Social Influence in Legal Deliberations*

Chaim Fershtman†  Uzi Segal‡
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Abstract

Juries, appellate courts, parole boards are all institutes that need to make collective decisions. What characterizes these institutes is that they are typically engage in deliberations prior to decision making. Beyond information exchange, such deliberations also aim to affect the opinions, preferences and votes of other members. Using a model of social influence, we demonstrate how deliberation and voting procedures affect the voting outcome even when the same information is available to all. We then demonstrate the ability of a “designer” to manipulate the deliberation procedure in order to increase the probability that the outcome he favors will be selected.

Keywords: Jury, parole board, social influence, deliberation.

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†Coller School of Management, Tel Aviv University & CEPR, fersht@post.tau.ac.il
‡Department of Economics, Boston College, segalu@bc.edu
1 Introduction

Deliberation characterizes most of collective decision making. Examples can be a jury, parole boards, congressional committees, multi-judge appellate courts, or in general, any committee that needs to make a collective decision. Different institutions may have different deliberation protocols. For example, deliberations among the Justices in the US Supreme Court proceed by order of seniority. During such deliberations individual decision makers exchange information, argue, exchange ideas, and try to persuade and convince each other regarding the “right” decision. Final decisions, which are often made by a vote, are the outcomes of the collective interactions during the deliberation process.

Starting from Condorcet (1785) there are numerous studies on collective decision making. However, most of this literature focuses on information aggregation and voting protocols. The meaning of deliberation in this literature is information sharing, information manipulation, and information aggregation. Given that committee members may have different information and preferences, the literature analyzes the incentives committee members have to disclose or distort their private information or to acquire new information.

While information aggregation is an important part of deliberation, it is irrelevant to many judicial procedures (e.g., to juries, judges, or members of parole boards), as members of such groups are not permitted to use information beyond what is revealed to them and which is, by definition, known to

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1See for example Iaryczower, Shi and Shum (2018) for an empirical analysis on the effect of deliberation on the decisions of the US appellate courts.


all of them. There is, however, another important aspect of interaction which has been largely ignored, even though it is very relevant to such procedures. When juries, judges, or members of parole boards deliberate they typically argue, convince, explain and express their opinion. Other members may be persuaded by their arguments or even just influenced by listening to other opinions. It is this aspect of deliberation which is the focus of our paper.

There is an extensive documentation in social psychology of the effect of social influence on individuals’ behavior and decisions. For a review of this literature, see Isenberg (1986), Myers and Lamm (1976), and Myers (1975, 1982). The effect of deliberation on jury decisions was discussed by Schade, Sunstein, and Kahneman (2000) and Mendelberg (2006). Political scientists argue that the composition of the discussion group changes the views expressed by those who participate in it (see Farrar, Green, Green, Nickerson and Shewfelt (2009, p. 616)). Aronson, Wilson, and Akert (2010) claim that group discussions may make people more risk taking than their initial tendencies. Finally, Hoff and Stiglitz (2016) claim that preferences and behavior are influenced by actions and beliefs of people around the decision maker.

The deliberation in a committee may follow a protocol that specifies when members of the committee are allowed to express their opinion and explain their vote or it may be free of any restrictions. But the protocol of deliberation may affect the way decision maker shape their opinions. Consider, for example, two different decision procedures that are part of academic life: recruiting a new faculty and the evaluation of papers. Both are important decisions that require the inputs, vote, and recommendations of several people. There are, however, important differences between the two. In the first, there is typically a meeting of the recruiting committee in which there is an open deliberation regarding the different candidates. Referees, on the other hand, often write their reports and recommendations directly to the editor without any discussion or deliberation among them, but here too, letters reveal not only information, but also the reviewers’ opinions and preferences.
In a previous paper (Fershtman and Segal (2018), hereafter FS) we modeled social influence by introducing a setup in which each individual is characterized by two sets of preferences: unobservable core preferences and observable behavioral preferences, where actual choice is determined by the latter. Each person has an individual social influence function that determines the way this individual is affected by the opinions of others. Formally, this influence function determines the individual’s behavioral preferences as a function of her core preferences and the observed behavioral preferences of others. In the present paper we capture the effect of deliberation by using this social influence procedure.

We consider a group of $n$ individuals who need to choose between two alternatives, differing with respect to two attributes. For example, a panel of judges has to determine an appropriate prison term of a convicted criminal, where the two relevant factors are the severity of the crime and the history of the convict. Another example is a jury that has to decide whether an accused person is guilty or not, based on forensic evidence and oral testimonies. The levels of these attributes are fully observable by all participants but panelists may differ in their preferences regarding the relative importance of these attributes. During the deliberation stage members may try to convince each other regarding the appropriate criteria for choosing an alternative. As the final behavioral preferences are the equilibrium of this adaptation process, our aim is to examine this equilibrium and the type of voting outcomes that emerge under different protocols of deliberations or for a different profile of members.

In Section 3 we discuss the effect of the committee’s profile of preferences on its final decision. The main insight is that since members are subject to social influence it is important to pay attention not just to their ordinal preferences (i.e., if they prefer alternative $A$ or $B$) but also to their cardinal preferences, (that is, by how much they prefer one alternative to the other), as these preferences affect the pattern of social influence. For example, we consider a jury that is expected to vote for alternative $A$ and analyze the
replacement of one of its members with another juror who in her core preferences prefers alternative $A$. We show that it is possible that such a switch will result in a shift of vote by the jury from $A$ to $B$. This may happen if the replaced juror has strong preferences for $A$ while the new juror’s preferences for $A$ are much milder and therefore she will be much less effective in the social influence process. In a similar way we show that if a committee is expected to vote for one of the alternatives then adding a member who in her core preferences prefers the same alternative may induce the committee to change its opinion. Finally, we show that as a result of social influence, deliberation may result in a violation of the unanimity property. That is, even when all committee members prefer in their core preferences alternative $A$ over alternative $B$, deliberation may lead them to vote for alternative $B$.

In Section 4 we analyze the effect of deliberation protocols on the final decision. We focus on two aspects of such protocols. First, we consider a sequential protocol in which there is a specific order of speaking. As early speakers influence the preferences of other members but are not influenced by them, the outcome of the deliberation depends on the order in which it is done. We also show that an individual in control of the order of deliberation may manipulate it to achieve his favorite outcome. Interestingly, there is a different optimal order under majority and unanimity rules and the designer may choose a different protocol-manipulation depending on the type of voting rule. In a similar way we show the difference between decision making in committees in which all individuals must express their opinion before voting and situations in which they may choose only to listen to others or even do not have to attend meetings at all and just choose to send their written ballots. All proofs appear in the appendix.
2 The Model

2.1 Preliminaries

A group of decision makers (jury, judges, a committee) consisting of \(n > 2\) members needs to choose one of two possible options \(\{A, B\}\). We assume that the decision of the group is based on two attributes that characterize the alternatives, denoted by \((a_1, a_2)\) for alternative \(A\) and \((b_1, b_2)\) for \(B\). These attributes can be the reliability of the forensic findings and the witnesses evidences when the two options are \{guilty, not guilty\}, the nature of the crime and the criminal history of the defendant in the sentencing stage or a prisoner’s behavior in jail and his likelihood of recidivism when an early parole is considered. We assume that these attributes are perfectly observable and there is no dispute regarding their levels.

We assume that members of the committee have their own core preferences \(\alpha_i \in [0, 1]\) over the relative importance of the two attributes. For example if the two relevant factors for the jury decision are the reliability of the forensic and of the witnesses evidence, then a higher \(\alpha_i\) implies that member \(i\) puts a higher emphasis on forensic relative to oral evidences. With such preferences, member \(i\) prefers \(A\) to \(B\) if and only if \(a_1 + \alpha_ia_2 \geq b_1 + \alpha_ib_2\). If \(a_1 \geq b_1\) and \(a_2 \geq b_2\), option \(A\) clearly dominates \(B\) and the choice is trivial. We therefore assume, without loss of generality, that \(a_1 > b_1\) but \(a_2 < b_2\) so that there is no clear choice between the two options. Let \(\alpha = (\alpha_1, \ldots, \alpha_n)\), and assume, w.l.g., that \(\alpha_1 \leq \ldots \leq \alpha_n\). Given our assumptions, for any given pair \(\{A, B\}\) with known and observable attributes there is a critical value \(\gamma = (b_1 - a_1)/(a_2 - b_2)\) such that member \(i\) prefers \(A\) to \(B\) if and only if \(\alpha_i \leq \gamma\).

We discuss throughout two major ways in which committees (like judges or juries) make decisions. Majority rule, in which alternative \(A\) is selected over alternative \(B\) if and only if more members prefer \(A\) to \(B\) then those who prefer \(B\) to \(A\) (and vice versa), and a unanimous rule, in which one alternative is selected if and only if all members prefer it to the other one. If
no alternative receives unanimous support, then typically there is a variety of options how to proceed, for example, instructing the juries to continue deliberation or declaring a mistrial (see e.g. LaFave et al. §29.4.d).

2.2 Deliberation and Social Influence

Following Fershtman and Segal (2018), we assume that each individual has two types of preferences; core preferences, which were described above and behavioral preferences that govern his behavior and which are given by $\beta_i$, $\beta_i \in [0,1]$. Without social influence behavioral preferences are identical to the core preferences. But with deliberation the behavioral preferences depend on the individual’s core preferences and the behavioral preferences of other individuals with whom he interacts. Formally, each individual has a social influence function that shapes his behavioral preferences such that $\beta_i = g^i(\alpha_i, \beta_{-i})$ where $\beta_{-i} = (\beta_1, \ldots, \beta_{i-1}, \beta_{i+1}, \ldots, \beta_n)$ and individual $i$ votes for $A$ over $B$ if and only if $\beta_i \leq \gamma$. Observe that individuals do not necessarily have the same $g^i$ function and may therefore be influenced in different ways. Some may be stubborn enough to ignore any opinion of others, having $g^i(\alpha_i, \beta_{-i}) = \alpha_i$ for every $\beta_{-i}$. On the other hand, some individuals’ convincing power may overcome the opinions of others. In the extreme case we’ll have an individual $j^*$ who has the ability to dominate discussion in such that for every $i \neq j^*$ we get $g^i(\alpha_i, \beta_{-i}) = \beta_{j^*}$. However, for most of the analysis we consider the case in which all members are subject to social influence.

There is a difference between our social influence effect and the more familiar social learning setting of DeGroot (1974) and more recently Golub and Jackson (2010). In the learning setup there is a group of individuals where each of them has some beliefs about the probability of some event. These individuals communicate with each other (or with their neighbors in the social network) and update their beliefs. Each person’s new beliefs are a weighted average of his and his neighbors’ (last period) beliefs. The question in this literature is under what conditions do these beliefs converge to a consensus and whether this consensus is the correct probability of the event. This type
of interaction is irrelevant to jury, judges, or parole-boards deliberations, as both are forbidden from using any private information. In contrast, the social influence we discuss is about affecting preferences, not information.

When juries need to determine conviction, and for this choice they need to weigh different types of evidence, it is not clear that reaching a consensus captures the richness of social influence. It is possible, for example, that at the end of the deliberation period, some juries will put a higher weight than others on forensic evidences. Our focus therefore is not on the possibility of reaching a consensus but rather on the equilibrium profile of preferences. That is, for every profile of core preferences, social influence functions, and a protocol of deliberation, we are looking for a profile of behavioral preferences and the voting outcome based on these preferences which are determined by the core preferences of each person and the behavioral preferences of others.

One familiar type of social influence is conformism, where individuals conform to the behavior and beliefs of their group (for example, the jury members). When there are two individuals in the group, conformism implies that the behavioral preferences $\beta_i = g^i(\alpha_i, \beta_j)$ are between $\alpha_i$ and $\beta_j$. In a larger group conformism implies that the behavioral preferences of individual $i$ satisfy $\beta_i \in [\min\{\alpha_i, \beta_{-i}\}, \max\{\alpha_i, \beta_{-i}\}]$.

**Claim 1** If $g$ is monotonic in both arguments, then conformism is equivalent to the requirement that $g(\alpha, \alpha, \ldots, \alpha) \equiv \alpha$.

In other words, conformism is equivalent to the requirement that if the behavioral preferences of everyone else are equal to person $i$’s core preferences, then his behavioral preferences too will be the same.

Claim 1 highlights the limitations of conformity by showing that an individual is content with his behavior if other people behave according to his core preferences. But consider again the example of juries weighting forensic and oral evidence. The parameter $\alpha_i$ describes the relative importance that agent $i$ attaches to the former versus the latter, where a higher $\alpha_i$ means a higher weight on forensic evidence. It is possible that agent $i$ views herself
as someone who puts an emphasis on scientific evidence, but when she discovers that her weight $\alpha$ is the same as the (behavioral) weights of all other members, she may behave and vote as if her weight on forensic evidence is higher than her core preferences.

Let $\alpha$ be the $n-1$ tuple $(\alpha, \ldots, \alpha)$. In the case described above, $g^i(\alpha, \alpha) > \alpha$ and we say that the social influence function has an Super Reinforcement (SR) property. Similarly, we call the property $g^i(\alpha, \alpha) < \alpha$ Under Reinforcement (UR).\textsuperscript{4} Note that SR and UR are local properties and the social influence function may have different characteristics at different values of the parameters. So it is possible to have $g^i(\alpha, \alpha) > \alpha$ for some values of $\alpha$ and $g^i(\alpha, \alpha) < \alpha$ for other values. In particular, it may happen that there is $\alpha^*$ such that $g(\alpha, \alpha) > \alpha$ iff $\alpha < \alpha^*$, in which case preferences gravitate towards the center. Or it may happened that $g(\alpha, \alpha) > \alpha$ iff $\alpha > \alpha^*$, in which case preferences are pushed to the extreme.

Claim 2 in FS offers an axiomatic framework under which the behavioral parameter $\beta$ depends only on one’s core preferences and the average of the observable behavioral parameters of everyone else. That is, $\beta_i = g^i(\alpha_i, \sum_{j \neq i} \beta_j/(n-1))$. We adopt this setup here too.\textsuperscript{5} We assumed there that the partial derivatives $0 < g_1, g_2 \leq 1$ and proved that if all agents have the same social influence function $g$, then (i) If all agents start with the same $\alpha$, then $\beta := \beta_1 = \ldots = \beta_n \geq \alpha \iff g(\alpha, \alpha) \geq \alpha$; and (ii) $\beta_i \geq \beta_j \iff \alpha_i \geq \alpha_j$. In this paper we maintain the assumption that $0 < g_1, g_2 \leq 1$, and assume further that $g_{12} < 0$. The assumptions $g_1 > 0$ and $g_2 > 0$ mean that behavioral preferences are consistent with the person’s preferences.

\textsuperscript{4}Note that there is some analogy between these two properties since by proper representation of our setting we can transform $\alpha$ — the weight of the second attribute — to $\hat{\alpha}$ which will be the weight of the first attribute. In this case whenever $g(\alpha, \alpha) > \alpha$ we would have $g(\hat{\alpha}, \hat{\alpha}) < \hat{\alpha}$ so SR with respect to the second attribute implies UR with respect to the first one.

\textsuperscript{5}This setup assumes that $g$ depends on one’s core preferences and the average behavioral preferences of the rest without specifying how many other individuals there are in the influence group. One can modify this assumption by indexing the $g$ function according to the number of individuals in the influence group.
own core preferences and that the dependence on other people’s observed behavior is of acceptance and not of rejection. The requirements $g_1 \leq 1$ and $g_2 \leq 1$ mean that the change in the behavioral parameter cannot be larger than the change in the relevant parameters, reflecting the fact that the other parameters which did not change mitigate the influence of changes of the parameter that did change. Finally, $g_{12} < 0$ suggests that the sensitivity of a person’s behavior to an increase in his core preferences is higher when these preferences are moving in an opposite direction to his observed environment as such a change is more indicative to him than when his core preferences move up together with the observed preferences of everyone else.

Interactions between individuals open the door to possible manipulations and strategic behavior. In the present context, there are two possible types of such behavior. Individuals may misrepresent their views, knowing that other members are influenced by their discourse and arguments, and those who control the procedures of deliberation may manipulate it in order to influence its outcome. In this paper we want to focus attention on the second type of strategic behavior. We assume that the organizers of the deliberation procedures understand the pattern of social influence and may manipulate the deliberation procedure in order to affect its outcome. We assume, however, that committee members themselves express their true opinions without any strategic motives. What we have in mind is a parole board or judges who make periodic decisions on various issues and its members cannot express different opinions at different meetings. On the other hand, the chair of the committee has the power to determine the procedure of deliberation, for example, its order, and it may change from one meeting to another.

### 2.3 Networks of Influence

When there is a jury in which there is a free discussion without any specific protocol of deliberation then we are looking for the equilibrium profile of behavioral preferences such that the behavioral preferences of each individual is derived from her core preferences and the behavioral preferences of other jury
members. But in many situations there is a specific deliberation protocol, which may take different ways with a rich set of possible scenarios. In order to capture the effect of different types of deliberation protocols we consider a model of network of influence such that the network structure captures the particular pattern of deliberation. We model a social influence induced by a protocol of deliberation as a directed network in which there is a directed link between agent $i$ and agent $j$ only when agent $j$ is socially influenced by agent $i$ and the preferences he observes are $i$’s behavioral preferences. Given this structure we need to look for equilibrium in the behavioral preferences which would depend on the network of social influence.\footnote{With such a structure it is very simple to capture sequential deliberation by a network in which there is a directed link between any individuals and those who deliberate and vote later according to the protocol.}

Note that while our structure allows only for direct interaction, indirect influence plays an important role in our model. Agent $i$ may never meet agent $k$ but he may still be indirectly influenced by her. This happens whenever agent $k$ socially interacts with agent $j$ and influences her preferences and then agent $j$ interacts with agent $i$.

Given a network of influence, we are looking for a social influence equilibrium, which is a vector of behavioral preferences $\beta = (\beta_1, \ldots, \beta_n)$ such that each agent $i$’s behavioral preferences are determined by his core preferences $\alpha_i$ and the equilibrium behavioral preferences of those agents by whom he is influenced. Formally, the social influence function takes the form $\beta_i = g_i(\alpha_i, \beta_{-i})$ where $\frac{\partial g_i}{\partial \beta_j} \equiv 0$ for all members $j$ who do not influence $i$. We assume that $g_i(\alpha_i, \beta_{-i})$ is continuous in all its arguments. For any profile of core utilities $\alpha = (\alpha_1, \ldots, \alpha_n)$ and social influence functions $g = (g^1, \ldots, g^n)$, we define equilibrium behavioral preferences for a given network as $\beta^*(\alpha) = (\beta^*_1(\alpha), \ldots, \beta^*_n(\alpha))$ such that for every $i$, $\beta^*_i(\alpha) = g^i(\alpha_i, \beta^*_{-i})$.

**Claim 2** Consider a given network of influence. For every profile of core utilities $\alpha$ and social influence functions $g^1, \ldots, g^n$, there is an equilibrium behavioral preferences vector $\beta^*(\alpha)$.\footnote{The proof is a special case of the proof of Claim 1 in FS, which considers a case in
3  Social influence in Jury Deliberation

Jury deliberation is typically done without specific protocol. There is a discussion without any specific order of speaking and all jury members may participate in the deliberation, which is then followed by voting. In this section we discuss the relationship between the profile of jury members and the final vote, and define several intuitive properties that the deliberation process may satisfy. These properties are satisfied when jury members vote according to their core preferences. But whether these properties are satisfied at the presence of social influence depends on the individual social influence functions as well as on the profile of their core preferences.

We start with a simple property of unanimous acceptance which requires that if there is a unanimous support for an alternative prior to the deliberation, then it will be chosen by the jury after the deliberation as well. Note that this is a weak notion of unanimous acceptance. A stronger version would imply that if prior to the deliberation all jury members prefer one alternative to another, then after deliberation they will still unanimously vote for the first. The justification for the stronger version is that if no jury member supports a certain alternative, then no one will be able to convince others to vote for it.

Property 1 (Unanimous Acceptance): If all jury members prefer one alternative (e.g. for all \( i, \alpha_i < \gamma \)), then social influence during the deliberation process results in an equilibrium behavioral preferences \( \beta \) such that the same alternative is chosen.

The second property deals with situations in which one of the jury members is replaced or a new member is added to a committee. The consistency which both the core and the behavioral preferences are utility function on \([0, 1]\) which are assumed to be bounded, continuous, and equi-Lipschitz on \([0, 1]\), but are not necessarily represented by a single parameter \( \beta \). In FS, the social influence is defined with respect to a complete undirected network but can be easily extended to any directed network.
property is very intuitive and clearly holds if members would vote according to their core preferences without any social influence.

**Property 2** (Consistency): (i) If an alternative is selected after deliberation and a new member supporting it is added to the committee, then the new committee will still vote for it. (ii) If an alternative is selected after deliberation and one of the jury members is replaced with a new juror who, in his core preferences, supports this alternative, then the new jury will continue to select the same alternative.

Our last property also deals with replacing a jury members but now the requirement takes into account not only the ordinal aspect of the juror’s preferences, as represented by his vote, but also the cardinal aspect of his preferences, as represented by the size of the coefficient $\alpha$.

**Property 3** (Monotonicity): If a jury votes for alternative $B$ and one of its members is replaced with a new member with the same social influence function who, in his core preferences, is more inclined to vote for $B$ (that is, has a higher value of $\alpha$), then the new jury will also vote for alternative $B$.

Our social influence setting implies that what matters for understanding the jury vote is not just whether its members prefer $A$ or $B$ but also the **intensity** of these preferences, as represented by the values of $\alpha$ and $\beta$. This intensity also determines the way jury members affect other members. It is therefore important to analyze the deliberation stage which reveals the intensity of preferences and not just the voting intentions declared by members which only reveal their ordinal preferences. It is this characteristic of our social influence setting which generates the possibility of inconsistencies in the juries’ decisions.

**Claim 3** Under both majority and unanimity rules, the deliberation process satisfies the monotonicity property, but it does not necessarily satisfy the unanimous acceptance and consistency properties.
The above claim provides a valuable information regarding the relationship between the types of jury members and their final vote. Its main message is that it is not enough to focus on the ordinal preference (i.e., which alternative the juror prefers) as the cardinal preferences (the intensity of the ordinal preferences) play an important role in determining the social influence and therefore the final vote.

In many situations there is a person — call her “designer” — who has some control over the identity of the members of the committee and sometimes over the deliberation procedures. This person may have her own preferences regarding the outcome of the vote. Claim 2 implies that this designer can sometimes apply her power to affect the decision of the committee. The designer may be the chair of a congressional committee, a judge, or a lawyer who may have an influence on jury selection.

Consider for example a designer who prefers alternative $A$ and has some control over the identity of the individuals in a parole board. The above claim implies that adding an individual that mildly prefers alternative $A$ may be counterproductive and it may convince other members of the board to move their support from alternative $A$ to $B$. The designer is better off adding one charismatic member with strong preferences for $A$ rather than adding more individuals that only mildly support this option.

4 Procedures of Deliberation

Juries, judges, or parole boards may have different deliberation procedures. In some cases members may choose not to express their opinion or to refrain from explaining the way they are going to vote. In other cases they must explain their decision before voting. There are committees in which members do not have to attend meetings, they may just send their written vote while in others the protocol insists on open discussion before any vote. In this section we show that procedures may affect the formation of the equilibrium of the behavioral preferences and the outcomes of the voting. We focus
on two aspects of the deliberation procedure: (i) the effect of the order of deliberation in a sequential procedure and (ii) the requirement to participate in the deliberation.

4.1 The Effect of the Order of Deliberation.

There are many situations in which deliberation is done sequentially. The order may be determined by seniority, rank, or even by a lottery. In this subsection we analyze the effect of the order of deliberation on the outcome of the debate. When behavioral preferences are formed during the deliberation then the order of the deliberation may play an important role in shaping those preferences as committee members are influenced only by individuals who have already expressed their opinions.

To illustrate, consider a committee consisting of three members expressing their views sequentially. Denote these players by the order in which they speak (assuming that each of them is allowed to express his views only once) as $i-j-k$. The first person is exposed to no other views, therefore $\beta_i = \alpha_i$ and he votes according to his core preferences. The second person is influenced only by the first and therefore

$$\beta_j = g(\alpha_j, \beta_i) = g(\alpha_j, \alpha_i)$$

The third person is influenced by the other two. But the order in which she is exposed to these views may affect this influence. Person $k$ may give a higher weight to the last person she heard, or to the first one, or she may treat their opinions equally. We will therefore consider the general case in which person $k$ weighs the opinions of the two other members at the ratio $\theta : 1 - \theta$, $\theta \in [0, 1]$. In other words,

$$\beta_k = g(\alpha_k, \theta \beta_i + (1 - \theta) \beta_j) = g(\alpha_k, \theta \alpha_i + (1 - \theta) g(\alpha_j, \alpha_i))$$

As before, we restrict our analysis to the case in which all members use the same function $g$, and require in addition that $g(\alpha, \alpha) \equiv \alpha$. Consider the

Claim 4 Let $\alpha_1 < \alpha_2 < \alpha_3$. If decision is made by the unanimity rule, then option $B$, which has an advantage in the second attribute, is most likely to be selected if the order of deliberation is 3-1-2, regardless of the value of $\theta$. Under majority rule, it is most likely to be elected if the order is 3-2-1.

This claim shows that the designer of the protocol, who has his own favorite choice, may benefits from manipulating the order in which deliberation takes place. However, the optimal order depends on the committee’s decision rule. There are different optimal orders under majority and unanimity rules.

Remark: It is clear from Table 1 in the proof of Claim 4 that the order of the behavioral parameters $\beta_1, \beta_2, \beta_3$ does not have to be the same as the order of the core parameters $\alpha_1, \alpha_2, \alpha_3$. For example, if $g(\alpha_3, \alpha_1) < \alpha_2$ and $\theta$ is sufficiently close to 0, then $\beta_2^{(ii)} > \beta_3^{(ii)}$ even though $\alpha_2 < \alpha_3$ (see Table 1).

4.2 The Effect of No Participation in the Deliberation

In many committees members do not have to participate in the deliberation. They can vote without explaining their opinion or they may even send their vote by mail without listening to the opinions of other committee members. In order to demonstrate the effect of such procedures we consider a decision making by a parole board, where (one of) the relevant factors is the safety of the community. Suppose for simplicity that the board specifies a benchmark $\gamma$ of a critical risk level such that any prisoner posing a higher risk to society will not be granted a parole. However, each board member has her own opinion regarding the risk associated with each prisoner. Although all members have the same information and are subject to the same guidelines, they may differ in the way they apply these guidelines and information to specific cases, following their different experiences or different backgrounds, or some other personal characteristics. We assume that these opinions are
captured by a single parameter $\alpha$ (which is equivalent in our terminology to the core preferences). Similarly, following the deliberation in the parole board each member may adjust her opinion to $\beta$ (which is equivalent to the behavioral preferences). Thus, members with $\beta \leq \gamma$ will vote in favor of a parole and those with $\beta > \gamma$ will vote against it.

We assume boards of three members and consider two possible decision rules. The first is a majority rule in which a parole decision requires the support of at least two members of the board. The second is a unanimity rule in which parole decision requires the support of all three members. If not all members support a parole, than the prisoner is not released from jail.

Suppose that the three members in the parole board have the same social influence function $g$, but they differ in their core preferences (risk assessment) which are given by $\alpha_1 < \alpha_2 < \alpha_3$ and that after deliberation their risk assessment (behavioral preferences) become $\beta_1, \beta_2, \beta_3$. Let $\gamma_m(\beta_1, \beta_2, \beta_3)$ be the smallest value greater than or equal to at least two of $\beta_1, \beta_2, \beta_3$ and let $\gamma_u(\beta_1, \beta_2, \beta_3) = \max\{\beta_1, \beta_2, \beta_3\}$ be the critical risk indexes under the majority and the unanimity rules, respectively, such that if these values are below $\gamma$ than the case will proceed and the prisoner will be released. Clearly, $\gamma_m \leq \gamma_u$, as any case that should lead to a parole according to all members should also imply a parole by at least two members.

When there is a deliberation with the participation of all committee members then by Claim 7 in FS, $\alpha_1 < \alpha_2 < \alpha_3$ implies $\beta_1 < \beta_2 < \beta_3$, hence $\gamma_m(\beta_1, \beta_2, \beta_3) = \beta_2$ and $\gamma_u(\beta_1, \beta_2, \beta_3) = \beta_3$. We simplify our analysis and assume that the social influence function used by all members is such that $g(\alpha, \alpha) \equiv \alpha$. Under this assumption (see Claim 8 in FS), equilibrium behavioral preferences move towards the average such that $\beta_1 > \alpha_1, \beta_3 < \alpha_3$, but the relationship between $\beta_2$ and $\alpha_2$ is unclear. Therefore, under the unanimity rule $\gamma_u(\beta_1, \beta_2, \beta_3) = \beta_3 < \alpha_3 = \gamma_u(\alpha_1, \alpha_2, \alpha_3)$, which implies that as a result of deliberation and social influence there is a larger set of cases that will be released by the committee. However, if a committee uses the majority rule then the effect of deliberation is unclear as both $\beta_2 > \alpha_2$ and $\beta_2 < \alpha_2$
are possible.

Suppose now that person $i$ does not take part in the deliberation process, and therefore $\beta_i = \alpha_i$. We continue to assume that all members have the same social influence function $g$ and that $g(\alpha, \alpha) \equiv \alpha$. Denote by $\beta^i_j$ the behavioral preferences of person $j$ when person $i$ does not take place in the deliberation. The analysis of this situation depends on the identity of the non-participating individual.

**Claim 5** If person 1 does not participate in the deliberation, then both unanimity and majority rules lead to less paroles than the case in which all members participate in the deliberation. But if player 3 does not participate, then the unanimity rule paroles less while the majority rule paroles more cases than under full deliberation. \(^8\)

Consider a designer who has some control over the deliberation process. Assume that this designer prefers to parole as many prisoners as possible. Assume further that the designer knows the preferences of all parole board members and has some control over the deliberations. Then Claim 4 implies the following corollary.

**Corollary 1** The designer should insist that person 1 participates in the deliberations. Under unanimity rule the designer should insist that player 3, the most cautious committee member, participates in the deliberation. But under majority rule the designer should approve person 3’s wish to vote without taking part in the deliberations.

## 5 Concluding Remarks

There are many decisions that are done by a group of individuals. But before making decisions these individuals typically deliberate, exchange information, argue with one another and try to convince each other regarding the

\(^8\)The case where person 2 does not participate is more involved and the analysis depends on whether $\beta_2$ is above or below the average of $\beta_1$ and $\beta_3$. 
right choice. This is true in juries, parole boards and even in most families. Our paper focuses on the effect of deliberation as a mechanism that changes preferences and opinions. This important aspect of deliberation implies that the deliberation process is not just an exchange of information (or manipulation of information). The convincing part is about changing opinions and attitude. Such effects open the door for strategic manipulations of committees’ work. In many situations there is an individual who may affect the choice of committee members (either jury members or members of the parole board) or affect the deliberation and voting procedure. This individual that we labeled as the “designer” may manipulate the procedure of deliberation or the identity of committee members in order to increase the probability of a decision that he favors.

The results of our paper were presented for the simple symmetric case but the setup can be easily extended to a variety of situations in which committee members are asymmetric in their ability to influence or by their tendency to be influenced by others. Such a setup may be used for the analysis of a variety of asymmetric situations and answer for example questions regarding the circumstances under which we would like to put the influential person at the end or at the beginning of the discussion, how to choose committee members, and what deliberation protocols to adopt that would provide the desired outcome.

Appendix: Proofs

Proof of Claim 1: Let $g$ be monotonic in all arguments and suppose first that $g(\alpha, \alpha, \ldots, \alpha) \equiv \alpha$. For a given $(\alpha_i, \beta_{-i})$, let $\gamma_i = \min\{\alpha_i, \beta_{-i}\}$ and $\gamma^* = \max\{\alpha_i, \beta_{-i}\}$. Then by monotonicity,

$$\gamma_* = g(\gamma_*, \gamma_*, \ldots, \gamma_*) \leq g(\alpha_i, \beta_{-i}) \leq g(\gamma^*, \gamma^*, \ldots, \gamma^*) = \gamma^*$$

If, on the other hand, $g(\alpha, \alpha, \ldots, \alpha) \neq \alpha$, for example, if for some $\alpha$, $g(\alpha, \alpha, \ldots, \alpha) < \alpha$, then by continuity, for sufficiently small $\varepsilon > 0$, $g(\alpha+\varepsilon, \alpha, \ldots, \alpha) <$
$\alpha$. Likewise, if $g(\alpha, \alpha, \ldots, \alpha) > \alpha$, then $\alpha < g(\alpha - \varepsilon, \alpha, \ldots, \alpha)$, a contradiction.

Proof of Claim 3:

Monotonicity: Consider the system $\beta_i = g^i(\alpha_i, \sum_{j \neq i} \beta_j / (n - 1))$, $i = 1, \ldots, n$. Take the total differential to obtain for $i = 1, \ldots, n$

$$g_i^i\left(\alpha_i, \sum_{j \neq i} \frac{\beta_j}{n - 1}\right) = \frac{d\beta_i}{d\alpha_i} - \frac{1}{n - 1} \sum_{j \neq i} g_i^j\left(\alpha_i, \sum_{j \neq i} \frac{\beta_j}{n - 1}\right) \frac{d\beta_j}{d\alpha_i} \quad (1)$$

Let the matrix $B$ be given by $b_{i,i} = 1$, and $b_{i,j} = -\frac{1}{n - 1} g_i^j\left(\alpha_i, \sum_{j \neq i} \frac{\beta_j}{n - 1}\right)$ whenever $i \neq j$. Let $C_j$ be obtained from $B$ by replacing column $j$ of $B$ with $(0, \ldots, 0, g_i^i\left(\alpha_i, \sum_{j \neq i} \frac{\beta_j}{n - 1}\right), 0, \ldots, 0)^T$. The matrices $B, C_1, \ldots, C_n$ all satisfy the conditions of theorem 4.1 in Takayama (1985, p. 392), and moreover, for $x^T = (1, \ldots, 1)$ and $A = B, C_1^T, \ldots, C_n^T$, $A \cdot x \geq 0$ (recall that $0 < g_2 \leq 1$). By the above theorem, $\det(B), \det(C_1), \ldots, \det(C_n) > 0$. It thus follows from the system of linear equations (1) that for all $i, j$, $\frac{d\beta_i}{d\alpha_i} > 0$.

All committee members are now more inclined to choose alternative $B$, and as it was preferred to $A$ before the shift, it is certainly preferred after.

Unanimous Acceptance: Suppose that all members have the same social influence function $g(\alpha, \beta)$ such that $\beta$ is the average preferences of everyone else. If all agents have the same core preferences $\alpha$ and the social preference function is SR (i.e., $g(\alpha, \alpha) > \alpha$), then the equilibrium occurs at $\beta > \alpha$ (see Claim 6 in FS). Let $\alpha'$ and $\beta'$ be such that $\beta' = g(\alpha', \beta')$. If $\alpha' < \gamma < \beta'$ then by their core preferences all agents prefer $A$ to $B$ (since $\alpha' < \gamma$), but by their behavioral preferences they would vote for $B$ since $\gamma < \beta'$.

Consistency: Suppose that all members have the same core preferences $\alpha > \gamma$, but as their preferences are UR (that is, $g(\alpha, \alpha) < \alpha$), their common

\[9\] This proof proves a stronger result than the claim itself, as it shows that it is possible that prior to the deliberation all members favor one alternative but as a result of the deliberation all of them favor the other one.
behavioral preferences $\beta$ are just above $\gamma$ and alternative $B$ is selected. Add a new committee member whose core preferences are just above $\gamma$ (but sufficiently below $\alpha$) and his preferences may push the behavioral preferences of all other agents below $\gamma$.

**Proof of claim 4**: We use the following table

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) 1-2-3</td>
<td>$\alpha_1$</td>
<td>$g(\alpha_2, \alpha_1)$</td>
<td>$g(\alpha_3, \theta \alpha_1 + (1 - \theta)g(\alpha_2, \alpha_1))$</td>
</tr>
<tr>
<td>(ii) 1-3-2</td>
<td>$\alpha_1$</td>
<td>$g(\alpha_2, \theta \alpha_1 + (1 - \theta)g(\alpha_3, \alpha_1))$</td>
<td>$g(\alpha_3, \alpha_1)$</td>
</tr>
<tr>
<td>(iii) 2-1-3</td>
<td>$g(\alpha_1, \alpha_2)$</td>
<td>$\alpha_2$</td>
<td>$g(\alpha_3, \theta \alpha_2 + (1 - \theta)g(\alpha_1, \alpha_2))$</td>
</tr>
<tr>
<td>(iv) 2-3-1</td>
<td>$g(\alpha_1, \theta \alpha_2 + (1 - \theta)g(\alpha_3, \alpha_2))$</td>
<td>$\alpha_2$</td>
<td>$g(\alpha_3, \alpha_2)$</td>
</tr>
<tr>
<td>(v) 3-1-2</td>
<td>$g(\alpha_1, \alpha_3)$</td>
<td>$g(\alpha_2, \theta \alpha_3 + (1 - \theta)g(\alpha_1, \alpha_3))$</td>
<td>$\alpha_3$</td>
</tr>
<tr>
<td>(vi) 3-2-1</td>
<td>$g(\alpha_1, \theta \alpha_3 + (1 - \theta)g(\alpha_2, \alpha_3))$</td>
<td>$g(\alpha_2, \alpha_3)$</td>
<td>$\alpha_3$</td>
</tr>
</tbody>
</table>

Table 1

Person $B$ is selected by a unanimity rule iff $M := \min\{\beta_1, \beta_2, \beta_3\} \geq \gamma$.

(i) 1-2-3: For $i = 1, 2, 3$, $\beta_i^{(v)} \geq \beta_i^{(i)}$, hence $M^{(v)} \geq M^{(i)}$.

(ii) 1-3-2: For $i = 1, 2, 3$, $\beta_i^{(v)} \geq \beta_i^{(ii)}$, hence $M^{(v)} \geq M^{(ii)}$.

(iii) 2-1-3: For $i = 1, 2, 3$, $\beta_i^{(iv)} \geq \beta_i^{(iii)}$, hence $M^{(iv)} \geq M^{(iii)}$.

(iv) 2-3-1: For $i = 1, 3$, $\beta_i^{(v)} > \beta_i^{(iv)}$. Consider two cases.

1. $g(\alpha_1, \alpha_3) < \alpha_2$. Then $\beta_1^{(v)} = g(\alpha_1, \alpha_3) = g(g(\alpha_1, \alpha_3), g(\alpha_1, \alpha_3)) < g(\alpha_2, g(\alpha_1, \alpha_3)) \leq g(\alpha_2, \theta \alpha_3 + (1 - \theta)g(\alpha_1, \alpha_3)) = \beta_2^{(v)} < \beta_3^{(v)}$, hence $M^{(v)} = \beta_1^{(v)} > \beta_1^{(iv)} \geq M^{(iv)}$.

2. $g(\alpha_1, \alpha_3) \geq \alpha_2$. Then $\beta_2^{(v)} \geq \beta_2^{(iv)}$ and since for $i = 1, 2, 3$, $\beta_i^{(v)} \geq \beta_i^{(iv)}$, it follows that $M^{(v)} \geq M^{(iv)}$. 

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Proof of Claim 5: If person 1 does not participate in the deliberation, then $\beta_1^1 = \alpha_1$. We show first that $\beta_2^1 > \beta_2$ and $\beta_3^1 > \beta_3$. Observe that by Claim 8 in FS, $\beta_2^1 > \alpha_2$. If $\beta_2 \leq \alpha_2$, then clearly $\beta_2^1 > \beta_2$, and since $\beta_2 > \beta_1$,

$$
\beta_3^1 = g(\alpha_3, \beta_2^1) > g(\alpha_3, \frac{1}{2}[\beta_1 + \beta_2]) = \beta_3
$$

(vi) 3-2-1: $\beta_3^{(vi)} > \beta_2^{(vi)} > \beta_1^{(vi)}$, hence $M^{(vi)} = \beta_1^{(vi)}$. Obviously $M^{(vi)} \leq \beta_1^{(v)} < \beta_3^{(v)}$ and by definition, $M^{(vi)} \leq \beta_1^{(v)}$. We show next that for all $\alpha_2 \in [\alpha_1, \alpha_3]$, $\beta_1^{(vi)} \leq \beta_2^{(v)}$. For $\alpha_2 = \alpha_1$ we get $\beta_1^{(vi)} = \beta_2^{(v)}$. Differentiate both with respect to $\alpha_2$:

$$
\frac{\partial \beta_1^{(vi)}}{\partial \alpha_2} = g_2(\alpha_1, \theta \alpha_3 + (1 - \theta)g(\alpha_2, \alpha_3)) \times (1 - \theta)g_1(\alpha_2, \alpha_3)
$$

$$
\frac{\partial \beta_2^{(v)}}{\partial \alpha_2} = g_1(\alpha_2, \theta \alpha_3 + (1 - \theta)g(\alpha_1, \alpha_3))
$$

We assumed that $g_{12} < 0$ (see end of subsection 2.2), hence $g_1(\alpha_2, \alpha_3) \leq g_1(\alpha_2, \theta \alpha_3 + (1 - \theta)g(\alpha_1, \alpha_3))$ and since $g_2(\cdot, \cdot) \leq 1$, $\frac{\partial \beta_1^{(vi)}}{\partial \alpha_2} \leq \frac{\partial \beta_2^{(v)}}{\partial \alpha_2}$, implying $\beta_1^{(vi)} \leq \beta_2^{(v)}$ for all $\alpha_2 \in [\alpha_1, \alpha_3]$. As $M^{(vi)} \leq \beta_i^{(v)}$ for $i = 1, 2, 3$, it follows that $M^{(vi)} \leq M^{(v)}$.

Person $B$ is selected by a majority rule if and only if $L$, the mid-value of $\beta_1, \beta_2, \beta_3$ satisfies $L \geq \gamma$. Using the above table we get

(i) 1-2-3: For $i = 1, 2, 3$, $\beta_i^{(vi)} > \beta_i^{(i)}$, hence $L^{(vi)} > L^{(i)}$.

(ii) 1-3-2: For $i = 1, 2, 3$, $\beta_i^{(vi)} > \beta_i^{(ii)}$, hence $L^{(vi)} > L^{(ii)}$.

(iii) 2-1-3: $L^{(iii)} = \min\{\beta_2^{(iii)}, \beta_3^{(iii)}\}$. As $\beta_2^{(vi)} > \beta_2^{(iii)}$ and $\beta_3^{(vi)} > \beta_3^{(iii)}$, it follows that $L^{(vi)} > L^{(iii)}$.

(iv) 2-3-1: $L^{(vi)} = \beta_2^{(vi)} > \max\{\beta_1^{(iv)}, \beta_2^{(iv)}\} > L^{(iv)}$.

(v) 3-1-2: $L^{(vi)} = \beta_2^{(vi)} > \max\{\beta_1^{(v)}, \beta_2^{(v)}\} > L^{(v)}$.  

Proof of Claim 5: If person 1 does not participate in the deliberation, then $\beta_1^1 = \alpha_1$. We show first that $\beta_2^1 > \beta_2$ and $\beta_3^1 > \beta_3$. Observe that by Claim 8 in FS, $\beta_2^1 > \alpha_2$. If $\beta_2 \leq \alpha_2$, then clearly $\beta_2^1 > \beta_2$, and since $\beta_2 > \beta_1$,

$$
\beta_3^1 = g(\alpha_3, \beta_2^1) > g(\alpha_3, \frac{1}{2}[\beta_1 + \beta_2]) = \beta_3
$$
Suppose that $\beta_2 > \alpha_2$ but $\beta_2^1 \leq \beta_2$. Since by FS $\beta_1^1 < \beta_3$ (see end of subsection 2.2),

$$\beta_2 = g(\alpha_2, \frac{1}{2}[\beta_1 + \beta_3]) \geq \beta_2^1 = g(\alpha_2, \beta_3^1) \implies$$

$$\frac{1}{2}[\beta_1 + \beta_3] \geq \beta_3^1 \implies$$

$$\beta_3 > \beta_3^1 \quad (2)$$

Also, since $g_2 < 1$,

$$\beta_2 = g(\alpha_2, \frac{1}{2}[\beta_1 + \beta_3]) > \begin{cases} 
\beta_2^1 = g(\alpha_2, \beta_3^1) 
\end{cases} \implies$$

$$\beta_2 - \beta_2^1 < \frac{1}{2}[\beta_1 + \beta_3] - \beta_3^1 \quad (3)$$

Similarly, using inequality (2)

$$\beta_3 = g(\alpha_3, \frac{1}{2}[\beta_1 + \beta_2]) > \begin{cases} 
\beta_3^1 = g(\alpha_3, \beta_2^1) 
\end{cases} \implies$$

$$\beta_3 - \beta_3^1 < \frac{1}{2}[\beta_1 + \beta_2] - \beta_2^1 \quad (4)$$

Combining inequalities (3) and (4) together and recalling that $\beta_1 < \beta_2$, we get

$$2\beta_3 - 2\beta_3^1 < \beta_1 + \beta_2 - 2\beta_2^1 < 2\beta_2 - 2\beta_2^1 < \beta_1 + \beta_3 - 2\beta_3^1 \implies$$

$$\beta_3 < \beta_1$$

A contradiction, hence $\beta_2^1 > \beta_2$. And since $\beta_1 < \beta_2$, it follows that $\beta_2^1 > \frac{1}{2}[\beta_1 + \beta_2]$, hence $\beta_3^1 > \beta_3$. It thus follows that both unanimity rule (determined by person 3) and majority (determined by person 2) rule accept less projects than the case in which all members participate in the deliberation.

Suppose now that person 3 does not participate. Then by Claim 8 in FS, $\beta_3^3 = \alpha_3 > \beta_3$ and the unanimity rule will accept less projects. Since $\beta_2^2 > \beta_3^3$ (Claim 7 in FS), in order to show that the majority rule will accept
more project it is enough to show that $\beta_2 > \beta_3^2$. Since by the aforementioned claim, $\alpha_2 > \beta_2^3$, this is clearly the case when $\beta_2 \geq \alpha_2$. We therefore prove the impossibility of $\alpha_2 > \beta_2^3 > \beta_2$. Otherwise,

$$\beta_2^3 = g(\alpha_2, \beta_1^3) \geq \beta_2 = g(\alpha_2, \frac{1}{2}[\beta_1 + \beta_3]) \implies \beta_1^3 > \beta_1$$

Since $g_2 < 1$, we get

$$\begin{align*}
\beta_2^3 - \beta_2 &< \beta_1^3 - \frac{1}{2}[\beta_1 + \beta_3] \\
\beta_1^3 - \beta_1 &< \beta_2^3 - \frac{1}{2}[\beta_2 + \beta_3] \\
2\beta_3 &< \beta_1 + \beta_2
\end{align*}$$

A contradiction to the fact that $\beta_3 > \beta_2 > \beta_1$. $\blacksquare$
References


