarter (the surveys published at the beginning of March, June, September, and December) to avoid overlapping observations. Because of the sparseness of data for five-quarter and six-quarter horizons (see the gaps in Figure 2), results are displayed only for horizons up to four quarters. Estimation is performed separately for each pair of sample period and horizon.

The most important message to take from Table 2 is that inflation risk, as defined by (7), is small. For both the full sample and the pre-Greenspan sample, the largest point estimate is only 0.2, and all of the ML asymptotic standard errors are tight. The only exceptions to this pattern are driven by the financial crisis. For the post-1987 sample, inflation risk for the one-quarter and two-quarter horizons is higher, exceeding 0.6 at the one-quarter horizon. This sample had the post-Lehman negative spike of inflation expectations and the ensuing active constraint of the zero lower bound (see Figures 2 and 4 respectively). Yet even these effects disappear for horizons greater than two quarters, where inflation risk remains below 0.2 for this more recent sample.

\(^5\)The relevant literature is vast. A relatively early contribution is Schwert (1989). He estimates the variability and persistence of conditional volatility for a variety of important financial and macroeconomic time series.
Table 3 reports similar results for the Survey of Professional Forecasters. Recall that for this survey the measure of inflation is the GDP price index. Here the maximum horizon is three quarters. Although the inflation measure is different, the results are consistent with the Blue Chip CPI survey results. All estimates for the two-quarter and three-quarter maturities are less than 0.2. As with Blue Chip forecasts, the outlier is the one-quarter horizon for the post-1987Q3 sample. Yet even the estimate for this maturity and sample is less than a third. (A comparison of Figures 2 and 3 reveal that the negative spike in the GDP inflation forecast is smaller than the corresponding spike in the CPI forecast.)

It is worth emphasizing that although news about expected future inflation is much more volatile in the pre-Greenspan sample than later, estimates of inflation risk are noticeably lower in the pre-Greenspan sample. The measure of inflation risk is a variance ratio, and yield innovations were relatively more volatile in the pre-Greenspan sample. Put differently, the higher volatility in inflation expectations in the pre-Greenspan sample was accompanied by higher volatility of expected future real rates, expected future excess returns, or both.

### 3.2 Longer-horizon estimates

Survey data on long-horizon expectations of inflation are sparse. Blue Chip and SPF participants are occasionally asked to predict inflation at the five to ten year horizon, but neither the frequency of the responses nor the precision of the inflation horizon is suitable for the exercise here. Therefore empirical implementation of (3) for bond maturities at horizons greater than a year requires a model.

The modeling approach here is motivated by a conclusion of Faust and Wright (2012). They describe a method that produces intermediate-range inflation forecasts “close to the frontier of predictive performance.” Simply use a glide path to connect survey forecasts of current inflation to survey forecasts of distant inflation. A corollary to their conclusion, verified below, is that longer-horizon forecasts can be extrapolated from the glide path on which short-horizon forecasts lie.

A trend-cycle model captures the intuition of the glide-path approach. The model assumes a unit root in inflation, consistent with models of inflation such as Stock and Watson (2007) and Cogley, Primiceri and Sargent (2010). Inflation is the sum of a random walk
component and a transitory component,

\[ \pi_t = \tau_t + \varphi_t, \quad (9) \]

\[ \tau_t = \tau_{t-1} + \xi_t, \quad E_{t-1}(\xi_t) = 0, \ Var_{t-1}(\xi_t) = \sigma_\xi^2, \quad (10) \]

\[ \varphi_t = \theta \varphi_{t-1} + \nu_t, \quad E_{t-1}(\nu_t) = 0, \ Var_{t-1}(\nu_t) = \sigma_\nu^2. \quad (11) \]

I follow Nason and Smith (2014) by assuming shocks are iid. Other trend-cycle models appear in the literature. Stock and Watson (2007) use a variant of (9) through (13) in which the transitory shocks are not persistent and volatilities vary over time. Cogley, Primiceri and Sargent (2010) allow the persistence of transitory shocks to vary over time. I use (9) through (11) because of its simplicity. Concerns about stability of parameters, including volatilities, are addressed through subsample estimation.

Combine this inflation model with a simple description of a long-term bond yield. A literature beginning with Duffee (2002) documents a random walk model of long-term bond yields produces more accurate forecasts than many formal term structure models. I therefore assume that

\[ y^{(m)}_t = y^{(m)}_{t-1} + \hat{y}^{(m)}_t \quad (12) \]

for some long maturity \( m \).

In this model both inflation expectations and long-term yields have unit roots. This raises the question of cointegration. The earliest comprehensive empirical analysis of cointegration among nominal yields is in Campbell and Shiller (1987). If both yields and inflation have unit roots, but they are not cointegrated, then either real rates or term premia must also have a unit root. Rose (1988) is the first to use cointegration logic to study the properties of real rates.

Research in the applied cointegration literature typically produces a double negative: we cannot reject the hypothesis that inflation and nominal yields are both nonstationary and not cointegrated. Examples include Lardic and Mignon (2004) and Hjalmarsson and Österholm (2010). In their critical review of the literature, Neely and Rapach (2008) conclude that “...studies [of real rates] often report evidence of unit roots, or—at a minimum—substantial persistence.”

\[ ^6 \text{Neely and Rapach note that it is hard to tell whether shocks to real rates are persistent or whether the} \]

\[ ^6 \text{Neely and Rapach note that it is hard to tell whether shocks to real rates are persistent or whether the} \]
In line with this evidence, I do not impose cointegration. The covariance matrix of shocks to inflation expectations and the shock to an \( m \)-maturity yield is

\[
(\xi_t, v_t, \tilde{y}_t^{(m)})' \sim MVN(0, \Omega'), \quad \Omega = \begin{pmatrix}
\Omega_{11} & 0 & 0 \\
\Omega_{21} & \Omega_{22} & 0 \\
\Omega_{31} & \Omega_{32} & \Omega_{33}
\end{pmatrix}.
\]

(13)

Forecasts of future inflation are formed with

\[
E_t(\pi_{t+j}) = \tau_t + \theta^j \varphi_t.
\]

The innovation from \( t \) to \( t + 1 \) in the expectation of inflation in period \( t + j \) is

\[
(E_{t+1} - E_t) \pi_{t+j} = \xi_{t+1} + \theta^{j-1} v_{t+1}.
\]

Similarly, the innovation from \( t \) to \( t + 1 \) in the expectation of average inflation from \( t + 2 \) to \( t + m + 1 \) is

\[
(E_{t+1} - E_t) \frac{1}{m} \sum_{i=1}^{m} \pi_{t+i+1} = \xi_{t+1} + \frac{1}{m} \beta \left( \frac{1 - \beta^m}{1 - \beta} \right) v_{t+1}.
\]

(14)

Variance risk is measured by the variance of (14) divided by the variance of the yield innovation in (13).

I estimate this model using observations of survey forecasts of inflation and a ten-year bond yield. The Kalman filter is a natural tool to apply because the two inflation components are unobserved. The observables are survey forecasts of inflation from the current quarter (nowcasts) up to the maximum available horizon. This maximum is seven quarters for the Blue Chip survey and four quarters for the Survey of Professional Forecasters. The Kalman filter is a straightforward method to handle the many missing observations. As with the short-horizon estimates in Table 3, Blue Chip survey forecasts are taken from the end of the second month of each quarter.

A stochastic singularity problem arises if the bond’s yield and all survey forecasts are assumed to be measured without error. There are only three state variables and more than three observables. Therefore I assume all observables are contaminated by measure-conditional mean of the real-rate process changes periodically.
ment error. The measurement error for a particular observable—say, three-quarter-ahead forecasts—is assumed to be iid. The standard deviations of observables’ measurement errors are free parameters.

Estimation results are in Table 4. These long-horizon results are largely consistent with the short-horizon results in Table 2 and 3. Point estimates of inflation risk are all less than 0.2. Asymptotic standard errors, which rely on the homoskedasticity assumption, are tight. One difference between these results and short-horizon results is that here, estimates of inflation risk in the pre-Greenspan sample are somewhat lower than in the post-Volcker sample.

It is important to verify that the long-run inflation forecasts from this dynamic model are accurate, in the sense that they capture investor expectations of long-run inflation. The solid line in Figure 6 displays filtered values of the random walk component of CPI inflation. The estimates are taken from the full-sample estimates. The circles are semiannual Blue Chip survey forecasts of CPI inflation over the period beginning five years and ending ten years from the survey date. These data are unavailable prior to 1984. The x’s are survey forecasts of long-run GDP inflation rather than CPI inflation.

A glance at the figure reveals two key features of the data. First, the two sets of forecasts closely correspond. The largest differences between the forecasts occur during the financial crisis, when model’s forecasts are more volatile than are the survey forecasts. Second, the outside of the monetarist experiment period of Volcker, these forecasts are smooth over time. In the post-Volcker period the standard deviation of quarterly shocks is in the neighborhood of ten basis points. Yet as Table 4 documents, the standard deviation of quarterly shocks to the ten-year yield is close to 40 basis points. Shocks to inflation are more volatile earlier in the sample, but as Table 4 shows, so are shocks to the ten-year yield.

The main conclusion of this section is that for maturities greater than three months, inflation risk accounts for at most a fifth of the overall variance of shocks to bond yields. This conclusion naturally leads to two follow-up questions. First, is this result interesting? Second, what—since it is not inflation news—drives bond yields?

\footnote{Values from subsample estimation are almost identical to those displayed.}
4 Inflation risk in workhorse dynamic models

Most asset pricing research relies on representative-agent consumption-based preferences grounded in either recursive utility or habit formation. This section discusses inflation risk in the context of some standard dynamic models. To briefly summarize, values implied by models with recursive utility are around one. In other words, volatilities of yield shocks roughly equal volatilities of news about average expected inflation. Values implied by habit formation are closer to the empirical estimates in the previous section.

Before getting into details it is helpful to highlight a few statistics from the previous section. Over the sample 1969 through 2013, standard deviations of quarterly shocks to nine-month and ten-year bond yields were about 90 and 50 basis points respectively. Forecasts from the Survey of Professional Forecasters imply that standard deviations of inflation news over the same horizons were about 35 and 20 basis points. Squaring and taking ratios of the latter numbers to the former numbers produces estimates of inflation risk less than 0.2.

4.1 Recursive utility preferences


Table 5 reports model-implied unconditional standard deviations of quarterly shocks to yields and news about expected inflation. The standard deviations are implied by the population properties of parameterized models in each of these papers. The table also reports the associated measures of inflation risk. Excluding Rudebusch and Swanson, all of the variance ratios are around one. We first discuss the papers excluding Rudebusch and Swanson, then complete the analysis by turning to their paper.

The basic problem shared by these papers is that there are no channels for volatile and persistent real rates, nor channels for volatile term premia. The real rate properties are embedded in the long-run risk intuition of Bansal and Yaron (2004). Two key features of the long-run risk approach are (a) variations in expected consumption growth are small but
persistent, and (b) the elasticity of intertemporal substitution (EIS) is high. In combination, these features imply that short-term real rates do not vary much over time. Thus from quarter to quarter, news about expected future ex-ante real rates is small.

Moreover, to the extent that expectations of short-term real rates vary, they go in the wrong direction. The possibility of stagflation is a key feature of these papers. Agents fear states of the world in which long-run expected consumption growth is low and long-run expected inflation is high. This fear generates positive risk premia on nominal bonds. But this same fear generates negative covariances between news about expected real rates and expected future inflation. Bad news about expected consumption growth lowers expected future real rates as investors attempt to save for future bad times. A negative covariance damps the volatility of shocks to nominal yields, raising the ratio of inflation-news volatility above yield-shock volatility: variance ratios in Piazzesi and Schneider all exceed one.

Matching the empirical volatility of yield shocks requires counterfactually high volatility of inflation expectations. Estimation splits the difference. Table 5 reports that the model of Piazzesi and Schneider produces population standard deviations of yield shocks and inflation news that are too low and too high, respectively.\(^8\)

The variance ratios implied by the model of Bansal and Shaliastovich (2013) are somewhat smaller than those of Piazzesi and Schneider because they include time-varying conditional variances. Risk premia change with changes in conditional covariances between expected consumption growth and yield shocks. Time-varying conditional variances also produce news about expected future short-term real rates through the precautionary savings channel. Finally, they induce time-varying convexity in yields, which shows up in the expected excess log returns component of the yield decomposition (3).

Nonetheless, this channel does not produce measures of inflation risk much lower than one. Two constraints limit the influence of time-varying conditional variances on the decomposition (6). First, neither risk premia nor convexity are important at short maturities. Second, the effect of variance shocks on risk premia is proportional to average yield spreads,\(^8\) The model of Piazzesi and Schneider examined here has homoskedastic shocks to aggregate consumption and inflation. They also study a model with learning, in which shocks (according to the investors’ filtration) are heteroskedastic. In the learning model the economy converges to full information in the long run. Since I study population properties of models here, I skip examination of their learning model. The parameters underlying the model calculations are from the model described as “large information set.” The calculations were made easier because the authors graciously provide their Matlab code on Monika’s web site.

\(^8\)
and average spreads are small. (As an extreme example, if investors were risk-neutral, changes in conditional variances would have no effect on risk premia.) The mathematics of this argument are in the paper’s appendix. The following example gives the intuition.

Assume that at quarter $t$, the yield spread between the ten-year yield and the one-year yield is 140 basis points. This is the sample mean from 1969 to 2013. Also assume that at $t$, all conditional variances are at their unconditional means. Then at $t + 1$, all conditional variances unexpectedly increase to 110 percent of their unconditional means. Assume investors at $t + 1$ anticipate this increase to fully persist for at least ten years.

Investors at $t + 1$ demand higher expected excess bond returns than they demanded at $t$. Expected excess returns are proportional to conditional covariances, thus they also jump to 110 percent of their unconditional means, and are expected to remain at this higher level for at least ten years. To provide investors with the required higher expected excess return, the yield spread must increase by one-tenth of its unconditional mean, or 14 basis points. Thus this large change in conditional variances produces a small change in the ten-year yield.

As with Piazzesi and Schneider, Bansal and Shaliastovich’s estimates split the difference in fitting volatilities of inflation news and volatilities of yield shocks. Table 5 reports the counterfactually high and values for the former and latter, respectively.

Kung (2015) endogenously generates stagflation in a New Keynesian model with investment. Positive shocks to productivity raise investment, raise expected productivity growth, and lower marginal costs so that monopolistically competitive firms cut prices in order to capture business. Relative to an endowment economy, the endogenous-capital economy has much higher short-term volatility of short-term real rates and lower persistence of real-rate shocks. The positive shock to productivity initially lowers consumption because investment demand is high. Expected consumption growth is steep for a few quarters (thus high real rates), then tapers off and turns negative. As in Piazzesi and Schneider, this short-term increase in real rates doesn’t contribute to the volatility of nominal yields because it is negatively correlated with news about expected inflation.

As with the other papers, Kung’s estimated model cannot simultaneously fit the volatilities of inflation news and yield shocks, thus it splits the difference. In Table 5, the former are too high and the latter are too low relative to empirical estimates. The net result is very high values of inflation risk—variance ratios well above one—for all maturities.

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9This exercise was possible because Howard graciously provided his Dynare++ code.
Rudebusch and Swanson (2012) also endogenously generate stagflation in a New Keynesian model. Yet the results in Table 5 appear to contrast sharply with those of Kung. The explanation lies in the differing methodologies used to parameterize the models. Rudebusch and Swanson calibrate their model without attempting to fit the behavior of bond yields. With their parameters, volatilities of news about expected future short-term real rates and expected future excess returns are small, as they are in the other models. But in their model, volatilities of news about expected future inflation are also small. Thus the model generates extremely small volatilities of yield shocks. Table 5 reports quarterly standard deviations of yield shocks around 25 to 30 basis points, depending on maturity. As noted at the beginning of this section, empirical counterparts are around 50 to 90 basis points. Thus the results of Rudebusch and Swanson are consistent with the conclusion that in recursive utility models, high yield-shock volatility cannot be generated without some mechanism to produce correspondingly high inflation-news volatility.

4.2 Habit formation

Asset-pricing investigations using habit formation usually follow the path of Campbell and Cochrane (1999) of emphasizing the role of surplus consumption dynamics. Surplus consumption is a state variable that, depending on the model specification, can affect both the short-term risk-free rate and prices of risk. Campbell and Cochrane’s work focuses on real prices and interest rates. Wachter (2006) adds a homoskedastic inflation process, while Ermolov (2015) adds heteroskedastic shocks to both consumption and inflation.

Table 6 reports model-implied unconditional standard deviations of quarterly shocks to yields and news about expected inflation. The standard deviations are implied by the population properties of parameterized models in each of these papers. The table also reports the associated measures of inflation risk. Both models produce variance ratios of about 0.6 at short maturities and less than 0.2 at long maturities. Thus the models generate measures of inflation risk for ten-year bonds that are close to empirically-observed measures. At shorter maturities the fit is much worse, although nonetheless better than the fit of recursive utility models.

Habit formation preferences have two related advantages over recursive utility preferences.

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10 Eric graciously produced for me a long time series of data generated by the model’s DGP.

11 Jessica graciously provided her Matlab code and helped me understand some of its features.
First, surplus consumption can drive volatile and persistent fluctuations in real short-term rates. When surplus is high, investors anticipate that surplus will slowly return to its mean. They want to save more to offset expected declining happiness over time. In the aggregate they cannot, and simply push down short-term real rates.

Second, news about expected inflation can be positively correlated with news about expected real rates, rather than negatively correlated as it is with stagflation. This positive correlation magnifies the variability of yield shocks, lowering the magnitude of inflation risk. With habit formation, this correlation is also consistent with an upward-sloped nominal term structure.\textsuperscript{12}

News about expected excess returns can also be large in habit formation models that tie risk pricing to a nonlinear function of surplus consumption. As in Campbell and Cochrane, expected excess returns can sometimes swing substantially with small changes in surplus. For example, in Wachter’s model, about half of the variance in quarterly shocks to the ten-year yield is related to shocks to expected excess returns.

5 A dynamic model of yields and expected inflation

What drives shocks to bond yields? Section 3 confirms that it is not news about inflation, but does not help disentangle other possibilities. In an accounting sense, yields must be driven primarily by news about expected future short-term real rates, news about expected future excess returns, or both. This section attempts to answer the question. To preview the results, there is not enough information in the data to tell.

5.1 The framework

Yield shocks are the sums of news about expected future inflation, news about expected future short-term real rates, and news about expected future excess returns. Section 3.2 uses a dynamic model of short-horizon inflation expectations to infer inflation news. Here

\textsuperscript{12}Assume shocks to surplus are negatively correlated with shocks to inflation and that shocks to inflation are persistent. Then nominal bonds are speculative assets because they tend to increase in value when marginal utility is unexpectedly low. This drives an upward-sloped term structure. Moreover, shocks to surplus are negatively correlated with news about expected future short-term real rates, inducing a negative correlation between news about expected future short-term real rates and news about expected future inflation.
we expand the dynamic model to include short-term and long-term nominal rates. News about expected future short-term real rates is inferred from the model. News about expected future excess returns is produced as a residual. The model necessarily requires much more structure than the models in Sections 3.1 and 3.2.

The dynamics of nominal yields and expected inflation are linked through their joint dependence on a state vector. Denote the length-$n$ state vector by $x_t$. State-space models are standard in the dynamic term structure literature.\(^\text{13}\) The state vector has Gaussian VAR(1) dynamics

$$x_{t+1} = \mu + Kx_t + \Sigma \epsilon_{t+1}, \quad \epsilon_{t+1} \sim MVN(0, I). \quad (15)$$

Nominal yields are affine functions of the state vector. The notation for the yield on an $m$-maturity bond is

$$y_t^{(m)} = A_m + B_m'x_t + \eta_{m,t}, \quad (16)$$

where $\eta_{m,t}$ represents measurement error or some other deviation from an exact affine representation. Similarly, expectations of future inflation are affine functions of the state vector,

$$E_t(\pi_{t+j}) = A_{\pi,j} + B_{\pi,j}'x_t. \quad (17)$$

As with the long-horizon forecasting model in Section 3.2, both bond yields and inflation expectations are observed with measurement error. This breaks the stochastic singularity problem when there are more observables than state variables.

Many researchers use similar frameworks to study the joint dynamics of inflation and bond yields. Notable examples include Campbell and Viceira (2001), who estimate two-factor Gaussian no-arbitrage models of nominal yields and inflation. Ang, Bekaert, and Wei (2008) estimate a four-factor model with time-varying risk premia and an additional factor that captures changes in regimes. Chernov and Mueller (2012) estimate a variety of four-factor and five-factor Gaussian models, and Haubrich, Pennacchi, and Ritchken (2012) estimate a seven-factor model with stochastic volatility.

This paper diverges from this earlier literature in both its objective and its estimation

\(^{13}\)The first application of these models to interest rates is Hamilton (1985), although his motivation differs from that in the dynamic term structure literature.
procedure. The earlier work investigates the risk compensation investors require to face shocks, thus they can put a price on the nominal risk embedded in nominal Treasury bonds. In this model the absence of arbitrage is not imposed. No-arbitrage places restrictions on the coefficients of (16). By itself, the assumption of no-arbitrage is unimportant in this VAR setting. Joslin, Le, and Singleton (2013) show that when risk premia dynamics are not constrained, Gaussian no-arbitrage macro-finance models are close to factor-VAR models such as (15) and (16). No-arbitrage restrictions matter only when they are coupled with restrictions on risk premia dynamics. Such restrictions can certainly affect inference about the variance decomposition (6). But I do not want to inadvertently impose restrictions on (6) that override information in the data.

I break from the usual estimation approach by not including realized inflation among the observables. There are two reasons. First, including realized inflation is unlikely to improve the model’s ability to describe expected inflation. Adding it to the observables simply increases the number of free parameters and raises the likelihood of overfitting. Second, I am not interested in estimating the compensation investors require to face shocks to inflation. If investors are not risk-neutral with respect to the shock $\pi_t - E_{t-1} \pi_t$, then the one-period nominal rate will include a risk premium. This risk premium cannot be pinned down without observing realizations of inflation. However, risk premia are not the focus of this analysis.

Calculations of news about expected future inflation and expected future short-term real rates use standard vector autoregression mathematics. Consider an $m$-period bond for which we have parameters of the affine mapping (16). Shocks to the yield (excluding measurement error) are then

$$\tilde{y}_t^{(m)} = B_m' \Sigma_t.$$

As of time $t$, the average expected value of the state vector from $t$ to $t + m - 1$ is

$$\frac{1}{m} \sum_{j=0}^{m-1} E_t(x_{t+j}) = \text{[constant term]} + \frac{1}{m} (K^0 + K^1 + \ldots K^{m-1}) x_t.$$

In the case of stationary dynamics, this can be written more simply as

$$\frac{1}{m} \sum_{j=0}^{m-1} E_t(x_{t+j}) = \text{[constant term]} + \frac{1}{m} (I - K^m) (I - K)^{-1} x_t.$$
Write this (either the general or stationary version) as

\[ \frac{1}{m} \sum_{j=0}^{m-1} E_t(x_{t+j}) = W_{m,0} + W_{m,1} x_t. \]

Then average expected inflation from \( t + 1 \) to \( t + m \) is

\[ \frac{1}{m} \sum_{j=0}^{n-1} E_t(\pi_{t+1+j}) = A_{\pi,1} + B'_{\pi,1} W_{m,0} + B'_{\pi,1} W_{m,1} x_t \]

and the news at \( t \) about this average expected inflation is

\[ \eta_{\pi,t} = B'_{\pi,1} W_{m,1} \Sigma \epsilon_t. \] (18)

Similarly, news at \( t \) about the average expected real short rate over the life of the bond is

\[ \eta_{r,t} = (B_1 - B_{\pi,1})' W_{m,1} \Sigma \epsilon_t. \]

Term premia shocks are calculated by subtracting shocks to average expected inflation and real rates from the yield shock. Population variances and covariances among these shocks are computed using the population covariance matrix of state-vector shocks (the identity matrix).

### 5.2 The data and estimation details

Statistical inference with this highly-parameterized model is improved with a longer sample. I therefore use only the quarterly consensus forecasts from the SPF, which begin in 1968Q4. The Blue Chip sample is too short to draw any firm conclusions. The final observation used here is 2013Q4, for a total of 181 quarters. Treasury bond yields are observed in the middle of the second month of each quarter, roughly aligned with the SPF forecast dates. Zero-coupon yields are for maturities of three months, one through five years, and ten years. Sources of yield data are described in Section 2.4.

In the model description of Section 5.1, the length of the state vector is not specified. Results presented here all use four latent factors. Versions with three and five factors were
explored, but the results were not sufficiently novel to present. The state vector is latent and thus unidentified. Normalizations are imposed in estimation to eliminate global and local underidentification. Parametric restrictions are imposed to satisfy (17). After normalizations, the four-factor model has 59 free parameters.

The likelihood function is given by the Kalman filter and the parameters are estimated by maximizing the likelihood. For convenience, stationarity is imposed. In practice there is no way to distinguish statistically a unit root in the $K$ matrix from an extremely persistent process. The covariance matrix of parameter estimates is constructed with the outer product of first derivatives. Confidence bounds on nonlinear functions of the parameters are calculated using Monte Carlo simulations, randomly drawing parameter vectors from a multivariate Gaussian distribution with a mean equal to the parameter estimates.

5.3 Variance decompositions

The most important message contained in these results is that inflation shocks account for a small fraction of the total variance of shocks to nominal yields. Table 7 presents detailed results for a four-factor model estimated over the entire sample 1968Q4 through 2013Q4. Variance decompositions are reported for bonds with maturities of one, five, and ten years.

There are three observations from the table worth emphasizing. First, for all the bonds, between 15 and 20 percent of the variance of nominal yield shocks is statistically explained by the direct contribution of news about inflation expectations. These results are mirror those presented in Section 3. The two-sided 95 percent confidence bounds are tight. For each bond, we confidently conclude that quarterly inflation news accounts for between 5 and 35 percent of yield-shock variance.

Second—and this is the main contribution of this section—there is insufficient information in the data to decompose accurately the remaining variance of long-maturity yields into news about expected future real rates and news about term premia. The point estimates suggest that the former is more important at maturities less than five years and the latter is more important for longer maturities. However, the confidence bounds are huge, and do not rule out the possibility that either source dominates the other.

Third, point estimates imply a positive covariance between news about expected real rates and news about expected excess returns. The estimates indicate that between 20 and
35 percent of the variance of yield shocks is attributable to this covariance. The confidence bounds are very large. Section 5.4 shows that these latter two observations are closely related.

5.4 Impulse responses

Impulse responses to shocks provide some additional information about the small contribution of inflation expectations to yield shocks. Figure 7 displays responses to a one standard deviation quarterly shock to the three-quarter-ahead inflation forecast, based on full-sample results. Panel A reveals that the shock is small—about 30 basis points—and highly persistent. There is too much uncertainty in the parameter estimates to pin down the covariation between the shock to expected inflation and shocks to ex ante real rates, long-term yields, and term premia.

This figure contrasts sharply with Figure 8, which displays responses to a one standard deviation shock to the real short rate. (The shocks in Figures 7 and 8 are not orthogonalized.) The initial shock is large—almost a full percentage point—and the point estimates imply that it dies out quickly. The immediate response of the five-year yield is a little less than 50 basis points. Because the shock dies out so quickly, this response of the five-year yield substantially exceeds the shock to the average expected real rate over the next five years. Thus the term premium for the bond also immediately jumps by 20 basis points.

The pattern of these responses accounts for the positive estimated covariance between average expected real rates and term premia. The confidence bounds on these responses account for the inability to distinguish statistically between the roles played by average expected real rates and term premia. The point estimates imply that shocks to real rates die out quickly, but the confidence bounds in Panel A allow for the possibility that real rates are actually highly persistent. If real rates are highly persistent, then the immediate response of the five-year yield to the shock to real rates is in line with the shock to the average expected real rate over the next five years. Hence the confidence bounds on the response of the term premium includes the possibility that term premia do not react at all.

Another way to say this is that in the sample, shocks to real rates are volatile and not persistent. Long-term bond yields covary strongly with these shocks and these responses die out quickly as well. There are two ways to explain this pattern. One is that term premia
are also volatile, covary strongly with real rates, and die out quickly. The other is that the sample pattern is at odds with the population properties of the data. If this explanation is correct, then shocks to real rates are truly highly persistent. Investors know this and price long-term bonds accordingly. This explanation implies that investors were surprised by the speed at which the shocks died out in the sample. There is not enough information in the sample to reject either explanation.

Neither theory nor the existing empirical literature offers much to help pin down the relative contributions of news about average expected real rates or shocks to term premia. If the shocks are primarily news about real rates, then shocks to real rates must be highly persistent. Such shocks arise naturally in settings where investors learn slowly about the dynamics of consumption, as in Johannes, Lochstoer, and Mou (2014). However, it is not clear that the amount of variation we see in real rates is consistent with learning. Other shocks may be more important. Hanson and Stein (2014) find that monetary policy shocks have substantial effects on long-term nominal and TIPS yields. They interpret these as term premia shocks rather than news about expected real rates, because standard theories of monetary policy do not allow policy shocks to have long-term effects on short-term real rates. Nakamura and Steinsson (2013) disagree about both the magnitude of shocks to long-maturity yields and their interpretation as primarily term premia.

5.5 Level, slope, and curvature

Litterman and Scheinkman (1991) show that almost all of the cross-sectional variation in bond returns can be characterized by level, slope, and curvature factors. Returns are closely related to yield shocks, thus it is not surprising that the same decomposition holds for shocks. This subsection asks whether this same decomposition holds for the components of yield shocks. Can news about average expected inflation also be summarized by level, slope, and curvature? What about the innovations in yields not attributable to news about inflation?

News about expected inflation for an \( m \)-maturity bond is defined by (18). The parameterized models imply a covariance matrix of inflation news for bonds with the seven maturities used in estimation. Figure 9 displays the first three principal components (PCs) of the covariance matrix constructed using the four-factor full-sample estimates. The first PC is the
blue line in the Panel A. The second and third PCs are in Panel B, illustrated with a blue solid line and a blue dashed line respectively.

The same decomposition could be produced separately for news about average expected real rates and term premia shocks. However, since these shocks are difficult to distinguish statistically, I sum these two shocks and produce a single PC decomposition. The first three PCs are displayed as red lines in Figure 9. All of the PCs are scaled to represent the effect of a unit standard deviation shock.

The figure shows that both sets of PCs can be described as level, slope, and curvature. For both sets, the first PC accounts for about 96 percent of the total variance. The most obvious difference between the two sets is that shocks are smaller for news about expected inflation than for combined shocks to expected future real rates and term premia.

The similarities motivate a more complicated description of yield shocks than we typically infer from Litterman and Scheinkman. There are two types of level shocks to yields. The smaller type is a shock to average expected inflation. The larger is a shock to the combination of expected real rates and term premia. Similarly, there are two types of slope and curvature shocks.

6 Modeling concerns

This section discusses in detail two potential problems with this paper’s approach. The first is that survey forecasts are sticky and the second is that inflation has no stationary components.

6.1 Sticky information

The model treats mean survey forecasts as true expectations, albeit possibly contaminated with measurement error. Coibion and Gorodnichenko (2012a,b) argue that empirically, mean survey forecasts exhibit patterns consistent with informational rigidities. These rigidities imply that individual forecasters update their predictions infrequently, inducing sluggishness in mean forecasts. If true, expectations about inflation impounded in bond prices likely differ from those extracted from mean forecasts, since market prices are determined by active buyers and sellers. These agents are those most likely to have recently updated their
This subsection reviews their model of forecasts. It concludes that a more plausible interpretation of the evidence is that forecasters update frequently, and any evidence of rigidities is an accident of the sample.

Notation in this discussion differs from notion used in previous sections to accommodate the lag in reporting of quarterly inflation. Date $t$ is the middle of calendar quarter $t$. At this time NIPA releases an estimate of inflation (GDP index) during quarter $t-1$. Denote this estimate by $\pi_{t|t-1}$, where the first part of the subscript refers to the quarter the information is revealed and the second refers to the quarter over which inflation is measured. Respondents to the Survey of Professional Forecasters predict $\pi_{t|t-1}$ in the middle of calendar quarters $t-1, \ldots, t-5$.

Denote the full information rational expectations (FIRE) expectations with the usual expectations operator. A rational agent updates her expectation of $\pi_{t|t-1}$ every period. These updates are uncorrelated over time, thus we can write the realization as the sum of news about $\pi_{t|t-1}$ at $t, t-1$, and so on:

$$\pi_{t|t-1} = \phi_{t}^{(t)} + \phi_{t-1}^{(t)} + \phi_{t-2}^{(t)} + \ldots, \quad E_{t-j-1} \left( \phi_{t-j}^{(t)} \right) = 0.$$  

(This equation ignores a constant term.) The supercript on the shock $\phi$ is the date of the inflation announcement and the superscript is the date the shock is revealed. The FIRE expectation of inflation of $t-j$ is

$$E_{t-j} \left( \pi_{t|t-1} \right) = \phi_{t-j}^{(t)} + \phi_{t-j-1}^{(t)} + \ldots \quad (19)$$

Coibion and Gorodnichenko (2012b) describe a simple model of sticky information. The discussion here adds some notation to their framework in order to consider forecast errors at different horizons. A fraction $1 - \rho$ of respondents immediately update their prediction of $\pi_{t|t-1}$ in response to news at $t-j$. Therefore the mean, across respondents, of the forecast of $\pi_{t-1}$ adjusts by only $(1 - \rho)$ of the true shock $\phi_{t-j}^{(t)}$. Of those who do not update immediately, $(1 - \rho)$ update a period later, and so on. Hence the mean survey forecast of $\pi_{t|t-1}$ made at $t-j$ is a sum of partial current and lagged shocks to the FIRE expectation. Denoting mean
survey forecasts with hats,

$$\hat{E}_{t-j} (\pi_{t|t-1}) = (1 - \rho)\phi_{t-j}^{(t)} + (1 - \rho^2)\phi_{t-j-1}^{(t)} + (1 - \rho^3)\phi_{t-j-2}^{(t)} + \ldots \quad (20)$$

Substituting (19) into (20) expresses mean survey forecasts as deviations from FIRE forecasts,

$$\hat{E}_{t-j} (\pi_{t|t-1}) = E_{t-j} (\pi_{t|t-1}) - \rho \sum_{i=0}^{\infty} \rho^i \phi_{t-j-i}^{(t)}.$$

Therefore the error in the mean survey forecast made at $t-j$ is the sum of the FIRE forecast error and the expectational error,

$$\pi_{t|t-1} - \hat{E}_{t-j} (\pi_{t|t-1}) = \left\{ \pi_{t|t-1} - E_{t-j} (\pi_{t|t-1}) \right\} + \rho \sum_{i=0}^{\infty} \rho^i \phi_{t-j-i}^{(t)}. \quad (21)$$

These forecast errors are closely related to lagged revisions in mean survey expectations. The revision in the mean survey expectation from $t-j-1$ to $t-j$ is

$$\left( \hat{E}_{t-j} - \hat{E}_{t-j-1} \right)\pi_{t|t-1} = (1 - \rho) \sum_{i=0}^{\infty} \rho^i \phi_{t-j-i}^{(t)}.$$

Plug this expression for forecast revisions into the survey forecast error (21) to produce the relation between forecast errors and forecast revisions,

$$\pi_{t|t-1} - \hat{E}_{t-j} (\pi_{t|t-1}) = \frac{\rho}{1 - \rho} \left( \hat{E}_{t-j} - \hat{E}_{t-j-1} \right)\pi_{t|t-1} + \left\{ \pi_{t|t-1} - E_{t-j} (\pi_{t|t-1}) \right\}. \quad (22)$$

The term in curly brackets is unforecastable (by both FIRE and survey respondents) as of $t-j$, and therefore is orthogonal to the first term on the right. Hence (22) can be interpreted as a regression equation.

Coibion and Gorodnichenko use a variant (22) to test for the presence of sticky information, focusing on annual forecasts of inflation. An implication of (22) explored here is that the coefficient on the forecast revision is independent of $j$. According to the model, the length of time between the forecast $t-j$ and the realization $t$ does not affect the coefficient on the forecast revision.
Table 8 reports results of estimating the regression

$$\pi_{t-1} - \hat{E}_{t-j}(\pi_{t-1}) = \beta_{0,j} + \beta_{1,i}(\hat{E}_{t-j} - \hat{E}_{t-j-1})\pi_{t-1} + e_{t,j}$$

for various choices of quarterly lags $j$ and two sample periods. The inflation measure is real-time GDP inflation, available from the Federal Reserve Bank of Philadelphia.\(^{14}\) For the full sample from 1969 through 2013, the point estimates are positive and significant, both economically and statistically. In this respect, the results support Coibion and Gorodnichenko’s interpretation.

However, two aspects of these results cast considerable doubt on this story. First, the regression coefficients rise substantially as the forecast horizon increases. The estimate for $j = 1$ corresponds to a value of $\rho$ less than 0.3, while the estimate for $j = 4$ corresponds to $\rho = 0.65$. Yet the theory does not accommodate shorter periods of inattention for near-term inflation.

Second, the statistical significance disappears after 1984. For the sample 1985 through 2013, forecast revisions are only weakly associated with survey forecast errors, both economically and statistically. For example, none the regression $R^2$s exceed three percent. Nason and Smith (2014) test the sticky information hypothesis for inflation expectations and arrive at a similar conclusion about the role of the sample period. If inattention to news about inflation accounts for these results, inattention during the sample period was concentrated in the subsample when inflation high and volatile. Casual intuition suggests that the opposite should be true.

An alternative and more perhaps plausible story is that the observed relation between survey forecasts of inflation and realized inflation is not representative of the population relation. It is well-known that the steady increase in inflation during the late 1970s and early 1980s surprised most forecasters, whether they were paying attention or not. For example, beginning with 1978Q2, mean survey forecasts of three-quarter-ahead inflation were revised upwards for nine straight quarters. The corresponding realized forecast errors (the left side of (23)) were positive for all nine.

Just as unusual was the energy price shock in 1973 and 1974. Three-quarter-ahead

\(^{14}\)Because of a Federal government shutdown, the estimate of GDP inflation for 1995Q4 was not published by the Bureau of Economic Analysis during 1996Q1. For these regressions, the published value in 1996Q2 is treated as known by participants in 1996Q1.
inflation forecasts were revised upwards for nine straight quarters beginning in 1973Q1. The corresponding three-quarter-ahead and four-quarter-ahead forecast errors were almost entirely positive and occasionally extremely large.\textsuperscript{15} The errors associated with predictions made during the first three quarters of 1973 cannot be attributed to forecaster inattention, since the Yom Kippur War did not begin until October 1973. Subsequent forecast errors are more reasonably associated with difficulties in predicting the peak of energy prices rather than inattention to the OPEC oil embargo.

\subsection*{6.2 Nonstationary components of inflation}

The measures of inflation risk described in Section 2 are borrowed from Campbell and Ammer’s decomposition of excess bond returns. However, they assume a different process for inflation and conclude that shocks to average expected inflation account for the vast majority of shocks to nominal yields. This subsection contracts their methodology with the approach taken here.

Their procedure assumes that expectations of future inflation and short-term nominal rates are determined by the stationary dynamics of a vector autoregression (VAR). The six variables included in the VAR are the ex-post real interest rate (short nominal rate at \( t \) less inflation at \( t + 1 \)), the change in the short nominal rate, the excess return to the aggregate stock market, the slope of the term structure, the dividend-price ratio, and the relative bill rate.\textsuperscript{16} Inflation enters only in the form of a linear combination with the short nominal rate, and the level of yields is not included. Therefore yields and inflation are assumed to be nonstationary and cointegrated.

Since neither the level of inflation nor the level of some yield appears by itself in the VAR, these variables are implicitly assumed to not have any stationary components. This assumption starkly contrasts with the trend-cycle model of inflation in Section 3.2. Real rates are assumed to be stationary.

Campbell and Ammer produce estimates of inflation risk that are substantially higher than for either the stationary model of Section 5.1 or the trend-cycle model. They conclude

\textsuperscript{15}Figure 1 displays the substantial swings in survey forecasts over this period. A couple of the residuals for (23) are more than four standard deviations above zero.

\textsuperscript{16}The slope is measured by the ten-year yield less the one-month bill rate. The relative bill rate is the one-month yield less the one-year backward moving average of the one-month yield.
that inflation risk accounts for effectively all of the variance of bond return shocks. To confirm and update their results, I apply their methodology to more recent data. I use their data definitions and frequency (monthly).

Table 9 reports the implied variance decompositions of shocks to a ten-year bond yield for the three sample periods studied in Section 3. The results are similar to those reported by Campbell and Ammer for different sample periods. The measure of inflation risk is roughly one for all three periods, including the post-Volcker era. In other words, the variance of news about expected average inflation over ten years is close to the variance of shocks to the ten-year yield. The confidence bounds all easily reject the null hypothesis that less than half of the yield variance is attributable to news about inflation.

When news about inflation is so large, expectations of long-run inflation must vary substantially over time. Figure 10 displays the model’s monthly forecasts of average inflation over the next 120 months. The forecasts fluctuate substantially over time, from more than 12 percent in the early 1980s to less than −1 percent in 2011. Corresponding survey forecasts are also displayed; they are identical to those in Figure 6. It is clear from the figure that the model’s estimates of long-run inflation are wildly at odds with both the survey forecasts and the forecasts produced by the models that allow some or all of the variability in inflation to be mean reverting.

7 Concluding comments

This paper studies the joint dynamics of nominal yields and inflation expectations from 1968 through 2013. For this sample as well as subsamples, quarterly shocks to nominal yields are primarily shocks to real rates and term premia. News about expected future inflation contributes relatively little to the variance of yield shocks.

This is a robust result that can be used to help evaluate dynamic term structure models. In particular, some estimated models in the literature imply that inflation shocks drive much of the variation in nominal yields. This feature of the models is inconsistent with the U.S. experience, even excluding the period of stable inflation during the Federal Reserve leadership of Greenspan and Bernanke.
References

Ang, Andrew, Geert Bekaert, and Min Wei, 2007, Do macro variables, asset markets or surveys forecast inflation better?, *Journal of Monetary Economics* 54, 1163-1212.


Coibion, Olivier, and Yuriy Gorodnichenko, 2012a, What can survey forecasts tell us about informational rigidities, *Journal of Political Economy* 120, 116-159.


Ermolov, Andrey, 2015, Time-varying risk of nominal bonds: How important are macroeconomic shocks, Working paper, Columbia GSB.


Nakamura, Emi, and Jón Steinsson, 2013, High frequency identification of monetary non-neutrality, Working paper, Columbia GSB.


Table 1. Predictions and realizations of inflation and bond yields, mid-1997

Panel A reports, for May and August 1997, consensus forecasts of four future quarter-to-quarter CPI inflation rates from the Blue Chip Financial Forecast. It also reports implied forecasts of average inflation over horizons of one, two, three, and four quarters. Differences between the August and May forecasts is the news about expected future inflation.

Panel B reports yields on short-term zero-coupon Treasury bonds for the same two months. The row labeled “Forecasted change” contains predicted changes in yields from May to August. Yield forecasts are from OLS regressions using yields on three-month, 12-month, and 18-month bonds as predictive variables. Yield innovations are realized changes from May to August less predicted changes.

Quarter-to-quarter forecasts are in annualized percent without compounding. Average inflation and bond yields are in annualized percent using continuous compounding.

Panel A. Inflation expectations

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>May</td>
<td>q-to-q inflation forecast</td>
<td>2.99</td>
<td>2.98</td>
<td>2.91</td>
<td>2.86</td>
</tr>
<tr>
<td>May</td>
<td>average inflation forecast</td>
<td>2.94</td>
<td>2.94</td>
<td>2.92</td>
<td>2.89</td>
</tr>
<tr>
<td>August</td>
<td>q-to-q inflation forecast</td>
<td>2.70</td>
<td>2.69</td>
<td>2.71</td>
<td>2.76</td>
</tr>
<tr>
<td>August</td>
<td>average inflation forecast</td>
<td>2.65</td>
<td>2.66</td>
<td>2.68</td>
<td>2.69</td>
</tr>
<tr>
<td>August</td>
<td>average inflation news</td>
<td>−0.28</td>
<td>−0.28</td>
<td>−0.25</td>
<td>−0.21</td>
</tr>
</tbody>
</table>

Panel B. Bond yields

<table>
<thead>
<tr>
<th>Month-end, 1997</th>
<th>Value</th>
<th>3 mon</th>
<th>6 mon</th>
<th>9 mon</th>
<th>12 mon</th>
</tr>
</thead>
<tbody>
<tr>
<td>May</td>
<td>level</td>
<td>4.85</td>
<td>5.42</td>
<td>5.53</td>
<td>5.72</td>
</tr>
<tr>
<td>May</td>
<td>forecasted change</td>
<td>−0.02</td>
<td>−0.14</td>
<td>−0.20</td>
<td>−0.25</td>
</tr>
<tr>
<td>August</td>
<td>level</td>
<td>5.13</td>
<td>5.30</td>
<td>5.40</td>
<td>5.66</td>
</tr>
<tr>
<td>August</td>
<td>yield innovations</td>
<td>0.30</td>
<td>0.03</td>
<td>0.07</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Quarterly shocks to average expected inflation over various maturities are from Blue Chip surveys completed at the end of the second month of each quarter. Corresponding shocks to bond yields are constructed from in-sample estimates of a forecasting model. The model assumes changes in yields are predictable with the shape of the short end of the yield curve. Shocks for a given maturity are assumed to be jointly normally distributed. Inflation risk is measured by the ratio of the variance of shocks to average expected inflation to the variance of yield shocks. Standard deviations are in annualized percent/quarter. Maximum likelihood standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1980Q1–2013Q4</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD of news about expected infl</td>
<td>0.45</td>
<td>0.35</td>
<td>0.31</td>
<td>0.26</td>
</tr>
<tr>
<td>SD of yield innovations</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>0.97</td>
</tr>
<tr>
<td>Variance ratio</td>
<td>0.202</td>
<td>0.121</td>
<td>0.096</td>
<td>0.074</td>
</tr>
<tr>
<td><em>(0.020)</em></td>
<td><em>(0.014)</em></td>
<td><em>(0.012)</em></td>
<td><em>(0.011)</em></td>
<td></td>
</tr>
<tr>
<td><strong>1980Q1–1987Q3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD of news about expected infl</td>
<td>0.72</td>
<td>0.59</td>
<td>0.54</td>
<td>0.49</td>
</tr>
<tr>
<td>SD of yield innovations</td>
<td>1.89</td>
<td>1.90</td>
<td>1.87</td>
<td>1.85</td>
</tr>
<tr>
<td>Variance ratio</td>
<td>0.145</td>
<td>0.096</td>
<td>0.084</td>
<td>0.070</td>
</tr>
<tr>
<td><em>(0.058)</em></td>
<td><em>(0.047)</em></td>
<td><em>(0.046)</em></td>
<td><em>(0.046)</em></td>
<td></td>
</tr>
<tr>
<td><strong>1987Q4–2013Q4</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD of news about expected infl</td>
<td>0.34</td>
<td>0.24</td>
<td>0.20</td>
<td>0.18</td>
</tr>
<tr>
<td>SD of yield innovations</td>
<td>0.42</td>
<td>0.45</td>
<td>0.48</td>
<td>0.50</td>
</tr>
<tr>
<td>Variance ratio</td>
<td>0.654</td>
<td>0.290</td>
<td>0.184</td>
<td>0.127</td>
</tr>
<tr>
<td><em>(0.115)</em></td>
<td><em>(0.046)</em></td>
<td><em>(0.030)</em></td>
<td><em>(0.023)</em></td>
<td></td>
</tr>
</tbody>
</table>
Quarterly shocks to average expected inflation over various maturities are from SPF surveys completed in the middle of each quarter. Corresponding shocks to bond yields are constructed from in-sample estimates of a forecasting model. The model assumes changes in yields are predictable with the shape of the short end of the yield curve. Shocks for a given maturity are assumed to be jointly normally distributed. Inflation risk is measured by the ratio of the variance of shocks to average expected inflation to the variance of yield shocks. Standard deviations are in annualized percent/quarter. Maximum likelihood standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968Q4–2013Q4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD of news about expected infl</td>
<td>0.46</td>
<td>0.40</td>
<td>0.35</td>
</tr>
<tr>
<td>SD of yield innovations</td>
<td>0.95</td>
<td>0.94</td>
<td>0.92</td>
</tr>
<tr>
<td>Variance ratio</td>
<td>0.228</td>
<td>0.180</td>
<td>0.146</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.019)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>1968Q4–1987Q3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD of news about expected infl</td>
<td>0.65</td>
<td>0.57</td>
<td>0.50</td>
</tr>
<tr>
<td>SD of yield innovations</td>
<td>1.36</td>
<td>1.32</td>
<td>1.27</td>
</tr>
<tr>
<td>Variance ratio</td>
<td>0.226</td>
<td>0.188</td>
<td>0.158</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.042)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>1987Q4–2013Q4</td>
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</tr>
<tr>
<td>SD of news about expected infl</td>
<td>0.23</td>
<td>0.19</td>
<td>0.18</td>
</tr>
<tr>
<td>SD of yield innovations</td>
<td>0.40</td>
<td>0.43</td>
<td>0.45</td>
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<tr>
<td>Variance ratio</td>
<td>0.329</td>
<td>0.199</td>
<td>0.164</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.034)</td>
<td>(0.028)</td>
</tr>
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</table>
Quarterly shocks to average expected inflation over a ten-year horizon are estimated from a model that assumes inflation is the sum of a random walk component and an AR(1) component. The model is estimated over various sample periods using survey forecasts of inflation over horizons ranging up to seven quarters ahead. Shocks to the ten-year yield are calculated assuming the ten-year yield is assumed to follow a random walk. Shocks to the inflation components and the bond yield are assumed to be jointly normally distributed. Inflation risk is measured by the ratio of the variance of shocks to average expected inflation to the variance of yield shocks. Standard deviations are in annualized percent/quarter. Maximum likelihood standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Survey</th>
<th>Sample Period</th>
<th>SD of inflation news</th>
<th>SD of yield shocks</th>
<th>Variance Ratio</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue Chip</td>
<td>1980Q1–2013Q4</td>
<td>0.21</td>
<td>0.60</td>
<td>0.120</td>
<td>(0.025)</td>
</tr>
<tr>
<td></td>
<td>1980Q1–1987Q3</td>
<td>0.35</td>
<td>1.11</td>
<td>0.098</td>
<td>(0.062)</td>
</tr>
<tr>
<td></td>
<td>1987Q4–2013Q4</td>
<td>0.10</td>
<td>0.38</td>
<td>0.071</td>
<td>(0.028)</td>
</tr>
<tr>
<td>Survey of Prof Forecasts</td>
<td>1968Q4–2013Q4</td>
<td>0.20</td>
<td>0.51</td>
<td>0.158</td>
<td>(0.032)</td>
</tr>
<tr>
<td></td>
<td>1968Q4–1987Q3</td>
<td>0.27</td>
<td>0.65</td>
<td>0.174</td>
<td>(0.061)</td>
</tr>
<tr>
<td></td>
<td>1987Q4–2013Q4</td>
<td>0.14</td>
<td>0.38</td>
<td>0.139</td>
<td>(0.054)</td>
</tr>
</tbody>
</table>
Table 5. Volatilities of shocks to bond yields and expected inflation implied by models with long-run risk and recursive utility

The table reports population standard deviations of quarterly shocks to yields and expected inflation over the same horizon as the bond maturities. The population values are implied by parameterized models described in the indicated source. Inflation risk is measured by the ratio of the latter variance to the former variance.

<table>
<thead>
<tr>
<th>Source</th>
<th>Maturity (quarters)</th>
<th>Std dev of inflation news</th>
<th>Std dev of yield innovations</th>
<th>Variance ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piazzesi and Schneider (2007)</td>
<td>1</td>
<td>60</td>
<td>58</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>56</td>
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<td></td>
<td>20</td>
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<td>1.50</td>
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<tr>
<td></td>
<td>40</td>
<td>30</td>
<td>25</td>
<td>1.45</td>
</tr>
<tr>
<td>Bansal and Shaliastovich (2013)</td>
<td>1</td>
<td>44</td>
<td>51</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>44</td>
<td>46</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>40</td>
<td>37</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>35</td>
<td>36</td>
<td>0.96</td>
</tr>
<tr>
<td>Kung (2015)</td>
<td>1</td>
<td>85</td>
<td>72</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>68</td>
<td>49</td>
<td>1.92</td>
</tr>
<tr>
<td></td>
<td>20</td>
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<td>39</td>
<td>1.94</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>46</td>
<td>32</td>
<td>2.11</td>
</tr>
<tr>
<td>Rudebusch and Swanson (2012)</td>
<td>1</td>
<td>61</td>
<td>28</td>
<td>4.79</td>
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<tr>
<td></td>
<td>20</td>
<td>26</td>
<td>32</td>
<td>0.67</td>
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<tr>
<td></td>
<td>40</td>
<td>16</td>
<td>24</td>
<td>0.46</td>
</tr>
</tbody>
</table>
Table 6. Volatilities of shocks to bond yields and expected inflation implied by models with habit formation

The table reports population standard deviations of quarterly shocks to yields and expected inflation over the same horizon as the bond maturities. The population values are implied by parameterized models described in the indicated source. Inflation risk is measured by the ratio of the latter variance to the former variance.

<table>
<thead>
<tr>
<th>Source</th>
<th>Maturity (quarters)</th>
<th>Std dev of inflation news</th>
<th>Std dev of yield innovations</th>
<th>Variance ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wachter (2006)</td>
<td>1</td>
<td>70</td>
<td>93</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>64</td>
<td>89</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>42</td>
<td>78</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>27</td>
<td>77</td>
<td>0.12</td>
</tr>
<tr>
<td>Ermolov (2015)</td>
<td>1</td>
<td>84</td>
<td>105</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>75</td>
<td>108</td>
<td>0.49</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>40</td>
<td>28</td>
<td>74</td>
<td>0.15</td>
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</table>
Table 7. Decompositions of population variances of yield innovations

The table reports model-implied standard deviations of quarterly shocks to nominal Treasury bond yields. Yields are expressed in percent per year. The table also reports decompositions of the corresponding variances. The model uses four factors to describe the joint dynamics of nominal yields and expected inflation. Yields are the sum of average expected inflation and short-term real rates over the life of the bond, plus a term premium. The contributions to total variance sum to one (aside from rounding). The sample period is 1968Q4 through 2013Q4. Brackets display [2.5% 97.5%] percentile bounds.

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Std dev</th>
<th>1. Average expected real rate</th>
<th>2. Average expected inflation</th>
<th>3. Term premium</th>
<th>2Cov([1],[2])</th>
<th>2Cov([1],[3])</th>
<th>2Cov([2],[3])</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>0.87</td>
<td>0.66</td>
<td>0.16</td>
<td>0.02</td>
<td>−0.06</td>
<td>0.25</td>
<td>−0.03</td>
</tr>
<tr>
<td></td>
<td>[0.80 1.60]</td>
<td>[0.49 1.13]</td>
<td>[0.06 0.25]</td>
<td>[0.00 0.11]</td>
<td>[−0.54 0.09]</td>
<td>[−0.04 0.48]</td>
<td>[−0.15 0.03]</td>
</tr>
<tr>
<td>Five</td>
<td>0.61</td>
<td>0.29</td>
<td>0.18</td>
<td>0.22</td>
<td>0.12</td>
<td>0.35</td>
<td>−0.16</td>
</tr>
<tr>
<td></td>
<td>[0.54 1.08]</td>
<td>[0.12 1.69]</td>
<td>[0.06 0.26]</td>
<td>[0.07 0.80]</td>
<td>[−0.44 0.29]</td>
<td>[−1.03 0.60]</td>
<td>[−0.48 0.10]</td>
</tr>
<tr>
<td>Ten</td>
<td>0.51</td>
<td>0.18</td>
<td>0.19</td>
<td>0.33</td>
<td>0.22</td>
<td>0.22</td>
<td>−0.15</td>
</tr>
<tr>
<td></td>
<td>[0.44 0.81]</td>
<td>[0.05 2.26]</td>
<td>[0.05 0.35]</td>
<td>[0.15 1.60]</td>
<td>[−0.50 0.54]</td>
<td>[−2.43 0.52]</td>
<td>[−0.80 0.24]</td>
</tr>
</tbody>
</table>
Table 8. Explaining survey forecast errors with forecast revisions

The table reports estimates of (23) in the text, which regresses inflation forecast errors on lagged revisions in mean survey forecasts. Mean survey forecasts of GDP inflation are from the Survey of Professional Forecasters. Forecast errors are the difference between the quarter-to-quarter GDP inflation rate observed at $t$ and the mean survey forecast as of $t - j$. Revisions in forecasts equal the mean survey forecast at $t - j$ less the forecast at $t - j - 1$. Asymptotic standard errors are adjusted for $j - 1$ lags of moving average residuals. A few observations of survey forecasts are missing.

<table>
<thead>
<tr>
<th>Forecast lag $j$ (quarters)</th>
<th>Number of obs</th>
<th>Coef  (Std err)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 1969Q1 to 2013Q4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>179</td>
<td>0.41 (0.17)</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>178</td>
<td>0.84 (0.40)</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>177</td>
<td>1.22 (0.48)</td>
<td>0.09</td>
</tr>
<tr>
<td>4</td>
<td>171</td>
<td>1.83 (0.47)</td>
<td>0.13</td>
</tr>
<tr>
<td>B. 1985Q1 to 2013Q4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>115</td>
<td>-0.29 (0.28)</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>114</td>
<td>0.10 (0.31)</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>113</td>
<td>0.07 (0.48)</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
<td>0.75 (0.45)</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Table 9. Variance decompositions using a model of nonstationary inflation and yields

The table reports model-implied variance decompositions of quarterly shocks to a ten-year nominal yield. The model is a first-order vector autoregression of monthly data. The observed data are described in Section 6.2. The model imposes cointegration on yields and inflation, while real rates are stationary. Yields are the sum of average expected inflation and short-term real rates over the life of the bond, plus a term premium. The contributions to total variance sum to one (aside from rounding). Brackets display $[2.5\% \ 97.5\%]$ percentile bounds.

<table>
<thead>
<tr>
<th>Sample period</th>
<th>1. Average expected real rate</th>
<th>2. Average expected inflation</th>
<th>3. Term premium</th>
<th>2Cov([1], [2])</th>
<th>2Cov([1], [3])</th>
<th>2Cov([2], [3])</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/1968–12/2013</td>
<td>0.02</td>
<td>1.27</td>
<td>0.60</td>
<td>−0.06</td>
<td>−0.02</td>
<td>−0.80</td>
</tr>
<tr>
<td></td>
<td>[0.01 0.10]</td>
<td>[0.90 2.78]</td>
<td>[0.23 1.98]</td>
<td>[−0.40 0.14]</td>
<td>[−0.28 0.24]</td>
<td>[−3.45 0.23]</td>
</tr>
<tr>
<td>10/1968–9/1987</td>
<td>0.02</td>
<td>0.97</td>
<td>0.85</td>
<td>−0.15</td>
<td>0.21</td>
<td>−0.89</td>
</tr>
<tr>
<td></td>
<td>[0.01 0.39]</td>
<td>[0.65 5.89]</td>
<td>[0.30 4.53]</td>
<td>[−2.10 0.05]</td>
<td>[−0.06 1.76]</td>
<td>[−9.21 0.21]</td>
</tr>
<tr>
<td>10/1987–12/2013</td>
<td>0.02</td>
<td>0.92</td>
<td>0.18</td>
<td>−0.07</td>
<td>−0.02</td>
<td>−0.02</td>
</tr>
<tr>
<td></td>
<td>[0.01 0.17]</td>
<td>[0.60 2.65]</td>
<td>[0.07 1.09]</td>
<td>[−0.78 0.09]</td>
<td>[−0.23 0.37]</td>
<td>[−2.37 0.25]</td>
</tr>
</tbody>
</table>
Figure 1. Time line for news about expected inflation over the life of a one-year bond

The theory of the news decomposition assumes that we observe agents’ predictions at dates $T_1$ and $T_2$ about the log change in the price level from date $T_2$ to date $T_3$. In practice, we observe survey forecasts as of $T_1$ and $T_2$ of the log change from the average price level in the quarter containing $T_2$, indicated by the first shaded box, to the average price of the quarter containing $T_3$, indicated by the second shaded box.
Figure 2. Realized quarterly news about average expected inflation from Blue Chip surveys.

Quarterly changes in consensus forecasts from Blue Chip surveys are used to construct innovations in forecasts of expected average inflation over horizons ranging from one to six quarters. The sample range is May 1980 through December 2013.
Quarterly changes in consensus forecasts from SPF surveys are used to construct innovations in forecasts of expected average inflation over horizons ranging from one to three quarters. The sample range is 1968Q4 through 2013Q4.
Figure 4. Realized quarterly innovations of Treasury yields, 1980–2013

OLS forecasting regressions are used to construct quarterly innovations of yields. The forecasting variables are yields for maturities of one, four, and six quarters. The sample range is May 1980 through December 2013.
Figure 5. Realized quarterly innovations of Treasury yields, 1968–2013

OLS forecasting regressions are used to construct quarterly innovations of yields. The forecasting variables are yields for maturities of one, four, and six quarters. The sample range is 1968Q4 through 2013Q4.
Figure 6. Long-horizon inflation forecasts from surveys and a model

The black line is the fitted forecast of average CPI inflation over the next ten years. The model assumes inflation is the sum of a random walk and an AR(1) process. The parameters are estimated using Blue Chip survey forecasts of CPI inflation up to seven quarters ahead. The circles are Blue Chip survey forecasts of inflation five to ten years ahead. The x’s are Blue Chip survey forecasts of GDP inflation five to ten years ahead. Neither the circles nor the x’s are used in model estimation.
A four-factor dynamic model of nominal yields and expected inflation is estimated over the sample 1968Q4 through 2013Q4. The figure displays model-implied impulse responses to a shock to three-quarter-ahead expected inflation. The initial shock is 33 basis points, which is the model-implied population standard deviation of the shock. The yield and term premium responses are for a five-year nominal bond. Also displayed are 95 percentile confidence bounds on the impulse responses.
Figure 8. Impulse responses for a shock to the ex ante real rate

A four-factor dynamic model of nominal yields and expected inflation is estimated over the sample 1968Q4 through 2013Q4. The figure displays model-implied impulse responses to a shock to the ex ante real rate, defined as the three-month nominal yield less expected inflation during the next quarter. The initial shock is 91 basis points, which is the model-implied population standard deviation of the shock. The yield and term premium responses are for a five-year nominal bond. Also displayed are 95 percentile confidence bounds on the impulse responses.
Figure 9. Principal component decompositions of shocks to bond yields

Shocks to nominal yields are decomposed into shocks to average expected inflation over the life of the bond and a remainder, which is the sum of a shock to average expected real rates and a term premium shock. The decomposition and the resulting principal components of the shocks are calculated with a four-factor model of yields and inflation expectations estimated over the period 1968Q4 through 2013Q4. Principal components for inflation shocks are in blue and principal components for the remainder are in red. The third principal component is displayed with a dashed line. The principal components all correspond to a one standard deviation shock to the respective component.
Figure 10. Ten-year inflation forecasts from surveys and a model of nonstationary inflation and yields

Section 6.2 describes a joint model of inflation and nominal yields that assumes no components of inflation are stationary. The solid black line represents the model’s prediction of average CPI inflation during the next ten years. The model’s parameters are estimated using the sample October 1968 through December 2013. The circles are Blue Chip survey forecasts of inflation five to ten years ahead. The x’s are Blue Chip survey forecasts of GDP inflation five to ten years ahead.
Appendix

This appendix provides intuition for the magnitude of news about expected excess bond returns in a no-arbitrage model such as Bansal and Shaliastovich (2013). Assume the one-step-ahead joint conditional distribution of the nominal stochastic discount factor (SDF) and nominal bond prices is log-normal. Then standard no-arbitrage arguments imply that the expected excess log return to an \( m \)-maturity bond is

\[
E_{t-1}\left( ex_t^{(m)} \right) = -\frac{1}{2} \text{Var}_{t-1}\left( ex_t^{(m)} \right) - \text{Cov}_{t-1}\left( \log SDF_t, ex_t^{(m)} \right).
\]

(24)

Consider the case in which both the conditional variance and covariance in (24) are proportional to a single state variable \( V \). The main example of this case is when the conditional standard deviations of both log returns and the log SDF are linear in the square root of the state variable, while the conditional correlation between log returns and the log SDF is constant. This is a simpler setting than that studied in Bansal and Shaliastovich (2013).

In this case we can rewrite (24) as

\[
E_{t-1}\left( ex_t^{(m)} \right) = k_{1,m} V_{t-1} + k_{2,m} V_{t-1} = k_m V_{t-1}.
\]

(25)

Now assume that the state variable is stationary, so unconditional expectations exist. Multiply and divide (25) by its unconditional expectation to produce

\[
E_{t-1}\left( ex_t^{(m)} \right) = E\left( ex_t^{(m)} \right) \frac{V_{t-1}}{E(V)}.
\]

(26)

We can iterate this equation forward to produce multi-step forecasts,

\[
E_{t-1}\left( ex_{t+i}^{(m)} \right) = E\left( ex_t^{(m)} \right) E_{t-1}\left( \frac{V_{t+i-1}}{E(V)} \right), \quad i \geq 0.
\]

(27)

Plug (27) into the equation for news about expected excess returns (4). The result is

\[
\eta_{ex,t}^{(m)} = \frac{1}{m} E\left( ex_{t+i}^{(m-i+1)} \right) \sum_{i=1}^{m} \frac{(E_t - E_{t-1})V_{t+i-1}}{E(V)}
\]

(28)

Equation (28) tells us the news about expected average excess returns depends on the
size of the variance shock relative to the average level of variance, the persistence of the variance shock, and the average compensation to holding nominal bonds. To get a sense of an upper bound on the magnitudes of news, consider a variance shock at $t$ that does not begin to die out until after the bond has matured. Denoting this shock with a tilde, the news about expected average excess returns is

$$\eta^{(m)}_{ex,t} = \frac{\tilde{V}}{E(V)} \frac{1}{m} \sum_{i=1}^{m} E\left(e^{x_t}_{t+i}\right). \tag{29}$$

Finally, plug (1) into (29), replacing the sum of expected excess log returns with the yield spread:

$$\eta^{(m)}_{ex,t} = \frac{\tilde{V}}{E(V)} E\left(y^{(m)}_t - y^{(1)}_t\right). \tag{30}$$

The intuition behind (30) is straightforward. Imagine that, say, conditional variances suddenly double and are expected to remain at the higher level through horizon $m$. Then conditional expectations of excess returns also double. If the average yield spread is, say, one percent, then the yield must jump another one percent to give investors the expected excess return they require. Therefore the news about expected future excess returns is negative one percent.