Rational Inattention and the Business Cycle Effects of Productivity and News Shocks*

Bartosz Maćkowiak
European Central Bank and CEPR

Mirko Wiederholt
Sciences Po and CEPR

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Abstract

In the standard neoclassical model, anticipated fluctuations in productivity fail to cause business cycle comovement. In response to news about higher future productivity, consumption rises but employment and investment fall. Suppose that firms are subject to rational inattention. They choose an optimal signal about the state of the economy. The optimal signal turns out to confound current with expected future productivity. Labor and investment demand rise after a news shock, causing an output expansion. Rational inattention also improves the propagation of a standard productivity shock, by inducing persistence.

Keywords: information choice, rational inattention, business cycles, news shocks, productivity shocks (*JEL*: D83, E32, E71).

*Maćkowiak: European Central Bank, 60640 Frankfurt am Main, Germany (e-mail: bartosz.mackowiak@ecb.int); Wiederholt: Sciences Po, 28 Rue des Saints-Pères, 75007 Paris, France (e-mail: mirko.wiederholt@gmail.com). The views expressed in this paper are solely those of the authors and do not necessarily reflect the views of the European Central Bank. The authors thank Christoph Görtz, Yulei Luo, Lumi Stevens, Christian Wolf, and numerous seminar participants for helpful discussions, and Romain Aumond for excellent research assistance.

1 Introduction

The basic challenge for any business cycle model is to specify an impulse and a propagation mechanism that produce business cycle comovement. This challenge is difficult, as Barro and King (1984) first explained.[1] A key insight from the Real Business Cycle model is that fluctuations in productivity generate comovement in the standard neoclassical economy. Employment, investment, output, and consumption move together after a productivity shock, as they do in the data in a business cycle expansion or contraction.[2]

This insight is fragile, however. In the real world, information about changes in productivity may become available some time before they occur. In the model, it matters if agents can learn in advance about changes in productivity. If agents can learn in advance, variables respond in ways inconsistent with a business cycle. Anticipated fluctuations in productivity do not cause comovement. Suppose productivity will rise in the future (while current productivity is unchanged). The news causes a wealth effect. Firms have no incentive to increase labor demand before productivity improves, while households reduce labor supply due to the wealth effect. As a result, hours worked fall. With capital predetermined and current productivity unchanged, output contracts. Lower saving due to the wealth effect causes a reduction in the capital stock over time. Investment declines while consumption rises. The model fails to produce comovement in response to news about future productivity. It predicts an output contraction after news that productivity will improve.[3]

It is convenient to model anticipated fluctuations in productivity as "news shocks about productivity" ("news shocks" for short). A shock drawn by nature in quarter t affects productivity in quarter $t+h$, where h is a strictly positive integer. The question is how the economy responds to a news shock before quarter $t+h$. In the standard neoclassical model, labor input, investment, and output fall while consumption rises. Labor input, investment, and output increase only once productivity improves. In the New Keynesian model, each firm commits to supply output at a

[1] Much more recently, Jaimovich and Rebelo (2009), p.1097, write that "the ability to generate comovement is a natural litmus test for macroeconomic models. It is a test that most models fail."

[2] Kydland and Prescott (1982), Hansen (1985), Prescott (1986), and King, Plosser, and Rebelo (1988) are classic references on the RBC model.

[3] With a high elasticity of intertemporal substitution, the model predicts a rise in employment and investment and a fall in consumption. The substitution effect due to an increase in the real interest rate dominates the wealth effect in this case, pushing consumption down and labor supply up.

fixed price, and therefore a rise in consumption exerts upward pressure on the demand for labor and investment. The response of the economy to a news shock depends on monetary policy. With optimal monetary policy the response is identical to the flexible-price neoclassical benchmark.[4]

In these models once information becomes available, agents absorb it completely. This paper asks how a single friction, rational inattention, changes the propagation of a news shock in the neoclassical setting. Rational inattention is the idea that people cannot process all available information (available information is not internalized information) and they allocate attention optimally (Sims, 2003). In a rational inattention model, an agent chooses an optimal signal about the state of the economy, recognizing that a more informative signal requires more attention, which is costly. The agent takes actions based on the optimal signal, rather than based on perfect information or some exogenous incomplete information set. How does a news shock propagate when people have a limited ability to process information and can choose what information to absorb?

We consider a baseline RBC model. Neoclassical firms produce homogeneous output with capital and labor. There are no adjustment costs. Households have standard preferences for consumption and leisure. The perfect information equilibrium is familiar. We focus on the equilibrium when firms are subject to rational inattention and households have perfect information.[5]

The main qualitative insight from the paper is that rational inattention induces an increase in the firms' demand for labor and investment on impact of a positive news shock. The reason is that the optimal signal confounds current with expected future productivity. Thus firms react on impact of a news shock, as if productivity has already changed with some probability. The intuition for the optimality of a confounding signal is twofold. First, noise in signals due to rational inattention introduces delay in actions. Paying attention to future productivity helps reduce this delay in actions. Second, a one-dimensional signal that confounds current productivity with expected future productivity requires less attention than a two-dimensional signal consisting of a signal on current productivity and a separate signal on future productivity. For these two reasons, a signal

[4] With suboptimal monetary policy a standard New Keynesian model (Smets and Wouters, 2007) produces comovement after news about future productivity, but the impulse response of employment turns negative once productivity improves. The same is true in a heterogeneous agent version of the model (we thank Christian Wolf for these observations). For a review of the literature on news shocks, see Lorenzoni (2011), Beaudry and Portier (2014), and Jaimovich (2017).

[5] We add rational inattention on the side of households in a later section of the paper.

confounding current with expected future productivity is optimal.[6]

The main quantitative insight from the paper is that the rational inattention effect on labor and investment demand is strong enough to change the responses of employment and output on impact of a news shock from negative to positive, and the response of investment from negative to zero. The rational inattention effect on labor demand more than offsets the wealth effect on labor supply. Thus, employment and output increase on impact of a news shock. The rational inattention effect on investment demand offsets the wealth effect on saving supply. As a result, the response of investment on impact of a news shock equals zero, as opposed to a sizable negative number in the standard model. To arrive at these quantitative results, we solve a dynamic stochastic general equilibrium model with rational inattention, which is a non-trivial task.

Hence, the single assumption of rational inattention by firms makes the model predict an output expansion after news that productivity will improve. By maintaining that households have perfect information, we stack the deck against us, because in this case the wealth effect that reduces the supply of labor and saving is fully operating. We also solve a version of the model with both firms and households subject to rational inattention. We find that comovement strengthens.

In addition, we ask if rational inattention improves the propagation of a standard productivity shock (a shock that affects productivity in the same period in which the shock is drawn). It has been a challenge for the RBC model to reproduce the persistence in the data. The first-order autocorrelations of employment, investment, and output growth are positive in the data but negative in the baseline model.[7] We find that when firms are subject to rational inattention, the impulse responses of employment, investment, and output to a productivity shock become hump-shaped. Since the optimal signal contains noise, the firms' beliefs are anchored on the steady state and evolve slowly. As a result, employment, investment, and output respond with delay to a productivity shock. The first-order autocorrelations of employment, investment, and output growth in the model become positive and are approximately in line with the data. This finding holds true even though rational inattention is the only source of inertia and the marginal cost of attention is

[6] In Lucas (1972), firms are assumed to observe a one-dimensional signal about nominal aggregate and relative demand. In the rational inattention RBC model with news shocks, firms choose to observe a one-dimensional signal about current and expected future productivity.

[7] This shortcoming of the RBC model was first noted by Cogley and Nason (1995) and Rotemberg and Woodford (1996).

small.

The literature has explored a number of ways to obtain a model that predicts comovement in response to news about future productivity. Jaimovich and Rebelo (2009) modify the baseline RBC model by adding three assumptions: investment adjustment costs, variable capital utilization, and a new class of preferences. Investment adjustment costs and variable capital utilization produce an increase in input demand in response to a news shock, whereas the new preferences control the wealth effect on input supply.[8] Beaudry and Portier (2004, 2007) move to a multi-sector neoclassical setting. They introduce a complementarity so that higher output in one sector makes production more efficient in other sectors, leading to a rise in input demand. Another approach has been to combine nominal stickiness with suboptimal monetary policy. Lorenzoni (2009) analyzes a New Keynesian economy with a Taylor rule where noise in a public signal about productivity causes comovement.[9] By contrast, we explore how a single new assumption, rational inattention, changes the propagation of a news shock in the baseline RBC model. The assumption of rational inattention seems well suited to apply to the question if people have an incentive to be perfectly aware of the timing of productivity changes.

Key papers in the DSGE literature on news shocks focus on changes in productivity in the near future, a few quarters ahead ($h = 3$ in Beaudry and Portier, 2004, $h = 2$ in Jaimovich and Rebelo, 2009). We adopt the same approach.[10] The aim is to show that the comovement insight from the neoclassical economy is not fragile; it is not sensitive to precisely when information becomes available, once one introduces rational inattention. There is also a VAR literature on news shocks. A common identification approach is to assume that a news shock does not affect productivity on impact and explains most of the variance in productivity at some very long horizon like 40 quarters. With this identification productivity typically responds to a news shock very slowly, sometimes remaining essentially unchanged for as long as 3 years.[11] We do not think that rational inattention by itself can explain the impulse responses from this identification. If productivity will

[8] Schmitt-Grohé and Uribe (2012) estimate a related augmented RBC model.

[9] Angeletos and La'O (2010) study a neoclassical model with strategic complementarity and dispersed information in which a similar noise shock causes comovement. On news and noise see also Blanchard, L'Huillier, and Lorenzoni (2013) and Chahrour and Jurado (2018).

[10] We focus on $h = 2$ and $h = 4$.

[11] There are variants of this identification approach. See Beaudry and Portier (2006, 2014), Barsky and Sims (2011), Kurmann and Sims (2019), Görtz, Gunn, and Lubik (2020), and Görtz, Tsoukalas, and Zanetti (2020).

change only in a long run, a rationally inattentive agent will be fairly confident that productivity has not yet changed. The short-run response of the agent's action will differ little from the perfect information response.[12]

Turning to standard productivity shocks, the literature has explored the idea that moving away from full information rational expectations can improve the propagation mechanism relative to the baseline RBC model. Eusepi and Preston (2011) abandon rational expectations altogether, replacing it by adaptive learning. They find that the first-order autocorrelations of employment, investment, and output growth in the model become positive. We add rational inattention, a form of incomplete information rational expectations, to the baseline RBC model. Surprisingly, the single assumption of rational inattention turns out to be sufficient to bring the first-order autocorrelations of employment, investment, and output growth in the model approximately into line with the data.[13]

Solving a DSGE model with rational inattention is challenging. One needs to solve attention problems (signal choice problems) of individual agents in a dynamic model. Furthermore, one needs to find a fixed point of an economy in which the optimal signal of an agent depends on the signals chosen by other agents. Several papers make progress solving attention problems of individual agents in a dynamic environment.[14] Maćkowiak and Wiederholt (2015) solve a DSGE model with rational inattention where the physical environment is similar to a simple New Keynesian model (for example, there is no capital).[15] By contrast, here the physical environment is a neoclassical business cycle model. We adopt a guess-and-verify method to find the fixed point, at each iteration using the results of Maćkowiak, Matějka, and Wiederholt (2018) to solve agents' attention problems.

[12] In response to news about the near future the model generates impulse responses of employment, investment, output, and consumption similar to the impulse responses in the VAR literature. For example, compare Figure 5 ($h = 4$) with Figure 1 in Görtz, Gunn, and Lubik (2020).

[13] Business cycle models face the challenge of matching the persistence in the macro data more generally, not only conditional on a productivity shock. See Sims (1998) for a general discussion, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) in the context of New Keynesian models, and Auclert, Rognlie, and Straub (2020) in the context of a heterogeneous agent New Keynesian model. Our finding may therefore be helpful also for model builders who allow for sources of fluctuations other than productivity.

[14] Sims (2003), Maćkowiak and Wiederholt (2009), Woodford (2009), Sims (2010), Steiner, Stewart, and Matějka (2017), Maćkowiak, Matějka, and Wiederholt (2018), Afrouzi and Yang (2020), Jurado (2020), Miao, Wu, and Young (2020), and Stevens (2020).

[15] See also Ellison and Macaulay (2019) who introduce rational inattention into a HANK model.

One issue in the literature on rational inattention is how to define equilibrium. We assume that prices, which all agents take as given, adjust to guarantee market clearing.[16]

The next section defines the physical environment. Section 3 introduces rational inattention. Section 4 develops intuition for the effects of rational inattention, by considering special cases of the model. Section 5 shows the effects of productivity shocks and news about future productivity in the complete model. Section 6 studies a version of the model in which all agents, firms and households, are subject to rational inattention. Section 7 concludes and outlines further research. There are four appendices with supplementary material.

2 Model – physical environment

We consider a baseline RBC model that allows for an additional factor of production ("an entrepreneurial input") in fixed supply. The production function is Cobb-Douglas and exhibits decreasing returns to scale in the variable factors, capital and labor. We introduce a third factor in fixed supply because to formulate the attention problem of a firm we need the firm's choice of capital and labor under perfect information, not only the capital-labor ratio, to be determinate.

Time is discrete. There is a continuum of firms indexed by $i \in [0,1]$. All firms produce the same good using an identical technology represented by the production function

$$Y_{it} = e^{a_t} K_{it-1}^{\alpha} L_{it}^{\phi} N_i^{1-\alpha-\phi}$$

where Y_{it} is output of firm i in period t, K_{it-1} is capital input, L_{it} is labor input, and e^{a_t} is total factor productivity, common to all firms. N_i is an entrepreneurial input, specific to firm i, in fixed supply. The parameters α and ϕ satisfy $\alpha \geq 0$, $\phi \geq 0$, and $\alpha + \phi < 1$.

The capital stock of firm i evolves according to the law of motion

$$K_{it} - K_{it-1} = I_{it} - \delta K_{it-1}$$

where $\delta \in (0,1]$ is the depreciation rate. The firm maximizes the expected discounted sum of profits or dividends. The dividend of firm i in period t, D_{it}, is given by

$$D_{it} = Y_{it} - W_t L_{it} - I_{it}$$

[16] In Maćkowiak and Wiederholt (2015), in each market one side of the market sets the price and the other side chooses the quantity.

where W_t is the wage rate. The dividends of all firms flow to a mutual fund. Households own and trade shares in the mutual fund.[17]

Total factor productivity is determined according to the law of motion

$$a_t = \rho a_{t-1} + \sigma \varepsilon_{t-h}$$

where ε_t follows a Gaussian white noise process with unit variance, $\rho \in (0,1)$, $\sigma > 0$, and $h \geq 0$. A shock drawn by nature in period t affects productivity in period $t+h$. We solve the model either with $h = 0$ (a standard productivity shock) or with $h \geq 1$ (a news shock).[18] For ease of exposition we abstract from long-run growth.

There is a continuum of households indexed by $j \in [0,1]$. Each household j maximizes the expected discounted sum of utility. The discount factor is $\beta \in (0,1)$. The utility function is

$$U\left(C_{jt}, L_{jt}\right) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{jt}^{1+\eta}}{1+\eta}$$

where C_{jt} is consumption by household j in period t, L_{jt} is hours worked, $\gamma > 0$ is the inverse of the elasticity of intertemporal substitution, and $\eta \geq 0$ is the inverse of the Frisch elasticity of labor supply. Typically, we will set $\gamma = 1$ and $\eta = 0$. The budget constraint in period t is

$$V_t Q_{jt} - V_t Q_{jt-1} = W_t L_{jt} + D_t Q_{jt-1} - C_{jt}$$

where V_t is the price of a share in the mutual fund in period t, Q_{jt} is household j's share in the mutual fund, and $D_t \equiv \int_0^1 D_{it} di$ is the dividend from the mutual fund.

Aggregate output is $Y_t \equiv \int_0^1 Y_{it} di$. Aggregate capital and investment are defined analogously. Aggregate consumption is $C_t \equiv \int_0^1 C_{jt} dj$.

In equilibrium in every period the wage adjusts so that labor demand equals labor supply, $\int_0^1 L_{it} di = \int_0^1 L_{jt} dj$, and the price of a share in the mutual fund adjusts so that asset demand equals asset supply normalized to one, $\int_0^1 Q_{jt} dj = 1$.

The non-stochastic steady state of this economy is described in Appendix A. To solve the model when firms and households have perfect information, we log-linearize their first-order conditions

[17] When firm i was sold to the mutual fund, the entrepreneurial input was paid the present value of its future marginal products and in return committed to supply its service without additional payments.

[18] We also consider the case when productivity is driven by two orthogonal shocks, a standard productivity shock and a news shock. See Appendix D.

and the other equilibrium conditions at the non-stochastic steady state. This yields the completely standard log-linear equilibrium conditions stated in Appendix B. We refer to the solution as the perfect information equilibrium.

3 Model – rational inattention by firms

A rationally inattentive individual cannot process all available information but can decide what information to focus on. The decision-maker in firm i chooses an optimal signal about the state of the economy. He or she maximizes the expected discounted sum of profits, recognizing that a more informative signal requires more attention, which is costly.[19] This section begins by deriving the agent's objective. We then state the agent's attention problem. Finally, we define the equilibrium in the economy in which firms are subject to rational inattention and households have perfect information.

3.1 Loss in profit from suboptimal actions

We derive an expression for the expected discounted sum of losses in profit when actions of firm i deviate from the profit-maximizing actions – the actions the firm would take if it had perfect information in every period. To obtain this expression, we compute the log-quadratic approximation to the expected discounted sum of profits at the non-stochastic steady state.

Recall that the profit of firm i in period t is given by $Y_{it} - W_t L_{it} + (1-\delta) K_{it-1} - K_{it}$. We assume that the mutual fund instructs each firm to value profits according to the marginal utility of consumption.[20] The profit function can be written in terms of log-deviations from the non-stochastic steady state:

$$C^{-\gamma} e^{-\gamma c_t} Y \left\{ e^{a_t + \alpha k_{it-1} + \phi l_{it}} - \phi e^{w_t + l_{it}} + \left(\frac{\alpha}{\beta^{-1} - 1 + \delta} \right) \left[(1-\delta) e^{k_{it-1}} - e^{k_{it}} \right] \right\}$$

where an upper-case letter without a time subscript denotes the value of a variable in the non-stochastic steady state, and a lower-case letter denotes the log-deviation of a variable from its value in the non-stochastic steady state. The term $C^{-\gamma} e^{-\gamma c_t}$ is the marginal utility of consumption.

[19] The optimal signal may follow a multivariate stochastic process.

[20] All households have the same consumption level so long as households have perfect information.

Taking the quadratic approximation to the expected discounted sum of profits, we obtain the following expression for the expected discounted sum of losses in profit from suboptimal actions:

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} \left(x_t - x_t^*\right)' \Theta_0 \left(x_t - x_t^*\right) + \left(x_t - x_t^*\right)' \Theta_1 \left(x_{t+1} - x_{t+1}^*\right) \right] \tag{1}$$

where $x_t \equiv (k_{it}, l_{it})'$, $x_t^* \equiv (k_{it}^*, l_{it}^*)'$, the matrices Θ_0 and Θ_1 are given by

$$\Theta_0 = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha(1-\alpha) & 0 \\ 0 & \phi(1-\phi) \end{bmatrix}$$

$$\Theta_1 = C^{-\gamma} Y \begin{bmatrix} 0 & \beta\alpha\phi \\ 0 & 0 \end{bmatrix}$$

and the stochastic process x_t^* satisfies the equations

$$E_t a_{t+1} - (1-\alpha) k_{it}^* + \phi E_t l_{it+1}^* = \frac{\gamma E_t \left(c_{t+1} - c_t\right)}{1 - \beta(1-\delta)} \tag{2}$$

$$a_t + \alpha k_{it-1}^* - (1-\phi) l_{it}^* = w_t \tag{3}$$

and the initial condition $k_{i,-1}^* = k_{i,-1}$. See Appendix C. The vector x_t^* is the *profit-maximizing* input choice when the decision-maker in the firm has perfect information in every period. Equations (2)-(3) are the usual optimality conditions for capital and labor, where E_t denotes the expectation operator conditioned on the entire history up to and including period t. Equation (2) states that the profit-maximizing capital input equates the expected marginal product of capital to the cost of capital, where the latter is proportional to the expected consumption growth rate. Equation (3) states that the profit-maximizing labor input equates the marginal product of labor to the wage. The vector x_t is an *alternative* input choice. Expression (1) gives the expected discounted sum of losses in profit when the stochastic process for the firm's actions, x_t, differs – for whatever reason – from the stochastic process for the profit-maximizing actions, x_t^*. After the quadratic approximation this loss is quadratic in $x_t - x_t^*$. The interaction term $\left(x_t - x_t^*\right)' \Theta_1 \left(x_{t+1} - x_{t+1}^*\right)$ appears because bringing too much capital into a period raises the optimal labor input in that period.[21]

Maćkowiak, Matějka, and Wiederholt (2018) derive analytical results for attention problems in a dynamic environment with Gaussian shocks. We now rewrite objective (1) so that it matches the objective in that paper. This requires only that we redefine x_t and x_t^*.

[21] Objective (1) is written so that the firm wants to maximize it.

We show in Appendix C that expression (1) is equivalent to

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} \left(x_t - x_t^* \right)' \Theta \left(x_t - x_t^* \right) \right] \tag{4}$$

where $x_t \equiv (k_{it}, l_{it} - \frac{\alpha}{1-\phi} k_{it-1})'$, $x_t^* \equiv (k_{it}^*, l_{it}^* - \frac{\alpha}{1-\phi} k_{it-1}^*)'$, the matrix Θ is given by

$$\Theta = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha \left(1 - \alpha - \frac{\alpha\phi}{1-\phi}\right) & 0 \\ 0 & \phi \left(1 - \phi\right) \end{bmatrix}$$

and the stochastic process x_t^* satisfies

$$x_t^* = \begin{pmatrix} \frac{1}{1-\alpha-\phi} \left[E_t a_{t+1} - \phi E_t w_{t+1} - (1-\phi) \frac{\gamma E_t (c_{t+1} - c_t)}{1-\beta(1-\delta)} \right] \\ \frac{1}{1-\phi} \left(a_t - w_t \right) \end{pmatrix}. \tag{5}$$

The first entry of the new vector x_t^* is the profit-maximizing capital stock to be carried into period $t+1$, k_{it}^*. The profit-maximizing capital stock, k_{it}^*, is proportional to the difference between expected productivity and a weighted average of the expected wage and the cost of capital. The second entry of the new vector x_t^* is the profit-maximizing labor input for a given capital stock, $l_{it}^* - [\alpha/(1-\phi)]\, k_{it-1}^*$. The profit-maximizing labor input for a given capital stock, $l_{it}^* - [\alpha/(1-\phi)]\, k_{it-1}^*$, is proportional to the difference between productivity and the wage. The advantage of rewriting equations (2)-(3) as equation (5) is that the right-hand side of equation (5) depends only on variables exogenous to the firm. Moreover, expression (1) collapses to expression (4) once it is written in terms of the new vectors x_t and x_t^*.

It follows from expression (4) that the best response of firm i in period t given any information set $\mathcal{I}_{it}$ is the conditional expectation of x_t^*, $x_t = E\left(x_t^* | \mathcal{I}_{it}\right)$.

3.2 The attention problem of a firm

In period $t = -1$, the decision-maker in firm i chooses the stochastic process for the signal to maximize the expected discounted sum of profits, (4), net of the cost of attention. In every period $t = 0, 1, 2, ...$, the decision-maker observes a realization of the optimal signal and takes actions – chooses capital and labor.

The statement of the attention problem can be simplified, without loss of generality, based on Maćkowiak, Matějka, and Wiederholt (2018). Let x_{1t}^* denote the first element and x_{2t}^* the second element of x_t^*, $x_{1t}^* = k_{it}^*$ and $x_{2t}^* = l_{it}^* - [\alpha/(1-\phi)]\, k_{it-1}^*$. Suppose that x_{1t}^* and x_{2t}^* each follows a finite-order ARMA process. The vector x_t^* has a first-order VAR representation

$$\xi_{t+1} = F\xi_t + v_{t+1}$$

where v_t is a Gaussian vector white noise process, F is a square matrix, and ξ_t is a vector containing x_{1t}^* and x_{2t}^* and, if appropriate, lags of x_{1t}^* and x_{2t}^* and current and lagged ε_t. The state vector ξ_t contains all information available in period t about the current and future profit-maximizing actions. The analytical results of Maćkowiak, Matějka, and Wiederholt (2018) imply that the optimal signal is a signal about the state vector ξ_t; furthermore, without loss of generality, one can restrict attention to signals that are at most two-dimensional.[22]

The decision-maker in firm i solves:

$$\max_{G,\Sigma_\psi} \sum_{t=0}^{\infty} \beta^t \left\{ E_{i,-1}\left[\frac{1}{2}(x_t - x_t^*)'\,\Theta\,(x_t - x_t^*)\right] - \lambda I\left(\xi_t; S_{it}|\mathcal{I}_{it-1}\right)\right\} \tag{6}$$

subject to

$$\xi_{t+1} = F\xi_t + v_{t+1} \tag{7}$$

$$x_t = E\left(x_t^*|\mathcal{I}_{it}\right) \tag{8}$$

$$\mathcal{I}_{it} = \mathcal{I}_{i,-1} \cup \{S_{i0}, \ldots, S_{it}\} \tag{9}$$

$$S_{it} = G'\xi_t + \psi_{it} \tag{10}$$

where

$$I\left(\xi_t; S_{it}|\mathcal{I}_{it-1}\right) = H\left(\xi_t|\mathcal{I}_{it-1}\right) - H\left(\xi_t|\mathcal{I}_{it}\right). \tag{11}$$

Expression (6) states that the decision-maker maximizes the expected discounted sum of profits net of the cost of attention. The cost of attention in any period t is proportional to mutual information $I\left(\xi_t; S_{it}|\mathcal{I}_{it-1}\right)$, where $\lambda > 0$ is the marginal cost of attention. Mutual information is

[22] In Maćkowiak, Matějka, and Wiederholt (2018) the optimal action x_t^* is a scalar, while in this model the optimal action x_t^* is a vector, but the proof of Proposition 1 in Maćkowiak, Matějka, and Wiederholt (2018), which states that the optimal signal is a signal about the state vector, extends in a straightforward way from the case of a scalar x_t^* to the case of a vector x_t^*. To obtain the result that the optimum can be attained with a two-dimensional signal, one can follow the steps in the proof of Proposition 2 in Maćkowiak, Matějka, and Wiederholt (2018). Note also that the optimal signal is Gaussian, because the objective is quadratic and the optimal action is Gaussian. See Maćkowiak, Matějka, and Wiederholt (2020).

defined below. The decision-maker takes as given the law of motion for the state vector (equation (7)). The agent's actions are equal to the conditional expectation of the profit-maximizing actions given the period t information set (equation (8)). The period t information set $\mathcal{I}_{it}$ consists of the sequence of signal realizations $S_{i0}, \ldots, S_{it}$ and initial information $\mathcal{I}_{i,-1}$ (equation (9)). The optimal signal is a signal about the state vector ξ_t (equation (10)), where the noise ψ_{it} follows a Gaussian vector white noise process with variance-covariance matrix Σ_ψ. The noise ψ_{it} is assumed to be independently distributed across firms.[23] The decision-maker chooses the signal weights G (the number of signals and what each signal is about) and the variance-covariance matrix of the noise Σ_ψ. Equation (11) states that mutual information (between the signal and the state vector) equals the difference between prior uncertainty and posterior uncertainty about the state vector in a given period. $H\left(\xi_t|\mathcal{I}_{i\tau}\right)$ denotes the entropy of ξ_t conditional on $\mathcal{I}_{i\tau}$, $\tau = t-1, t$. $H\left(\xi_t|\mathcal{I}_{it-1}\right)$ is the prior uncertainty, before receiving the period t signal, and $H\left(\xi_t|\mathcal{I}_{it}\right)$ is the posterior uncertainty.

Both the expected discounted sum of profits and the cost of attention in expression (6) depend on conditional second moments.[24] The conditional second moments can in principle vary over time, because the decision-maker conditions on more signal realizations as time passes. To abstract from transitional dynamics in the conditional second moments, we assume that after choosing the signal process in period -1, the agent receives a sequence of signals in period -1 such that the conditional second moments are independent of time. The conditional second moments can then be computed using the steady-state Kalman filter, with state equation (7) and observation equation (10), and problem (6)-(11) can be solved numerically in a straightforward way.[25]

3.3 Equilibrium

We focus on the equilibrium when decision-makers in firms are subject to rational inattention and households have perfect information. For simplicity, until Section 6 we refer to this equilibrium as

[23] Woodford (2003) and Maćkowiak and Wiederholt (2015) make the same assumption. This assumption implies that information is dispersed: in every period, each firm i has a different conditional expectation $E\left(x_t^*|\mathcal{I}_{it}\right)$.

[24] There is a well-known, closed-form expression for mutual information in the Gaussian case. See Maćkowiak, Matějka, and Wiederholt (2018).

[25] Maćkowiak, Matějka, and Wiederholt (2018) make the same assumption. We relax this assumption in Appendix D. Woodford (2003) also uses the steady-state Kalman filter to compute conditional second moments in a model in which agents observe exogenously given signals.

the rational inattention equilibrium.[26]

The rational inattention equilibrium can be defined as follows. In period -1, each firm solves problem (6)-(11). In every period $0, 1, 2, \ldots$, firms and households maximize given their information sets, and markets clear: The wage w_t adjusts so that labor demand equals labor supply, $\int_0^1 l_{it} di = \int_0^1 l_{jt} dj$, and the price of a mutual fund share v_t adjusts so that asset demand equals asset supply, $\int_0^1 q_{jt} dj = 0$.

Some details about firms' and households' maximization are helpful. The choice of G and Σ_ψ by firm i together with equations (8)-(10) yield the input choices of the firm, k_{it} and l_{it}. Firm-level output, investment, and profit satisfy: $y_{it} = a_t + \alpha k_{it-1} + \phi l_{it}$, $\delta i_{it} = k_{it} - (1-\delta) k_{it-1}$, and $(D/Y) d_{it} = y_{it} - (WL/Y)(w_t + l_{it}) - (I/Y) i_{it}$.[27] Aggregate variables are given by: $y_t = \int_0^1 y_{it} di$, $k_t = \int_0^1 k_{it} di$, $i_t = \int_0^1 i_{it} di$, and $d_t = \int_0^1 d_{it} di$.[28] Since households have perfect information, they satisfy the usual first-order conditions

$$\gamma E_t (c_{t+1} - c_t) = \beta E_t v_{t+1} - v_t + (1-\beta) E_t d_{t+1} \tag{12}$$

and

$$w_t - \gamma c_t = \eta l_t. \tag{13}$$

Households are identical, implying that $c_{jt} = c_t$ and $l_{jt} = l_t$ for each j. Finally, the resource constraint reads[29]

$$y_t = (C/Y) c_t + (I/Y) i_t. \tag{14}$$

4 Developing intuition

How does rational inattention affect the propagation of productivity shocks and news about future productivity? To develop intuition, this section studies special cases of the model. In the first special case, labor is the only variable input. In the second special case, capital is the only variable input. Section 5 analyzes the rational inattention equilibrium of the complete model.

[26] In Section 6 we add rational inattention on the side of households.

[27] These equations follow from log-linearization of the production function, the law of motion of capital, and the definition of profit. See Section 2. All relevant steady-state ratios appear in Appendix A.

[28] These equations follow from log-linearization of the definitions of the aggregate variables. See Section 2.

[29] To obtain the resource constraint, we log-linearize the flow budget constraint of household j and we aggregate, imposing market clearing and plugging in the equation for the dividend from the mutual fund.

4.1 The case with labor only

Suppose that labor is the only variable input, $\alpha = 0$. The attention problem of a firm simplifies. The firm's action (labor input choice) is one-dimensional with $x_t = l_{it}$, $x_t^* = l_{it}^*$, and $\Theta = -C^{-\gamma} Y \phi (1 - \phi)$. Labor supply is governed by equation (13). Households live hand-to-mouth because there is no capital and all households are identical.

The perfect information equilibrium can be solved for analytically. Equating labor demand, $\int_0^1 l_{it} di = [1/(1-\phi)](a_t - w_t)$, and labor supply, which follows from $\eta l_t = w_t - \gamma c_t$ and $c_t = y_t = a_t + \phi l_t$, yields

$$l_t = \left(\frac{1-\gamma}{1-\phi+\gamma\phi+\eta} \right) a_t. \tag{15}$$

Labor input is proportional to productivity. The impulse response of labor input to a news shock is zero until productivity changes. Firms have no incentive to change labor demand until productivity changes. Similarly, households have no incentive to change labor supply in this special case of the model. The wealth effect on labor supply vanishes, because hand-to-mouth households cannot vary saving and consumption in response to a news shock.

Consider the rational inattention equilibrium. To find the fixed point, we guess that in equilibrium the profit-maximizing labor input l_{it}^* follows a finite-order ARMA process. We solve the attention problem of firm i. We compute the firm's labor input from the equation $l_{it} = E\left(l_{it}^* | \mathcal{I}_{it}\right)$ and the solution of the attention problem. We verify the guess from the optimality condition $l_{it}^* = [1/(1-\phi)](a_t - w_t)$, where w_t is the market-clearing wage that depends on the optimal signal, labor demand, and labor supply. One period in the model equals one quarter. As an example, we assume $\gamma = 0.5$, $\eta = 0$, $\phi = 0.6$, $\beta = 0.99$, $\rho = 0.9$, $\sigma = 0.01$, and $\lambda = (4/100,000)C^{-\gamma}Y$.[30]

The upper-left panel in Figure 1 shows the impulse response of aggregate labor input l_t to a productivity shock ($h = 0$).[31] In the perfect information equilibrium, labor input is proportional to productivity and thus the impulse response peaks on impact and then declines monotonically (line with points). The impulse response is hump-shaped in the rational inattention equilibrium (line with circles). This is the usual result that rational inattention introduces dampening and delay due to noise in signals about the state of the economy. For a similar figure, see for instance Figure 1 in Sims (2003).

[30] Section 5 discusses the choice of the value for the marginal cost of attention λ.

[31] In all figures, an impulse response of 1 is a 1 percent deviation from the non-stochastic steady state.

The upper-right panel in Figure 1 shows the impulse response of l_t to a news shock ($h = 4$). The shock is drawn in period 0 while productivity changes in period $h = 4$. In the perfect information equilibrium, labor input is proportional to productivity (equation (15)) and thus the impulse response is zero until productivity changes (line with points). Under rational inattention labor demand rises on impact of a news shock. The reason is that the optimal signal of firms confounds current with expected future productivity. The resulting increase in labor demand puts upward pressure on the wage. Labor supply is still governed by $\eta l_t = w_t - \gamma c_t$ and $c_t = y_t = a_t + \phi l_t$. In equilibrium, labor input rises on impact of a news shock (line with circles) and keeps going up thereafter.

To see analytically that a confounding signal is optimal, consider the following special case. Suppose that a measure zero of firms are subject to rational inattention. Since a measure one of firms have perfect information, the equilibrium employment is given by equation (15) and the equilibrium wage is given by $w_t = [(\gamma + \eta)/(1 - \gamma)]l_t$, implying that the profit-maximizing labor input of an individual firm is proportional to productivity: $l_{it}^* = [1/(1-\phi)](a_t - w_t) = [(1-\gamma)/(1-\phi+\gamma\phi+\eta)]a_t$. Furthermore, suppose that $a_{t+1} = \rho a_t + \sigma \varepsilon_t$ (that is, $h = 1$). In this case, the profit-maximizing labor input has a first-order VAR representation with state vector $\xi_t = (l_{it}^*, \varepsilon_t)'$, or equivalently $\xi_t = (a_t, \varepsilon_t)'$. The optimal signal then follows from Propositions 1, 2, and 5 in Maćkowiak, Matějka, and Wiederholt (2018). Proposition 1 states that the optimal signal is about the state vector, $S_{it} = G'\xi_t + \psi_{it}$. Proposition 2 states that with a one-dimensional action (here, the labor input), the optimal signal is a *one-dimensional* signal about the state vector, $S_{it} = a_t + g\varepsilon_t + \psi_{it}$. Proposition 5 states that $g \neq 0$. Hence, the optimal signal confounds current with expected future productivity. It turns out that this result still holds when all firms are subject to rational inattention and $h > 1$.

To gain intuition for the optimality of a confounding signal, compare rational inattention with an alternative model. Continue to assume that $h = 1$ and a measure zero of firms are subject to rational inattention. In the alternative model, a measure zero of firms solve the same attention problem *subject to the restriction* that one must obtain a two-dimensional signal consisting of a signal on current productivity and a separate signal on future productivity. A signal process of this form does not confound current with future productivity. We find that firms in the alternative model set to zero the precision of the signal on future productivity (they decide to observe only a signal on current productivity). Furthermore, firms in the alternative model do worse, for the same amount of attention, than firms in the rational inattention model (the expected profit loss is larger in the alternative model than in the rational inattention model).[32] The lower-left panel in Figure 1 reports the impulse response of labor input to a news shock by rationally inattentive firms (line with circles) and by firms in the alternative model (line with squares). Before productivity changes (in period 0), firms in the alternative model make no labor input mistake conditional on a news shock. Once productivity has changed (in period 1 and subsequent periods), firms in the alternative model make larger labor input mistakes conditional on a news shock than rationally inattentive firms. The signal in the rational inattention model smooths the action compared with the alternative model, which lowers the overall expected profit loss.

The lower-right panel in Figure 1 shows what happens when the marginal cost of attention λ changes. The impulse response of the action (here, the labor input) on impact of a news shock, in period 0, is non-monotonic in λ. With a λ near zero, the solution is close to the perfect information equilibrium in which the impulse response on impact is zero. With a high λ, the solution is close to a "no information" model in which the impulse response in all periods is zero.

Table 2 and Appendix D show that the more distant is the change in productivity, the weaker is the short-run response of the action to news. The impulse response of labor demand on impact of a news shock declines with h (approaching zero as h rises). If productivity will change in the near future, a rationally inattentive agent believes that productivity has already changed with a non-trivial probability. The short-run response of the action can then be strong (even though the perfect information response is zero). If productivity will change only in a longer run, the agent is fairly confident that productivity has not yet changed. The short-run response of the action approaches the perfect information response.

4.2 The case with capital only

Suppose that capital is the only variable input, $\phi = 0$. The attention problem of a firm is analogous to Section 4.1. The firm's action (capital input choice) is one-dimensional with $x_t = k_{it}$, $x_t^* = k_{it}^* = \frac{1}{1-\alpha}\left[E_t a_{t+1} - \frac{\gamma E_t(c_{t+1}-c_t)}{1-\beta(1-\delta)}\right]$, and $\Theta = -C^{-\gamma} Y \beta\alpha(1-\alpha)$. Consumption-saving behavior of households is governed by equation (12).

[32] To hold the amount of attention equal in the two models, we increase the marginal cost of attention λ in the alternative model relative to the rational inattention model.

Assume log utility from consumption, $\gamma = 1$, and full capital depreciation, $\delta = 1$. The perfect information equilibrium can be solved for analytically: $k_t = \alpha k_{t-1} + a_t$, $k_t = i_t = y_t = c_t = d_t = v_t$. In this special case the model can produce some positive autocorrelation in investment and output growth. However, the impulse responses of all variables to a news shock are zero until productivity changes. An increase in expected productivity creates an incentive to invest in the period before productivity improves, but this incentive is completely offset by a rise in the cost of capital.

Consider the rational inattention equilibrium. To find the fixed point, we guess that in equilibrium the profit-maximizing capital input k_{it}^* follows a finite-order ARMA process. We solve the attention problem of firm i. We compute the firm's capital input from the equation $k_{it} = E\left(k_{it}^* | \mathcal{I}_{it}\right)$ and the solution of the attention problem. Aggregating across firms yields k_t, i_t, y_t, and d_t, while the budget constraint implies that $c_t = d_t$. We verify the guess from the optimality condition $k_{it}^* = [1/(1-\alpha)]\left[E_t a_{t+1} - E_t\left(c_{t+1} - c_t\right)\right]$, where the expected consumption growth rate depends on the optimal signal, investment demand, and saving supply. The market-clearing mutual fund share price v_t can be calculated from equation (12) and the solution for c_t. As an example, we assume $\gamma = 1$, $\alpha = 0.33$, $\beta = 0.99$, $\delta = 1$, $\rho = 0.9$, $\sigma = 0.01$, and $\lambda = (8/100,000)C^{-\gamma}Y$.

The top panel in Figure 2 displays the impulse response of aggregate investment i_t to a productivity shock ($h = 0$). The rational inattention equilibrium (line with circles) features more first-order autocorrelation in the growth rate of investment compared with the perfect information equilibrium (line with points).

The middle panel in Figure 2 shows the impulse response of i_t to a news shock ($h = 4$). In the perfect information equilibrium, the impulse response of investment is zero until productivity changes in period 4 (line with points). In the rational inattention equilibrium, investment rises in period 0 and keeps going up thereafter (line with circles). Rational inattention induces an increase in investment demand on impact of a news shock. As a result, investment rises in equilibrium.

Since the attention problem of a firm is analogous to Section 4.1, the intuition for what happens to investment demand is the same as the intuition given there. The forward-looking attention choice leads investment demand to react immediately to a news shock, as if productivity has already changed with some probability. As in Section 4.1, the bottom panel in Figure 2 compares the rational inattention model with the alternative model (the model with a restricted signal process of the form "a separate signal on each element of the state vector") with $h = 4$. The alternative model yields no capital input mistakes conditional on a news shock from period 0 through period $h-1$, followed by larger mistakes than in the rational inattention model. By smoothing the action, the signal in the rational inattention model lowers the overall expected profit loss.[33]

Let us summarize Section 4. In a dynamic environment rational inattention induces a combination of delay in actions and forward-looking actions. As a result, the impulse responses to productivity shocks and news about future productivity change significantly. Employment and investment react with delay to a productivity shock. They *rise* in response to news that productivity will improve.

5 Predictions of the model

What does rational inattention imply about the business cycle effects of productivity shocks and news about future productivity? We return to the complete model with variable capital and labor, $\alpha > 0$ and $\phi > 0$.

Throughout this section we set $\gamma = 1$, $\eta = 0$, $\alpha = 0.33$, $\phi = 0.65$, $\beta = 0.99$, $\delta = 0.025$, $\rho = 0.9$, and $\sigma = 0.01$. Thus, we assume log utility from consumption and linear disutility from work, $\alpha + \phi$ close to 1, a depreciation rate of 2.5 percent per quarter, and a persistent productivity process with an innovation of 1 percent.[34]

5.1 The effects of productivity shocks

Let $h = 0$. Consider the perfect information equilibrium. Figure 3 shows the impulse responses to a productivity shock (lines with points). Aggregate labor input, investment, output, and consumption move in the same direction, consistent with a business cycle. The impulse responses of labor input,

[33] We emphasized that rational inattention makes very different predictions than the alternative model in the case of news shocks ($h \geq 1$). With $h = 0$ actions based on the optimal signal are also different from actions based on the restricted signal, except when the optimal action follows an AR(1) process. How much difference there is depends on the details of the model. In this model the difference turns out to be modest. Consider the partial equilibrium analysis with $h = 0$ and the same parameter values. The profit-maximizing capital input follows an AR(2) process. The investment growth rate of rationally inattentive firms has a serial correlation of 0.67. With the restricted signal the serial correlation rises to 0.72.

[34] Below we state the value of the marginal cost of attention λ. Only the ratio σ^2/λ matters for the equilibrium impulse responses because the first term in objective (6) is linear in σ^2 and the second term is linear in λ.

investment, and output peak on impact and then decline monotonically. They inherit the shape of the impulse response of exogenous productivity.

Following common practice, we compare unconditional second moments in the model and in the data. Table 1 reports selected unconditional moments for the model (column "Perfect information") and for the quarterly post-war data from the United States.[35] The comparison is familiar. Let us focus on the persistence of growth rates. The first-order autocorrelations of employment, investment, and output growth are positive in the data but negative in the model. In the model these variables inherit the autocorrelation of exogenous productivity growth.[36]

Consider the rational inattention equilibrium. Searching for the fixed point is more difficult than in Section 4 because we must consider two inputs, capital and labor, and two factor prices, the cost of capital and the wage. In equilibrium the factor prices depend on the optimal signal, input demand, and input supply. To find the fixed point, we guess that in equilibrium consumption c_t follows a finite-order ARMA process. With $\gamma = 1$ and $\eta = 0$, the optimality condition (13) simply states that the wage process w_t equals the consumption process c_t. Therefore, a guess about consumption implies a guess about both factor prices, the cost of capital (the expected consumption growth rate) and the wage. We calculate the implied ARMA representations of the optimal inputs $x_{1t}^* = k_{it}^*$ and $x_{2t}^* = l_{it}^* - [\alpha / (1-\phi)]\, k_{it-1}^*$ from equation (5). We solve the attention problem of firm i. We compute the firm's capital and labor inputs from the equations $x_t = E\left(x_t^* | \mathcal{I}_{it}\right)$, $k_{it} = x_{1t}$, $l_{it} = x_{2t} + [\alpha/(1-\phi]k_{it-1}$, and the solution of the attention problem. Aggregating across firms yields k_t, i_t, l_t, y_t, and d_t. We verify the guess by solving for c_t from the resource constraint (14). The market-clearing share price v_t can be calculated from equation (12) and the solution for d_t and c_t.

[35] The unconditional moments of the data come from Eusepi and Preston (2011), Table 2. The sample period is 1955Q1-2007Q4. The unconditional moments from the model are computed from the equilibrium MA representation of each variable. As a measure of labor input, Eusepi and Preston (2011) use the measure of hours worked by Francis and Ramey (2009). Productivity is measured as real GDP divided by hours worked. See the Data Appendix in Eusepi and Preston (2011).

[36] The model matches well the standard deviation of consumption, investment, and productivity relative to output, while underpredicting the volatility of hours worked. The model matches well the correlation of consumption, hours worked, and investment with output, while overstating the correlation of productivity with output. Finally, the model matches well the first-order autocorrelation of consumption growth. It turns out that rational inattention has little effect on these predictions of the model. See Table 1.

What are the effects of rational inattention on the propagation of a productivity shock? We set $\lambda = (1/10,000)C^{-\gamma}Y$, which means that the per period marginal cost of attention is equal to $1/10,000$ of steady-state output.[37] In the rational inattention equilibrium, the impulse responses of employment, investment, and output become hump-shaped (Figure 3, lines with circles). These impulse responses are hump-shaped even though there are no adjustment costs. The first-order autocorrelations of employment, investment, and output growth become positive (Table 1, column "Rational inattention"). The model matches well the first-order autocorrelation of employment growth in the data, even though rational inattention is the only source of inertia and the marginal cost of attention is small. The model underpredicts somewhat the serial correlation of output and investment growth.

In Figure 3 note also that consumption declines somewhat when firms become subject to rational inattention. Households consume less because rationally inattentive firms underestimate productivity and produce less than in the perfect information equilibrium.

Section 4 explained the effects of rational inattention one input at a time. In this section the new feature is that rational inattention induces delay in the demand for both inputs, capital and labor, at the same time. Figure 3 shows the impulse response of the conditional expectation of productivity by firms to a productivity shock. The impulse response is hump-shaped, indicating that the firms' beliefs are anchored on the steady state and evolve slowly. The rational inattention effect turns out to be sufficient to bring the first-order autocorrelations of employment, investment, and output growth in the model approximately into line with the data.

The amount of inattention in the model, governed by the parameter λ, can be compared to survey data on expectations. Coibion and Gorodnichenko (2015) show that models with an informational friction predict a regression relationship between the average forecast error and forecast revision in a cross-section of agents. Suppose that firms in this model report their forecasts of output. Let $\hat{y}_{t+\tau|t}$ denote the period t average forecast of output in period $t+\tau$, where τ is a positive integer. The average forecast error, $y_{t+\tau} - \hat{y}_{t+\tau|t}$, is positively related to the average forecast revision, $\hat{y}_{t+\tau|t} - \hat{y}_{t+\tau|t-1}$. The regression coefficient increases in the size of the informational friction, in this model governed by the value of λ. Coibion and Gorodnichenko (2015) and Bordalo,

[37] In the rational inattention equilibrium, we can compute the expected profit loss of firm i from suboptimal actions. This is equal per period to $4/100,000$ of steady-state output, even less than the marginal cost of attention.

Gennaioli, Ma, and Shleifer (2019) estimate this regression relationship using survey data on forecasts of a number of variables. Typically, these authors report coefficients in the range of 0.3-1.4.[38] We repeat their estimation using quarterly data on median forecasts of output (real GDP) from the U.S. Survey of Professional Forecasters for the period 1968Q4-2019Q4 obtained from the Federal Reserve Bank of Philadelphia. Focusing on $\tau = 3$, we estimate a regression coefficient of 0.76 with a standard error of 0.30.[39] Next, we simulate data from our model with the parameter values used in this section, including the value of λ. When we run the same regression on the simulated data, on average across the simulations we obtain a coefficient of 1.07. We conclude that the amount of inattention in the model is consistent with the survey data on expectations. It is remarkable that the model produces persistent growth rates of employment, investment, and output, as in the macro data, while being consistent with the survey data on expectations.

5.2 The effects of news about future productivity

Let $h \geq 1$. We focus on $h = 2$ (the same case that Jaimovich and Rebelo, 2009, focus on) and $h = 4$ (one of two cases in Schmitt-Grohé and Uribe, 2012).

Consider the perfect information equilibrium. Figures 4 and 5 show the impulse responses with $h = 2$ and $h = 4$, respectively (lines with points). The shock is drawn in period 0 while productivity changes in period h. A news shock causes a wealth effect. Consumption and leisure are normal goods, and therefore households want to consume more (save less) and work less after a positive news shock. Firms have no incentive to increase labor demand before productivity improves, while households reduce labor supply due to the wealth effect. As a result, employment falls. With capital predetermined and current productivity unchanged, output contracts. On impact firms have no incentive to increase investment, while the wealth effect reduces desired saving by households. Investment declines while consumption rises. The model fails to produce comovement in response to news about future productivity. It predicts an output contraction after news that productivity

[38] See in particular Coibion and Gorodnichenko (2015), Table 1 and Figures 1-2, and Bordalo, Gennaioli, Ma, and Shleifer (2019), Table 3.

[39] Coibion and Gorodnichenko (2015) and Bordalo, Gennaioli, Ma, and Shleifer (2019) also focus on $\tau = 3$. Both papers report results for forecasts of output *growth* but not output *level*. Some observations on forecasts of the level of output cannot be used due to base year changes in the dataset; furthermore, we remove as outliers the top 1 percent of forecast errors and revisions.

will improve. Note also that, after decreasing on impact, employment, investment, and output keep falling between when the news arrives (period 0) and when productivity changes (period h). This is particularly clear in Figure 5 ($h = 4$). An increase in expected productivity creates an incentive to invest in the period before productivity improves (period $h - 1$), but this incentive is more than offset by a rise in the cost of capital. Employment, investment, and output increase only once productivity improves.

With a high elasticity of intertemporal substitution, the model predicts a fall in consumption and a rise in labor input and investment. The substitution effect due to an increase in the real interest rate dominates the wealth effect in this case, pushing consumption down and labor supply up. "However, no combination of parameters can generate a joint increase in consumption, investment, and employment." (Lorenzoni, 2011, p.539.)

Consider the rational inattention equilibrium (Figures 4-5, lines with circles).[40] In both figures employment rises in period 0 and keeps going up thereafter. The conditional expectation of productivity by firms increases on impact, which pushes up labor demand. In general equilibrium, the desire of households to reduce labor supply is pulling employment down. It turns out that the rational inattention effect on labor demand is strong enough to *more than offset* the wealth effect on labor supply. As a result, employment rises in equilibrium.

Figures 4-5 show the impulse response of investment in general equilibrium ("RI general equilibrium") and the impulse response of investment by rationally inattentive firms of measure zero when other firms have perfect information ("RI partial equilibrium," line with asterisks). In partial equilibrium, investment rises in period 0 and keeps going up thereafter. The conditional expectation of productivity by rationally inattentive firms increases on impact, which pushes up investment demand. In general equilibrium, the desire of households to reduce saving for a given level of output is pulling investment down. We find that the rational inattention effect on investment demand approximately offsets the wealth effect on saving supply. The response of investment on impact of a news shock is close to zero (whereas it is below -2 percent in the perfect information equilibrium). Note also that investment rises between period 0 and period h. This is particularly clear in Figure

[40] To find the fixed point we proceed as in Section 5.1. In the economy with $h = 2$, we assume the same value of λ as in Section 5.1. This yields a per period expected profit loss equal to $6/100,000$ of steady-state output. With $h = 4$ we set $\lambda = (3.5/10,000)C^{-\gamma}Y$, which means that the per period marginal cost of attention is equal to $3.5/10,000$ of steady-state output. The per period expected profit loss turns out to equal $2/10,000$ of steady-state output.

5 ($h = 4$).

With capital predetermined and an increase in employment in period 0, the impulse response of output on impact of a news shock is positive. Output increases further between period 0 and period h, as employment and investment rise. The rational inattention effect on input demand induces an output expansion in response to a news shock.

Consider in more detail what affects investment in general equilibrium. Investment rises on impact of a positive news shock relative to the perfect information equilibrium. The cost of capital increases (the expected consumption growth rate rises). The profit-maximizing capital input of an individual firm falls. See the first line in equation (5). Capital is a strategic substitute. An individual firm demands less capital when other firms invest more. This general equilibrium feedback effect turns out to be very strong. The coefficient on the expected consumption growth rate in the first line of equation (5) equals -504.[41] The coefficient on the expected consumption growth rate increases in the depreciation rate, δ, and decreases in the elasticity of output with respect to labor, ϕ. In Section 4.2, with full capital depreciation and without labor input ($\delta = 1$, $\phi = 0$), this coefficient decreases in absolute value by more than two orders of magnitude, to -1.5, implying that the strategic substitutability is much weaker. The impulse response of investment on impact of a news shock is positive in this case.

To summarize, rational inattention induces an increase in the demand for labor and investment in response to news that productivity will improve. The rational inattention effect on labor demand more than offsets the wealth effect on labor supply. Thus, employment and output rise on impact. The rational inattention effect on investment demand offsets the wealth effect on saving supply. As a result, the response of investment on impact equals zero, as opposed to a sizable negative number in the perfect information equilibrium.

In Figures 4-5 note also that consumption increases somewhat when firms become subject to rational inattention. Households consume more because rationally inattentive firms overestimate productivity and produce more than in the perfect information equilibrium.

What is the optimal signal? In problem (6)-(11) the firm can in principle choose a multi-dimensional signal process, consisting of signals on elements of the state vector ξ_t, signals on linear

[41] Labor is also a strategic substitute. However, the general equilibrium dampening of labor demand due to a higher wage is weak. The coefficient on the wage in the second line of equation (5) equals -2.9.

combinations of the elements of ξ_t, or both. We find that a one-dimensional signal on all elements of the state vector is optimal. A signal on all elements of the state vector confounds current with expected future productivity.[42] The left panel in the top row in Figure 6 shows the impulse response of the optimal signal to a news shock in the rational inattention equilibrium with $h = 4$. The signal rises after a news shock. To simplify, the message to firms from a positive signal realization is: "Hire and invest, productivity is either already up or about to rise (and it is not that important precisely when productivity rises)."[43]

Again, we can compare the amount of inattention in the model to the SPF data. With $h = 2$ we assume the same value of λ as in Section 5.1, $\lambda = (1/10,000)C^{-\gamma}Y$. When we run the Coibion-Gorodnichenko regression on data simulated from the economy with $h = 2$ (with $\tau = 3$), on average we obtain a coefficient of 1.23. This amount of inattention is consistent with the survey data on expectations.[44] With $h = 4$ the model requires more inattention to produce an increase in employment after a positive news shock. We raise the marginal cost of attention to $\lambda = (3.5/10,000)C^{-\gamma}Y$. When we run the Coibion-Gorodnichenko regression on data simulated from this economy (with $\tau = 3$), on average we obtain a coefficient of 2.87. This amount of inattention is somewhat greater than in the SPF data.[45]

6 Rational inattention by firms and households

We focused on the equilibrium when decision-makers in firms are subject to rational inattention and households have perfect information. To obtain comovement in response to a news shock, it

[42] We find that a univariate signal process is optimal in this model even though the optimal action follows a bivariate process (recall the definition of x_t^* in equation (5)). We write that the optimal signal is on "all elements" of the state vector bearing in mind that an element can be dropped if it can be written as a linear combination of the other elements.

[43] The optimal signal is analogous to an asset price. An asset price is also a statistic that combines information about the current and expected future state of the economy. The left panel in the bottom row in Figure 6 shows the impulse response of the price of a mutual fund share v_t to a news shock in the economy with $h = 4$. In the rational inattention equilibrium (line with circles), the stock price rises on impact of a news shock, like the optimal signal, even though productivity has not yet improved.

[44] Recall that in the SPF data the analogous regression coefficient is 0.76 with a standard error of 0.30.

[45] It seems plausible that in the real world decision-makers in small and medium firms pay less attention to the aggregate economy than professional forecasters.

seems critical to find a mechanism leading to a shift in labor demand and investment demand for a given level of productivity. Rational inattention *by firms* is such a mechanism. To illustrate in the most transparent way the effects of rational inattention by firms, we assumed that households have perfect information.

Finding a fixed point of an economy in which firms and households are subject to rational inattention is more difficult than what we have considered so far. Now equilibrium depends on the signals chosen by firms and on the signals chosen by households. In a special case of the model, however, we can solve for equilibrium in which all agents, firms and households, are subject to rational inattention. We assume that labor is the only variable input ($\alpha = 0$, as in Section 4.1) and households cannot trade shares in the mutual fund.[46] Each household j chooses a signal about the state of the economy to maximize the expected discounted sum of utility. The household recognizes that a more informative signal requires more attention. The attention problem of each firm i is unchanged.[47]

We substitute the flow budget constraint into the utility function of household j and rewrite the utility function in terms of log-deviations from the non-stochastic steady state. Taking the quadratic-approximation to the expected discounted sum of utility, following the same steps as in Appendix C and assuming the same regularity conditions, we obtain an expression for the expected discounted sum of losses in utility from suboptimal actions. Suppose that the utility-maximizing labor supply l_{jt}^* follows a finite-order ARMA process. This process has a first-order VAR representation in terms of the state vector $\tilde{\xi}_t$. The optimal signal is a one-dimensional signal about the state vector $\tilde{\xi}_t$ (Maćkowiak, Matějka, and Wiederholt, 2018). Household j solves:

$$\max_{\tilde{g},\tilde{\sigma}_\psi^2} \sum_{t=0}^{\infty} \beta^t \left\{ E_{j,-1} \left[\frac{-C^{1-\gamma}\phi\left(\phi\gamma+\eta\right)}{2} \left(l_{jt} - l_{jt}^*\right)^2 \right] - \mu I\left(\tilde{\xi}_t; S_{jt}|\mathcal{I}_{jt-1}\right) \right\}$$

subject to the law of motion for the state vector $\tilde{\xi}_t$, $l_{jt} = E(l_{jt}^*|\mathcal{I}_{jt})$, $\mathcal{I}_{jt} = \mathcal{I}_{j,-1} \cup \{S_{j0}, \ldots, S_{jt}\}$, and $S_{jt} = \tilde{g}'\tilde{\xi}_t + \psi_{jt}$, where $\mu > 0$ and $I(\tilde{\xi}_t; S_{jt}|\mathcal{I}_{jt-1}) = H(\tilde{\xi}_t|\mathcal{I}_{jt-1}) - H(\tilde{\xi}_t|\mathcal{I}_{jt})$. The household's problem is analogous to problem (6)-(11) except that the household takes a single action (decides how much to work), and thus the signal weights $\tilde{g}$ form a vector rather than a matrix and the noise

[46] The latter assumption ensures that households live hand-to-mouth even though they hold different beliefs about the state of the economy.

[47] Households no longer have the same consumption level in this version of the model. We assume that firm i values profits according to the marginal utility of consumption of the representative (average) household.

ψ_{jt} simply follows a univariate Gaussian white noise process with variance $\tilde{\sigma}_{\psi}^2$. The noise ψ_{jt} is assumed to be independently distributed across households. We compute the conditional second moments using the steady-state Kalman filter.

To find the fixed point, we guess that in equilibrium the wage w_t follows a finite-order ARMA process. We calculate the implied ARMA representation of the profit-maximizing labor input from the optimality condition $l_{it}^* = [1/(1-\phi)](a_t - w_t)$. We solve the attention problem of firm i, and we compute the firm's labor input from the equation $l_{it} = E(l_{it}^* | \mathcal{I}_{it})$ and the solution of the attention problem. We then calculate the firm's dividend d_{it} from the equation $(1-\phi)\, d_{it} = y_{it} - \phi(w_t + l_{it})$, where $y_{it} = a_t + \phi l_{it}$. Next, we compute the ARMA representation of the utility-maximizing labor supply from the guess for the wage and the optimality condition $\eta l_{jt}^* = w_t - \gamma c_{jt}$, where the budget constraint implies that $c_{jt} = \phi(w_t + l_{jt}^*) + (1-\phi)\int_0^1 d_{it} di$. We solve the attention problem of household j, and we compute the household's labor input from the equation $l_{jt} = E(l_{jt}^* | \mathcal{I}_{jt})$ and the solution of the attention problem. We adjust the guess for the wage so that labor demand equals labor supply, $\int_0^1 l_{it} di = \int_0^1 l_{jt} dj$, in every period.

This appears to be the first time in the literature that a general equilibrium model is solved in which all agents are subject to rational inattention and prices, which the agents take as given, adjust so that markets clear (here, the wage adjusts to equate labor demand and supply in every period).[48]

How does rational inattention by households affect the dynamics of employment? Figure 6 shows the new equilibrium with firms and households subject to rational inattention (middle column, lines with asterisks).[49] The equilibrium studied in Section 4.1, with firms subject to rational inattention and perfectly informed households, is reported for comparison (middle column, lines with circles). Consider a productivity shock, $h = 0$ (Figure 6, middle column, top row). With inattentive households the wage response must be stronger for labor supply to change by a given amount. The stronger responsiveness of the wage reduces the profit-maximizing labor input. Employment in the new equilibrium is lower than in the old equilibrium (while the wage is higher). The autocorrelation of employment growth rises.

[48] In Maćkowiak and Wiederholt (2015), all firms and households are also subject to rational inattention. In each market one side of the market sets the price and the other side chooses the quantity.

[49] We assume the same parameter values as in Section 4.1. In addition we set $\mu = (2/100,000)C^{1-\gamma}$, which means that the per period marginal cost of attention to a household is equal to $2/100,000$ of steady-state consumption.

Consider a news shock with $h = 6$ (Figure 6, middle column, bottom row). With inattentive households the labor supply decision becomes forward-looking, which makes households more willing to supply labor in response to a news shock. The payoffs from future work rise, and the optimal signal of households confounds the payoff from current work with the payoffs from future work. This effect reduces the responsiveness of the current wage, raising the profit-maximizing labor input. Employment in the new equilibrium is *higher* in the short run than in the equilibrium with perfectly informed households (while the wage is lower).

The model with all agents subject to rational inattention produces a rise in labor input after a news shock, just like in the main part of the paper. Furthermore, rational inattention by households *strengthens* this result (labor input is even higher in the short run).

7 Conclusions and outlook

In the neoclassical business cycle model, it matters if agents can learn in advance about changes in productivity. If agents can learn in advance, variables respond in ways inconsistent with a business cycle. Employment, investment, and output fall while consumption rises. Under rational inattention firms choose to be imperfectly aware of the timing of productivity changes. This effect alone causes an expansion in employment and output after news that productivity will improve. Rational inattention also improves the propagation of standard productivity shocks, by inducing persistence. The same single friction, rational inattention, changes the model in two distinct, helpful ways.

It would be interesting to solve for general equilibrium of the model with variable capital and labor and rational inattention by firms and households. By dampening the wealth effect, rational inattention by households could help the model produce an increase in investment immediately after news that productivity will improve. It would also be interesting to think about the identification of news shocks, when rational inattention causes an increase in expectations of productivity in the near future while productivity remains unchanged for some time.

A Non-stochastic steady state

The non-stochastic steady state is the solution of the model when total factor productivity e^{a_t} is equal to 1 in every period and this is common knowledge.

Let an upper-case letter without a time subscript denote the value of a variable in the non-stochastic steady state. Profit maximization implies that $\alpha K_i^{\alpha-1} L_i^{\phi} N_i^{1-\alpha-\phi} = \beta^{-1} - 1 + \delta$ and $\phi K_i^{\alpha} L_i^{\phi-1} N_i^{1-\alpha-\phi} = W$ for each firm i, which determines K_i and L_i as functions of W and parameter values (including N_i):

$$K_i = \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{1-\phi}{1-\alpha-\phi}} \left(\frac{\phi}{W}\right)^{\frac{\phi}{1-\alpha-\phi}} N_i$$

$$L_i = \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha-\phi}} \left(\frac{\phi}{W}\right)^{\frac{1-\alpha}{1-\alpha-\phi}} N_i.$$

Suppose that N_i is constant across i, $N_i = N$. It follows that K_i and L_i are constant across i, $K_i = K$, $L_i = L$. Moreover, Y_i, I_i and D_i are also constant across i, $Y_i = Y = K^{\alpha} L^{\phi} N^{1-\alpha-\phi}$, $I_i = I = \delta K$, $D_i = D = Y - WL - I$.

Utility maximization implies that $V = [\beta/(1-\beta)]\, D$ and $W C_j^{-\gamma} = L_j^{\eta}$ for each household j. Suppose that in the non-stochastic steady state each household holds an equal share of the mutual fund, $Q_j = 1$ for each j. C_j and L_j are then constant across j, $C_j = C$, $L_j = L$, and the budget constraint implies that $C = WL + D$. Combining this equation with $D = Y - WL - I$ yields the resource constraint $Y = C + I$.

One can solve the system of equations:

$$K = \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{1-\phi}{1-\alpha-\phi}} \left(\frac{\phi}{W}\right)^{\frac{\phi}{1-\alpha-\phi}} N$$

$$L = \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha-\phi}} \left(\frac{\phi}{W}\right)^{\frac{1-\alpha}{1-\alpha-\phi}} N$$

$$W = L^{\eta} \left(K^{\alpha} L^{\phi} N^{1-\alpha-\phi} - \delta K\right)^{\gamma}$$

for K, L and W for given parameter values (including N). The last equation comes from combining the equilibrium condition $WC^{-\gamma} = L^{\eta}$ with the resource constraint. One can then compute the other endogenous variables (Y, I, C, D, and V) from the equations $Y = K^{\alpha} L^{\phi} N^{1-\alpha-\phi}$, $I = \delta K$, $C = Y - I$, $D = Y - WL - I$, $V = [\beta/(1-\beta)]\, D$.

The following steady-state ratios are useful: $WL/Y = \phi$, $I/Y = \alpha\beta\delta/[1 - \beta(1-\delta)]$, $C/Y = 1 - I/Y$, $D/Y = 1 - WL/Y - I/Y$, $WL/C = (WL/Y)(Y/C)$, $V/C = [\beta/(1-\beta)]\,(D/Y)(Y/C)$.

B Perfect information benchmark

Suppose that all agents have perfect information. Let a lower-case letter denote the log-deviation of a variable from its value in the non-stochastic steady state. The firms' first-order conditions imply that

$$a_t + \alpha k_{t-1} - (1-\phi)\, l_t = w_t$$

and

$$E_t a_{t+1} - (1-\alpha)\, k_t + \phi E_t l_{t+1} = \frac{\gamma\,(E_t c_{t+1} - c_t)}{1-\beta\,(1-\delta)}.$$

From the production function, the law of motion of capital and the profit function, we have

$$y_t = a_t + \alpha k_{t-1} + \phi l_t$$

$$\delta i_t = k_t - (1-\delta)\, k_{t-1}$$

and

$$(D/Y)\, d_t = y_t - (WL/Y)\,(w_t + l_t) - (I/Y)\, i_t.$$

The households' first-order conditions imply that

$$\gamma E_t\,(c_{t+1} - c_t) = \beta E_t v_{t+1} - v_t + (1-\beta)\, E_t d_{t+1}$$

and

$$w_t - \gamma c_t = \eta l_t.$$

Finally, the resource constraint reads

$$y_t = (C/Y)\, c_t + (I/Y)\, i_t.$$

C Expected loss in profit from suboptimal actions

Proposition 1 *Let $E_{i,-1}$ denote the expectation operator conditioned on information of the decision-maker of firm i in period -1. Let g denote the functional that is obtained by multiplying the profit function by β^t and summing over all t from zero to infinity. Let $\tilde{g}$ denote the second-order Taylor approximation of g at the non-stochastic steady state. Let x_t, z_t and v_t denote the following vectors*

$$x_t = \begin{pmatrix} k_{it} \\ l_{it} \end{pmatrix} \quad z_t = \begin{pmatrix} a_t \\ w_t \\ c_t \end{pmatrix} \quad v_t = \begin{pmatrix} x_t \\ z_t \\ 1 \end{pmatrix}.$$

Suppose that the decision-maker of firm i knows in period -1 the firm's initial capital stock, $k_{i,-1}$. Suppose also that there exist two constants $\delta < (1/\beta)$ and $A \in \mathbb{R}$ such that, for each period $t \geq 0$, for $\tau = 0, 1$, and for all $m, n \in \{1, 2, 3, 4, 5, 6\}$,

$$E_{i,-1} \left| v_{m,t} v_{n,t+\tau} \right| < \delta^t A. \tag{16}$$

Here $v_{m,t}$ and $v_{n,t}$ denote the mth and nth element of the vector v_t. Then the expected discounted sum of losses in profit when the law of motion for the actions differs from the law of motion for the optimal actions under perfect information equals

$$\begin{aligned} & E_{i,-1} \left[\tilde{g} \left(k_{i,-1}, x_0, z_0, x_1, z_1, \ldots \right) \right] - E_{i,-1} \left[\tilde{g} \left(k_{i,-1}, x_0^*, z_0, x_1^*, z_1, \ldots \right) \right] \\ = & \sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} \left(x_t - x_t^* \right)' \Theta_0 \left(x_t - x_t^* \right) + \left(x_t - x_t^* \right)' \Theta_1 \left(x_{t+1} - x_{t+1}^* \right) \right]. \end{aligned} \tag{17}$$

The matrices Θ_0 and Θ_1 are given by

$$\Theta_0 = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha(1-\alpha) & 0 \\ 0 & \phi(1-\phi) \end{bmatrix} \quad \Theta_1 = C^{-\gamma} Y \begin{bmatrix} 0 & \beta\alpha\phi \\ 0 & 0 \end{bmatrix}. \tag{18}$$

The optimal actions under perfect information are given by

$$x_t^* = \begin{pmatrix} k_{it}^* \\ l_{it}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{1-\alpha-\phi} \left[E_t a_{t+1} - \phi E_t w_{t+1} - (1-\phi) \frac{\gamma E_t (c_{t+1} - c_t)}{1-\beta(1-\delta)} \right] \\ \frac{\alpha}{1-\phi} k_{it-1}^* + \frac{1}{1-\phi} \left(a_t - w_t \right) \end{pmatrix}, \tag{19}$$

where E_t denotes the expectation operator conditioned on the entire history up to and including period t, and the initial capital stock is given by the initial condition $k_{i,-1}^* = k_{i,-1}$.

Proof. First, we introduce notation. The profit of firm i in period t depends on three sets of variables: (i) variables that the decision-maker of firm i chooses in period t (here k_{it} and l_{it}), (ii) variables that the decision-maker chose in the past (here k_{it-1}), and (iii) variables that the decision-maker takes as given (here a_t, w_t and c_t). The first set of variables is collected in the vector x_t, the second set of variables is an element of x_{t-1} for all $t \geq 0$ once we define the vector $x_{-1} = (k_{i,-1}, 0)'$, and the third set of variables is collected in the vector z_t.

The next steps are word for word identical to steps "Second" to "Seventh" in proof of Proposition 2 in online Appendix D of Maćkowiak and Wiederholt (2015). The reason is that these steps only require that the payoff in period t depends only on own current actions (collected in x_t), own previous-period actions (collected in x_{t-1}), and variables that the decision-maker takes as given (collected in z_t) and that the initial condition $k_{i,-1}$ and the vector v_t satisfy conditions (40)-(42) in online Appendix D of Maćkowiak and Wiederholt (2015). The payoff in period t in Proposition 1 is profit, whereas the payoff in period t in online Appendix D of Maćkowiak and Wiederholt (2015) is period utility, but in both cases this payoff depends only on own current actions (x_t), own previous-period actions (x_{t-1}), and variables that the decision-maker takes as given (z_t). Conditions (40)-(41) in online Appendix D of Maćkowiak and Wiederholt (2015) are satisfied because of the assumption in Proposition 1 that the decision-maker knows the initial condition x_{-1}. Condition (42) in online Appendix D of Maćkowiak and Wiederholt (2015) is equal to condition (16) in Proposition 1. These steps "Second" to "Seventh" yield equation (17), where Θ_0 is defined as the Hessian matrix of second derivatives of g with respect to x_t evaluated at the non-stochastic steady state and divided by β^t, Θ_1 is defined as the Hessian matrix of second derivatives of g with respect to x_t and x_{t+1} evaluated at the non-stochastic steady state and divided by β^t, and x_t^* is defined as the actions that the decision-maker would take if he or she had perfect information in every period $t \geq 0$.

Eighth, the functional g in Proposition 1 is the discounted sum of profit

$$g\left(x_{-1}, x_0, z_0, x_1, z_1, \ldots\right) = \sum_{t=0}^{\infty} \beta^t f\left(x_t, x_{t-1}, z_t\right),$$

where the function f is the profit function

$$f\left(x_t, x_{t-1}, z_t\right) = C^{-\gamma} e^{-\gamma c_t} Y \left\{ e^{a_t + \alpha k_{it-1} + \phi l_{it}} - \phi e^{w_t + l_{it}} + \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right) \left[(1-\delta) e^{k_{it-1}} - e^{k_{it}} \right] \right\}.$$

Computing the matrices Θ_0 and Θ_1 for this functional g yields equation (18).

Ninth, we characterize the optimal actions under perfect information. Formally, the process $\{x_t^*\}_{t=0}^{\infty}$ is defined by the initial condition $x_{-1}^* = (k_{i,-1}, 0)'$ and the optimality condition

$$\forall t \geq 0 : E_t \left[\theta_0 + \Theta_{-1} x_{t-1}^* + \Theta_0 x_t^* + \Theta_1 x_{t+1}^* + \Phi_0 z_t + \Phi_1 z_{t+1} \right] = 0. \tag{20}$$

Here θ_0 is defined as the vector of first derivatives of g with respect to x_t evaluated at the non-stochastic steady state and divided by β^t, Θ_{-1} is defined as the matrix of second derivatives of g with respect to x_t and x_{t-1} evaluated at the non-stochastic steady state and divided by β^t, Φ_0 is defined as the matrix of second derivatives of g with respect to x_t and z_t evaluated at the non-stochastic steady state and divided by β^t, Φ_1 is defined as the matrix of second derivatives of g with respect to x_t and z_{t+1} evaluated at the non-stochastic steady state and divided by β^t, and E_t denotes the expectation operator conditioned on the entire history up to and including period t. See the step "Fourth" in proof of Proposition 2 in online Appendix D of Maćkowiak and Wiederholt (2015). Computing the vector θ_0 and the matrices Θ_{-1}, Φ_0, and Φ_1 for the functional g defined in the previous step yields

$$\begin{array}{cc} \theta_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \Theta_{-1} = C^{-\gamma} Y \begin{bmatrix} 0 & 0 \\ \alpha\phi & 0 \end{bmatrix} \\ \Phi_0 = C^{-\gamma} Y \begin{bmatrix} 0 & 0 & \gamma \frac{\alpha}{\beta^{-1} - 1 + \delta} \\ \phi & -\phi & 0 \end{bmatrix} & \Phi_1 = C^{-\gamma} Y \begin{bmatrix} \alpha\beta & 0 & -\gamma \frac{\alpha}{\beta^{-1} - 1 + \delta} \\ 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Substituting the equations for θ_0, Θ_{-1}, Φ_0, Φ_1, Θ_0, Θ_1, x_t and z_t into equation (20) yields

$$E_t a_{t+1} - (1 - \alpha) k_{it}^* + \phi E_t l_{it+1}^* = \frac{\gamma E_t (c_{t+1} - c_t)}{1 - \beta (1 - \delta)} \tag{21}$$

$$a_t + \alpha k_{it-1}^* - (1 - \phi) l_{it}^* = w_t. \tag{22}$$

Equations (21)-(22) are the usual optimality conditions for capital and labor. Equation (21) states that the profit-maximizing capital input equates the expected marginal product of capital to the cost of capital. Equation (22) states that the profit-maximizing labor input equates the marginal product of labor to the wage. Rearranging equations (21)-(22) yields the closed-form solution (19) for the actions that the firm would take in period t if the firm had perfect information in every period $t \geq 0$. ■

Proposition 2 *Under condition (16), we have*

$$\sum_{t=0}^{\infty}\beta^t E_{i,-1}\left[\frac{1}{2}\left(x_t-x_t^*\right)'\Theta_0\left(x_t-x_t^*\right)+\left(x_t-x_t^*\right)'\Theta_1\left(x_{t+1}-x_{t+1}^*\right)\right]$$
$$= -C^{-\gamma}Y\sum_{t=0}^{\infty}\beta^t E_{i,-1}\left[\frac{\beta\alpha\left(1-\alpha-\frac{\alpha\phi}{1-\phi}\right)}{2}\left(k_{it}-k_{it}^*\right)^2+\frac{\phi\left(1-\phi\right)}{2}\left(\zeta_{it}-\zeta_{it}^*\right)^2\right], \tag{23}$$

where $\zeta_{it}\equiv l_{it}-\frac{\alpha}{1-\phi}k_{it-1}$ *and* $\zeta_{it}^*\equiv l_{it}^*-\frac{\alpha}{1-\phi}k_{it-1}^*$.

Proof. First, we have

$$\frac{1}{2}\left(x_t-x_t^*\right)'\Theta_0\left(x_t-x_t^*\right)+\left(x_t-x_t^*\right)'\Theta_1\left(x_{t+1}-x_{t+1}^*\right)$$
$$= C^{-\gamma}Y\left[-\frac{\beta\alpha\left(1-\alpha\right)}{2}\left(k_{it}-k_{it}^*\right)^2-\frac{\phi\left(1-\phi\right)}{2}\left(l_{it}-l_{it}^*\right)^2+\beta\alpha\phi\left(k_{it}-k_{it}^*\right)\left(l_{it+1}-l_{it+1}^*\right)\right] \tag{24}$$

because $x_t=\left(k_{it},l_{it}\right)'$, $x_t^*=\left(k_{it}^*,l_{it}^*\right)'$ and the matrices Θ_0 and Θ_1 are given by equation (18). Second, define

$$\zeta_{it}=l_{it}-\frac{\alpha}{1-\phi}k_{it-1},$$

and

$$\zeta_{it}^*=l_{it}^*-\frac{\alpha}{1-\phi}k_{it-1}^*.$$

This definition implies

$$l_{it}-l_{it}^*=\zeta_{it}-\zeta_{it}^*+\frac{\alpha}{1-\phi}\left(k_{it-1}-k_{it-1}^*\right).$$

Substituting the last equation into equation (24) yields

$$\frac{1}{2}\left(x_t-x_t^*\right)'\Theta_0\left(x_t-x_t^*\right)+\left(x_t-x_t^*\right)'\Theta_1\left(x_{t+1}-x_{t+1}^*\right)$$
$$= C^{-\gamma}Y\left[\begin{array}{c}-\frac{\beta\alpha(1-\alpha)}{2}\left(k_{it}-k_{it}^*\right)^2-\frac{\phi(1-\phi)}{2}\left(\zeta_{it}-\zeta_{it}^*\right)^2\\ -\alpha\phi\left(\zeta_{it}-\zeta_{it}^*\right)\left(k_{it-1}-k_{it-1}^*\right)-\frac{\alpha^2\phi}{2(1-\phi)}\left(k_{it-1}-k_{it-1}^*\right)^2\\ +\beta\alpha\phi\left(k_{it}-k_{it}^*\right)\left(\zeta_{it+1}-\zeta_{it+1}^*\right)+\frac{\beta\alpha^2\phi}{1-\phi}\left(k_{it}-k_{it}^*\right)^2\end{array}\right].$$

Multiplying by β^t and summing over all t from zero to T yields

$$\sum_{t=0}^{T}\beta^t\left[\frac{1}{2}\left(x_t-x_t^*\right)'\Theta_0\left(x_t-x_t^*\right)+\left(x_t-x_t^*\right)'\Theta_1\left(x_{t+1}-x_{t+1}^*\right)\right]$$
$$= C^{-\gamma}Y\left[\begin{array}{c}-\frac{\beta\alpha}{2}\left(1-\alpha-\frac{\alpha\phi}{1-\phi}\right)\sum_{t=0}^{T}\beta^t\left(k_{it}-k_{it}^*\right)^2-\frac{\phi(1-\phi)}{2}\sum_{t=0}^{T}\beta^t\left(\zeta_{it}-\zeta_{it}^*\right)^2\\ +\beta^T\frac{\beta\alpha^2\phi}{2(1-\phi)}\left(k_{iT}-k_{iT}^*\right)^2+\beta\alpha\phi\beta^T\left(k_{iT}-k_{iT}^*\right)\left(\zeta_{iT+1}-\zeta_{iT+1}^*\right)\end{array}\right],$$

where we have used $k_{i,-1}^{*}=k_{i,-1}$ and the fact that several terms on the right-hand side cancel. Taking the expectation $E_{i,-1}$ and the limit as $t\to\infty$ yields

$$
\begin{aligned}
&\sum_{t=0}^{\infty}\beta^{t}E_{i,-1}\left[\frac{1}{2}\left(x_{t}-x_{t}^{*}\right)'\Theta_{0}\left(x_{t}-x_{t}^{*}\right)+\left(x_{t}-x_{t}^{*}\right)'\Theta_{1}\left(x_{t+1}-x_{t+1}^{*}\right)\right]\\
=\;& C^{-\gamma}Y\left[\begin{array}{c}
-\frac{\beta\alpha}{2}\left(1-\alpha-\frac{\alpha\phi}{1-\phi}\right)\sum_{t=0}^{\infty}\beta^{t}E_{i,-1}\left(k_{it}-k_{it}^{*}\right)^{2}-\frac{\phi(1-\phi)}{2}\sum_{t=0}^{\infty}\beta^{t}E_{i,-1}\left(\zeta_{it}-\zeta_{it}^{*}\right)^{2}\\
+\frac{\beta\alpha^{2}\phi}{2(1-\phi)}\lim_{T\to\infty}\beta^{T}E_{i,-1}\left(k_{iT}-k_{iT}^{*}\right)^{2}+\beta\alpha\phi\lim_{T\to\infty}\beta^{T}E_{i,-1}\left(k_{iT}-k_{iT}^{*}\right)\left(\zeta_{iT+1}-\zeta_{iT+1}^{*}\right)
\end{array}\right].
\end{aligned}
$$

Under condition (16) the two infinite sums on the right-hand side of the last equation converge to an element in $\mathbb{R}$ and the third and fourth term on the right-hand side of the last equation equal zero. ■

D Additional numerical results

In this appendix we report additional numerical results that help develop intuition about optimal signals in an economy with news shocks. We consider the version of the model from Section 4.1 with labor as the only variable input, $\alpha = 0$. All parameters have the same values as in Section 4.1 unless otherwise indicated. We solved for the rational inattention equilibrium in Section 4.1. In this appendix we suppose that a measure zero of firms are subject to rational inattention and other firms (and all households) have perfect information. We study the attention problem of the rationally inattentive firms.

In the first experiment, we vary h in the law of motion for productivity, $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-h}$, between $h = 0$ and $h = 6$. Table 2 reports how the optimal signal changes. With $h = 0$ the optimal signal is on the current optimal action l_{it}^*, $S_{it} = l_{it}^* + \psi_{it}$, where ψ_{it} follows a Gaussian white noise process with standard deviation $\sigma_\psi = 0.0191$. With $h \geq 1$ the optimal signal has non-zero weights on $\varepsilon_t, ..., \varepsilon_{t-(h-1)}$ because all elements of the state vector $\xi_t = \left(l_{it}^*, \varepsilon_t, ..., \varepsilon_{t-(h-1)}\right)'$ help predict future optimal actions. For example, with $h = 2$ the optimal weight on ε_t is 0.0045 and the optimal weight on ε_{t-1} is 0.0059 (with the weight on l_{it}^* normalized to one). The largest weight is always on $\varepsilon_{t-(h-1)}$, the innovation useful for predicting the optimal action in period $t+1, t+2, ...$ The weights decline monotonically from $\varepsilon_{t-(h-1)}$ to ε_t, the innovation useful for predicting the optimal action in period $t+h, t+h+1, ...$ For a given marginal cost λ, the chosen amount of attention falls with h (because the marginal benefit of attention falls with h). Table 2 also reports the impulse response of labor input on impact as a fraction of the maximum response under perfect information. The impulse response on impact decreases with h, approaching zero as h rises. The more distant is the change in productivity, the weaker is the response of the action on impact of a news shock.

In the second experiment, we set $h = 1$ ($a_t = \rho a_{t-1} + \sigma\varepsilon_{t-1}$) and we vary ρ, adjusting σ to keep constant the unconditional variance of the optimal action l_{it}^* (as ρ rises the optimal action becomes more persistent while its unconditional variance remains unchanged). The state vector is $\xi_t = (l_{it}^*, \varepsilon_t)'$. The optimal signal has a non-zero weight on ε_t, and we find that the weight on ε_t falls with ρ. See Table 3. With a more persistent productivity process, learning about the innovation ε_t becomes less important relative to learning about the current state of productivity a_t. In addition, (i) the expected profit loss declines with ρ, and (ii) there is a non-monotonic relation between ρ and the chosen amount of attention (as ρ falls the quality of tracking deteriorates, but the marginal value of attention may go up or down). See also Proposition 3 in Maćkowiak and Wiederholt (2009).

In the third experiment, we add another MA term in the law of motion for productivity. As an example, we suppose that productivity follows the process $a_{t+1} = \rho a_t + \pi\varepsilon_{t-1} + \sigma\varepsilon_{t-3}$. Information about productivity becomes available $h = 4$ periods in advance and, if $\pi \neq 0$, additional information becomes available in an intermediate period (two periods in advance). Table 4 shows what happens to the optimal signal as we vary π (we adjust σ to keep constant the unconditional variance of the optimal action). The state vector is $\xi_t = (l_{it}^*, \varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3})'$. As π rises, it is optimal to increase the weights on ε_t and ε_{t-1} and to decrease the weights on ε_{t-2} and ε_{t-3} in the signal. Table 4 also reports the response of labor input on impact of a news shock as a fraction of the maximum response under perfect information. This ratio rises with π, indicating that the action becomes *more* front-loaded.

In the fourth experiment, we assume that productivity is driven by two orthogonal shocks, a standard productivity shock and a news shock: $a_t = \rho a_{t-1} + \sigma_1\varepsilon_{1t} + \sigma_2\varepsilon_{2,t-h}$, where ε_{1t} and ε_{2t} follow independent Gaussian white noise process with unit variance, $\sigma_1 > 0$, $\sigma_2 > 0$, and $h \geq 1$. The profit-maximizing labor input l_{it}^* is still proportional to a_t (equation (15)). Focus on $h = 1$. The optimal signal is a one-dimensional signal about the vector $(l_{it}^*, \varepsilon_{2t})'$, or equivalently $(a_t, \varepsilon_{2t})'$.[50] The only difference to the case when productivity follows the process $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-1}$ is that the signal is on $(a_t, \varepsilon_{2t})'$ instead of $(a_t, \varepsilon_t)'$. Let us split the variance of productivity equally between the component driven by the first shock (ε_{1t}) and the component driven by the second shock (ε_{2t}), and solve the attention problem. We find that labor input l_{it} rises on impact of a positive news shock. The impulse response of l_{it} to ε_{2t} is very similar to the impulse response of l_{it} to ε_t in the lower-left panel in Figure 1 (the case with $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-1}$). The response of l_{it} on impact of a news shock equals 0.16 as a fraction of the maximum response under perfect information – the same ratio equals 0.15 in the case when productivity follows the process $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-1}$.

So far we have assumed that, after chosing the signal process in period -1, the agent receives a sequence of signals in period -1 such that the conditional second moments are independent of time. See the discussion of problem (6)-(11) in Section 3.2. In the fifth experiment, we resolve problem

[50] For any $h \geq 1$ we can write $l_{it}^* = l_{i1t}^* + l_{i2t}^*$, where l_{i1t}^* follows an AR(1) process driven by the standard productivity shock and l_{i2t}^* follows an ARMA(1,h) process driven by the news shock. The optimal signal is a one-dimensional signal about the state vector with equal weights on l_{i1t}^* and l_{i2t}^*. See also Maćkowiak, Matějka, and Wiederholt (2018), Section 4.4.

(6)-(11) having dropped this assumption. We use the methodology of Afrouzi and Yang (2020) to compute the steady-state conditional variance of ξ_t given $\mathcal{I}_{it-1}$, the steady-state conditional variance of ξ_t given $\mathcal{I}_{it}$, and the implied action. This approach allows for transitional dynamics in the conditional second moments. As before in this appendix, we study the version of the model from Section 4.1 (we also continue to suppose that only a measure zero of firms are subject to rational inattention). We compare: (i) the impulse response of labor input to a news shock with the assumption about the initial sequence of signals made earlier, and (ii) the analogous impulse response without this assumption. The two impulse responses are *identical* in the limit as the discount factor β approaches 1 (see Afrouzi and Yang, 2020, for a proof). The question is by how much the two impulse responses differ in this model for the value of the discount factor we have assumed, $\beta = 0.99$. As an example, let $h = 1$. The right panel in the top row in Figure 6 shows the comparison (the baseline is the line with circles, the solution allowing for transitional dynamics is the line with asterisks). The two impulse responses are *almost identical*: the line with asterisks is slightly closer to the perfect information equilibrium than the line with circles. As another example, let $h = 6$ (Figure 6, right column, bottom row). Again, the impulse responses are almost identical (it is now more readily apparent that the line with asterisks is slightly closer to the perfect information equilibrium than the line with circles). Thus, the two solutions differ very little quantitatively and the qualitative result is unchanged: Labor input rises on impact of a positive news shock.

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Table 1: Business cycle statistics

	Data	Model, *h = 0*	
		Perfect information	Rational inattention
Relative standard deviation			
σ_c/σ_y	0.55	0.56	0.59
σ_l/σ_y	0.92	0.66	0.57
σ_i/σ_y	2.88	3.05	2.93
σ_a/σ_y	0.52	0.46	0.51
Correlation			
$\rho_{c,y}$	0.78	0.78	0.81
$\rho_{l,y}$	0.85	0.85	0.83
$\rho_{i,y}$	0.90	0.93	0.92
$\rho_{a,y}$	0.40	1.00	0.99
First-order serial correlation			
Δc	0.27	0.23	0.28
Δl	0.41	-0.06	0.46
Δi	0.35	-0.06	0.14
Δy	0.30	-0.05	0.13
Δa	-0.06	-0.05	-0.05

Data: United States, 1955Q1-2007Q4, from Eusepi and Preston (2011).
Model: Unconditional moments computed from the equilibrium MA representation of each variable.

Table 2: Varying *h* in the law of motion for productivity

	Coefficient in the optimal signal on								
h	ε_t	ε_{t-1}	ε_{t-2}	ε_{t-3}	ε_{t-4}	ε_{t-5}	σ_ψ	Attention, bits per period	Labor input on impact as fraction of maximum labor input with perfect information
0	0	0	0	0	0	0	0.0191	0.216	0.26
1	0.0055	0	0	0	0	0	0.0195	0.211	0.15
2	0.0045	0.0059	0	0	0	0	0.0200	0.201	0.09
3	0.0037	0.0051	0.0063	0	0	0	0.0205	0.192	0.06
4	0.0031	0.0044	0.0056	0.0065	0	0	0.0212	0.183	0.04
5	0.0027	0.0039	0.0050	0.0060	0.0067	0	0.0219	0.175	0.03
6	0.0022	0.0034	0.0045	0.0056	0.0064	0.0069	0.0227	0.168	0.02

Productivity follows the law of motion $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-h}$.

The only variable input is labor, $\alpha = 0$, and parameter values are as in Section 4.1.

A measure zero of firms are subject to rational inattention. Other firms and all households have perfect information.

The table reports how the optimal signal and the labor input based on the optimal signal vary with *h* .

Table 3: Varying the persistence of productivity

ρ	Coefficient in the optimal signal on ε_t	σ_ψ	Attention, bits per period	Expected profit loss
0.5	0.0110	0.0303	0.216	2.00E-05
0.6	0.0099	0.0271	0.234	1.79E-05
0.7	0.0087	0.0244	0.242	1.55E-05
0.8	0.0074	0.0219	0.239	1.29E-05
0.9	0.0055	0.0195	0.211	9.55E-06
0.95	0.0041	0.0181	0.172	7.14E-06

Productivity follows the law of motion $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-h}$ with $h = 1$.

The only variable input is labor, $\alpha = 0$, and parameter values are as in Section 4.1 except as otherwise indicated.

A measure zero of firms are subject to rational inattention. Other firms and all households have perfect information.

The table reports how the optimal signal varies with ρ.

The value of σ is adjusted so that the unconditional variance of the optimal action is constant across the rows.

The last column gives the per period expected profit loss at the solution as a fraction of steady-state output.

Table 4: An extra term in the law of motion for productivity*

	Coefficient in the optimal signal on						
π	ε_t	ε_{t-1}	ε_{t-2}	ε_{t-3}	σ_ψ	Attention, bits per period	Labor input on impact as fraction of maximum labor input with perfect information
0	0.0031	0.0044	0.0056	0.0065	0.0212	0.183	0.04
0.002	0.0036	0.0051	0.0052	0.0062	0.0203	0.220	0.05
0.004	0.0040	0.0057	0.0046	0.0057	0.0197	0.247	0.06
0.006	0.0043	0.0062	0.0038	0.0049	0.0194	0.265	0.07
0.008	0.0046	0.0065	0.0027	0.0036	0.0192	0.270	0.08

***Productivity follows the law of motion $a_t = \rho a_{t-1} + \pi\varepsilon_{t-2} + \sigma\varepsilon_{t-4}$.**

The only variable input is labor, $\alpha = 0$, and parameter values are as in Section 4.1 except as otherwise indicated.

A measure zero of firms are subject to rational inattention. Other firms and all households have perfect information.

The table reports how the optimal signal and the labor input based on the optimal signal vary with π .

The value of σ is adjusted so that the unconditional variance of the optimal action is constant across the rows.

Figure 1: Impulse responses with α = 0

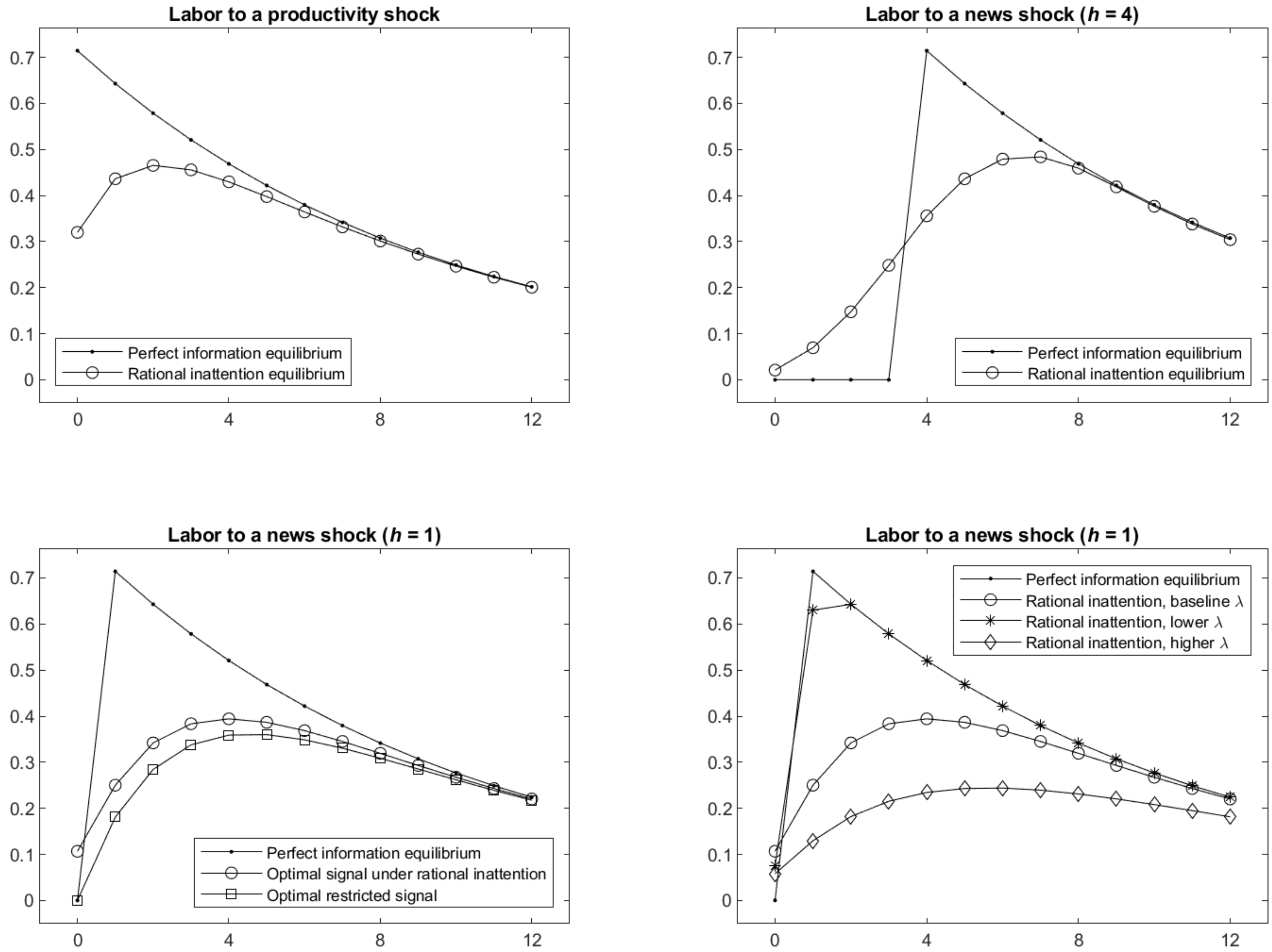

Figure 2: Impulse responses with ϕ = 0

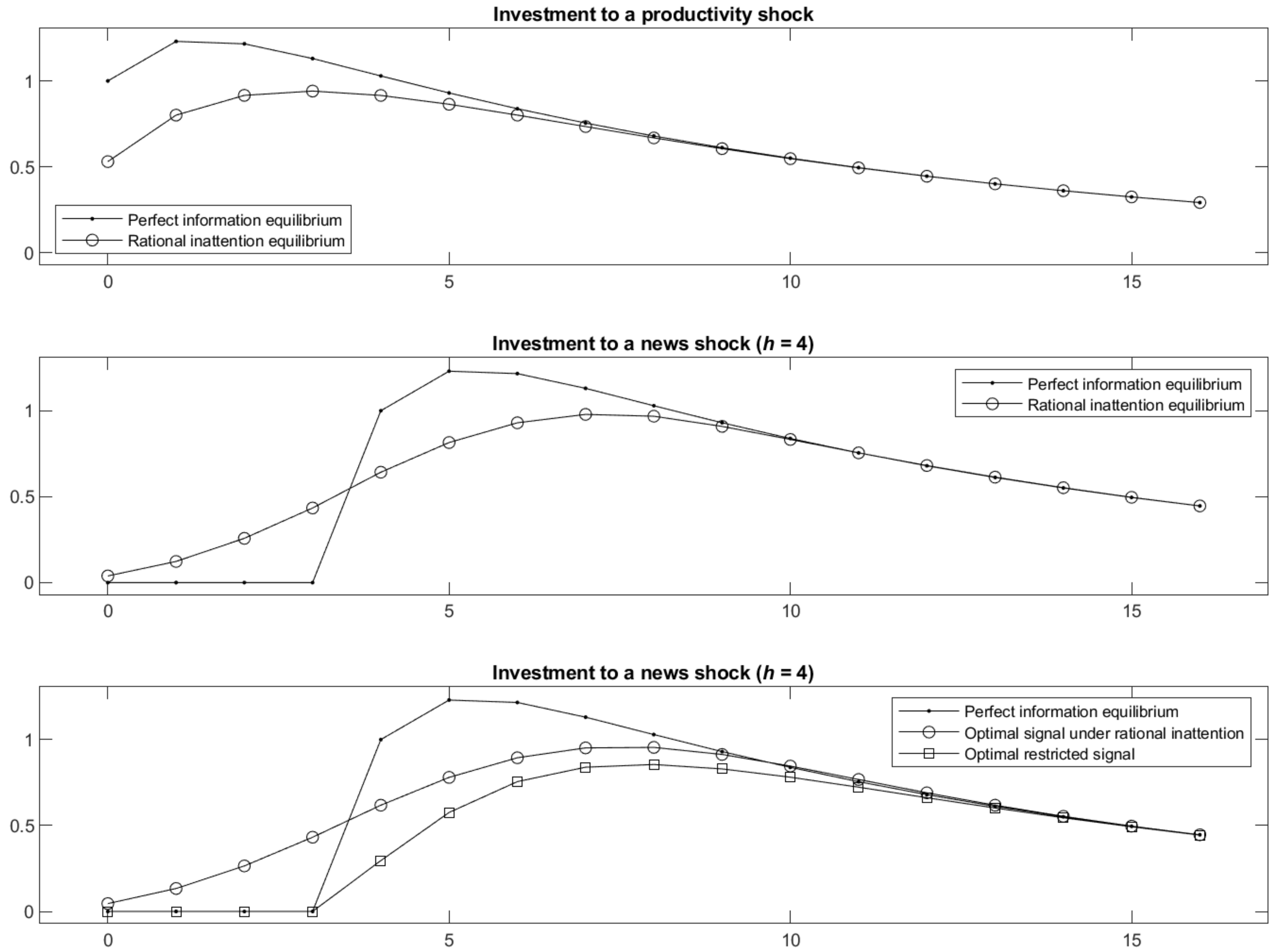

Figure 3: Impulse responses to a productivity shock

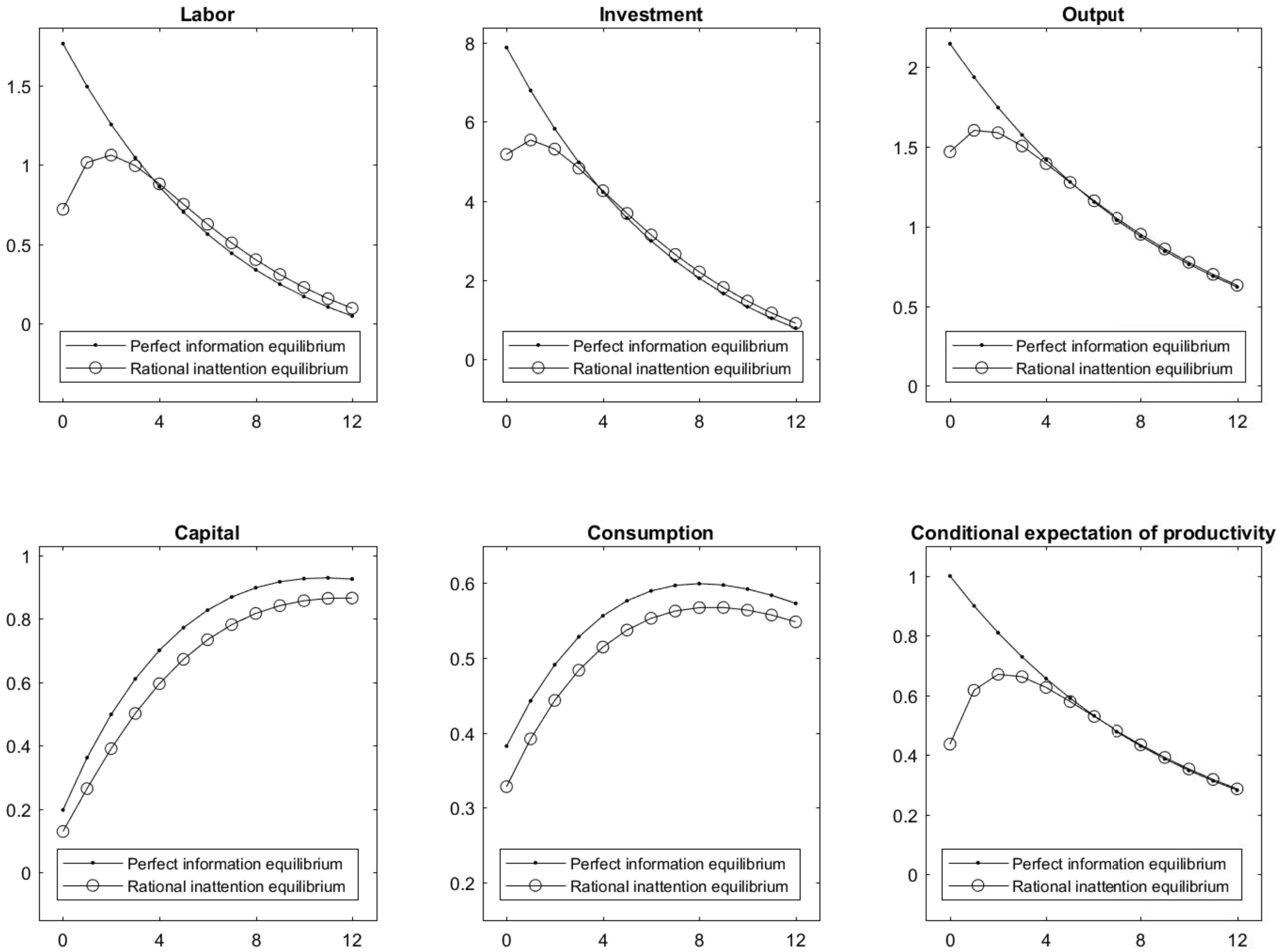

Figure 4: Impulse responses to a news shock (h = 2)

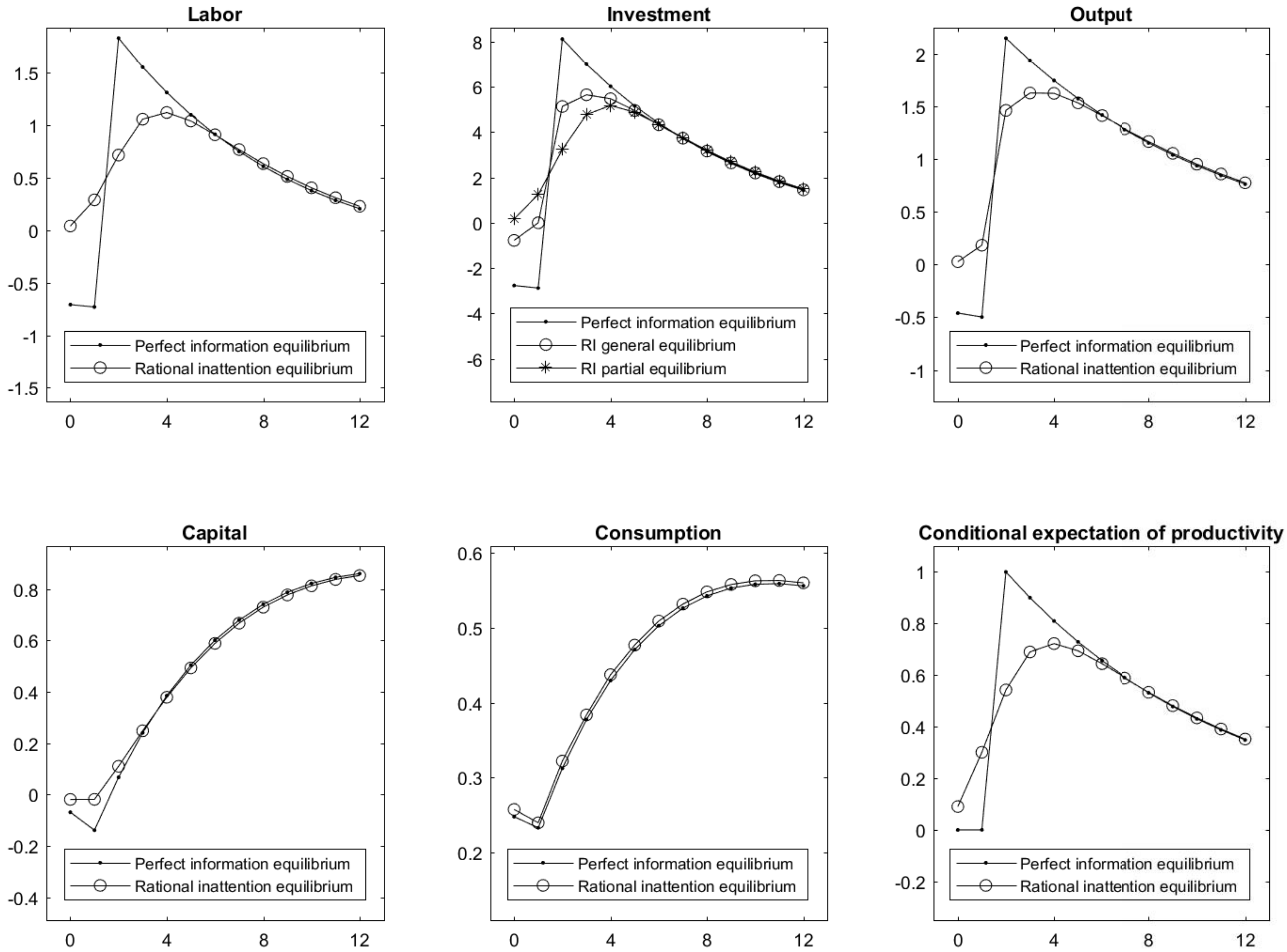

Figure 5: Impulse responses to a news shock (h = 4)

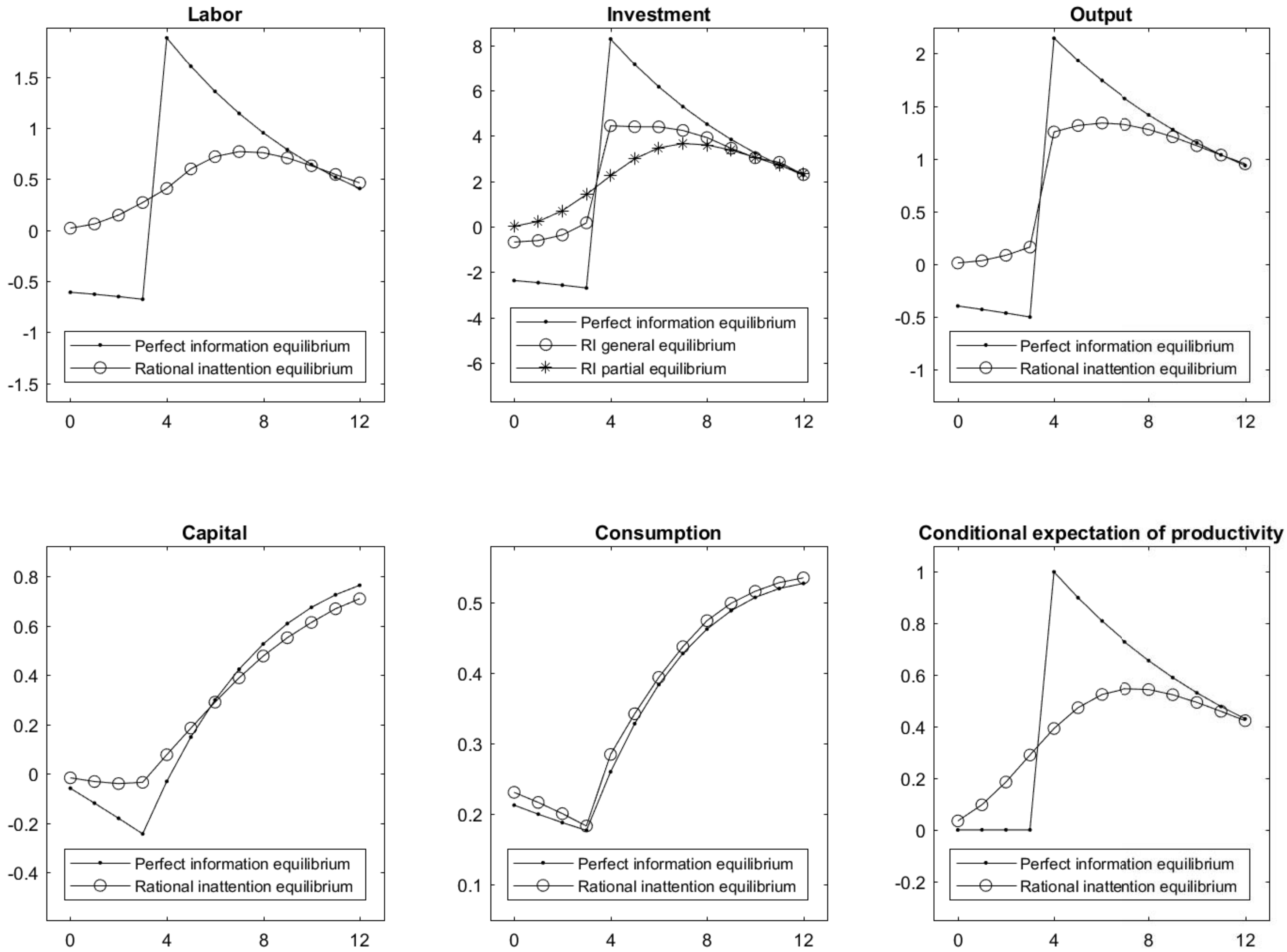

Figure 6: Additional impulse responses