Abstract Expressionism and Fractal Geometry

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1 Matter Paintings

The recent discovery of the trove of paintings in Herbert Matter’s collection has, among other unforeseen consequences, brought the debate surrounding the fractal nature of Pollock’s work to the fore. Although the circumstances related to their discovery establish, by themselves, a compelling provenance for the newly found pieces, the question of their authenticity remains unresolved, a predicament drawing scholars from outside the traditional orbit of art historical analysis into the fray.¹

The Pollock/Krasner foundation provided the catalyst. Representative of the Pollock Estate, the foundation (as articles published in The New York Times² and Nature³ revealed) commissioned physicist Richard Taylor to analyze six of the new paintings using sophisticated image-processing techniques. Taylor had claimed to have discovered fractal patterns in Pollock’s poured abstractions in the past, and even declared such patterns to be so exclusive to the artist’s production as to provide a conclusive means of resolving questions of attribution.⁴ Intriguingly, although Taylor and his collaborators published extensively on Pollock and fractals since 1999, the art historical community had all but ignored his findings. Apprehensive, perhaps, about treading on the domain of science, art historians left Taylor’s invitation to ponder the broader implications of his discovery unanswered.⁵ Once the Matter controversy erupted, however, fractal analyses were suddenly seen as highly relevant; and, even then, only because of their perceived efficacy in determining whether a given work may or may not be authenticated as a Pollock.

At the behest of the foundation, Taylor thus set out to determine whether fractals were also manifest in the Matter pieces. Fractals, as will be elucidated in greater detail below, are
mathematically regular, visually complex structures of unusual geometry. As of this writing, the full extent of Taylor’s analysis has not been made public, but, in the press, Taylor offered his opinion that “significant deviations” existed between the Matter Paintings and already authenticated Pollocks.

A flurry of commentary in newspapers and magazines ensued. While Randy Kennedy titled a New York Times article: “Computer Analysis Suggests Paintings are Not Pollocks,” Alison Abbot, writing in Nature, concluded: “The results may be enough to cast doubt on the value of Matter’s finds...” In the New York Times Op-Ed pages, moreover, Don Foster tentatively summarized the implications of Taylor’s statements this way: “computer-assisted analysis...discovered that the paintings may well be fakes.” Extrapolating from the same fragmentary information, a number of art critics also felt empowered to view fractal analysis as decisive in authenticating works of art. Not everyone was convinced, however, that such an issue could be so easily resolved. “What determines authenticity for me,” Peter Schjeldahl suggested in The New Yorker, “is a hardly scientific, no doubt fallible intuition...” Along similar lines, John Haber cautioned that “technical analysis...can lead eager defenders of art or science to mistaken conclusions.”

Given that this debate does indeed lie at the intersection of art and science, perhaps a certain degree of controversy and confusion was inevitable. But the paucity of information available and the narrow scope of the discussion have only compounded the problem, leaving crucial questions unanswered. Which six Matter Paintings were analyzed? What were the actual results? Is fractal analysis a strictly codified mode of investigation? How reliably does it function as an authentication tool? What can fractal analysis contribute, specifically, to our understanding of works of art in general, and Pollock’s abstractions in particular? Is a mathematical concept such as “fractal” even applicable to a material object such as a painted pattern? And, more importantly, are there broader implications of fractal analyses over and beyond the sole issue of authentication?

These questions are of serious concern to scholars and academics; but they are also of considerable interest to a wider public. A number of collectors, gallery owners, and even lawyers have, for instance, contacted the authors of this essay, some seeking to understand the meaning of the term “fractal,” some wondering about the significance of Taylor’s findings, and others anxious over the possibility that hitherto secured attributions will now be open to revision. The whole inquiry into the fractal nature of Pollock’s work opens, therefore, a conversation well beyond the scope of the commentary to which it has been restricted thus far.

II Fractal Authentication

In order to answer these questions, it is imperative to define, from the outset, what mathematicians and physicists mean by the term “fractal.” The concept is a subtle one. To qualify as fractal, a pattern on a flat surface must display three distinct properties. The first is progressively fine detail (i.e., new features appearing at ever smaller scales). The second is “self-similarity” (i.e., the emergence of similar features, at least in a statistical sense, at increasing magnifications). And the third is non-Euclidean character, specifically, a geometrical dimension greater than 1 but smaller than 2, a fraction like 3/2 or 7/4—a fractal dimension. Fractal dimension, in effect, measures a pattern’s complexity, or, to be more precise, a pattern’s density at various scales, with a simple line as the lower limit of the spectrum, having a Euclidean (integer) dimension of 1, and a plane in its entirety as the higher limit, having a Euclidean dimension of 2.

Mathematicians have defined, constructed, and studied the properties of fractal objects from the late nineteenth century onward; among the most famous examples is the Koch curve devised in 1904—an infinitely repeating form resembling the edge of a hyperfine snow flake. Within the last two decades, both mathematicians and physicists have paid increasing attention to fractals since they display intricate configurations and often resemble forms found in nature: trees with branches of decreasing size sprouting in seemingly random directions; ocean waves, from huge swells down to tiny water ripples; or coastlines with ever smaller topographical features. “Fractals are of great interest,” Steven Strogatz, an applied mathematician, writes, “because of their exquisite combination of beauty, complexity, and endless structure.” They typically arise in systems—whether natural or mechanical—whose dynamics (or evolution) is governed by a few deterministic rules, but also inflected by unpredictable variations in external circumstances. This class of phenomena is the purview of a new branch of physics called Chaos Theory (and, even more recently, complexity theory). These theories aim, in particular, to explain qualitatively the dynamics of seemingly intractable natural processes—for example, how tree branches grow, following biologically encoded information, but also responding to minute fluctuations of temperature, water supply, and light (this phenomenon is termed “sensitive dependence on initial conditions”). And the same description, significantly, applies to Pollock’s poured technique, a process that combines the deterministic physics of paint falling under gravity with both the improvisational character of the gestures the artist makes while pouring, and the inherent fluid instabilities of his streams of paint. This convergence suggests that the analytic tools of Chaos Theory may indeed be brought to bear upon the study of Pollock’s work. Although he died years before fractal
geometry was invented, and fractals were perceived in nature, the artist—merely by incorporating chaotic dynamics in his painting process, and crafting “allover” compositions without conspicuous centers of attention—was already creating conditions conducive to the emergence of fractals.

Even so, is it possible for the totality of Pollock’s production to qualify as fractal? Probably not. First, the majority of Pollock’s paintings are conventionally painted rather than poured. Second, the three criteria mentioned above are particularly exacting and difficult to satisfy in a painted image, even in the poured abstractions. Unlike mathematically-generated fractals, moreover—which display fine details ad infinitum—paintings and naturally occurring structures qualify as fractal only in an approximate way. Beyond a certain level of magnification, after all, the visual patterns in a work of art (or natural form) will no longer reveal new details. In the computer analyses hitherto published, including those by another group of physicists, J. R. Mureika and his collaborators, some license was perf ormed applied whenever Pollock’s patterns were analyzed (a point to be elaborated upon later).

But irrespective of the detailed algorithm adopted to discern fractals in art, or of the different strategies employed by Taylor and Mureika to accommodate art’s finite complexity, not all of Pollock’s abstractions will betray characteristics required of fractals, no matter how loosely the definition is applied. Looking through the artist’s Catalog Raisonné, one may conjecture that at least 10 percent of the poured abstractions—and an even larger percentage for the poured works on paper—do not even display sufficiently fine detail, the first of three conditions required of fractals (nor visually-discernable self-similarity, the second condition). Not surprisingly, among the seventeen authenticated Pollocks Taylor had previously selected for analysis, four apparently yielded inconclusive results. More recently, Katherine Jones-Smith and Harsh Mathur have found that Wooden Horse: Number 10A, 1948, an accepted work in the Pollockian canon, “fails to satisfy the fractal authentication criteria” employed by Taylor. Our own recent analysis adds another such example (see below) and surely more can be found. Along the same lines, it is evident—simply from their appearance—that at least eight of the Matter Paintings would not reveal new details at any magnification and thus fail to meet the conditions for fractality. Accordingly, even if the six Taylor analyzed turn out not to be fractal, such a determination, by itself, would hardly settle the issue of their authenticity. And since that number comprises approximately one third of the lot, the results obtained cannot necessarily be construed as emblematic of the entire set, especially as Jones-Smith and Mathur have found at least one Matter Painting (fig. 1) that, by Taylor’s standards, does qualify as fractal.

Questions concerning the authorship of the Matter Paintings have also been raised because the six pieces analyzed—and, presumably, their fractal characteristics—are purportedly “extremely varied” (our own preliminary analysis is presented below). On this basis, Taylor surmises that they may have been created by different hands. Given the wide visual and material disparity among the newly found pieces (figs. 2a and 3a; figs. 2b and 3b will be discussed below), this is a distinct possibility. But it should also be remembered that Pollock’s entire production is also “extremely varied.” Even within the tiny sample of authenticated Pollocks he examined, Taylor himself reported their fractal dimensions to span nearly the entire possible range, roughly from 1.1 to 1.9. In fact, he readily conceded that the “extreme” variation among the Matter Paintings may not necessarily be anomalous. “It could be, for example,” he conjectured, “that Pollock chose to deviate with these paintings.” All the same, the meaning of “deviate” was never explained. And “deviate” from what? Depending on the criteria applied, and the set of paintings considered “typical,” many a well-known but “atypical” Pollock canvas might also be considered “deviant.” The artist’s production is in fact so variegated that even paintings created during any single year of Pollock’s mature phase would likely yield a broad range of measurements—consider, for example, the remarkable diversity during 1950 alone.

As already related in “Cutting Pollock Down to Size” in this volume, Taylor’s group reported that the Pollock canvases they analyzed were not just fractal, but also double-fractal, possessing not just one, but two characteristic dimensions (each extending across a different range of scales). This point is of considerable significance. The discrete dimensions are not just numerically different; they arguably “record” the two different physical aspects of Pollock’s creative process mentioned above (the artist’s arm movements and the instabilities of his paint). Since Taylor draws conclusions about authenticity on the basis of his fractal measurements, it would have been crucial to know whether he found the limited number of Matter Paintings he examined to be non-fractal, fractal, double-fractal, or any combination thereof.
Fig. 2a. *Untitled no. 4*, poured media on board, 8 7/8 × 9 3/4 in. Private Collection

Fig. 2b. *Untitled no. 4* Scaling Plot

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Fig. 3a. *Untitled no. 18*, poured media on board, 7 3/16 × 9 13/16 in. Private Collection

Fig. 3b. *Untitled no. 18* Scaling Plot
But reliance on detecting double-fractals to authenticate Pollock’s paintings faces another daunting obstacle. Combining an approach comparable to Taylor’s with other modes of computer-based analysis, Mureika and his group have detected fractal patterns in the work of other abstract painters such as the Québec Automatiste Jean-Paul Riopelle. The inescapable conclusion, they argue, is that “the fractal dimension of drip paintings is not unique to any one artist and cannot be used for any...authentication scheme.”

In order to differentiate Pollock’s production from that of contemporary artists using comparable techniques, it is therefore necessary, Mureika and his collaborators submit, to take an additional step: a detailed study of what they calls “edges.” By “edges,” they mean luminance gradients between adjacent colors, essentially a measure of the rate with which brightness varies as position changes along the canvas. Although this method seems to provide additional information beyond conventional fractal analysis, it has yet to be adopted by other researchers.

Against the backdrop outlined above, any disparities discerned among the Matter Paintings, or between these works and accepted Pollocks (which may be attributable to their comparatively small dimensions), cannot, by themselves, be considered determinative in resolving the issue of their authenticity. Since many accepted Pollock paintings do not have fractal structure, and, conversely, since some paintings by other artists ostensibly do, fractal characteristics cannot be considered exclusive to Pollock.

### III Scaling Analysis

This conclusion does not automatically mean that fractal, or, more broadly, scaling analyses, are irrelevant to, or cannot enrich, our understanding of Pollock’s poured abstractions. It does mean, however, that any attempt to address the questions raised at the outset involves separating the broader conversation on empirical fractals from the narrower issue of authentication. It also means that the tools of fractal geometry need some recalibration before they can be applied to finite, material objects.

Perhaps the most fundamental challenge in analyzing paintings or naturally occurring patterns lies in obtaining reliable data. Counterintuitive as it may sound, computational analyses designed to search for scaling regularities (including fractals) are not entirely free of subjectivity, particularly when it comes to the problems of color separation and differentiating pattern from background. The standard approach, the so-called box-counting method, proceeds as follows. A computer program superimposes successively finer grids on the pattern in question (a Pollock abstraction, a snowflake, or a tree), and, at each step, “counts” the number of grid cells (or boxes) containing any part, however small, of the pattern. This process, as applied to a photograph of a tree, is illustrated in fig. 4 (a red rectangle indicates the fragment selected).

The data is then graphed on a plot designed to reveal the scaling properties, if any, of the pattern—basically a plot of the number of “occupied” cells against the dimension (or scale) of the individual cells. Logarithmic scales are employed on each axis of these plots in order to amplify any regularity in the pattern’s scaling behavior. This approach, however, requires that a criterion be established to differentiate foreground (pattern) from background (field) by setting a threshold intensity value: namely, the lowest (or lightest pigment) value used to determine whether a pixel is “noticeably” part of the pattern (for more details, see Appendix). In our own analyses, we found that, even for monochromatic images, the data obtained varied considerably depending on the threshold chosen. Similarly, Mureika, Dyer, and Cupchik have stressed that their fractal readings also hinged on the threshold selected, which they designate the “cutoff value.” Depending on the cutoff value chosen, the pattern yielded different measurements and, hence, the fractal dimension for each individual piece fluctuated by as much as 20%. The “selection criteria,” they point out, “must be extremely well defined. Otherwise, the results risk becoming meaningless.” In other words, while computer analyses, in general, are often touted as “a more objective approach... than [say] the conflicting opinions of art historians,” the entire process is not completely devoid of subjectivity.

Investigating the scaling properties of multi-layered, polychrome paintings involves overcoming additional impediments. Different color layers may be analyzed separately, in which case one faces the challenge of interpreting a set of discrete readings, or in combination, in which case much of the detailed information is lost. Since Pollock’s compositions comprise a complex interweaving of overlapping skeins of identical or different colors, additional problems arise. If Pollock reapplied the same color during a subsequent campaign, different layers may amalgamate to such an extent as to become nearly indistinguishable; and, if two different colors overlap, one will invariably obscure the other. In either case, the scaling analysis will be affected to some degree. As Jones-Smith and Mathur have pointed out, a partially covered fractal pattern, just as the amalgamation of two or more overlapping fractal patterns, may no longer prove fractal. Separate layers of the same composition, furthermore, need not have the same scaling behavior as the composition in its entirety, nor the union of two patterns the same regularity as its constituent parts. Thus, quantitative investigations of Pollock’s abstractions call for well-defined strategies, with all their intrinsic limitations acknowledged, and clarity regarding precisely what is being measured.
The next—perhaps even more delicate—step in pattern analysis consists of interpreting the plots obtained via the box-counting procedure described above. If the data points fall exactly on a straight line (corresponding to a simple “power-law” scaling behavior\(^{35}\)), the pattern is deemed to be fractal, and the slope of that line is interpreted as its fractal dimension. Consider, for example, a perfect fractal called Sierpinski Gasket, created by a computer (fig. 5a). This pattern is generated by starting with an equilateral triangle (here, black on white background), dividing it into four similar equilateral sub-triangles, removing the central sub-triangle from the pattern (i.e., converting it from black to white), then subdividing the remaining three smaller triangles, each into four sub-triangles, removing the central sub-triangle in each of the four, and so on ad infinitum. The resulting pattern has, by virtue of this recursive construction, infinitely fine detail and is manifestly self-similar since it repeats ever smaller equilateral triangles. For such a mathematical structure, the box-counting plot shown in blue in fig. 5b appears to form the straight line requisite for fractals.

In principle, one could stop the analysis here, especially in the case of mathematically generated fractals. But, since deviations from simple, straight-line regularity may be difficult to discern in plots for other patterns, the present study introduces a reploting of the data in a way that amplifies any such deviations by directly indicating the slope (steepness) of the curve at each point. To this end, a “derivative” plot, here shown in black, is employed. It indicates the slope along the original line (in blue), or, to put it differently, gives the steepness of the line at each local position.\(^{36}\) The black curve, in other words, although extrapolated in two steps from the original pattern, nevertheless allows for a more readily intelligible display of the actual trends in the data. In the case of the Sierpinski Gasket, the derivative curve is a horizontal line whose (constant) slope is about 1.585 (see the coordinate on the left axis of the plot), indicating the pattern’s fractal dimension—notably, a fractional number between 1 and 2. The slight “bumpiness” of the black line is a result of approximations inherent to the computer calculations (some technical details about the algorithm employed can be found in the appendix).

The fractality of any pattern, therefore is discerned only after the computer-generated data is shown to obey a uniquely simple scaling regularity—represented by a horizontal derivative line, as in fig. 5b—not simply verified, as one would the presence of a certain chemical element in a material sample. Scaling plots for exact mathematical fractals will, by definition, form perfectly straight lines (ignoring any anomalies stemming from the computational algorithm). But a material, empirical object that may display fractal characteristics—such as a well-developed (leafless) tree or a Pollock abstraction—will result in a scaling plot with data, at best, falling approximately along a single straight line. Extrapolating a configuration from the data thus involves something of a judgment call. The closer the empirical object is to a fractal, the more exactly the corresponding data will fall on a straight line; the farther, the least likely it is to do so. In fact, a pattern may exhibit a scaling behavior of an altogether different kind, i.e., a line of varying curvature.\(^{37}\) Although such an inconstant scaling plot indicates that the pattern in question is not fractal, it nonetheless establishes a particular “scaling signature,” if by “signature” we mean a recognizable class, not a portent of authenticity. Naturally occurring shapes of some complexity (e.g., edges of clouds, tree branches, coastlines, etc.) are in fact more likely to be characterized by scaling plots forming curved rather than perfectly straight lines.

Deciding whether these forms can legitimately be designated as fractal, therefore, involves assessing how closely the plots approximate straight lines.

But how close is close enough? The answer will invariably be a matter of subjective judgment. The photograph of a tree in fig. 6a, a two-dimensional representation of a three-dimensional object, results, for example, in a scaling regularity with a noticeable deviation from the best fit continuous straight line shown in blue in fig. 6b. The
best fit straight line is established by the five data points at the smallest scales, shown as blue asterisks in fig. 6b. In this case, the derivative—or local fractal dimension—graph (in black) is no longer a clear horizontal line, but a gently rounded curve with two plateaus, one at small and one at large scales. Not a perfect fractal with a single fractal dimension to be sure, but a curve with the local fractal dimension falling in the relatively small range between 1.7 and 2. Full-grown trees, in general, are likely to yield qualitatively similar scaling plots, as will other natural patterns formed by sufficiently complex networks of ever smaller branches and sub-branches, such as vascular systems, neural networks, or river tributaries.

In order to interpret and differentiate individual derivative plots from other such plots, it is essential to determine the limits within which derivative plots may vary depending on their underlying regularity; or, to compare them to plots at the two extremes of the available range: highly organized fractal structures (with exact self-similarity) on the one hand, and completely disorganized structures (without any discernable regularity), on the other. The contrast will perhaps be most evident if one selects “disorder” itself, in other words, a pattern created by randomly distributed dots or disks, such as those shown in fig. 7a. This haphazard distribution does not have any underlying order beyond a mathematically-defined unpredictability. Nevertheless, the corresponding fractal dimension graph in fig. 7b is smooth and distinct. Since a scaling plot represents a global, statistical measurement of the rate at which pattern density varies with scale, it will tend to “smooth out” any actual irregularities of the pattern, as is the case here. All the same, the local fractal dimension varies greatly, from about 0.7 to 2, a range that may yet be increased by reducing the size of the dots. Regardless, while it is not possible for a real, empirical pattern to betray a perfectly fractal scaling behavior, one may posit, by comparing plots in figs. 5b, 6b, and 7b, that the scaling property of a tree resembles that of a fractal—witness the small range of local fractal dimensions—but with some inflection reminiscent of noise or randomness. For the lack of an appropriate term, we will call this kind of scaling signature an arcfactal, referencing the “arc” formed by the derivative plot.

Admittedly, the presence or kind of regularity underlying the data is, at least to some degree, in the eye of the beholder, leaving the question of interpreting the pattern’s scaling behavior open to debate. In this context, it should be reiterated that scaling plots are always smooth, descending curves (i.e. monotonic functions), most often with slopes limited to a narrow range. As a result, interpreting any scaling curve as a straight line (or a combination of two straight lines) and thus designating the pattern as fractal (or double-fractal) would be unconvincing, because even a decidedly non-fractal pattern generates a nearly linear scaling curve. To provide a more discriminating diagnostic tool, derivative plots (black curves) were added in the graphs published here. Such plots supply the very information required of scaling analysis: namely, how the local fractal dimension varies continually across scales. Given these advantages, we suggest that derivative plots be included as a standard mode of analyzing patterns in art or nature.

These refinements notwithstanding, computational analyses of empirical patterns—either of paintings or naturally occurring forms—may still meet with skepticism. As already indicated, fractals require a simple scaling behavior, manifested by a straight line in the scaling plot. But a bona fide scaling behavior, whether fractal, arcfactal, or any other, requires that the same regularity be manifest in the scaling plot over a wide range of scales, or spatial dimensions. And, just as fractal structures are expected to harbor new details at finer and finer magnification, reliable scaling signatures must likewise extend from the larger to the smaller features of the pattern. The level of fine detail actually needed to ascertain the consistency of the scaling behavior for a limited-range empirical form has been the subject of some debate within the physics community, particularly in the 1990’s. Even so, a consensus seems to have emerged that scaling behavior can usefully be claimed only for objects displaying features spanning at least three orders of magnitude—in other words, patterns whose largest units are at least 1000 times bigger than their smallest.

In this context, the key question to ask about Pollock’s patterns is whether they do indeed reveal well-resolved scaling regularity over a sufficiently wide range of scales to be usefully described as having a distinct scaling signature. And, if so, what does that signature suggest? These are delicate questions. Pollock’s most imposing compositions include features spanning some four orders of magnitude in size. The largest whiplash curves in Number 32 or Autumn Rhythm, for example, are multi-meter in length, whereas the thinnest skeins are submilimeter in width (this range, roughly from 5 meters down to 0.5 millimeters, corresponds to four orders of magnitude, since 0.5 mm × 10,000 is 5,000 mm or 5 meters). But Pollock’s work is typically much smaller (see “Cutting Pollock Down to Size” in this volume), usually ranging over only three orders of magnitude. More typical dimensions would limit the largest features of the work to under 0.5 meters.

The extent of these parameters bears directly on the discussion at hand. If Pollock’s abstract patterns can barely accommodate a single scaling behavior, can they be rigorously described as double-fractal? In a recent note in Nature, Katherine Jones-Smith and Harsh Mathur argue that Pollock’s paintings “exhibit fractal
characteristics over too small a range to be usefully considered as [double] fractal" since “less than two orders of magnitude” are available to establish each scaling rule.\textsuperscript{39} In other words, while both Taylor’s and Mureika’s groups report discovering discernable scaling behavior in some of the selected Pollocks they analyzed, and while there are physical reasons to expect different scaling behavior at the smallest and the largest scales (determined by the fluid instabilities of the paint and the artist’s gestural dynamics, respectively), claiming his paintings to be fully-developed double-fractals stretches the definition beyond its accepted limits.

IV Fractal Affinities

Ten years after the first paper on the subject, the debate on the scaling characteristics of Pollock’s patterns, far from being settled, is just now beginning in earnest. Only three groups of researchers have published on the artist’s poured abstractions thus far, and their results pertain only to a dozen works or so, barely 5% of his production in this genre (see “Cutting Pollock Down to Size”). And even for this small number of select paintings, very few scaling plots have appeared in print. Any generalizations, therefore—either about the implications of these plots, or whether they are representative of Pollock’s pourings as a whole—would be premature. Restricting the extent of the debate to whether Pollock’s abstractions are fractal or not, has hardly been conducive to reaching any sort of consensus. For some, the artist’s poured abstractions unambiguously qualify as double-fractals whose dimensions can be reliably measured (and even used for authentication or dating purposes). For others, patterns whose elements span only three or fewer orders of magnitude (including the majority of Pollock’s works as well as a large number of natural forms hitherto described as fractal) hardly meet the conditions required of fractals.

Although these positions seem irreconcilable, this much is clear. On the one hand, the objection that Pollock’s compositions exhibit a range too limited to accommodate two separate fractal scalings is sound. And, as was noted earlier, a certain percentage of Pollock’s abstractions clearly do not qualify as fractal, however loosely the term is defined. On the other, since perhaps as many as one quarter of the poured abstractions display sufficiently fine detail and span at least three orders of magnitude, they may, at least in principle, generate meaningful scaling plots. Based on the study recently undertaken by the authors, a new approach—straddling, as it were, a middle ground—offers a compromise. Even if it is highly unlikely for all works in the fraction of Pollock’s production just mentioned to exhibit an easily interpretable regularity, let alone prove unambiguously fractal, deciphering their particular scaling properties, comparing the plots qualitatively to one another—as well as to other works of art—and determining whether they bear any affinity to fractals, might set a fruitful dialogue into motion.

In this spirit, it may be advantageous to shift the terms of the conversation from whether any particular subset of Pollock’s production can be usefully described as fractal, to the broader and more open-ended inquiry into the scaling properties of poured abstractions, irrespective of the label one may want to assign to them. To put this new approach into practice, and in order to test its effectiveness, it is advisable to chose a large, fully-developed and resolved poured abstraction, one that adheres strictly to the dictates of all-over composition and displays fine detail over the cascade of ever-smaller scales. Further, to avoid problems arising from multilayered, polychrome patterns—the need to separate overlapping skeins of different color, or the dilemma of how to treat results obtained for each color layer—a monochromatic painting satisfying all the requirements just listed was chosen: \textit{Number 32 (1950)} at the Kunstmuseum Nordrhein-Westfalen in Düsseldorf (fig. 8a). The scaling plot and the corresponding local fractal dimension

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig8a.png}
\caption{Pollock at Work, 1950 (Number 32 is clearly visible in the background) Photo: Hans Namuth, Courtesy Center for Creative Photography, University of Arizona. ©1991 Hans Namuth Estate}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig8b.png}
\caption{Number 32 Scaling Plot}
\end{figure}
graph are shown in fig. 8b, where a straight line fit (defined, as before, by five data points for small scales) is also included.

The plots for Number 32 invite the following observations. First, the scaling curve shows a pronounced deviation from a straight-line fit, indicating that the pattern is clearly departing from an unambiguously fractal regularity. Second, the fractal dimension (or slope) curve confirms that the pattern is not exactly fractal, but nonetheless displays a marked regularity, a scaling signature for the painting. Third, the scaling curves resemble those of fig. 6b, with a similar deviation from a straight-line scaling plot, and a similar fractal dimension curve as that for a tree—in other words, an arcfractal with its characteristic two plateaus smoothly connected by a gently sloping line. By introducing such a sensitive analytical tool as the derivative plot, it is thus possible, even on the basis of one painting, to ponder the affinities between Pollock’s work and the patterns created in nature. Since it would be premature to draw over-reaching conclusions from the limited evidence presented here, more studies, clearly, are required before this connection may be fully explored. Still, the preliminary results are encouraging, and, in retrospect, even opposite. Like Pollock’s abstractions, naturally occurring “fractal” forms, after all (such a tree with ever smaller branches, or a snowflake with ever smaller dendrites), also span about three orders of magnitude, and will, in any case, likely yield a curving rather than a perfectly straight scaling plot. Further, since the artist’s practice of pouring paint incorporated fluid dynamical processes commonly occurring in natural phenomena, it should come as no surprise if Pollock’s patterns were only as “regular” as those already discerned in nature. This correlation was hardly lost on the artist himself. Over and beyond the technique he employed, a connection to nature is also consistently evoked in many of the titles he assigned to (or accepted for) his canvases: Autumn Rhythm, Watery Paths, Sea Change, The Nest, Comet, and so on. On this score, the affinity between the scaling regularity found in his work and that underlying many naturally occurring patterns is all the more striking.

Even so, one should not expect all poured paintings to display an arcfractal scaling signature similar to that of Number 32. As was already mentioned, Pollock’s works display remarkable variation. To make this point all the more emphatic, consider a manifestly non-fractal poured painting, such as Untitled 1948-49 (fig. 9a). Qualitatively different from those obtained from Number 32, the plots in fig. 9b confirm our initial visual impression. In fact, they are very nearly diametrically opposite, with the scaling graph in fig. 9b rising above the straight-line fit (both in blue), and the fractal dimension curve sloping down in the direction of increasing scales. Intriguingly, a similar range of variation may also be discerned among the Matter Paintings. The scaling and derivative plots in fig. 2b, for example, resemble the graphs for Number 32 (fig. 8b), whereas those for fig. 3b are more akin to the graphs for Untitled 1948-49 (fig. 9b).

Number 32 and Untitled 1948-49 are not simply different; they lie close to the two extremes of Pollock’s repertoire, at least insofar as monochromatic pattern complexity and scaling behavior are concerned. Given the artist’s penchant for innovation and experimentation, the totality of his poured abstractions will likely span much of this range. Before this conjecture may be verified, however, and more general conclusions drawn, a systematic study of a large number of paintings must be undertaken. Indeed, any particular implementation of box-counting analysis should be tested on an array of computer-generated “test-case” images, and applied consistently to all works selected for study. These two steps would go a long way to help separate research groups compare their findings, and allow the broader meaning of scaling analyses to emerge in sharper focus. At that point, art historians might even feel encouraged to draw upon such scholarship. Not because these kind of analyses validate Pollock’s work or provide a reliable method of authentication. But because scaling regularities underpin the equilibrium of order and chaos in Pollock’s compositions. And because their apparent affinities to natural phenomena align, compellingly, with the artist’s own way of constructing meaning.
Appendix: Computational Approach

This appendix, including some technical details, describes the computational procedure employed to produce the scaling plots used in this essay, figs. 2b, 3b, and 5b-9b. We use a variation on the standard box-counting method as described in general terms in Section III.

The first step of the procedure is to convert the image to be analyzed into a binary representation, whereby each pixel is valued either 0 (black, “pattern”) or 1 (white, “background”). If the input image is computer generated, as in figs. 5 and 7, it is generated directly as a binary image. If the input image is a photograph, then it is converted to a binary image by thresholding 41 pixel distances in RGB space. For example, if the target pattern in the photograph is black, as in figs. 6, 8, and 9, then pixels are thresholded based on their distance to the RGB origin. We have chosen images for which thresholding is sensible, meaning images for which a threshold can be found such that the resulting loss of detail in the pattern is visually negligible. A suitable value of the threshold for each image is chosen manually. In the case of paintings with multiple layers of different colors in the foreground (for example, red, white, and two grays on a black background in fig. 2a, or red, orange, and yellow on a blue-gray background in fig. 3a), each color layer is extracted separately via suitable thresholding, and the resulting masks are superimposed to create a single agglomerate binary image of the foreground.

After thresholding the image to separate the foreground pattern from the background, we measure scaling properties of the pattern with the box-counting procedure using square grids. Let \( k \) be the width of the square boxes, in pixels. We consider a range of integer values for \( k \) from 1 pixel to \( \frac{1}{4} \times \min(w, h) \) pixels, where \( w \) and \( h \) denote the width and the height of the image rectangle, respectively. For a specific \( k \), box counting proceeds as follows. A grid of \( k \times k \) pixel boxes is laid over the image, and the number of boxes containing any part of the pattern—as opposed to the background—is counted. In other words, we count the number \( N(k) \) of \( k \times k \) boxes containing at least one pixel assigned the value of 0.

Fig. 4 shows the box counting operation for a range of values of \( k \). The plot of \( \log(N) \) as a function of \( \log(k) \) is the scaling curve for the image, shown with blue crosses in figs. 2b, 3b, and 5b-9b. 42 The discrete derivative of the scaling curve (strictly, its negative) gives the fractal dimension curve, which is shown in black in figs. 2b, 3b, and 5b-9b.

It should be noted that a simple box-count measurement is systematically biased, and also exhibits anomalous variances. The systematic bias is toward over-counting and arises because a given box is counted as long as it contains any part of the pattern, whether just one pixel or many. The anomalous variances arise because the box count varies depending on how efficiently the pattern is covered by the grid, given the grid’s specific position and orientation.

To minimize the variance of the measurement, we calculate the mean box count over all possible grid positions. There are \( k^2 \) possible positions for a grid of \( k \times k \) pixel cells superimposed on any image; our measure \( N(k) \) is the average of all \( k^2 \) corresponding box counts. To shorten computation time, we accelerate the naïve box-counting algorithm without any loss of precision by a factor of \( k^2 \) using the method of integral images developed by Viola and Jones, 13 so that, even with averaging, the time complexity is linear in the number of image pixels. One might also average over grid orientations, but we find this to be unnecessary if the pattern is irregular (as in figs. 2, 3, 6-9) or if the pattern is rotated away from the grid orientation (as in fig. 5).

The mean box count measurement exhibits small variance, but may still be biased. The over-counting bias becomes particularly significant for larger boxes, and is especially severe for boxes that overlap the image border. To rectify the second of these effects, we count boxes situated at the image boundary fractionally, assigning to them values proportional to the part of their area that overlaps the image rectangle. We find that this procedure effectively eliminates the over-counting bias due to the image border. An inherent over-counting bias remains, but it appears constant enough between adjacent scales to not affect the scaling measurement in a significant way. 44

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Endnotes


5 One notable exception is Michael Schreyach, “I am Nature’s Science and Jackson Pollock,” *Apollo* 545 (2007): 35-43. Although Schreyach discusses Taylor’s work at length, it is in the service of a broader question: namely, why is it important to project meaning, in this case, the relationship between Pollock’s work and nature, upon abstract canvases. Taylor’s claims, that Pollock’s work is fractal, that his works can be authenticated and dated according to their fractal dimension, Schreyach leaves untested.


7 As Steven Litt wrote in the *Cleveland Plain Dealer*: “By authorizing only journalistic summaries of Taylor’s work, the Pollock Krasner Foundation has prevented disinterested scholars from reviewing his analysis. Instead, it has bolstered a negative view of the newly discovered paintings without giving anyone else a fair shot at critiquing Taylor’s study.” Steven Litt, “Experts Square Off Over Authenticity of Possible Pollocks,” *Cleveland Plain Dealer*, 19 February 2006.

8 Abbott, “In the Hands of a Master.”


10 Abbott, “In the Hands of a Master.”


12 Michael Kimmelman, for example, declared that Pollock’s paintings cannot be faked: “Now fractal science helps prove the point.” “A Drip by Any Other Name,” *New York Times*, The Week in Review, 12 February 2006: 16.


15 There is still some debate about the most useful or general definition of fractals. In this essay, however, we are using the authoritative definition employed by Steven H. Strogatz in *Nonlinear Dynamics and Chaos* (Cambridge, MA: Perseus Books, 1994) 402.

16 Strictly speaking, it is possible for an exact (mathematical) fractal embedded in a plane to have an integer fractal dimension (e.g., 2). This would be an “anomalous” case, however, and cannot occur for any painted image.


18 Strogatz 398.

19 Ibid. 320.


23 Katherine Jones-Smith, Lawrence Krauss, and Harsh Mathur, “Press Release.” In the present study, we analyze two other Matter Paintings.

24 Kennedy, Section B1, B7.

25 Richard Taylor, email communication to the authors, February 2004.

26 Richard Taylor, cited in Douglas, 32.

27 Katherine Jones-Smith and Harsh Mathur have questioned the possibility of
discerning two fractal structures in Pollock’s work in general on the grounds that his paintings span only about three orders of magnitude (the ratio of the canvas size to the size of the smallest feature is about one thousand). This leaves less than two orders of magnitude in size for each fractal, too narrow a range to establish a fractal pattern with mathematical rigor. (Katherine Jones-Smith and Harsh Mathur, “Revisiting Pollock’s Drip Paintings,” Nature 444 [November 2006]: E9-E10).

28 “Multifractal Fingerprints in the Visual Arts” 54.

29 See “Multifractal Structure in Nonrepresentational Art.”

30 See Claude Cernuschi and Andrzej Herczynski, “Cutting Pollock Down to Size: The Boundaries of the Poured Technique,” in this volume.


33 Alison Abbott, “In the Hands of a Master.”


35 Power law refers here to expressions such as $x^2$ or $x^3$.

36 Strictly speaking, it is the negative of the derivative (or minus the slope) that is plotted here, in accordance with the definition of fractal dimension (see Strogatz 409).

37 This curve must be a single-valued function.

38 Strictly speaking, the disks are placed according to a Poisson process; they are randomly distributed in the sense that their individual positions lack any spatial coherence.


40 See also, Schreyach.

41 Thresholding converts a real-valued image (where each pixel is a real number) to a binary image (where each pixel is either 0 or 1) by setting all pixels below the threshold to 0, and all pixels above the threshold to 1.

42 For clarity we use a logarithmic scale on the horizontal axis for $k$ rather than regular log(k) axis.


44 The scaling measurement, which is the plot of local fractal dimension versus scale, is the discrete derivative of the box count measurement. It is consequently impervious to any locally uniform bias in the box counts.