# Electoral incentives: the interaction between candidate selection and electoral rules*

Benoit S Y Crutzen† Nicolas Sahuguet‡

Erasmus University Rotterdam HEC Montréal

Latest version: October 2018

**Abstract**

Politicians are rational actors who respond to incentives. We study how the interplay between intra- and interparty competition generates incentives for politicians to cater to the voters' interests. We show that the rules governing the intraparty game to become a party candidate have a first order effect on the link between nationwide political institutions such as the electoral rule and electoral outcomes. Our findings also point to a causal link between the electoral rule and the way political parties organize. Thus, empirical exercises in political economy should not disregard the characteristics of intraparty game.

*Valuable feedback and suggestions on different parts of this project were provided by participants at APET 2017, CETC 2017, EPSA 2016, Lancaster GTC 2016, Laval, Mannheim, Mont Saint Anne 2018, MPSA 2016, Petralia 2016, Rotterdam, Rovira i Virgili, SAET 2017, UQAM and in particular by Peter Buisseret, Micael Castanheira, Torun Dewan, Olle Folke, Thomas Konig, Kai Konrad, Andrea Mattozzi, Nicola Persico, Carlo Prato, Johanna Rickne, Gerard Roland, Francesco Squintani, Otto Swank and Galina Zudenkova.

†Department of Economics, Erasmus School of Economics, Erasmus Universiteit Rotterdam. crutzen@ese.eur.nl.

‡Department of Applied Economics, HEC Montréal. nicolas.sahuguet@hec.ca

# 1 Introduction

Incentives matter, even for politicians. Competition for office is the obvious source of such incentives. This competition is waged not only across parties, but also within parties. In this paper, we develop a novel theory of the interplay between intra- and interparty incentives, to understand the impact of intra- and interparty competition on electoral outcomes.

Our model generates two fundamental implications. First, we highlight the importance of the intraparty stage. For example, all else equal, the rules governing intraparty competition affect the set of candidates voters can choose from. As the set of available candidates impacts the electorate's voting decisions, our results suggest we cannot understand the mapping from the rules governing interparty competition to electoral outcomes without taking into account the rules governing intraparty competition. If the rules governing the intraparty game are not accounted for, researchers may wrongly attribute any observed differences in electoral outcomes to the rules of the interparty game, whereas these differences may be driven by differences at the intraparty level. Could the differences in electoral outcomes observed between France and the US (two democracies relying on plurality rule in single member districts for their legislative elections) be driven at least in part by differences in the intraparty game? Could differences in the way parties organize also explain (part of) the differences observed in Iceland and Israel (two democracies in which the electoral rule is closed list proportional representation)?

The second prediction of our results, which reinforces the first, is about the existence of a causal link from the electoral rule to the way parties organize, as different electoral rules impact differentially the incentives politicians face. Then, a rational, strategic party leadership should take the electoral rule into account when designing the internal organization of their party. This appears to be indeed the case. Political parties exhibit many different forms. They go from the American model of the party as a decentralized and candidate-centered organization – Katz and Kolodny (1994) refer to the two main American parties as "empty

vessels" given the dominant position individual incumbents seem to have in US politics – to European parties which are highly hierarchical and structured organizations that regulate public affairs, such as all the mainstream parties in The Netherlands or Japan.[1]

Our model focuses on the electoral rule as the main institution governing incentives generated by competition between parties, as this is the main institution governing legislative elections. We consider the two electoral rules that dominate elections worldwide: US- or British-style plurality rule in single member districts (FPTP hereafter) in which voters can cast their ballot for the candidate they prefer, and Israeli-style closed-list proportional representation in a single nationwide district (PR hereafter), in which voters can only vote for the list of candidates parties put forward, and not for any individual politician.

We concentrate on the candidate selection process as the key driver of competition inside parties. Our choice is due to the fact that, as stated by Norris (2006), "[T]he process of recruitment to elected and appointed office is widely regarded as one of the most important residual functions of parties, with potential consequences for [...] the accountability of elected members". Gallagher and Marsh (1988) even refer to this intraparty stage as "the secret garden of politics". The profound effects the introduction of primaries in the US between the late 1890s and 1915 had on electoral competition and outcomes is a key example of how intra- and interparty competition interact in impacting policy outcomes. Jackson, Mathevet and Mattes (2007), Adams and Merrill (2008), Castanheira, Crutzen and Sahuguet (2010a, b) and Snyder and Ting (2011) are some contributions focusing on this landmark reform in US politics.[2] Similarly, there are countless real world examples of how different party

[1] Our results complement Carey and Shugart (1995)'s ranking of electoral rules in terms of how candidate- or party-centered they are. Their theory is about how different electoral rules impact the incentives of candidates to base their campaign on the party platform as opposed to a more personal message. Their focus is thus on the voter-candidate relationship. Our prediction focuses on the incentive map running from parties to candidates (across electoral rules).

[2] Ranney (1975: 121, quoted by Ware 2002: 1, 95) states that "[T]he adoption of the direct primary by the states from the early 1900s onward is [. . .] the most radical of all the party reforms adopted in the

actors rely on the intraparty selection stage to have the 'right' candidates on the party's electoral list under PR. However, there is no model to study the mechanisms driving how incentives within and across parties impact candidate incentives under PR.[3] In the paper, we fill this gap and introduce a model that is tractable enough under PR to allow us to compare electoral outcomes across both electoral rules and candidate selection processes.

In our model, we follow Caillaud and Tirole (2002) and assume that parties play the role of intermediaries between voters and candidates. Two parties select their candidates before the general election takes place. Candidates exert costly effort to improve the quality of their electoral platform can and influence their chances of being selected by the party and o winning the election.

We model the general election as an imperfectly discriminating contest (Tullock, 1980) between candidates for the available legislative seats. Under FPTP, in every district, there is a contest between the candidates selected by the two parties. The probability of winning of a candidate is proportional to the ratio of his or her effort over the sum of all efforts of all party candidates running in that district. Under PR, there is country-wise contest between two party lists. The aggregate effort of all candidates on the party list determine the party's 'electoral output'. These outputs then determine each party's probability of winning a certain share of the available legislative seats via a standard contest success function. The technology governing the allocation of seats is the one introduced by Crutzen, Flamand and Sahuguet (2017) in their analysis of a team contest for multiple indivisble seats. This technology is especially useful in our context as it generates the intuitive feature that it is easy for each party to win a few legislative seats, but it is hard for any party to win

whole course of American history".

[3]The few existing theoretical contributions do not focus on incentives but, rather, on selection issues centred around the quality of politicians or the alignment of their preferences with those of the electorate. See for example Galasso and Nannicini (2015), Beath, Christia, Egorov and Enikolopov (2016), Besley, Folke Persson and Rickne (2017) or Buisseret and Prato (2018).

nearly all seats. Buisseret, Folke, Prato and Rickne (2018) estimate the distribution of the probabilities that a party wins any number of seats for municipal elections in Sweden and find that this distribution has a shape that is isomorphic to the one our model generates.

Let us turn to the selection of candidates. Under FPTP, the party must select a candidate for each district it runs in. Under PR, the party sets up a list of candidates on which voters vote in the election. In the real world, under both FPTP and PR, candidates can be imposed by the party leadership or can be selected by primaries or caucuses in which the pool of voters varies from a single person (the party leader) to a very wide population (possibly any voter in society). We conflate the different characteristics of the intraparty candidate selection process and the different criteria used during the selection into a single dimension: the degree of competitiveness of the candidate selection process. We contrast the equilibrium of our model under a fully uncompetitive process – in which candidates are selected before they exert effort and on the basis of criteria such as sex, gender, seniority or closeness to the current party leadership – with that under a fully competitive case – in which selection is solely based on the effort decisions of candidates. The competitive case under FPTP corresponds to a primary election that selects a candidate in an unbiased and fair fashion based on the efforts of the candidates (as in Caillaud and Tirole (2002), Jackson et al. (2007) or Castanheira et al. (2010)). The competitive case under PR corresponds to a fair process that determines which candidates are included on the party list and in which position, depending on their effort choice.

We first compare the two electoral rules when selection is not competitive. In line with the previous literature, we show that FPTP provides stronger incentives than PR.[4] To quote Persson et al. (2003, p. 961) on the drawbacks of PR, "politicians' incentives are [...] diluted by two effects. First, a free-rider problem arises among politicians on the same list. Under

[4]Candidates still spend some effort under both electoral rules, but only to the extend that such a decision improves their probability of election, as effort does not impact the probability of being selected by one's party.

proportional representation, the number of seats depends on the votes collected by the whole list, rather than the votes for each individual candidate. Second, [as] the list is closed and voters cannot choose their preferred candidate, an individual's chance of re-election depends on his rank on the list, not his individual performance". This implies that candidates on the top and bottom of the list have little incentive to exert effort. The candidates on the top of the list are nearly certain of getting elected. The candidates at the bottom of the list know their chances are nearly zero. Finally, the efforts chosen by the individuals in the middle of the list, those with the strongest incentives, are not sufficient to compensate the near-zero effort provision of the other list members.

We then turn to the case of competitive selection. Under FPTP, competitive selection inside each party takes the form of a primary. We first solve for equilibrium efforts for a given number of candidates in each party's primary and then show that having two internal candidates per party in each district maximizes incentives. Next, we turn to PR. We model the competitive selection process as a contest between individuals for multiple prizes, following Clark and Riis (1996). The prizes candidates compete for are a position on the list. Thus, under the competitive candidate selection process, candidates exert effort not only to improve their party's electoral success but also their own chance to win a seat by getting a safer slot on the party list. The extra incentives generated by the intraparty competition for slots on the party list implies that effort provision is now larger under PR.

Overall, accounting for the incentives generated by the candidate selection process reverses the ranking of electoral rules in terms of average individual efforts chosen by party candidates. Of course, competitive selection improves incentives under both electoral rules. However, too much competition can also dilute incentives to exert effort for the general election, as increasing the number of candidates a party can choose from must eventually dent individual incentives to provide effort, because the chances of being selected shrink with the size of the pool of candidates. This dilution effect is particularly important under FPTP,

because to generate intraparty competition a party must let more than one candidate run in each district primary. To the contrary, under PR, a party can create internal competition without increasing the size of its candidate pool: it simply conditions the positions on the list to the candidates' effort choices. This channel turns out to be particularly effective and is in itself sufficient to generate the reversal in effort rankings between electoral rules.

We then consider various extensions to show that this reversal of the ranking of electoral rules is robust to more realistic scenarios. We first allow for more than two parties under PR. Increasing the number of parties dilutes incentives to exert effort; however our results survive provided the number of parties under PR is not too large.[5] This extension also highlights that PR is more immune to noise or randomness in the election than FPTP, again because of the powerful intraparty incentive levers available under PR. We then let the electorate be characterised by a non degenerate distribution of ideological preferences. Once again, our key result survives. Finally, we also let candidates care about their party winning a majority of seats, so as to control government. Our results go through as long as candidates also care about themselves getting elected, irrespective of how important to candidates their party winning control of government is.

Our findings generate two key predictions. First, they imply that any empirical comparative politics exercise should include data at the party level, to avoid endogeneity issues due to an omitted variable problem. Ideally, this data should focus not only on the characteristics of the candidate selection process but also on the party leadership's goals and strength. Indeed, if it would be rational for a farsighted party leadership to choose the selection procedure that maximizes the party's electoral success, we could expect the leadership to use their power to select candidates for other reasons, such as favoring their friends or factions or returning favors to other members of the party.[6] Unfortunately, data on the characteris-

[5]This results points to a trade off between the provision of incentives to politicians and a democracy's desire to see its legislature represent faithfully the preferences and characteristics of society.

[6]See Hirano and Snyder (2014) and Hassell (2016) for recent accounts of such issues in US politics.

tics of candidate selection methods, party organization and party leadership is scarce, even though some contributions have made some progress on these fronts.[7] Collecting more data at the level of parties is thus a pressing issue for the profession.

Our results also hint to a causal relationship between electoral rules and the way political parties organize. One way to interpret our results is that, all else equal, under FPTP, the general election on its own incentivizes politicians already substantially, while these incentives are less powerful in elections under PR. As a consequence, under FPTP, relative to under PR, parties have less need to control and manage intraparty matters. Thus, PR should be associated with stronger incentives for parties to become and remain well-organized and hierarchical machines. This offers one rationale for the US parties being 'empty vessels'. Differences in electoral rules could thus explain differences in party organizations observed worldwide. Of course, other important institutions and aspects of politics influence how parties organize.

# 2 Related Literature

The paper contributes to several strands of the literature in political economy and, from a methodological point of view, to the theory of contests.

We contribute to the theoretical political economy literature on the incentive consequences of different party organizations that takes the set of electoral institutions as given. Caillaud and Tirole (2002) is arguably the first contribution to model party organization as

[7]For example, Schumacher et al. (2013) use survey data to build a proxy for "the degree to which a party is activist dominated or leadership dominated". Bille (2011), Lundell (2004) and Shomer (2014) focus on the degree of centralization (around the party leadership) and inclusiveness (how large is the body of individuals who get to select candidates) in the candidate selection process. Yet, all these contributions focus on the degree of centralization and the inclusiveness of the candidate selection process but are silent about its degree of competition, a feature of the process that our results suggest is fundamental.

a source of incentives for individual politicians.[8] Castanheira, Crutzen and Sahuguet (2010) extend the analysis of Caillaud and Tirole (2002) to allow for different political motives and general equilibrium effects. Persico, et al.(2011) and Dewan and Squintani (2016) are two contributions that focus on the role of factions. The former presents a theory of how intra-party competition between rival factions to control party resources affect the incentives of all factional members and in turn electoral outcomes. Dewan and Squintanti (2016) present a theory of factional interaction and their consequences for factional incentives to share privately held information that is relevant to improve a party's electoral platform. Crutzen, Flamand and Sahuguet (2017) extend of the classical Tullock (1980) contest model to build a model of closed list elections under PR to highlight that the use of closed lists can be optimal for incentive provision under some conditions. We build on their approach to analyse non-competitive selection under PR and extend their model to include competition for better spots on the list, using the framework of Clark and Riis (1996).

We also open the theoretical literature on the incentive effects of electoral rules to the effects of having different party organizations. The extant literature disregards what is going on inside parties. Also, its predictions do not all point in the same direction. Taking into account the way parties organize can rationalize this heterogeneity in predictions. Key contributions in this strand of the literature include Bawn and Thies (2003), Cox (1987a, 1990), Lizzeri and Persico (2001, 2005), Myerson (1993, 1999), Persson and Tabellini (1999, 2000) and Seror and Verdier (2018). Lizzeri and Persico (2001), Myerson (1993, 1999) and Seror and Verdier (2018) suggest that candidates' incentives are more aligned with voter preferences under PR. Their argument is based on lower barriers to entry due to the large electoral districts (Cox, Myerson), smaller incentives to target subgroups of the population for electoral success (Lizzeri and Persico 2001) or more costly rent extraction under PR (Seror and Verdier). In contrast, Persson and Tabellini (1999) hold the opposite view: incentives are

[8] Alesina and Spear (1988) analyse a moral hazard problem between a party and its candidates without an explicit focus on the role of different party organizations.

strongest in elections under US- or British-style FPTP because voters can more effectively discipline individual politicians and make them accountable for their deeds. Lizzeri and Persico (2005) add that having too many parties – something that is arguably more likely under PR than under FPTP – can also lead to suboptimal outcomes. Our model is closer in spirit to Persson and Tabellini (1999) as we have a model based on costly effort choice.[9]

Our findings suggest the existence of a causal mapping from electoral rules to the way parties organize. There is an important literature on party organization. On top of the contributions already mentioned, see Bille (2001), Farrell and Holliday (2002), Francois, et al. (2017). Gallagher and Marsh (1988), Katz and Mair (1994, 2002), Li and Li-An (2005), Lundell (2004), Luther and Muller-Rommel (2002), Potguntke, Scarrow and Webb (2016), Shomer (2014) and Webb and White (2007). With the exception of Francois et al. (2018), this literature is either not formalized or exclusively empirical (as are Lundell 2004 and Shomer 2014). Our model suggest that the characteristics of the candidate selection process, and especially its degree of competitiveness and fairness has a first order effect on electoral outcomes. To the best of our knowledge, the causes and consequences of the degree of competitiveness of the candidate selection process have not be analyzed yet in the literature in either political science or economics. Some very recent empirical works – Besley et al. (2017), Dal Bo et al. (2017), Folke et al. (2016, 2018), Galasso and Nannicini (2011, 2015) and Hangartner et al. (2018) – offer some promising advances on this front, but they focus more on selection than incentives. Hangartner et al. (2018) is the only contribution that devotes some attention to incentives, and finds that candidates at the bottom of party lists (when these are closed) exert less effort than other candidates, as predicted by our theory.

Our findings also speak to the existing empirical comparative politics literature. Key contributions here include Persson and Tabellini (1999, 2003), Persson, Tabellini and Trebbi

[9]There is also a theoretical literature on the mapping from the political regime to incentives, which once again disregards any action taking place within parties. See for example Myerson (1999), Milesi-Ferretti, Perotti and Rostagno (2002), Morelli (2004) and Persson, Roland and Tabellini (1997, 2000).

(2003), Raffler (2016) and Tavitz (2007). We believe that the lack of clearcut incentive effects across PR and FPTP in Persson et al. (2003) could be due to their not taking into account differences in how parties organize. The same shortcoming is found in Tavitz (2007): the variables used to proxy the clarity of responsibility of politicians do not include any intraparty aspect. Yet, some candidate selection processes do very much put the accent on the mapping from candidate responsibility to (re-)selection.

Our analysis compares the effects of an unbiased and fair selection process based on individual candidates' choices to one in which such choices do not matter. This raises the thorny and largely still unresolved issue of the origins and consequences of leadership. Hermalin (1998) develops the notion of leading by example. Canes-Wrone, Herron, and Shotts (2001) draw a distinction between "leadership" and "pandering" to a majority. Relatedly, Canes-Wrone (2006) develops the notion of "transformative leadership" in which a leader influences a group's decision by modifying the set of issues to be discussed. Dewan and Myatt (2007, 2008) develop the notion of "focal leadership" in which the leaders' characteristics, views and acts help the rest of a group take a decision. Dewan and Squintani (2018) develop the notion of "relational leadership" found in the social psychology literature in which better leaders are those who manage to be backed by better associates. Egorov and Sonin (2011) are quite close to the question of what makes selection unbiased and fair as they focus on the tradeoff between competence and loyalty to the leader.

The last paper we just mentioned leads us naturally to the theoretical and empirical literature on selection based on competence or quality and preferences of politicians as opposed to incentives. This literature is surveyed by Besley (2005) and Dal Bo and Finan (forthcoming). Important recent contributions include Beath et al. (2016), Besley et al. (2017), Buisseret et al. (2018), Buisseret and Prato (2018a, b), Carrillo and Mariotti (2001), Dal Bo et al. (2017), Folke et al. (2016), Galasso and Nannicini (2011, 2015), Hangartner et al. (2018), Hortala-Vallve and Mueller (2015), Mattozzi and Merlo (2015), Serra (2011) and

Snyder and Ting (2002, 2011). Contrary to the political economy literature on incentives, most contributions on selection do take into account the intraparty dimension, at least to some extent. For example, Mattozzi and Merlo (2015) and Snyder and Ting (2002, 2011) focus explicitly on the interplay between intra- and interparty competition. Buisseret and Prato (2018b) endow the party leadership with control rights over the party list, like us.

Turning to our contribution to the theoretical literature on team contests, we use a novel way to model team contests for multiple homogeneous prizes, building on the work of Crutzen, Flamand and Sahuguet (2018). The number of prizes won by a party follows a binomial distribution with a success probability that corresponds to the Tullock contest function based on the electoral outputs of the two parties. Crutzen et al. (2018) prove that the use of a closed list under PR modelled with such a binomial-Tullock contest success function can be optimal for incentive provision under a weak monotonicity constraint provided individual efforts are substitutable enough, something we impose in this paper.[10] Competing mechanisms to model competition between parties are the well known probabilistic model of Enelow and Hinich (1982) and Lindbeck and Weibull (1987) or the mechanism put forward by Nalebuff and Stiglitz (1983). The main advantage of our binomial-Tullock mechanism is its tractability and the fact that it generates the intuitive feature that it is easy for each party to win a few legislative seats, but it is hard for both parties to win nearly all seats. For example, this feature is recovered empirically by Buisseret et al. (2018) in their empirical analysis of Swedish municipal elections. With regard to our specific modelling choices for the allocation of the legislative seats across parties and for the candidate selection procedure, our model contributes to the theory of team contests and to the theory of contests with multiple prizes. Starting with the latter, the selection stage within each party corresponds to a contest for multiple prizes (for a recent survey on contests with multiple prizes, see Sisak (2009)). To model this intraparty contest, we build on the allocation mechanism of Clark

[10]See also Crutzen, Flamand, Konishi and Sahuguet (2018).

and Riis (1996), which has been shown to hold all the desired axiomatic properties for this type of contests; see Fu and Lu (2009).

# 3 The Model

Consider a society with a continuum of voters of mass $K$, $K$ being an odd natural number. We consider two electoral rules: US- or British-style plurality rule in single member districts (FPTP) and Israeli-style proportional representation in a single nationwide district (PR). Under FPTP, society is divided into $K$ identical electoral districts with a unit mass of citizens. Each district elects one representative to the legislature. Under PR, there is a single nationwide electoral district which elects all $K$ legislators.

Politicians exert effort prior to the election to improve the quality of their platform. The "quality" of a candidate's platform corresponds to the platform characteristics that the electorate values (see Caillaud and Tirole (2002) or Castanheira et al. (2010) for similar models of elections).

Candidates belong to one of two parties, are office-motivated and choose their effort to maximize their expected utility. If elected, a candidate earns a payoff equal to $V$. If not elected, a candidate earns 0. We assume a quadratic effort cost function $C(e) = \frac{1}{2}e^2$. The objective function of a candidate is thus:

$$\Pr(\text{elected})\, V - \frac{1}{2}e^2 \tag{1}$$

Under FPTP, each party selects one candidate per district and each district-specific election is a contest between two party representatives; the outcome of the election depends of the efforts chosen by the two candidates. Under PR, each party selects the $K$ candidates and their place on the electoral list. Voters choose which list to vote for. ListS are evaluated by voters on the basis of the sum of all efforts the candidates on the list exerted previously. We are thus focusing on PR with closed lists as in Iceland, Spain or Argentina.

We also consider two types of candidate selection processes. The first process is non-competitive: the party chooses the candidates who will represent it in the election before candidates decide how much effort to exert. Under FPTP, this means that the party selects K candidates, one for each district. Under PR, the party chooses K candidates and their order on their list. We also allow for a competitive candidate selection process. Under FPTP, in each district, $N \geq 2$ candidates compete in a primary for the right to run in the general election. Under PR, $S \geq K$ candidates compete nation-wide for the right to be on the party list and for their position on the list. Under both rules, candidates are selected in an unbiased and fair fashion on the basis of the effort they exerted.

## 3.1 FPTP

**Non-competitive selection**

In each district, parties first select a representative for the election. This individual then decides how much effort to exert. Voters in district $d$ then consider the efforts of the two party candidates, $e_d^L$ and $e_d^R$. We assume that the probability that the candidate of party $L$ wins the seat in district $d$ is given by a modified Tullock (1980) contest success function:

$$P_d^L\left(e_d^L, e_d^R\right) = \beta\left(\frac{e_d^L}{e_d^L + e_d^R}\right) + \frac{1-\beta}{2}. \tag{2}$$

Parameter $\beta$ represents the importance of effort in the result of the election. A high $\beta$ means that effort plays an important role in determining the result of the election. The case $\beta = 1$ corresponds to the standard Tullock contest function. The case $\beta = 0$ corresponds to a random election result. As will be clear from equations (5) and (10) below, parameter $\beta$ thus determines the relative importance of intra- and interparty competition for electoral incentives.

**Competitive selection**

The intra-party candidate selection process is modelled as a Tullock contest between $n \geq 2$ party candidates. The contest is based on the effort decisions of the candidates. The

probability that candidate $i$ of party $L$ in district $d$ is selected to represent their party in their district general election is given by :

$$Q_d^{iL} = \frac{e_d^{iL}}{e_d^{iL} + \sum_{k \neq i} e_d^{kL}}, \tag{3}$$

where $e_d^{kL}$ denotes the effort of candidate $k$ among the other $n-1$ candidates in $i$'s party.

Let $e_d^R$ be the effort of the representative of party $R$ in district $d$. Candidate $i$ in district $d$ chooses effort $e_d^{iL}$ to maximize:

$$Q_d^{iL} P_d^L \left(e_d^L, e_d^R\right) - \frac{1}{2}\left(e_d^{iL}\right)^2 \tag{4}$$

$$= \left(\frac{e_d^{iL}}{e_d^{iL} + \sum_{k \neq i} e_d^{kL}}\right)\left(\beta \frac{e_d^{iL}}{e_d^{iL} + e_d^R} + \frac{1-\beta}{2}\right) V - \frac{1}{2}\left(e_d^{iL}\right)^2 \tag{5}$$

## 3.2 PR

Under PR, a party is representetd an ordered list of $K$ candidates. Seats are allocated to candidates in the order of the list. Thus, if a party wins $m$ seats, the first $m$ candidates on the list get elected while the $K-m$ remaining candidates don't get elected.

The electorate bases its voting decision on the quality of the parties' platforms. The quality of a party's platform is the sum of the party candidates' individual efforts: $E^L = \sum_{m=1}^K e_m^L$ and $E^R = \sum_{m=1}^K e_m^R$, where $e_m^P$ is effort by the candidate of party $P$, $P = L, R$, who is in $m$th position on the party list.

**Non competitive selection**

The party first picks $K$ candidates and orders them on the list. Then, the seelcted candidates decide how much effort to exert. The quality of the two parties' platforms map into the parties' seat shares in the legislature according to a Binomial-Tullock distribution that is a natural extension of the Tullock contest function to the case of multiple prizes. We assume that the probability that a party wins a given seat is given by the following Tullock

contest success function: $P_L\left(E_L, E_R\right) = \beta\left(\frac{E^L}{E^L+E^R}\right) + \frac{1-\beta}{2}$. All seats are attributed to parties using independent draws from this distribution. The total number of seats won by party $L$ thus follows a binomial distribution with parameters $K$ and $P_L$. The probability that party $L$ wins $k$ seats is given by:

$$\mathcal{P}_L(k) = C_k^K P_L^k \left(1 - P_L\right)^{K-k} \tag{6}$$

Under the noncompetitive candidate selection procedure, each politician on the party electoral list chooses his level of effort to maximize their chances of getting elected, as they do not have to worry about selection. The candidate in $mth$ position on the electoral list of party $L$ chooses effort $e_m^L$ to maximize:

$$\sum_{k=m}^{K} \mathcal{P}_L(k)V - \frac{1}{2}\left(e_m^L\right)^2. \tag{7}$$

**Competitive selection**

If the selection procedure is competitive, $S \geq K$ candidates compete for the $K$ positions on the party list. Candidates compete on the basis of the effort they provide. Given that within each party legislative seats are allocated to candidates in the order of the list, the first position on the list is more valuable than the second position and so on and so forth. We thus model the competitive selection procedure as a contest between $S$ candidates for $K$ prizes of different values. In particular, we rely on the imperfectly discriminating contest model of Clark and Riis (1996). Denoting $e_i$ the effort of politician $i$, the probability that $i$ ends up in position $k$ or higher on the party list is given by:

$$Q_i(k) = p_1 + \sum_{j=2}^{k} p_j \left[\prod_{s=1}^{j-1}(1 - p_s)\right] \tag{8}$$

where $p_j$ is the probability that $i$ ends up in position $j = 1, ..., k$ on the list and is given by the standard Tullock ratio contest success function among the candidates who have not yet been attributed a slot on the list (thus, for the $j$th prize, there are $S - j + 1$ candidates competing with each other):

$$p_j = \frac{e_i}{e_i + \sum_{k \neq i} e_k}, \#k = S - j. \tag{9}$$

One can interpret these probabilities as the result of a sequential process. The contestants make one contribution that is valid for the entire contest. The winner of the first prize (the first spot the list) is decided using the Tullock contest function with the contributions of the $S$ contestants. The winner and his contribution disappear and the winner of the second prize is then decided using the Tullock contest function with the contributions of the remaining $S-1$ contestants. This process continues until all spots on the list have been filled. The value of each spot on the party list is then endogenously determined by the probability that the party wins a given number of seats in the election. These probabilities are computed using the parties' aggregate efforts, which are the total sum of the efforts of the candidates on each party list

Under the competitive candidate selection process, an individual candidate chooses effort $e_i^L$ to maximize:

$$\sum_{m=1}^{K} \mathcal{P}_L(m) Q_i(m) V - \frac{1}{2}\left(e_i^L\right)^2, \tag{10}$$

where $Q_i(m) = p_1 + \sum_{j=2}^{m} p_j \left[\prod_{s=1}^{j-1}(1-p_s)\right]$ and $\mathcal{P}_L(m) = C_m^K P_L^m \left(1-P_L\right)^{K-m}$.

### 3.3 Discussion of the model

We model political competition as contests in which candidates only differ in their effort choice. While elections are often modelled as games in which parties (or candidates) choose ideological positions, candidates do spend considerable efforts and resources to improve their electoral platforms. In section 4 we extend the model to allow for ideological preferences among voters and show that our key comparative politics results still hold.

The assumption that candidates make a single effort decision allows us to make direct comparisons of the incentives generated by the interplay of the candidate selection process

and the electoral rule. This assumption also simplifies the analysis. Still, Hirano and Snyder (2014) offer evidence that the candidates' choices in their intra-party primaries matters for the general election too. The single effort decision problem also allows us to differentiate parsimoniously between the competitive and the noncompetitive candidate selection process

The assumption that candidates only care about their individual result is a strong assumption that we relax it in section 4. Candidates most likely also care about their party winning a majority of legislative seats, as this guarantees control of government. We show that when candidates also care about their party winning a majority of the seats, the main results still obtain provided candidates also care about themselves being elected.

Our modelling of the inter-party competition via a binomial-Tullock imperfectly discriminating contest is more novel. We build on Crutzen, Flamand and Sahuguet (2018). The binomial-Tullock technology delivers a realistic prediction about how easy it is for parties to win seats: Buisseret, Folke, Prato and Rickne (2018) estimate the probability that a party wins any number of seats on data from Swedish municipal elections and find that the distribution has a shape that is isomorphic to the one generated by the Binomial-Tullock function.

The model of Clark and Riis (1996) that we use to model the competitive candidate selection process under PR is arguably the horsemodel for contests with multiple prizes. Its main advanatge is its tractability as it allows for closed-form solutions. The model also presents many desirable axiomatic properties of contest success functions, as shown by Fu and Lu (2012).

## 4 Equilibrium individual and aggregate efforts

We now solve for equilibrium effort of candidates under each electoral rule and selection procedure. In the next section, we then use those results to compare the different scenarios and derive comparative politics predictions.

## 4.1 Non-competitive candidate selection procedure

### 4.1.1 FPTP

Party $L$'s candidate in district $d$ chooses $e_d^L$ to maximize equation (2) above. The first-order condition associated to this problem is:[11]

$$\frac{e_d^R}{\left(e_d^L+e_d^R\right)^2}\beta V - e_d^L = 0.$$

At the symmetric Nash equilibrium, we have:

$$e_d^{L*} = e_d^{R*} = \frac{\sqrt{\beta V}}{2}.$$

**Proposition 1** *Under plurality rule with no competitive selection, candidates exert effort* $e^* = \frac{\sqrt{\beta V}}{2}$. *Effort at the party level is thus equal to* $E^* = \frac{K\sqrt{\beta V}}{2}$.

### 4.1.2 PR

When the candidate selection procedure is non-competitive, party candidates exert effort to increase their party's success. The position on the list is then key to understand the incentives to exert effort, as is clear from equation (7). The first order condition to the problem of candidate in position $m$ in party $L$ is given by:

$$\begin{aligned} e_m^L = {} & \beta V \sum_{k=m}^{K} C_k^K P_L^{k-1} \left(1-P_L\right)^{K-k} k \frac{E^R}{\left(E^L+E^R\right)^2} \\ & - \beta V \sum_{k=m}^{K} C_k^K \left(P_L\right)^k \left(1-P_L\right)^{K-k-1} \left(K-k\right) \frac{E^R}{\left(E^L+E^R\right)^2} \end{aligned}$$

[11] It is immediate to check that the second order condition is satisfied.

In the symmetric equilibrium, effort choices of candidates in the same position on the list are equal in both parties parties and parties total effort are also equal, thus $E^{L*} = E^{R*} = E^*$. We can simplify the above first order condition to find:

$$e_m^L = \frac{\beta V}{4E^*}\left(\frac{1}{2}\right)^{K-1}\sum_{k=m}^{K}(2k-K)\,C_k^K$$

As $\sum_{k=m}^{K}(2k-K)\,C_k^K = mC_m^K$, the above first order condition thus boils down to:

$$e_m^L = \frac{\beta V}{4E^*}m\left(\frac{1}{2}\right)^{K-1}C_m^K.$$

Summing these optimal effort decisions over all party list members and exploiting the fact that $\sum_{m=1}^{K} m\left(\frac{1}{2}\right)^{K-1}C_m^K = K$ , we get:

$$E^* = \frac{\sqrt{\beta V K}}{2}, \tag{11}$$

and:

$$e_{mL}^* = mC_m^K\left(\frac{1}{2}\right)^K\sqrt{\frac{\beta V}{K}}. \tag{12}$$

**Proposition 2** *Under PR with a noncompetitive candidate selection process, the effort exerted by the candidate in mth position is* $e_m^L = mC_m^K\left(\frac{1}{2}\right)^K\sqrt{\frac{\beta V}{K}}$. *Aggregate party effort is* $E^L = E^R = \frac{\sqrt{\beta V K}}{2}$.

As anticipated, candidates' position on the list determines their level of effort. The distribution of individual efforts along the list is bell-shaped and symmetric about the median list member. Candidates at the top and bottom of the party list exert very little effort (this first because they are certain or nearly certain of getting elected independently of the effort they exert, the last because they are certain or nearly certain of not being elected) whereas those in the middle of the list are the candidates who exert most effort. The median list member is the candidate exerting the highest amount of effort. Indeed, this is the position where the marginal benefit of effort, driven by the binomial-Tullock mapping, is greatest.

## 4.2 Competitive candidate selection procedure

### 4.2.1 FPTP

With a competitive selection procedure, candidates in each district now exert effort to be first selected by their party and then to win the general district election. In each district, the party organizes a primary with $N$ candidates. Candidate $i$ from party $L$ in district $d$, chooses effort $e_d^{iL}$ to maximize equation (5) above. The first order condition yields:

$$e_d^{iL*} = V\left\{\frac{\sum_{k\neq i} e_d^{kL} - e_d^{iL*}}{\left[\sum_{j=1}^{N} e_d^{jL}\right]^2}\left(\beta\frac{e_d^{iL*}}{e_d^{iL*}+e_d^R}+\frac{1-\beta}{2}\right)+\frac{e_d^{iL*}}{\sum_{j=1}^{n} e_d^{jL}}\frac{\beta e_d^R}{(e_d^{iL*}+e_d^R)^2}\right\}.$$

The first argument within the curly bracket corresponds to the marginal benefit of effort on a candidate's chances of being selected. The the second term corresponds to the marginal benefit of effort on a candidate's general election prospects, given that they were selected by their party, which happens with probability $\frac{e_d^{iL*}}{\sum_{j=1}^{N} e_d^{jL}}$.

In the symmetric equilibrium in which all candidates exert the same effort, we get:

$$e_d^{iL*} = e_d^{R*} = e^* = \sqrt{V\left(\frac{N-1}{2N^2}+\frac{\beta}{4N}\right)}. \tag{13}$$

Thus:

**Proposition 3** *Under plurality rule, with $N$ candidates competing in the primary elections in each district, in the symmetric equilibrium, candidates exert effort equal to: $e^* = \frac{1}{2N}\sqrt{V\left((2+\beta)N-2\right)}$.*

Thus, if $\beta < 1$, $N = 2$ maximises individual effort provision. If $\beta = 1$, $N = 1$ and $N = 2$ both maximize individual effort provision. The intuition behind this finding is as follows. Intra-party competition has two effects on candidates incentives: a dilution and a competition effect. The optimal number of candidates in the primary election trades off

these two effects optimally. In the present model, the dilution effect counterbalances the competition effect quickly as the number of candiddates in the primary election increases. This implies that it is optimal for parties to restrict competition quite severely and parties do not choose to have more than 2 candidates in the primary.

### 4.2.2 PR

When the candidate selection procedure is competitive, the selection process creates two news sources of incentives. First, candidates want to be among the set of candidates who are selected by their party; second, conditional on being selected, each candidate would like to end up as high as possible on the party list, as this increases one's chances of getting elected to the legislature.

As candidates are ex-ante identical, they all have the same objective function. Taking the first order condition associated with maximizing equation (10) , and evaluating it at the symmetric equilibrium, we get:

**Proposition 4** *When $S$ candidates compete in each party for one of the $K$ spot on the list, the equilibrium effort is given by:*

$$e^* = \sqrt{V} * \sqrt{\frac{\beta}{4S} + \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left(1 - \frac{m}{S}\right) \left[\sum_{j=1}^{m} \frac{1}{S-j+1}\right]}$$

**Proof.** See appendix. ■

The first term under the square root represents the marginal effect of effort on the party's electoral success given that every candidate has the same probability of getting a seat. The second term represents the marginal effect of effort on improving the candidate's position on the list.

Unfortunately, pinning down the optimal number of candidates is difficult. Direct computation shows that the optimal number of candidates is equal to $K+1$ for values of $K$ below 5 while numerical simulations suggest that it is equal to $K$ for values of $K$ larger than

5. In what follows, we thus simply assume that $S = K$. As this may not be the optimal number of internal candidates a party should have under the competitive candidate selection process, the restriction we impose reinforces, if anything, the results we derive below.

# 5 Comparative Politics

We now compare the effort of candidates in each of the four scenarios considered in the previous section, we have the follwoing proposition:

**Theorem 5** *Comparing effort provision across electoral rules:*
*When the candidate selection rule is non-competitive, aggregate candidate effort is higher under FPTP than under PR.*
*When candidate selection is competitive, parties choose the degree of competition optimally under FPTP and have $S = K$ internal candidates under PR, aggregate effort is higher under PR than under FPTP.*

**Proof.** See appendix. ■

This first part of the theorem is in line with previous results in the literature (see for example Persson and Tabellini (2000) and Persson et al. (2003) and the references therein). The intuition behind this result is as follows. FPTP gives direct incentives to candidates to exert effort. PR creates a free-riding problem and dilutes incentives . Candidates work for the success of the party and only indirectly work for the chance to get a seat. There is thus a public good aspect to the party's success and this leads to underprovision of effort.

The main intuition behind the second part of the theorem is that, under FPTP, the benefit of a competitive candidate selection process is compromised by the dilution effect, while under PR, the benefits of competitive selection are obtained without the need to increase the number of candidates and thus the dilution of incentives can be avoided. Intra-party competition for the best spots on the party list turns out to be a strong enough

incentive device.

Our findings also complement those of Myerson (1993) and Buisseret and Prato (2018) on the effects of district magnitude. Both papers conclude that increasing the size of electoral districts leads to outcomes that increase voters' utility, because larger districts are either associated to more *inter-party* competition, which gives voters more freedom of choice (Myerson) or allow for a better balancing of the objectives of voters and parties (Buisseret and Prato). We add that there is also an *intra-party* dimension to these incentive problems, which reinforces the positive, inter-party incentives effects of larger districts.

# 6 Discussion and implications

*Data.* Our findings suggest that the candidate selection process is a major source of incentives. Thus, any empirical exercise mapping political incentives into electoral or policy outcomes should take intraparty incentives into account. This point was already put on the agenda before us. For example Jun and Hix (2010, 153) argue that "any theory of how electoral systems shape individual parliamentary behavior needs to look beyond the opportunities provided by the electoral rules...to the structure of candidate selection inside parties...". If intraparty rules matter for parliamentary behavior, they must also matter for electoral behavior. A key aspect of such ruls is their degree of fairness and competitiveness. To the best of our knowledge, data proxying this aspect of intraparty competition is still not available. The data in Shomer (2014) speaks only to the relative importance of the party leadership and the size of the pool of voters in the selection process. Yet Hassell (2016) and Fowler (2016), for example, show that a selection process can be quite uncompetitive and biased even if, on paper, selection is through a primary in which a large pool of voters can express their preferences. The data in Schumacher et al. (2013) is also silent on the competitiveness and fairness of intraparty matters. They distinguish between leadership-dominated and activist-dominated parties, but this distinction is not sufficient to capture the degree

of competitiveness of candidate selection or intraparty decision making more generally. The Political Party Database Project (Poguntke et al. 2016; https://www.politicalpartydb.org) suffers again from the same limitation: it focuses on the degree of centralization (leadership control) of the candidate selection process, but is silent about its degree of competitiveness and fairness. Also, information on the ways parties select their candidates was not provided to researchers in this project by a vast majority of parties, highlighting that this aspect on intraparty politics is indeed regarded by most parties as their 'secret garden'.

Some promising works (focusing more on selection than incentives though) are slowly emerging; see for example Besley et al. (2017), Dal Bo et al. (2017), Folke et al. (2016, 2018), Galasso and Nannicini (2011, 2015) and Hangartner et al. (2018). As the data in these contributions focuses on single countries only (Colombia, Italy and Sweden) and offers only indirect information on the inner workings of parties, a pressing issue is to gather more high quality cross-country data on all intraparty aspects of selection, and especially its degree of fairness and competitiveness.

*How parties organize.* Our findings also suggest the existence of a causal link from electoral rules to the way parties organize and thus, ultimately, electoral outcomes. Indeed, as there is both more room and more to be gained from the strategic management of party organizational choices under PR compared to FPTP, the prediction of our model is that parties should be more centrally organized and hierarchical under PR than under FPTP. This is driven by the presence of the party list under PR and the smaller magnitude of districts under FPTP, which reinforces the benefit of local ties under FPTP and thus introduces a cost of central control that is less present under PR. Benedetto and Hix (2007, p.777) put forward this argument quite forcefully: "[Under plurality rule, O]nce an MP has been selected in a constituency, the MP is difficult to remove if he or she is supported by his or her local party elite. Contrast this with a national-based, party-list proportional representation system, which gives the national party leadership the power to move a candidate down the

party list in the next election and so reduce his or her chances of being reelected."

Whereas this prediction of our model still awaits a thorough empirical test, some causal observations appear to be consistent with it. For example, legislative elections are run under FPTP in the US and the two main national parties have been described as "empty vessels" that leave ample room for individual candidates and especially incumbents to manage their fortunes in the way they see fit; see for example Katz and Kolodny (1994), Boatright (2015) or Hassell (2016). To the contrary, in countries using PR such as Spain, The Netherlands or Israel, the candidate selection processes (and other key party activities) are firmly in the hands of the top brass of the party who allocate the right to represent the party in a more or less safe position to the candidates whose characteristics best meet the objectives of the top brass; see for example Montabes and Ortega (1999) for Spain and Portugal.

Casual examination of the candidates filling the top electoral spots across electoral rules also reveals some patterns consistent with different party structures. Under PR, the party leader is typically leading the party list, and the other top slots are allocated to other prominent party members. These are also the candidates who win the top political offices available to a party (be them internal, legislative or executive jobs) after the election. To the contrary, in the US, the key legislative and internal positions, such as the committee chairs and the position of party leader in the House and Senate, are allocated following rules that are heavily biased towards incumbents, as seniority is the number one criteria.[12] Also, the large incumbency advantage found in the US not only in general elections but also in intraparty primaries – see for example Ansolabehere et al. (2007) or Hirano and Snyder (2014) – is consistent with the fact that incumbents are by and large firmly in control of their electoral fortunes. Nevertheless, we must await the collection of better and more comprehensive data to be able to test properly this prediction of our model.

[12]Since 1975, parties have regained some weight in the procedure regarding assignments to committees, given that formal nominations now happen via caucuses and are not purely based on seniority anymore.

# 7 Extensions

## 7.1 Candidates care about their party winning a majority of seats

We assumed so far that candidates only care about getting elected. We now extend the model to analyse the consequences of candidates' caring not only about getting a seat but also about their party getting a majority in the election so as to gain control of the executive. We focus on the two-party case and assume that a candidate enjoys payoff $M$ when their party gets at least a majority of the legislative seats, namely at least $k = (n+1)/2$ seats. Every candidate's effort matters for the party's probability of winning at least a majority of the legislative seats.

Under plurality rule, the event that the party of a candidate wins control of the executive is relevant for a candidate's effort decision if and only if this candidate is pivotal for this event to happen, namely when party $L$ wins in exactly $\frac{K-1}{2}$ districts out of the $K-1$ districts in which candidate $d$ is not running. In equilibrium, this happens with probability $C^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1}$. Under proportional representation, candidates' effort increases the total party output and improves the chances that the party gets a majority of seats.

How will their effort decisions be affected by this change in their utility function? We should expect that the positive effect of the incentives on effort of proportional representation with a competitive candidate selection process shrink as the value candidates attribute to their party winning the executive grows relative to their utility from getting elected themselves. Indeed, in the limit, if candidates care only about their party winning, their position on the party list is immaterial to them. We show that our results are robust to this change in the candidates objective function.

We then have:

**Proposition 6** *When candidates care about their party winning a majority of seats, the ranking of efforts of theorem 5 remain the same.*

**Proof.** See the appendix ■

## 7.2 Ideology

We assumed so far that ideology plays no role in the model. Voters all have the same preferences and care only about effort as parties are not ideologically differentiated. We now extend the model to allow for ideological differences between parties generated by voter-specific ideological biases.

Assume that the population is composed of voters with ideological preferences. Under FPTP, we allow for biased districts with bias parameters $\left(\tilde{b}_d^L, \tilde{b}_d^R\right)$. To generate such districts, we redistribute voters from one district to another one. The case considered in sections 4-5 would then correspond to a situation with identical unbiased districts in which the number of voters biased in favor of one party cancels out exactly the number of voters biased against that party. Still, we restrict such redistribution of voters to be symmetric, in the sense that for one district biased in favor of party R, there is another district with the opposite bias for party D. These biases are common knowledge to all players of the game.

Our symmetry restriction implies that no party has an advantage at the country-level and under PR the probability of winning a seat is given by the symmetric Tullock contest success function analysed previously. We can thus study the effect of ideology by analysing its relative impact under FPTP only.

Given that the following biased inter-party binomial-Tullock contest success function

$$\frac{\tilde{b}_d^L e_d^L}{\tilde{b}_d^L e_d^L + \tilde{b}_d^R e_d^R}$$

can always be rewritten as

$$\frac{b_d^L e_d^L}{b_d^L e_d^L + e_d^R}$$

or

$$\frac{e_d^L}{e_d^L + b_d^R e_d^R}$$

with $b_d^L = \frac{\tilde{b}_d^L}{\tilde{b}_d^R}$, and $b_d^R = \frac{\tilde{b}_d^R}{\tilde{b}_d^L}$, there is no loss of generality in focusing on only one random district among the biased ones, like the one in which $L$'s probability of winning the seat is given by:

$$P_d^L\left(e_d^L, e_d^R\right) = \beta\left(\frac{b_d^L e_d^L}{b_d^L e_d^L + e_d^R}\right) + \frac{1-\beta}{2} \tag{14}$$

In that district, if the selection process is non-competitive, the first order condition of the candidate is given by

$$\frac{e_d^R}{\left(b_d^L e_d^L + e_d^R\right)^2}\beta b_d^L V - e_d^L = 0$$

for the representative of party $L$ and

$$\frac{e_d^L}{\left(b_d^L e_d^L + e_d^R\right)^2}\beta b_d^L V - e_d^R = 0,$$

for the representative of party $R$, which yield:

$$e_d^{L*} = e_d^{R*} = \frac{\sqrt{\beta b_d^L V}}{2} \tag{15}$$

Thus, if the selection process is non-competitive, whenever a district is biased in favour of either $L$ or $R$, both candidates exert less effort than what they would if the district was unbiased, an effect of the biases in the contest success function that is well known in the literature on the theory of contests.

If the selection process is competitive, the first order condition for any candidate in party $L$ boils down to, in equilibrium:

$$e_d^{iL*} = V\left\{\frac{n-1}{n^2 e_d^{iL*}}\left(\beta\frac{b_d^L e_d^{iL*}}{b_d^L e_d^{iL*} + e_d^R} + \frac{1-\beta}{2}\right) + \frac{1}{n}\frac{\beta b_d^L e_d^R}{\left(b_d^L e_d^{iL*} + e_d^R\right)^2}\right\},$$

whereas that for any candidate in party $R$ boils down to

$$e_d^{iR*} = V\left\{\frac{n-1}{n^2 e_d^{iR*}}\left(\beta\frac{e_d^{iR*}}{e_d^{iR*} + b_d^L e_d^L} + \frac{1-\beta}{2}\right) + \frac{1}{n}\frac{\beta b_d^L e_d^L}{\left(e_d^{iR*} + b_d^L e_d^L\right)^2}\right\}.$$

Remark now that $e_d^{iR*} = b_d^L e_d^{iL*}$ is a possible solution to the above system of equations. Following this conjecture, we get that

$$e_d^{iR*} = b_d^L e_d^{iL*} < e_d^{iL*} = \sqrt{V\left(\frac{n-1}{2n^2} + \frac{\beta}{4n}\right)}. \tag{16}$$

implying that the candidates in the advantaged party exert less effort than those in the disadvantaged one if selection is competitive, so as to equalise across parties the probability of winning the general election.

More importantly for our purposes, it follows immediately that theorem 5 is actually reinforced by the introduction of ideology, as the effort disadvantage of PR is reduced when selection is non-competitive and the advantage of PR is magnified when selection is competitive.

We then have:

**Proposition 7** *When parties are ideologically differentiated, the effort under FPTP decreases with the ideological bias, while effort remains unchanged under PR.*

### 7.3 Number of parties

A stylized facts of electoral competition is that the number of parties is typically higher under PR than under FPTP. In line with this, let us allow for $T \geq 2$ parties under PR. In the appendix, we show that equilibrium effort provision with $T \geq 2$ parties is given by:

$$\sqrt{V} * \sqrt{\frac{\beta\,(T-1)}{T^2 S} + \sum_{m=1}^{K} C_m^K \left(\tfrac{1}{T}\right)^m \left(\tfrac{T-1}{T}\right)^{K-m} \left(1 - \tfrac{m}{S}\right) \left[\sum_{j=1}^{m} \tfrac{1}{S-j+1}\right]} \tag{17}$$

In the figure below, we illustrate that, as long as the number of parties under PR does not grow excessively large, our key comparative politics result survives ($K = 300$, $V = 1$; effort under PR is given by the downward-sloping series of blue dots for $\beta = 2/3$ and the black dots for $\beta = 1/3$).

The figure also illustrates that the degree of noisiness of the election, parametrized by $\beta$ in our model, plays a key role. Indeed, the more noisy the election is, the larger is the number

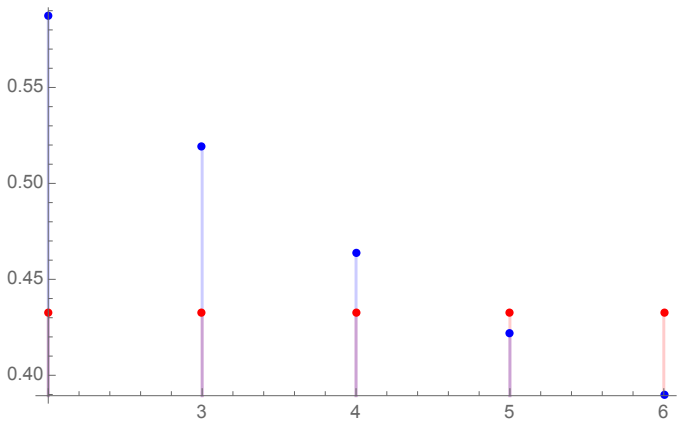


Figure 1: Effort provision with T teams under PR and two under FPTP

of parties under PR that is still associated to higher effort provision under PR compared to FPTP. Comparing (13) for $N = 2$ with (17) for $S = K$, it is immediate to see that the difference in efforts grows larger as we let $\beta$ shrinks towards 0. The stronger 'resistance' of effort provision incentives under PR compared to under FPTP is another aspect of electoral rules that appears to have been missed by the extant literature, but is an obvious side effect of our finding that PR offers more intraparty incentive levers than FPTP.

Analyzing (17), we can also prove that, as we let the number of parties grow large, the incentive advantage of PR over FPTP disappears.[13]. This is of course quite intuitive: once the number of parties grows very large under PR, the dilution effect dominates and candidates have very little incentives to exert effort. This suggests a novel trade off under PR between the desire to offer the best possible representation of citizens preferences – which speaks for the presence of many parties – and the need to keep electoral incentives at work – which favors having a small number of parties. Lizzeri and Persico (2005) propose a related trade-off – centered around the need to balance general, country-wide goals and local or particularistic ones – which also points to a cost of having an 'excessive' number of parties under PR.

[13]The proof of this result is also in the Appendix

# 8 Conclusion

We presented a new model of the interplay between intra- and inter party incentives, to understand the impact of intra- and interparty competition on electoral outcomes. A distinctive feature of the model is that we let the candidate selection process inside parties take center stage. This process shapes the different politicians' incentives to improve their electoral appeal, thereby impacting voters' decisions and ultimately electoral outcomes. We also showed that outcomes differ across electoral rules, underscoring the interplay between competition within and across parties.

Besides suggesting that intraparty rule should feature prominently in any analysis of electoral incentives, our findings also suggest that there could be a causal link between electoral rule and the way parties organize. Causal evidence appears not to contradict this prediction of the model.

We view this paper as a first step in extending electoral competition models to take more into account the incentives and decisions of the party leadership. Given the scope of such an agenda, we believe there is is plenty of room to advance our knowledge in this area. In particular, there appear to be large gaps in our understanding of the workings and implications of the way parties select their candidates under PR, especially if we allow incentives to interact with selection by letting candidates differ in ability or competence. The tractability of our modelling of interparty competition seems to make the present model well suited to analyze this question.

Yet our theory raises other and new important questions. For example, what makes the party leadership be strong enough and have a long enough horizon to ensure that the candidate selection process remains unbiased and fair, as opposed to being turned into a machine that works to protect entrenched interests and existing incumbents? Crutzen and Flamand (2018) and Dewan and Squintani (2018) have a first pass at this question, but much remains to be done. In particular, there appears to a clear lacunae in the literature

when it comes to understanding (political) leadership, its origins and its consequences. Also, fair competition is easier to ensure between similar-minded competitors. Yet parties are not ideological monoliths. Thus, to what extent are mainstream parties, with their ideological more diverse base, more prone to unfair and biased selection processes than more ideologically narrow parties such as niche and extreme parties?

## References


Amegashie, J. Atsu. "A contest success function with a tractable noise parameter." Public Choice 126.1-2 (2006): 135-144.

Bawn, Kathleen and Michael F. Thies. (2003). "A comparative theory of electoral incentives" *Journal of theoretical politics* 15: 5-32

Beath, A., F. Christia, G. Egorov, and R. Enikolopov. (2016). "Electoral Rules and Political Selection: Theory and Evidence from a Field Experiment in Afghanistan." *The Review of Economic Studies* 83: 932-968.

Benedetto, Giacomo and Simon Hix. (2007). "The Rejected, the Ejected, and the Dejected: Explaining Government Rebels in the 2001-2005 British House of Commons." *Comparative Political Studies* 40: 755-781

Besley, Timothy, Olle Folke, Torsten Persson, and Johanna Rickne. (2017). "Gender Quotas and the Crisis of the Mediocre Man: Theory and Evidence from Sweden". *American Economic Review* 107: 2204-2242

Bille, Lars. (2001). "Democratizing a Democratic Procedure: Myth or Reality?" *Party Politics* 7: 363-380

Buisseret, Peter and Carlo Prato. (2017). "Electoral Accountability in Multi-Member Districts," mimeo University of Chicago School of Public Policy.

Buisseret, Peter, Olle Folke, Carlo Prato and Johanna Rickne. (2018). "Party Nomination Strategies in Closed and Flexible List PR," mimeo University of Chicago School of

Public Policy.

Caillaud, Bernard and Tirole, Jean. (2002). "Parties as Political Intermediaries". *The Quarterly Journal of Economics* 117: 1453-1489.

Canes-Wrone, Brandice. 2006. Who Leads Whom? Presidents, Policy, and the Public. Chicago: Chicago University Press.

Canes-Wrone, Brandice, Michael C. Herron, and Kenneth W. Shotts. (2001). "Leadership and Pandering: A Theory of Executive Policy- making." *American Journal of Political Science* 45: 332?50.

Carey, John M. and Simon Hix. (2011). "The Electoral Sweet Spot: Low-Magnitude Proportional Electoral Systems." *American Journal of Political Science* 55: 383-397.

Castanheira, Micael, Benoit S Y Crutzen and Nicolas Sahuguet. (2010). "Party Organization and Electoral Competition." *Journal of Law, Economics and Organization* 26: 212-242.

Clark, Derek J., and Christian Riis. (1996). "A multi-winner nested rent-seeking contest." *Public Choice* 87.1-2 : 177-184.

Corchon, L. C. (2007): "The theory of contests: a survey," *Review of Economic Design*, 11, 69-100.

Crutzen Benoit and Sabine Flamand (2018). "Leaders, Factions and Party Unity." Mimeo, Erasmus School of Economics

Crutzen, Benoit S Y, Sabine Flamand and Nicolas Sahuguet (2018). "On the optimality of closed list PR", mimeo, Erasmus School of Economics.

Dal Bo, Ernesto, Frederico Finan, Olle Folke, Torsten Persson, and Johanna Rickne (2017). "Who Becomes a Politician?" *Quarterly Journal of Economics,* 132(4): 1877–1914

Dewan, Torun, and David P. Myatt. 2007. "Leading the Party: Coordination, Direction and Communication." *American Political Science Review* 101: 825?43.

Dewan, Torun, and David P. Myatt. 2008. "The Qualities of Leader- ship: Communica-

tion, Direction and Obfuscation." *American Political Science Review* 102: 351?68.

Dewan, Torun and Francesco Squintani (2018). "Leadership with Trustworthy Associates", American Political Science Review

Egorov, Georgy, and Konstantin Sonin. 2011. "Dictators and Their Viziers: Endogenizing the Loyalty-Competence Trade-off." *Journal of the European Economic Association* 9: 903?30.

Eliaz Kfir and Qinggong Wu. (2017). "A Simple Model of Competition Between Teams", working paper.

Enelow, James M. and Melvin J. Hinich (1982). "Ideology, issues and the spatial theory of elections," *American Political Science Review*, 76, 493-501.

Francois, Patrick, Francesco Trebbi and Kairong Xiao (2017). "Factions in nondemocracies: theory and evidence from the Chinese Communist party". Mimeo, the University of British Columbia.

Fu Qiang and Jingfeng Lu. (2009). "The beauty of "bigness": On optimal design of multi-winner contests", *Games and Economic Behavior* 66: 146–161

Galasso, Vincenzo and Tommaso Nannicini (2011). "Competing on Good Politicians." *American Political Science Review* 105: 79-99.

Galasso, Vincenzo and Tommaso Nannicini (2015). "So closed: Political selection in proportional systems". *European Journal of Political Economy*, 40, 260-273.

Hangartner, Dominik, Nelson A. Ruiz, and Janne Tukiainen (2018). "Open or closed: How List Type Affects Electoral Performance, Candidate Selection, and Campaign Effort". Mimeo, VATT Helsinki

Hazan, Reuven Y. and Paul Pennings (eds.). (2001). *Democratizing Candidate Selection: Causes and Consequences.* Special Issue of *Party Politics.*

Hazan, Reuven Y. and Gideon Rahat. (2006). "Candidate Selection: Methods and Consequences." in Richard S. Katz and William Crotty (eds.). Handbook of Party Politics,

London: SAGE.

Hazan, Reuven Y. and Gideon Rahat. (2010). *Democracy within Parties: Candidate Selection Methods and their Political Consequences.* Oxford University Press.

Hazan, Reuven Y and Gerrit Voerman. (2006). "Electoral Systems and Candidate Selection" *Acta Politica* 41: 146-162.

Hermalin, Benjamin E. (1998). "Toward an Economic Theory of Leadership: Leading by Example." *American Economic Review* 88: 1188?206.

Jun, Hae-Won and Simon Hix. (2010). "Electoral Systems, Political Career Paths and Legislative Behavior: Evidence form South Korea?s Mixed-Member System." *Japanese Journal of Political Science*: 153-171.

Katz, Richard S. et al. (1992). "The Membership of Political Parties in European Democracies, 1960–1990" European Journal of Political Research 22: 329–45.

Katz, Richard S. and Robin Kolodny (1994). "Party organisation as an empty vessel: Parties in American politics"; in Katz, Richard S. and Peter Mair. 1994. *How parties organize: change and adaptation in party organizations in Western democracies.* SAGE publications Ltd.

Katz, Richard S. and Peter Mair. (1994). *How parties organize: change and adaptation in party organizations in Western democracies.* SAGE publications Ltd.

Kenig, Ofer, Gideon Rahat, and Reuven Y. Hazan. (2013). "Leadership Selection Versus Candidate Selection in Parliamentary Democracies: Similarities and Differences." Working Paper, available at https://ecpr.eu/Filestore/PaperProposal/d730a6f6-7e80-4f6f-a3df-a0590f645d79.pdf.

Kunicova, Jana and Susan Rose-Ackerman. (2005). "Electoral Rules and Constitutional Structures as Constraints on Corruption." *British Journal of Political Science* 35: 573-606.

Konrad, K. A. (2009): Strategy and Dynamics in Contests, Oxford UK: Oxford University Press.

Laver, Michael, and W. Ben Hunt. (1992). *Policy and Party Competition.* London: Routledge.

Li, Hongbin, and Li-An Zhou. (2005). "Political Turnover and Economic Performance: the Incentive Role of Personnel Control in China". *Journal of Public Economics*: 1743-62.

Lindbeck, Assar and Jörgen W. Weibull (1987). "Balanced-budget redistribution as the outcome of political competition," *Public Choice*, 52(3), 273-297.

Lizzeri, A. and Persico, N., (2001). "The provision of public goods under alternative electoral incentives." *American Economic Review*, pp.225-239.

Lizzeri, A. and Persico, N., (2005). "A drawback of electoral competition." *Journal of the European Economic Association*, 3(6), pp.1318-1348.

Lundell Krister. (2004). "Determinants of Candidate Selection." *Party Politics* 10: 25-47.

Mair, Peter, and Ingrid van Biezen. (2000). "Party Membership in Twenty European Democracies, 1980–2000." Party Politics 7(1): 5–21.

Mattozzi, Andrea and Antonio Merlo (2015). "Mediocracy". *Journal of Public Economics* 130: 32-44

Maublanc François and Sébastien Rouillon. 2017 "Contests with an uncertain number of prizes". mimeo university of Bordeaux.

Milesi-Ferretti, Gian-Maria, Roberto Perotti and Massimo Rostagno (2002). "Electoral systems and public spending". *Quarterly Journal of Economics* 609-657

Montabes, Juan and Ortega, Carmen. (1999). "Candidate Selection in Two Rigid List Systems: Spain and Portugal." Unpublished manuscript, presented at the ECPR Joint Sessions, Mannheim

Morelli, Massimo (2004). "Party formation and policy outcomes under different electoral systems". *Review of Ecopnomic Studies* 71: 829-853

Myerson, Roger (1993). "Effectiveness of Electoral Systems for Reducing Government

Corruption: A Game-Theoretic Analysis." *Games and Economic Behavior* 5: 118-132

Nalebuff, Barry J. and Joseph E. Stiglitz (1983). "Prizes and incentives: towards a general theory of compensation and competition," *The Bell Journal of Economics*, 21-43.

Norris, Pippa. (1997). *Passages to power: legislative recruitment in advanced democracies*. Cambridge University Press.

Norris, Pippa. (2006). "Recruitment." In: Richard S. Katz and William Crotty (eds.). Handbook of Party Politics. London: Sage.

Persson, Torsten, Gerard Roland and Guido Tabellini (2000). "Comparative Politics and Public Finance". *Journal of Political Economy* 108: 1121-1161.

Persson, Torsten and Guido Tabellini. (1999). "The size and scope of government: Comparative politics with rational politicians". Alfred Marshall Lecture. *European Economic Review* 43: 699-735.

Persson, Torsten and Guido Tabellini. (2000). Political Economics: Explaining Economic Policy. The MIT Press.

Persson, Torsten and Guido Tabellini. (2003). The Economic Effect of Constitutions. The MIT Press.

Persson, Torsten, Guido Tabellini and Francesco Trebbi. (2003). "Electoral rules and Corruption." *The Journal of the European Economic Association* 1: 958-989.

Raffler, Pia (2016). "Accountability and Electoral Systems". Mimeo, Yale university.

Rahat, Gideon and Reuven Y. Hazan. (2001). "Candidate selection methods: An analytical framework". *Party Politics* 7: 297-322.

Sahuguet Nicolas and Nicola Persico (2006). "Campaign spending regulation in a model of redistributive politics." *Economic Theory*, 28(1), p. 95-124.

Scarrow, Susan E. , Paul D. Webb, and Thomas Poguntke (2017). *Organizing Political Parties: Representation, Participation, and Power* Oxford University Press.

Schumacher Gijs, Catherine E. de Vries and Barbara Vis. (2013). "Why Do Parties

Change Position? Party Organization and Environmental Incentives." *The Journal of Politics*, 75(2) p. 464–477.

Shomer, Yael, (2014). "What affects candidate selection processes? A cross-national examination" *Party Politics* 20: 533-546

Snyder, James M. Jr and Michael Ting (2011). "Electoral selection with parties and primaries". *American Journal of Political Science*

Spivey, Michael Z. (2007). "Combinatorial sums and finite differences", *Discrete Mathematics,* 307: 3130-3146,

Tavits, Margit (2007). "Clarity of Responsibility and Corruption". *American Journal of Political Science* 51: 218-229.

Tullock, Gordon (1980). "Efficient rent-seeking." in J.M. Buchanan, R.D. Tollison and G. Tullock (eds.) *Toward a theory of the rent-seeking society.* College Station Texas A&M Press.

## Appendix : Proofs

### Proof of proposition 4

The first order condition to the problem faced by any politician $i$ (in party $L$, say) is:

$$\left[\sum_{m=1}^{K}\frac{\partial P_L(l)}{\partial e_i}Q_i(m)+\sum_{m=1}^{K}P_L(m)\frac{\partial Q_i(m)}{\partial e_i}\right]V=e_i^*.$$

and, in the symmetric equilibrium, we have that:

As there are $N$ candidates competing for one of the list slots, the equilibrium probability of being offered slot $m$ on the list is $Q_i^*(m)=\frac{m}{N}$;

We also have

$$\frac{\partial P_L(m)}{\partial e_i}=\beta\frac{C_m^K}{4E^*}\left(2m-K\right)\left(\frac{1}{2}\right)^{K-1}=\beta\frac{C_m^K}{Ke_i^*}\left(2m-K\right)\left(\frac{1}{2}\right)^{K+1}$$

Finally:

$$\frac{\partial Q_i(m)}{\partial e_{iL}}=\frac{1}{e_i^*}\left(1-\frac{m}{N}\right)\sum_{j=1}^{m}\frac{1}{N-j+1}$$

and thus the FOC boils down to in the symmetric equilibrium:

$$e_i^*=\beta V\sum_{m=1}^{K}\frac{C_m^K}{Ke_i^*}\left(2m-K\right)\left(\frac{1}{2}\right)^{K+1}\frac{m}{N}+V\sum_{m=1}^{K}C_m^K\left(\frac{1}{2}\right)^{K}\left[\frac{1}{e_i^*}\left(1-\frac{m}{N}\right)\sum_{j=1}^{m}\frac{1}{N-j+1}\right]$$

We start by proving a useful lemma.

**Lemma 2**

$\sum_{k=m}^{n}C_k^n\left[kp^{k-1}\left(1-p\right)^{n-k}-\left(n-k\right)\left(1-p\right)^{n-k-1}p^k\right]=mC_m^np^{m-1}\left(1-p\right)^{n-m}$.

**Proof:**

We show that terms in the sum cancel. Consider the second term within the summation sign: $\left(n-k\right)C_k^n\left(1-p\right)^{n-k-1}p^k$. Using the identity $\left(n-k\right)C_k^n=\left(k+1\right)C_{k+1}^n$ we can write it as $\left(n-k\right)C_k^n\left(1-p\right)^{n-k-1}p^k=\left(k+1\right)C_{k+1}^n\left(1-p\right)^{n-k-1}p^k$, which corresponds exactly to the first term within the summation sign for the index $k+1$. These two terms cancel leaving

only the first and last term of the sum. The first term is $mC_m^n p^{m-1}(1-p)^{n-m}$. The last term is equal to zero.

■

Using the above lemma, equilibrium effort $e_i^*$ is given by:

$$\sqrt{V} * \sqrt{\frac{\beta}{4N} + \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left(1 - \frac{m}{N}\right) \left[\sum_{j=1}^{m} \frac{1}{N-j+1}\right]}.$$

**Proof of Theorem 5**

We compare equilibrium effort under FPTP when there are two candidates in each primary with equilibrium effort under PR when the number of candidates is $N = K$.

We need to compare $\sqrt{V\frac{(1+\beta)}{8}}$ and $\sqrt{V\left(\frac{\beta}{4K} + \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left(1 - \frac{m}{K}\right) \left[\sum_{j=1}^{m} \frac{1}{K-j+1}\right]\right)}$.

Effort under FPTP does not change with $K$.

It is straightforward to check by direct computation that, for $K = 3$, equilibrium effort under PR – $\sqrt{V}\sqrt{\frac{\beta}{12} + \frac{5}{24}}$ – is greater than that under FPTP – $\sqrt{V}\sqrt{\frac{1+\beta}{8}}$ – as $\sqrt{\frac{\beta}{12} + \frac{5}{24}} - \sqrt{\frac{1+\beta}{8}} = \sqrt{\frac{1}{12} - \frac{\beta}{12}} \geq 0, \forall \beta \in [0,1]$ .

The first term making up effort under proportional representation, $\frac{\beta}{4K}$, is decreasing in $K$.

We now show that second term, $\sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left(1 - \frac{m}{K}\right) \left[\sum_{j=1}^{m} \frac{1}{K-j+1}\right] = \lambda(K)$ is increasing in $K$, and that it is increasing faster than $\frac{\beta}{4K}$ is decreasings for any $K$.

The following result about combinatorial sums of finite differences will prove useful. Identity 14 in Spivey (2007) shows that $\sum_{m=1}^{K} 2^{-K} C_m^K (H_K - H_{K-m}) = \sum_{m=1}^{K} \frac{1}{m2^m}$ with $H_K = \sum_{j=1}^{K} \frac{1}{j}$ is the $Kth$ harmonic number.

We have

$$\lambda(K) = \sum_{m=1}^{K} \left(\frac{1}{2}\right)^K C_m^K (1 - m/K) \left(\sum_{j=1}^{m} \frac{1}{K+1-j}\right) = \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^K C_m^{K-1} (H_K - H_{K-m})$$
$$= \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^K C_m^{K-1} \left(H_{K-1} - H_{K-1-m} + \frac{1}{K} - \frac{1}{K-m}\right).$$

Thus, exploiting identity 14 in Spivey (2007):

$$\lambda(K) = \frac{1}{2} \sum_{m=1}^{K-1} \frac{1}{m2^m} + \frac{1}{2} \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^{K-1} C_m^{K-1} \left(\frac{1}{K} - \frac{1}{K-m}\right).$$

The second term in $\lambda(K)$ simplifies to:

$$\frac{1}{K} \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^{K-1} C_m^{K-1} \left(\frac{-m}{(K-m)}\right) = -\frac{1}{K^2} \sum_{m=1}^{K-1} \left(\frac{1}{2}\right)^{K-1} C_m^K m = -\frac{1}{K} \left(1 - \left(\frac{1}{2}\right)^{K-1}\right)$$

Thus

$$\lambda(K) = \frac{1}{2} \left(\sum_{m=1}^{K-1} \left(\frac{1}{m2^m}\right) - \frac{1}{K} + \frac{1}{K} \left(\frac{1}{2}\right)^{K-1}\right).$$

This implies in turn that the second term is indeed increasing in $K$:

$$\lambda(K+1) - \lambda(K) = \left(\frac{1}{(K)\,2^K} - \frac{1}{K+1} + \frac{1}{K+1} \left(\frac{1}{2}\right)^K + \frac{1}{K} - \frac{1}{K} \left(\frac{1}{2}\right)^{K-1}\right)$$
$$= \frac{1}{2^K K} \frac{2^K - 1}{K+1} > 0.$$

To see that the second term grows faster than the speed at which the first term shrinks, simple algebra implies that:

$$\frac{\beta}{4(K+1)} + \lambda(K+1) - \left(\frac{\beta}{4(K)} + \lambda(K)\right) = \beta \left(\frac{1}{4(K+1)} - \frac{1}{4K}\right) + \frac{1}{2^K K} \frac{2^K - 1}{K+1}$$
$$= \frac{1}{K(K+1)} \left(\frac{2^K - 1}{2^K} - \frac{\beta}{4}\right).$$

For $K \geq 3$ and $\beta \in [0,1]$, $\frac{2^K-1}{2^K} \geq 7/8 > \beta/4$

Finally, for completeness, effort under proportional representation approaches $\sqrt{log(2)/2} = 0.58$ for $K$ approaching infinity. This is obviously higher than equilibrium effort under plurality rule, as $\sqrt{(1+\beta)/8} < 1/2, \forall \beta \in [0,1]$.

■

## Proof of Proposition 6

We first derive equilibrium effort under the non-competitive selection rule

**Plurality Rule**

Let us focus on candidate of party $L$ in district $d$. The outcome in district $d$ is the only outcome candidate $dL$ can influence. The candidate is pivotal when party $L$ wins in exactly $\frac{K-1}{2}$ districts out of the $K-1$ districts in which candidate $d$ is not running. in equilibrium, this happens with probability $C^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1}$.

The candidate chooses effort $e_{dL}$ to maximize:

$$\left(V + C^{K-1}_{\frac{K-1}{2}}\left(\tfrac{1}{2}\right)^{K-1} M\right)\left(\tfrac{e_{dL}}{e_{dL}+e_{dR}}\right) - \frac{e_{dL}^2}{2}.$$

Taking the first order condition and evaluating at the symmetric equilibrium, we get

$$e^*_{dL} = \frac{1}{2}\sqrt{V + C^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1} M}.$$

We thus get:

$$E^* = \frac{K}{2}\sqrt{V + C^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1} M}.$$

**Proportional Representation**

Every candidate's effort matters for the party's probability of winning at least a majority of the legislative seats. Candidate in position $m$ on party $L$ list chooses $e_m$ to maximize:

$$V\sum_{l=m}^{K} C_l^K \left(\tfrac{E_L}{E_L+E_R}\right)^l \left(1-\left(\tfrac{E_L}{E_L+E_R}\right)\right)^{K-l} + M\sum_{j=\frac{K+1}{2}}^{K} C_j^K \left(\tfrac{E_L}{E_L+E_R}\right)^j \left(1-\left(\tfrac{E_L}{E_L+E_R}\right)\right)^{K-j} - \frac{e_m^2}{2}$$

The first order condition evaluated in the symmetric equilibrium yields:

$$\begin{aligned} e_m^* &= \frac{V}{4E^*}\left(\frac{1}{2}\right)^{K-1}\sum_{l=m}^{K}(2l-K)\,C_l^K + \frac{M}{4E^*}\left(\frac{1}{2}\right)^{K-1}\sum_{j=\frac{K+1}{2}}^{K}(2j-K)\,C_j^K \\ &= \frac{1}{4E^*}\left(mC_m^K\left(\frac{1}{2}\right)^{K-1}V + \left(\frac{1}{2}\right)^{K-1}\left(\frac{K+1}{2}\right)C_{\frac{K+1}{2}}^K M\right). \end{aligned}$$

where the second line exploits the fact that $\sum_{l=j}^{K}(2l-K)\,C_l^K = jC_j^K$.

Then, summing the effort over all candidates on the list, we get $E^* = \sum_{m=1}^{K} e_m$ or:

$$\begin{aligned} E^* &= \frac{V}{4E^*}\sum_{m=1}^{K}\left(\frac{1}{2}\right)^{K-1} mC_m^K + \frac{M}{4E^*}\left[\left(\frac{K+1}{2}\right)\left(\frac{1}{2}\right)^{K-1} C_{\frac{K+1}{2}}^K M\right] \\ &= \frac{1}{4E^*}\left(VK + MK\left(\frac{K+1}{2}\right)C_{\frac{K+1}{2}}^K\left(\frac{1}{2}\right)^{K-1}\right) \end{aligned}$$

and thus using the fact that $\frac{K+1}{2}C_{\frac{K+1}{2}}^K = KC_{\frac{K-1}{2}}^{K-1}$

$$E^* = \sqrt{VK + MK^2 C_{\frac{K-1}{2}}^{K-1}\left(\frac{1}{2}\right)^{K-1}}.$$

Comparing (14) and (15) it is easy to verify that total effort is always higher under the majoritarian system if $V > 0$, via the incentives regarding the candidates' individual reward:

$\frac{1}{2}\sqrt{VK^2 + MK^2 C_{\frac{K-1}{2}}^{K-1}\left(\frac{1}{2}\right)^{K-1}} > \frac{1}{2}\sqrt{VK + MK^2 C_{\frac{K-1}{2}}^{K-1}\left(\frac{1}{2}\right)^{K-1}}$.

Indeed, the part of the incentive problem linked to the party's winning at least a majority generates the same incentives across the two electoral rules. Thus, proposition 1 carries through to the case in which candidates care about their party winning the executive office too as long as candidates care at least marginally about themselves getting elected too, as otherwise the electoral rule is immaterial for incentives.

We now derive the effort under the competitive candidate selection procedure.

**Plurality Rule**

As we have seen before, the optimal number of candidates is 2.

Equilibrium effort is thus the same as in the non-competitive case:

$$e^* = \sqrt{\frac{V}{4} + C^{K-1}_{\frac{K-1}{2}} \left(\frac{1}{2}\right)^{K-1} \frac{M}{4}}.$$

**Proportional Representation**

A candidate from party $L$ chooses effort $E_L$to maximise:

$$V \sum_{m=1}^{K} \left\{ \left\{ p_1 + \sum_{j=2}^{m} p_j \left[ \prod_{s=1}^{j-1} (1 - p_s) \right] \right\} \left\{ C^K_m \left( \tfrac{E_L}{E_L+E_R} \right)^m \left( 1 - \tfrac{E_L}{E_L+E_R} \right)^{K-m} \right\} \right\}$$
$$+ M \sum_{j=\frac{K+1}{2}}^{K} C^K_j \left( \tfrac{E_L}{E_L+E_R} \right)^j \left( 1 - \tfrac{E_L}{E_L+E_R} \right)^{K-j} - \frac{e_L^2}{2}.$$

where the second term is party $L$'s probability of winning at least a majority of legislative seats which, from the perspective of candidate $i$ when they choose their effort level, is independent on where on the list candidate $i$ ends up.

The FOC is thus given by:

$$\begin{aligned} e^* = & V \sum_{m=1}^{K} C_m^K \frac{1}{Ke^*} (2m-K) \left(\frac{1}{2}\right)^{\mathcal{L}+1} \frac{m}{K} \\ & + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left[\frac{1}{e^*}\left(1-\frac{m}{n}\right) \sum_{j=1}^{m} \frac{1}{n-j+1}\right] \\ & + \frac{M}{4Ke^*} \left(\frac{1}{2}\right)^{K-1} \sum_{j=\frac{\mathcal{L}+1}{2}}^{K} (2j-K) C_j^K \end{aligned}$$

which yields

$$e^* = \sqrt{\frac{V}{4K} + V \sum_{m=1}^{K} C_m^K \left(\tfrac{1}{2}\right)^K \left[\tfrac{1}{e^*}\left(1-\frac{m}{K}\right) \sum_{j=1}^{m} \tfrac{1}{K-j+1}\right] + \frac{M}{4} C_{\frac{K-1}{2}}^{K-1} \left(\tfrac{1}{2}\right)^{K-1}} \tag{18}$$

Comparison of (19) and (20) together with that of (14) and (15) implies that theorem 5 thus holds unless $M > V = 0$, in which case neither the CSP nor the electoral rule matter anymore for incentives.

■

## Optimal effort provision with $T$ parties under competitive and non-competitive candidate selection

Equilibrium individual effort under FPTP is equal to $K\frac{\sqrt{\beta V}}{2}$.

Let there be $T$ identical parties under PR. The probability that team $i$ wins $l$ prizes is given by:

$$P_i(l) = \frac{K!}{(K-l)!l!} (P_p)^l (1-P_p)^{K-l}, \tag{19}$$

with $P_p = \frac{1-\beta}{T} + \beta \frac{E_p}{\sum_{j=1}^{T} E_j}$.

The problem for the candidate in position $m$ on the list of party $p$ is to maximise with respect to their own effort $e_m^p$:

$$\sum_{k=m}^{K} P_p(k) V - \frac{1}{2} (e_m^p)^2 . \tag{20}$$

The first order condition to the problem of the candidate in position $m$ on the list of party $p$ is given by:

$$e_m^p = \beta V \sum_{k=m}^{K} C_k^K \left(P_p\right)^{k-1} \left(1-P_p\right)^{K-k} k \frac{\sum_{j=1}^{T} E_j - E_p}{\left(\sum_{j=1}^{T} E_j\right)^2} - \beta V \sum_{k=m}^{K} C_k^K \left(P_p\right)^{k} \left(1-P_p\right)^{K-k-1} \left(K-k\right) \frac{\sum_{j=1}^{T} E_j - E_p}{\left(\sum_{j=1}^{T} E_j\right)^2}$$

In the symmetric equilibrium, effort choices of candidates in the same position on the list are equal across parties and thus $E^{1*} = ... = E^{P*} = E^*$ and $P_p = 1/T$. We can simplify the above first order condition to find:

$$\begin{aligned} e_m^L &= \frac{\left(T-1\right)\beta V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[ k \left(\frac{1}{T}\right)^{k-1} \left(\frac{T-1}{T}\right)^{K-k} - \left(K-k\right) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k-1} \right] \\ &= \frac{\left(T-1\right)\beta V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[ \left( Tk - \frac{T\left(K-k\right)}{T-1} \right) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \\ &= \frac{\left(T-1\right)\beta V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[ \frac{T}{T-1} \left(Tk-K\right) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \\ &= \frac{\beta V}{T E^*} \sum_{k=m}^{K} C_k^K \left[ \left(Tk-K\right) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \end{aligned}$$

Exploiting the fact that $\forall T, \sum_{k=m}^{K} C_k^K \left[ \left(Tk-K\right) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] = \left(\frac{1}{T}\right)^{K} \left(T-1\right)^{K-m+1} m C_m^K$, this simplifies further to:

$$e_m^L = \frac{\beta V}{T E^*} \left(\frac{1}{T}\right)^{K} \left(T-1\right)^{K-m+1} m C_m^K.$$

Summing these optimal effort decisions over all party list members and exploiting the fact that $\sum_{m=1}^{K} \left(\frac{1}{T}\right)^{K} \left(T-1\right)^{K-m+1} m C_m^K = K \frac{T-1}{T}$, we get:

$$E^* = \frac{\beta V}{T E^*} K \frac{T-1}{T} \iff E^* = \frac{\sqrt{K\left(T-1\right)\beta V}}{T} \tag{21}$$

Comparing the above party output to that under FPTP, $K\frac{\sqrt{\beta V}}{2}$ , we see that the result of the first part of theorem 5 is reinforced.

We now turn to the competitive selection.

Nothing changes under FPTP. Under PR the first order condition to the problem faced by any politician $i$ (in party $L$, say) is:

$$\left[\sum_{m=1}^{K}\frac{\partial P_p(l)}{\partial e_i}Q_i(m)+\sum_{m=1}^{K}P_p(m)\frac{\partial Q_i(m)}{\partial e_i}\right]V=e_i^*$$

and, in the symmetric equilibrium, we have that:

1. As there are $S$ candidates competing for one of the $K$ list slots, the equilibrium probability of being offered slot $m$ on the list is $Q_i^*(m)=\frac{m}{S}$;
2. Also, as $P_p(m)=C_m^K\left(\frac{1-\beta}{T}+\beta\frac{E_i}{\sum_{j=1}^{T}E_j}\right)^m\left(1-\frac{1-\beta}{T}-\beta\frac{E_i}{\sum_{j=1}^{T}E_j}\right)^{K-m}$, in equilibrium we have that, exploiting some of the algebra above:

$$\frac{\partial P_p(m)}{\partial e_i} = \frac{\beta V}{TKe^*}C_m^K\left[(Tm-K)\left(\frac{1}{T}\right)^m\left(\frac{T-1}{T}\right)^{K-m}\right]$$

3. Also:

$$\frac{\partial Q_i(m)}{\partial e_{iL}}=\frac{1}{e_i^*}\left(1-\frac{m}{S}\right)\sum_{j=1}^{m}\frac{1}{S-j+1}$$

4. Finally:

$$P_p^*(m)=C_m^K\left(\frac{E_i^*}{\sum_{j=1}^{T}E_j^*}\right)^m\left(1-\frac{E_i^*}{\sum_{j=1}^{T}E_j^*}\right)^{K-m}=C_m^K\left(\frac{1}{T}\right)^m\left(\frac{T-1}{T}\right)^{K-m} \tag{22}$$

and thus the FOC boils down to in the symmetric equilibrium:

$$\begin{aligned} e_i^* &= \sum_{m=1}^{K}\frac{\beta V}{TKe_i^*}C_m^K\left[(Tm-K)\left(\frac{1}{T}\right)^m\left(\frac{T-1}{T}\right)^{K-m}\right]\frac{m}{S} \\ &\quad +V\sum_{m=1}^{K}C_m^K\left(\frac{1}{T}\right)^m\left(\frac{T-1}{T}\right)^{K-m}\left[\frac{1}{e_i^*}\left(1-\frac{m}{S}\right)\sum_{j=1}^{m}\frac{1}{S-j+1}\right] \\ &= \frac{\beta V}{T^2e_i^*}\frac{T-1}{S} \\ &\quad +V\sum_{m=1}^{K}C_m^K\left(\frac{1}{T}\right)^m\left(\frac{T-1}{T}\right)^{K-m}\left[\frac{1}{e_i^*}\left(1-\frac{m}{S}\right)\sum_{j=1}^{m}\frac{1}{S-j+1}\right] \end{aligned}$$

which implies therefore that equilibrium effort $e_i^*$ is given by:

$$\sqrt{V} * \sqrt{\frac{\beta\left(T-1\right)}{T^2 S} + \sum_{m=1}^{K} C_m^K \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m} \left(1-\frac{m}{S}\right) \left[\sum_{j=1}^{m} \frac{1}{S-j+1}\right]}. \tag{23}$$

To prove that equilibrium effort tends to 0 as we let $T$ tend to infinity, we now show that the RHS of the first order condition above, namely the marginal benefit of exerting effort, must eventually shrink to 0 as the number of parties grows large enough.

As there are $S$ candidates competing for one of the $K$ list slots, the equilibrium probability of being offered slot $m$ on the list is $Q_i^*(m) = \frac{m}{S}$;

Also, $P_p(m) = C_m^K \left(\frac{1-\beta}{T} + \beta\frac{E_i}{\sum_{j=1}^{T} E_j}\right)^m \left(1 - \frac{1-\beta}{T} - \beta\frac{E_i}{\sum_{j=1}^{T} E_j}\right)^{K-m}$. Then, exploiting some of the algebra above:

$$\begin{aligned}\frac{\partial P_p(m)}{\partial e_i} &= \frac{\beta V}{TKe^*} C_m^K \left[(Tm-K)\left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m}\right] \\ &= \frac{\beta V}{Ke^*} C_m^K \left[\left(m-\frac{K}{T}\right)\left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m}\right]\end{aligned}$$

Thus, $lim_{T\to\infty} \frac{\beta V}{Ke^*} C_m^K \left[\left(m-\frac{K}{T}\right)\left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m}\right] = 0$.

Also:

$$\frac{\partial Q_i(m)}{\partial e_{iL}} = \frac{1}{e_i^*}\left(1-\frac{m}{K}\right)\sum_{j=1}^{m} \frac{1}{K-j+1}$$

does not depend on $T$ and

$$P_p^*(m) = C_m^K \left(\frac{E_i^*}{\sum_{j=1}^{T} E_j^*}\right)^m \left(1 - \frac{E_i^*}{\sum_{j=1}^{T} E_j^*}\right)^{K-m} = C_m^K \left(\frac{1}{T}\right)^m \left(\frac{T-1}{T}\right)^{K-m} \tag{24}$$

is also decreasing in $T$.

Thus, as an increase in $T$ must eventually depress the marginal benefit of effort, there must be some threshold $T^* \geq 2$ such that for any $T < T^*$, equilibrium effort under PR is greater than that under FPTP but the converse is true for $T > T^*$.