Social Influence in Committee Deliberation

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Abstract

Committee protocols typically involve deliberations in which committee members try to influence and convince each other regarding the "right" choice. Such deliberations do not involve only information exchange, but their aim is also to affect the preferences and the votes of other members. This aspect of social influence and committee deliberation is the focus of this paper. Using a model of social influence we demonstrate how deliberation procedures affect the voting outcome and how different protocols of consultation by committees' chairs may affect the chairs' final decisions. We then analyze the ability of a "designer" to control the deliberation protocol and to manipulate the deliberation procedure to increase the probability that the outcome he favors will be selected.

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1 Introduction

Consider two different decision procedures that are part of academic life: recruiting a new faculty and the evaluation of papers. Both are important decisions that require the inputs, vote, and recommendations of several people. There are, however, some differences between the two. In the first, there is typically a meeting of the recruiting committee in which there is an open deliberation regarding the different candidates. Such deliberations typically go far beyond providing additional information as they involve the expression of preferences and priorities while trying to convince and persuade other committee members regarding the attractiveness of each of the candidates. Referees, on the other hand, often write their reports and recommendations directly to the editor without any discussion or deliberation among them, but here too, letters reveal not only information ("Theorem 1 is known," "the proof of Lemma 2 is wrong"), but also the reviewers' opinions and preferences ("Claim 3 is interesting," "Section 4 adds little to the paper").[1]

In discussing the effect of committee deliberations the focus of the literature so far has been its role in information sharing. (For a recent survey see Li and Suen (2009) and the related literature subsection below). Given that committee members may have different information and preferences, the literature considers the incentives committee members have to disclose or distort their private information or to acquire new information.

There is however another important dimension of committees' work which has been largely ignored. When committee members explain and express their opinion, other committee members may be persuaded by their arguments or even just being influenced by listening to other opinions. In many committee discussions members indeed try to convince and persuade other members regarding the attractiveness of different alternatives. In this type of deliberation committee members influence the preferences of other committee members and do not just provide relevant information. This type of influence is the focus of this paper.

There are different ways in which recruit committees make decisions and there are different types of possible articles' reviewing processes. Recruit decisions may be made by a majority rule, may require unanimous support, or may be done by the department chair (or by the dean) based on the

[1] In the first round the referees cannot influence each other. In many journals, however, when there is a second round of "revise and resubmit," referees see each other's first round reports, and may therefore be influenced by them.

recommendation of faculty members. Editors may assign a committee for each article, asking its members to deliberate and to evaluate the paper and to send back their final recommendation, in the same way candidates are chosen.[2] Alternatively, one may adopt a sequential refereeing procedure in which the first referee writes his report which is then sent to the second referee who adds his own report. Both are then sent to a third referee or to the editor for a final decision. We show in this paper that such procedures generate different decisions regarding candidates or different profile of published papers. Moreover, if the deliberation protocol affects decisions, then individuals in control of the protocol, for example, chairs of recruit committees, may manipulate them to their advantage.

In a previous paper (Fershtman and Segal (2018)) we modeled social influence by introducing a setup in which each individual is characterized by two sets of preferences: unobservable core preferences and observable behavioral preferences, where actual choice is determined by the latter. Each person has an individual social influence function that determines behavioral preferences as a function of core preferences and the observed behavioral preferences of others. In the present paper we capture the effect of deliberation by using this social influence procedure.

We consider a committee of n individuals who need to choose between two candidates (or alternatives), differing with respect to two attributes. The levels of these attributes are observable by all committee members and there is no disagreement among them regarding the candidates' types. But committee members differ in their preferences regarding the relative importance of these attributes. Such situations occur when choosing between two candidates for a faculty position who have different research and teaching abilities, choosing between investment projects with different expected returns and different levels of risk, choosing a location for a new facility with a trade-off between convenience and price, etc. During the deliberation stage committee members argue, express and explain their opinions, and try to convince each other regarding the appropriate criterion for choosing an alternative. Such deliberations create social influence and our aim is to examine the equilibrium of this process and the type of voting outcomes that emerge under different architectures of committee decision making.

There is a difference between our social influence effect and the more

[2] This is how papers are accepted for presentation in some computer science conferences, e.g. the ACM Conferences on Economics and Computation.

familiar social learning setting of DeGroot (1974) and more recently Golub and Jackson (2010). In the learning setup there is a group of individuals where each of them has some beliefs about the probability of some event. These individuals communicate with each other (or with their neighbors in the social network) and update their beliefs. Each person's new beliefs are a weighted average of his and his neighbors' (last period) beliefs. The question in this literature is under what conditions these beliefs converge to a consensus and whether this consensus is the correct probability of the event. But social influence is about affecting preferences and not just information. For example, Ariely and Levav (2000) showed that when individuals observe the choices of dishes of other individuals in a restaurant, the variance of chosen dishes goes up. When a committee needs to choose a new faculty member, and for this choice they need to weigh teaching and research abilities, it is not clear that adopting a weighted average of the preferences of committee member capture the richness of social influence. It is possible, for example, that some would like to have the higher weight on research than others. Therefore our focus is not on the possibility of reaching a consensus but rather on the equilibrium profile of preferences. That is, for every profile of core preferences, social influence functions and architecture of deliberation, we are looking for a profile of social behavioral preferences, where the behavioral preferences of each member are determined by his core preferences and the behavioral preferences of committee members that affect him given the protocol of deliberations. In this setup we will examine the effect of the architecture of deliberation on the equilibrium behavioral preferences and the resultant decision by the committee.

We start by analyzing a committee with an open deliberation procedure in which all members participate in the deliberation and influence each other, and examine different properties of the equilibrium of this process. For example, deliberation may violate the unanimity property. Even when all committee members prefer in their core preferences one alternative, deliberation may lead them to vote for the other alternative.

Committees have different protocols of deliberation and voting. In some cases members do not have to express their opinion before the voting and may choose only to listen to others. In other cases they have to explain their decision (e.g., judges siting together on the bench). There are committees in which members do not have to attend meetings and may choose just to send their written ballots. Deliberation and voting can be done simultaneously or sequentially, and in different orders. The order of a sequential deliberation implies that early speakers influence the preferences of other committee members but they will not be influenced by them. The outcome of the deliberation and voting thus depends on the order in which it is done. We capture these types of deliberation and voting protocols by modeling it as a directed social network in which individuals influence others only if they are connected and the direction of the network determines the direction of the social influence. The designer, who has his own favorite choice, may manipulate the order in which deliberation is carried out to achieve his favorite outcome. Interestingly, there is a different optimal order under majority and unanimity rules and the designer may choose a different protocol manipulation depending on the decision rule.

We also consider situations in which there is one person who is making all the decisions, but he may consult with a group of "advisors," e.g., a CEO of a firm with a board of directors. We distinguish between situations in which the advisors only consult the chair and situations in which there is an open deliberation. The pattern of consulting may be bilateral, where each advisor talks only with the chair but not with other advisors, or it can be multilateral such that advisors also talk with each other. Each such procedure determines a different pattern of social influence and may lead to different decisions. Consequently, here too the designer may benefit from choosing the procedure of consultation that will yield his preferred outcome.[3]

1.1 Related Literature

This paper relates to several branches of literature. Starting from Condorcet (1785) there are numerous studies on decision making by committees.[4] Following his work most of the literature focuses on information aggregation and voting protocols. For a survey of this literature, see Li and Suen (2009). The meaning of deliberation in this literature is information sharing, manipulation, and information aggregation. There is no arguing, convincing, and social influence in the formal deliberation process unless it involves provid-

[3]In this paper we do not argue that deliberations helps the committee to overcome shortcoming of the voting procedure (see Coughlan (2000) for a model of private information voting games in which deliberation may overcome shortcoming of voting rules).

[4]Condorcet (1785) showed that the quality of decisions made by committees with diverse information is increasing with the group size.

ing additional information.[5] It is these aspects of deliberation which are the focus of our paper.

Our analysis considers also the effect of deliberation protocol on committee decisions and it relates to the literature concerning dynamic committee decision making. See for example Moldovanu and Shi (2013), Dekel and Piccione (2000, 2014), Damiano, Li and Suen (2009) and Chan, Lizzeri, Suen and Yariv (2018). Recently Iaryczower, Shi and Shum (2018) studied the effect of deliberation on the probability of incorrect decisions by a jury in the context of criminal cases.

There is an extensive documentation of the effect of social interaction on individuals' behavior and opinions. For a review of this literature in social psychology, see Isenberg (1986), Myers and Lamm (1976), and Myers (1975, 1982). The effect of deliberation on jury decisions was discussed by Schade, Sunstein, and Kahneman (2000) and Mendelberg (2006). Political scientists too argue that "the composition of the discussion group changes the views expressed by those who participate in it." (See Farrar, Green, Green, Nickerson and Shewfelt (2009, p. 616)). Ariely and Levav (2000) provide an experiment suggesting that social influence affects the choice of dishes in restaurants. Specifically, they found a larger variance in the dishes ordered by individuals who were part of a group than by individuals sitting by themselves. Aronson, Wilson, and Akert (2010) claim that group discussions may make people more risk taking than their initial tendencies. In a recent article, Hoff and Stiglitz (2016) claim that preferences and behavior are influenced by actions and beliefs of people around the decision maker. An alternative formulation of social influence was introduced by Cuhadaroglu (2017). In this paper individuals are assumed to have incomplete preferences and are socially influenced by other individuals only on issues which they cannot resolve by themselves.

Our paper is also related to the literature on social preferences (see, for example, Charness and Kuhn (2011), Fehr and Gächter (2000), Sobel (2005), and Fehr and Schmidt (1999)). Social preferences imply that the utility of an individual depends on other people's outcomes, on the distribution of payoffs, or on the actions taken by other people. The literature focuses, for example, on altruism, fairness concerns, reciprocity, and inequality aversion. There is

[5] We assume that committee members have heterogeneous preferences and therefore there is a room for social influence and there is no career concern or reputation effect. Such committees of experts were discussed by Visser and Swank (2007), Levy (2007), and Gersbach and Hahn (2012).

however a difference between "social preferences" and our concept of "social influence." We assume that when individuals need to make decisions, their preferences may be altered by the interaction with other people, but these preferences may or may not take other people's welfare into consideration. But even when there is a social concern like fairness, this concern may change as a result of social influence.

Finally, the paper is related to the literature on endogenous preferences. For a survey of the literature on evolutionary sociobiology, see Becker (1970) and Dawkins (1976) and for a survey of applications in economics see Bowles (1998), Samuelson (2001), and more recently Alger and Weibull (2013). This literature assumes that preferences change over time but the assumed dynamics follows a simple evolutionary rule such that people adopts the preferences of "successful" (high fitness) individuals. The second approach to endogenous preferences is the dynamic cultural transmission framework (see Bisin and Verdier (2001), Boyd and Richerson (1985), and Cavalli-Sforza and Feldman (1973)). Our approach does not assume endogenous preferences but that choices made by individuals are affected by the choices made by other people with whom they interact.

2 The Model

2.1 Preliminaries

We consider a committee consisting of $n > 2$ members that needs to choose one of two possible options $\{A, B\}$. We assume that the decision is based on two attributes of the alternatives, denoted (a_1, a_2) for A and (b_1, b_2) for B. These attributes can be a candidate's ability and willingness to work, research and teaching performance, the expected return and the risk of a project, etc. We assume that these attributes are perfectly observable and there is no dispute among committee members regarding their levels.

Members of the committee have their own core preferences $\alpha_i \in [0, 1]$ over the relative importance of the two attributes. For example if the two attributes of candidates for a faculty position are teaching skills and research abilities, then a higher α_i implies that member i puts a higher emphasis on research ability relative to teaching skills. With such preferences, i prefers B to A if and only if $b_1 + \alpha_i b_2 \geqslant a_1 + \alpha_i a_2$. If $a_1 \geqslant b_1$ and $a_2 \geqslant b_2$, candidate A dominates candidate B and the choice is trivial. We therefore assume, without loss of generality, that $a_1 > b_1$ but $a_2 < b_2$ so that there is no clear choice between the two candidates. Let $\alpha = (\alpha_1, \ldots, \alpha_n)$, and assume, without loss of generality, that $\alpha_1 \leqslant \ldots \leqslant \alpha_n$. Given our assumptions, for any given pair of candidates $\{A, B\}$ with known and observable attributes there is a critical value $\gamma = (b_1 - a_1)/(a_2 - b_2)$ such that member i prefers A to B if and only if $\alpha_i \leqslant \gamma$.

We discuss throughout two major ways in which committees make decisions. Majority rule, in which option A is selected over option B if and only if more members prefer A to B then those who prefer B to A (and vice versa), and a unanimous rule, in which an option is selected if and only if all members prefer it to the alternative. If no option receives unanimous support, then there is typically a default option (e.g., postpone hiring till next year) which is then selected.

2.2 Social Influence in Committee Deliberation

When there is no deliberation and individuals cast their votes without any social interaction, then they vote according to their core preferences $(\alpha_1, \ldots, \alpha_n)$. But under deliberation, there is some social influence which may affect their vote. Following Fershtman and Segal (2018), we assume that each individual has two types of preferences; core preferences, which were described above and behavioral preferences that govern his behavior. Without social influence behavioral preferences are identical to the core preferences. But when there is deliberation among committee members the behavioral preferences depend on the individual's core preferences and the behavioral preferences of other individuals with whom he interacts. We assume that voting after deliberation would be according to a behavioral parameter β_i, $\beta_i \in [0, 1]$. Formally, each individual has a *social influence function* that shapes his behavioral preferences such that $\beta_i = g^i(\alpha_i, \beta_{-i})$ where $\beta_{-i} = (\beta_1, \ldots, \beta_{i-1}, \beta_{i+1}, \ldots, \beta_n)$ and individual i votes for A over B if and only if $\beta_i \leqslant \gamma$. Observe that individuals do not necessarily have the same g^i function and may therefore be influenced in different ways. Some may be stubborn enough to ignore any opinion of other individuals, having $g^i(\alpha_i, \beta_{-i}) = \alpha_i$ for every β_{-i}. On the other hand, some individuals' convincing power may overcome the opinions of others. In the extreme case we'll have an individual j^* who has the ability to dominate the discussion in the committee such that for every $i \neq j^*$ we get $g^i(\alpha_i, \beta_{-i}) = \beta_{j^*}$. However, for most of the analysis we consider the case in which all committee members are subject to social influence.

One type of social influence is conformism, where individuals conform to the behavior and beliefs of their group (in our case, the committee members). When there are two individuals in the group, conformism implies that the behavioral preferences $\beta_i = g^i(\alpha_i, \beta_j)$ are between α_i and β_j. In a larger group conformism implies that the behavioral preferences of individual i would be a weighted average of the behavioral preferences of the other group members and his own core preferences. This type of social influence resembles the (naive) learning models of DeGroot (1974) and Golub and Jackson (2010). In this learning setup an individual has beliefs about the probability of some event. After observing the beliefs of other individuals he updates his beliefs by taking the weighted average of his and the other individuals' beliefs from previous period. But conformism is only one form of social influence and as we show below, our social influence function g is more general than conformist attitude.

If g is monotonic in both arguments, then conformism is equivalent to the requirement that $g(\alpha, \alpha) \equiv \alpha$.[6] In this case, when individual i interacts with another person j with behavioral preferences which are the same as i's core preferences, then the behavioral preferences of i will be the same as his core preferences. But social influence function does not necessarily has this property. Consider again the example of hiring a new faculty member. The parameter α_i describes the relative importance that agent i attaches to research versus teaching abilities, where a higher α_i means a higher weight on research ability. It is possible that agent i views himself as someone who puts an emphasis on research, but when he discovers that his weight on research is the same as the (behavioral) weights of all other members, he behaves and votes as if his weight on research is higher than his core preferences. In this case $g^i(\alpha, \alpha) > \alpha$ and we say that the social influence function has an *Super Reinforcement* (SR) property. Similarly, we call the property $g^i(\alpha, \alpha) < \alpha$ *Under Reinforcement* (UR).[7] Note that SR and UR are local properties and the social influence function may have different characteristics at different values of the parameters. So it is possible to have $g^i(\alpha, \alpha) > \alpha$ for some values of α and $g^i(\alpha, \alpha) < \alpha$ for other values.

[6] If $g(\alpha, \alpha) \equiv \alpha$ then for $\alpha < \beta$, $\alpha = g(\alpha, \alpha) \leqslant g(\alpha, \beta) \leqslant g(\beta, \beta) = \beta$.

[7] Note that there is some analogy between these two properties since by proper representation of our setting we can transform α — the weight of the second attribute — to $\hat{\alpha}$ which will be the weight of the first attribute. In this case whenever $g(\alpha, \alpha) > \alpha$ we would have $g(\hat{\alpha}, \hat{\alpha}) < \hat{\alpha}$ so SR with respect to the second attribute implies UR with respect to the first one.

Fershtman and Segal (2018, Claim 2) offer an axiomatic framework under which the behavioral parameter β depends only on one's core preferences and the average of the observable behavioral parameters of everyone else. That is, $\beta_i = g^i(\alpha_i, \sum_{j\neq i}\beta_j/(n-1))$.[8] We assumed there that $0 < g_1, g_2 \leqslant 1$ and proved that if all agents have the same social influence function g, then (i) If all agents start with the same α, then $\beta := \beta_1 = \ldots = \beta_n \geqslant \alpha \iff g(\alpha,\alpha) \geqslant \alpha$; (ii) $\beta_i \geqslant \beta_j \iff \alpha_i \geqslant \alpha_j$. In this paper we maintain the assumption that $0 < g_1, g_2 \leqslant 1$, and assume further that $g_{12} < 0$.

2.3 Networks of Influence

Deliberation and voting procedures may take different ways with a rich set of possible scenarios. Social influence may occur as a direct outcome of any meeting and discussion protocol. Note that while our structure allows only for direct social influence, indirect influence may play an important role in the model. Agent i may never meet agent k but he may still be indirectly influenced by her. This happens whenever agent k socially interact with agent j and influence his preferences and then agent j interact with agent i. Given this structure we can model any social influence pattern by a directed network in which there is a directed link between agent i and agent j only when agent j is socially influenced by agent i and the preferences he observes are i's behavioral preferences. Given this structure we need to look for equilibrium in the behavioral preferences which would depend on the network of social influence.

A deliberation network is a pair (N,Γ), such that N is the set of players and Γ is a directed social network on the nodes N. The network $\Gamma \subseteq \{(i,j)|\, i,j \in N, i \neq j\}$ denotes the pairs of nodes that are socially connected and the direction of social influence, where $(i,j) \in \Gamma$ indicates that agent i is influenced by player j.[9] The set Γ is not necessarily symmetric and it is possible that $(i,j) \in \Gamma$ but $(j,i) \notin \Gamma$. In this case agent j influences

[8] Our setup assumes that g depends on one's core preferences and the average behavioral preferences of the rest without specifying how many other individuals there are in the influence group. One can modify this assumption by indexing the g function according to the number of individuals in the influence group. Claims 3 and 5 below only require that the vector β_{-i} can be replaced with a parameter $\beta \in [\min_{j\neq i}\beta_j, \max_{j\neq i}\beta_j]$.

[9] As an alternative we can let Γ be an $n \times n$ adjacency matrix, with entry $\Gamma_{ij} \in \{0,1\}$ denoting whether i interacts with player j. In this formulation one can permit $\Gamma_{ij} \in [0,1]$, indicating intensities of influence, so it is possible to consider committees in which some individuals have a greater social influence than others.

agent i but is not influenced by him. Such situations occur for example when deliberation and voting are sequential and if agent j's turn precedes i's.

Given the directed network Γ, we are looking for a social influence equilibrium, which is a vector of behavioral preferences $\beta = (\beta_1, \ldots, \beta_n)$ such that each agent i's behavioral preferences are determined by his core preferences α_i and the equilibrium behavioral preferences of those agents by whom he is influenced. Formally, the social influence function takes the form of $\beta_i = g^i(\alpha_i, \beta_{-i}|\Gamma)$ which means that agent i is influenced only by agents that are defined by the directed network Γ. We assume that $g^i(\alpha_i, \beta_{-i}|\Gamma)$ is continuous in all its arguments. For any profile of core utilities $\alpha = (\alpha_1, \ldots, \alpha_n)$, social influence functions $g = (g^1, \ldots, g^n)$, and a directed network Γ, we define *equilibrium behavioral preferences* as $\beta^*(\alpha, \Gamma) = (\beta_1^*(\alpha, \Gamma), \ldots, \beta_n^*(\alpha, \Gamma))$ such that for every i, $\beta_i^*(\alpha, \Gamma) = g^i(\alpha_i, \beta_{-i}^*|\Gamma)$.

Claim 1 For every profile of core utilities α, social influence functions g^1, $\ldots, g^n$, and a directed social network Γ, there is an equilibrium behavioral preferences vector $\beta^*(\alpha, \Gamma)$.

The proof is a special case of the proof of Claim 1 in Fershtman and Segal (2018) (hereafter FS), which considers a case in which both the core and the behavioral preferences are utility function on $[0, 1]$ which are assumed to be bounded, continuous, and equi-Lipschitz functions on $[0, 1]$, but are not necessarily represented by a single parameter β. In FS, the social influence is defined with respect to a complete undirected network but can be easily extended to any directed network.[10]

3 The Effect of Deliberation in Complete Networks

In this section we analyze the effect of deliberation on committees' decisions when there is no specific deliberation protocol and all committee members participate in the deliberation, which is then followed by voting. We start by defining several properties that the deliberation and voting process may satisfy. Whether these properties are satisfied or not depends on the profile of core preferences and the individual social influence functions.

[10] We can extend the existence claim to the case of a general (bounded and equi-Lipschitz) preferences over $[0, 1]$ following the same step of the proof in FS.

Property 1 (Unanimous Acceptance): If all committee members prefer one candidate (e.g. for all i, $\alpha_i < \gamma$), then social influence during the deliberation process results in an equilibrium behavioral preferences β which implies that the same candidate is chosen.

Note that this is a weak notion of unanimous acceptance. A stronger version would imply that if prior to the deliberation all committee members prefer candidate A then after the deliberation they still unanimously vote for A. The justification for such a strong version of unanimous acceptance is that if no committee member supports alternative B, then no one can convince others to vote for B. Note however that the social influence is with respect to preferences, represented by β, and not directly with respect to the candidates. Our weaker concept only requires that if there is a unanimous support for candidate A prior to the deliberation, then A will be chosen by the committee after the deliberation.

Property 2 (Consistency): If B is selected after deliberation and a new member supporting B is added to the committee, then the new committee will still vote for B.

Property 3 (Monotonicity): If a committee votes for alternative B and one of its members is replaced with a new member with the same social influence function who, in his core preferences, is more inclined to vote for B, then the new committee will also vote for alternative B.

Our social influence setting implies that what matters for understanding the committee voting is not just whether its members prefer A or B but also the intensity of these preferences. This intensity also determines the way committee members affect other members. It is therefore important to analyze the deliberation stage which reveals the intensity of preference and not just the voting stage which reveals members' ordinal preferences. It is this characteristic of our social influence setting which generates the possibility of inconsistencies in the committees' decisions.

Claim 2 Under both majority and unanimity rules, the deliberation process satisfies the monotonicity property, but it does not necessarily satisfy unanimous acceptance and consistency.

All proofs appear in the appendix.

In many situations there is a person — call him "designer" — who has some control over the identity of committee members and over the deliberation procedures of the committee. This person may have his own preferences regarding the choice of the committee. Claim 2 implies that this person can sometimes apply his power to affect the decision of the committee. The designer may be the chair of the committee (for example, in academic promotion committees), but he may be a non-member of the committee, as is sometimes the case with investment committees, where protocols of discussion and deliberation are decided by boards of shareholders.

Corollary 1 Consider a designer who prefers candidate A.

1. If he knows that all (or most) of the committee members prefer candidate A over candidate B, then he should go directly to voting without deliberation. As claim 2 shows, deliberation may change the committee vote.

2. If he has some control over the identity of the individuals in the committee, then the designer is better off putting in the committee individuals that have strong preferences for candidate A (and may strongly influence other committee members) rather than adding more individuals that mildly support this candidate.

4 Star Network: Committees with a Chair

There are situations in which there is one individual, the "chair," who is making the decisions, but there is a group of "advisors" with whom he may consult and deliberate. This person can be a CEO of a company consulting with his board of directors or a president of a country deliberating or consulting with his cabinet members before making a decision. This kind of a committee can be modeled as a star network such that the chair is the agent in the center who is directly connected to all other agents who may or may not be connected among themselves.

Such advisory networks can function in different ways. First we distinguish between a "deliberation process," in which the chair and the advisors deliberate the decision problem among themselves, and a "consultation process," in which the chair asks his advisors for their opinion but does not express his own. The main difference between the two processes is that in the former the chair participates in the deliberation and affects the preferences of his advisors while in the latter his role is more passive as he only listens to his advisors without expressing his own opinion so that he does not affect his advisors' opinions. We also distinguish between a "bilateral procedure" in which the chair discusses (or consults) the issue separately with each of his advisors and a "multilateral procedure" in which there is an open discussion among all agents.

The difference between the procedures of this section and those used before is that here the decision is dictatorial and is made by the chair without any voting. We are interested in the effect of possible deliberation among committee members on the decision of the chair even though the other members have no voting power.

Formally, we consider the following network. There is a chair (agent 0) and a group of n committee members labeled $N = \{1, \ldots, n\}$. We assume for simplicity that all $n+1$ agents (chair and members) have the same social influence function g. The core preference parameter of the chair is given by α_0, and we assume that the n committee members have the same core preferences $\alpha_i = \alpha < \alpha_0$. There is a link between agent 0 and each of the n committee members. When there is a deliberation the link is not directed but consultation is modeled as a directed link (from the agent to the chair). When there is a multilateral procedure all the agents are also linked while in the bilateral procedure there are no links between any two of the n committee members.

For the interpretation of our results we will continue to use our benchmark example of choosing between two candidates with two attributes such that candidate A is stronger in the first attribute and candidate B is stronger in the second. Therefore, when the chair ends up with a higher β, it makes candidate B more attractive.

We start by considering the consultation process, in which the chair listens to his team but does not express his own view, and the effect of a deliberation on the outcome of such a process. That is, we compare the behavioral preferences of the chair when there is a bilateral consultation process, denoted β_0^{cb}, with his behavioral preferences when there is a multilateral consultation process, denoted β_0^{cm}.

Claim 3 (Consultation): If $\alpha < \alpha_0$, then $\beta_0^{cm} > \beta_0^{cb}$ if and only if the social influence function is SR. That is, under a multilateral consultation procedure candidate B looks more attractive (to the chair) than under the bilateral consultation procedure if and only if $g(\alpha, \alpha) > \alpha$.

Claim 3 implies that when $g(\alpha, \alpha) > \alpha$, letting members of the board to deliberate with one another prior to consulting the chair will make candidate B more likely to be selected as $\beta_0^{cm} > \beta_0^{cb}$, and when $g(\alpha, \alpha) < \alpha$, deliberation among the agents makes A's candidacy stronger.

As all committee members have the same core value, if $g(\alpha, \alpha) > \alpha$, then each will adopt behavioral preferences $\beta > \alpha$. As deliberations among committee members before they consult the chair lead to a higher β, consultation will push the chair to a higher value of β_0 then where there is no deliberation among committee members and their behavioral preferences are α.

Consider now the case of deliberation, in which the chair does not only listen to his team, but also reveals his views to them. In the bilateral procedure the chair deliberates with each of the committee members separately but they do not deliberate among themselves. Denote by $\beta^{db} = g(\alpha, \beta_0^{db})$ and $\beta_0^{db} = g(\alpha_0, \beta^{db})$ the equilibrium behavioral preferences of this procedure. In the multilateral deliberation procedure on the other hand, the chair and his advisors openly deliberate among themselves. Let β^{dm} and β_0^{dm} be the equilibrium behavioral preferences of this multilateral deliberation procedure, where $\beta_0^{dm} = g(\alpha_0, \beta^{dm})$ and $\beta^{dm} = g(\alpha, \frac{\beta_0^{dm} + (n-1)\beta^{dm}}{n})$.

Claim 4 (Deliberation): If $\alpha < \alpha_0$, then $\beta_0^{db} > \beta_0^{dm}$. That is, multilateral deliberations of the chair and the committee result in lower emphasis on the second attribute than in a procedure in which the deliberation is with each advisor separately.

Note that the effect of multilateral vs. bilateral process is different in the case of consultation than in the case of deliberation. Multilateral deliberation leads to a higher β for the chair while in the consultation case the effect depends on $g(\alpha, \alpha)$.

Next, we compare the consultation and the deliberation procedure assuming they both adopt the bilateral protocol.

Claim 5 Assume that all the committee members have SR social influence function. Then $\alpha < \alpha_0$ implies $\beta_0^{db} > \beta_0^{cb}$. That is, when there is a bilateral protocol the deliberation procedure yields a higher β then the consultation procedure, which implies that candidate B looks relatively stronger to the chair under deliberation than under consultation.

Consider now a situation in which there is a designer who needs to decide what kind of decision making process the chair and the advisors will adopt. Assume further that this designer puts a high emphasis on the second attribute. That is, he would always prefer candidate B to A. The next corollary shows how this designer can set the rules that would favor his preferred candidate.

Corollary 2 Assume $\alpha < \alpha_0$ and that the social influence function is SR. (i) When there is a bilateral procedure such that all the advisors interact with each other, then the designer is better off with the deliberation procedure rather than the consultation procedure. (ii) When there is a deliberation procedure the designer should set the bilateral deliberation rule, and (iii) In the consultation case the designer is better off adopting the multilateral procedure.

5 Procedures of Deliberation and Voting

Committees may have different voting and deliberation procedures. In some cases members may choose not to express their opinion or to refrain from explaining their vote. In other cases they must explain their decision (e.g., judges that sit together on the bench). There are committees in which members do not have to attend meetings, they may just send their written vote. Voting can be done simultaneously or sequentially (and in a different order). In this section we demonstrate that procedures may affect the formation of the equilibrium behavioral preferences and the outcomes of committees' voting. We focus on two aspects of the voting procedure: (i) the requirement to participate in the deliberation and (ii) the effect of the order of deliberation and voting in a sequential procedure. These rules affect the formation of behavioral preferences. Members who do not express their opinions do not influence other committee members and those who do not participate in the debate at all are not influenced by others.

5.1 The Effect of No Participation in the Deliberation

In order to demonstrate these effects we consider an investment committee consisting of three members who need to vote on whether to accept or reject risky projects. We consider two possible decision rules. The first is a majority rule in which a project is accepted only when it gains the approval of at least two members of the committee. The second is a unanimity rule in which acceptance requires the support of all three members of the committee. Risk aversion is captured by a single parameter: α (for the core preferences) and β (for the behavioral preferences).[11] Higher values of α and β imply higher levels of risk aversion. Each new project is characterized by a risk index γ such that individuals with $\beta \leqslant \gamma$ vote to accept the project and those with $\beta > \gamma$ reject it. A higher γ implies that the project is less risky as even individuals with a higher level of risk aversion will vote to accept it.

Suppose that the three members have the same social influence function g, but they differ in their core preferences which are given by $\alpha_1 < \alpha_2 < \alpha_3$ with the behavioral preferences $\beta_1, \beta_2, \beta_3$. Denote by $\gamma_m(\beta_1, \beta_2, \beta_3)$ and $\gamma_u(\beta_1, \beta_2, \beta_3)$ the critical risk indexes under the majority and the unanimity rules, respectively, such that all projects characterized by values of γ higher than these values will be accepted. Clearly, $\gamma_m \leqslant \gamma_u$, as any project that is accepted by all members is also accepted by at least two members.

When there is a deliberation with the participation of all committee members then by Claim 7 in FS, $\beta_1 < \beta_2 < \beta_3$, therefore $\gamma_m(\beta_1, \beta_2, \beta_3) = \beta_2$ and $\gamma_u(\beta_1, \beta_2, \beta_3) = \beta_3$. We simplify our analysis and assume that the social influence function used by all members is such that $g(\alpha, \alpha) = \alpha$. Under this assumption (see Claim 8 in FS), equilibrium behavioral preferences move towards the average such that $\beta_1 > \alpha_1$, $\beta_3 < \alpha_3$, but the relationship between β_2 and α_2 is unclear. Therefore, under the unanimity rule $\gamma_u(\beta_1, \beta_2, \beta_3) < \gamma_u(\alpha_1, \alpha_2, \alpha_3)$, which implies that as a result of deliberation and social influence there is a larger set of projects that will be acceptable by the committee. However, if a committee uses the majority rule then the effect of deliberation is unclear as both $\beta_2 > \alpha_2$ and $\beta_2 < \alpha_2$ are possible.

Suppose now that person i does not take part in the deliberation process, and therefore $\beta_i = \alpha_i$. We continue to assume that all members have the same social influence function g and that $g(\alpha, \alpha) \equiv \alpha$. Denote the behavioral preferences of person j when person i does not take place in the deliberation by β_j^i. The analysis of this situation depends on the identity of the non-participating individual.

Claim 6 If person 1 does not participate in the deliberation, then both unanimity and majority rules accept less projects than the case in which all

[11] For example, all members are expected utility maximizers with the vNM utility $\alpha u + (1-\alpha)\tilde{u}$, where u is a concave transformation of $\tilde{u}$.

members participate in the deliberation. But if player 3 does not participate, then the unanimity rule accepts less while the majority rule accepts more projects than the case of full deliberation.[12]

Consider a designer who has some control over the deliberation process. Assume that this designer, being risk neutral, prefers the committee to accept as many projects as possible (note that all the project have identical positive expected profits and they vary only with respect to their risk). Assume further that the designer knows the preferences of all committee members and has some control over the deliberations. For example the designer needs to approve absentee voting etc. Then Claim 6 implies the following corollary.

Corollary 3 The designer should insist that person 1 participates in the deliberations. Under unanimity rule the designer should insist that player 3, the most risk averse committee member, participates in the deliberation. But under majority rule the designer should approve player 3's wish to vote without taking part of the deliberations.

5.2 The Effect of the Order of Deliberation.

There are many situations in which deliberation and voting are done sequentially. The order may be according to seniority, rank, or even by a lottery. In this subsection we analyze the effect of the order of deliberation on the outcome of the debate. When behavioral preferences are formed during the deliberation then the order of the deliberation may play an important role in shaping those preferences as committee members are influenced only by individuals who have already expressed their opinions.

To illustrate this effect consider a committee consisting of three members expressing their views and voting sequentially. Denote these players by the order in which they vote as i-j-k. The first person is exposed to no other views, therefore $\beta_i = \alpha_i$ and he votes according to his core preferences. The second person is influenced only by the first and therefore

$$\beta_j = g(\alpha_j, \beta_i) = g(\alpha_j, \alpha_i)$$

The third person is influenced by the other two. But the order in which he is exposed to these views may affect this influence. Person k may give a

[12] The case where person 2 does not participate is more involved and the analysis depends on whether β_2 is above or below the average of β_1 and β_3.

higher weight to the last person he heard, or to the first one, or he may treat their opinions equally. We will therefore consider the general case in which person k weighs the opinions of the two other members at the ratio $\theta : 1-\theta$, $\theta \in [0,1]$. In other words,

$$\beta_k = g(\alpha_k, \theta\beta_i + (1-\theta)\beta_j) = g(\alpha_k, \theta\alpha_i + (1-\theta)g(\alpha_j, \alpha_i))$$

As before, we restrict our analysis to the case in which all members use the same function g and θ, and $g(\alpha, \alpha) \equiv \alpha$. Consider the six orders (i): 1-2-3, (ii): 1-3-2, (iii): 2-1-3, (iv): 2-3-1, (v): 3-1-2, and (vi): 3-2-1.

Claim 7 Let $\alpha_1 < \alpha_2 < \alpha_3$. If selection is done by the unanimity rule, then person B, who has an advantage in the second attribute, is most likely to be selected if the order of deliberation is 3-1-2, regardless of the value of θ. If selection is done by a majority rule, then she is most likely to be elected if the order is 3-2-1.

This claim shows that the designer of the protocol, who has his own favorite choice, may benefits from manipulating the order in which deliberation and voting take place. However, the optimal order depends on the committee's decision rule. There are different optimal orders under majority and unanimity rules.

Remark: It is clear from Table 1 in the proof of Claim 7 that the order of the behavioral parameters $\beta_1, \beta_2, \beta_3$ does not have to be the same as the order of the core parameters $\alpha_1, \alpha_2, \alpha_3$. For example, if $g(\alpha_3, \alpha_1) < \alpha_2$ and θ is sufficiently close to 0, then $\beta_2^{(ii)} > \beta_3^{(ii)}$ even though $\alpha_2 < \alpha_3$.

6 Concluding Remarks

There are many decisions that are done by committees. But before making decisions committee members typically deliberate, exchange relevant information, and try to convince each other regarding the right choice. Our paper focuses on the effect of deliberation as a mechanism that changes preferences and opinions. Such effects open the door for strategic manipulations of committees' work and decision procedures. Our setup assumes for simplicity that committee members are symmetric in the their ability to influence and be influenced by others. But our setup may be used for the analysis of a variety of asymmetric situations and answer questions regarding circumstances under which we would like to put the influential person at the end or at the beginning of the discussion, how to choose committee members, and what deliberation protocols to adopt that would provide the desired outcome.

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Appendix

Proof of Claim 2:

Monotonicity: Consider the system $\beta_i = g^i(\alpha_i, \sum_{j\neq i}\beta_j/(n-1))$, $i = 1,\ldots,n$. Take the total differential to obtain for $i = 1,\ldots,n$

$$g_1^i\left(\alpha_i, \sum_{j\neq i}\frac{\beta_j}{n-1}\right) = \frac{\mathrm{d}\beta_i}{\mathrm{d}\alpha_i} - \frac{1}{n-1}\sum_{j\neq i} g_2^i\left(\alpha_i, \sum_{j\neq i}\frac{\beta_j}{n-1}\right)\frac{\mathrm{d}\beta_j}{\mathrm{d}\alpha_i} \qquad (1)$$

Let the matrix B be given by $b_{i,i} = 1$, and $b_{i,j} = -\frac{1}{n-1}g_2^i\left(\alpha_i, \sum_{j\neq i}\frac{\beta_j}{n-1}\right)$ whenever $i \neq j$. Let C_j be obtained from B by replacing column j of B with $\left(0,\ldots,0, g_1^i\left(\alpha_i, \sum_{j\neq i}\frac{\beta_j}{n-1}\right), 0,\ldots,0\right)^T$. The matrices $\mathrm{B}, \mathrm{C}_1, \ldots, \mathrm{C}_n$ all satisfy the conditions of theorem 4.D.1 in Takayama (1985, p. 392), and moreover, for $x^T = (1,\ldots,1)$ and $\mathrm{A} = \mathrm{B}, \mathrm{C}_1^T, \ldots, \mathrm{C}_n^T$, $\mathrm{A}\cdot x \geqq 0$ (recall that $0 < g_2 \leqslant 1$). By the above theorem, $\det(\mathrm{B}), \det(\mathrm{C}_1), \ldots, \det(\mathrm{C}_n) > 0$. It thus follows from the system of linear equations (1) that for all i, j, $\frac{\mathrm{d}\beta_j}{\mathrm{d}\alpha_i} > 0$. All committee members are now more inclined to choose candidate B, and as he was preferred to A before the shift, he is certainly preferred after.

Unanimous Acceptance: Suppose that all members have the same social influence function $g(\alpha, \beta)$ such that β is the average preferences of everyone else. If all agents have the same core preferences α and the social preference function is SR (i.e., $g(\alpha, \alpha) > \alpha$), then the equilibrium occurs at $\beta > \alpha$ (see Claim 6 in FS). Let α' and β' be such that $\beta' = g(\alpha', \beta')$. If $\alpha' < \gamma < \beta'$ then by their core preferences all agents prefer A to B (since $\alpha' < \gamma$), but by the behavioral preferences they would vote for B, that is $B \succ A$ since $\gamma < \beta'$.[13]

Consistency: Suppose that all members have the same core preferences $\alpha > \gamma$, but as their preferences are UR (that is, $g(\alpha, \alpha) < \alpha$), their common behavioral preferences β are just above γ and candidate B is selected. Add a new committee member whose core preferences are just above γ (but sufficiently below α) and his preferences may push the behavioral preferences of all other agents below γ. ■

Proof of Claim 3: If committee members do not deliberate among themselves, then there is no change in their preferences and therefore $\beta^{cb} = \alpha$ and $\beta_0^{cb} = g(\alpha_0, \alpha)$. When the n agents deliberate among themselves their equilibrium behavioral preference is given by $\beta^{cm} = g(\alpha, \beta^{cm})$. By Claim 6 in FS, $\beta^{cm} = g(\alpha, \beta^{cm}) > \alpha$ if and only if $g(\alpha, \alpha) > \alpha$. And since $g_2 > 0$, $\beta_0^{cm} = g(\alpha_0, \beta^{cm}) > g(\alpha_0, \alpha) = \beta_0^{cb}$. ■

Proof of Claim 4: We start with the case *db* in which the chair deliberates with each advisor separately. Consider the behavioral preferences of each advisor which is a function of his core preferences and the behavioral preferences of the chair, $\beta(\alpha, \beta_0) \equiv g(\alpha, \beta_0)$. Given that $g_2 > 0$, this function is increasing in β_0. Also, $\beta_0(\alpha_0, \beta) \equiv g(\alpha_0, \beta)$ are the behavioral preferences of the chair given his core preferences and the behavioral preferences of the advisors. Figure 1 depicts both functions in a $(\beta \times \beta_0)$ space where α, α_0 are fixed parameters. The intersection $(\beta^{db}, \beta_0^{db})$ (point s in Figure 1) is an equilibrium of the bilateral deliberation process as $\beta^{db} = g(\alpha, \beta_0^{db})$ and $\beta_0^{db} = g(\alpha_0, \beta^{db})$. Note also that since $\alpha < \alpha_0$, it follows that the equilibrium point is above the 45° line because in equilibrium $\beta^{db} < \beta_0^{db}$ (see point (ii) at the end of subsection 2.2).

Examine now the equilibrium of the multilateral deliberation process. The function $\beta_0(\alpha_0, \beta)$ is the same as before. The function that determines the behavioral preferences of the advisors given the behavioral preferences of the chair is given now by $\beta = g(\alpha, \frac{\beta_0 + (n-1)\beta}{n})$.[14] This curve is a left

[13] This proof is stronger than the claim itself, as it shows that it is possible that prior to the deliberation all members favor one candidate but as a result of the deliberation all of them favor the other candidate.

[14] We can put a higher weight on β_0 if the opinion of the chair is more influential.

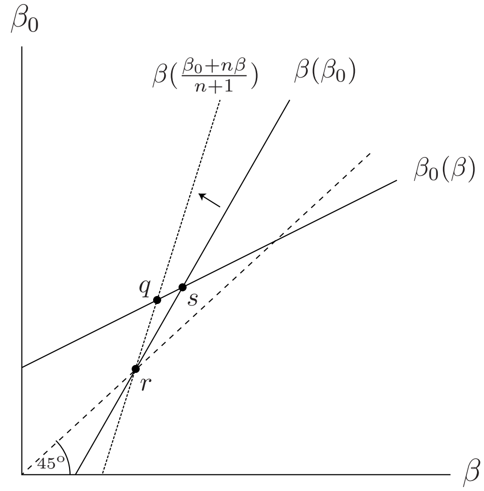

Figure 1: Proof of Claim 4

(anticlockwise) rotation of the curve $\beta(\alpha, \beta_0)$ around the symmetric point r (which is the intersection of the 45° line and the $\beta(\alpha, \beta_0)$ curve). The reason is that when $\beta = \beta_0$, the two functions imply the same behavioral preferences. It is a left rotation because for all the points above the 45° line $\beta < \beta_0$ and therefore the function for the multilateral procedure yields a lower value. The intersection q of the two curves, $\beta_0(\alpha_0, \beta)$ and $\beta = g(\alpha, \frac{\beta_0+(n-1)\beta}{n})$ yields the equilibrium for the multilateral deliberation case. As depicted in Figure 1, comparing the two equilibrium points yields that $\beta_0^{db} > \beta_0^{dm}$. ■

Proof of Claim 5: Under the consultation procedure, $\beta^{cb} = \alpha$ and $\beta_0^{cb} = g(\alpha_0, \alpha)$, while under the deliberation procedure $\beta^{db} = g(\alpha, \beta_0^{db})$ and $\beta_0^{db} = g(\alpha_0, \beta^{db})$. We show first that $\beta^{db} > \alpha$. If the core preference of the chair is α, then the advisors and the chair have the same core preferences and consequently the same behavioral preferences β. Given our SR assumption, β must be larger than α. They can obviously not equal α, because $g(\alpha, \alpha) > \alpha$. If $\beta < \alpha$, then since $g(\alpha, \alpha) > \alpha$ and $g(\alpha, \beta) = \beta$, we get

$$\frac{g(\alpha, \alpha) - g(\alpha, \beta)}{\alpha - \beta} > \frac{\alpha - \beta}{\alpha - \beta} = 1$$

In contradiction to the assumption that $g_2 < 1$. As $\alpha_0 > \alpha$, the β value of the chair is higher than when his core preferences are α, and consequently, so is the β values of the advisors.

Since $\beta_0^{b0} = g(\alpha_0, \beta^{db})$ and $\beta^{db} > \alpha$, it follows that $\beta_0^{db} > g(\alpha_0, \alpha) = \beta_0^{cb}$, hence the claim. ■

Proof of Claim 6: If person 1 does not participate in the deliberation, then $\beta_1^1 = \alpha_1$. We show first that $\beta_2^1 > \beta_2$ and $\beta_3^1 > \beta_3$. Observe that by Claim 8 in FS, $\beta_2^1 > \alpha_2$. If $\beta_2 \leqslant \alpha_2$, then clearly $\beta_2^1 > \beta_2$, and since $\beta_2 > \beta_1$,

$$\beta_3^1 = g(\alpha_3, \beta_2^1) > g(\alpha_3, \tfrac{1}{2}[\beta_1 + \beta_2]) = \beta_3$$

Suppose that $\beta_2 > \alpha_2$ but $\beta_2^1 \leqslant \beta_2$. Since by FS (2018) $\beta_1 < \beta_3$ (see end of subsection 2.2),

$$\begin{aligned} &\beta_2 = g(\alpha_2, \tfrac{1}{2}[\beta_1 + \beta_3]) \geqslant \beta_2^1 = g(\alpha_2, \beta_3^1) \Longrightarrow \\ &\tfrac{1}{2}[\beta_1 + \beta_3] \geqslant \beta_3^1 \Longrightarrow \\ &\beta_3 > \beta_3^1 \end{aligned} \tag{2}$$

Also, since $g_2 < 1$,

$$\left.\begin{aligned} &\beta_2 = g(\alpha_2, \tfrac{1}{2}[\beta_1 + \beta_3]) > \\ &\beta_2^1 = g(\alpha_2, \beta_3^1) \end{aligned}\right\} \Longrightarrow$$

$$\beta_2 - \beta_2^1 < \tfrac{1}{2}[\beta_1 + \beta_3] - \beta_3^1 \tag{3}$$

Similarly, using inequality (2)

$$\left.\begin{aligned} &\beta_3 = g(\alpha_3, \tfrac{1}{2}[\beta_1 + \beta_2]) > \\ &\beta_3^1 = g(\alpha_3, \beta_2^1) \end{aligned}\right\} \Longrightarrow$$

$$\beta_3 - \beta_3^1 < \tfrac{1}{2}[\beta_1 + \beta_2] - \beta_2^1 \tag{4}$$

Combining inequalities (3) and (4) together and recalling that $\beta_1 < \beta_2$, we get

$$\begin{aligned} &2\beta_3 - 2\beta_3^1 < \beta_1 + \beta_2 - 2\beta_2^1 < 2\beta_2 - 2\beta_2^1 < \beta_1 + \beta_3 - 2\beta_3^1 \Longrightarrow \\ &\beta_3 < \beta_1 \end{aligned}$$

A contradiction, hence $\beta_2^1 > \beta_2$. And since $\beta_1 < \beta_2$, it follows that $\beta_2^1 > \frac{1}{2}[\beta_1 + \beta_2]$, hence $\beta_3^1 > \beta_3$. It thus follows that both unanimity rule (determined by person 3) and majority (determined by person 2)rule accept less projects than the case in which all members participate in the deliberation

Suppose now that person 3 does not participate. Then by Claim 8 in FS, $\beta_3^3 = \alpha_3 > \beta_3$ and the unanimity rule will accept less projects. Since $\beta_2^3 > \beta_1^3$ (Claim 7 in FS), in order to show that the majority rule will accept more project it is enough to show that $\beta_2 > \beta_2^3$. Since by the aforementioned claim, $\alpha_2 > \beta_2^3$, this is clearly the case when $\beta_2 \geqslant \alpha_2$. We therefore prove the impossibility of $\alpha_2 > \beta_2^3 > \beta_2$. Otherwise,

$$\beta_2^3 = g(\alpha_2, \beta_1^3) \geqslant \beta_2 = g(\alpha_2, \tfrac{1}{2}[\beta_1 + \beta_3]) \Longrightarrow \beta_1^3 > \beta_1$$

Since $g_2 < 1$, we get

$$\left.\begin{array}{l} \beta_2^3 - \beta_2 < \beta_1^3 - \frac{1}{2}[\beta_1 + \beta_3] \\ \beta_1^3 - \beta_1 < \beta_2^3 - \frac{1}{2}[\beta_2 + \beta_3] \end{array}\right\} \Longrightarrow$$

$$2\beta_3 < \beta_1 + \beta_2$$

A contradiction to the fact that $\beta_3 > \beta_2 > \beta_1$. ■

Proof of claim 7: We use the following table

	β_1	β_2	β_3
(i) 1-2-3	α_1	$g(\alpha_2, \alpha_1)$	$g(\alpha_3, \theta\alpha_1 + (1-\theta)g(\alpha_2, \alpha_1))$
(ii) 1-3-2	α_1	$g(\alpha_2, \theta\alpha_1 + (1-\theta)g(\alpha_3, \alpha_1))$	$g(\alpha_3, \alpha_1)$
(iii) 2-1-3	$g(\alpha_1, \alpha_2)$	α_2	$g(\alpha_3, \theta\alpha_2 + (1-\theta)g(\alpha_1, \alpha_2))$
(iv) 2-3-1	$g(\alpha_1, \theta\alpha_2 + (1-\theta)g(\alpha_3, \alpha_2))$	α_2	$g(\alpha_3, \alpha_2)$
(v) 3-1-2	$g(\alpha_1, \alpha_3)$	$g(\alpha_2, \theta\alpha_3 + (1-\theta)g(\alpha_1, \alpha_3))$	α_3
(vi) 3-2-1	$g(\alpha_1, \theta\alpha_3 + (1-\theta)g(\alpha_2, \alpha_3))$	$g(\alpha_2, \alpha_3)$	α_3

Table 1

Person B is selected by a unanimity rule if and only if $M := \min\{\beta_1, \beta_2, \beta_3\} \geqslant \gamma$.

(i) 1-2-3: For $i = 1, 2, 3$, $\beta_i^{(v)} \geqslant \beta_1^{(i)}$, hence $M^{(v)} \geqslant M^{(i)}$.

(ii) 1-3-2: For $i = 1, 2, 3$, $\beta_i^{(v)} \geqslant \beta_1^{(ii)}$, hence $M^{(v)} \geqslant M^{(ii)}$.

(iii) 2-1-3: For $i = 1, 2, 3$, $\beta_i^{(iv)} \geqslant \beta_1^{(iii)}$, hence $M^{(iv)} \geqslant M^{(iii)}$.

(iv) 2-3-1: For $i = 1, 3$, $\beta_i^{(v)} > \beta_i^{(iv)}$. Consider two cases.

1. $g(\alpha_1, \alpha_3) < \alpha_2$. Then $\beta_1^{(v)} = g(\alpha_1, \alpha_3) = g(g(\alpha_1, \alpha_3), g(\alpha_1, \alpha_3)) < g(\alpha_2, g(\alpha_1, \alpha_3)) \leqslant g(\alpha_2, \theta\alpha_3 + (1-\theta) g(\alpha_1, \alpha_3)) = \beta_2^{(v)} < \beta_3^{(v)}$, hence $M^{(v)} = \beta_1^{(v)} > \beta_1^{(iv)} \geqslant M^{(iv)}$.
2. $g(\alpha_1, \alpha_3) \geqslant \alpha_2$. Then $\beta_2^{(v)} \geqslant \beta_2^{(iv)}$ and since for $i = 1, 2, 3$, $\beta_i^{(v)} \geqslant \beta_i^{(iv)}$, it follows that $M^{(v)} \geqslant M^{(iv)}$.

(vi) 3-2-1: $\beta_3^{(vi)} > \beta_2^{(vi)} > \beta_1^{(vi)}$, hence $M^{(vi)} = \beta_1^{(vi)}$. Obviously $M^{(vi)} \leqslant \beta_1^{(v)} < \beta_3^{(v)}$ and by definition, $M^{(vi)} \leqslant \beta_1^{(vi)}$. We show next that for all $\alpha_2 \in [\alpha_1, \alpha_3]$, $\beta_1^{(vi)} \leqslant \beta_2^{(v)}$. For $\alpha_2 = \alpha_1$ we get $\beta_1^{(vi)} = \beta_2^{(v)}$. Differentiate both with respect to α_2:

$$\frac{\partial \beta_1^{(vi)}}{\partial \alpha_2} = g_2(\alpha_1, \theta\alpha_3 + (1-\theta) g(\alpha_2, \alpha_3)) \times (1-\theta) g_1(\alpha_2, \alpha_3)$$

$$\frac{\partial \beta_2^{(v)}}{\partial \alpha_2} = g_1(\alpha_2, \theta\alpha_3 + (1-\theta) g(\alpha_1, \alpha_3))$$

We assumed that $g_{12} < 0$ (see end of subsection 2.2), hence $g_1(\alpha_2, \alpha_3) \leqslant g_1(\alpha_2, \theta\alpha_3 + (1-\theta) g(\alpha_1, \alpha_3))$ and since $g_2(\cdot, \cdot) \leqslant 1$, $\frac{\partial \beta_1^{(vi)}}{\partial \alpha_2} \leqslant \frac{\partial \beta_2^{(v)}}{\partial \alpha_2}$, implying $\beta_1^{(vi)} \leqslant \beta_2^{(v)}$ for all $\alpha_2 \in [\alpha_1, \alpha_3]$. As $M^{(vi)} \leqslant \beta_i^{(v)}$ for $i = 1, 2, 3$, it follows that $M^{(vi)} \leqslant M^{(v)}$.

Person B is selected by a majority rule if and only if L, the mid-value of $\beta_1, \beta_2, \beta_3$ satisfies $L \geqslant \gamma$. Using the above table we get

(i) 1-2-3: For $i = 1, 2, 3$, $\beta_i^{(vi)} > \beta_i^{(i)}$, hence $L^{(vi)} > L^{(i)}$.

(ii) 1-3-2: For $i = 1, 2, 3$, $\beta_i^{(vi)} > \beta_i^{(ii)}$, hence $L^{(vi)} > L^{(ii)}$.

(iii) 2-1-3: $L^{(iii)} = \min\{\beta_2^{(iii)}, \beta_3^{(iii)}\}$. As $\beta_2^{(vi)} > \beta_2^{(iii)}$ and $\beta_3^{(vi)} > \beta_3^{(iii)}$, it follows that $L^{(vi)} > L^{(iii)}$.

(iv) 2-3-1: $L^{(vi)} = \beta_2^{(vi)} > \max\{\beta_1^{(iv)}, \beta_2^{(iv)}\} \geqslant L^{(iv)}$.

(v) 3-1-2: $L^{(vi)} = \beta_2^{(vi)} > \max\{\beta_1^{(v)}, \beta_2^{(v)}\} \geqslant L^{(v)}$. ■