An Augmented Plane Wave Plus Local Orbital Program for Calculating Crystal Properties

Peter Blaha
Karlheinz Schwarz
Georg Madsen
Dieter Kvasnicka
Joachim Luitz
Robert Laskowski
Fabien Tran
Laurence Marks

November 2001
Vienna, AUSTRIA
Vienna University of Technology

http://www.wien2k.at
General remarks on WIEN2k

- WIEN2k consists of many independent F90 programs, which are linked together via C-shell scripts.
- Each "case" runs in his own directory ./case
- The "master input" is called case.struct
- Initialize a calculation: init_lapw
- Run scf-cycle: run_lapw (runsp_lapw)
- You can run WIEN2k using any www-browser and the w2web interface, but also at the command line in an xterm.
- Input/output/scf files have endings as the corresponding programs:
  - case.output1...lapw1; case.in2...lapw2; case.scf0...lapw0
- Inputs are generated using STRUCTGEN(w2web) and init_lapw
Based on www

- *WIEN2k can be managed remotely via w2web*

Important steps:

- **start w2web on all your hosts**
  - login to the desired host (ssh)
  - w2web (at first startup you will be asked for username/password, port-number, (master-)hostname. creates ~/.w2web directory)

- **use your browser and connect to the (master) host:portnumber**
  - firefox  http://fp98.zserv:10000

- **create a new session on the desired host (or select an old one)**
w2web GUI (graphical user interface)

- Structure generator
  - spacegroup selection
  - import cif or xyz file
- step by step initialization
  - symmetry detection
  - automatic input generation
- SCF calculations
  - Magnetism (spin-polarization)
  - Spin-orbit coupling
  - Forces (automatic geometry optimization)
- Guided Tasks
  - Energy band structure
  - DOS
  - Electron density
  - X-ray spectra
  - Optics
Spacegroup $P4_2/mnm$

**Structure given by:**
- **spacegroup**
- **lattice parameter**
- **positions of atoms**
  (basis)

**Rutile TiO$_2$:**
- **Spacegroup** $P4_2/mnm$ (136)
- **a** = 8.68, **c** = 5.59 bohr
- **Ti**: (0,0,0)
- **O**: (0.304,0.304,0)
Structure generator

- Specify:
  - Number of nonequivalent atoms
  - Lattice type (P, F, B, H, CXY, CXZ, CYZ) or spacegroup symbol
    - If existing, you must use a SG-setting with inversion symmetry:
      - Si: ±(1/8,1/8,1/8), not (0,0,0)+(1/4,1/4,1/4)!
  - Lattice parameters a, b, c (in Å or bohr)
  - Name of atoms (Si) and fractional coordinates (position)
    - As numbers (0.123); fractions (1/3); simple expressions (x-1/2,...)
    - In fcc (bcc) specify just one atom, not the others in (1/2,1/2,0; ...)
- "save structure"
  - Updates automatically Z, r0, equivalent positions
- "set RMT and continue": (specify proper "reduction" of NN-distances)
  - Non-overlapping "as large as possible" (saves time, may require $L^{vns}=6(8)$)
  - RMT for sp (d) - elements 10-20 % smaller than for d (f) elements
  - Largest spheres not more than 50 % larger than smallest sphere
  - Exception: H in C-H or O-H bonds: RMT~0.6 bohr (RKMAX~3-4)
  - Do not change RMT in a "series" of calculations, RMT equal for same atoms
- "save structure – save+cleanup"
Program structure of WIEN2k

- **init_lapw**
  - *step-by-step* or *batch* initialization
  - symmetry detection (*F*, *I*, *C*-centering, inversion)
  - input generation with recommended defaults
  - quality (and computing time) depends on *k-mesh* and *R.Kmax* (determines #PW)

- **run_lapw**
  - *scf-cycle*
  - optional with SO and/or LDA+U
  - different convergence criteria (energy, charge, forces)

- **save_lapw tic_gga_100k_rk7_vol0**
  - cp case.struct and clmsum files,
  - mv case.scf file
  - rm case.broyd* files
The convergence criterion in APW is the product of $R_{MT} \cdot K_{max}$

$$\Psi = \sum_{K_n}^{K_{MAX}} c_{K_n} e^{iK_n r}$$

http://www.wien2k.at/reg_user/faq/rkmax.html

Medium quality convergence for smallest atom:

- Basis set scales with $R_{Kmax}^3$
- Cputime scales with $N_{PW}^3$

Increasing $R_{Kmax}$ by 10% doubles cputime

Start with SMALL $R_{Kmax}$ (relaxation), increase/test later
BZ integration, “FERMI”-methods

- Replace the “integral” of the BZ by a finite summation on a mesh of “k-points”
  \[ \rho(r) = \sum_{n} \int_{E_n < E_F} \psi_{k,n}^{*} \psi_{k,n} \, d^3k = \sum_{k,n} w_{k,n} \psi_k^* \psi_k \]

- Weights \( w_{k,n} \) depend on \( k \) and bandindex \( n \) (occupation)
  - For full “bands” the weight is given by “symmetry”
    - \( w(\Gamma) = 1, \ w(x) = 2, \ w(\Delta) = 4, \ w(k) = 8 \)
  - Shifted “Monkhorst-Pack” mesh

- For partially filled bands (metals) one must find the Fermi-energy (integration up to NE) and determine the weights for each state \( E_{k,n} \)
  - Linear tetrahedron method (TETRA, eval=999)
  - Linear tetrahedron method + “Bloehl” corrections (TETRA)
  - “Broadening methods”
    - Gauss-broadening (GAUSS 0.002)
    - Temperature broadening (TEMP/TEMPS 0.002)
  - Broadening useful to damp SCF oscillations, but dangerous (magnetic moment)
k-mesh generation

- **x kgen** (generates k-mesh and reduces to irreducible wedge using symmetry)
  - *automatically “adds inversion”*
    - time inversion holds and $E(k) = E(-k)$
    - except in magnetic spin-orbit calculations (x –so kgen; uses case.ksym file)
  - x –fbz kgen (generates „full mesh“ in BZ)
- **always “shift” the mesh for scf-cycle**
  - gaps often at $\Gamma$ ! (might not be in your mesh)
- **small unit cells and metals** require large k-mesh (1000-100000)
- **large unit cells and insulators** need only 1-10 k-points
- use at first a fairly **coarse** mesh for scf/relaxations
- continue **later** with **finer** mesh
  - mesh was good if nothing changes and scf terminates after few (3) iterations
- use even finer meshes for DOS, spectra, optics, ...
Program execution:

- All programs are executed via the "master" shell-script `x_lapw x_lapw2 -up -orb`
- This generates a "def" file: `lapw2.def`
  
  ```
  5, 'tin.in2c', 'old', 'formatted'
  6, 'tin.output2up', 'unknown', 'formatted'
  8, 'tin.clmvalup', 'unknown', 'formatted'
  10, './tin.vectorup', 'unknown', 'unformatted'
  ```
- and executes: `lapw2c lapw2.def`
- All WIEN2k-shell scripts have long and short names:
  
  ```
  x_lapw; runsp_lapw, runfsm_lapw ➔ x; runsp; runfsm
  ```
- All scripts have a "help" switch "-h", which explains flags and options (without actually execution)
  
  `x -h     x lapw1 -h`
**scf-cycle**

- **run_lapw [options]** (for nonmagnetic cases)
  - `-ec 0.0001` convergence of total energy (Ry)
  - `-cc 0.0001` convergence of charge distance (e⁻)
  - `-fc 1.0` convergence of forces (mRy/bohr)
  - `-it (-it1,-it2 , -noHinv)` iterative diagonalization (large speedup)
  - `-p` parallel calculation (needs .machines file)
  - `-so` add spin-orbit (only after "init_so")
  - *Spacegroups without inversion use automatically lapw1c, lapw2c (case.in1c,in2c)*

- **case.scf**: master output file, contains history of the scf-cycle
  - *most information is stored with some "labels" (grep :label case.scf)*
    - :ENE  :DIS  :FER  :GAP  :CTO001  :NTO001  :QTL001
    - :FOR002: 2.ATOM 19.470 0.000 0.000 19.470
    - :FGL002: 2.ATOM 13.767 13.767 0.000 total forces
    - :LAT  :VOL  :POSxxx
Getting help

- `*_lapw --h` "help switch" of all WIEN2k-scripts
- `help_lapw`:
  - `opens usersguide.pdf`; Use `^f keyword to search for an item ("index")`
- html-version of the UG: (`$WIENROOT/SRC_usersguide/usersguide.html`)
- [http://www.wien2k.at/reg_user](http://www.wien2k.at/reg_user)
  - FAQ page with answers to common questions
  - Update information: *When you think the program has an error, please check newest version*
  - Textbook section: *DFT and the family of LAPW methods by S.Cottenier*
- Mailing-list:
  - subscribe to the list (always use the same email)
  - full text search of the "digest" (your questions may have been answered before)
  - posting questions: Provide sufficient information, locate your problem (case.dayfile, *.error, case.scf, case.outputX).
  - "My calculation crashed. Please help." This will most likely not be answered.
most common problems

- "QTL-B" value too large - STOP (or :WARN): "ghostbands"
  - identify for which eigenvalue, atom and ℓ it happens, check $E_F$
    (case.scf2, case.output2)
  - identify the corresponding linearization energies in case.scf1
  - change the corresponding linearization energy in case.in1
    - compare and check with :EPL and :EPH lines in case.scf2
    - default E-parameters are adapted automatically but may need changes for
      surfaces, molecules (negative EF) or heavy elements (EF often larger than 1.0)
    - add a local orbital (or adjust its energy)
  - if QTL-B occurs for an atom with large RMT, reduce RMT
    - this may happen for larger RKMAX ("numerical linear dependency")

- scf-cycle diverges (grep :DIS case.scf):
  - check structure (most likely a wrong structure caused divergence);
  - check E-parameters (see above), check :NEC01 (correct number of $\epsilon$)
  - rm *.broyd* case.scf; x dstart
**case.in1**

Set $E_i$ to $E_F - 0.2$ Ry

### WFFIL

- **EF = 0.634** (WFPRIL, SUPWF)

### Parameters

- **7.00** 10 4 (R-MT*K-MAX; MAX L IN WF, V-NMT)
- **0.30** 5 0 (global E-param with N other, napw)

### Energy Levels

- **0** 0.30 0.000 CONT 1 (Es)
- **0** -3.72 0.005 STOP 1 (Es-LO with search)
- **1** -2.07 0.010 CONT 1 (Ep with search)
- **1** 0.30 0.000 CONT 1 (Ep-LO)
- **2** 0.30 0.010 CONT 1 (0/1..LAPW/APW+lo)

### K-Vectors

- **K-VECTORS FROM UNIT: 4**

### Wave Functions

$$\Psi = \sum_{K_n}^{KMAX} c_{K_n} e^{iK_n r}$$

$$\Phi_{K_n} = \sum_{l}^{l_{max}} A_{lm} u_{l}(E, r) Y_{lm}$$

$$H^{NS}_{n,m} = \langle \Phi_{l} | V^{NS}_{LM} | \Phi_{l} \rangle$$
f (d) wavefunctions have a large E-dependency in cases with large RMT

For high precision calculations extend the basis set with a HDLO (high derivative LO):

\[ \Phi_{K_n} = \sum_l A_{lm}(K_n)u_l(E_l, r)Y_{lm} \]
\[ \phi_{l,\text{atom}} = (A_{lm}u_{lm} + B_{lm}\dot{u}_l)Y_{lm} \]
\[ \phi_{l,\text{atom}} = (A_{lm}u_{lm} + C_{lm}\ddot{u}_l)Y_{lm} \]

- 2 0.30 0.010 CONT 1
- 2 0.30 0.010 CONT 2
- F. Karsai et al., CPC 220, 230 (2017)
case.klist, case.in2

- GAMMA  
  0 0 0 40 1.0 IX, IY, IZ, IDIV, WEIGHT
  1 0 0 40 6.0
- ...
- X  
  40 0 0 40 3.0
- END

---

**case.in2:**

- TOT (TOT, FOR, QTL, EFG, FERMI)
  -9.0 16.0 0.50 0.05 EMIN, NE, ESEPARMIN, ESEPAR0
- TETRA 0.000 (GAUSS, ROOT, TEMP, TETRA, ALL eval)
  - 00 40 44 60 64
  - 00 40 44 60 64
  - 14. GMAX (for small H set it to 20-24)
- FILE FILE/NOFILE write recprlist

\[
\rho(r) = \sum_{LM} \rho_{LM}(r) Y_{LM}(\hat{r}) \\
\rho(r) = \sum_{G} \rho_{G} e^{iGr}
\]
Properties with WIEN2k - I

- **Energy bands**
  - *classification of irreducible representations*
  - ´character-plot´ (emphasize a certain band-character)

- **Density of states**
  - *including partial DOS with l and m- character (eg. p_x, p_y, p_z)*

- **Electron density, potential**
  - *total-, valence-, difference-, spin-densities, \( \rho \) of selected states*
  - *1-D, 2D- and 3D-plots (Xcrysden)*
  - *X-ray structure factors*
  - *Bader ´s atom-in-molecule analysis, critical-points, atomic basins and charges (\( \nabla \rho \cdot \bar{n} = 0 \))*
  - *spin+orbital magnetic moments (spin-orbit / LDA+U)*

- **Hyperfine parameters**
  - *hyperfine fields (contact + dipolar + orbital contribution)*
  - *Isomer shift*
  - *Electric field gradients*
partial charges “qtl” + DOS

- be sure to have case.vector on a dense tetrahedral mesh after a scf calculation
  - eventually:
    - x kgen
    - edit case.in1 (larger Emax)
    - x lapw1
  - x lapw2 –qtl
    \( \Psi_n^* \Psi_n = 1 = q_{out} + \sum_{at} \sum_{l} q_{t,l} \)
  - case.output
    - integrated DOS
  - case.dos1ev (3ev)
    - text-file for plotting
    - E-zero at \( E_F \)
partial charges:

- **local rotation matrix:**
  - transfers $z$ ($y$) into highest symmetry
  - reduces terms in LM series
  - “chemical” interpretation
    - $p_x$ is different from $p_y$

- see case.struct and case.outputs

- **x qtl** (instead of x lapw2 -qtl)
  - **f-orbitals**
  - qtls for **different coordinate system** (eg. “octahedral” in $\text{TiO}_2$)
  - relativistic basis ($p_{1/2}$-$p_{3/2}$ or $d_{3/2}$-$d_{5/2}$ splitting in so calculation)
  - for angular dependend TELNES (ISPLIT 88, 99)
Properties with WIEN2k - II

- **Total energy and forces**
  - *optimization of internal coordinates, (MD, BROYDEN)*
  - *cell parameter only via $E_{tot}$ (no stress tensor)*
  - *elastic constants for cubic, hexagonal, and tetragonal cells*
  - *Phonons via supercells*
    - interface to PHONON (K.Parlinski) – bands, DOS, thermodynamics, neutrons
    - interface to PHONOPY (A. Togo)
      - http://www.wien2k.at/reg_user/unsupported

- **Spectroscopy**
  - *core level shifts*
  - *X-ray emission, absorption, electron-energy-loss (with core holes)*
    - core-valence/conduction bands including matrix elements and angular dep.
  - *optical properties (dielectric function in RPA approximation, JDOS including momentum matrix elements and Kramers-Kronig)*

- **fermi surface: 2D, 3D (using XcrysDen)**
Fermi surfaces

- `xcrysden --wien_fermisurface tin.struct`
  - choose a good k-mesh (eg. 10000 points)
  - plot the FS for all bands which cross $E_F$ and compare to band structure

- for 2D plots there is also a WIEN2k-tool „fsgen“ (see UG)
- SKEAF ([www.wien2k.at/reg_users/unsupported](http://www.wien2k.at/reg_users/unsupported)): quantum oscillations
Properties with WIEN2k - II

- **Total energy and forces**
  - optimization of internal coordinates, \((MD, \text{BROYDEN})\)
  - cell parameter only via \(E_{\text{tot}}\) (no stress tensor)
  - elastic constants for cubic, hexagonal, and tetragonal cells
  - Phonons via supercells
    - interface to PHONON (K.Parlinski) – bands, DOS, thermodynamics, neutrons
    - interface to PHONOPY (A. Togo)
      - http://www.wien2k.at/reg_user/unsupported

- **Spectroscopy**
  - core level shifts
  - X-ray emission, absorption, electron-energy-loss (with core holes)
    - core-valence/conduction bands including matrix elements and angular dep.
  - optical properties (dielectric function in RPA approximation, JDOS including momentum matrix elements and Kramers-Kronig)
  - fermi surface: 2D, 3D (using XcrysDen)
Cohesive energy

\[ E_{A_x B_y}^{\text{cohes.}} = E^{\text{crystal}} - x E_A^{\text{atom}} - y E_B^{\text{atom}} \]

- \( E^{\text{crystal}} \): scalar-relativistic valence (or approx. SO)

- \( E^{\text{atom}} \): LSTART: fully-relativistic ➔ inconsistent description

➔ for heavier elements (2\textsuperscript{nd} row):

supercell with one atom in a \( \sim 30 \) bohr distorted FCC box (identical RMT, equivalent RKmax, 1 k-point, spinpolarized)
Structural optimizations:

- Lattice parameters, volume, c/a ratio only via total energies:
  - *x optimize: creates a series of “struct” files + script “optimize.job”*
    - select volume or c/a, ...
    - select number of cases and desired changes in volume (in % of \( V_0 \))
  - *edit optimize.job*
    - adapt to your need: change / uncomment various lines, eg.:
      - select different convergence parameters, parallelization, more iterations (-i 40)
      - modify “save_lapw” line (with more specific names)
      - replace “run_lapw” by “runsp_lapw” or add options (-min –fc 1 –orb)
  - *execute optimize.job*
  - *plot (analyse) the results*

- *combinations of volume and c/a are possible: 2Doptimize*
  - “x optimize” always uses case_initial.struct (if present)
  - do a “volume” optimization to create case_vol_xx.struct files
  - copy the respective case_vol_xx.struct file to case_initial.struct
  - x optimize with “c/a” for this particular volume and proceed as above.
Symmetry:

- WIEN „preserves“ symmetry:
  - \( c/a \) optimization of „cubic“ TiC:
    - change \( c \) lattice parameter in TiC.struct (tetragonal distortion, \#sym.op=0)
    - init_lapw
    - change \( c \) back to cubic
    - x optimize ...
  - „Jahn-Teller“ distortion:
    - when you start with a perfect octahedra, you will never get any distortion
    - \( \rightarrow \) start with slightly distorted positions
Supercells (impurities, vacancies, alloys)

2x2x2 = 8 atoms

(0,0,0) \( \rightarrow \) 8 atoms
(0,0,0) \( \rightarrow \) (.5,0,0) (.5,.5,0) (.5,.5,.5)
(0,.5,0) (.5,0,.5)
(0,0,.5) (0,.5,.5)

B \( \rightarrow \) 4 atoms
yes yes no no

F \( \rightarrow \) 2 atoms
yes no no yes

4x4x4 supercells: P (64), B (32), F (16) atoms

\( \sqrt{2} \times \sqrt{2} \) supercells (1 \( \rightarrow \) 2 atoms)
Supercells

- Program „supercell“:
  - start with „small“ struct file
  - specify number of repetitions in x,y,z (only integers, e.g. 2x2x1)
  - specify P, B or F lattice
  - add „vacuum“ for surface slabs (only (001) indexed surfaces)
  - shift all atoms in cell

- You must break symmetry !!! (otherwise sgroup will restore your original struct file)
  - replace (impurities, vacancies) or
  - displace (phonons) or
  - label at least 1 atom (core-holes, specific magnetic order; change “Fe” to “Fe1”; this tells the symmetry-programs that Fe1 is NOT a Fe atom!!)

- „supercell“ works only along unit-cell axes!!!
Structeditor (by R. Laskowski)

- requires octave (matlab) and xcrysden (visualization)
- allows complex operations on struct-files

```
octave
s=loadstruct("GaN.struct")
# make an orthorhombic supercell and visualize it
a=[1 0 0; 1 1 0; 0 0 2]
sout=makesupercell (s,a);
showstruct(sout);
# save it as test.struct
savestruct (sout,"test.struct");
# get help on all commands
helpstruct
```
Surfaces

- 2D-slabs with finite number of layers with „vacuum“ in 3rd dimension

**bcc (001) 7 layers:**

- (0 0 6z) (.5 .5 +/-3z) with lattice parameters: a, a, c=(3a+15-20bohr vacuum)
- (.5 .5 5z) (0 0 +/-2z) a, a, c=(3a+15-20bohr vacuum)
- (0 0 4z) shift to (.5 .5 +/-z)
- (.5 .5 3z) (0 0 0) z= a/2c
- (0 0 2z) inversion
- (.5 .5 z)
- (0 0 0)

**bcc (110):**

- orthorhombic CXY-lattice: a, \( \sqrt{2}a \), c
- (0 0 0) z=\( a/\sqrt{2}a \) c
- (0 .5 +/-z)
- (0 0 +/-2z)
Work function

\[ \text{WF} = \text{VZERO} - \text{FER} \]
(check convergence with vacuum)
Total energies and atomic forces

(Yu et al.; Kohler et al.)

- **Total Energy:**
  - *Electrostatic energy*
  - *Kinetic energy*
  - *XC-energy*

- **Force on atom $\alpha$:**

- **Hellmann-Feynman-force**
- **Pulay corrections**
  - Core
  - Valence

- expensive, contains a summation of matrix elements over all occupied states

\[
U[\rho] = \frac{1}{2} \int d^3\vec{r} \ \rho(\vec{r}) V_{es}(\vec{r}) + \frac{1}{2} \sum_\alpha Z_\alpha V_{es}^\alpha (\vec{r})
\]

\[
T[\rho] = \sum_i n_i \varepsilon_i - \int d^3\vec{r} \ \rho(\vec{r}) V_{eff} (\vec{r})
\]

\[
E_{xc}[\rho] = \int d^3\vec{r} \ \rho(\vec{r}) \varepsilon_{xc} (\vec{r})
\]

\[
\vec{F}_\alpha = -\frac{dE_{tot}}{d\vec{R}_\alpha} = F_{HF}^\alpha + F_{core}^\alpha + F_{val}^\alpha
\]

\[
F_{HF}^\alpha = Z_\alpha \lim_{m \rightarrow 0} \sum_{r_\alpha} \frac{V_{1m}^{es}(r_\alpha)}{r_\alpha} \nabla_\alpha [r_\alpha V_{1m} (\hat{r})]
\]

\[
F_{core}^\alpha = -\int \rho_{core} (r) \nabla_\alpha V_{eff} (r) \ d\vec{r}
\]

\[
F_{val}^\alpha = \int \alpha V_{eff} (r) \nabla_\alpha \rho_{val} (r) \ d\vec{r} + \sum_i n_i \sum_{K,K'} c_i^*(K') c_i(K) \times
\]

\[
\left[ (K^2 - \varepsilon_i) \int \phi_{K'}^*(r) \phi_K (r) \ dS_\alpha - i(K-K') \langle \phi_K, H - \varepsilon_i | \phi_K \rangle_\alpha \right]
\]
Optimization of internal parameters using “forces”

- Forces only for “free” structural parameters:
  - NaCl: (0,0,0), (0.5,0.5,0.5) : all positions fixed by symmetry
  - TiO$_2$: Ti (0,0,0), O (u,u,0): one free parameter (u,x,y,z)

- Forces are only calculated when using “-fc”:
  - run_lapw –fc 1.0  (mRy/bohr)
    - grep :fgl002 case.scf
      - 200. partial
      - -130. partial
      - 140. partial
      - 135 partial only $F_{HF} + F_{core}$
      - 120 partial
      - 122 partial forces converging
      - 121 partial $\rightarrow$ changes “TOT” to “FOR” in case.in2
      - -12.3 total $F_{HF} + F_{core} + F_{val}$, only this last number is correct

- Forces are useful for
  - structural optimization (of internal parameters)
  - phonons
Structure optimization (atomic positions)

- Atomic Positions
- Density
- Potential
- Solve eigenvectors values
- New Density
- Mix Density
- Converged?
- Yes
  - Forces Small
  - No
- No

Traditional way:

- Inner loop: obtain fixed-point for given atom positions
- Outer loop: optimize atomic positions
Traditional algorithm:

- Calculate SCF mapping, time $T_0$
- Broyden expansion for fixed-point problem, self-consistent density, $N_{SCF}$ iterations
- BFGS is most common for optimizing the atomic positions (Energy), $N_{BFGS}$
- Time scales as $N_{SCF} \cdot N_{BFGS} \cdot T_0$

Each step is a full SCF calculation producing accurate forces.

Structural optimization of internal parameters using “PORT”

- `/home/pblaha/tio2> min_lapw [-p -it -sp] [-j “run -fc 1 -p -it”] [-NI]`
- **performs scf-cycle for fixed positions**
- **get forces and move atoms along forces (building an approximate Hessian) and writing a new case.struct file**
- **extrapolate density (case.clmsum)**
- **perform next scf cycle and loop until forces are below „tolf“**

**CONTROL FILES:**
- `.minstop` stop after next structure change
- `tio2.inM` (generated automatically by “pairhess” at first call of min_lapw)
  - PORT 2.0 # (NEW1, NOSE, MOLD, **tolf** (a4,f5.2))
  - 0.0 1.0 1.0 1.0 # Atom1 (0 will **constrain** a coordinate)
  - 1.0 1.0 1.0 1.0 # Atom2 (NEW1: 1,2,3:delta_i, 4:eta (1=MOLD, damping))

**monitor minimization in file case.scf_mini**
- contains **last** iteration of each **geometry** step
- **each step N is saved as case_N.scf** (overwritten with next min_lapw !)
  - grep :ENE case.scf_mini
  - grep :FGLxxx case.scf_mini (:POSxxx)
Optimization of atomic positions (E-minimization via forces)

- damped Newton mechanics scheme (NEW1: with variable step)
- quite efficient quasi-Newton (PORT) scheme
  - minimizes E (using forces as gradients and construct approx. Hessian)
  - If minimizations gets stuck or oscillates: (because E and $F_i$ are inconsistent):
    - touch .minstop; min –nohess (or rm case.tmpM .min_hess)
    - improve scf-convergence (-ec), Rkmax, k-mesh, ...
    - change to NEW1 scheme

W impurity in Bi (2x2x2 supercell: Bi$_{15}$W)
Alternative method: **Fused Loop**

- Treat the **density** and **atomic positions** *all* at the same time.
- No restrictions to “special” cases, general algorithm has to work for insulators, metals, semiconductors, surfaces, defects, hybrids....
- Few to no user adjustable parameters

each step is a **single** scf cycle producing only **approximate** forces
Broyden Fixed-Point Methods

- Solve \((\rho(r,x)-F(\rho(r,x)), G)=0\)
- \(s_k = (\rho, x)_{k+1} - (\rho, x)_k; y_k = (F(\rho, x), G)_{k+1} - (F(\rho, x), G)_k\)
- Broyden’s “Good Method”
  \[B_{k+1} = B_k + \frac{(y_k - B_k s_k)s_k^T}{s_k^T s_k} \quad H_{k+1} = H_k + \frac{(s_k - H_k y_k)s_k^T}{s_k^T y_k}\]
- Broyden’s “Bad Method”
  \[H_{k+1} = H_k + \frac{(s_k - H_k y_k)y_k^T}{y_k^T y_k}\]
- Generalizable to multisecant method (better,


Comparison of the 2 methods

Larger Problems:
52 atoms, MgO (111)+H₂O               108 atoms AlFe


Lyudmila V. Dobysheva (2011)

J. Chem. Theory
Comput, DOI: 10.1021/ct4001685
run_lapw -min -fc 1.0 -cc 0.001 -ec 0.0001 [-it -noHinv -p ]
modifies case.inm and sets „MSR1a“

This runs ONE big scf-calculations optimizing the density and the positions (forces towards zero) simultaneously (may need hundreds of iterations).

Monitor: :ENE and :FR (av. and max forces, movements)

it continues until all :FR quantities are below „tolf“ (case.inM) and switches then automatically to MSR1 for a final charge optimization (with fixed positions).

quite efficient, **recommended** method, still under development by L.Marks (Northwestern Univ).
Calculations of Phonons: The Direct Method

WIEN2k + Phonon

Copyright by K. Parlinski

http://wolf.ifj.edu.pl/phonon/

Alternatively use A. Togo's PHONOPY code
(see www.wien2k.at/unsupported)
**THEORY OF DIRECT METHOD**

System energy $E$ (at $T = 0$) as a function of atomic positions $\mathbf{R}(n, \mu)$ is

$$E(\mathbf{R}(n, \mu), \ldots \mathbf{R}(m, \nu), \ldots) = E_0 + \frac{1}{2} \sum_{n, \mu, m, \nu} \Phi(n, \mu; m, \nu) \mathbf{U}(n, \mu) \mathbf{U}(m, \nu)$$

where the force constant matrix is

$$\Phi_{i,j}(n, \mu; m, \nu) = \left. \frac{\partial^2 E}{\partial \mathbf{R}_i(n, \mu) \partial \mathbf{R}_j(m, \nu)} \right|_0$$

is defined at $\frac{\partial E}{\partial \mathbf{R}_i(n, \mu)} |_0 = 0$.

The dynamical matrix is defined as

$$\mathbf{D}(k; \mu, \nu) = \frac{1}{\sqrt{M_\mu M_\nu}} \sum_{m} \Phi(0, \mu; m, \nu) \exp\{2\pi i \mathbf{k} \cdot [\mathbf{R}(0, \mu) - \mathbf{R}(m, \nu)]\}$$

$m$ runs over all atoms. Diagonalization of the dynamical matrix

$$\omega^2(k, j) \mathbf{e}(k, j) = \mathbf{D}(k) \mathbf{e}(k, j)$$

gives phonon frequencies $\omega^2(k, j)$ and polarization vectors $\mathbf{e}(k, j)$.

Any atomic displacement $\mathbf{U}(m, \nu)$ generates forces

$$\mathbf{F}(n, \mu) = -\frac{\partial E}{\partial \mathbf{R}(n, \mu)}$$

on all other atoms. Hence

$$F_i(n, \mu) = -\sum_{m, \nu, j} \Phi_{i,j}(n, \mu; m, \nu) U_j(m, \nu)$$

Master equation of direct method.
CUMMULANT FORCE CONSTANTS

Displace an atom by \( U(m, \nu) \)

\[
F_i(n, \mu) = - \sum_L \Phi_{i,j}(n, \mu, m + L, \nu)U_j(m, \nu)
\]

\( L = (L_a, L_b, L_c) \) are the indices of supercell lattice constants.

or

\[
F_i(n, \mu) = -\Phi^{(\Sigma)}_{i,j}(n, \mu, m, \nu)U_j(m, \nu)
\]

where the **cummulant force constant** is

\[
\Phi^{(\Sigma)}_{i,j}(n, \mu, m, \nu) = \sum_L \Phi_{i,j}(n, \mu, m + L, \nu)
\]

\( L \) runs over all supercell images.
Supercell dynamical matrix. Exact wave vectors.

Conventional dynamical matrix:

\[ D(k; \mu, \nu) = \frac{1}{\sqrt{M_\mu M_\nu}} \sum_m \Phi(0, \mu; m, \nu) \exp\{-2\pi i k \cdot [R(0, \mu) - R(m, \nu)]\} \]

Supercell dynamical matrix:

\[ D^{(SC)}(k; \mu, \nu) = \frac{1}{\sqrt{M_\mu M_\nu}} \sum_{m \in SC} \Phi^{(SC)}(0, \mu; m, \nu) \exp\{-2\pi i k \cdot [R(0, \mu) - R(m, \nu)]\} \]

These two matrices are equal if

\[ D^{(SC)}(k; \mu, \nu) = D(k; \mu, \nu) \]

- **interaction range** is confined to **interior** of supercell (supercell is big enough)
- wave vector is **commensurate with the supercell** and fulfills the condition (independent of interaction range):

\[ \exp\{-2\pi i k_s \cdot L\} = 1 \]

At wave vectors \( k_s \) the phonon frequencies are “exact”, provided the supercell **contains the complete list of neighbors**.

Wave vectors \( k_s \) are commensurate with the supercell size.
Exact wave vectors

1x1x1

Exact: $\Gamma$

2x2x2

Exact: $\Gamma, X, M, R$

3x3x3

Exact: $\Gamma$

Exact: $X$

Exact: $\Gamma$

Exact: $M$

Exact: $\Gamma$

Exact: $\Gamma$

Exact: $\Gamma$

Exact: $\Gamma$
Phonon dispersions + density of states

GeO$_2$ P4$_2$/mnm

Frequency $\omega$

Total + Germanium

Total + Oxygen
Thermodynamic functions of phonon vibrations

Internal energy:
\[ E = \frac{1}{2} r \int_0^\infty d\omega \, g(\omega) \left( \frac{\hbar \omega}{2k_B T} \right) \coth \left( \frac{\hbar \omega}{2k_B T} \right) \]

Free energy:
\[ F = r k_B T \int_0^\infty d\omega \, g(\omega) \ln \left[ 2 \sinh \left( \frac{\hbar \omega}{2k_B T} \right) \right] \]

Entropy:
\[ S = r k_B \int_0^\infty d\omega \, g(\omega) \left\{ \left( \frac{\hbar \omega}{2k_B T} \right) \left( \frac{\hbar \omega}{2k_B T} \right) - 1 \right\} - \ln \left[ 1 - \exp \left( - \frac{\hbar \omega}{k_B T} \right) \right] \]

Heat capacity \( C_v \):
\[ C_v = r k_B \int_0^\infty d\omega \, g(\omega) \left( \frac{\hbar \omega}{k_B T} \right)^2 \frac{\exp \left( \frac{\hbar \omega}{k_B T} \right)}{\left[ \exp \left( \frac{\hbar \omega}{k_B T} \right) - 1 \right]^2} \]

Thermal displacements:
\[ B_{ij}(\mu) = \langle U_i(\mu) U_j(\mu) \rangle \]
\[ B_{ij}(\mu) = \frac{\hbar r}{2M_\mu} \int_0^\infty d\omega \, g_{ij,\mu}(\omega) \frac{1}{\omega} \coth \left( \frac{\hbar \omega}{2k_B T} \right) \]
**PHONON-I**

- **PHONON**
  - by K.Parlinski (Crakow)
  - Linux or MS-windows
  - uses a „direct“ method to calculate Force-constants with the help of an ab initio program
  - with these Force-constants phonons at arbitrary k-points can be obtained

- Define your spacegroup
- Define all atoms

Phonons:

- **selects symmetry adapted atomic displacements** (4 displacements in cubic perovskites)

  ![Displacement pattern for cubic perovskite](image)

- **select a supercell:** (eg. 2x2x2 atom P-type cell)

- **calculate all forces for these displacements with high accuracy** (WIEN2k)

  - → **force constants** between all atoms in the supercell
  - → **dynamical matrix** for arbitrary q-vectors
  - → **phonon-dispersion** (“bandstructure”) using PHONON (K.Parlinski)
PHONON-II

- Define an interaction range (supercell)
  - Create displacement file
  - Transfer case.d45 to Unix
- Calculate forces for all required displacements
  - init_phonon_lapw
    - For each displacement a case_XX.struct file is generated in an extra directory
    - Runs nn and lets you define RMT values like:
      - 1.85 1-16

- init_lapw: either without symmetry (and then copies this setup to all case_XX)
  or with symmetry (must run init_lapw for all case_XX) (Do NOT use SGROUP)
- run_phonon: run_lapw –fc 0.1 –i 40 for each case_XX
analyze_phonon_lapw

- reads the forces of the scf runs
- generates "Hellman-Feynman" file case.dat and a "symmetrized HF-file case.dsy (when you have displacements in both directions)
  - check quality of forces:
    - sum $F_x$ should be small (0)
    - abs($F_x$) should be similar for +/- displacements

- transfer case.dat (dsy) to Windows
- Import HF files to PHONON
- Calculate force constants
- Calculate phonons, analyze phonons eigenmodes, thermodynamic functions
Applications:

- phonon frequencies (compare with IR, Raman, neutrons)
- identify dynamically unstable structures, describe phase transitions, find more stable (low T) phases.
- free energies at T>0; quasiharmonic approximation

Pyrochlore structure of Y$_2$Nb$_2$O$_7$: strong phonon instabilities → phase transition