Intransitivity in the Small and in the Large

Sushil Bikhchandani* Uzi Segal†‡

April 11, 2021

Abstract

We propose a regret-based model that allows the separation of attitudes towards transitivity on triples of random variables that are close to each other and attitudes towards transitivity on triples that are far apart. This enables a theoretical reinterpretation of evidence related to intransitive behavior in the laboratory. When viewed through this paper's analysis, the experimental evidence need not imply intransitive behavior for large risky decisions such as investment choices and insurance.

Keywords: Intransitivity, regret, local preferences, preference reversal

1 Introduction

Transitivity is a fundamental assumption of decision theory, both at the individual and at the social level. The requirement that if A precedes B and B

*UCLA Anderson School of Management (`sbikhcha@anderson.ucla.edu`)

†Department of Economics, Boston College (`segalu@bc.edu`)

‡We are grateful to Manshu Khanna for helpful research assistance.

precedes C then A precedes C seems almost obvious. Yet we know that not all decision rules satisfy transitivity. The voting paradox of Condorcet (see, e.g., Muller [19]) shows that a majority rule may lead to a violation of transitivity. Nor do decision makers always behave according to this rule. Many experiments show that individual preferences are often intransitive, especially preferences over lotteries. Consider for example the following experiment reported by Loomes, Starmer, and Sugden [15]. There are three possible states of nature with the probabilities $\Pr[s_1] = \Pr[s_2] = 0.3$ and $\Pr[s_3] = 0.4$ and three random variables,

$$\begin{aligned} A &= (18, s_1; 0, s_2; 0, s_3) \\ B &= (8, s_1; 8, s_2; 0, s_3) \\ C &= (4, s_1; 4, s_2; 4, s_3) \end{aligned}$$

A significant number of subjects exhibited the intransitive cycle $A \succ C \succ B \succ A$. For other documented violations of transitivity, see e.g. Lichtenstein and Slovic [13], Grether and Plott [10], Starmer [22], Birnbaum and Schmidt [5], and Regenwetter, Dana, and Davis-Stober [21].

Obviously, experiments cannot reconstruct real market behavior, as budget constraints restrict the size of the gambles involved, typically to less than \$50, while real world financial decisions usually involve sums in the thousands, if not much more. Arguably, some psychological violations of "rationality" (as in the axiomatic approach to decision making) become more effective when the size of the gambles involved is large. For instance, if an experiment reveals Allais-type violations of expected utility in the small, this behavior will become even more persistent in the large.[1]

There are however arguments in the literature that violations of expected utility may disappear if prizes are sufficiently large. To quote Hey [11], "if you

[1] For experiments demonstrating that Allais-paradox behavior is less prevalent when the size of prizes is reduced, see Conlisk [7], Fan [9], MacCrimmon and Larsson [16] and the references cited in footnote 31 of Cerreia-Vioglio, Dillenberger, and Ortoleva [6].

put enough zeroes on the ends of the payoffs ..., you will observe subjective expected utility behavior.” In our view, this argument is especially relevant for violations of transitivity because such violations may be due to insufficient consideration by decision makers. That is, cycles may be observed with respect to small gambles, but as decision makers will pay more attention and exert more effort when making big financial decisions, cycles are less likely to happen. If this is the case, then unlike violations of the independence axiom and its alternatives, where experiments may reveal insight into real-world decision making, observing violations of transitivity in experiments does not necessarily indicate such behavior in the large. Our aim is to provide a formal framework for such a behavior. For this, we use variations of regret theory, which is the most widely used model to analyze violations of transitivity. [2]

Bell [2] and Loomes and Sugden [14] offered a simple idea to explain violations of transitivity. Unlike standard models of economics, where the value of an outcome depends only on the outcome itself, regret theory postulates that decision makers evaluate each possible outcome they may receive by comparing it to the alternative outcome they could have received by choosing differently. When comparing the random variable $X = (x_1, s_1; \ldots; x_n, s_n)$ with $Y = (y_1, s_1; \ldots; y_n, s_n)$, the decision maker computes the expected value of a (subjective) elation/regret function $\psi(x, y)$ and will choose X over Y if and only if this expected value is positive. Formally, X is preferred to Y if and only if $\sum_i \Pr[s_i]\, \psi(x_i, y_i) \geqslant 0$.

Regret theory often assumes that decision makers are *regret averse*. That is, if $x > y > z$ then $\psi(x, z) > \psi(x, y) + \psi(y, z)$. The justification for this assumption is that large differences between what one obtained and what one would have obtained from an alternative choice give rise to disproportionately greater regret and elation. Observed violations of transitivity can be

[2]Tserenjigmid [23] provides a related model of intransitivity using intra-dimensional comparisons. See also Nishimura [20].

explained by regret aversion. [3] In fact, regret-based preferences are transitive if and only if they are expected utility. Loomes et al. [15] justified the cycle discussed earlier by arguing, for instance, that in state s_2 the regret/elation difference between 8 and 0 is significant, while the differences between 8 and 4 and between 4 and 0 are not. But suppose that the payoffs are multiplied by 1,000. Can one really argue that these differences are insignificant? The argument that $\psi(8,0)$ is significantly greater than $\psi(8,4)+\psi(4,0)$ seems less convincing when the outcomes are 8,000, 4,000, and 0.

The purpose of the present paper is two fold. First, we formalize a regret model in which a violation of transitivity on random variables that are close to each other does not imply a violation of transitivity on random variables that are far apart. In this we do not claim that regret theory is not a valid theory. What we claim is that applying this theory to decisions involving "large" random variables (like insurance or investment decisions) cannot be justified based on these experiments.

Our second objective is to establish that even if regret theory applies only to random variables that are close to each other, it is still a very powerful theory. It implies that preferences in different neighborhoods are not independent of each other. Formally, suppose that for each random variable W there is a neighborhood around W on which preferences are induced by regret, but the regret function may change from one random variable W to another. We show that to a certain extent, all these "local" regret functions are tightly knitted to each other. Therefore, even if experiments showing intransitivity in the small do not prove intransitivity in the large, they should still indicate a strong connection between local behavior around all random variables.

The paper is organized as follows. Section 2 shows that with linear regret,

[3] See e.g. Loomes, Starmer, and Sugden [15] or Starmer [22]. For an axiomatization of regret aversion, see Diecidue and Somasundaram [8].

intransitivity in one part of the domain implies intransitivity everywhere. A more general regret model, which permits a decoupling of attitudes towards transitivity in the small and transitivity in the large, is considered in Section 3. All proofs are in Appendix A while Appendix B contains several examples.

2 Linear Regret

Consider a set $\mathcal{L}$ of finite-valued random variables X of the form $X = (x_1, s_1; \ldots; x_n, s_n)$ where the outcomes are monetary payoffs (which may be positive or negative).[4] The events $s_1, \ldots, s_n$ partition the sure event and the probability of s_i is p_i. The set $\mathcal{L}$ is endowed with the L^2 norm. Thus,[5] for $X = (x_1, s_1, \ldots; x_n, s_n)$ and $W = (w_1, s_1; \ldots; w_n, s_n)$, we have $||X - W|| = \sqrt{\sum_{i=1}^{n} p_i(x_i - w_i)^2}$. An ε-neighborhood of W is the set $\mathcal{B}(W, \varepsilon) = \{X : ||X - W|| < \varepsilon\}$.

The decision maker has complete but not necessarily transitive preferences $\succeq$ on $\mathcal{L}$. Preferences are intransitive if there exists even one triplet $X, Y, Z \in \mathcal{L}$ such that $X \succeq Y$, $Y \succeq Z$, yet $Z \succ X$. We are interested in more complex situations, where preferences may be transitive on some parts of the domain but intransitive on other parts. More importantly, we want to investigate how pervasive are such intransitive cycles, and if they exist, whether they are sporadic or must they appear everywhere.

Regret is a convenient way to model intransitive preferences. Bell [2] and Loomes and Sugden [14] suggested a model of *linear regret*: for two random

[4]The random variables X are defined over an underlying probability space (S, Σ, P) where $S = [0, 1]$, Σ is the standard Borel σ-algebra on S, and P is the Lebesgue measure. We assume an unbounded domain of payoffs, but our results hold for the case of bounded domain as well.

[5]Two random variables can be written on the same list of events without loss of generality. See Appendix A.

variables $X = (x_1, s_1; \ldots; x_n, s_n)$ and $Y = (y_1, s_1; \ldots; y_n, s_n)$ over the same set of events,

$$X \succeq Y \ \text{ if and only if } \ \sum_i \Pr[s_i]\, \psi(x_i, y_i) \geqslant 0 \tag{1}$$

where ψ is a *regret function* which is continuous and for all x and y,

(i) $\psi(x, y) = -\psi(y, x)$,

(ii) ψ is increasing in its first and decreasing in its second argument.

The function ψ represents the feelings of the decision maker when he wins x, knowing that had he chosen differently his outcome would have been y. If $x > y$ he will be elated (and $\psi(x, y) > 0$), but if $x < y$ he will be disappointed and regretful (hence $\psi(x, y) < 0$). Condition (i) simply says that the elation from winning x greater than y equals the regret of winning y less than x. The other condition asserts that elation is increasing with the winning outcome and decreasing with the foregone one. A consequence of the first condition is that $\psi(x, x) = 0$.

Bell [2] and Loomes and Sugden [14] assumed linearity in probabilities, that is, they evaluated regret by taking expected values of a regret function. We show that with linear regret, intransitivity is pervasive in the sense that the existence of one intransitive cycle implies the existence of intransitive cycles everywhere. The basic linear model was extended by Bikhchandani and Segal [3] to more general evaluations. Let $\Psi(X, Y) = (\psi(x_1, y_1), p_1; \ldots; \psi(x_n, y_n), p_n)$ be a regret lottery. Define

$$X \succeq Y \ \text{ if and only if } \ V(\Psi(X, Y)) \geqslant 0 \tag{2}$$

where the function V is any general function evaluating regret lotteries Ψ. If $\succeq$, represented as in eq. (2), is transitive, then it must be expected utility (see [3]). Thus regret-based behavior is not consistent with any transitive non-expected utility choice.[6] For clarity, we refer to preferences as defined in eqs. (1) or (2) as *universal regret*, as later we define local regret.

Assume linear regret. If for some x_1, x_2, x_3 and s_1, s_2, s_3 such that $\Pr[s_1] = \Pr[s_2] = \Pr[s_3] = \frac{1}{3}$, $(x_1, s_1; x_2, s_2; x_3, s_3) \sim (x_3, s_1; x_1, s_2; x_2, s_3) \sim (x_2, s_1; x_3, s_2; x_1, s_3) \sim (x_1, s_1; x_2, s_2; x_3, s_3)$, then eq. (1) implies that $\psi(x_1, x_3) = \psi(x_1, x_2) + \psi(x_2, x_3)$. If the above indifferences hold for all x_1, x_2, x_3, it follows from the proof of Lemma 7 of [3] that there exists $u : \Re \to \Re$ such that $\psi(x, y) = u(x) - u(y)$ which yields expected utility. Therefore, if linear regret preferences are non-expected utility then there exists at least one intransitive cycle of the form[7]

$$\begin{aligned}(x_1, s_1; x_2, s_2; x_3, s_3) \;&\succ\; (x_3, s_1; x_1, s_2; x_2, s_3) \\ &\succ\; (x_2, s_1; x_3, s_2; x_1, s_3) \succ (x_1, s_1; x_2, s_2; x_3, s_3)\end{aligned} \tag{3}$$

As the next theorem shows, the existence of one cycle implies that in the neighborhood of each random variable there are cycles. We call this *intransitivity in the small.* Moreover, there exist intransitive cycles in which the random variables are far apart from each other. We call this *intransitivity in the large.*

Theorem 1 *Suppose that non-expected utility preferences $\succeq$ can be represented by linear regret. Then*

(i) For every W and $\varepsilon > 0$ there are $X, Y, Z \in \mathcal{B}(W, \varepsilon)$ such that $X \succ Y \succ Z \succ X$.

(ii) For any $M > 0$, there exist random variables, X_i, $i = 1, 2, 3$, with $||X_i - X_j|| \geq M$ such that $X_1 \succ X_2 \succ X_3 \succ X_1$.

[6] However, regret over pairs of independent lotteries (rather than random variables) is compatible with betweenness and other transitive non-expected utility models. See Machina [18] and Bikhchandani and Segal [4].

[7] Each of the three preferences is strict because they generate the same regret lottery.

The proof of Theorem 1(i) makes specific predictions that can be checked experimentally. A violation of transitivity implies a cycle as in eq. (3). Then for every y there is a sufficiently small $\varepsilon > 0$ such that for $s_0, \ldots, s_3$ where $\Pr[s_0] = 1 - \varepsilon$ and $\Pr[s_1] = \Pr[s_2] = \Pr[s_3] = \frac{\varepsilon}{3}$,

$$(y, s_0; x_1, s_1; x_2, s_2; x_3, s_3) \succ (y, s_0; x_3, s_1; x_1, s_2; x_2, s_3) \succ \\ (y, s_0; x_2, s_1; x_3, s_2; x_1, s_3) \succ (y, s_0; x_1, s_1; x_2, s_2; x_3, s_3)$$

Theorem 1 strongly depends on the assumption that regret is linear in probabilities, but it does not hold for non-linear models of regret. Example 1 in Appendix B provides a regret relation that is transitive in the large, yet violates transitivity in the small. Once regret is not linear in probabilities, the opposite is also possible. Example 2 in Appendix B presents a non-linear model of regret which is expected utility (and therefore transitive) in every small neighborhood, yet has intransitive cycles in the large.

Although non-linear regret permits a separation between attitudes towards transitivity in the small and in the large, it nevertheless imposes some strict restrictions over preferences in small neighborhoods. We analyze such preferences in the next section.

3 Local preferences and regret

To facilitate a distinction between intransitive cycles where random variables are far away from each other and cycles where random variables are all in small neighborhoods, define preferences to be *locally regret-based* if they can be represented as in eq. (2) above in a neighborhood around each random variable W, albeit possibly with different functions ψ and V. Formally, a binary relation is locally regret-based if for every W there is $\varepsilon > 0$ such that for all $X, Y \in \mathcal{B}(W, \varepsilon)$,

$$X \succeq Y \quad \text{if and only if} \quad V_W(\Psi_W(X, Y)) \geqslant 0$$

As we show below, local regret does not imply universal regret, yet it does impose restrictions on ψ_W and V_W across different values of W (Theorem 2). First, all the local-regret functions ψ_W can be taken to be the same. Second, any two local-regret functionals V_W, $V_{W'}$ with domains $\mathcal{R}_W, \mathcal{R}_{W'}$ are *concordant* in the sense that if they have the same sign on regret lotteries common to both domains. That is, for all $R \in \mathcal{R}_W \cap \mathcal{R}_{W'}$,[8] $V_W(R) \gtreqqless 0$ if and only if $V_{W'}(R) \gtreqqless 0$. In particular, concordant regret functionals have the same indifference curve through zero.

Theorem 2 *If preferences are locally regret-based, then:*

(i) All the ψ_W functions are ordinally equivalent and can be taken to be the same.

(ii) Any pair of local-regret functionals V_W and $V_{W'}$ are concordant.

The proof of the theorem uses the fact that the line segment connecting any two random variables W and W' is covered by a finite number of open neighborhoods of random variables $W_1, \ldots, W_n$ on the line segment. Local-regret evaluations must be equivalent on the intersection of neighborhoods of W_i and W_{i+1} for each i. This is shown to imply that local-regret evaluations must also be equivalent at W and W'.

Theorem 2 makes some simple behavioral predictions. In particular, preferences between two random variables that are close to each other do not depend on outcomes that are common to both. To formalize this, let $W^j = (w_1^j, s_1; \ldots; w_n^j, s_n)$, $j = 1, 2$ where $\Pr[s_1] = \Pr[s_2] = \delta$. Let $X^j = (x_1, s_1; x_2, s_2; w_3^j, s_3; \ldots; w_n^j, s_n)$ and $Y^j = (y_1, s_1; y_2, s_2; w_3^j, s_3; \ldots; w_n^j, s_n)$. Let $\varepsilon > 0$ and let δ be small enough such that $X^j, Y^j \in \mathcal{B}(W^j, \varepsilon)$, $j = 1, 2$.

[8] The regret lottery that yields 0 with probability 1 belongs to the set $\mathcal{R}_W \cap \mathcal{R}_{W'}$. Hence, $\mathcal{R}_W \cap \mathcal{R}_{W'}$ is non-empty.

Proposition 1 *If preferences are locally regret-based, then* $X^1 \sim Y^1$ *if and only if* $X^2 \sim Y^2$, *where* X^j, Y^j *are defined above.*

This proposition is related to Savage's sure-thing principle. The difference is that in Proposition 1 the common parts of X^j and Y^j are "large" while these is no such restriction in the sure-thing principle.

If each local-regret functional is linear in probabilities, then we have a stronger result than Theorem 2.

Proposition 2 *If preferences are locally regret-based and each local-regret functional* V_W *is linear in probabilities, then local regrets are identically linearly evaluated. That is, each local regret is the expected value of a common (up to positive multiplication) local-regret function* ψ *for all* W.

In the proof of Theorem 2(i), the ordinal equivalence of the ψ_W functions is obtained by adjusting the regret functionals. The adjusted regret functional will in general be non-linear, even if the initial regret functional is linear. Thus, Proposition 2 does not follow from Theorem 2(i).

Remark 1 If preferences are locally regret-based then Theorem 2 implies that either there are intransitive cycles in every neighborhood or there is no intransitive cycle in any neighborhood. Intransitivity in some but not all neighborhoods are possible when preferences do not satisfy local regret.

Remark 2 Our distinction between preferences in the small and in the large should not be confused with Machina's [17] model of Fréchet differentiable representations, where preferences violate the independence axiom while converging at each point to expected utility. Intransitive regret models of the type discussed in this paper do not permit a representation function (which necessarily implies transitivity), hence are orthogonal to Machina's analysis.

4 Discussion

Example 1 in Appendix B shows that violations of transitivity when random variables are close to each other do not imply the existence of intransitive cycles when random variables are far apart from each other. As experiments are done with "small" random variables, it is questionable to what extent one may deduce from these experiments that individuals violate transitivity in "big" decisions like financial investments, real-estate transactions, or retirement planning.

But isn't this true for all experimental results? For example, when real payments are involved, experiments regarding the Allais paradox (Allais [1]; see also MacCrimmon and Larsson [16], Kahneman and Tversky [12], and Starmer [22]) are conducted, for obvious reasons, with small amounts of money. Will an argument similar to the one made in the paper lead to the conclusion that we cannot learn from these experiments that the Allais paradox really exists?

There is however an important difference between experiments on transitivity and experiments on phenomena like the Allais paradox. A standard presentation of this decision problem asks the decision maker to choose between $A = (5M, 0.1; 0, 0.9)$ and $B = (1M, 0.11; 0, 0.89)$, and then between $C = (5M, 0.1; 1M, 0.89; 0, 0.01)$ and $D = (1M, 1)$. The commonly observed preferences $A \succ B$ together with $D \succ C$ violate expected utility maximization. The psychological rationale behind these preferences is that A and B offer similar probabilities of success, but A offers a much higher payoff. This argument applies to C and D as well, but there is another factor that tilts the scales in favor of D, and this is the possibility of winning zero in C. Before making the choice the decision maker knows that he will be devastated if after choosing C he were to win zero, when he could have avoided all risk by choosing D (and receiving $1M$).

This argument becomes less powerful if all outcomes are scaled down, yet such preferences persist even after such modifications (see for example problems 1 and 2 in [12]). The actual experimental data regarding the Allais paradox therefore supports that hypothesis that this phenomenon exists not only in the small but also in the large.

Regret theory, on the other hand, is based on the intuition that for $x > y > z$, the elation from obtaining x when the alternative is z is greater than the sum of the two smaller elations, from receiving x when the alternative is y and from receiving y when the alternative is z. This intuition is convincing when there is a certain threshold above which the decision maker's feelings of elation or regret become relevant. But it is much less obvious that this property also holds for large numbers. Therefore, even if it is true that $\psi(8000, 0) > \psi(8000, 4000) + \psi(4000, 0)$, it may well happen (and indeed, is quite reasonable to expect) that the propensity for intransitivity weakens in the sense that

$$\frac{\psi(8000,0)}{\psi(8000,4000)+\psi(4000,0)} < \frac{\psi(8,0)}{\psi(8,4)+\psi(4,0)}$$

To summarize, our argument that a certain behavior in the small may not necessarily indicate a similar behavior in the large can be formally extended to other violations of expected utility theory. However, with respect to other phenomena, violations in the small are less likely to happen than violations in the large, and therefore, experiments showing violations in the small correctly predict violations in the large. In contrast, with respect to intransitive behavior the opposite may be true. Intransitive behavior may be due to insufficient reflection by the decision maker. Consequently, intransitivity is more likely to happen in the small than in the large, and therefore experiments showing violations of transitivity in the small do not necessarily indicate similar violations in the large.

Our notion of local regret is weaker than the original formulation of Bell [2] and Loomes and Sugden [14] in that it applies only to random variables that are close to each other. Nevertheless, local regret implies that preferences in different neighborhoods are not independent of each other. We show that in our formulation, all local regret functions are ordinally equivalent and local regret functionals have the same indifference curve through zero. Thus, even if experiments showing local regret do not imply universal regret, they should still indicate a strong connection between local behaviors around different random variables.

Appendix A: Proofs

First, we show that a finite number of finite-valued random variables may be written on the same list of events. For two random variables

$$X^j = (x_1^j, s_1^j; \ldots; x_{i_j}^j, s_{i_j}^j), \quad j = 1, 2$$

define $s_{1,1}, \ldots, s_{1,i_2}, \ldots, s_{i_1,1}, \ldots, s_{i_1,i_2}$ by $s_{i,j} = s_i^1 \cap s_j^2$. Note that

$$\begin{aligned} X^1 &= (x_1^1, \cup_k s_{1,k}; \ldots; x_{i_1}^1, \cup_k s_{i_1,k}) \\ X^2 &= (x_1^2, \cup_k s_{k,1}; \ldots; x_{i_2}^2, \cup_k s_{k,i_2}) \end{aligned}$$

Therefore, we can assume without loss of generality that any finite number of random variables can be defined on the same list of events.

Second, any event may be partitioned into two sub-events with any probability ratio. For an event s_i and $\alpha \in [0, 1]$, define $\beta(s_i, \alpha)$ such that $\Pr[s_{i,\alpha}] := \Pr[s_i \cap [0, \beta(s_i, \alpha)]\,] = \alpha \Pr[s_i]$, and let $s'_{i,1-\alpha} = s_i \backslash s_{i,\alpha}$. (Note that $\beta(s_i, \alpha)$ exists because the probability measure is atomless.)

Proof of Theorem 1:
(i): As preferences are non-expected utility and represented by a linear functional, there exist x_1, x_2, x_3 which admit the intransitive cycle of eq. (3). Hence

$$V(\psi(x_1, x_3), \tfrac{1}{3}; \psi(x_2, x_1), \tfrac{1}{3}; \psi(x_3, x_2), \tfrac{1}{3}) \; = \tag{4}$$

$$\frac{\psi(x_1,x_3)+\psi(x_2,x_1)+\psi(x_3,x_2)}{3} \; > \; 0$$

Let $W=(w_1,t_1;\ldots;w_\ell,t_\ell)\in\mathcal{L}$. For any $m>\frac{1}{\varepsilon}$, let $s_1,\ldots,s_{3m}$ be pairwise disjoint with the probabilities $\frac{1}{3m}$ each. The random variables X,Y,Z defined below are in an ε-neighborhood of W:

$$\begin{aligned}
X &= (x_1,s_1;x_2,s_2;x_3,s_3;w_1,t_1\cap(\cup_{j=4}^{3m}s_j);\ldots;w_\ell,t_\ell\cap(\cup_{j=4}^{3m}s_j))\\
Y &= (x_3,s_1;x_1,s_2;x_2,s_3;w_1,t_1\cap(\cup_{j=4}^{3m}s_j);\ldots;w_\ell,t_\ell\cap(\cup_{j=4}^{3m}s_j))\\
Z &= (x_2,s_1;x_3,s_2;x_1,s_3;w_1,t_1\cap(\cup_{j=4}^{3m}s_j);\ldots;w_\ell,t_\ell\cap(\cup_{j=4}^{3m}s_j))
\end{aligned}$$

That $X\succ Y\succ Z\succ X$ follows from

$$\begin{aligned}
&V(\psi(x_1,x_3),\tfrac{1}{3m};\psi(x_2,x_1),\tfrac{1}{3m};\psi(x_3,x_2),\tfrac{1}{3m};0,\tfrac{m-1}{m})\\
=\;&\frac{\psi(x_1,x_3)+\psi(x_2,x_1)+\psi(x_3,x_2)}{3m}>0
\end{aligned}$$

where the inequality follows from (4).

(ii): Let

$$\begin{aligned}
X_1 &= (a,t_1;0,t_2;0,t_3;x_1,s_1;x_2,s_2;x_3,s_3)\\
X_2 &= (0,t_1;a,t_2;0,t_3;x_3,s_1;x_1,s_2;x_2,s_3)\\
X_3 &= (0,t_1;0,t_2;a,t_3;x_2,s_1;x_3,s_2;x_1,s_3)
\end{aligned}$$

where x_1,x_2,x_3 are in the intransitive cycle of eq. (3), $\Pr[t_i]=\frac{1}{3}-\varepsilon$ and $\Pr[s_i]=\varepsilon$, where $\varepsilon>0$ is small. Select $a>2M$ so that $||X_i-X_j||\geq M$. Then

$$V(X_1,X_2)=V(X_2,X_3)=V(X_3,X_1)=\frac{\psi(x_1,x_3)+\psi(x_2,x_1)+\psi(x_3,x_2)}{3}\varepsilon>0$$

Hence, $X_1\succ X_2\succ X_3\succ X_1$. ■

Proof of Theorem 2:

(i): First, we show that for any regret functional V,

$$V\left(r,\tfrac{1}{\ell};-r,\tfrac{1}{\ell};0,\tfrac{\ell-2}{\ell}\right)=V\left(-r,\tfrac{1}{\ell};r,\tfrac{1}{\ell};0,\tfrac{\ell-2}{\ell}\right)=0 \qquad (5)$$

for any regret level r and integer $\ell \geq 2$. The first equality is true as $(r, \frac{1}{\ell}; -r, \frac{1}{\ell}; 0, \frac{\ell-2}{\ell})$ and $\left(-r, \frac{1}{\ell}; r, \frac{1}{\ell}; 0, \frac{\ell-2}{\ell}\right)$ are the same regret lottery. Suppose that $V\left(r, \frac{1}{\ell}; -r, \frac{1}{\ell}; 0, \frac{\ell-2}{\ell}\right) > 0$. Let x_1, x_2 be such that $\psi(x_1, x_2) = r$. By skew symmetry, $\psi(x_2, x_1) = -r$. With equiprobable events $s_1, \ldots, s_\ell$, we have

$$\begin{aligned}(x_1, s_1; x_2, s_2; x_3, s_3; \ldots; x_\ell, s_\ell) &\succ (x_2, s_1; x_1, s_2; x_3, s_3; \ldots; x_\ell, s_\ell) \\ &\succ (x_1, s_1; x_2, s_2; x_3, s_3; \ldots; x_\ell, s_\ell)\end{aligned}$$

which is a violation of irreflexivity. Hence (5).

Define the $\oplus$ operation as follows. Let $X = (x_1, s_1; \ldots; x_n, s_n)$ and $Y = (y_1, s_1; \ldots; y_n, s_n)$. Then

$$\alpha X \oplus (1-\alpha)Y = (x_1, s_{1,\alpha}; \ldots; x_n, s_{n,\alpha}, y_1, s'_{1,1-\alpha}, \ldots; y_n, s'_{n,1-\alpha})$$

where $\Pr[s_{i,\alpha}] = \alpha \Pr[s_i]$ and $\Pr[s'_{i,1-\alpha}] = (1-\alpha)\Pr[s'_i]$.

Let $[W, W'] = \{\alpha W \oplus (1-\alpha)W' : \alpha \in [0,1]\}$. The set $\{\alpha \in [0,1] : \alpha W \oplus (1-\alpha)W' \in \mathcal{B}(\beta W \oplus (1-\beta)W', \varepsilon)\}$ is open. As $[0,1]$ is compact, there is a finite sequence of overlapping neighborhoods $\mathcal{B}(W, \varepsilon) = \mathcal{B}(W_1, \varepsilon), \ldots, \mathcal{B}(W_n, \varepsilon) = \mathcal{B}(W', \varepsilon)$ covering $[W, W']$. Let ψ_i be the local-regret function at W_i. We show that for $i = 1, \ldots, n-1$, ψ_i and ψ_{i+1} are ordinally equivalent. Suppose not. Then there are $(x, y), (x', y')$ such that $\psi_i(x, y) > \psi_i(x', y')$ but $\psi_{i+1}(x, y) \leqslant \psi_{i+1}(x', y')$. Let $Z \in \mathcal{B}(W_i, \varepsilon) \cap \mathcal{B}(W_{i+1}, \varepsilon)$. As this intersection is open, there is a sufficiently small β such that

$$X = \beta(x, H; y', T) \oplus (1-\beta)Z, \quad Y = \beta(y, H; x', T) \oplus (1-\beta)Z$$

and $X, Y \in \mathcal{B}(W_i, \varepsilon_i) \cap \mathcal{B}(W_{i+1}, \varepsilon_{i+1})$. With $\ell \geqslant \frac{2}{\beta}$ we obtain that

$$\begin{aligned}V_i(\Psi_i(X, Y)) &= V_i\left(\psi_i(x, y), \frac{1}{\ell}; \psi_i(y', x'), \frac{1}{\ell}; 0, \frac{\ell-2}{\ell}\right) \\ &= V_i\left(\psi_i(x, y), \frac{1}{\ell}; -\psi_i(x', y'), \frac{1}{\ell}; 0, \frac{\ell-2}{\ell}\right)\end{aligned}$$

$$
\begin{aligned}
&> V_i\left(\psi_i(x,y),\frac{1}{\ell};-\psi_i(x,y),\frac{1}{\ell};0,\frac{\ell-2}{\ell}\right)\\
&= 0
\end{aligned}
$$

where we use the fact that ψ is skew symmetric, $-\psi_i(x,y) < -\psi_i(x',y')$, the monotonicity of $V(\cdot)$, and equation (5). Therefore, $X \succ Y$. Similarly,

$$
\begin{aligned}
V_{i+1}(\Psi_{i+1}(X,Y)) &= V_{i+1}\left(\psi_{i+1}(x,y),\frac{1}{\ell};\psi_{i+1}(y',x'),\frac{1}{\ell};0,\frac{\ell-2}{\ell}\right)\\
&= V_{i+1}\left(\psi_{i+1}(x,y),\frac{1}{\ell};-\psi_{i+1}(x',y'),\frac{1}{\ell};0,\frac{\ell-2}{\ell}\right)\\
&\leqslant V_{i+1}\left(\psi_{i+1}(x,y),\frac{1}{\ell};-\psi_{i+1}(x,y),\frac{1}{\ell};0,\frac{\ell-2}{\ell}\right)\\
&= 0
\end{aligned}
$$

and therefore $X \preceq Y$, a contradiction.

Therefore, for all $(x,y),(x',y')$ such that $\psi_i(x,y) > \psi_i(x',y')$ we have $\psi_{i+1}(x,y) > \psi_{i+1}(x',y')$. Hence there exists a well-defined increasing function $h_{i+1,\,i}$ such that $h_{i+1,\,i}(\psi_i(x,y)) = \psi_{i+1}(x,y)$, for all x,y. Since $\psi_1,\ldots,\psi_n$ are ordinally equivalent, so are ψ_W and $\psi_{W'}$.

Fix a random variable W_0 and let ψ_0 be the local-regret function at W_0. Ordinal equivalence of local-regret functions implies that for each $W \in \mathcal{L}$ there is an increasing function $h_W : \Re \to \Re$ such that $\psi_W = h_W \circ \psi_0$, where ψ_W is the local-regret function at W. Define a new local-regret functional at W

$$
\begin{aligned}
&\hat{V}_W(\psi_0(x_1,y_1),p_1;\ldots;\psi_0(x_n,y_n),p_n) \equiv\\
&\qquad V_W(h_W\circ\psi_0(x_1,y_1),p_1;\ldots;h_W\circ\psi_0(x_n,y_n),p_n)
\end{aligned}
$$

Thus, for each W, local-regret preferences may be represented by $\psi_0,\hat{V}_W$. This completes the proof of Theorem 2(i).

(ii): Consider $W,W' \in \mathcal{L}$. We know from the proof of part (i) that there exists a regret function ψ such that preferences are locally regret with ψ and V_W on $\mathcal{B}(W,\varepsilon)$ and locally regret with ψ and $V_{W'}$ on $\mathcal{B}(W',\varepsilon)$.

Let $\mathcal{R}_W$ be the set of regret lotteries generated by $X, Y \in \mathcal{B}(W, \varepsilon)$ and $\mathcal{R}_{W'}$ be the set of regret lotteries generated by $X', Y' \in \mathcal{B}(W', \varepsilon)$. The set $\mathcal{R}_W \cap \mathcal{R}_{W'}$ is non-empty as $(0, 1)$ belongs to it. As $\mathcal{R}_W$ and $\mathcal{R}_{W'}$ are open sets so is $\mathcal{R}_W \cap \mathcal{R}_{W'}$. Therefore, we may take $R \in \mathcal{R}_W \cap \mathcal{R}_{W'}$ such that $R \neq (0, 1)$. Thus, there exist $X, Y \in \mathcal{B}(W, \varepsilon)$, $X \neq Y$ and $X', Y' \in \mathcal{B}(W', \varepsilon)$, $X' \neq Y'$ such that $R = \Psi(X, Y) = \Psi(X', Y')$. Without loss of generality we may write X, X', W, and W' on the same list of events $s_1, \ldots, s_n$. We can partition each s_i into two sub-events, $s_{i,\alpha}$ and $s_{i,1-\alpha}$, such that $\Pr[s_{i,\alpha}] = \alpha \Pr[s_i]$ and $\Pr[s_{i,1-\alpha}] = (1 - \alpha) \Pr[s_i]$. Thus,

$$\begin{aligned}
\alpha X \oplus (1-\alpha)X' &= (x_1, s_{1,\alpha}; \ldots; x_n, s_{n,\alpha}; x'_1, s_{1,1-\alpha}; \ldots; x'_n, s_{n,1-\alpha}) \\
\alpha Y \oplus (1-\alpha)Y' &= (y_1, s_{1,\alpha}; \ldots; y_n, s_{n,\alpha}; y'_1, s_{1,1-\alpha}; \ldots; y'_n, s_{n,1-\alpha}) \qquad (6) \\
\alpha W \oplus (1-\alpha)W' &= (w_1, s_{1,\alpha}; \ldots; w_n, s_{n,\alpha}; w'_1, s_{1,1-\alpha}; \ldots; w'_n, s_{n,1-\alpha})
\end{aligned}$$

and

$$\begin{aligned}
& ||\alpha X \oplus (1-\alpha)X' - [\alpha W \oplus (1-\alpha)W']|| \\
= \; & \sum_{i=1}^{n} (x_i - w_i)^2 \Pr[s_{i,\alpha}] + \sum_{i=1}^{n} (x'_i - w'_i)^2 \Pr[s_{i,1-\alpha}] \\
= \; & \alpha \sum_{i=1}^{n} (x_i - w_i)^2 \Pr[s_i] + (1-\alpha) \sum_{i=1}^{n} (x'_i - w'_i)^2 \Pr[s_i] \\
= \; & \alpha ||X - W|| + (1-\alpha)||X' - W'||
\end{aligned}$$

Consequently, if $X \in \mathcal{B}(W, \varepsilon)$ and $X' \in \mathcal{B}(W', \varepsilon)$ then

$$\alpha X \oplus (1-\alpha)X' \in \mathcal{B}(\alpha W \oplus (1-\alpha)W', \varepsilon)$$

Similarly, $\alpha Y \oplus (1-\alpha)Y' \in \mathcal{B}(\alpha W \oplus (1-\alpha)W', \varepsilon)$ if $Y \in \mathcal{B}(W, \varepsilon)$ and $Y' \in \mathcal{B}(W', \varepsilon)$. From eq. (6), it follows that

$$\Psi(\alpha X \oplus (1-\alpha)X', \alpha Y \oplus (1-\alpha)Y') = R$$

Hence, if $R \in \mathcal{R}_W \cap \mathcal{R}_{W'}$ then $R \in \mathcal{R}_{\alpha W \oplus (1-\alpha)W'}$ for any $\alpha \in (0, 1)$. That is, if the regret lottery R is locally generated in the neighborhoods of W and W', then R is locally generated in the neighborhood of each random variable on the line segment joining W and W'.

Suppose that V_W and $V_{W'}$ are not concordant. In particular, $V_W(R) > 0$ and $V_{W'}(R) \leq 0$. Let

$$\underline{\alpha} \; = \sup\{\, \alpha \in [0,1] \,:\, V_{\alpha W \oplus (1-\alpha) W'}(R) \leq 0 \,\}$$

As the regret lottery R is locally generated at each $\alpha W \oplus (1-\alpha)W'$, $\underline{\alpha}$ is well-defined. Further, $V_{W'}(R) \leq 0$ implies that the sup is taken over a non-empty set.

From the continuity of $\succeq$ it follows that $\underline{\alpha} < 1$, $V_{\underline{\alpha} W \oplus (1-\underline{\alpha}) W'}(R) = 0$, and that

$$V_{\alpha W \oplus (1-\alpha) W'}(R) \;\; > \;\; 0, \quad \forall \alpha \in (\underline{\alpha}, 1]$$

We know that

$$\underline{\alpha} X \oplus (1-\underline{\alpha}) X', \underline{\alpha} Y \oplus (1-\underline{\alpha}) Y' \in \mathcal{B}(\underline{\alpha} W \oplus (1-\underline{\alpha}) W', \varepsilon) \tag{7}$$

As

$$R \;=\; \Psi(\underline{\alpha} X \oplus (1-\underline{\alpha}) X', \underline{\alpha} Y \oplus (1-\underline{\alpha}) Y')$$

eq. (7) together with $V_{\underline{\alpha} W \oplus (1-\underline{\alpha}) W'}(R) = 0$ implies that

$$\underline{\alpha} X \oplus (1-\underline{\alpha}) X' \sim \underline{\alpha} Y \oplus (1-\underline{\alpha}) Y$$

For α_1 close to $\underline{\alpha}$,

$$\underline{\alpha} X \oplus (1-\underline{\alpha}) X', \underline{\alpha} Y \oplus (1-\underline{\alpha}) Y' \;\; \in \;\; \mathcal{B}(\alpha_1 W \oplus (1-\alpha_1) W', \varepsilon)$$

Take such a $\alpha_1 > \underline{\alpha}$. Then $V_{\alpha_1 W \oplus (1-\alpha_1) W'}(R) > 0$ implies that

$$\underline{\alpha} X \oplus (1-\underline{\alpha}) X' \succ \underline{\alpha} Y \oplus (1-\underline{\alpha}) Y$$

Contradiction. ■

Proof of Proposition 1: We first prove that if $X^1 \sim Y^1$, then $\psi_{W^1}(x_1, y_1) = -\psi_{W^1}(x_2, y_2)$.

By the skew-symmetry of the functions ψ,

$$\begin{aligned} X^1 \sim Y^1 \iff & V_{W^1}(\psi_{W^1}(x_1, y_1), \delta;\ \psi_{W^1}(x_2, y_2), \delta;\ 0, 1-2\delta) = 0 \\ \iff & V_{W^1}(-\psi_{W^1}(y_1, x_1), \delta;\ -\psi_{W^1}(y_2, x_2), \delta;\ 0, 1-2\delta) = 0 \end{aligned}$$

Therefore, $\psi_{W^1}(x_1, y_1) \geqslant -\psi_{W^1}(y_2, x_2)$ iff $\psi_{W^1}(x_2, y_2) \leqslant -\psi_{W^1}(y_1, x_1)$. However, $\psi_{W^1}(x_1, y_1) > -\psi_{W^1}(x_2, y_2)$ implies

$$\begin{aligned} & -\psi_{W^1}(x_1, y_1) = \psi_{W^1}(y_1, x_1) < -\psi_{W^1}(y_2, x_2) = \psi_{W^1}(x_2, y_2) \Longrightarrow \\ & V_{W^1}(\psi_{W^1}(y_1, x_1), \delta;\ \psi_{W^1}(y_2, x_2), \delta;\ 0, 1-2\delta) < \\ & \qquad V_{W^1}(\psi_{W^1}(x_1, y_1), \delta;\ \psi_{W^1}(x_2, y_2), \delta;\ 0, 1-2\delta) = 0 \end{aligned}$$

A contradiction to $X^1 \sim Y^1$.

By Theorem 2(i), ψ_{W^2} is an increasing ordinal transformation of ψ_{W^1}. Therefore, $\psi_{W^1}(x_1, y_1) = -\psi_{W^1}(x_2, y_2)$ implies that $\psi_{W^2}(x_1, y_1) = -\psi_{W^2}(x_2, y_2)$ and thus $X^2 \sim Y^2$. ■

Proof of Proposition 2: Let $W = (w_1, s_1; w_2, s_2; \ldots; w_n, s_n)$. Let ψ_W be a local-regret function at W. First, we show that the linearity of V_W implies that each ψ_W is unique up to positive multiples.

For two regret levels $r_1, r_2 > 0$, let $x, y > 0$ be monetary outcomes such that $r_1 = \psi_W(x, -x)$ and $r_2 = \psi_W(y, -y)$. Define

$$\begin{aligned} X &= (x, s_{1,\varepsilon_1}; -y, s_{1,\varepsilon_2}; w_1, s_{1,1-\varepsilon_1-\varepsilon_2}; w_2, s_2; \ldots; w_n, s_n) \\ Y &= (-x, s_{1,\varepsilon_1}; y, s_{1,\varepsilon_2}; w_1, s_{1,1-\varepsilon_1-\varepsilon_2}; w_2, s_2; \ldots; w_n, s_n) \end{aligned}$$

where $\varepsilon_1, \varepsilon_2 > 0$, $\varepsilon_1 + \varepsilon_2 < 1$, $\varepsilon_2/\varepsilon_1 = r_1/r_2$, and $\Pr[s_{1,\varepsilon_\ell}] = \varepsilon_\ell$, $\ell = 1, 2$. Choose $\varepsilon_1, \varepsilon_2$ small enough so that $X, Y \in \mathcal{B}(W, \varepsilon)$. Thus,

$$V_W(\Psi_W(X, Y)) \;=\; \psi_W(x, -x) \Pr[s_{1,\varepsilon_1}] + \psi_W(-y, y) \Pr[s_{1,\varepsilon_2}]$$

$$
\begin{aligned}
&= r_1\varepsilon_1 - r_2\varepsilon_2 \\
&= 0
\end{aligned}
$$

Hence, $X \sim Y$.

Let $\hat{\psi}_W$ be another local-regret function at W for which the regret functional $\hat{V}_W$ is linear in probabilities. From $X \sim Y$ we conclude

$$
\begin{aligned}
\hat{V}_W(\hat{\Psi}_W(X,Y)) &= \hat{\psi}_W(x,-x)\Pr[s_{1,\varepsilon_1}] + \hat{\psi}_W(-y,y)\Pr[s_{1,\varepsilon_2}] \\
&= \hat{r}_1\varepsilon_1 - \hat{r}_2\varepsilon_2 \\
&= 0
\end{aligned}
$$

Thus, there exists a such that $\hat{\psi}_W(x,-x) = a\psi_W(x,-x) = ar_1$ and $\hat{\psi}_W(y,-y) = a\psi_W(y,-y) = ar_2$. That, $a > 0$ follows from the fact that $\hat{\psi}_W(x,-x) > 0$. By varying r_2 (and y, ε_2 in the above construction), we conclude that $\hat{\psi}_W \equiv a\psi_W$ for some $a > 0$.

Let $W, W' \in \mathcal{L}$. We show that ψ_W and $\psi_{W'}$ are positive multiples of each other. As in the proof of Theorem 2, there is a finite sequence of random variables $W = W_1, W_2, \ldots, W_n = W'$ such that the open neighborhoods $\mathcal{B}(W_j, \varepsilon)$, $j = 1, \ldots, m$ cover the line segment $[W, W']$. Thus, for each j, there exists

$$
\hat{W}_j = (\hat{w}_{1j}, s_1; \hat{w}_{2j}, s_2; \ldots; \hat{w}_{nj}, s_n)
$$

on the line segment joining W_j and W_{j+1} such that $\hat{W}_j \in \mathcal{B}(W_j, \varepsilon) \cap \mathcal{B}(W_{j+1}, \varepsilon)$.

Let ψ_j be the local-regret function at W_j. Define

$$
\begin{aligned}
X_j &= (x, s_{1,\varepsilon_{1j}}; -y, s_{1,\varepsilon_{2j}}; \hat{w}_{1j}, s_{1,1-\varepsilon_1-\varepsilon_2}; \hat{w}_{2j}, s_2; \ldots; \hat{w}_{nj}, s_n) \\
Y_j &= (-x, s_{1,\varepsilon_{1j}}; y, s_{1,\varepsilon_{2j}}; \hat{w}_{1j}, s_{1,1-\varepsilon_1-\varepsilon_2}; \hat{w}_{2j}, s_2; \ldots; \hat{w}_{nj}, s_n)
\end{aligned}
$$

where $\varepsilon_1, \varepsilon_2 > 0$, $\varepsilon_2/\varepsilon_1 = \psi_j(x,-x)/\psi_j(y,-y)$, and $\Pr[s_{1,\varepsilon_\ell}] = \varepsilon_\ell$, $\ell = 1, 2$. Choose $\varepsilon_1, \varepsilon_2$ small enough so that $X, Y \in \mathcal{B}(W_j, \varepsilon) \cap \mathcal{B}(W_{j+1}, \varepsilon)$. Thus, local regret evaluated at W_j implies that $X_j \sim Y_j$. Hence, the local regret evaluated at W_{j+1} also implies $X_j \sim Y_j$. As x and y are arbitrary, we conclude that ψ_{j+1} is a positive multiple of ψ_j. Consequently, $\psi_{W'}$ is a positive multiple of ψ_W. ■

Appendix B: Examples

Example 1 INTRANSITIVE IN THE SMALL, TRANSITIVE IN THE LARGE
Let $\psi(x, y) = x - y$ be a regret function. For $X = (x_1, s_1; \ldots; x_n, s_n)$, $Y = (y_1, s_1; \ldots; y_n, s_n)$, $\Pr[s_i] = p_i$ define

$$V(\Psi(X,Y)) = \begin{cases} \sum_{i=1}^{n} p_i \psi(x_i, y_i) & \text{if } ||\Psi(X,Y)|| \geq \varepsilon \\ \\ \alpha_{XY} \sum_{i=1}^{n} p_i \psi(x_i, y_i) + \\ \quad (1-\alpha_{XY}) \sum_{i=1}^{n} p_i [\psi(x_i, y_i)]^3 & \text{otherwise} \end{cases}$$

where $\alpha_{XY} \equiv \min\{1, ||\Psi(X,Y)||/\varepsilon\}$. Note that α_{XY} increases continuously from 0 to 1 as $||\Psi(X,Y)||$ increases from 0 to ε. Hence, V is a continuous functional.

Take $a > 0$. Define $X = (-a, s_1; 0, s_2; a, s_3)$, $Y = (0, s_1; a, s_2; -a, s_3)$, and $Z = (a, s_1; -a, s_2; 0, s_3)$, where s_1, s_2, s_3 are equiprobable, disjoint events with $s_1 \cup s_2 \cup s_3 = S$. By symmetry, $\alpha_{XY} = \alpha_{YZ} = \alpha_{ZX} \equiv \alpha$. For small enough a, $||\Psi(X,Y)|| < \varepsilon$, $||\Psi(Y,Z)|| < \varepsilon$, and $||\Psi(Z,X)|| < \varepsilon$ and $\alpha < 1$. Therefore,

$$V(\Psi(X,Y)) = V(\Psi(Y,Z)) = V(\Psi(Z,X)) = (1-\alpha)2a^3 > 0$$

which implies $X \succ Y \succ Z \succ X$. Therefore, for each W local regret in $\mathcal{B}(W, 0.5\varepsilon)$ is intransitive.

Transitivity in the large follows from $V(\Psi(X,Y)) = \mathrm{E}[X] - \mathrm{E}[Y]$ whenever $||\Psi(X,Y)|| > \varepsilon$. ■

As preferences are transitive in the large in Example 1, Theorem 1(ii) implies that the regret functional V in this example is not linear in probabilities. That is, there exist regret lotteries R_1, R_2, and a constant $c \in (0,1)$, such that

$$V\Big(cR_1 + (1-c)R_2\Big) \neq cV(R_1) + (1-c)V(R_2)$$

Take two regret lotteries, R_1, R_2, with $||R_1|| > \varepsilon$ and $||R_2|| < \varepsilon$. Let c be sufficiently close to 1 so that $||cR_1 + (1-c)R_2|| > \varepsilon$. Then

$$\begin{aligned} V\Big(cR_1 + (1-c)R_2\Big) &= c\mathrm{E}[R_1] + (1-c)\mathrm{E}[R_2] \\ &\neq cV(R_1) + (1-c)V(R_2) \end{aligned}$$

as $\mathrm{E}[R_1] = V(R_1)$ but $\mathrm{E}[R_2] \neq V(R_2)$.

Example 2 TRANSITIVE IN THE SMALL, INTRANSITIVE IN THE LARGE
Let X, Y, and $\psi(x,y)$ be as in Example 1. Define

$$V(\Psi(X,Y)) = \begin{cases} \sum_{i=1}^n p_i \psi(x_i, y_i) & \text{if } \ ||\Psi(X,Y)|| < \varepsilon \\ & \\ (1-\beta_{XY}) \sum_{i=1}^n p_i \psi(x_i, y_i) + & \\ \qquad \beta_{XY} \sum_{i=1}^n p_i \left[\psi(x_i, y_i)\right]^3 & \text{if } \ \varepsilon \leqslant ||\Psi(X,Y)|| < 1 \\ & \\ \sum_{i=1}^n p_i \left[\psi(x_i, y_i)\right]^3 & \text{if } \ 1 \leqslant ||\Psi(X,Y)|| \end{cases}$$

where $\beta_{XY} \equiv \frac{||\Psi(X,Y)|| - \varepsilon}{1-\varepsilon}$. An argument similar to that in Example 1 implies that preferences are transitive in the small but intransitive in the large. ■

Local-regret preferences do not uniquely determine preferences in the large. For instance, $V(\Psi(X,Y)) = \mathrm{E}[X] - \mathrm{E}[Y]$ for all $\Psi(X,Y)$ has the same local-regret preferences as Example 2.

References

[1] Allais, M., 1953. "Le comportement de l'homme rationnel devant le risque: Critique des postulates et axiomes de l'ecole Americaine," *Econometrica* 21:503–546.

[2] Bell, D., 1982. "Regret in decision making," *Operations Research* 30:961–981.

[3] Bikhchandani, S. and U. Segal, 2010. "Transitive regret," *Theoretical Economics* 6:95–108.

[4] Bikhchandani, S. and U. Segal, 2014. "Transitive regret over statistically independent lotteries," *Journal of Economic Theory* 152, 237–248.

[5] Birnbaum, M.H. and U. Schmidt, 2008. "An experimental investigation of violations of transitivity in choice under uncertainty," *Journal of Risk and Uncertainty* 37:77–91.

[6] Cerreia-Vioglio, S., D. Dillenberger and P. Ortoleva, 2015. "Cautious expected utility and the certainty effect," *Econometrica* 83:693–728.

[7] Conlisk, J., 1989. "Three variants on the Allais example," *American Economic Review* 79:392-–407.

[8] Diecidue, E. and J. Somasundaram, 2017. "Regret theory: A new foundation," *Journal of Economic Theory* 172:88–119.

[9] Fan, C.-P., 2002. "Allais Paradox in the Small," *Journal of Economic Behavior and Organization* 49: 411-–421.

[10] Grether, D. and Plott, C., 1979. "Economic theory of choice and the preference reversal phenomenon," *American Economic Review* 69:623–638.

[11] Hey, J., 1991. *Experiments in Economics*. Basil Blackwell.

[12] Kahneman, D. and A. Tversky, 1979. "Prospect theory: An analysis of decision under risk," *Econometrica* 47:263–291.

[13] Lichtenstein, S. and Slovic, P., 1971. "Reversal of Preference Between Bids and Choices in Gambling Decisions," *Journal of Experimental Psychology* 89:46–55.

[14] Loomes, G. and R. Sugden, 1992. "Regret theory: An alternative theory of rational choice under uncertainty," *Economic Journal* 92:805–824.

[15] Loomes, G., Starmer, C., and Sugden, R., 1991. "Observing Violations of Transitivity by Experimental Methods," *Econometrica* 59:425–439.

[16] MacCrimmon, K.R., and S. Larsson, 1979. "Utility Theory: Axioms Versus 'Paradoxes'," in M. Allais and O. Hagen, Eds.: *Expected Utility Hypotheses and the Allais Paradox*. Dordrecht: D. Reidel.

[17] Machina, M.J., 1982. " 'Expected utility' analysis without the independence axiom," *Econometrica* 50:277–323.

[18] Machina, M.J., 1987. "Choice under uncertainty: Problems solved and unsolved," *Journal of Economic Perspectives* 1:121–154.

[19] Mueller, D.C., 2003. *Public Choice III*. Cambridge University Press.

[20] Nishimura, H., 2018. "The transitive core: inference of welfare from nontransitive preference relations," *Theoretical Economics* 13:579–606.

[21] Regenwetter, M., J. Dana, and C.P. Davis-Stober, 2011. "Transitivity of preferences," *Psychological Review* 118:42-56.

[22] Starmer, C., 2000. "Developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk," *Journal of Economic Literature* 38:332–382.

[23] Tserenjigmid, G., 2015. "Theory of decisions by intra-dimensional comparisons," *Journal of Economic Theory* 159:326–338.