Who gets the bonus? Affirmative Action Reforms in High School Admissions in China *

Umut Dur[†] Onur Kesten[‡] Tong Wang[§] Congyi Zhou[¶]

Abstract

China has implemented affirmative action reforms to improve access to quality high school education for students from underperforming middle schools by awarding bonus points to a select group of students. Our study reveals significant flaws in practice due to challenges in determining how bonuses should be distributed. We propose a choice rule that "endogenously" identifies bonus-recipients and show that it is the unique acceptant and fair choice rule that efficiently assigns the bonus. Embedded in the deferred acceptance mechanism, it ensures stability, strategy-proofness, and constrained optimality. Empirical analysis shows that our proposal significantly improves representation for underperforming schools and effectively assigns the bonus to the "right" students.

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[†]Umut Dur, Poole College of Management, North Carolina State University, udur@ncsu.edu

[‡]Onur Kesten, School of Economics, The University of Sydney, onur.kesten@sydney.edu.au

[§]Tong Wang, College of International Relations, Ritsumeikan University, tongw@fc.ritsumei.ac.jp

[¶]Congyi Zhou, the Wilf Family Department of Politics, New York University, zhoucongyi@nyu.edu

1 Introduction

China has the world's largest centralized school choice system, encompassing admissions from elementary school to college. Each year, around 16 million students graduate from middle schools in China. However, only about half of these graduates are able to attend high school, exhibiting significant disparities in admission rates across provinces.[1] Meanwhile, minority students and those from rural areas tend to have considerably lower admission rates compared to their urban counterparts.[2] Such educational inequality has long been a controversial issue in China. The Chinese government has implemented diverse education reforms to address this issue.[3] One widely implemented policy to reduce this inequality is affirmative action within school admission systems.

Since the beginning of the reform era in the 1980s, various forms of affirmative action have been introduced at different levels of the admission process. In 2002, the Ministry of Education reformed the primary and secondary education admission system to improve access for students from lower socioeconomic districts to quality urban schools.[4] In 2014, the State Council introduced a guideline for comprehensive nationwide reforms in examination and admission systems. Among these reforms, the introduction of the Zhibiaosheng (ZBS) policy, which implements affirmative action in high school admissions, is considered one of the most significant nationwide initiatives.[5] Some cities have explicitly outlined the purpose of this policy. For example, the Education Bureau of Dezhou in Shandong Province stated:

> "The purpose of the ZBS policy is to uphold educational equity by ensuring that every middle school has a certain proportion of students admitted to regular high schools... it is a key initiative to promote balanced development in compulsory education, aiming to provide students from underperforming schools with opportunities to access high school educational resources."[6]

[1] See http://www.moe.gov.cn/jyb_sjzl/moe_560/jytjsj_2019/gd/202006/t20200610_464550.html; http://www.moe.gov.cn/jyb_sjzl/moe_560/jytjsj_2019/gd/202006/t20200610_464605.html.

[2] See https://www.sohu.com/a/207893982_112404.

[3] See https://www.globaltimes.cn/content/1192288.shtml.

[4] In 2002, the Ministry of Education stated: "Relying solely on the entrance examination scores as the main criterion for enrollment is worth re-evaluation... It is imperative an imperative to actively explore systems that allocate enrollment quotas... promote balanced development among schools during the compulsory education phase." See https://www.gov.cn/gongbao/content/2003/content_62173.htm.

[5] See https://www.gov.cn/zhengce/content/2014-09/04/content_9065.htm.

[6] See https://sdxw.iqilu.com/share/YS0yMS0xMjk0NzUyNw==.html.

High school admissions in Chinese cities are based on local high school entrance exams. The ZBS policy is implemented through a so-called *privilege system*, in which each high school offers more favorable admission standards to select students from designated middle schools, either by adding bonus points to their exam scores or by lowering admission thresholds. Although privilege systems may appear similar and serve comparable purposes, they differ from *reserve systems*, which have gained significant attention in recent market design applications, including school choice (e.g., Dur et al. 2020), pandemic rationing (e.g., Pathak et al. 2023), affirmative action in India (e.g., Sönmez and Ünver 2022), and H1-B visa allocation (e.g., Pathak et al. 2022). In a reserve system, intended beneficiaries (e.g., students from designated middle schools) receive unconditional priority for reserved resources (e.g., designated high school seats), often resulting in exclusive access. Differently, a privilege system provides a more flexible advantage, offering a "bump up" in priority or a conditional admission guarantee, subject to meeting a (endogenously determined) minimum standard. A privilege system generalizes the reserve system, as any reserve system can be implemented with an appropriate allocation of bonus scores, but not vice versa. These notable differences render privilege systems as a hybrid approach between priority-based (i.e., score-subsidy) and reserve-based systems, enabling high schools to admit students with similar aptitude while maintaining overall quality. This "filtering flexibility" likely influenced Chinese officials to adopt privilege systems over purely reserve- or priority-based models.

Privilege systems are conceptually innovative and flexible, but their integration with student assignment algorithms remains unsettled. By analyzing China's current high school admissions, we highlight the challenges of implementing privilege systems, identify flaws in existing mechanisms, and propose a solution to achieve their intended goals.

Currently, various types of privilege systems have emerged across Chinese cities. The core of the ZBS policy debate centers on determining which subset of a middle school's students should benefit from the designated privileges for that school. Cities with privilege systems generally adopt one of two mechanisms: privileged students are determined either *autonomously* by each middle school or *automatically* by the assignment algorithm. We refer to the former as the *Chinese Early Selection* (CES) mechanism and the latter as the *Chinese Automated Selection* (CAS) mechanism. A CES mechanism consists of two stages.

In the first stage, each middle school has full autonomy to select which of its students will be eligible to receive the privilege benefit, such as bonus scores. After privilege eligibility is determined in the early selection stage, all students proceed to the second stage, participating in a Chinese parallel mechanism (Chen and Kesten 2017), in which high schools apply easier admission standards for students eligible for privileges.

A major shortcoming of CES mechanisms is the lack of transparency in the early selection stage, where outcomes are often shaped by internal coordination between school administrators and students. To boost their school's representation at top high schools, officials may prioritize granting privileges to students who need them most, rather than high achievers likely to secure admission without additional support. This practice has sparked major controversy, as reflected in the following interview from a public debate on the reform:

> "An education expert supporting the change [to CAS] told a journalist, "I support the change in [implementation of the] ZBS policy, primarily because of transparency." She explained that the traditional method of determining ZBS eligibility, based on middle school recommendations and criteria set by the schools, creates opportunities for *black-box manipulation*, a concern frequently raised by parents."[7]

A second major shortcoming of the CES mechanisms is the suboptimal assignment of privileges. Even with the best intentions, middle schools distributing privileges ad hoc, without knowing students' relative rankings based on exam scores, may result in privileges being granted to the "wrong" students. This issue is highlighted by Chinese Education News:

> "In many cities [using CES], 50% of the total quota of good high schools is privilege capacity, but only 35% is effectively used. However, students who need bonus points may not receive them and end up in less undesirable schools."[8]

To address the lack of transparency in the CES mechanism, some Chinese cities have adopted the CAS mechanism, which eliminates the early selection stage. CAS automatically awards the designated privileges to the top applicants (based on raw exam scores) from each middle school in the form of lump-sum bonus points on top of their raw scores. While CAS resolves CES's transparency issues, it still results in wasted privileges. Intuitively, privileges can be wasted in two ways: (1) the privilege is assigned to a high-achiever who would gain

[7] `http://www.hebjy.com/html/2014/08/221108217563.htm`

[8] `https://www.edu.cn/edu/yiwujiaoyu/201305/t20130510_941017.shtml`

admission without it (observed under both the CES and CAS mechanisms); and (2) the privilege is assigned to a low-achiever who fail to gain admission and cause a high-achiever not to be admitted (observed under the CES mechanism).

In addition to falling short of an ideal privilege distribution, mechanisms in both classes lack strategy-proofness. This stems from two factors: first, student assignments rely on the Chinese parallel mechanisms, which are well-known for their strategic vulnerabilities; second, privileges are effective only if the corresponding high school is ranked first. Worse still, the CES and some versions of CAS mechanisms may also lead to unstable outcomes. Moreover, these shortcomings cannot be rectified by a simple transition to a mechanism such as the deferred acceptance (DA), as the choice rules associated with these mechanisms fail to be substitutable.

In this paper, we ask whether a privilege system can systematically and transparently assign privileges to the "right" students while ensuring fairness, stability and strategy-proofness. We define a context-appropriate *fairness* as: (1) a student has a rightful claim to a seat if her score exceeds the (privilege-adjusted) score of the current assignee, and (2) a student has a rightful claim to a privilege if it is not already utilized by a higher-scoring student from her middle school. We further introduce the concept of "efficient allocation of privileges" to formalize the goal of assigning privileges to the "right" students.

Toward the goal of assigning privileges in the most efficient and fair way, we design a *privilege choice rule* that ensures that a student who would be chosen based on her raw score, is never granted the privilege. In essence, our choice rule "endogenously" determines who should receive the privilege designated for a middle school by iteratively "simulating" scenarios where no privileges are initially assigned but may be potentially allocated later, only if they improve the outcome for the beneficiary. We show that this choice rule is essentially the unique acceptant and fair choice rule that efficiently allocates privileges (Theorem 1). Furthermore, the DA mechanism embedded with this choice rule, called DA-PCR, is stable, strategy-proof, fair, and efficient in privilege allocation (Theorem 2). More importantly, DA-PCR Pareto dominates any other fair mechanism that efficiently allocates privileges (Theorem 3).

The theoretical properties of these mechanisms motivate an investigation into real-world student behavior and welfare outcomes. We use a dataset on high school admissions from a large Chinese city in 2014.[9] By combining admission records with survey data from the same year, we address the challenge of analyzing student preferences based solely on observed rank-ordered lists (ROLs). This is particularly relevant under *non*-strategy-proof mechanisms, where students may misreport their true preferences. We detect at least 798 unfair matches where privileges were "misallocated", impacting 747 out of 5,254 students. Although we cannot directly observe early selection processes for evidence of coordination, we infer potential coordination by analyzing students' ROLs and assignments. Our findings suggest that 3.9% of students from top middle schools are likely to engage in coordination within their schools, compared to less than 2% from other schools. This suggests that top middle schools, with more high-achieving students, have stronger incentives to coordinate to secure more spots in prestigious high schools.

Using survey data, we estimate students' true preferences without considering students' strategic behaviors in ROLs and conduct counterfactual experiments to evaluate different matching mechanisms. The ZBS policy aims to improve access for students of lower-achieving middle schools to top high schools. Our analysis shows that replacing the CES mechanism with the CAS mechanism reduces top middle schools' placements in top high schools by 0.9% to 4.5%, with marginal gains for other tier schools. While the CAS mechansim improves transparency, it does little to enhance representation for lower-achieving schools. In contrast, replacing the CES mechanism with DA-PCR significantly benefits median and bottom-tier schools, with bottom-tier schools placing 7.6% to 10.5% more students in top high schools.

A notable feature of DA-PCR is its ability to allocate privileges to the students who need them most. We evaluate privilege allocation effectiveness by measuring the proportion of students admitted due to privilege bonuses—students who would otherwise be rejected without this privilege. Under DA-PCR, over 95% of privileges are used effectively, with 84% to 100% of low-tier middle schools' privilege capacities directly aiding admissions.[10] In contrast, CES allocates less than 26% and CAS less than 35% of privilege capacities to students who genuinely benefit from these bonuses.

[9] Confidentiality restrictions prevent disclosure of the city's name.

[10] Less than full utilization occurs when students' scores are too low to gain admission even with bonuses.

The rest of this paper proceeds as follows. Section 2 defines the school choice problem with privileges. Section 3 discusses the current mechanisms for implementing affirmative action policies in China. In Section 4, we introduce our mechanism design approach to this market. Sections 5 and 6 describe our data and analyze students' behaviors in practice, respectively. Section 7 presents the empirical model and our estimates of student preferences, while Section 8 conducts counterfactual experiments across mechanisms. Section 9 reviews the relevant literature, and Section 10 concludes with a summary of our findings.

2 School Choice Problem with Privileges

Our model is based on the standard school choice framework (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003), with a finite set of students I and a finite set of high schools H. Each high school $h \in H$ has a capacity of q_h, and $q = (q_h)_{h\in H}$ represents the capacity vector. Unlike the standard model, each student has a type defined by the type set M, with a type function $\tau : I \to M$. Consistent with the Chinese high school admission context, M represents the set of middle schools. For each $m \in M$, the set of students currently attending middle school m is $I^m = \{i \in I : \tau(i) = m\}$. Since students can be only enrolled one middle school, $I = \cup_{m\in M} I^m$ and $I^m \cap I^{m'} = \emptyset$ for all $m, m' \in M$.

Every student $i \in I$ has a strict preference order P_i over schools and the option of being unassigned, $\emptyset$. Let $P = (P_i)_{i\in I}$ represent the preference profile, and R_i the associated weak preference order of student i. Every student $i \in I$ has a raw score $\pi(i) \in [0, \pi_{max}]$, where π_{max} is the maximum raw score. Conditional on eligibility and meeting certain criteria, a student may receive an additional bonus score $\beta \in [0, \pi_{max} + 1]$, referred to as the *privilege*. For simplicity, we assume $\pi(i) \neq \pi(j)$ and $\pi(i) \neq \pi(j) + \beta$ for any two students $i, j \in I$.[11]

Each high school h offers the privilege only to a certain number of students from each middle school, denoted by p_h^m, the *privilege capacity* of middle school m at high school h. We allow for the possibility that the total privilege capacity of a high school may exceed its capacity. For $h \in H$, let $p_h^n = max\{0, q_h - \sum_{m\in M} p_h^m\}$ denote its undesignated capacity. Let $p_h = (p_h^m)_{m\in M}$ and $p = (p_h)_{h\in H}$.

[11] In practice, exogenous tie-breaking rules are used. Moreover, bonus points can be easily arranged in a way that does not cause a tie in the scores.

Typically, not every student from a given middle school is eligible for a bonus. Let $E_h^m \subseteq I^m$ be the set of eligible students for bonus from middle school m at high school h, with $E_h = (E_h^m)_{m \in M}$ and $E = (E_h)_{h \in H}$. Eligibility alone does not guarantee receiving a bonus, i.e., an eligible student may need to meet additional criteria to receive a bonus.

A school choice problem is a tuple $G = (I, H, M, q, p, \pi, P, \beta, E)$. We fix all elements except P, β, and E, and simply denote a problem with (P, β, E).

A **matching** is a function $\mu : I \to H \cup \{\emptyset\}$ such that $|\mu^{-1}(h)| \leq q_h$ for all $h \in H$. In the context of Chinese high school admissions, a student can be admitted to a school either with or without bonus scores. To track admission scores, we pair a given matching with an admission score profile. Let $\alpha(i) \in \{\pi(i), \pi(i) + \beta\}$ denote the admission score of student i to a school, possibly $\emptyset$, and $\alpha = (\alpha(i))_{i \in I}$ be an **admission score** profile. An **assignment** is a matching-admission-score pair (μ, α) such that for each $h \in H$ and $m \in M$, $\{i \in \mu^{-1}(h) \cap I^m \ : \ \alpha(i) = \pi(i) + \beta\} \subseteq E_h^m$, and $|\{i \in \mu^{-1}(h) \cap I^m \ : \ \alpha(i) = \pi(i) + \beta\}| \leq p_h^m$. That is, only eligible students can receive a bonus score, and the number of bonus recipients from middle school m for high school h never exceeds its privilege capacity, p_h^m.

Next, we define the desirable properties of an assignment. An assignment (μ, α) is **non-wasteful** if no school-student pair (h, i) exists such that $h \ P_i \ \mu(i)$ and $|\mu^{-1}(h)| < q_h$. An assignment (μ, α) is **individually rational** if $\mu(i) \ R_i \ \emptyset$ for all $i \in I$.

In the standard school choice model, fairness requires that a student with a higher score does not envy a student with a lower score. In our context, affirmative action allows for this possibility if the lower-scoring student receives the privilege and her raw score is "sufficiently high". Our fairness notion incorporates this feature as follows:

An assignment (μ, α) is **fair** if whenever there exists a student-school pair (i, h) such that $i \in I^m$ and $h \ P_i \ \mu(i)$, then

(a) $\alpha(j) > \pi(i)$ for all $j \in \mu^{-1}(h)$; and

(b) if $i \in E_h$ and there exists $j \in \mu^{-1}(h)$ such that $\pi(i) + \beta > \alpha(j)$, then $|\{i' \in \mu^{-1}(h) \cap I^m \ : \ \pi(i') > \pi(i) \text{ and } \alpha(i') = \pi(i') + \beta\}| = p_h^m$.

Condition (a) means that any student assigned to a school preferred by i to $\mu(i)$ has a higher admission score than the raw score of i. Condition (b) means that if i is eligible for a bonus and would have a higher score with bonus than the admission score of another student assigned to h, then there are at least p_h^m other eligible students in I^m assigned to h who already received a bonus, and their raw scores are higher than the raw score of i.

An assignment (μ, α) **efficiently allocates privileges** if there does not exist a high school h and a student pair $(i, j) \in E_h \times E_h$ such that

(a) $\tau(i) = \tau(j)$, $h = \mu(j)\ P_i\ \mu(i)$,

(b) $\alpha(j) = \pi(j) + \beta$, and

(c) $\pi(j) > \alpha(k)$ and $\pi(i) + \beta > \alpha(k)$ for some k with $\mu(k) = \mu(j) = h$.

In words, under an assignment that efficiently allocates privileges, a student j assigned to some school h with bonus does not get this privilege if she could have already been assigned to h without it when that privilege would have helped with the admission to h of another eligible student i from the same middle school.

An assignment (μ, α) **Pareto dominates** another assignment (μ', α') if $\mu(i)\ R_i\ \mu'(i)$ for all $i \in I$ and $\mu(j)\ R_j\ \mu'(j)$ for some $j \in I$.

A **mechanism** Φ is a procedure that selects an assignment for every problem. The assignment selected by Φ in problem (P, β, E) is denoted as $\Phi[P, \beta, E] = (\Phi^\mu[P, \beta, E], \Phi^\alpha[P, \beta, E])$, where $\Phi[P, \beta, E](i) = (\Phi^\mu[P, \beta, E](i), \Phi^\alpha[P, \beta, E](i))$ represents the assignment for student i. Let $\Phi[P, \beta, E](h)$ be the set of students assigned to h under Φ for (P, β, E). A mechanism Φ is fair {individually rational} [non-wasteful], if $\Phi[P, \beta, E]$ is fair {individually rational} [non-wasteful] for any problem (P, β, E). A mechanism Φ efficiently allocates privileges, if $\Phi[P, \beta, E]$ efficiently allocates privileges. A mechanism Φ is strategy-proof if there is no problem (P, β, E), some student i, and a preference order P_i' such that $\Phi^\mu[(P_i', P_{-i}), \beta, E](i)\ P_i\ \Phi^\mu[P, \beta, E](i)$, i.e., no student can benefit from misreporting her true preferences.

3 Affirmative Action Mechanisms in China

In this section, we describe two types of mechanisms used across China to implement the ZBS policy. Table 1 summarizes the distribution of these mechanisms in some major cities, including provincial capitals and economically developed large cities.[12]

Table 1: Affirmative action mechanisms used in some major Chinese cities

City	Full score/Bonus	Mechanism	Privilege capacity	Position in the ROL
Changchun	750/30	CES	80% or 60%†	1st Choice
Chengdu	710/threshold	CES	50% or 30%†	1st Choice
Hangzhou	600/50	CES	50% or 40%†	1st Choice
Our focal city	650/30, 360/15‡	CES until 2014, CAS after	65%	1st Choice
Tianjin	800/20	CES	50%	1st Choice
Wuhan	550/30	CES	50%	1st Choice
Changsha	720/♯	CAS	60%	Choose one school
Fuzhou	800/35	CAS	50% or 40%†	1st Choice
Lanzhou	740/50	CAS	75%	1st Choice
Nanchang	670/40 or 50♯	CAS	70%	Choose one school
Shenzhen	610/20	CAS	50%	Choose one school
Shijiazhuang	640/50	CAS	80%	Choose one school
Taiyuan	660/50	CAS	60%	1st Choice
Xiamen	800/25	CAS	10% to 50%†	Choose one school
Xining	770/50	CAS	50%	1st Choice
Zhengzhou	600/50	CAS	60%	1st Choice

Notes: This table shows mechanisms used in some large Chinese cities to implement the ZBS policy in 2022. The privilege capacity refers to the proportion of privilege capacity relative to a high school's total capacity. ‡ denotes that the *full score/bonus points* was 650/30 until 2014, then it switched to 360/15 after. ♯ denotes that the bonus point is school dependent. † represents that various types of high schools offer differing capacities for privileges.

3.1 The Class of Chinese Early Selection Mechanisms

A typical CES mechanism has two stages: "the early selection stage" and "the admission stage". In the early selection stage, each middle school determines a subset of its students eligible for the privilege. In the admission stage, students are assigned to high schools using a modified Chinese parallel mechanism, in which those eligible students who score above a (fixed or endogenously-determined) cutoff receive a bonus score on top of their raw scores. Variations in the second stage exist across cities, so we broadly refer to these as the class of CES mechanisms. To isolate the effects of privilege assignment from mechanism-specific effects, for the admission stage, our analysis assumes a version of the Chinese parallel mechanism that is equivalent to DA to eliminate the fairness and incentive issues arising in

[12] Other provincial capitals and major cities, such as Beijing and Shanghai, lack clear explanations of high school admission mechanisms in official documents, and are thus excluded from the table.

other versions of Chinese parallel mechanisms even in standard school choice settings.[13] Moreover, we focus on a general version of these mechanisms where the cutoff for eligible students to receive a bonus is set endogenously, and an eligible student $i \in E_h$ for any high school h can receive bonus regardless of h's position in P_i.

Early Selection Stage:

- *Each middle school $m \in M$ autonomously determines the subset $E_h^m \subseteq I^m$ with $|E_h^m| = p_h^m$ of students who will be eligible to receive the privilege of each high school $h \in H$ in the admission stage. We refer to students in $E^m = (E_h^m)_{h \in H}$ as* **eligible students.**[14]

Due to its ad-hoc nature and variation across middle schools, we do not formalize this stage. Eligibility is often determined through a mix of predetermined rules and informal negotiations. Generally, students with higher GPAs or notable extracurricular achievements are prioritized for the privilege. However, as high-achieving students are more likely to gain admission to top high schools without the privilege, administrators may persuade them to relinquish their claims, as suggested by our empirical analysis (see Section 6.3).

Admission Stage:

The matching algorithm in this stage is the DA mechanism. Recall that, $p_h^n = \max\{0, q_h - \sum_{m \in M} p_h^m\}$ is the undesignated capacity of high school $h \in H$.[15] We calculate the outcome through the following steps:

- **Step 1:** *Each student applies to her first choice. Each high school h considers its normal (i.e., non-eligible) applicants. If the number of normal applicants is less than p_h^n or $p_h^n = 0$, set $c_h = 0$. Otherwise, let c_h be the p_h^n-th highest-scoring normal applicant's raw score. For each eligible applicant $i \in E_h$ such that $\pi(i) + \beta \geq c_h$, set her admission score as $\alpha(i) = \pi(i) + \beta$. For other applicants, set their admission score equal to the raw score. Each high school h tentatively holds the top q_h applicants with the highest admission scores, and rejects the rest.*

[13]For brevity, we skip a detailed description of the Chinese parallel mechanisms. These mechanisms are hybrids between the immediate acceptance and DA mechanisms. As such, they might fail stability and strategy-proofness. See Chen and Kesten (2017) for their incentive, stability, and welfare properties.

[14]An eligible student has the potential (but not the guarantee) to receive the privilege. In China, these students are also referred as *ZBS* or *privileged* students.

[15]In practice, every high school has undesignated capacity under the CES mechanism.

In general, for $k > 1$

- **Step $k > 1$:** *Each student applies to her most preferred choice which has not rejected her yet. Each high school h considers its normal (i.e., non-eligible) applicants. If the number of normal applicants is less than p_h^n or $p_h^n = 0$, set $c_h = 0$. Otherwise, let c_h be the p_h^n-th highest-scoring normal applicant's raw score. For each eligible applicant $i \in E_h$ such that $\pi(i) + \beta \geq c_h$, set her admission score as $\alpha(i) = \pi(i) + \beta$. For other applicants, set their admission score equal to the raw score. Each high school h tentatively holds the top q_h applicants with the highest admission scores, and rejects the rest.*

  The algorithm terminates in Step K in which no student is rejected. All tentative assignments at this step are final. The admission scores of the assigned students are the scores calculated in this termination step. The admission scores of the unassigned students are their raw scores.

The CES mechanisms have several deficiencies. We have previously discussed the public concerns raised due to the non-transparent early selection stage. Additionally, these mechanisms encounter two major incentive issues: (1) the use of Chinese parallel mechanisms, other than DA, which are nonstrategy-proof and compel students to carefully strategize their school preferences for each round; and (2) the requirement in some cases for eligible students to rank the corresponding high school as their first choice to exercise the privilege. Both practices also undermine fairness. It is tempting to imagine that switching to the DA mechanism, where eligible students retain their privileges for all choices, could potentially solve these issues. This is indeed the version of the CES mechanism described above. However, as Example 1 illustrates, such a switch does not completely resolve fairness and incentive problems.

Example 1. *Let $H = \{h_1, h_2\}$, $M = \{m_1, m_2\}$, $I^{m_1} = \{i_1, i_2\}$ and $I^{m_2} = \{i_3, i_4, i_5, i_6\}$. The school capacities are $q_{h_1} = 3$ and $q_{h_2} = 1$, with privilege capacities $p_{h_1}^{m_1} = 2$ and $p_{h_1}^{m_2} = p_{h_2}^{m_1} = p_{h_2}^{m_2} = 0$. Suppose $\beta = 10$, and the students' raw scores are: $\pi(i_1) = 73$, $\pi(i_2) = 75$, $\pi(i_3) = 82$, $\pi(i_4) = 81$, $\pi(i_5) = 86$, and $\pi(i_6) = 87$. Let $E_{h_1}^{m_1} = I^{m_1} = \{i_1, i_2\}$.*

The preferences of students are given as:

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	P_{i_6}
h_1	h_1	h_1	h_1	h_2	h_2
$\emptyset$	h_2	h_2	h_2	h_1	h_1
	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

We run the CES mechanism as described above, using the DA algorithm in the admission stage without requiring students to rank a school as their top choice to exercise their privileges.

In the admission stage, in Step 1, students i_1, i_2, i_3, i_4 apply to h_1, while i_5 and i_6 apply to h_2. For h_1, set $c_{h_1} = 82$. Since the raw scores of i_1 and i_2, when added with bonus, exceed the cutoff of 82, they receive bonus scores. Then, based on admission scores, i_1, i_2, and i_3 are tentatively held, while i_4 is rejected by h_1. For h_2, i_6 is tentatively held, and i_5 is rejected.

In Step 2, students i_1, i_2, i_3, i_5 apply to h_1, while i_4 and i_6 apply to h_2. For h_1, the cutoff is set at $c_{h_1} = 86$. Since the raw scores of i_1 and i_2, even added with bonus, do not exceed 86, they do not receive bonus scores. Then, based on the admission scores, i_2, i_3, and i_5 are tentatively held, and i_1 is rejected by h_1. For h_2, i_6 is tentatively held and i_4 is rejected.

The algorithm terminates in Step 3, i_1 and i_4 apply to $\emptyset$, while all other students apply to the schools where they were tentatively held in Step 2.

Finally, CES selects the assignment (μ, α) such that:

$$\mu(i_1) = \emptyset,\ \mu(i_2) = h_1,\ \mu(i_3) = h_1\ \mu(i_4) = \emptyset,\ \mu(i_5) = h_1,\ \mu(i_6) = h_2$$

$$\alpha(i_1) = 73,\ \alpha(i_2) = 75,\ \alpha(i_3) = 82,\ \alpha(i_4) = 81,\ \alpha(i_5) = 86,\ \alpha(i_6) = 87.$$

Assignment (μ, α) is not fair because i_4 prefers h_1 to her assignment, $\emptyset$, and she has a higher raw score than i_2's admission score. Additionally, i_4 could have been assigned to h_1 by misreporting her preferences as $h_2 - h_1 - \emptyset$.

In addition to the persistent issues with CES in Example 1, the challenge that lies in selecting the "right" set of eligible students often leads to wasteful usage of privileges. We illustrate this problem in Example A.1 in Appendix A.

3.2 The Class of Chinese Automated Selection Mechanisms

A second class of mechanisms eliminates the early selection stage and instead determines privilege recipients from each middle school automatically during the matching stage. These are referred to as the class of Chinese Automated Selection (CAS) mechanisms.

There are two types of CAS mechanisms. The first type uses the same admission stage as a CES mechanism but considers all students from a middle school m eligible for the privilege at high school h when $p_h^m > 0$. Namely, it sets $E_h^m = I^m$ for all $m \in M$ and $h \in H$ where $p_h^m > 0$, awards bonuses to at most p_h^m students, and follows the CES admission process. We refer to this mechanism as a CAS-1 mechanism. By eliminating the early selection stage, CAS-1 mechanisms overcome the nontransparency issue in CES. However, they fail to satisfy fairness and strategy-proofness.[16] Moreover, since all students from middle schools with privilege capacity are de facto privilege-eligible at the corresponding high schools, the admission round automatically favors higher-scoring students for privileges. This makes these mechanisms prone to the inefficient use of privileges, which we will explore further with the second type of CAS mechanisms.

Under the second type of CAS mechanisms, all students from middle school m are eligible for privilege at high school h when $p_h^m > 0$.[17] Whether an eligible student i receives a bonus at high school h depends on her relative ranking among the eligible applicants from her middle school. Specifically, the top p_h^m applicants from m automatically have the bonus score added to their raw scores. More precisely, the CAS-2 mechanism works as follows:[18]

- **Step 1:** *Each student applies to her first choice. For each high school h and middle school m, any applicant $i \in I^m$ ranked among the top p_h^m from m receives an additional β privilege bonus points, and set her admission score $\alpha(i)$ to $\pi(i) + \beta$. For other applicants, set their admission score equal to the raw score. Each high school h tentatively holds the top q_h applicants with the highest admission scores, and rejects the rest.*

  In general, for $k > 1$

[16] In particular, CAS-1 and CES mechanisms work exactly in the same way when applied to Example 1, illustrating these failures.

[17] In some versions, eligibility also requires ranking the relevant school as first choice.

[18] As with CES, CAS-2 is based on the DA mechanism, and privileges are not limited to first-choice schools.

- **Step $k > 1$:** *Each student applies to her most preferred choice which has not rejected her yet. For each high school h and middle school m, any applicant $i \in I^m$ ranked among the top p_h^m from m receives an additional β privilege bonus points, and set her admission score $\alpha(i)$ to $\pi(i) + \beta$. For any other applicants, the admission score is set equal to their raw score. Each high school h tentatively holds the top q_h applicants with the highest admission scores, and rejects the rest.*

  The algorithm terminates when no student is rejected. All tentative assignments at the termination step become final. The admission scores of assigned students are those calculated in the termination step, while unassigned students retain their raw scores as admission scores.

CAS-2, while innovative, still leads to privilege misallocation. High-scoring students who don't need bonuses may use up a middle school's privilege capacity, depriving marginal students of the opportunity to gain admission to better schools. This issue, common to both types of CAS mechanisms, is illustrated in Example 2.

Example 2. *Let $H = \{h_1, h_2\}$, $M = \{m_1, m_2\}$, $I^{m_1} = \{i_1, i_2\}$ and $I^{m_2} = \{i_3\}$. The school capacities are $q_{h_1} = 2$ and $q_{h_2} = 1$, with privilege capacities $p_{h_1}^{m_1} = 1$ and $p_{h_1}^{m_2} = p_{h_2}^{m_1} = p_{h_2}^{m_2} = 0$. Suppose, and students' raw scores are: $\pi(i_1) = 86$, $\pi(i_2) = 75$, and $\pi(i_3) = 82$.*

The preferences of students are given as:

P_{i_1}	P_{i_2}	P_{i_3}
h_1	h_1	h_1
h_2	h_2	h_2
$\emptyset$	$\emptyset$	$\emptyset$

We first consider CAS-1, which follows the CES admission stage to selects its outcome. When all students apply to h_1, i_3 is selected for the undesignated capacity, and i_1 receives a bonus score. Hence, i_1 and i_3 are tentatively held by h_1, and i_2 is rejected. In the next step, i_2 applies to h_2 and is tentatively held.

Next, we consider CAS-2. When all students apply to h_1, i_1 receives a bonus score. Thus, i_1 and i_3 are tentatively held by h_1, while i_2 is rejected. In the next step, i_2 applies to h_2 and is tentatively held.

Then, both CAS-1 and CAS-2 select the assignment (μ, α) *such that:*

$$\mu(i_1) = h_1,\ \mu(i_2) = h_2,\ \mu(i_3) = h_1$$
$$\alpha(i_1) = 96,\ \alpha(i_2) = 75,\ \alpha(i_3) = 82.$$

Although assignment (μ, α) *is fair, the privilege capacity of middle school* m_1 *at* h_1 *is "wasted" on* i_1*, who does not need it. Indeed, a fair alternative could achieve better representation of middle school* m_1 *at* h_1*:*

$$\mu'(i_1) = h_1,\ \mu'(i_2) = h_1,\ \mu'(i_3) = h_2$$
$$\alpha'(i_1) = 86,\ \alpha'(i_2) = 85,\ \alpha'(i_3) = 82.$$

4 A Mechanism Design for Privilege Systems

We consider a mechanism satisfying strategy-proofness, fairness, non-wastefulness, and efficient allocation of privileges as a *first-best.* In Section 3, Examples 1 and 2 demonstrate that the current mechanisms fail to meet these criteria. This result is summarized in Proposition 1. All proofs are provided in Appendix B.

Proposition 1. *(a) The class of CES and CAS-1 mechanisms violate fairness, strategy-proofness, or efficient allocation of privileges.*

(b) The class of CAS-2 mechanisms violate efficient allocation of privileges.

Additionally, the CES mechanisms are highly nontransparent, adding further complications and practical challenges (see Example A.1 in Appendix A). Therefore, our main goal is to explore an alternative mechanism that satisfies all the desired properties should such a mechanism exist.

Our design relies on a two-step approach: first, identifying the appropriate "student selection procedure" for high schools, and then embedding it into the "right" mechanism. In CES and CAS mechanisms, a primary difference lies in how high schools (tentatively) select a subset of applicants during each step of the DA mechanism. In the literature, such a selection procedure is commonly known as a *choice rule*, which determines which students are retained and their admission scores for a given set of applicants. The inclusion of the choice rule into our framework also enables us to define the property of stability: An assignment is **stable** if it is individually rational and no school prefers to select a student, who also prefers that school over their current match, from the set of its current assignees and the said student.[19] We begin by constructing a suitable choice rule to guarantee fairness and efficient privilege allocation. To guarantee stability, strategy-proofness, and non-wastefulness in the resulting DA mechanism, this construction must also meet a set of necessary conditions, which we define next.

4.1 Choice Rules in Privilege Systems and Stability

Given a set of applicants $J \subseteq I$, let $C_h^s(J;\beta,E_h) \subseteq J$ and $C_h^\alpha(J;\beta,E_h) \in \mathbb{R}^{|J|}$, respectively, represent the chosen set of applicants and the admission score profile under the choice rule C_h, where β is the bonus score and E_h is the set of eligible students. Let $C_h(J;\beta,E_h) = (C_h^s(J;\beta,E_h), C_h^\alpha(J;\beta,E_h))$. The admission score of student $i \in J$, denoted $C_h^\alpha(J;\beta,E_h)[i]$, is such that $C_h^\alpha(J;\beta,E_h)[i] \in \{\pi(i), \pi(i)+\beta\}$ for all $i \in J$ and $|\{i \in J \cap I^m \ : \ C_h^\alpha(J;\beta,E_h)[i] = \pi(i)+\beta\}| \leq p_h^m$ for all $i \in J$ and $m \in M$. This formulation assigns admission scores to all applicants, regardless of whether they are selected by high school h.

First, we define the key properties of suitable choice rules to meet our objectives. These properties are typically specified only for the chosen set of applicants; we adopt them to our setting.

A choice rule C_h satisfies **law of aggregate demand** (LAD) if $|C_h^s(J;\beta,E_h)| \leq |C_h^s(K;\beta,E_h)|$ for each subset of students $J \subset I$ and $K \subseteq I$ such that $J \subset K$.

A choice rule C_h is q_h**-acceptant** if $|C_h^s(J;\beta,E_h)| = min\{q_h, |J|\}$ for each subset of students $J \subseteq I$. Note that q_h-acceptance implies the law of aggregate demand.

A choice rule C_h is **substitutable** if $i \notin C_h^s(J \cup \{i\};\beta,E_h)$ implies $i \notin C_h^s(J \cup \{i,j\};\beta,E_h)$ for any subset of students $J \subset I$ and any pair of students $i,j \in I \setminus J$.

Next, we define two key properties to ensure fairness in the student selection procedure and efficient use of privileges. These properties apply to both the chosen applicants and

[19] We formalize the choice rule and stability in the following subsection.

their admission score profiles.

A choice rule C_h is **fair**[20] if for any $J \subseteq I$ and $i \in J \setminus C_h^s(J;\beta,E_h)$ the following holds

(a) $C_h^\alpha(J;\beta,E_h)[i] < C_h^\alpha(J;\beta,E_h)[j]$ for all $j \in C_h^s(J;\beta,E_h)$; and

(b) if $i \in E_h$ and there exists $j \in C_h^s(J;\beta,E_h)$ such that $\pi(i)+\beta > C_h^\alpha(J;\beta,E_h)[j]$, then $|\{i' \in J \cap E_h^{\tau(i)} \ : \ \pi(i') > \pi(i) \text{ and } C_h^\alpha(J;\beta,E_h)[i'] = \pi(i')+\beta\}| = p_h^m$.

Condition (a) says that a student not selected by a school has a lower admission score than any selected student. Condition (b) says that if an eligible student is not selected but would have been with a bonus, then the privilege capacity of her middle school must be fully exhausted by higher-scoring students from the same middle school.

Next, we define our property that ensures optimal usage of privileges for choice rules, capturing the idea that bonuses are never allocated to students who do not need them, while another student from the same middle school could achieve a better assignment with a bonus.

A choice rule C_h **efficiently allocates privileges** if for any $J \subseteq I$, there does not exist a student pair $i, j \in J \cap E_h$ such that

(a) $\tau(i) = \tau(j)$, $i \in C_h^s(J;\beta,E_h)$ and $j \notin C_h^s(J;\beta,E_h)$;

(b) $C_h^\alpha(J;\beta,E_h)[i] = \pi(i)+\beta$ and $C_h^\alpha(J;\beta,E_h)[j] = \pi(j)$; and

(c) $\pi(i) > C_h^\alpha(J;\beta,E_h)[k]$ and $\pi(j)+\beta > C_h^\alpha(J;\beta,E_h)[k]$ for some $k \in C_h^s(J;\beta,E_h)\setminus\{i,j\}$.

A choice rule that allocates privileges efficiently avoids the following situation:

- Among two eligible students i and j from the same middle school, only i receives the bonus and is selected by the choice rule while j, without the bonus, is not selected (conditions a and b), and
- The student receiving the bonus, i, has a raw score higher than the admission score of at least one selected student, while the student not receiving the bonus, j, would have a higher admission score than at least one selected student if given the bonus.

We begin with a general class of choice rules that encompass many commonly used in practice.[21] A typical choice rule in this class involves two main steps: (1) setting the ad-

[20] A fair choice rule is the natural analog of a fair assignment in this setting. Indeed, one can consider the outcome of a choice rule as the assignment in a problem composed of only one school.

[21] Examples of these choice rules are provided in Appendix C.

mission score profile by determining which applicants receive bonus points, and (2) selecting a subset of applicants based on the admission scores. We refer to these as **2-step choice rules**.

2-step Choice Rules:

Given a subset of students $J \subseteq I$ the admission score profile, $C_h^\alpha(J;\beta,E_h)$, and the chosen set of students, $C_h^s(J;\beta,E_h)$, are determined following the two steps:

Step 1: Admission Score Profile Setting. Consider students in J and determine which eligible students, i.e., $J \cap E_h$, receive bonus scores accordingly, by respecting the following conditions:[22]

(a) $C_h^\alpha(J;\beta,E_h)[i] \in \{\pi(i), \pi(i)+\beta\}$ for all $i \in J$,

(b) $\{i \in J \ : \ C_h^\alpha(J;\beta,E_h)[i] = \pi(i)+\beta\} \subseteq E_h$, and

(c) $|\{i \in J \ : \ C_h^\alpha(J;\beta,E_h)[i] = \pi(i)+\beta\} \cap I^m| \leq p_h^m$ for all $m \in M$.

Step 2: Applicant Selection. Once the subset of applicants receiving bonus scores is determined, select the highest-scoring q_h students under $(C_h^\alpha(J;\beta,E_h)[i])_{i\in J}$ and add them to $C_h^s(J;\beta,E_h)$.

By the construction of Step 2, it is evident that, either all applicants are selected or all seats are filled regardless of the outcome of Step 1. Thus, all choice rules that can be represented as a 2-step choice rule, including those used in practice, satisfy q_h-acceptance and, consequently, the law of aggregate demand. However, none of the choice rules associated with existing mechanisms meet the remaining properties. Table 2 summarizes the performance of these choice rules.[23]

We end this subsection with the definition of stability. Given a list of choice rules for all high schools, $(C_h)_{h\in H}$, an assignment (μ,α) is **stable** if it is it is individually rational and

(a) $\mu^{-1}(h) = C_h^s(\mu^{-1}(h);\beta,E_h)$ for all $h \in H$, and

[22]The three conditions follow from our model. Conditions (a) and (c) follow from the definition of a choice rule whereas (b) comes from the feasibility of an assignment.

[23]Proposition 5 in Appendix C formally states the results shown in this table.

Table 2: Performances of Choice Rules Associated with the Mechanisms in Practice

	Substituability	q_h-acceptant	Fairness	Efficient Allocation of Privilege
CES	×	✓	×	×
$CAS1$	×	✓	×	×
$CAS2$	✓	✓	✓	×

(b) there does not exist a high school-student pair (h, i) such that $h \; P_i \; \mu(i)$ and $i \in C_h^s(\mu^{-1}(h) \cup \{i\}; \beta, E_h)$.

It is important to note that the stability of an assignment depends on the choice rules, whereas its fairness does not. Therefore, stability and fairness are distinct properties that do not necessarily imply one another. A natural question arises: does a fair choice rule used with the DA mechanism guarantee a stable matching? The answer is negative. Fairness does not imply substitutability or the LAD property for a choice rule, both of which are essential for achieving stable matching. This point is illustrated in Example 3.

Example 3. *Let $H = \{h\}$, $M = \{m\}$, $I = I^m = \{i_1, i_2\}$, $q_h = 1$, $\beta = 0$, and $p_h^m = 0$. Both students consider h acceptable. Consider a choice rule C_h such that $C_h^s(\{i_1\}; \beta, E_h) = \{i_1\}$, $C_h^s(\{i_2\}; \beta, E_h) = \{i_2\}$, $C_h^s(I; \beta, E_h) = \emptyset$, and C_h selects $\emptyset$ for any other instance. This choice rule is fair.*

Applying the DA mechanism equipped with this choice rule yields the outome: $\mu(i_1) = \mu(i_2) = \emptyset$. Here, μ is not stable given choice rule C_h.

Since substitutability and LAD are independent of scores, while fairness is score-dependent, a stable outcome can be achieved by using the DA mechanism with substitutable choice rules that satisfy LAD, even if they fail fairness.

4.2 Privilege Choice Rule

We design a new 2-step choice rule that aims to ensure fairness and efficient allocation of privilege capacities. To achieve stability and deter strategic behavior, it must also satisfy q_h-acceptance and substitutability.

Intuitively, the first step of our choice rule determines which eligible students receive privileges. All privilege assignments are *tentative* until the process converges. Initially, the top q_h students are selected based on raw scores, and the rest are rejected. In each subsequent step, students who has been rejected at *any* previous step are reconsidered for privileges. For each $m \in M$, privileges are tentatively assigned to the highest-scoring p_h^m students in this group. As privileges are reassigned and scores updated (e.g., a privilege granted at one step may be revoked later), a student's score can fluctuate. The top q_h students are then reselected based on updated scores, potentially leading to new rejections. The process continues until no further rejections occur, where tentative privileges become permanent. In Step 2, the highest-scoring q_h students are finalized based on updated scores.

Now, we are ready to formally present our choice rule.

Privilege Choice Rule (PCR):

Let $PCR_h = (PCR_h^s, PCR_h^\alpha)$ denote the choice rule for high school h. Given a set of applicants $\bar{I}$, it works as follows:

Step 1: Admission Score Profile Setting.

Step 1.0**:** Let $\bar{I}^m = \bar{I} \cap I^m$ for all $m \in M$. If $|\bar{I}| \leq q_h$, then set $PCR_h^\alpha(\bar{I}; \beta, E_h)[i] = \pi(i)$ for all $i \in \bar{I}$ and continue with Step 2. Otherwise, set $\pi_1^b(i) = \pi(i)$ for all $i \in \bar{I}$, $A^1 = B^1 = \emptyset$, and continue with Step 1.1.[24]

Step 1.1**:** Add q_h students one by one to A^1 by considering their scores under π_1^b. Let $R^1 = \bar{I} \setminus A^1$ and $T^1 = R^1$.[25] For each $i \in \bar{I} \setminus (T^1 \cup E_h)$, set $\pi_2^b(i) = \pi(i)$. For each $i \in T^1 \cap E_h$, if $|\{j \in \bar{I}^{\tau(i)} \cap T^1 \cap E_h^{\tau(i)} \ : \ \pi(j) > \pi(i)\}| < p_h^{\tau(i)}$, then set $\pi_2^b(i) = \pi(i) + \beta$. Otherwise, set $\pi_2^b(i) = \pi(i)$. Set $A^2 = \emptyset$ and $B^2 = \{i \in \bar{I} \ : \pi_2^b(i) = \pi(i) + \beta\}$.[26]

In general, for $k > 1$

Step $1.k$**:** Add q_h students one by one to A^k by considering their scores under π_k^b. Let $R^k = \bar{I} \setminus A^k$ and $T^k = T^{k-1} \cup R^k = R^1 \cup \ldots \cup R^k$. We have two cases:

[24] As it will be apparent below, in Step $1.k$, A^k is the q_h highest-scoring applicants based on the updated scores in the previous step, and B^k is the set of students to whom bonus points are added in Step $1.k-1$.

[25] Here, R^k denotes the set of rejected students in Step $1.k$. Whereas, T^k denotes the set of rejected students in Steps $1.1 - 1.k$.

[26] Notice that, we provide bonus scores to the top p_h^m scoring students from each m who have been rejected so far.

- $T^k = T^{k-1}$. We set $PCR_h^\alpha(\bar{I};\beta,E_h)[i] = \pi(i) + \beta$ for all $i \in B^k$. We set $PCR_h^\alpha(\bar{I};\beta,E_h)[i] = \pi(i)$ for all $i \in \bar{I} \setminus B^k$. We continue with Step 2.

- $T^k \neq T^{k-1}$. For each $i \in \bar{I} \setminus (T^k \cup E_h)$, set $\pi^b_{k+1}(i) = \pi(i)$. For each $i \in T^k \cap E_h$, if $|\{j \in \bar{I}^{\tau(i)} \cap T^k \cap E_h^{\tau(i)} \; : \; \pi(j) > \pi(i)\}| < p_h^{\tau(i)}$, then set $\pi^b_{k+1}(i) = \pi(i) + \beta$. Otherwise, set $\pi^b_{k+1}(i) = \pi(i)$. Set $A^{k+1} = \emptyset$ and $B^{k+1} = \{i \in \bar{I} \; : \; \pi^b_{k+1}(i) = \pi(i) + \beta\}$. We continue with Step 1.$k+1$.

Step 2: Applicant Selection

Select the highest q_h scoring students under $(PCR_h^\alpha(\bar{I};\beta,E_h)[i])_{i\in\bar{I}}$ and add them to $PCR_h^s(\bar{I};\beta,E_h)$.

Example A.2 in Appendix A illustrates how PCR works.

Since the number of students is finite, Step 1 (admission score profile setting) is guaranteed to terminate in a finite number of steps. Let Step 1.K denote the termination step. From the definitions of Step 1.K and Step 2 of the choice rule, it follows that $PCR_h^s(\bar{I};\beta,E_h) = A^K$. With this observation, we focus on the set A^K to show that PCR satisfies our desiderata.

We next show that PCR satisfies all the attractive properties discussed in Section 4.1. Moreover, we show that it is essentially the unique choice rule that satisfies a subset of these properties.

Theorem 1. *Fix h, β, and E_h.*

(a) PCR_h satisfies the q_h-acceptance (and therefore LAD), substitutability, fairness, and efficient allocation of privilege capacities.

(b) Suppose D_h is a q_h-acceptant and fair choice rule that efficiently allocates privilege capacities. Then, for any $\bar{I} \subseteq I$, $D_h^s(\bar{I};\beta,E_h) = PCR_h^s(\bar{I};\beta,E_h)$.

In addition, under PCR_h, if a student can be admitted without a bonus score, then she does not receive a bonus score. In the next result, we highlight an implicit consequence of the efficient allocation of privileges property on the number of selected students eligible for privilege. Before presenting our result, we introduce a weaker fairness property. A choice rule C_h is **weakly fair** if for any $J \subseteq I$ and $i \in J \setminus C_h^s(J;\beta,E_h)$ the following holds

(a) $C_h^\alpha(J;\beta,E_h)[i] < C_h^\alpha(J;\beta,E_h)[j]$ for all $j \in C_h^s(J;\beta,E_h)$; and

(b) $i \in E_h$ implies that $\pi(j) > \pi(i)$ for all $j \in C_h^s(J;\beta,E_h) \cap I^{\tau(i)}$.

Note that fairness implies weak fairness for choice rules. .

Proposition 2. *Fix $\bar{I}$, h, β and E_h. Let $\hat{I} = \{i \in \bar{I}^m \cap E_h \ : \ p_h^m > 0\}$ and D_h be a weakly fair choice rule. If C_h is fair, q_h-acceptant, and satisfies efficient allocation of privileges, then $|C_h^s(\bar{I};\beta,E_h) \cap \hat{I}| \geq |D_h^s(\bar{I};\beta,E_h) \cap \hat{I}|$.*

All choice rules associated with mechanisms used in practice satisfy weak fairness.[27] Therefore, Theorem 1, Proposition 2, and the weak fairness of these choice rules imply that PCR_h selects more privilege-eligible students than other choice rules used in practice.

Next, we examine how changes in the bonus score level affect the set of eligible students chosen under PCR and show that this set monotonically increases with the size of the bonus.

Proposition 3. *Fix $\bar{I}$, h and E_h. Let $\hat{I} = \{i \in \bar{I}^m \cap E_h \ : \ p_h^m > 0\}$ and $\beta > \hat{\beta}$. Then $|PCR_h^s(\bar{I};\beta,E_h) \cap \hat{I}| \geq |PCR_h^s(\bar{I};\hat{\beta},E_h) \cap \hat{I}|$.*

We conclude this section by showing that slot-specific priorities (Kominers and Sönmez, 2016) cannot be used to implement a privelege system without violating fairness. Consider a high school h. Suppose $\sum_{m \in M} p_h^m \leq q_h$. Let S_h denote the set of seats at h such that $|S_h| = q_h$. Then, the slot-specific priorities framework can simply be adopted to our context as follows: for each $m \in M$, we choose p_h^m seats. We construct the priorities of each of these seats by adding β on top of the raw score of each student in $I^m \cap E_h$ and keeping the raw scores (and hence priorities) of all other students the same. Let $\triangleright_h$ be the processing order over S_h. Then, for a given set of students, we consider seats in S_h one-by-one following $\triangleright_h$ and fill the seats according to the constructed slot-specific priorities. If a student i is selected for some seat s, then her admission score is equal to her score calculated for seat s. Otherwise, the admission scores of unselected students are set to their raw scores. We call this choice rule as **score-elevated choice rule** and denote it with SEC_h (Sönmez and Ünver, 2022). The following example illustrates that SEC_h (with a given processing order) violates fairness.

[27]See Appendix C for definitions of these choice rules.

Example 4. *Let* $H = \{h\}$*,* $M = \{m_1, m_2, m_3\}$*,* $I^{m_1} = \{i_1\}$*,* $I^{m_2} = \{i_2\}$*,* $I^{m_3} = \{i_3\}$*,* $q_h = 2$ *and* $p_h^{m_1} = p_h^{m_2} = 1$*. The raw scores of the students are* $\pi(i_1) = 60$*,* $\pi(i_2) = 50$*, and* $\pi(i_3) = 100$*. Let* $\beta = 20$*.*

Now consider the case in which the first seat gives privilege to m_1 *and the second seat gives privilege to* m_2*. Then, the priority orders for the first and second seats are* $i_3 - i_1 - i_2$ *and* $i_3 - i_2 - i_1$*, respectively.*

The score-elevated choice rule selects i_2 *and* i_3 *with the admission scores* 70 *and* 100*, respectively. That is,* $SEC_h^s(I; \beta, E_h) = \{i_2, i_3\}$*,* $SEC_h^\alpha(I; \beta, E_h)[i_1] = 60$*,* $SEC_h^\alpha(I; \beta, E_h)[i_2] = 70$*, and* $SEC_h^\alpha(I; \beta, E_h)[i_3] = 100$*. However, this outcome is not fair:* m_1*'s privilege capacity is not binding and* $\pi(i_1) + \beta > SEC_h^\alpha(I; \beta, E_h)[i_2]$*.*[28]

Although SEC_h fails to satisfy fairness; it is weakly fair. Hence, Theorem 1 and Proposition 2 imply that PCR_h selects weakly more privilege eligible students than SEC_h.

Next, we focus on a restricted case in which there is only one middle school with privilege capacity. Then, PCR_h and SEC_h select the same set of students when open seats are processed first under SEC_h.

Proposition 4. *Fix* $\bar{I}$*,* h*,* β*, and* E_h*. Suppose* $p_h^m > 0$ *for some* $m \in M$*,* $p_h^{m'} = 0$ *for all* $m' \neq m$ *and open seats are processed first under* SEC_h*. Then,* $PCR_h^s(\bar{I}; \beta, E_h) = SEC_h^s(\bar{I}; \beta, E_h)$*.*

4.3 Proposed Solution

When stability and strategy-proofness are desired in a matching market, the DA mechanism is the go-to solution. If the choice rules embedded in the DA mechanism satisfy substitutability and the LAD property, no student can benefit from misreporting their true preferences, and the selected outcome is stable. Since PCR satisfies substitutability and LAD (Theorem 1), the DA mechanism equipped with the PCR choice rule ensures stability and strategy-proofness. Furthermore, it satisfies fairness and efficient allocation of privileges, as defined in Section 2. It is worth noting that the standard DA mechanism only considers current applicants in each step and produces a matching. To align with our model, we adopt the

[28] One could argue that the fairness violation in this example could be avoided if the processing order of the seats were reversed. In this case, a new problem where the scores of the students were switched would lead to the same conclusion.

mechanism to allow for high schools to consider all applicants who have proposed so far, as in the cumulative offer process (Hatfield and Milgrom, 2005).[29]

We define the DA mechanism with PCR as follows.

DA Mechanism with PCR:

Step 1: Each student i applies to her most preferred school. For each school h, we denote the set of applicants in Step 1 as I_h^1. Each school h tentatively holds applicants in $PCR_h^s(I_h^1; \beta, E_h)$ and rejects the rest of the applicants.

In general:

Step $k > 1$: Each student i applies to her most preferred school that has not rejected her yet. For each school h, we denote the set of applicants in Step k as I_h^k. Each school h tentatively holds applicants in $PCR_h^s(\cup_{k' \leq k} I_h^{k'}; \beta, E_h)$ and rejects the rest of the applicants.

The procedure terminates when no more students are rejected, and each student is assigned to the school, tentatively holding her in the terminal step with the admission score calculated via the PCR. If a student i is not tentatively held by any school, then she is matched to $\emptyset$ with an admission score of $\pi(i)$.

We illustrate how DA-PCR works in Example A.3 in Appendix A. Now, we present our results on DA-PCR. First, we show that it satisfies all desired properties.

Theorem 2. *DA-PCR is fair, individually rational, non-wasteful, stable, strategy-proof, and efficiently allocates privileges.*

In addition to satisfying all the appealing properties, DA-PCR provides welfare gains over any other mechanism that satisfies fairness and efficient allocation of privileges, as shown in the following theorem.

Theorem 3. *Let Φ be a fair mechanism that efficiently allocates privileges. Then DA-PCR (weakly) Pareto dominates Φ.*

[29] This modification does not change the DA mechanism's outcome when the choice rules satisfy substitutability (Hatfield et al., 2020).

5 Local Background and Data Description

5.1 High School Admission in Our Focal City

Our empirical analysis uses data from high school admissions in a large Chinese city.[30] Every March, the City Education Bureau (hereafter referred to as "the Bureau") announces an admissions plan detailing the total and privilege capacities for each school. In mid-May, students submit their ROLs with up to three schools. In mid-June, students take the high school entrance exam.[31] Then, a centralized matching mechanism assigns students to schools.

The admission procedure has two parts. The first part focuses on admission to local public high schools. The Bureau sets and announces a public high school admission threshold (hereafter "the threshold") based on the score distribution and the availability of seats. Only students who score above this threshold, which was 535 in 2014, are eligible for admission to public high schools. The second part of the admission procedure is for other types of high schools, such as vocational and private high schools.

Our study focuses on the first part of the admission procedure for public high schools. Before 2015, the Bureau adopted the CES mechanism, using the Chinese parallel mechanism with a permanency execution vector (2,1). Students who received privileges from high schools need to list those schools as their first choices. They also need to indicate whether they would accept random assignments if rejected by all listed schools in ROLs. Unmatched students open to random assignment are placed in public high schools with available seats,[32] while others explore alternatives, such as the second part of the admission to other schools or joining the workforce. In 2015, the Bureau replaced the CES with the CAS-2 mechanism, maintaining the same underlying Chinese parallel mechanism. All schools use the same strict priority based on exam scores and include a 30-point privilege bonus.

[30]The city's urban population was 3.7 million in 2014.

[31]In 2014, the maximum score that can be received was 665

[32]In 2014, around 7.3% of students are assigned through this channel.

5.2 Data Description

Our data set consists of two parts: administrative data and survey data. The former was collected from the Bureau, and it comprises admission records in 2014. Those records include the students' ROLs, exam scores, final assignments, whether a student was admitted as a normal student or as a privileged student, and each student's middle school and home address. In 2014 administrative records, a total of 14,194 students were included. After excluding unqualified students, the final sample size was 5,254 students.[33]

In early May 2014, we conducted a survey of middle school graduates that asked each student to list five high schools they might attend and to rank them according to her true preferences. Our design aims to compare students' ROLs. Therefore, limiting the selection to five schools is sufficient to cover the required length of the ROL (three schools), while also avoiding an overload of options that could reduce response accuracy. We surveyed 6,980 middle school graduates. After matching the survey data with the final administrative data sample just described above, we were left with 2,611 survey observations for the subsequent analysis. Thus, our survey covers 49.7% of the selected sample. The validity of this survey has been addressed in Wang and Zhou (2024).[34]

5.3 School Characteristics

In the administrative data, all non-public high schools were coded by the Bureau with a single number, so we treated them as a single group without distinguishing among them. Table F.1 in Appendix F summarizes the characteristics of public high schools. There were 13 public high schools and six special classes in 2014,[35] and they charge a flat tuition of 1,600 Yuan ($260) per year for all students. Eight high schools provide middle school-specific privilege capacities. These high schools offer privileges to every middle school, except for one small middle school. On average, the privilege capacity accounts for 61% of a high school's

[33]We excluded students whose exam scores were below the threshold (60.2% of all students), as well as those admitted under separate procedures (2.8%), such as sports or art scholarships.

[34]We provide discussion in Appendix D.

[35]Special classes admit gifted students. They have their own admission quotas and do not allocate privilege capacities to middle schools.

total capacity.[36] Additionally, each middle school, on average, has a privilege capacity that accounts for 14% of its total graduates. The reputation of public high schools is measured by the college admission rates of the schools in 2014, which is the most commonly used measure by Chinese students and parents to gauge school quality (Lai et al. 2009).[37] The average reputation of the schools is 82.4 over 100.[38]

For the convenience of subsequent analysis, we classify high schools into three categories: top high schools, which include two schools with the highest admission cutoffs and notable gaps relative to other schools; leftover schools, which consist of three schools with cutoffs equal to the high school admission threshold, indicating they have more available seats than acceptable applications received (i.e., the number of students listing them in ROLs); and moderate high schools, which encompass the remaining high schools.[39] Middle schools are classified into four groups based on their average graduates' exam scores: top middle schools, consisting of the ten middle schools with the highest average scores; upper median middle schools, which include the next eleven middle schools with average scores lower than the top ten; lower median middle schools, comprising another eleven middle schools with average scores lower than those of the upper median group; and bottom middle schools, which include the eleven middle schools with the lowest average scores.

6 Students' Behaviors under the CES

In this section, we characterize the behavior of students and detect deficiencies under the CES mechanism used in 2014.

6.1 Students' Strategic Behaviors in ROLs

Table F.2 in Appendix F summarizes students' ROLs and their assignments. The majority of both normal and privileged students submitted complete lists with three schools. Among

[36]The distribution of privilege capacity to middle schools mainly depends on the ratio of the high school entrance exam takers in a middle school to all the exam takers. Also, it may depend on factors such as the education and management quality of a middle school.

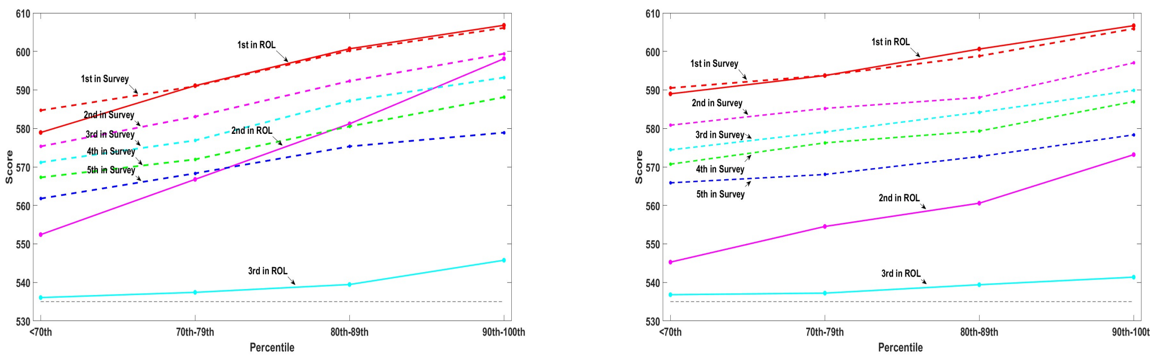
[37]The college admission rate includes the admissions to both four-year colleges (benke) and three-year specialized postsecondary colleges (dazhuan).

[38]To scale the measurement, we multiply the college admission rate by 100.

[39]Note that the demand for top and moderate high schools exceeds their capacities.

privileged students, 90.4% were assigned to their first choice, and only 3.7% of them were rejected by all three choices. In contrast, only 26.4% of normal students secure their first choice, while 17.5% were rejected by all choices they listed.

Figure 1: Average Admission Cutoffs of Schools in the Survey and ROLs



(a) Normal Students (b) Privileged Students

Notes: These figures show the average admission cutoffs for schools chosen by students in the survey and ROLs. The y-axis shows cutoff scores, and the x-axis groups students by their score percentiles: above the 90th, 80th–90th, 70th–80th, and below the 70th. Dashed lines represent survey responses, solid lines represent ROLs.

To analyze student strategies in their ROLs, we compare students' survey responses with their ROLs in Figure 1, which shows average admission cutoffs for schools selected by students, grouped into four categories based on their score percentiles.

For normal students (Figure 1a), according to survey responses, the average cutoff gap between consecutive choices within each student percentile group is around six points. Additionally, the average cutoff for the same choices decreases as exam scores decrease, with this trend of declining cutoffs for each additional choice, being consistent across all groups. In the ROLs, the average cutoffs for students' first-choice schools closely align with those in the survey. As exam scores decrease, the gap between the first and second choices widens considerably, with the average cutoffs for third choices hovering around the threshold across all groups. The larger gaps between consecutive choices in the ROLs compared to the survey suggest students' strategic behaviors to improve their admission chances.

For privileged students, their choices in the survey and ROLs follow a similar pattern to those normal students, with one notable difference shown in Figure 1b. The gap between their first and second choices in the ROLs (more than 30 points) is significantly larger than for normal students (7 to 25 points). This reflects the tendency of privileged students to maintain a larger gap between their choices because their 30-point bonus is added only to their first choice. All these comparisions show that both normal and privileged students acted strategically when submitted their ROLs.

6.2 Who Become the privileged students?

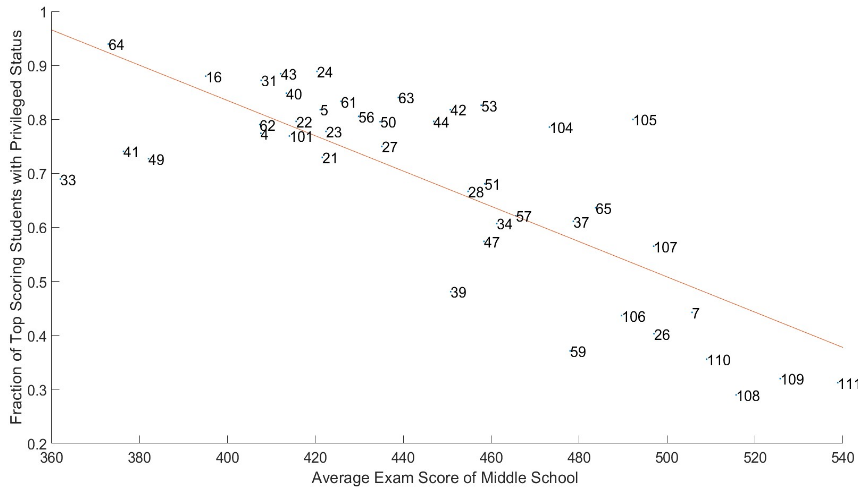
Under the CES mechanism used in 2014, middle schools generally use a merit-based criterion to rank students during the early selection stage, though specific criteria vary across schools. While we cannot directly observe students' rankings and decisions during this stage, we use their exam scores as a proxy to analyze their choices of becoming privileged students. We consider the students with highest exam scores based on each middle school's privilege capacity and calculate the proportion who chose privilege status. Figure 2 shows this proportion decreases with the middle school's average exam score. For instance, at the best middle school (School 111) with a privilege capacity of 64, only 31% of its top 64 students become privileged students. Conversely, at a lower-ranked school (School 62) with a privilege capacity of 62, 79% of its top 62 students accept the privilege bonuses.

The variation in privileged student choices across middle schools likely stems from differences in student quality. Under the version of CES used in 2014, a privileged student must rank that high school as their top choice in the ROL. Therefore, top students at good middle schools may decline privilege status if their preferred high schools' privilege slots are filled early. At School 111, 140 out of 478 students score above 598, and the exam scores of its privileged students range from 590 to 640 (Figure F.1a in Appendix F). In contrast, at School 62, where only 9 out of 412 graduates score above 598, nearly all students scoring over 580 opt for privilege status.

6.3 Potential Strategic Coordination

Another reason for opting out of privilege status could be strategic coordination among students from the same middle school, potentially orchestrated by the schools. A middle school might boost its representation in prestigious high schools by allowing high-scoring

Figure 2: Fraction of Top Students Attain Privilege Status



Notes: This figure shows the percentage of top-ranked students who become privileged students in middle schools. Each data point corresponds to one middle school. The x-axis is the average exam score of all graduates from each middle school, while the y-axis is the proportion of top-ranked students who became privileged students.

students to enter as normal students while allocating privilege bonuses to students with scores on marginal. Although we cannot definitively confirm coordination due to the lack of early selection information, we use students' ROLs and exam scores as proxies to approximate the likelihood of such behavior.

To detect students potentially involved in coordination, we apply the following criteria: First, they are normal students but assigned to their first-choice school in the ROL. Second, their exam scores exceed the school admission cutoffs by a significant margin (13 points, i.e., 2% of the full score). Third, within their middle schools, their scores are notably higher (by 13 points) than the lowest scores among privileged students receiving privileges at their assigned high schools. The first criterion ensures that these students are placed in their most preferred attainable school. The second criterion reduces the uncertainty from entrance exam results, as students confident in securing admission to their preferred school are more likely to participate in coordination. The third criterion indicates that these students' academic performances surpass some privileged students, suggesting they might have the opportunity to become privileged students at their assigned schools.

Table 3 shows that 3.9% of students from top middle schools may pontential engage in coordination, primarily (2.1%) targeting the top two high schools. The likelihood decreases to 1.8% for upper median middle schools, 0.5% for lower median middle schools, and is nonexistent for bottom middle schools. This trend suggests that top middle schools, with more high-scoring students, are more likely to coordinate to enhance their presence in prestigious high schools.

Table 3: Potential Coordination (%)

	Total	Top high schools	Moderate high schools
	(1)	(2)	(3)
Top middle schools	3.9	2.1	1.8
Upper median middle schools	1.8	0.7	1.1
Lower median middle schools	0.5	0.0	0.5
Bottom middle schools	0.0	0.0	0.0

Notes: This table shows the percentage of students who may have forfeited their privilege bonus by coordinating with peers during the early selection stage. Column 2 indicates the percentage of students who forfeited their privilege bonus at top high schools, while Column 3 shows the same for moderate high schools.

6.4 Unfair Privilege Assignment

Our theoretical analysis indicates that the CES mechanism may result in unfair matching outcomes. Since our study focuses on affirmative action, we limit our analysis to unfair outcomes related to the allocation of privilege bonuses.

We detect an unfair student-school pair (i, h) as follows: Student i from middle school m_i is assigned to high school h_i but ranks high school h over h_i and does not receive a privilege bonus from h. If i received the bonus, her score would exceed h's cutoff, allowing her to attend. Meanwhile, another student j from m_i, with a lower raw score than i, receives the privilege bonus from h. After comparing matching outcomes and ROLs, we detect 798 unfair student-school pairs involving 747 students. Our findings likely underestimates the true number of unfair matches, as the ROLs include only 3 choices.

7 Empirical Model and Preference Estimate

Our empirical estimation of students' preferences is a simplified version of the approach described in Wang and Zhou (2024). We briefly outline the process in this section.

Student i's (indirect) utility from being assigned to school h is

$$u_{i,h} = \sum_{l} \beta^l y_h^l + \sum_{w} \beta^w x_i^w y_h^w + \beta^D d_{ih} + \varepsilon_{ih}. \quad (1)$$

Here $\{y_h\}$ is a vector of school h's observed characteristics; $\{x_i\}$ is a vector of student i's observed characteristics; d_{ih} is the home–school distance, and we normalize the coefficient of d_{ih} to be -1 for female students. ε_{ih} is i's idiosyncratic errors for school h. Following Abdulkadiroğlu et al. (2017), we do not explicitly model an outside option.[40] We assume that ε_{ih} is independent of the explanatory variables and follows a type I extreme value distribution with cumulative distribution function F_ε.[41]

We use survey data to estimate student preferences without considering students' strategic behavior in their ROLs. Since each surveyed student ranked five schools they believed they could attend, these survey responses reflect students' true relative preferences. We focus on the ranks of the listed schools in the survey, ignoring the unlisted schools, and use the rank-ordered logit model (Beggs et al. 1981) to estimate coefficients. A detailed discussion of the validity of using this survey for estimation is provided in Appendix E.

7.1 Estimation Results and Model Fit

Table 4 presents the estimated results. We focus on Columns 5 and 6, which correspond to the full model with school fixed effects for normal and privileged students, respectively.

Rows 2-4 of Columns 5 and 6 report students' preferences for school reputation by exam scores: high (above the 90th percentile), medium (70th to 90th percentile), and low (below 70th percentile but above the threshold). privileged students across all groups are similarly

[40] This is because, as mentioned in Section 5.1, no outside option can be observed in the current admission record.

[41] In addition, when i attends a nonpublic high school, we simplify the utility function as $u_{i,o} = F_o + \varepsilon_{io}$. Here F_o is represents the fixed effect of nonpublic high schools.

willing to travel further for higher-reputation schools: a 1-unit increase in reputation leads girls to travel an extra 0.32 to 0.38 km, and boys 0.9 to 1 km. High-scoring normal students show the greatest sensitivity, with a 1-unit increase leading girls to travel 0.59 km and boys 3.02 km. Medium-scoring girls and boys will travel 0.21 km and 1.1 km, respectively, while low-scoring girls and boys will travel 0.18 km and 0.89 km, respectively. All students generally prefer smaller schools. A reduction of 100 seats in school capacity prompts privileged medium-scoring students to travel an additional 2.44 km, while their high-scoring counterparts are willing to travel 1.37 km. Meanwhile, normal students with similar scores are less willing to travel these additional distances compared to their privilege counterparts. High-scoring students, both normal and privilege, generally have a negative attitude towards special classes. Conversely, these classes are favored by medium- and low-scoring students.

Next, we examine how well our estimations from the survey data match the administrative data. We perform out-of-sample tests to validate the aggregate-level matching results, using the estimated coefficients from Columns (5) and (6) of Table 4 to simulate students' behaviors and compare it with administrative data.[42] First, we compare the actual and predicted admission cutoffs of each high school and special class (Table F.5 in Appendix F).The gaps between the actual and predicted cutoff are all less than 6 points (less than 1% of the full score). Second, we explore the aggregate-level matching outcomes for students' first two school choices (Table F.6 in Appendix F). The administrative data show that 26.45% of normal students were admitted to their first-choice schools, while our prediction is 28.59%. We underpredicted by 6 percentage points the number of normal students admitted to their second choices. For privileged students, the data show 90.44% were admitted to their first choices, and our prediction is 93.8%. We also predict 1.28% of privileged students would be admitted to their second choices, whereas the actual percentage is 3%.

8 Counterfactual Analysis

In this section, we conduct a counterfactual analysis comparing the CES and CAS-2 mechanisms used in our focal city with our proposed mechanism DA-PCR. Recall that in our

[42]We use all students in our sample in the test. The simulation procedure of students' behaviors in the sample test is in Appendix G.

Table 4: Preference Parameters

	No student interactions		With student interactions	
	Normal	Privilege	Normal	Privilege
	(1)	(2)	(3)	(4)
Reputation	0.296	0.401		
	(0.020)	(0.027)		
Reputation × H			0.592	0.384
			(0.147)	(0.099)
Reputation × M			0.209	0.368
			(0.037)	(0.035)
Reputation × L			0.175	0.324
			(0.029)	(0.067)
Special class	-2.121	5.218		
	(0.917)	(2.363)		
Special class × H			-6.925	-2.975
			(1.931)	(2.382)
Special class × M			0.597	2.880
			(1.586)	(2.585)
Special class × L			6.365	11.609
			(6.116)	(10.028)
Same district			-1.905	-1.518
			(0.234)	(0.290)
Same district × Male			1.759	2.075
			(0.296)	(0.431)
Distance	-1	-1	-1	-1
Distance × Male			0.804	0.645
			(0.036)	(0.056)
Dorm	4.253	0.205	4.389	-0.201
	(0.968)	(1.613)	(1.029)	(1.066)
Dorm × Male			0.633	1.169
			(0.275)	(0.570)
Capacity	-1.969	-2.554		
	(0.122)	(0.219)		
Capacity × H			-0.999	-1.374
			(0.854)	(0.525)
Capacity × M			-1.489	-2.443
			(0.301)	(0.380)
Capacity × L			-1.064	-2.321
			(0.242)	(0.550)
Indexed High School				3.664
				(0.281)
Non-public high school	2.005	3.372	1.052	3.096
	(0.439)	(0.573)	(0.580)	(0.802)
School Fixed Effect	Y	Y	Y	Y

Notes: This table reports the estimated parameters of students' preferences. The first four columns do not involve any interaction terms between students and school characterises. Column 5–8 include these interaction terms. Standard errors in parentheses. Distance is measured by kilometer. The coefficient of distance for female students is nomralized to -1. School capacities are normalized to 100 seats. H, M and L represent high-scoring, medium-scoring and low-scoring students respectively.

focal city, the CES and CAS-2 mechanisms are based on non-strategy proof alghorithms. Differently, we use the DA algorithm as the basis for both CES and CAS-2.[43] This allows our analysis to focus on the effects of different selection rules in affirmative action policies, rather than the intricacies of the Chinese parallel mechanism.

In this counterfactual analysis, utilizing estimated preferences and profiles of students and schools from administrative data, we simulate students' ROLs for all high schools and calculate the resulting matches. Since the DA-PCR and CAS-2 are strategy-proof, we assume students report their true preferences in ROLs under both mechanisms. For CES, to simplify the calculation, we assume that students use the truth-telling strategy (see Appendix G for details). Further, we consider three experiments involving different values of privilege capacities, which account for 20%, 40%, or 60% of the total capacity. Each middle school receives privileges proportional to its real privilege proportion in each high school.

8.1 Representation of Middle Schools in High Schools

The ZBS policy aims to provide opportunities for students from low-performing middle schools to attend prestigious high schools. Using the CES mechanism as the benchmark, we test whether an alternative mechanism can improve representation in high schools by admitting more students from lower median and bottom middle schools when replacing CES with this alternative.

In Table 5, Column (1) shows that replacing CES with CAS-2 results in a slight decrease (0.9% to 4.5%) in the number of students from top middle schools attending top high schools. While the representation of upper median and lower median middle schools varies with privilege capacities. The impact on bottom middle schools are larger than other middle schools. Column (3) reveals that with 20% privilege capacities, top and upper median middle schools have slightly increased representation in moderate high school seats. Conversely, with 40% and 60% privilege capacity, top and upper median middle schools gain less seats in moderate high schools, while lower median and bottom middle schools gain seats.

When CES is replaced with DA-PCR, top middle schools experience a more significant loss (5.3% to 13.5%) of seats in top high schools under all scenarios (Column 2). In contrast,

[43]Note that despite using DA as the base matching algorithm, the CES is still not strategy-proof.

other middle schools benefit by placing more students into these top high schools. Specifically, bottom middle schools place 10.5% to 14.7% more students in top high schools with 20% and 40% privilege capacities, and 7.6% more with a 60% capacity. Column (4) shows that top middle schools compensate for their losses in top high schools by increasing their shares in moderate high schools. Meanwhile, upper median middle schools, despite gaining more seats in top high schools, place fewer students in moderate high schools, but their gains in top high schools outweigh these losses.

In summary, although CAS-2 is more transparent than CES due to the lack of an early selection stage, it does not significantly improve diversity in high schools. Our proposed mechanism, DA-PCR, effectively provides lower-performing middle schools with more opportunities to send their students to prestigious high schools.

Table 5: Change of Representation (%)

	Top High Schools		Moderate High Schools	
	CES–CAS-2 (1)	CES–DA-PCR (2)	CES–CAS-2 (3)	CES–DA-PCR (4)
20%				
Top middle Schools	-0.9	-5.3	0.9	2.4
Upper median middle schools	-0.4	9.0	0.3	-3.7
Lower median middle schools	2.5	3.3	-0.8	2.3
Bottom middle schools	3.9	10.5	2.4	6.3
40%				
Top middle Schools	-2.7	-9.8	-0.3	2.1
Upper median middle schools	1.1	13.1	-2.9	-7.5
Lower median middle schools	5.8	11.7	1.7	2.1
Bottom middle schools	6.5	14.7	12.0	20.1
60%				
Top middle Schools	-4.5	-13.5	-1.7	3.7
Upper median middle schools	0.4	19.2	-5.6	-10.9
Lower median middle schools	8.9	14.4	8.6	8.4
Bottom middle schools	10.5	7.6	12.9	8.4

Notes: This table shows the impact on diversity when CES is replaced. Columns (1) and (3) report the percentage change in students assigned to top or moderate high schools with CAS-2, while columns (2) and (4) show the change with the DA-PCR. Results are presented for 20%, 40%, and 60% privilege capacities across the three panels.

8.2 Usage of Privileges

In this section, we examine whether the privileges are used to improve students' assignment rather than being wasted. Privileges can be wasted in two ways:

- A student receiving a bonus would still be assigned to the same school without it, while there are lower-scoring students from the same middle school rejected but could have attended this high school with the bonus.

- A student receives a bonus but is still rejected by the high school, while there are higher-scoring students from the same middle school also rejected but could have attended this high school with the bonus.

Table 6 shows that the DA-PCR allocates privileges much more efficiently than CES and CAS-2, causing less waste. With a privilege capacity of 20%, 100% of privileges are effectively used under the DA-PCR, compared to only 2.5% under CES and 11% under CAS-2. Although the effectiveness of privilege usage improves under CES and CAS-2 as the capacity increases, the DA-PCR remains superior. Both CES and CAS-2 tend to allocate privileges to top students in each middle school, resulting in most privileges being wasted.

We also examine the ratio of students admitted due to privileges over the total admitted students in different types of middle schools. With a 20% privilege capacity, 48.4% of students from bottom middle schools are admitted to better high schools using privileges under the DA-PCR, compared to 5.7% under CES and 15.8% under CAS-2. The DA-PCR continues to outperform CES and CAS-2 as the privilege capacity increases. In short, the DA-PCR is more effective in allocating privilege bonuses to students who genuinely need them to access better high schools.

9 Literature Review

This paper contributes to the growing literature on affirmative action policies in matching markets, especially in school choice. Two main affirmative action policies discussed in the literature are *score subsidies* and *reserves* for targeted groups.

The idea of score subsidies in matching markets was first discussed by Balinski and Sönmez (1999) in the form of improvement for a single student's test scores. Kojima (2012) studies the score subsidies in the form of improvement for a group of students' test scores.

Table 6: Usage of Privilege Capacities (%)

	CES			CAS-2			DA-PCR		
	Waste	Used	Used/Admitted	Waste	Used	Used/Admitted	Waste	Used	Used/Admitted
20%									
Top middle School	100.0	0.0	0.0	98.5	1.5	0.2	0.0	100.0	11.6
Upper median middle schools	99.8	0.2	0.0	98.4	1.6	0.3	0.0	100.0	19.3
Lower median middle schools	97.8	2.2	0.8	82.3	17.7	6.2	0.0	99.9	34.1
Bottom middle schools	89.1	10.9	5.7	68.7	31.3	15.8	0.0	100.0	48.4
Total	97.5	2.5	0.5	89.0	11.0	2.2	0.0	100.0	20.0
40%									
Top middle School	100.0	0.0	0.0	97.9	2.1	0.5	0.0	100.0	22.2
Upper median middle schools	99.2	0.8	0.3	96.3	3.7	1.4	0.0	99.3	39.1
Lower median middle schools	84.4	15.6	11.1	62.5	37.4	25.9	0.0	98.2	66.4
Bottom middle schools	57.8	41.5	39.8	42.7	57.3	50.0	0.0	97.4	79.2
Total	88.3	11.6	4.6	78.2	21.8	8.6	0.0	98.9	39.2
60%									
Top middle School	98.3	1.7	0.6	94.7	5.3	1.8	0.0	100.0	34.9
Upper median middle schools	92.8	6.6	3.9	88.6	10.9	6.7	0.0	98.8	60.4
Lower median middle schools	50.6	46.3	46.8	37.0	61.5	57.2	0.0	94.6	86.6
Bottom middle schools	25.0	63.6	73.1	19.8	75.8	77.9	0.0	84.7	90.2
Total	71.1	25.9	15.4	64.3	34.4	20.4	0.0	95.6	56.6

Notes: This table presents the usage of privilege bonuses. "Used" refers to the percentage of privileges utilized by students to gain admission to high schools, where they would otherwise be rejected without those privileges. "Used/Admitted" refers to the percentage of students admitted with privilege bonuses, who would otherwise be rejected, relative to the total number of students admitted from this level of middle school. The first panel shows results for a 20% privilege capacity, with the second and third panels covering 40% and 60% capacities, respectively.

He shows that for some markets, all students receiving score subsidies might be hurt.[44]

Abdulkadiroğlu and Sönmez (2003) introduce the type-specific quotas in the context of school choice. As in score subsidies, Kojima (2012) show the adverse effects of majority quotas on the welfare of minority students. As a welfare-enhancing alternative to the majority quota, Hafalir et al. (2013) introduce reserves for minority students. Dur et al. (2018) highlight the importance of the processing order of reserved seats using the slot-specific priority model introduced by Kominers and Sönmez (2016). In particular, when every individual has at most one reserve eligible type, then the highest representation of a target type can be achieved when the open seats are processed after the reserved seats for all other types and before the reserved seats of the corresponding type (Dur et al., 2020; Pathak et al., 2022, 2023). As discussed, in our context, slots are not designated for bonus recipients and doing so leads to a violation of fairness[45]

The affirmative action policy in our paper generalizes both score subsidies and reserve systems. Unlike traditional score subsidies, which grant equal bonuses to the same group members, we generalize this policy by limiting the number of recipients. This approach

[44]Jiao et al. (2022) and Dur and Xie (2023) focus on the restrictions that guarantee students receiving score subsidies will not be hurt.

[45]Papers such as Ehlers et al. (2014), Aygün and Turhan (2020), Delacrétaz (2021), Aygün and Bó (2021), Sönmez and Yenmez (2022) have studied different aspects of reserve systems in various other contexts.

enables analysis of selection procedures. Reserve policies allocate a set number of seats to specific student groups, prioritizing reserve-eligible students over others. This prioritization can be viewed as assigning sufficiently high bonus scores, applied to a limited number of seats. Privilege systems generalize reserve systems by allowing a subset of reserve-eligible students to receive bonus scores without necessarily prioritizing them over others. It eliminates the need to specify a processing order and makes it possible to generate an equivalent representation to any reserve system by assigning sufficiently high bonuses to chosen students.

In a recent paper inspired by applications in India, Sönmez and Ünver (2022) propose a variant of the reserve system, the score-elevated VR protected policy, where a reserve-eligible student is prioritized over a higher-scoring student if the score difference is below a specific threshold. Like our policy, it does not grant exclusive seat rights. While their focus is on reserves and the role of the processing order, our goal is to optimize the selection of students receiving bonus scores, without designating specific slots for them.

Our policy, which uses bonus scores and restricts their allocation to a subset of students, strikes a balance between meritocracy and diversity. It limits priority violations by ensuring that bonus recipients may not surpass all other students. Under reserve policies, Abdulkadiroglu and Grigoryan (2021) formalize the compromise by minimizing priority violations and provide an axiomatic characterization. Imamura (2020) introduces a choice rule for reserve systems to balance meritocracy and diversity, allowing policymakers to compare students based on priorities. He characterizes this rule with an axiom requiring greater merit in the chosen set as diversity constraints are relaxed. Hafalir et al. (2022) propose choice rules that maximize merit while meeting diversity goals, using an index satisfying ordinal concavity to achieve optimal diversity. In a continuum model, Celebi and Flynn (2022) develop mechanisms to optimize an authority's expected utility, balancing student scores with diversity objectives. This paper examines a discrete economy where policymakers lack explicit diversity objectives. Our proposed choice rule, characterized by axioms, ensures stable matching and the efficient, fair distribution of bonus scores. In a recent study, Dur and Zhang (2025) examine choice rules in Chinese high school admissions, focusing on two types of score constraints for reserve eligibility. They show that these rules lack substitutability and propose an alternative substitutable choice rule. Unlike our analysis, their study does not model privilege systems or consider properties such as fairness and efficient privilege allocation.

Additionally, our study relates to papers addressing distributional constraints in matching markets (c.f.,Abdulkadiroğlu (2005), Westkamp (2013), Echenique and Yenmez (2015), Kamada and Kojima (2015), Doğan (2016), Fragiadakis and Troyan (2017), and Hafalir et al. (2022)).

Our research also contributes to a growing body of empirical work on school choice mechanisms (see Agarwal and Somaini 2020 and Agarwal and Budish 2021 for two reviews of this literature). One strand of that literature uses preferences reported under non–strategy-proof mechanisms to estimate students' preferences. The challenge of these studies is to recover students' preferences from their manipulated ROLs. From the supply and demand approach, Agarwal and Somaini (2018) develop a method to identify preferences through the choice environment variation. Calsamiglia et al. (2020) identify the sophisticated and naive types of households using data from Barcelona. Other papers focus on strategy-proof mechanisms. Abdulkadiroğlu et al. (2017) treat preferences reported under the DA mechanism as students' true preferences and then use those preferences to analyze the demand for particular schools in New York City. Fack et al. (2019) propose an approach to estimate preferences that does not require truth-telling to be the unique equilibrium under the DA mechanism. Several empirical papers (e.g., Burgess et al. 2014; Akyol and Krishna 2017; Ajayi 2022) bear similarities to our strict priority setting, such as exam scores.

Using reported ROLs to identify preferences is subject to the fact that students can make mistakes, even under the strategy-proof mechanisms (Fack et al. 2019). Therefore, there is an increasing use of survey data to explore strategic behavior under matching mechanisms. Budish and Cantillon (2012) conduct a survey on students' preferences over offered courses to study the course allocation mechanism at Harvard Business School, and Rees-Jones (2018) provides survey-based evidence of preference misrepresentation. Burgess et al. (2014) use survey data to directly assess student preferences regarding schools. Surveys are also used by Kapor et al. (2020) to study heterogeneous beliefs in the school choice problem and by De Haan et al. (2023) to analyze students' mistakes and deficiencies of the Boston mechanism.

10 Conclusion

We examine the allocation of middle school students to high schools in China as a prominent case of affirmative action policy reforms in school choice. These policies operate through what we have termed "privilege systems," which blend elements of priority-based and reserve-based assignment, allowing authorities to screen candidates rather than providing unconditional admission guarantees. However, our research indicates that current privilege systems in China fall short of their intended goals.

We propose that the allocation of privileges should be determined internally to achieve optimal distribution and fairness. Our empirical analysis suggests that employing the DA mechanism, coupled with a unique choice rule, can significantly enhance the representation of middle schoolers from disadvantaged socioeconomic backgrounds in quality high schools.

Privilege systems represent an innovative approach, bridging priority-based and reserve-based assignment models, offering greater flexibility compared to traditional methods. Therefore, it would be worthwhile to reevaluate existing implementations of these systems from a privilege-basedstandpoint. Further investigation into these matters is left for future research.

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Appendices

A Omitted Examples

In Example A.1, we consider three possible cases that illustrate the difficulty of choosing the privileged (eligible) student a prior under the CES mechanism. In each case, the bonus is added to the "wrong" student either rendering the bonus useless while some student from the same middle school could have benefited or a student with a higher raw score prefers the assignment of another student from the same middle school with lower raw score.

Example A.1. *Let* $H = \{h_1, h_2\}$, $M = \{m_1, m_2\}$, $I^{m_1} = \{i_1, i_2\}$ *and* $I^{m_2} = \{i_3, i_4\}$. *Each school has two seats, i.e.,* $q_{h_1} = q_{h_2} = 2$. *Let* $p^{m_1}_{h_1} = 1$ *and* $p^{m_2}_{h_1} = p^{m_1}_{h_2} = p^{m_2}_{h_2} = 0$. *Suppose* $\beta = 10$, *and we apply the DA mechanism without ranking restriction in the admission stage. The preferences of students are given as:*

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}
h_1	h_1	h_1	h_1
h_2	h_2	h_2	h_2
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Case 1: The added bonus does not improve the assignment of an under-achieving student, although it could have helped someone else from the same middle school. *The raw scores of the students are* $\pi(i_1) = 70$, $\pi(i_2) = 75$, $\pi(i_3) = 82$, *and* $\pi(i_4) = 84$. *Suppose, in the early selection stage, student* i_1 *is selected to be eligible to enjoy the privilege, i.e.,* $E^{m_1}_{h_1} = \{i_1\}$.

In the admission stage of CES, all students apply to h_1. *As the normal applicants, students* i_2, i_3, *and* i_4 *are first considered for the single undesignated capacity. Since* i_4 *has the highest raw score, we set* $c_h = 84$. *The raw score of the unique eligible student,* i_1, *does not exceed* 84 *when the bonus score is added* $(70 + 10 < 84)$. *Hence, no student will receive a bonus score. Then, admission scores of all students are set to their raw scores, and* i_3 *and* i_4 *are tentatively held by* h_1. *In the next step, both rejected students apply to* h_2, *and no student is rejected.*

CES selects the assignment (μ, α) *such that:*

$$\mu(i_1) = h_2,\ \mu(i_2) = h_2,\ \mu(i_3) = h_1,\ \mu(i_4) = h_1$$
$$\alpha(i_1) = 70,\ \alpha(i_2) = 75,\ \alpha(i_3) = 82,\ \alpha(i_4) = 84.$$

Notice that the only eligible student from m_1*, i.e.,* i_1*, does not receive bonus. However,* i_2 *would have been receiving a bonus and assigned to* h_1 *if she had been selected as eligible by* m_1*.*

Case 2: The added bonus improves the assignment of an underachieving student although it could have helped a more deserving student from the same middle school. *Consider the same problem with the following changes. The raw score of* i_1 *is* 80*, i.e.,* $\pi(i_1) = 80$*, and* $E^{m_1}_{h_1} = \{i_2\}$*.*

In the admission stage of CES, all students apply to h_1*. Among the normal applicants,* i_1*,* i_3*, and* i_4*,* i_4 *has the highest raw score. Hence, we set* $c_h = \pi(i_4) = 84$*. The raw score of the unique eligible student,* i_2*, exceeds* 84 *when bonus score is added* $(75 + 10 > 84)$*. Hence, she will receive a bonus score, and her admission score will be 85. The admission scores of all other students are set to their raw scores. Then, based on the admission scores,* i_2 *and* i_4 *are tentatively held. In the next step, both rejected students apply to* h_2*, and no student is rejected.*

CES selects the assignment (μ, α) *such that:*

$$\mu(i_1) = h_2,\ \mu(i_2) = h_1,\ \mu(i_3) = h_2,\ \mu(i_4) = h_1$$
$$\alpha(i_1) = 80,\ \alpha(i_2) = 85,\ \alpha(i_3) = 82,\ \alpha(i_4) = 84.$$

Notice that, i_1 *prefers* h_1 *to her assigned school and has a higher raw score than* i_2*, who receives the privilege.*

Case 3: The added bonus has no effect on the assignment of a high-achieving student, but it could have helped a low-achiever from the same middle school who does not get it. *The previous two cases might suggest that determining the higher-scoring student as eligible would have led to the ideal usage of the privilege capacity. We next* *show that this is not the case in general. Consider now the same problem with the following changes. The raw score of i_1 is 88, i.e., $\pi(i_1) = 88$, and $E_{h_1}^{m_1} = \{i_1\}$.*

In the admission stage of CES, all students apply to h_1. Among the normal applicants, i_2, i_3, and i_4, i_4 has the highest raw score. Hence, we set $c_h = \pi(i_4) = 84$. The raw score of the unique eligible student, i_1, exceeds the cutoff of 84 even without the bonus score. Hence, she will receive a bonus score, and her admission score will be 98. The admission scores of all other students are set to their raw scores. Then, based on the admission scores, i_1 and i_4 are tentatively held. In the next step, both rejected students apply to h_2, and no student is rejected.

CES selects the assignment (μ, α) such that:

$$\mu(i_1) = h_1,\ \mu(i_2) = h_2,\ \mu(i_3) = h_2,\ \mu(i_4) = h_1$$
$$\alpha(i_1) = 98,\ \alpha(i_2) = 75,\ \alpha(i_3) = 82,\ \alpha(i_4) = 84.$$

The privilege of m_1 is "wasted" on student i_1 who would have gotten into h_1 without any privilege, and i_2 would have been assigned together with i_1 if she had received the bonus score.

Example A.2. *Let $\bar{I} = \{i_1, i_2, i_3, i_4, i_5\}$, $M = \{m_1, m_2, m_3\}$, $\bar{I}^{m_1} = \{i_1, i_2\}$, $\bar{I}^{m_2} = \{i_3, i_4\}$, $\bar{I}^{m_3} = \{i_5\}$, $q_h = 3$, $p_h^{m_1} = p_h^{m_2} = 1$, and $p_h^{m_3} = 0$. Students' raw scores are $\pi(i_1) = 74$, $\pi(i_2) = 70$, $\pi(i_3) = 65$, $\pi(i_4) = 69$, and $\pi(i_5) = 81$. Let $\beta = 10$.*

We first apply the privilege selection step, i.e., Step 1 of the PCR.

Step 1:

Step 1.0: *Since $|\bar{I}| > q_h$, we set $\pi_1^b(i) = \pi(i)$ for all $i \in \bar{I}$ and $A^1 = B^1 = \emptyset$.*

Step 1.1: *We add $\{i_1, i_2, i_5\}$ to A^1, i.e., $A^1 = \{i_1, i_2, i_5\}$. We set $T^1 = R^1 = \bar{I} \setminus A^1 = \{i_3, i_4\}$. Then, we add bonus scores to i_4 and obtain the updated scores as follows: $\pi_2^b(i) = \pi(i)$ for all $i \in \bar{I} \setminus \{i_4\}$ and $\pi_2^b(i_4) = \pi(i_4) + \beta = 69 + 10 = 79$. We set $A^2 = \emptyset$ and $B^2 = \{i_4\}$.*

Step 1.2: *We add $\{i_1, i_4, i_5\}$ to A^2, i.e., $A^2 = \{i_1, i_4, i_5\}$. We set $R^2 = \bar{I} \setminus A^2 = \{i_2, i_3\}$ and $T^2 = T^1 \cup R^2 = \{i_2, i_3, i_4\}$. Since $T^2 \neq T^1$, we add bonus scores to i_2 and i_4 and obtain the updated scores as follows: $\pi_3^b(i) = \pi(i)$ for all $i \in \bar{I} \setminus \{i_2, i_4\}$, $\pi_2^b(i_2) = \pi(i_2) + \beta = 70 + 10 = 80$ and $\pi_2^b(i_4) = \pi(i_4) + \beta = 69 + 10 = 79$. We set $A^3 = \emptyset$ and $B^3 = \{i_2, i_4\}$.*

Step 1.3: *We add* $\{i_2, i_4, i_5\}$ *to* A^3*, i.e.,* $A^3 = \{i_2, i_4, i_5\}$*. We set* $R^3 = \bar{I} \setminus A^3 = \{i_1, i_3\}$ *and* $T^3 = T^2 \cup R^3 = \{i_1, i_2, i_3, i_4\}$*. Since* $T^3 \neq T^2$*, we add bonus scores to* i_1 *and* i_4 *and obtain the updated scores as follows:* $\pi_3^b(i) = \pi(i)$ *for all* $i \in \bar{I} \setminus \{i_1, i_4\}$*,* $\pi_3^b(i_1) = \pi(i_1) + \beta = 74 + 10 = 84$ *and* $\pi_2^b(i_4) = \pi(i_4) + \beta = 69 + 10 = 79$*. We set* $A^4 = \emptyset$ *and* $B^4 = \{i_1, i_4\}$*.*

Step 1.4: *We add* $\{i_1, i_4, i_5\}$ *to* A^4*, i.e.,* $A^4 = \{i_1, i_4, i_5\}$*. We set* $R^4 = \bar{I} \setminus A^4 = \{i_2, i_3\}$ *and* $T^4 = T^3 \cup R^4 = \{i_1, i_2, i_3, i_4\}$*. Since* $T^4 = T^3$*, we calculate the admission scores as follows:* $PCR_h^\alpha(\bar{I}; \beta, E_h)[i] = \pi(i)$ *for all* $i \notin B^4 = \{i_1, i_4\}$*, and* $PCR_h^\alpha(\bar{I}; \beta, E_h)[i_1] = 84$ *and* $PCR_h^\alpha(\bar{I}; \beta, E_h)[i_4] = 79$*.*

Step 2: *Since* $\{i_1, i_4, i_5\}$ *have the highest* $q_h = 3$ *scores under* $(PCR_h^\alpha(\bar{I}; \beta, E_h)[i])_{i \in \bar{I}}$*, we have* $PCR_h^s(\bar{I}; \beta, E_h) = \{i_1, i_4, i_5\}$*.*

Example A.3. *Let* $H = \{h_1, h_2, h_3\}$*,* $M = \{m_1, m_2, m_3, m_4\}$*,* $I^{m_1} = \{i_1, i_2\}$*,* $I^{m_2} = \{i_3, i_4\}$*,* $I^{m_3} = \{i_5\}$ *and* $I^{m_4} = \{i_6\}$*. Schools* h_1 *and* h_3 *have two seats and* h_2 *has one seat. The raw scores of the students are* $\pi(i_1) = 50$*,* $\pi(i_2) = 40$*,* $\pi(i_3) = 55$*,* $\pi(i_4) = 46$*,* $\pi(i_5) = 47$ *and* $\pi(i_6) = 42$*. Let* $E_h = I$ *for all* $h \in H$*,* $\beta = 10$*,* $p_{h_1}^{m_2} = p_{h_2}^{m_3} = p_{h_3}^{m_1} = p_{h_3}^{m_4} = 1$*, and all other privilige capacities be 0.*

Students' preferences over schools are as follows:

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	P_{i_6}
h_1	h_1	h_2	h_1	h_3	h_3
$\emptyset$	h_3	h_1	h_2	h_2	h_1
	h_2	h_3	h_3	h_1	h_2
	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Note that the second favorite school of student i_1 *is* $\emptyset$*, i.e., only* h_1 *is acceptable for* i_1*.*

The admission process based on DA-PCR will proceed as follows:

Step 1: *Students* i_1*,* i_2 *and* i_4 *apply to high school* h_1*; student* i_3 *applies to high school* h_2*; students* i_5 *and* i_6 *apply to* h_3*. Then, we have*

$$PCR_{h_1}^s(\{i_1, i_2, i_4\}; \beta, E_h\}) = \{i_1, i_4\}$$

$$PCR_{h_2}^s(\{i_3\}; \beta, E_h\}) = \{i_3\}$$

$$PCR_{h_3}^s(\{i_5, i_6\}; \beta, E_h\}) = \{i_5, i_6\}$$

Step 2: *Students* i_1 *and* i_4 *apply to high school* h_1*; student* i_3 *applies to high school* h_2*; students* i_2*,* i_5 *and* i_6 *apply to* h_3*. Then, we have*

$$PCR^s_{h_1}(\{i_1, i_2, i_4\}; \beta, E_h\}) = \{i_1, i_4\}$$
$$PCR^s_{h_2}(\{i_3\}; \beta, E_h\}) = \{i_3\}$$
$$PCR^s_{h_3}(\{i_2, i_5, i_6\}; \beta, E_h\}) = \{i_2, i_6\}$$

Step 3: *Students* i_1 *and* i_4 *apply to high school* h_1*; student* i_3 *and* i_5 *apply to high school* h_2*; students* i_2 *and* i_6 *apply to* h_3*. Then, we have*

$$PCR^s_{h_1}(\{i_1, i_2, i_4\}; \beta, E_h\}) = \{i_1, i_4\}$$
$$PCR^s_{h_2}(\{i_3, i_5\}; \beta, E_h\}) = \{i_5\}$$
$$PCR^s_{h_3}(\{i_2, i_5, i_6\}; \beta, E_h\}) = \{i_2, i_6\}$$

Step 4: *Students* i_1*,* i_3 *and* i_4 *apply to high school* h_1*; student* i_5 *applies to high school* h_2*; students* i_2*, and* i_6 *apply to* h_3*. Then, we have*

$$PCR^s_{h_1}(\{i_1, i_2, i_3, i_4\}; \beta, E_h\}) = \{i_3, i_4\}$$
$$PCR^s_{h_2}(\{i_3, i_5\}; \beta, E_h\}) = \{i_5\}$$
$$PCR^s_{h_3}(\{i_2, i_5, i_6\}; \beta, E_h\}) = \{i_2, i_6\}$$

Step 5: *Students* i_3 *and* i_4 *apply to high school* h_1*; student* i_5 *applies to high school* h_2*; students* i_2 *and* i_6 *apply to* h_3*; student* i_1 *applies to* $\emptyset$*. Then, we have*

$$PCR^s_{h_1}(\{i_1, i_2, i_3, i_4\}; \beta, E_h\}) = \{i_3, i_4\}$$
$$PCR^s_{h_2}(\{i_3, i_5\}; \beta, E_h\}) = \{i_5\}$$
$$PCR^s_{h_3}(\{i_2, i_5, i_6\}; \beta, E_h\}) = \{i_2, i_6\}$$

Since no more students are rejected, the procedure terminates. The assigned schools and *admission scores of the students are as follows:*

$$\mu(i_1) = \emptyset,\ \mu(i_2) = h_3,\ \mu(i_3) = h_1,\ \mu(i_4) = h_1,\ \mu(i_5) = h_2,\ \mu(i_6) = h_3$$
$$\alpha(i_1) = 50,\ \alpha(i_2) = 52,\ \alpha(i_3) = 55,\ \alpha(i_4) = 56,\ \alpha(i_5) = 57,\ \alpha(i_6) = 52$$

B Proofs

Proof of Proposition 1. **E**xample 1 shows that CES and CAS-1 mechanisms fail to satisfy fairness and strategy-proofness. Example 2 shows that CAS-1 and CAS-2 mechanisms fail to satisfy efficient allocation of privileges. In the next example, we show that CES mechanisms fail to satisfy the efficient allocation of privileges.

Example A.4. *Let $H = \{h_1\}$, $M = \{m_1, m_2\}$, $I^{m_1} = \{i_1, i_2\}$, $I^{m_2} = \{i_3, i_4\}$ and $q_{h_1} = 3$. Let $\beta = 10$, $p^{m_1}_{h_1} = 2$, $p^{m_2}_{h_1} = 0$, and $E_{h_1} = \{i_1, i_2\}$.*

The raw scores of the students are $\pi(i_1) = 90$, $\pi(i_2) = 84$, $\pi(i_3) = 95$, and $\pi(i_4) = 86$. Suppose all students consider h_1 acceptable.

In this problem, CES selects the assignment (μ, α) such that:

$$\mu(i_1) = h_1,\ \mu(i_2) = \emptyset,\ \mu(i_3) = h_1,\ \mu(i_4) = h_1$$
$$\alpha(i_1) = 100,\ \alpha(i_2) = 84,\ \alpha(i_3) = 95,\ \alpha(i_4) = 86$$

Assignment (μ, α) does not satisfy the efficient allocation of privileges.

□

Before providing the proof of Theorem 1, we state and prove a lemma, which is referred to in the proof.

Lemma 1. *Suppose Step 1 of PCR terminates in Step $1.K$ and $K > 1$ when the set of applicants is J. Let $\sigma_k = \min\{\pi^b_k(i) \ : \ i \in A^k\}$ for all $k \in \{1, \dots, K\}$. Then, for all $k \in \{1, \dots, K-1\}$, $\sigma_k \leq \sigma_{k+1}$.*

Proof of Lemma 1. Since Step 1 of PCR terminates in Step $1.K$ where $K > 1$, we have $|J| > q_h$. We first consider Steps 1.1 and 1.2. Since π^b_2 is obtained from π^b_1 adding bonus score to some students and keeping the raw score for the others, we have $\pi_1^b(i) \leq \pi_2^b(i)$ for all $i \in J$. Let i^1 be the lowest scoring student in A^1 based on π_1^b. If $i^1 \in A^2$, then she is the lowest scoring student in A^2 based on π_2^b. If $i^1 \notin A^2$, then every student in A^2 has higher score than $\pi_2^b(i^1)$. Since, $\pi_1^b(i^1) \leq \pi_2^b(i^1)$, we have $\sigma_1 \leq \sigma_2$.

Suppose that $\sigma_{\bar{k}-1} \leq \sigma_{\bar{k}}$ for all $\bar{k} \leq k$ where $k < K$. Now we compare σ_k and σ_{k+1}. Let i^k be the lowest scoring student in A^k based on π_k^b. If $i^k \in A^{k+1}$, then she is the lowest scoring student in A^{k+1} based on π_{k+1}^b. This follows from the following observation: By definition, $T^{k+1} \supseteq T^k$, and therefore, for each student who has been added a bonus score at the end of Step 1.k, there exists a student with a weakly higher raw score, and bonus score is added at the end of Step 1.$k+1$. If $i^k \notin A^{k+1}$, then every student in A^{k+1} has higher score than $\pi_{k+1}^b(i^1)$. Since, $\pi_k^b(i^k) \leq \pi_{k+1}^b(i^k)$, we have $\sigma_k \leq \sigma_{k+1}$. □

Proof of Theorem 1. We prove these two statements separately.

Proof for Part a:

q_h**-acceptance:** Consider a set of applicants $\bar{I} \subseteq I$. First, if $|\bar{I}| \leq q_h$, every student in $\bar{I}$ will be selected. If $|\bar{I}| > q_h$, then at the end of Step 2 q_h student will be selected. As a result, $|PCR_h^s(\bar{I};\beta, E_h)| = min\{q_h, |\bar{I}|\}$.

As discussed in the main text, for any $\bar{I} \subseteq I$, $PCR_h^s(\bar{I};\beta, E_h) = A^K$ where Step 1.K is the termination step of the privilege selection step. In the rest of the proof, we focus on A^K instead of $PCR_h^s(\bar{I};\beta, E_h)$.

Substituability: Consider a set of applicants $\bar{I} \subseteq I$. Since PCR satisfies q_h-acceptance, no student will be rejected when the number of applicants is less than or equal to the capacity of high school h. Therefore, we cannot observe a violation of substitutability when $|\bar{I}| \leq q_h$. We consider a case in which $|\bar{I}| > q_h$. Let $\tilde{I} = \bar{I} \cup \{j\}$ and $i \in \bar{I} \setminus PCR_h^s(\bar{I};\beta, E_h)$. We will show that $i \notin PCR_h^s(\tilde{I};\beta, E_h)$.

We denote the corresponding variable of X in the calculation of $PCR_h(\tilde{I};\beta, E_h)$ and $PCR_h(\bar{I};\beta, E_h)$ with $\tilde{X}$ and $\bar{X}$, respectively.

We first compare Step 1.1 of the choice rule applied to $\bar{I}$ and $\tilde{I}$. Since $\tilde{I} \supset \bar{I}$ and $|\bar{I}| > q_h$, we have $\tilde{R}^1 \supset \bar{R}^1$ and $\bar{\sigma}_1 < \tilde{\sigma}_1$. Moreover, since $\tilde{R}^1 \supset \bar{R}^1$, $\bar{T}^1 = \bar{R}^1$ and $\tilde{T}^1 = \tilde{R}^1$, we have $\tilde{T}^1 \supset \bar{T}^1$. Notice that, $|\tilde{R}^1 \setminus \bar{R}^1| = |\tilde{T}^1 \setminus \bar{T}^1| = 1$. This follows from the fact that $|\tilde{I} \setminus \bar{I}| = 1$

and $\bar{\pi}_1^b(k) = \tilde{\pi}_1^b(k) = \pi(k)$ for all $k \in \bar{I}$. By definition, we will continue with Step 1.2 under both applications of the choice rule to $\bar{I}$ and $\tilde{I}$.

Next, we consider Step 1.2. Since $\tilde{T}^1 \supset \bar{T}^1$, for every student receiving bonus score under the calculation of $\bar{\pi}_2^b$, there exists a separate student with weakly higher raw score receiving bonus score under the calculation of $\tilde{\pi}_2^b$. Hence, we have $\tilde{\sigma}_2 \geq \bar{\sigma}_2$. Consider a student $k \in \bar{I}$. If $k \notin \bar{R}^1 = T^1$ and $k \in \bar{R}^2$, then $\bar{\pi}_2^b(k) = \bar{\pi}(k)$, and either $k \in \tilde{R}^1$ or $k \in \tilde{A}^1$ and $\tilde{\pi}_2^b(k) = \bar{\pi}_2^b(k) = \bar{\pi}(k)$. Since, $\bar{\pi}(k) < \bar{\sigma}_2 \leq \tilde{\sigma}_2$, we have $k \in \tilde{R}_1 \cup \tilde{R}_2$. Then, combining this with $\bar{R}^1 \subseteq \tilde{R}^1$, we have $\bar{T}^2 = \bar{R}^1 \cup \bar{R}^2 \subseteq \tilde{R}^1 \cup \tilde{R}^2 = \tilde{T}^2$.

We consider three cases based on the relation between $\bar{T}^1$, $\bar{T}^2$, $\tilde{T}^1$ and $\tilde{T}^2$.

Case 1: $\bar{T}^1 = \bar{T}^2$. Since $\bar{T}^1 = \bar{R}^1$, $\bar{T}^2 = \bar{T}^1 \cup \bar{R}^2$ and $|\bar{R}^1| = |\bar{R}^2|$, $\bar{T}^1 = \bar{T}^2$ implies $\bar{R}^1 = \bar{R}^2$. By definition of the PCR, Step 1.2 is the last substep of Step 1 when PCR is applied to $\bar{I}$. Then, $\bar{A}^2 = PCR_h^s(\bar{I}; \beta, E_h)$. Hence, by supposition, $i \notin \bar{A}^2$. Since $i \notin \bar{A}^2$ and $\bar{R}^1 = \bar{R}^2$, we have $i \in \bar{R}^1 \subset \tilde{R}^1$ and $i \in \bar{R}^2$. Moreover, $\bar{R}^1 = \bar{T}^1 \subset \tilde{T}^1 = \tilde{R}^1$ and $\bar{\sigma}_2 \leq \tilde{\sigma}_2$ and $i \in \bar{R}^2$ imply that $i \in \tilde{R}^2$. Lemma 1 implies that any student rejected in two consecutive steps will be rejected in all further steps. Hence, $i \notin PCR_h^s(\tilde{I}; \beta, E_h)$.

Case 2: $\bar{T}^1 \neq \bar{T}^2$ and $\tilde{T}^1 = \tilde{T}^2$. Since $\tilde{T}^1 = \tilde{R}^1$, $\tilde{T}^2 = \tilde{T}^1 \cup \tilde{R}^2$ and $|\tilde{R}^1| = |\tilde{R}^2|$, $\tilde{T}^1 = \tilde{T}^2$ implies $\tilde{R}^1 = \tilde{R}^2$. Since $\bar{T}^1 \subset \tilde{T}^1$, $|\tilde{T}^1 \setminus \bar{T}^1| = 1$, $\bar{T}^1 \subset \bar{T}^2$, and $\bar{T}^2 \subseteq \tilde{T}^2$, we have $\bar{T}^2 = \tilde{T}^2$. Since $j \notin \bar{I}$ and $\tilde{T}^2 = \bar{T}^2 \subseteq \bar{I}$, we have $j \notin \tilde{T}^2$ and therefore $j \in PCR_h^s(\tilde{I}; \beta, E_h) = \tilde{A}^2$. Moreover, $PCR_h^s(\tilde{I}; \beta, E_h) = \tilde{A}^1 = \tilde{A}^2$, $PCR_h^s(\tilde{I}; \beta, E_h) \setminus \bar{A}^1 = \{j\}$. Let $\{k\} = \bar{A}^1 \setminus \tilde{A}^1 = \tilde{R}^1 \setminus \bar{R}^1$. Since $\bar{T}^2 = \tilde{T}^2 = \tilde{R}^1$, $\bar{T}^2 = \bar{R}^1 \cup \bar{R}^2$, and $\tilde{R}^1 \setminus \bar{R}^1 = \{k\}$, we have $\{k\} = \bar{R}^2 \setminus \bar{R}^1$. Then, $|\bar{R}^1| = |\bar{R}^2|$ and $|\bar{R}^2 \setminus \bar{R}^1| = 1$ imply that $|\bar{R}^1 \setminus \bar{R}^2| = 1$. Let $\{k'\} = \bar{R}^1 \setminus \bar{R}^2$. Since $\tilde{R}^1 = \tilde{R}^2 = \bar{T}^2 = \bar{R}^1 \cup \bar{R}^2$, we have $k, k' \in \tilde{R}^1 = \tilde{R}^2$. Then, $\tilde{T}^1 = \tilde{T}^2 = \bar{T}^2$ implies that $\bar{B}^2 = \bar{B}^3$. Then, none of the students in $\bar{B}^3$ can replace students in $PCR_h^s(\tilde{I}; \beta, E_h) \setminus \{j\}$ in Step 1.3 of PCR when applied to $\bar{I}$. Otherwise, we cannot have $\tilde{R}^2 = \tilde{R}^1$. Moreover, it is easy to see that $(PCR_h^s(\tilde{I}; \beta, E_h) \setminus \{j\}) \subset \bar{A}^2$ and $(PCR_h^s(\tilde{I}; \beta, E_h) \setminus \{j\}) \subset \bar{A}^3$. Then, we have either $k' \in \bar{R}^3 \setminus \bar{R}^2$ or $\bar{R}^2 = \bar{R}^3$. Since, $k' \in \bar{R}^1$, $\bar{T}^3 = \bar{R}^1 \cup \bar{R}^2 \cup \bar{R}^3$, $\bar{T}^2 = \bar{R}^1 \cup \bar{R}^2$, either case implies that $\bar{T}^2 = \bar{T}^3$. Hence, $\bar{A}^3 = PCR_h^s(\bar{I}; \beta, E_h)$. Since $i \notin \bar{A}^3$ and $(PCR_h^s(\tilde{I}; \beta, E_h) \setminus \{j\}) \subset \bar{A}^3$, we have $i \notin PCR_h^s(\tilde{I}; \beta, E_h)$.

Case 3: $\bar{T}^1 \neq \bar{T}^2$ and $\tilde{T}^1 \neq \tilde{T}^2$. Then, we continue with Step 3 under both cases. As explained for Step 2, $\tilde{\sigma}_3 \geq \bar{\sigma}_3$ and $\bar{T}^3 \subseteq \tilde{T}^3$. We consider the possible relations between $\tilde{T}^3$,

$\bar{T}^3$, $\tilde{T}^2$, and $\bar{T}^2$ as we did for the previous step.

For the rest of the steps of the privilege selection procedure, by following the same arguments, we can show that $i \notin PCR^s_h(\tilde{I};\beta,E_h)$.

Fairness: Consider a set of students $\bar{I} \subseteq I$. If $|\bar{I}| \leq q_h$, then $PCR^s_h(\bar{I};\beta,E_h) = \bar{I}$. Hence, PCR_h satisfies fairness when $|\bar{I}| \leq q_h$. Suppose $|\bar{I}| > q_h$ and $i \in \bar{I} \setminus PCR^s_h(\bar{I};\beta,E_h)$. By the definition of Step 2 of PCR, $PCR^\alpha_h(\bar{I};\beta,E_h)[j] > PCR^\alpha_h(\bar{I};\beta,E_h)[i]$ for all $j \in PCR^s_h(\bar{I};\beta,E_h)$. Moreover, if $PCR^\alpha_h(\bar{I};\beta,E_h)[i] = \pi(i)$, then, by the definition of Step 1 of PCR, $|\{j \in \bar{I}^{\tau(i)} \ : \ PCR^\alpha_h(\bar{I};\beta,E_h)[j] = \pi(j)+\beta \text{ and } \pi(j) > \pi(i)\}| = p^{\tau(i)}_h$. This concludes that PCR_h is fair.

Efficient Allocation of Privilege Capacity: Consider a set of students $\bar{I} \subseteq I$. If $\bar{I} \leq q_h$, then $PCR^s_h(\bar{I};\beta,E_h) = \bar{I}$ and $PCR^s_h(\bar{I};\beta,E_h)[i] = \pi(i)$ for all $i \in \bar{I}$. Hence, PCR_h efficiently allocates privilige capacities when $|\bar{I}| \leq q_h$. Suppose $|\bar{I}| > q_h$ and the privilege selection procedure terminates in Step 1.K. Suppose $i \in PCR^s_h(\bar{I};\beta,E_h)$. First note that if $i \notin R^k$ for all $k \leq K$, then $PCR^\alpha_h(\bar{I};\beta,E_h)[i] = \pi(i)$. Hence, if $PCR^\alpha_h(\bar{I};\beta,E_h)[i] = \pi(i)+\beta$, then $i \in R^k$ for some $k < K$. Then, by definition, $\pi(i) < \sigma_k$ where $\sigma_k = min\{\pi^b_k(j) \ : \ j \in A^k\}$. Lemma 1 implies that admission score of all students in $PCR^s_h(\bar{I};\beta,E_h)$ are weakly greater than σ_k, and therefore $\pi(i)$. Hence, there does not exist a student $k \in PCR^s_h(\bar{I};\beta,E_h)$ such that $\pi(i) > PCR^\alpha_h(\bar{I};\beta,E_h)[k]$. This concludes that PCR satisfies efficient allocation of privileges.

Proof for Part b:

On the contrary, suppose D_h be a choice rule satisfying all these three properties and it selects a different outcome for some set of applicants $\bar{I}$, i.e., $D^s_h(\bar{I};\beta,E_h) \neq PCR^s_h(\bar{I};\beta,E_h)$.

First, notice that, if $|\bar{I}| \leq q_h$, then any q_h-acceptant choice rule selects all applicants in $\bar{I}$. Hence, $D^s_h(\bar{I};\beta,E^D_h) \neq PCR^s_h(\bar{I};\beta,E_h)$ implies $|\bar{I}| > q_h$.

By following the substeps of the privilege selection step of PCR, we will show the contradiction. Let 1.K be the last substep of the privilege selection under PCR.

We start with Step 1.1 of PCR. Since $|\bar{I}| > q_h$, $R^1 = T^1 \neq \emptyset$ and $B^2 \neq \emptyset$. Let $\chi^1 = T^1 \setminus B^2$, i.e., the set of rejected students who did not get bonus points at the end of Step 1.1. By the definition of PCR and Lemma 1, students in χ^1 will not be included in A^k in any further Step 1.k. Since $PCR^s_h(\bar{I};\beta,E_h) = A^K$, $\chi^1 \cap PCR^s_h(\bar{I};\beta,E_h) = \emptyset$. Moreover, any fair choice rule will not select a student from χ^1. To see that, on the contrary, suppose $i \in D_h^s(\bar{I};\beta,E_h) \cap \chi^1$. Then, there exists at least one student $j \notin D_h^s(\bar{I};\beta,E_h)$ and $j \in A^1$. Notice that, $\pi(j) > \pi(i)$. Hence, in order not to violate fairness, $D_h^\alpha(\bar{I};\beta,E_h)[i] = \pi(i) + \beta$. In particular, $D_h^\alpha(\bar{I};\beta,E_h)[i'] = \pi(i') + \beta$ for any student $i' \in D_h^s(\bar{I};\beta,E_h) \cap T^1$. Recall that, $i \in T^1 \setminus B^2$, i.e., i did not receive bonus points at the end of Step 1.1 of PCR. Hence, either $p_h^{\tau(i)} = 0$ or there is another student $\hat{i}$ such that $\tau(\hat{i}) = \tau(i)$, $\tau(\hat{i}) = B^2 \subset T^1$ and $D_h^\alpha(\bar{I};\beta,E_h)(\hat{i}) = \pi(\hat{i})$. As a result, $\pi(\hat{i}) > \pi(i)$ and $\hat{i} \notin D_h^s(\bar{I};\beta,E_h)^s$. This violates fairness. Hence, $\chi^1 \cap D_h^s(\bar{I};\beta,E_h) = \chi^1 \cap PCR_h^s(\bar{I};\beta,E_h) = \emptyset$.

Next, we consider Step 1.2 of PCR. As explained above, all students in χ^1 will be rejected in this step. Let $\Gamma_2 = B^2 \cap R^2$, i.e., the set of students who received bonus points at the end of Step 1.1 but rejected in Step 1.2 of PCR. Let $\kappa_2 = B^2 \cap A^2$, i.e., the set of students who received bonus points at the end of Step 1.1 and included in A^2 in Step 1.2. Let $\Omega_2 = A^1 \cap R^2$, i.e., the set of students who were selected in Step 1.1 but rejected in Step 1.2.

First note that, for every student $i \in A^2 \setminus B^2$, $j \in \Gamma_2$, and $k \in \kappa_2$, we have the following relations: $\pi(i) > \pi(k) > \pi(j)$, $\pi(i) > \pi(j) + \beta$, and $\pi(k) + \beta > \pi(j) + \beta$. By Lemma 1, $A^k \cap \Gamma_2 = \emptyset$ for $k > 2$ (and therefore, $PCR_h^s(\bar{I};\beta,E_h) \cap \Gamma_2 = \emptyset$). Moreover, in order not to violate fairness, if there exists $j \in \Gamma_2 \cap D_h^s(\bar{I};\beta,E_h)$, then $D_h^\alpha(\bar{I};\beta,E_h)[j] = \pi(j) + \beta$. Also notice that, $A^2 \setminus B^2 \subseteq D_h^s(\bar{I};\beta,E_h)$. Then, there exists at least $|\Gamma_2 \cap D_h^s(\bar{I};\beta,E_h)|$ students in κ_2 who are not included in $D_h^s(\bar{I};\beta,E_h)$. Let $\hat{\kappa} = |\kappa_2 \setminus D_h^s(\bar{I};\beta,E_h)|$. Then, each such student cannot receive bonus to avoid fairness violation. Moreover, in order not to violate efficient allocation of privilege if $i \in A^2 \setminus B^2$, $k \in \kappa_2 \setminus D_h^s(\bar{I};\beta,E_h)$ and $\tau(i) = \tau(k)$, then i cannot receive bonus when D_h is applied. Then, at least $\hat{\kappa}$ students in Ω_2 receive bonus scores when D_h is applied to $\bar{I}$ in order not to violate fairness. Since all such students have higher raw score than students in Γ_2, they are all included in $D_h^s(\bar{I};\beta,E_h)$. To sum up, if $\Gamma_2 \cap D_h^s(\bar{I};\beta,E_h) \neq \emptyset$, then $|D_h^s(\bar{I};\beta,E_h)| > q_h$, which is a contradition.

Also, notice that any student in κ_2 cannot be selected by a fair choice rule without receiving bonus points. This follows from the fact that $|A^1| = q_h$ and all students in A^1 have higher raw scores than the ones in κ_2. In particular, if a student in κ_2 does not receive bonus points at the end of Step 1.2 of PCR, then she will not be included in A^k for $k > 2$. For Step

1.2, we next consider students in Ω_2. First, notice that if a student $i \in \Omega_2$ does not receive bonus points at the end of Step 1.2, then she will not be included in any further substeps of PCR. Suppose that $i \in D_h^s(\bar{I}; \beta, E_h)$ and $D_h^\alpha(\bar{I}; \beta, E_h)[i] = \pi(i)$. Then, all students accepted in both Step 1.1 and Step 1.2 will be in $D_h^s(\bar{I}; \beta, E_h)$ and they will be admitted without bonus points. Then, some of the students in κ_2 will not be in $D_h^s(\bar{I}; \beta, E_h)$. As explained above, there are at least $|\kappa_2 \setminus D_h^s(\bar{I}; \beta, E_h)|$ in $\Omega_2 \cap D_h^s(\bar{I}; \beta, E_h)$ admitted with bonus points. Then, $|D_h^s(\bar{I}; \beta, E_h)| > q_h$, which is a contradiction. To sum up, if a student who is rejected in Step 1.2 of PCR and does not receive bonus points at the end of this step will not be in $PCR_h^s(\bar{I}; \beta, E_h)$ and $D_h^s(\bar{I}; \beta, E_h)$.

Since for any Step 1.k where $k > 2$, we only need to consider the students included in A^k and students start to receive bonus points in the previous step, we can apply the same reasoning as above. All other students cannot be included in $D_h^s(\bar{I}; \beta, E_h)$. Moreover, as explained above in Step 1.K students who are not in A^K and start to receive bonus points in Step 1.$K-1$ cannot be in $D_h^s(\bar{I}; \beta, E_h)$. Then, we can show that any student not included in $PCR_h^s(\bar{I}; \beta, E_h)$ cannot be included in $D_h^s(\bar{I}; \beta, E_h)$.

□

Proof of Proposition 2. On the contrary, suppose $|C_h^s(\bar{I}; \beta, E_h) \cap \hat{I}| < |D_h^s(\bar{I}; \beta, E_h) \cap \hat{I}|$. Since C_h is q_h-acceptant, $|C_h^s(\bar{I}; \beta, E_h)| = \min\{q_h, |\bar{I}|\}$. If $|\bar{I}| \leq q_h$, then $|C_h^s(\bar{I}; \beta, E_h) \cap \hat{I}| = |\hat{I}| \geq |D_h^s(\bar{I}; \beta, E_h) \cap \hat{I}|$. Hence, we consider the case $q_h < |\bar{I}|$. When $q_h < |\bar{I}|$, we have $|C_h^s(\bar{I}; \beta, E_h)| = q_h$. Then, we have at least two students $i \in \hat{I}$ and $j \in \bar{I} \setminus \hat{I}$ such that $i \in D_h^s(\bar{I}; \beta, E_h) \setminus C_h^s(\bar{I}; \beta, E_h)$ and $j \in C_h^s(\bar{I}; \beta, E_h) \setminus D_h^s(\bar{I}; \beta, E_h)$. Without loss of generality, let i and j be such students with the lowest raw scores. Then, weak fairness and the fact that $j \notin \hat{I}$ imply that $\pi(j) > C_h^\alpha(\bar{I}; \beta, E_h)[i] = \pi(i)$ and $D_h^\alpha(\bar{I}; \beta, E_h)[i] = \pi(i) + \beta > \pi(j)$. Since C_h is fair and efficiently allocates privileges, there exist $p_h^{\tau(i)}$ students from $\tau(i)$ in $\bar{I} \cap E_h$ with higher raw scores than $\pi(i)$ and lower raw scores than $\pi(j)$ who receive bonus scores when C_h is applied to $\bar{I}$. Given i receives bonus under D_h at least one student k from $\tau(i)$ who receives bonus under C_h does not receive bonus under D_h. Recall that, $\pi(k) > \pi(i)$ and $k \in E_h$. Since $\pi(j) > \pi(k) = D_h^s(\bar{I}; \beta, E_h)[k]$ and $j \notin D_h^s(\bar{I}; \beta, E_h)$, $k \notin D_h^s(\bar{I}; \beta, E_h)$. However, selecting i with a lower score than k violates the weak fairness of D_h. □

Proof of Proposition 3. Since PCR is q_h-acceptant, if $|\bar{I}| \leq q_h$, then $|PCR^s_h(\bar{I};\beta,E_h) \cap \hat{I}| = |PCR^s_h(\bar{I};\hat{\beta},E_h) \cap \hat{I}|$.

Suppose $|\bar{I}| > q_h$. Since PCR is q_h-acceptant, $|PCR^s_h(\bar{I};\beta,E_h)| = |PCR^s_h(\bar{I};\hat{\beta},E_h)| = q_h$. On the contrary, suppose that $|PCR^s_h(\bar{I};\beta,E_h) \cap \hat{I}| < |PCR^s_h(\bar{I};\hat{\beta},E_h) \cap \hat{I}|$. Then, there exists at least two students $i \in \bar{I} \setminus \hat{I}$ and $j \in \hat{I}$, such that $i \in PCR^s_h(\bar{I};\beta,E_h)$ but $i \notin PCR^s_h(\bar{I};\hat{\beta},E_h)$, and $j \in PCR^s_h(\bar{I};\hat{\beta},E_h)$ but $j \notin PCR^s_h(\bar{I};\beta,E_h)$.

Since $i \notin \hat{I}$, $PCR^\alpha_h(\bar{I};\beta,E_h)[i] = \pi(i)$. Hence, fairness implies that $\pi(i) > \pi(j)$ and $\pi(j)+\hat{\beta} > \pi(i)$. Moreover, since $j \notin PCR^s_h(\bar{I};\beta,E_h)$, $i \in PCR^s_h(\bar{I};\beta,E_h)$, $\beta > \hat{\beta}$, and $\pi(j)+\hat{\beta} > \pi(i)$, we have $PCR^\alpha_h(\bar{I};\beta,E_h)[j] = \pi(j)$. Then, fairness implies that there exists at least one student k such that $\tau(k) = \tau(j)$, $\pi(k) > \pi(j)$, $PCR^\alpha_h(\bar{I};\beta,E_h)[k] = \pi(k)+\beta$, $PCR^\alpha_h(\bar{I};\hat{\beta},E_h)[k] = \pi(k)$, and $k \in PCR^s_h(\bar{I};\hat{\beta},E_h)$. Efficient allocation of privileges implies that $\pi(i) > \pi(k)$. Then, $\pi(i) > \pi(k)$, $k \in PCR^s_h(\bar{I};\hat{\beta},E_h)$, $i \notin PCR^s_h(\bar{I};\hat{\beta},E_h)$, and $PCR^\alpha_h(\bar{I};\hat{\beta},E_h)[k] = \pi(k)$ contradict the fairness. □

Proof of Proposition 4. Let $\hat{I} \subseteq \bar{I}$ be the set of students who have the top $q_h - p^m_h$ raw scores among students in $\bar{I}$. Under SEC_h, students in $\hat{I}$ are selected to fill the first $q_h - p^m_h$ seats, i.e., open seats. Under the PCR, independent of who gets the bonus score, students in $\hat{I}$ belong to the top q_h scoring students based on the updated scores after bonus scores are added to the raw scores. Hence, by definition of PCR, students in $I^m \cap \hat{I}$ do not receive a bonus score under PCR. In particular, students in $\hat{I}$ are never rejected in Step 1 of PCR.

Let $\tilde{I} \subseteq \bar{I} \setminus \hat{I}$ be the set of students who have the top p^m_h updated scores among students in $\bar{I} \setminus \hat{I}$ such that under updated scores every student in $\bar{I} \cap I^m$ receives bonus scores. Under SEC_h, students in $\tilde{I}$ are selected to fill the remaining p^m_h seats. Let $i \in \tilde{I} \setminus I^m$. Independent of who gets the bonus score when PCR is applied, student i has one of the top q_h students based on the updated scores in Step 2. Hence, all students in $\tilde{I} \setminus I^m$ will be selected by PCR. Suppose $i \in \tilde{I} \cap I^m$ and i is not selected by PCR. However, this contradicts the fairness of PCR. □

Proof of Theorem 2. Since PCR satisfies the LAD and substitutability conditions (Theorem 1), by Hatfield and Milgrom (2005), the DA-PCR is strategy-proof and stable.

Since no student applies to an unacceptable school and PCR satisfies q_h-acceptance, the DA-PCR is individually rational and nonwasteful.

Notice that in the last step of the DA, each high school h considers all students weakly preferring h to their match. Definitions of fairness for a choice rule and an assignment and Theorem 1 imply fairness of the DA-PCR. Similarly, definitions of efficient allocation of privileges for a choice rule and an assignment and Theorem 1 imply efficient allocation of privileges of the DA-PCR. □

Proof of Theorem 3. Consider an arbitrary problem (P, β, E). Let $(\bar{\mu}, \bar{\alpha})$ and $(\hat{\mu}, \hat{\alpha})$ be the assignments of $DA - PCR$ and Φ under problem (P, β, E), respectively. We show that, under $\hat{\mu}$, a student cannot be assigned to a school that has rejected her in the calculation of $DA - PCR[P, \beta, E] = (\bar{\mu}, \bar{\alpha})$.

Consider Step 1 of the calculation of $DA - PCR[P, \beta, E]$. If there is no student rejected in this step, then our desired result follows, i.e., under $\hat{\mu}$, no student is assigned to a school that has rejected her in the calculation of $DA - PCR[P, \beta, E]$. Suppose some student i is rejected by some school h in Step 1. We will show that $\hat{\mu}(i) \neq h$. On the contrary suppose that $\hat{\mu}(i) = h$. We consider the application of PCR to the students applying to h in Step 1 of DA-PCR. Let $\bar{I}$ denote that students. Since PCR is q_h-acceptant, $|\bar{I}| > q_h$. Let $\bar{I}^a$ be the selected students by PCR, i.e., $\bar{I}^a = PCR^s_h(\bar{I}; \beta, E_h)$. Due to fairness of PCR, all students in $\bar{I}^a \cap \bar{I}^{\tau(i)}$ have higher raw score than $\pi(i)$. As a result, since all such students rank h as top choice, fairness of Φ implies that all such students are also in $\hat{\mu}^{-1}(h)$.

Consider a student $j \in \bar{I}^a$ such that $PCR^\alpha_h(\bar{I}; \beta, E_h)[j] = \pi(j)$. Then, any student who is not selected or selected with bonus score under PCR has raw score less than $\pi(j)$. Therefore, $\pi(i) < \pi(j)$. If $PCR^\alpha_h(\bar{I}; \beta, E_h)[i] = \pi(i) + \beta$, then $\pi(j) > \pi(i) + \beta$ and fairness of Φ implies that $\hat{\mu}(j) = h$. If $PCR^\alpha_h(\bar{I}; \beta, E_h)[i] = \pi(i)$ and $\hat{\alpha}(i) = \pi(i)$, then, since $\pi(j) > \pi(i)$, fairness of Φ implies that $\hat{\mu}(j) = h$. If $PCR^\alpha_h(\bar{I}; \beta, E_h)[i] = \pi(i)$ and $\hat{\alpha}(i) = \pi(i) + \beta$, then there exists at least one student k such that $\tau(k) = \tau(i)$, $PCR^\alpha_h(\bar{I}; \beta, E_h)[k] = \pi(k) + \beta$, $\hat{\alpha}(k) = \pi(k)$ and $\hat{\mu}(k) = h$. By definition of PCR, $\pi(j) > \pi(k)$. Then, fairness of Φ implies that $\hat{\mu}(j) = h$. As a result, $\hat{\mu}(i) = h$ implies that any student in $\bar{I}$ who is accepted by PCR without bonus is in $\hat{\mu}^{-1}(h)$. Then, there exists at least one student $j \in PCR^s_h(\bar{I}; \beta, E_h)$ such that $PCR^{\alpha}_h(\bar{I};\beta,E_h)[j]=\pi(j)+\beta$ and $j\notin\hat{\mu}^{-1}(h)$. As explained above, $\tau(j)\neq\tau(i)$. We consider the following cases:

Case 1: $PCR^{\alpha}_h(\bar{I};\beta,E_h)[i]=\pi(i)+\beta$**.** Then, $\pi(j)>\pi(i)$. Hence, if $\hat{\alpha}(i)=\pi(i)$, then Φ cannot be fair. Suppose $\hat{\alpha}(i)=\pi(i)+\beta$. Since $\pi(j)+\beta>\pi(i)+\beta$, in order not to violate fairness, there are at least $p_h^{\tau(j)}$ students in $\hat{\mu}^{-1}(h)\cap I^{\tau(j)}$ receiving bonus scores. Moreover, if there exists a student $k\in PCR^{s}_h(\bar{I},\beta,E_h)$ such that $\tau(k)=\tau(j)$ and $PCR^{\alpha}_h(\bar{I},\beta,E_h)[k]=\pi(k)$, then $k\in\hat{\mu}^{-1}(h)$ and $\hat{\alpha}(k)=\pi(k)$. This implies that the number of students from $\tau(j)$ and in $PCR^{s}_h(\bar{I},\beta,E_h)$ is weakly less than the one in $\hat{\mu}^{-1}(h)$. Since this is true for any such j, we have $|\hat{\mu}^{-1}(h)|>q_h$.

Case 2: $PCR^{\alpha}_h(\bar{I};\beta,E_h)[i]=\pi(i)$**.** There are at least $p_h^{\tau(i)}$ students with type $\tau(i)$ who receive bonus scores in the calculation of $PCR_h(\bar{I};\beta,E_h)$ and $\pi(j)+\beta>\pi(i)$. We consider two subcases:

Case 2.1: $\hat{\alpha}(i)=\pi(i)$**.** Since $\pi(j)+\beta>\pi(i)$, to avoid violation of fairness, there are at least $p_h^{\tau(j)}$ students in $\hat{\mu}^{-1}(h)\cap I^{\tau(j)}$ receiving bonus scores. Moreover, due to efficient privilege allocation, if there exists a student $k\in PCR^{s}_h(\bar{I},\beta,E_h)\cap I^{\tau(j)}$ who does not receive a bonus, then she is in $\hat{\mu}^{-1}(h)$ and still does not receive bonus. This implies that the number of students from $\tau(j)$ and in $PCR^{s}_h(\bar{I},\beta,E_h)$ is weakly less than the one in $\hat{\mu}^{-1}(h)$. Since this is true for any such j, we have $|\hat{\mu}^{-1}(h)|>q_h$.

Case 2.2: $\hat{\alpha}(i)=\pi(i)+\beta$**.** There is at least one student $k\in\bar{I}$ with type $\tau(i)$ such that $k\in\hat{\mu}^{-1}(h)$, $\hat{\alpha}(k)=\pi(k)$, and $\pi(k)>\pi(i)$. Since PCR allocates privileges efficiently, $\pi(j)+\beta>\pi(k)$. To avoid violation of fairness, there are at least $p_h^{\tau(j)}$ students from $\tau(j)$ in $\hat{\mu}^{-1}(h)$ who receive bonus scores. Moreover, due to efficient privilege allocation, if a student who is in $PCR^{s}_h(\bar{I},\beta,E_h)$ and from $\tau(j)$ does not receive a bonus, then she is in $\hat{\mu}^{-1}(h)$ and still does not receive bonus. This implies that the number of students from $\tau(j)$ and in $PCR^{s}_h(\bar{I},\beta,E_h)$ is weakly less than the one in $\hat{\mu}^{-1}(h)$. Since this is true for any such j, we have $|\hat{\mu}^{-1}(h)|>q_h$.

This concludes that there does not exist a student who is assigned to the school that rejected her in the first step of DA-PCR under $\hat{\mu}$. Then, we can apply the steps described above the following steps one by one and show the desired result. □

C Choice Rules of Current Mechanisms

In this section, we define choice rules that mimic the selection procedures of the high schools in each step of the mechanisms used in practice. We represent these choice rules as 2-step choice rules. As explained before, there might be different variants of these choice rules. Below, we provide the definitions corresponding to the variants emphasized in Section 3.

CES: Recall that, under the CES mechanism, each middle school m selects a subset of its own students, with size p_h^m, as eligible for the privilege by high school h. Let $CES_h = (CES_h^s, CES_h^\alpha)$ denote the choice rule for the CES mechanism. Given a set of applicants $\bar{I}$, CES_h works as follows:

> **Step 1: Admission Score Profile Setting.** Select p_h^n highest score students from $\bar{I} \setminus E_h$ and denote it by $\bar{I}^o$. If $|\bar{I}^o| < p_h^n$ or $p_h^n = 0$, we set $c_h = 0$. Otherwise, set c_h to the minimum score of students in $\bar{I}^o$. Let $\bar{I}^p = \{i \in E_h \cap \bar{I} : \pi(i) + \beta \geq c_h\}$. For each $i \in \bar{I}^p$, set $CES_h^\alpha(\bar{I}; \beta, E_h)[i] = \pi(i) + \beta$. For each $i \in \bar{I} \setminus \bar{I}^p$, set $CES_h^\alpha(\bar{I}; \beta, E_h)[i] = \pi(i)$.
>
> **Step 2: Applicant Selection.** Select the highest p_h^n scoring students under $(CES_h^\alpha(\bar{I}; \beta, E_h)[i])_{i \in \bar{I}}$ and add them to $CES_h^s(\bar{I}; \beta, E_h)$.

CAS-1: Under CAS-1, each middle school m with $p_h^m > 0$ sets all its students as eligible for the privilege by high school h. That is, under CAS-1, $E_h^m = I^m$ for every middle school m with $p_h^m > 0$ and $E_h^m = \emptyset$ for every middle school m with $p_h^m = 0$. Let $CAS1_h = (CAS1_h^s, CAS1_h^\alpha)$ denote the choice rule for CAS-1 mechanism. Given a set of applicants $\bar{I}$, $CAS1_h$ works as follows:

> **Step 1: Admission Score Profile Setting.** Select p_h^n highest score students from the set of normal applicants, i.e., $\bar{I} \setminus E_h$, and denote it by $\bar{I}^o$. If $|\bar{I}^o| < p_h^n$ or $p_h^n = 0$, we set $c_h = 0$. Otherwise, set c_h to the minimum score of students in $\bar{I}^o$. For each middle school m, denote the subset of students in $\{i \in \bar{I}_m \setminus \bar{I}^o : \pi(i) + \beta \geq c_h\}$ and have highest p_h^m raw score among such students with $\bar{I}_m^p$. Let $\bar{I}^p = \cup_{m \in M} \bar{I}_m^p$. For each $i \in \bar{I}^p$, set $CAS1_h^\alpha(\bar{I}; \beta, E_h)[i] = \pi(i) + \beta$. For each $i \in \bar{I} \setminus \bar{I}^p$, set $CAS1_h^\alpha(\bar{I}; \beta, E_h)[i] = \pi(i)$.

Step 2: Applicant Selection. Select the highest q_h scoring students under $(CAS1_h^{\alpha}(\bar{I};\beta,E_h)[i])_{i\in\bar{I}}$ and add them to $CAS1_h^{s}(\bar{I};\beta,E_h)$.

CAS-2: Under CAS-2, each middle school m with $p_h^m > 0$ sets all its students as eligible for the privilege by high school h. That is, under CAS-2, $E_h^m = I^m$ for every middle school m with $p_h^m > 0$ and $E_h^m = \emptyset$ for every middle school m with $p_h^m = 0$. Let $CAS2_h = (CAS2_h^s, CAS2_h^{\alpha})$ denote the choice rule for CAS-2 mechanism. Given a set of applicants $\bar{I}$, $CAS2_h$ works as follows:

Step 1: Admission Score Profile Setting. For each middle school m, denote the subset of students in $\bar{I}_m \cap E_h$ and have highest p_h^m raw score among such students with $\bar{I}_m^p$. Let $\bar{I}^p = \cup_{m\in M}\bar{I}_m^p$. For each $i \in \bar{I}^p$, set $CAS2_h^{\alpha}(\bar{I};\beta,E_h)[i] = \pi(i)+\beta$. For each $i \in \bar{I}\setminus\bar{I}^p$, set $CAS2_h^{\alpha}(\bar{I};\beta,E_h)[i] = \pi(i)$.

Step 2: Applicant Selection. Select the highest q_h scoring students under $(CAS2_h^{\alpha}(\bar{I};\beta,E_h)[i])_{i\in\bar{I}}$ and add them to $CAS2_h^{s}(\bar{I};\beta,E_h)$.

We emphasize that these choice rules differ based on the privilege selection step. We state the properties of these choice rules in the following proposition.

Proposition 5. *Consider a high school h.*

a) CES_h, $CAS1_h$, and $CAS2_h$ satisfy q_h-acceptance, and therefore LAD.

b) $CAS2_h$ satisfies fairness and substitutes condition.

c) CES_h, $CAS1_h$, and $CAS2_h$ fail to satisfy efficient allocation of privileges. In addition, CES_h and $CAS1_h$ fail to satisfy fairness and substitutes condition.

Proof. a) Recall that, independent of the privilege selection procedure, any 2-step choice rule is q_h-acceptant and, therefore, satisfies LAD. Since we represent these three choice rules as 2-step choice rules, they satisfy q_h-acceptance, and therefore LAD.

b) By definition, $CAS2_h$ selects students by their admission scores in Step 2. Moreover, if a student from some middle school m does not receive bonus score, then p_h^m students from m with higher raw scores receive bonus score. Hence, $CAS2_h$ is fair.

Consider an arbitrary set of students $\bar{I}$. Suppose $i \in (\bar{I} \setminus CAS2^s_h(\bar{I};\beta,E_h))$. Let $\hat{I} = \bar{I} \cup \{j\}$ and $j \notin \bar{I}$. A student not receiving bonus scores in Step 1 under the calculation of $CAS2_h(\bar{I};\beta,E_h)$ will not receive bonus scores under the calculation of $CAS2_h(\hat{I};\beta,E_h)$. Moreover, the minimum admission score of the selected students will weakly increase when we apply $CAS2_h$ to $\hat{I}$. Hence, $i \notin CAS2^s_h(\hat{I};\beta,E_h)$, i.e., the $CAS2$ choice rule is substitutable.

c) Example A.5 shows neither $CAS1_h$ nor $CAS2_h$ satisfies efficient allocation of privileges. Example A.6 shows CES_h does not satisfy efficient allocation of privileges. Example A.7 shows both CES_h and $CAS1_h$ fail to satisfy fairness and substitutes condition.

Example A.5. *Let* $I = \{i_1, i_2, i_3\}$, $I^m = \{i_1, i_2\}$, $I^{m'} = \{i_3\}$, $I^{m''} = \emptyset$, $p^m_h = p^{m''}_h = 1$, $p^{m'}_h = 0$ *and* $q_h = 2$. *Students' raw scores are:* $\pi(i_1) = 100$, $\pi(i_2) = 40$, *and* $\pi(i_3) = 50$. *Let* $\beta = 20$.

We apply $CAS1_h$ *and* $CAS2_h$ *choice rules to* I. *Under both choice rules, in Step 1,* i_1 *will receive bonus score. As a result, in Step 2,* i_1 *and* i_3 *will be selected. It is easy to see that* i_1 *will have the highest admission score regardless of whether she receives bonus scores or not. We could have selected one more student from* m, *which has a privilege capacity at* h, *if we gave the bonus score to* i_2 *instead of* i_1.

Example A.6. *Let* $I = \{i_1, i_2, i_3, i_4\}$, $I^m = \{i_1, i_2\}$, $I^{m'} = \{i_3, i_4\}$, $p^m_h = 2$, $p^{m'}_h = 0$ *and* $q_h = 3$. *Students' raw scores are:* $\pi(i_1) = 100$, $\pi(i_2) = 40$, $\pi(i_3) = 80$, *and* $\pi(i_4) = 50$. *Let* $\beta = 20$. *Let* $E^m_h = \{i_1, i_2\}$.

We apply CES_h *choice rule to* I. *In Step 1,* i_1 *will receive bonus score. As a result, in the application selection step,* i_1, i_3, *and* i_4 *will be selected. It is easy to see that* i_1 *will have the highest admission score no matter whether she receives bonus scores or not. We could have selected one more student from* m, *if we gave the bonus score to* i_2.

Example A.7. *Let* $M = \{m, m', m''\}$, $I = \{i_1, i_2, i_3, i_4, i_5\}$, $I^m = \{i_1\}$, $I^{m'} = \{i_2\}$, $I^{m''} = \{i_3, i_4, i_5\}$, $p^m_h = p^{m'}_h = 1$, $p^{m''}_h = 0$, $q = 3$, *and* $\beta = 30$. *Students' raw scores are:* $\pi(i_1) = 50$, $\pi(i_2) = 40$, $\pi(i_3) = 55$, $\pi(i_4) = 60$, $\pi(i_5) = 90$. *Under CES,* m *and* m' *select their students as eligible for privilege.*

We first consider $\bar{I} = \{i_1, i_2, i_3, i_4\}$*. Under* CES_h *and* $CAS1_h$*,* i_1 *and* i_2 *receive bonus scores in Step 1. Then, we have* $CES^s_h(\bar{I}; \beta, E_h) = CAS1^s_h(\bar{I}; \beta, E_h) = \{i_1, i_2, i_4\}$*.*

Now, consider $I = \bar{I} \cup \{i_5\}$*. Under* CES *and* $CAS1_h$*, neither* i_1 *nor* i_2 *receives bonus scores in Step 1. Then, we have* $CES^s_h(I; \beta, E_h) = CAS1^s_h(I; \beta, E_h) = \{i_3, i_4, i_5\}$*. Hence, both* $CAS1_h$ *and* CES_h *fail to be substituable.*

Notice that, under CES *and* $CAS1$*, when* I *is considered, no student from* m *receives bonus scores and* $\pi(i_1) + \beta$ *exceeds* i_3*'s admission score. Hence, these choice rules fail to be fair.*

□

D Survey Validity

In this section, we discuss the validity of the survey data (also see Wang and Zhou 2024). We conducted the survey in early May 2014, just two weeks before student submitted their ROLs, so a shift in their preferences within that brief window seems unlikely. Students were explicitly asked to report their true preferences over their attainable schools, and there was no compelling reason for them not to honor this request.

Unlike typical surveys that aim to uncover students' preferences regarding their favorite choices (Budish and Cantillon 2012, Kapor et al. 2020), we did not ask students to simply rank their favorite schools. Instead, we asked them to rank schools to which they thought they might be admitted based on their true preferences.

Our survey design aims to prevent low-scoring students from ranking schools where they have no chance of admission, despite the fact that they could include three schools with low cutoffs in their ROLs. This approach seeks to minimize the underreporting of lower-tier schools. However, if underreporting occurs, it may result in less accurate estimates of preferences for lower-tier schools, making it difficult to distinguish between students' preferences over these schools.[46] Furthermore, we cannot ask students to rank too many schools

[46] For example, if there are four schools, namely A, B, C and D. Most students prefer school A over B. Therefore, we can only infer preferences between A and B but not between lower-tier schools C and D.

in the survey. In the admission procedure, students can choose from over 50 schools, including public high schools, fine arts schools, and vocational schools.[47] Generally, when asking respondents to rank items according to their preferences, presenting respondents with too many choices can lead to cognitive overload, resulting in lower response rates and decreased accuracy (Groves et al. 2009). Therefore, asking students to rank five attainable schools is a reasonable design. Table D.1 shows that all public high schools received substantial representation in students' responses. In particular, each of the three leftover schools was chosen by more than 100 low-scoring students, while these schools were seldom mentioned by high-scoring students.

The validity of our survey can also be confirmed from students' responses. Figure 1 shows the average admission cutoffs of schools chosen by students both in the survey and in their ROLs. In the survey, the average cutoff gap between consecutive choices within each student percentile group is around six points. The consistency between the first choices in the survey and the ROLs implies that students prefer to apply to their favorite attainable schools. This consistency, along with the small cutoff gaps among choices reported in the survey, provides further evidence that the surveyed students accurately reported their five favorite attainable schools.

E Details of the Empirical Estimation

In this section, we describe the approach to estimate students' preferences (also see Wang and Zhou 2024). The utility function of students in Equation (1) is similar to that in Abdulkadiroğlu et al. (2017) and Agarwal and Somaini (2018), except that we do not present the random coefficient model for estimating students' heterogeneous preferences for observed school characteristics, owing to our data's limited variation. As a robustness check, we present an alternative random coefficient model, which performs worse than the non-random coefficient model in the out-of-sample tests. Note that the high school admission process

[47] Our analysis focuses on students whose scores exceed the threshold for admission to public high schools. Students with scores below the threshold participate in a separate admission process, which takes place after the public high school admissions are finalized. Since our survey was conducted before the entrance exam, students could list any school in their survey responses.

Table D.1: Schools' Frequencies of Occurrence in the Survey

		Students' Score in Percentile			
		Above 90th	90th-80th	80th-70th	Below 70th
School ID	Total Freq.	Freq.	Freq.	Freq.	Freq.
141	904	415	249	148	92
142*	153	6	12	43	92
147	421	31	79	126	185
165†	98	38	47	11	2
166†	82	11	33	33	5
167	1330	387	380	326	237
169†	152	57	70	21	4
173	129	9	20	43	57
177†	133	30	69	27	7
179	1080	100	231	358	391
180†	90	5	38	27	20
181*	193	10	23	45	115
183	677	331	152	102	92
184*	409	27	71	128	183
185	1652	322	415	485	430
186	1219	168	302	393	356
187	1155	393	342	251	169
188	412	53	92	129	138
200†	57	19	31	7	0

Notes: This table indicates the frequency of occurrence of each school in the survey. * indicates the leftover schools. † indicates the special classes. The second column shows the total frequency of occurrence, the third to sixth columns show the frequency of occurrence in different student scoring groups by categorized by score percentile.

in this city is citywide and does not consider the locations of school districts or homes. Therefore, we assume that the school choice mechanism does not directly influence residential decisions in the city.

We use the survey data to estimate student preferences. The advantage of survey data is that our estimates can proceed without having to account for students' strategic behavior when they submit their ROLs. Recall that each surveyed student ranked five schools that she believed herself capable of attending. This procedure implies that the student first selects the schools for where admission is possible and then ranks them. That process complicates our construction of a model showing how these students select schools in the first place. For example, if a school with a high admission cutoff does not make the surveyed student's list, then it is difficult to distinguish between (a) her preferring the listed schools to the focal school and (b) her thinking that admission to the high–cutoff school is not possible. From the evidence presented in Appendix D, we conclude that the survey responses represent students' true relative preferences—that is, conditional on their belief in the possibility of admission. To simplify the estimation process, we focus on the rankings of the listed schools (i.e., without considering the unlisted schools). In other words, we do not attempt to infer the relative ranks of listed and unlisted schools. This approach renders the estimate less efficient, but the estimation is consistent when surveyed students report their true relative rankings. For example, suppose student i lists high school h_1 before h_2 in her survey; these two high schools' admission cutoffs are S_{h_1} and S_{h_2}, respectively. Then the probability that i prefers h_1 to h_2 conditional on these two schools being attainable to her, is $Pr(u_{i,h_1} > u_{i,h_2}|\tilde{\alpha}_{i,h_1} > S_{h_1} \cap \tilde{\alpha}_{i,h_2} > S_{h_2})$, where $\tilde{\alpha}_{i,h}$ denotes i's estimated admission score to school h. This conditional probability equals to the unconditional probability, i.e.,

$$\begin{aligned} Pr(u_{i,h_1} > u_{i,h_2}|\tilde{\alpha}_{i,h_1} > S_{h_1} \cap \tilde{\alpha}_{i,h_2} > S_{h_2}) &= \frac{Pr(u_{i,h_1} > u_{i,h_2} \cap \tilde{\alpha}_{i,h_1} \geq S_{h_1} \cap \tilde{\alpha}_{i,h_2} \geq S_{h_2})}{Pr(\tilde{\alpha}_{i,h_1} \geq S_{h_1} \cap \tilde{\alpha}_{i,h_2} \geq S_{h_2})} \\ &= \frac{Pr(u_{i,h_1} > u_{i,h_2})Pr(\tilde{\alpha}_{i,h_1} \geq S_{h_1} \cap \tilde{\alpha}_{i,h_2} \geq S_{h_2})}{Pr(\tilde{\alpha}_{i,h_1} \geq S_{h_1} \cap \tilde{\alpha}_{i,h_2} \geq S_{h_2})} \\ &= Pr(u_{i,h_1} > u_{i,h_2}). \end{aligned} \quad (2)$$

The second equality follows from the fact that students' beliefs about the admission probability only affect whether researchers can observe students' preferences in the survey, but do not affect the relative positions of these preferences. For example, suppose that student i's true preference over all schools is $h_1 P_i h_2 P_i h_3 P_i \dots$, i.e., she prefers h_k to $h_{k'}$ when $k < k'$. If the selected set of schools in the survey is $h_5 P_i h_8 P_i h_{12}$, then the relative rank of any two of these schools still preserves the relationship: $h_k P_i h_{k'}$ with $k < k'$, regardless of how these schools are selected into the survey.

In short, our approach is based on the assumption that the process of selecting the feasible set is independent of preferences and the preference ranking among schools satisfies the independence of irrelevant alternatives (IIA). Thus, the rankings are independent of the set of schools selected, as long as the top five schools in a feasible set are chosen. Eq.(2) indicates that the relative preference between any two schools is independent of the set of schools picked in the survey. Note that we do not assume the selection of feasible schools in the survey is independent of students' scores or that students with the same scores select the same set of schools.

Using the survey data, we apply the rank-ordered logit model to estimate coefficients. Given a surveyed student i's ranked school list $(h_1, \dots, h_l)$ of length $l \leq 5$, we conclude that $h_1 P_i h_2 \cdots P_i h_l$. The joint probability of these choices is

$$\Pr(u_{i,h_1} > u_{i,h_2} > \cdots > u_{i,h_l}) = \prod_{k=1}^{l-1} \frac{e^{\hat{u}_{i,h_k}}}{e^{\hat{u}_{i,h_k}} + e^{\hat{u}_{i,h_{k+1}}} + \cdots + e^{\hat{u}_{i,h_l}}},$$

where $\hat{u}_{i,h}$ is the deterministic component of $u_{i,h}$ or $u_{i,o}$; and the log-likelihood function can be written as

$$\log L(\boldsymbol{\beta}) = \sum_{i=1}^{n} \sum_{k=1}^{l-1} \hat{u}_{i,h_k} - \sum_{i=1}^{n} \sum_{k=1}^{l-1} \log \left(\sum_{s=k}^{l} e^{\hat{u}_{i,h_s}} \right).$$

Then we can estimate coefficients using maximum likelihood estimation.

F Supplement Summary Statistics and Empirical Results

Table F.1: School Characteristics

	Number	Mean	S.D.	Max	Min
Number of high schools	13				
Number of special classes*	6				
Number of high schools provide privileges	8				
Admission capacity of high schools		400	136.6	600	80
Privilege capacity of high schools†		247.25	104.72	390	26
Privilege capacity/total capacity†		0.61	0.11	0.65	0.33
Reputation		82.4	11.2	97	65.7
Number of Middle schools	44				
Number of Middle schools receive privileges	43				
Privilege capacity of middle schools‡		45.3	19.2	79	11
Received privilege/total graduates‡		0.14	0.01	0.18	0.12
Total privilege capacity	1,946				
Total admission capacity	5,560				

Notes: * the capacity of every special class is 40. † only considers the high schools providing privilege capacities. ‡ only considers the middle schools receiving privilege capacities.

Table F.2: ROLs and Assignments

	Privileged Students		Normal Students	
	No.	Percent	No.	Percent
Rank Ordered Lists				
3 Schools	1,789	91.93%	3100	93.71%
2 Schools	148	7.61%	189	5.71%
1 Schools	9	0.46%	19	0.57%
Assignment Results				
1st Choice	1,760	90.44%	875	26.45%
2nd Choice	60	3.08%	1,290	39%
3rd Choice	55	2.83%	565	17.08%
Rejected by all 3	71	3.65%	578	17.47%
Total observations	1,946		3,308	

Table F.3: Whole Sample vs. Survey Sample

	Whole Sample			Survey Sample		
Score Distributions						
High-Scoring (>90th)	24.7%			23.5%		
Medium-Scoring (80th-90th)	50.5%			52.2%		
Low-Scoring (<70th)	24.8%			24.4%		
	Mean	S.D.		Mean	S.D.	
Total	579.5	25.1		576.9	576.5	
Assignment Results						
1st Choice	51%			51.6%		
2nd Choice	26.2%			26%		
3rd Choice	11.4%			11.7%		
Rejected by all three choices	11.4%			10.6%		
Average School Cutoffs in ROLs						
	1st Choice	2nd Choice	3rd Choice	1st Choice	2nd Choice	3rd Choice
>90th	607.4	587.2	542.6	606.9	582.3	543.3
80th-90th	601.1	572.3	539.1	600.8	569.4	539.3
70th-80th	592.8	563.7	537.4	592.3	562.4	537.4
<70th	581	553.3	535.9	580.2	551.8	536.4

Notes: Due to scheduling conflicts, some middle schools did not participate in our survey. In this table, we compare the survey sample with the whole sample to determine if our survey accurately represents the whole sample. The first panel shows the score distribution of the whole sample and the survey population. Students are categorized by score percentile. The second panel indicates the assignment results of the whole sample and the survey sample. Third panel indicates the average admission cutoffs of schools chosen by the whole sample and the survey sample in ROLs. Students are grouped into four categories by score percentile.

Table F.4: Survey Length

	Privileged Students Freq.	Normal Students Freq.	Percent
5 schools	683	900	63.75%
4 schools	130	242	14.98%
3 schools	97	130	9.14%
2 schools	126	175	12.12%
Total	1036	1447	

Notes: This table indicates how many schools are listed by surveyed students.

Table F.5: Admission Cutoffs

School ID	True Cutoffs (1)	Predicted (2)	Diff. (3)
141	605.0	600.5	4.5
142*	535.0	535.0	0.0
147	558.0	562.2	-4.2
165†	609.0	612.2	-3.2
166†	595.0	598.9	-3.9
167	593.5	590.3	3.2
169†	604.0	608.7	-4.7
173	552.0	555.0	-3.0
177†	600.5	600.8	-0.3
179	573.5	572.7	0.8
180†	584.5	590.4	-5.9
181*	535.0	535.0	0.0
183	611.0	607.5	3.5
184*	535.0	535.0	0.0
185	583.0	579.9	3.1
186	576.0	578.4	-2.4
187	596.5	593.4	3.1
188	580.0	580.5	-0.5
200†	607.0	612.2	-5.2

Notes: This table indicates the out-of-sample test for the schools' cutoffs. The full mark is 665. The threshold is 535 in 2014. * represents the leftover school with cutoff equal to the threshold. † represents the special class.

Table F.6: Admission Patterns (%)

	True Data	Predicted	Diff.
Normal 1st Choice	26.45	28.59	-2.14
Normal 2nd Choice	39	32.96	6.04
Indexed 1st Choice	90.44	93.8	-3.36
Indexed 2nd Choice	3.08	1.28	1.8

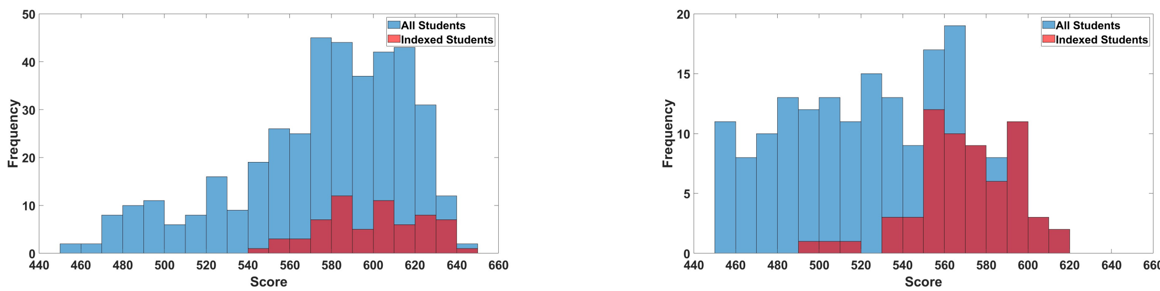
Notes: This table indicates the out-of-sample tests of the matching patterns for the 1st and 2nd choices for both normal and privileged students in the ROLs.

Table F.7: Admission under Different Matching Mechanism in the Counterfactual Analysis

	CES		CAS-2		DA-PCR	
	Top high	Moderate high	Top high	Moderate high	Top high	Moderate high
20%						
Top middle schools	578.8	706.5	573.8	712.4	548.3	723.0
Upper median middle schools	221.1	601.2	220.3	602.8	240.9	578.9
Lower median middle schools	145.1	283.6	148.7	281.3	149.9	290.1
Bottom middle schools	56.1	120.6	58.2	123.4	61.9	128.1
40%						
Top middle schools	568.2	715.9	552.9	713.8	512.7	731.3
Upper median middle schools	217.4	604.5	219.8	587.2	245.8	558.9
Lower median middle schools	148.9	277.6	157.5	282.4	166.3	283.4
Bottom middle schools	66.5	122.0	70.8	136.6	76.3	146.4
60%						
Top middle schools	533.9	711.7	510.1	699.9	461.6	737.7
Upper median middle schools	222.3	573.7	223.1	541.6	265.0	511.1
Lower median middle schools	164.0	282.5	178.6	306.9	187.5	306.3
Bottom middle schools	80.8	152.1	89.2	171.6	86.9	164.9

Notes: This table shows the number of students admitted to top and moderate high schools from middle schools of different levels in the counterfactual analysis. The results are presented for privilege capacities of 20%, 40%, and 60% across the three panels.

Figure F.1: Score distributions of the privileged students vs. all students in two schools



(a) A top-ranked middle school (School 111)

(b) A bottom-ranked middle school (School 62)

Notes: These histograms display the raw scores of all students and the raw scores of privileged students from the same middle school. These two schools have comparable size of graduates.

G Simulations in Counterfactual Analysis

In this section, we outline the simulation procedure used for both the counterfactual analysis and the out-of-sample test. For each mechanism, the reported results are based on the average of 3,000 simulations. To calculate matching outcomes under different mechanisms, the procedure is described as follows.[48]

[48]We would like to highlight that the CES used in the counterfactual analysis differs from that in the out-of-sample test. Therefore, the simulation procedure for the former is presented in Part 2, while that for the latter is described in Part 3.

Part 1. Generate utility functions

For each student i, we draw a value of $|H|$ dimensional vector of errors ε_i from type I extreme value distribution. Note that $|H|$ is the number of public high schools in the students' choice set. Label the draw ε_i^d with d and the elements of the draw as $\varepsilon_{i1}^d, ..., \varepsilon_{i|H|}^d$, where d denotes the dth draw from a total number of draws. Then we calculate the utility function as, $u_{i,h}^d = \hat{u}_{i,h} + \varepsilon_{ih}^d$, where $\hat{u}_{i,h}$ is the deterministic part of the utility function, i.e., student i's utility from getting into public high school h (the parameters come from Column (3) and (4) of Table 4).

Part 2. Run the matching mechanisms in the counterfactual analysis

Since our theoretical results are based on a complete information environment and apply to the general case—where the privilege bonus can be applied to any choice in the ROL—we adopt this setup in the counterfactual analysis.

The DA-PCR and the CAS-2:

Since both the DA-PCR and CAS-2 mechanisms are strategy-proof, we treat students' true preferences over all schools as their reported ROLs.[49] Each student is allowed to list all high schools in their ROL. We then apply the matching algorithm outlined in Section 4.3 for DA-PCR and the algorithm in Section 3.2 for CAS-2 to match students to schools.

The CES in the counterfactual analysis:

To align with our theoretical results, we use the DA as the base mechanism and allow the privilege bonus to apply to any choice in the ROL. Note that CES is not strategy-proof even if the CES choice rules are embedded to DA, thus it is difficult to calculate a pure strategy of all students under the complete information environment. Instead, we use the following way to simplify the simulation.

- Step 1. In the early selection stage, each middle school follows a sequential order based on students' exam scores,[50] allowing students to choose their privileges in turn. When

[49] Under CAS-2, we allow students to use their privilege bonus for any choice.

[50] As noted in Section 6.2, since students' rankings are not observable at this stage, we use exam scores as a proxy for their rankings within their middle schools.

student i makes her decision, she lists all high schools according to her true preferences and calculates the expected payoff for each still-available privilege from these schools.

From student i's perspective, we assume she uses the schools' admission cutoffs from the previous year as her prior belief. She anticipates her exam score to be $m_i + \eta_i$, where m_i represents either her mock exam score or her true ability (used to estimate her exam score), and η_i captures the uncertainty. We also assume η_i is i.i.d. and follows a normal distribution, $N(0, \delta)$. Since m_i cannot be directly observed in the data, we use the student's actual exam score, $\pi(i)$, as a proxy for m_i. For simplicity, we set $\delta = 20$, which corresponds to 3% of the full score. Student i then selects the privilege from the high school that offers the maximum expected payoff.

- Step 2. In the normal admission stage, each student reports her true preferences over all high schools and receives the privilege bonus from the high school that is selected from the early selection stage. Then students are matched to high schools following the CES mechanism.

Note that, due to the complexity of calculating the pure strategy equilibrium under our theoretical setup, the resulting matching outcome is not an equilibrium outcome. However, since our goal is to highlight the weaknesses of the CES in the matching process, this simplified analysis does not undermine any of our conclusions or counterfactual analysis.

Part 3. Run the CES in the out-of-sample test

In the out-of-sample test in Section 7.1, we aim to mimic students' strategic behavior in reality. To achieve this, we simulate an incomplete information environment (e.g., Calsamiglia et al. 2020; Wang and Zhou 2024), where students face uncertainty about their exam scores and maximize their expected payoffs.

Similar as described in Part 2. From the perspective of student i, he anticipates her exam score to be $\tilde{\pi}_i + \eta_i$, where $\tilde{\pi}_i$ is i's expected exam score, such as her mock exam score, and η_i is i.i.d. and follows a normal distribution $N(0, \delta)$ with $\delta = 20$. In the simulation, we use her true score $\pi(i)$ as a proxy. Given any school's admission cutoff, students can then calculate their admission probability and the expected payoff for any potential ROL.

To align with the CES used in our focal city, we adopt the Chinese parallel mechanism with a permanency execution vector of (2,1) as the base mechanism. privileged students, who receive a bonus from school h, are required to list h as their first choice in their ROL. We then calculate the equilibrium outcomes as follows:

- Step 1. Use the admission cutoffs from the previous year as the first prior belief for all the students.
- Step 2. For a given belief of the admission cutoffs, calculate each student's optimal strategy as follows:
  - *Step 2.1* Calculate the expected payoff of each student's potential ROLs as a normal student without any privileges, then choose the optimal ROL for each student with the highest payoff. If the optimal ROL is not unique, then randomly select one of these optimal ROLs. Here the admission probability of each school used to calculate the expected payoff comes from the belief of the admission cutoffs, which is the true school cutoff from the previous iteration (See Step 3), and students' expected scores are their true exam scores.
  - *Step 2.2* In the early selection stage, for each high school h, each student i calculates the expected payoff if she becomes a privileged student of h. This payoff is calculated as follows: Given the belief of the admission cutoffs, i must indicate h as her first choice in the ROL, then calculate the optimal option for the rest choices in the ROL, considering that she will receive bonus points for her first choice. When the optimal choice is not unique, we randomly select one.
  - *Step 2.3* Each student will form an optimal strategy as follows: In the early selection stage, student i ranks all the high schools based on the expected payoffs calculated in Step 2.2 as a privileged student, together with her optimal ROL as a normal student calculated in Step 2.1. In the normal admission stage, if i becomes a privileged student of h, then she will submit the optimal ROL calculated in Step 2.2 for h. If i is a normal student, then she will submit the optimal ROL calculated in Step 2.1.

- Step 3. Given students' optimal strategies, run the matching algorithm of the CES to match students to high schools. The matching outcome from this step will generate new admission cutoffs for schools. Use these cutoffs as the new belief to replace the previous one.

- Step 4. Rank all the students by their exam scores. Start this step with the first student, and let $k = 1$. Fix all the other students' strategy, and calculate the k-th student's optimal response (using the method in Step 2) to the belief generated in Step 3. If at least one deviation exists, either in the early selection stage or in the normal admission stage, for this student to strictly increase her expected payoff, then proceed to Step 5. If no deviation exists for this student to become strictly better off, then repeat Step 4 for the $k+1$-th student and set $k = k + 1$ when $k < N$. When $k = N$, the algorithm will proceed to Step 6.

- Step 5. Choose the optimal strategy of the k-th student in Step 4 as the new strategy in the submitted ROL. If the optimal strategy is not unique, then randomly choose one of optimal strategies. Thereafter, proceed to Step 3.

- Step 6. The current students' strategies are the equilibrium strategies. The matching outcome is the equilibrium outcome.