On the Relationship between the Rate of Change of Total Electron Content Index (ROTI), Irregularity Strength (CkL) and the Scintillation Intensity Index (S4)

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• We present a new quantitative theory for the rate of change of total electron content index (ROTI) by noting its straightforward relationship to the phase structure function of ionospheric turbulence.

• The theory explains several dependencies not previously accounted for quantitatively, including the sampling interval, satellite motion, propagation geometry, and the spectral shape, strength, anisotropy, and drift of the ionospheric irregularities.

• We validate the theory using GPS scintillation measurements collected at Ascension Island, an equatorial station, during solar maximum conditions.

• Our results suggest the possibility of using dense networks of inexpensive total electron content (TEC) monitors to provide quantitative diagnostics of ionospheric scintillation and the irregularities that cause them.

**Reference**: Carrano, Groves, and Rino, *On the Relationship between the Rate of Change of Total Electron Content Index (ROTI), Irregularity Strength (CkL) and the Scintillation Index (S4)*, Submitted to JGR November 2018.
• The rate of change of TEC index (ROTI):

\[ ROTI^2(\delta t) \equiv \left( \frac{|TEC(t + \delta t) - TEC(\delta t)|^2}{\delta t^2} \right) \]

• Rino’s scintillation model (1979) relates the structure function of phase fluctuations

\[ D_{\phi}(y) = \left( |\varphi(r) - \varphi(r + y)|^2 \right) \]

to the irregularity strength \((C_kL)\) and the scintillation index \((S_4)\). Spatial separations \(y\) are related to temporal differences \(\delta t\) through an effective scan velocity \(V_{eff}: \ y = V_{eff} \delta t\)

• If we neglect diffraction effects on the phase:

\[ ROTI^2(\delta t) \approx \left( \frac{|TEC(r) - TEC(r + V_{eff}\delta t)|^2}{\delta t^2} \right) = \frac{c^2}{\delta t^2} \left( |\varphi(r) - \varphi(r + V_{eff}\delta t)|^2 \right) = \frac{c^2}{\delta t^2} D_{\phi}(V_{eff}\delta t) \]

where \(c\) = -0.1865 TEC/rad converts a phase measurement at L1 (rad) to TECU.
A Quantitative Theory for ROTI

- An analytic theory for ROTI follows, valid for weak scintillation:

\[
\text{ROTI}^2(\delta t) = \frac{c^2}{\delta t^2} \left\{ r_e^2 \lambda^2 \sec \theta \left( \frac{2\pi}{1000} \right)^{2\nu+1} C_k L \right\} G
\]

\[
\Gamma(\nu - 1/2) \left[ 1 - 2 \left| q_0 V_{\text{eff}} \delta t / 2 \right|^{\nu-1/2} K_{\nu-1/2} \left( q_0 V_{\text{eff}} \delta t / \Gamma(\nu - 1/2) \right) \right]
\]

\[
\frac{1}{2\pi} \frac{2\Gamma(3/2 - \nu)}{\Gamma(\nu + 1/2)(2\nu - 1)^{2\nu-1}} \cdot \left| V_{\text{eff}} \delta t \right|^{2\nu-1}, \quad \frac{1}{2} < \nu < \frac{3}{2}
\]

where

- \( r_e \) – classical electron radius
- \( \lambda \) - wavelength
- \( C_k L \) – irregularity strength
- \( \nu \) - related to irregularity spectral index as \( p^{(3)}=2\nu+1 \)

- Scale-free approximation \( q_0 \rightarrow 0 \)

\[
\text{ROTI}^2(\delta t) \sim \frac{c^2}{\delta t^2} \left\{ r_e^2 \lambda^2 \sec \theta \left( \frac{2\pi}{1000} \right)^{2\nu+1} C_k L \right\} G
\]

\[
\frac{1}{2\pi} \frac{2\Gamma(3/2 - \nu)}{\Gamma(\nu + 1/2)(2\nu - 1)^{2\nu-1}} \cdot \left| V_{\text{eff}} \delta t \right|^{2\nu-1}, \quad \frac{1}{2} < \nu < \frac{3}{2}
\]

 ROTI depends on effective scan velocity to the power \( \nu+1/2 \). Explains how ROTI depends on irregularity drift, and propagation geometry w.r.t. magnetic field.
• Weak scatter theory for $S_4$ (Rino, Radio Sci., 1979)

\[
S_{4w}^2 = \left\{ r_e^2 \lambda^2 \sec \theta \left( \frac{2\pi}{1000} \right)^{2v+1} C_k L \right\} \rho_F^{2v-1} \varphi(v) \frac{\Gamma[(5/2-v)/2]}{2^{v+1/2} \sqrt{\pi} \Gamma[v/2 + 1/4](v-1/2)}
\]

where

- $\theta$ – propagation zenith angle
- $z_R$ – reduced vertical propagation distance

• Empirical strong scatter correction (Rician statistics)

\[
S_4^2 \approx 1 - \exp(-S_{4w}^2)
\]
ROT I and the Scintillation Index ($S_4$)

- Upon computing the ratio $\text{ROT I}/S_4$, irregularity strength cancels:

\[
\frac{\text{ROT I}^2(\delta t)}{S_{4w}^2} = \frac{c^2}{\delta t^2} \frac{1}{\rho_F^{2\nu-1} F_S(\nu) \varphi(\nu)} \frac{G}{2\pi} \frac{\Gamma(\nu - 1/2)}{\nu} \left[ 1 - 2|q_0 V_{\text{eff}} \delta t|/2)^{\nu-1/2} K_{\nu-1/2}(q_0 V_{\text{eff}} \delta t)/\Gamma(\nu - 1/2) \right]
\]

- Scale-free approximation $q_0 \to 0$:

\[
\frac{\text{ROT I}^2(\delta t)}{S_{4w}^2} \sim \frac{c^2}{\delta t^2} \frac{G}{\varphi(\nu)} \left\{ \frac{1}{F_S(\nu)} \frac{1}{2\pi} \frac{2\Gamma(3/2 - \nu)}{\Gamma(\nu + 1/2)(2\nu - 1)2^{2\nu-1}} \right\} \cdot \frac{V_{\text{eff}} \delta t}{\rho_F} \right|^{2\nu-1}, \quad \frac{1}{2} < \nu < \frac{3}{2}
\]

- Infinite axial ratio limit (equatorial field-aligned irregularities) & assuming $\nu=1.25$:

\[
\frac{\text{ROT I}(\delta t)}{S_{4w}} \sim 0.25 \frac{V_{\text{eff}} \delta t}{\rho_F} \right|^{0.75}
\]

**Approximate Theory**

$V_{\text{eff}} \delta t / \rho_F = \text{spatial scale to which ROTI is sensitive divided by the (Fresnel) scale to which } S_4 \text{ is sensitive. The former is *chosen by the analyst*, the latter is *imposed by the propagation physics*.}$
Scintillation Data to Validate the Theory

**Date:** 5-18 March 2002 (solar maximum)

**Location:** Ascension Island
(7.96°S, 14.41°W, dip latitude 12.4°S)

**GPS Receiver:** Ashtech Z-XII
(dual-frequency survey grade, choke ring ant.)

**Data:** 20 Hz Intensity and Phase

**VHF Spaced Receivers**

**Data:** Irregularity Drift, used to compute GPS $V_{eff}$
Predicted and Measured Scintillation Metrics

\[ \alpha_{\text{eff}} = \sin^{-1} \left( \frac{V_{\text{eff}}}{V_h} \right) \]

Red = from 1 Hz TEC
Black = from 20 Hz Intensity

Loss of Lock
Field-Alignment

UT (hours)
Predicted and Measured Scintillation Metrics

\[ \alpha_{\text{eff}} = \sin^{-1} \left( \frac{V_{\text{eff}}}{V_h} \right) \]

Red = from 1 Hz TEC
Black = from 20 Hz Intensity

Loss of Lock

Field-Alignment

\(C, L\)

\(S\)

UT (hours)

03/12/2002 PRN 08
Effect of Satellite Scan w.r.t. Magnetic Field

Cross-field Scan

Field-aligned Scan

Field aligned scans require model to extrapolate farther from $\delta t$ to Fresnel time-scale
Validation Results, 11 Day Campaign

\( V_{\text{eff}} (\text{m/s}) \)

- Weak: \( 0.07 < S_4 < 0.3 \)
- Moderate: \( 0.3 < S_4 < 0.6 \)
- Strong: \( 0.6 < S_4 < 1.0 \)

Slope increases with \( V_{\text{eff}} \)

### Graphs

1. Scatter plot showing \( 1 \text{ Hz ROTI} \) vs. \( S_4 \) (from 20 Hz C/No) with color gradient indicating \( V_{\text{eff}} \) values.
2. Scatter plot showing \( C_{\text{KL}} \) (from 1 Hz ROTI) vs. \( C_{\text{KL}} \) (from 20 Hz C/No) with different colors for different angle ranges and PCC values.
3. Probability distribution of \( S_4 \) error.
4. Scatter plot showing \( S_4 \) (from 1 Hz ROTI) vs. \( S_4 \) (from 20 Hz C/No) with different colors for different angle ranges and PCC values.
We developed a new theory for ROTI by noting its relationship to the structure function of phase fluctuations, which has been modeled previously (Rino, Radio Sci, 1979).

Aside from scaling and geometrical factors, we find that ROTI/$S_4$ depends on the spatial scale ($V_{\text{eff}}\delta t$) of ROTI divided by the Fresnel scale to the power $\nu^{-1/2}$.

For validation we compared predictions of $C_kL$ and $S_4$ made from 1 Hz ROTI samples with calculations made from 20 Hz intensity records. Spaced-receiver measurements of zonal drift were used to provide the effective scan velocity.

The theory produced accurate predictions for cross-field scans (more than 20° from the magnetic meridian). This geometrical requirement may be evaluated a-priori.

Some remaining errors are likely due to the dependence of ROTI/$S_4$ on Fresnel scale, which depends on irregularity altitude. Thankfully, this dependence is relatively weak.

ROTI can be more than just an irregularity indicator. 1 Hz TEC can provide quantitative diagnostics of ionospheric scintillation and the irregularities that cause them.