Market Power and Exchange Rate Dynamics

Marco Errico Luigi Pollio*

Boston College

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Abstract

The huge trading volumes in the foreign exchange rate markets are highly concentrated among few financial players. The presence of large investors has been advocated for rejecting the assumption of perfectly competitive financial markets. We develop an international portfolio choice model with noise shocks and traders' heterogeneity in market power. Large non-competitive traders internalize the impact of their portfolio decisions on the determination of prices. We find that higher market power i) amplifies (dampens) the response of the exchange rate to non-fundamental (fundamental) shocks; ii) destabilizes the exchange rate, increasing its volatility; iii) increases exchange rate predictability; iv) makes the exchange rate more disconnected to fundamentals. Our theoretical predictions are empirically confirmed on a cross-section of 18 currencies. Welfare analysis suggests that the consolidation in the financial sector in the last three decades increased investors' welfare by 30%.

JEL Codes: F31, G11, G15

Keywords: Exchange Rate, Market Power, Price Impact, Noisy Traders, Exchange Rate Puzzles, Excess Predictability, Exchange Rate Disconnect, Welfare.

*Contact: Marco Errico, erricom@bc.edu; Luigi Pollio, polliol@bc.edu. We would like to thank Susanto Basu, Ryan Chahrour, Vito Cormun, Luca Gemmi, Danial Lashkari, Jaromir Nosal, Rosen Valchev, and all participants at the Boston College Dissertation Workshop and Boston College Macro Lunch for the helpful comments.

1 Introduction

In June 2013, Bloomberg News trilled that "*traders at some of the world's biggest banks colluded to manipulate the benchmark foreign-exchange rates used to set the value of trillions of dollars of investments in Pensions Funds and money managers globally*". After extensive investigations, banks pleaded guilty and paid more than $10 billion in fines.[1]

We develope an international portfolio choice model with noise shocks and impefectly competitive markets. In our model, exchange rate dynamics are deeply influenced by the presence of traders that internalize the impact of their portfolio decisions on the determination of prices. The underlying institutional setting is a first role determinant of exchange rate and helps rationalize the presence of several puzzle in international finance. Despite extensive evidence that foreign exchange rate markets are highly concentrated and perfectly competition hardly holds in this market, the literature has ignored this feature of the underlying market organization when studying the dynamics of exchange rate.

The huge trading volume in the currency markets is highly concentrated among the market-making desks of banks and other large financial institutions.[2] The presence of large investors in the financial markets has been advocated by Kyle (1989) and subsiquent literature as an important reason to model traders as imperfect/strategic competitors. The price-taking assumption is also in contrast with the collusive misconducts unveiled in 2013 which suggest that large players behave as price setter, internalizing the effects of their tradings on the determination of the exchange rate.[3] Moreover, the opaque over-the-counter nature of the foreign exchange market favors specialization and concentration as extendedly reported in the literature, casting doubt upon the assumption of perfectly competitive markets.[4]

Our model captures this element of reality by taking seriously the competitive dynamics

[1]Investigations later revealed that traders were exchanging information about the type and volume of client orders in private chatrooms called 'The Cartel" or "The Mafia" in order to manipulate the WMR/Reuter fix for their own financial gain.

[2]In 2019, the average daily global volume the foreign exchange market was about $6.6 trillion. 80% of all transactions took place in the six major markets, UK, USA, Japan, Singapore, Switzerland and Hong Kong. Within each market, 75% of tradings is concentrated in the hand of few financial players, e.g. 4 in the US. Source: BIS Triennial Survey of Foreign Exchange Markets, 2019; NY Fed FX report, 2019.

[3]Despite significant institutional reforms in 2015, market manipulation might not be over (see Cochrane (2015) or Osler et al. (2016)).

[4]The finance and market microstructure literature stress that the OTC structure is prone to the rise of market power among market makers due to, for instance, search frictions (see Duffie et al. (2005), Lester et al. (2018), Allen et al. (2019)). Pieces of empirical evidence have been provided also for the foreign exchange market, as in Lyons (1995).

and the underlying market organization. We augment a standard monetary model of exchange rate determination in the spirit of Mussa (1982) and Bacchetta and Van Wincoop (2006) with imperfectly competitive financial markets. We depart from the standard assumption of perfectly competitive agents assuming there is a continuum of investors that differ in their degree of market of power. A fraction of traders act as competitive traders, taking prices as given. The complementary fraction is populated by a finite number of large traders acting as an oligopolist, internalizing the effects of their trading decision on equilibrium prices. Our theory of exchange rate determination with imperfectly competitive markets introduces institutional features as key determinants of exchange rate dynamics. The exchange rate is determined as a weighted average of fundamental (interest rate differential) and noise components. The weight on fundamental component represents the information loading factor of the exchange rate and depends on the level of competition.

In less competitive markets, investors' position is lower and less elastic, reducing exchange rate informativeness. Consequently, the response of the exchange rate to non-fundamental (fundamental) shocks is amplified (dampened) by the presence of non-competitive traders. The main implication of this amplification mechanism is that, for the same volatility in fundamentals, a less noisy process of non-fundamental trading is required to match the volatility of the exchange rate. This complements the common belief that exchange rate fluctuations are mostly driven by noise, shifting the focus on the role of the underlying market structure and its interaction with non-fundamental trading.

Based on standard parametrizations, the unconditional volatility of exchange rate monotonically increases when investors are less competitive. As market power increases, exchange rate price informativeness falls and the noise component acquires more relevance in the dynamics of exchange rate. Since the process of fundamentals is less volatile than the noise process, the variance of exchange rate increases.

We study the implications that market power has for some of the major puzzles identified in the literature. Non competitive financial markets can explain why high interest rate currencies have higher expected returns over the near future (Fama puzzle, Fama (1984)). The model accounts for the failure of the UIP because the presence of non-competitive traders reduces the risk appetite of market. This raises the required risk premium associated to foreign assets in order to clear the market. Our model predicts that currencies towards which traders have more market power tend to be more predictable. Moreover, the amplification of non-fundamental shocks due to the presence of imperfectly competitive agents rationalizes also the weak explanatory power of macroeconomics variables in predicting exchange rates, known as exchange rate disconnect puzzle (Meese and Rogoff (1983)). Since exchange rate price informativeness is decreasing in market power, the information content of exchange rate about fundamentals shrinks, explaining the disconnection between the two.

Based on this framework, we derive a set of cross-sectional predictions. We test our model on a set of 18 currencies exploting data on market power in trading volume for each currency pair provided by the Biannual FXC Report run by New York Federal Reserve Bank. Results validate all the qualitative predictions derived through the lens of the model, corroborating the relevance of institutional features in the determination of exchange rate dynamics.

The BIS Triannual surveys show that the FX market has experienced strong consolidation pattern in the last three decades.[5] We evalute the welfare consequences of the increase in market power through the lens of our model. Investors' welfare (indirect utility) has increased by 30% since 1990, with no particular redistribution across small and large investors. However, it is hard to draw unambiguous policy implications because our measure of welfare refers only to investors, disregarding that other important channels could affect the economy.[6]

1.1 Related literature

Our hybrid modeling approach integrates two branch of studies. It is inspired by the macro approach to exchange rate fluctuations in our choice of a monetary model in the spirit of Mussa (1982) and subsequent papers. As in these papers, our theoretical framework is based on the building blocks of a standard monetary model of exchange rate determination: i) purchasing power parity, ii) interest rate arbitrage and iii) money market equilibrium. It is also reminiscent of the market microstructure literature in the attention devoted to the underlying market organization. Similar to this literature, we model the presence of non-competitive traders as a fraction of investors that internalize the effect of their tradings on the equilibrium prices.[7] However, we deeply differ in the little role played by other pivotal

[5]This is not particularly surprising considering that FX is strictly related to the financial industry and the same consolidation trend appears in the financial sector, as document by Corbae and D'Erasmo (2020) and others.

[6]In particular, a less competitive FX market increases exchange rate volatility, which could have welfare consequences through price adjustment, consumption and FDI volatility, etc... Our stylized model does not account for these additional GE forces.

[7]In the microstructure literature, there is consensus in considering the assumption of perfect competition to be unrealistic in highly concentrated markets like the FX (see footnote 4 for theoretical and empirical arguments).

elements in this strand of literature, like information and trading process.[8]

This paper contributes to the literature that studies exchange rate determination in the presence of frictions. The literature has focused on different forms of frictions: informational frictions (Evans and Lyons (2002) and Bacchetta and Van Wincoop (2006)), infrequent portfolio adjustment (Bacchetta and Van Wincoop (2010) and Bacchetta and Van Wincoop (2019)), imperfect markets (Gabaix and Maggiori (2015)). To the best of our knowledge, our model is the first to focus on a specific feature of the market structure, the presence of non-competitive traders, for the determination of the exchange rate.

Our work is broadly inspired by the role that investor's heterogeneity can play in the determination of exchange rate, as highlighted in Bacchetta and Van Wincoop (2006). They focus on heterogeneous information among investors and, similarly to us, the response of the exchange rates to noise shock is magnified. We focus on another dimension of heterogeneity (market power), in a more tractable and directly testable way.

An implication of our model is that the presence of large non competitive traders increases exchange rate volatility. This result connects to a broad literature on the stabilizing role of large players that traces back to Friedman and Friedman (1953). Our finding is in line with Gabaix et al. (2006) and Wei and Kim (1997), which shows that large trader positions explain volatility (the latter with a specific focus to the FX market).

Finally, our theoretical analysis also relates to the vast literature trying to rationalize the major puzzles in international economics. We contribute here providing a new rationale based on imperfectly competitive markets for systematic UIP violations (Fama (1984)) and exchange rate disconnect (Meese and Rogoff (1983)). Related to the latter, we have a different approach than Evans and Lyons (2002). Bacchetta and Van Wincoop (2006) and this paper borrow elements from the microstructure literature to rationalize the failure of macrofundamental to predict exchange rate; on the other hand, Evans and Lyons (2002) borrow from the same literature to identify elements that can actually predict the exchange rate. Consistently with our theory, we document that, from a cross-sectional perspective, currencies traded in less competitive markets tend to have a more predictable excess return and be more disconnected from fundamentals. To the best of our knowledge, there are very few studies trying to address cross-sectional differences in exchange rate puzzles.

The remainder of the paper is organized as follow. Section 2 presents the theoretical framework, the equilibrium concept and solution method. In section 3, we derive all theoretical and numerical results and discuss key economics intuitions. Section 4 tests model

[8]In our macro approach, we abstract from information heterogeneity or detailed trading dynamics.

predictions on the data, presenting the results of the cross-currency analysis. Section 5 discusses welfare and potential policy implications. Section 6 concludes. Any omitted proofs and derivations are in the Appendixes.

2 A Monetary Model with Non-Competitive Traders

The model contains the standard elements of an exchange rate monetary model together with i) non-fundamentals trade in the form of noise traders, as in Bacchetta and Van Wincoop (2010), and ii) heterogeneity in investors' market power.

2.1 Basic Set-up

We develop a two-country, discrete time, stochastic general equilibrium model. We assume that agents have rational expectations on the dynamics of the exchange rate. We assume that one economy is infinitesimally small (Foreign country) so that market equilibrium is entirely determined by the market participants in the large country (Home country). Variables referring to foreign are indicated with a star. In the large economy, there is a continuum of traders of mass one. There are overlapping generations of agents that live for two periods and make only one investment decision.[9] Portfolio choices differ across agents and depend upon their market power: a segment $1-\lambda$ is composed by standard atomistic/competitive traders; the complementary segment λ of investors are strategic, in the sense that they internalize their effect on prices. λ is assumed to be divided into finite N strategic investors with mass λ_i acting as an oligopoly.[10] Investors are born with an exogenous endowment ω and can purchase one-period nominal bonds of both countries with interest rates i_t and $i_t^\star$, respectively, and a technology with fixed real return r. The latter is infinitely supplied while bonds are in fixed supply in the respective currency.

Each economy produces the same good and we assume PPP holds. In log terms:

$$p_t = p_t^\star + s_t$$

where s_t is the log of the nominal exchange rate. The exchange rate is the value of the foreign

[9]We abstract from saving decisions by assuming that investors derive utility from their wealth at the end of life.

[10]Obviosly, $\sum_i^N \lambda_i = \lambda$. For simplicity, we assume the oligopoly to be symmetric, meaning that $\lambda_i = \lambda/N$. Qualitative results do not change.

currency in term of domestic currency, so that an increase in the exchange rate reflects an appreciation (depreciation) of the foreign (domestic) currency.

We assume asymmetric monetary rules across countries: the Home central bank commits to a constant price level $p_t = 0$ so that $i_t = r$ while the monetary policy in Foreign is stochastic, $i_t^\star = -u_t$ where

$$u_t = \rho_u u_{t-1} + \sigma_u \epsilon_t^u \qquad \epsilon_t^u \sim N(0,1)$$

is the Foreign monetary policy structural shock. The interest rate differential is then $i_t - i_t^\star = u_t + r$ meaning that only the Foreign country influences the dynamics of the exchange rate through its monetary policy, because the interest rate in the domestic country is fixed. [11] In our model, we refer to a shock in the Foreign monetary policy as a fundamental shock.

2.2 Portfolio problem

Each investor $j \in [0,1]$ maximizes mean-variance preferences over next period wealth w_{t+1}^j, with a rate of risk aversion ρ:

$$\max_{b_t^j} E_t(w_{t+1}^j) - \frac{\rho}{2} Var_t(w_{t+1}^j)$$

$$\text{s.t. } w_{t+1}^j = (\omega - b_t^j) i_t + (i_t^\star + s_{t+1} - s_t) b_t^j$$

where the initial endowment is normalized to one and allocated between domestic and foreign bonds (b_t^j defines the foreign bond holdings).

i_t and $i_t^\star + s_{t+1} - s_t$ are the log-linearized returns of domestic and foreign bonds, respectively. Notice that, under the monetary policy assumptions and PPP, we have that $p_t^\star = -s_t$ and both returns are expressed in real terms. The only difference between the two assets is that the return on foreign bonds is stochastic.[12]

Investors can either exert market power or not. In the former case, they internalize the effects that their demand has on equilibrium prices or, more precisely, on the equilibrium exchange rate. Investor' type has implications for foreign asset demand and portfolio alloca-

[11] Bacchetta and Van Wincoop (2010) specifies a simplified Wicksellian rule $i_t^\star = \psi(p_t^\star - \bar{p^\star}) - u_t$ where ψ is set equal to zero, consistently with the low estimates reported by Engel and West (2005). Bacchetta and Van Wincoop (2010) shows that an exogenous interest rate rule, as in our case, does not compromise the existence of a unique stochastic steady state for the exchange rate.

[12] $p_t = 0$ implies $i_t = r$ and $p_t^\star = -s_t$ implies that the foreign bond return $i_t^\star + s_{t+1} - s_t$ is real as well.

tion. Define an economic agent $j = C$, an investor that is atomistic and acts competitively; similarly, let $j = S$ be the case in which the investor is strategic and acts oligopolistically. Appendix B shows that the optimal demand for foreign bonds by investor j is:

$$b_t^C = \frac{E_t(s_{t+1}) - s_t + i_t^\star - i_t}{\rho\sigma_t^2} \qquad \text{if } j = C$$

$$b_t^S = \frac{E_t(s_{t+1}) - s_t + i_t^\star - i_t}{\rho\sigma_t^2 + \frac{\partial s_t}{\partial b_t^s}} \qquad \text{if } j = S$$

where σ_t^2 is the variance of next's period excess return conditional to the information set at time t. When we solve for the stochastic steady state, we assume that the variance is time-invariant and the information structure homogeneous across investors. The optimal portfolio choice depends positively on the excess return $q_{t+1} \equiv E_t(s_{t+1}) - s_t + i_t^\star - i_t$ and negatively on its variance and investors' risk aversion.

As standard result in a non-competitive portfolio allocation, market power reduces investors' demand for every level of excess return as if non-competitive traders were more risk averse. This is captured by $\frac{\partial s_t}{\partial b_t^S}$, which represents investors' own price impact.

In addition to the agents described above, we introduce another set of agents, the noisy traders. Their behavior allows us to match observed exchange rates moments in the data. As in Bacchetta and Van Wincoop (2010), the noisy demand for foreign bonds is exogenously given by:

$$X_t = (\bar{x} + x_t)\bar{W}$$

where $\bar{W}$ is the aggregate financial wealth in Home in steady state, $\bar{x}$ is a constant and x_t follows an exogenous process:

$$x_t = \rho_x x_{t-1} + \sigma_x \epsilon_t^x \qquad \epsilon_t^x \sim N(0,1)$$

The demand of foreign assets absorbed by noise traders in the stochastic steady state is equal to $\bar{x}\bar{W}$. Any deviation from the steady state is driven by a random shock x_t which can be interpreted as both supply or demand shock orthogonal to fundamental. Positive shocks to x_t increase the desirability of the foreign assets leading the foreign currency to appreciate without movements in the interest rate differential.

2.3 Market clearing

We close the model imposing the market clearing condition in the foreign bond market.[13]. The market clearing condition implies that at any point in time the demand is equal to the supply of foreign bond denominated in the same currency:

$$(1-\lambda)b_t^C + \sum_i^N \lambda_i b_t^{S,i} + X_t = Be^{s_t}$$

where B is the fixed supply of foreign bonds in foreign currency and $b_t^{S,i}$ and b_t^C are multiplied by the corresponding investor size.

Manipulating the market clearing condition as shown in Appendix B we derive an expression for the price impact of oligopolist i, $\frac{\partial s_t}{\partial b_t^{S,i}}$:

$$\frac{\partial s_t}{\partial b_t^{S,i}} = \frac{\lambda_i \rho \sigma_t^2}{B\rho\sigma_t^2 + (1-\lambda)} = \frac{1}{N}\frac{\lambda \rho \sigma_t^2}{B\rho\sigma_t^2 + (1-\lambda)} > 0$$

which is positive for all values of λ and λ_i. Intuitively, oligopolist i's price impact inversely depends on the fraction of atomistic traders, *ceteris paribus*. Similarly, the price impact of i depends positively on its own size, *ceteris paribus*. Price impact is maximized in case of a monopolist with positive mass, $\lambda_i = \lambda$. The demand of foreign assets of a non-competitive investor decreases when the competitive fringe shrinks or its own size increases. The last equality holds in case of a symmetric oligopoly.[14] The magnitude of the individual price impact depends on the number of strategic traders, N, and the size of the non-competitive segment, λ. The larger the number of investors, the lower the price impact that they have on the exchange rate (as N goes to infinity, the price impact converges to zero in which we recover perfect competition). We interpret an increase in λ or a decline in N as an increase in market power since both increase the price impact of strategic investors. In the following sections, we study comparative statics in market power mainly through the lens of changes

[13] The market clearing for the domestic bond is not particularly relevant because the bond is perfectly substitutable for the risk free technology, which is infinitely supplied. Similarly, a monetary model would also require a market clearing condition for the money market. Bacchetta and Van Wincoop (2006) or Bacchetta and Van Wincoop (2010) assume that investors generate a money demand (independently of their portfolio decision) and that money supply accomodates it under the exogenous rule for interest rates. We do not explicitly model a money market in order to limit notation, leaving it in the background.

[14] Comprehensive market share data are not avaiable. The NY Fed FX report provides information on the aggregate market share of a certain number of players. See Appendix A. Symmetry is the best simplifying assumption we can do in this case. Notice that qualitative predictions are not altered.

in λ. The same qualitative predictions hold for changes in N.[15]

In international portfolio models, the supply of asset in foreign currency is subject to valuation effects due to exchange rate movements. Non-competitive traders account also for variations in the value of the supply of assets when they internalize the effect that their demand has on the exchange rate in equilibrium. This explains the presence of B at the denominator, reflecting a peculiarity of a portfolio of international bonds.

Lemma 1. *In foreign asset market, strategic traders have a lower price impact on the equilibrium price of an asset because the supply of assets is subject to valuation effects.*

Proof. See Appendix C. ■

A positive price impact means that the higher the demand, the higher the exchange rate (the price of the asset). However, an increase in the exchange rate implies that the supply of foreign assets acquires higher value, that is, a positive valuation effect. The supply shift dampens the initial increase in price, reducing the magnitude of the price impact. In other words, the residual net demand faced by non-competitive traders is more elastic. The main implication is that non-competitive traders have lower demand for foreign assets but not as low as in the case there was no valuation effect.

2.4 Equilibrium

Before we derive an explicit equation for the exchange rate, it is useful to define a concept of equilibrium in this model.

Definition 1. *For an history of shocks $\{\varepsilon_t^x,\ \varepsilon_t^{\Delta i}\}_{t=0}^{-\infty}$, an equilibrium path is a sequence of quantities $\{b_t^C,\ \{b_t^{S,i}\}_{i=1}^N\}$ and foreign currency (asset) price $\{s_t\}$ such that investors optimally choose their portfolio and market clearing condition holds.*

The model is simple enough to let us derive an explicit solution for the exchange rate, which can be solved for from the foreign bond market equilibrium condition.

$$s_t = \underbrace{(1-\mu)\left(\frac{\bar{x}}{b}-1\right)}_{\text{constant}} + \underbrace{\mu\left(E_t s_{t+1} + i_t^\star - i_t\right)}_{\text{fundamental}} + \underbrace{(1-\mu)\,\frac{1}{b}x_t}_{\text{noise}} \tag{1}$$

[15] Our choice is forced by the limitation of data. Cross-currency data are available only for λ, as described in the calibration section.

where $\mu = \frac{1}{1+\Phi(\lambda,N)}$ and $\Phi(\lambda, N) = \frac{B\rho \operatorname{Var}_t(s_{t+1})\left(1+B\rho \operatorname{Var}_t(s_{t+1})-\lambda\frac{N-1}{N}\right)}{\left(1+B\rho \operatorname{Var}_t(s_{t+1})-\lambda\frac{N-1}{N}\right)-\frac{\lambda^2}{N}}$, where the latter is increasing function of λ and decreasing in N. The ratio $b = \frac{B}{W}$ represents the share of foreign assets on the aggregate domestic wealth in steady state and it can be interpreted as an inverse measure of home bias.

As usual in this class of models, the exchange rate follows a forward looking autoregressive process with drift where the constant term depends on a set of parameters and the stochastic component depends on future fundamental and noise shocks. The main distinction with standard models is that the weight on the noise component depends on the size of traders that have market power. μ represents the informativeness of prices, a measure of how well the variation in exchange rate predicts the variation in fundamentals. Price informativeness ultimately depends on the market structure: exchange rate informativeness decreases when foreign bond markets are less competitive because the weigth on the noise process falls (formally, higher λ/lower N implies higher Φ and, thus, lower μ).[16] Intuitively, as the market becomes less competitive, total demand from non-noisy traders declines monotonically. Thus, lower demand implies that prices have a lower information content and the relative importance of noisy traders' demand in the determination of the exchange rate increases.[17]

2.5 Parameterization

We consider 18 exchange rate pairs, all defined against the USD, from 1993 to 2019 at daily frequency.[18] Without loss of generality, we set $\bar{r} = 0$, so that the $i_t - i_t^\star = u_t$. The interest rate differential is defined as the difference between the 1-month forward and the spot exchange rate, assuming covered interest rate parity holds. The volatility, σ_u, and the persistence, ρ_u, of the fundamental shock are calibrated to the cross country of the estimated AR(1) processes. This yields $\sigma_u = 0.012$ and $\rho_u = 0.8$.[19]

The perceived variance of the excess return, σ_t^2, is assumed to be time-invarying and

[16] Our price informativeness index μ compares to the magnification factor in Bacchetta and Van Wincoop (2006), which is microfounded from information dispersion, in the amplification mechanism.

[17] The result is well known from Kyle (1989) and subsequent literature: when traders recognize that the residual supply curve is upward-sloped, quantities are restricted and also less elastic. Price is then less informative. The same intuition applies here.

[18] The set of currencies includes: Euro, Japanese Yen, Argentinian Peso, Brazilian Real, Canadian Dollar, Swiss Franc, Australian Dollar, Chilean Peso, Indian Rupee, Mexican Peso, British Pound, South African Rand, Russian Ruble, Swedish Krona, Turkish Lira, New Zealand Dollar, Singapore Dollar, Norwegian Krone.

[19] We use daily data averaged at monthly frequency. These values fall inside the set of parametrization used in the previous literature.

homogeneous across investors. We approximate it to the average variance of the one-period exchange rate change, which is $\sigma(\Delta s_{t+1}) = 0.029$ in the data.

The process of noisy demand x_t is cannot be observed. The persistence of the noise shock, ρ_x, is set high enough such that the exchange rate behavior is sufficiently close to a random walk. We choose $\rho_x = 0.9$. The volatility of the process is chosen to match the volatility of the one-period change in exchange rate. However, from the model, the volatility of the one-period change in exchange rate $\sigma(\Delta s_{t+1})$ also depends on the market structure,λ and N. A proper parametrization of the size of non-competitive traders and the number of oligopostic investors in the market would require private data on the trading behavior of market participants, which are not available. Therefore, we use the NY Fed Biannual FX report to calibrate λ to the average market share of the top players in the US exchange rate market (dealers in the top quintile of size distribution), which is 70%, and N equal to the average number of dealers that account for that share of the market, which is 4.[20] Thus, σ_x is chosen to match $\sigma(\Delta s_{t+1})$ given the benchmark value of λ and N. This yields a value of $\sigma_x = 0.0998$.

Consistently with the behavior of price informativeness, there is a negative relationship between market power and σ_x, for a given value of $\sigma(\Delta s_{t+1})$. Everything else equal, an higher level of λ (or lower N) implies a lower volatility of noise shock because the dynamics of noise demand are amplified by the presence of larger traders.[21]

This has important implications for the dynamics of the exchange rate. We need less noise to match the volatility of the exchange rate in the data when investors have market power. Not taking into account the underlying market structure could mistakenly attribute additional noise to the non-fundamental component.

In the benchmark parametrization we set b, the inverse home bias measure, equal to 0.33, meaning that foreign assets account for one third of the total domestic financial wealth. This is an approximate average obtained from the IMF IIPS dataset as in Bacchetta and Van Wincoop (2019).

$\bar{x}$ is calibrated such that the value of the exchange rate in the stochastic steady state is zero.[22] This is guaranteed by the following condition, $\bar{x} = b$. For simplicity, the supply of

[20] See Appendix A for a better description of the mapping between data and model in terms of market power.

[21] In case of a perfectely competitive market, $\sigma_x = 0.12$, which is an order of magnitude higher than the fundamental shock, in line with the literature.

[22] This assumption excludes any trend in the dynamics of exchange rate but does not affect the results of our model.

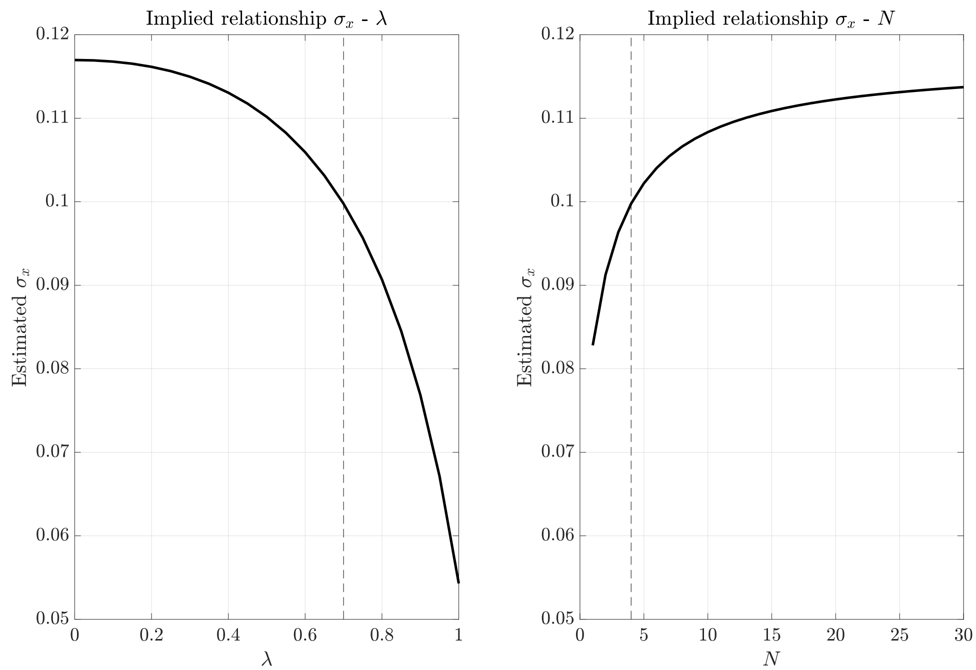

Figure 1: Relationship between λ and σ_x (left) and between N and σ_x (right).

foreign assets, B, is normalized to one. In order to consistently close the model, we need to set ω, the initial endowment of each investor, equal to 3.[23]

Finally, the rate of relative risk aversion ρ is set to 50 as in Bacchetta and Van Wincoop (2019). In the model, risk aversion is the only source of currency premia, which would be very small for standard rates of risk aversion. Our results are nevertheless qualitatively robust to different values of different risk aversion coefficients.[24]

3 Results

We study the implication of market power for i) the conditional response of the exchange rate to exogenous shock, ii) conditional and unconditional variance and iii) exchange rate puzzles, with particular focus on excess return predictability and exchange rate disconnect. In this section, an increase in market power is assumed to take the form of an increase in the size of non-competitive traders λ instead of a decrease in the number of the oligopolistic

[23]This comes from the fact that $b = \frac{B}{\bar{W}}$. Calibrating b and normalizing B mean that $\bar{W} = 3$. Total financial wealth in equilibrium is equal to the initial endowment, ω.

[24]Bacchetta and Van Wincoop (2019) also avoids to introduce other features to increase large premia, e.g. disaster risk, because they would distract from the main focus of the paper. Moreover, notice that ρ and B enter multiplicative in the model, and $\rho = 50$ could be different is B was normalized in a different way.

traders, N. The results do not depends on the dimension we look at.

Response to exogenous shock. Given the law motion of the exchange rate in equation (1), the effect of market power on the exchange rate conditional to a shock can be summarized as follow:

Proposition 1. *An increase in market power amplifies (dampens) the response of the exchange rate to noise (fundamental) shock.*

Proof. See Appendix C. ■

Appendix C shows that the result is independent of the parameterization of the model. Figure 2 plots the impulse response function to one standard deviation shock in fundamental (first row) and noise shock (second row).

A positive noise shock can be interpreted either as a positive demand shock or a negative supply shock. Either way, it increases the price of the foreign assets without any change in fundamentals. The excess return falls below the steady state because the exchange rate (the price of the foreign bond) increases. Lower excess return pushes investors to purchase less foreign assets, re-balancing in favor of domestic ones (and partially offsetting the increase in exchange rate). In a world where traders internalize the effect of their tradings on the exchange rate, the larger the non-competitive investors, the stronger the effect on exchange rate. Non-noisy investors' total demand reacts less when concentration is higher (see bottom right panel). A lower decline in investors' demand means that their demand is higher. The demand of foreign assets is therefore higher, exerting upward pressure on the exchange rate, which jumps more at impact for higher lambdas.[25] Even if non-competitive investors decreases their position by less, the excess return drops by more. This is due to the fact that non-competitive traders act as if they were more risk averse. The risk compensation per unit of asset is therefore higher.

A contraction in monetary policy in the foreign country leads the interest differential to drop, increasing the excess return and calling investors' demand of foreign asset to increase. This results in the appreciation of the foreign currency. In a world in which investors are heterogeneous, the more investors have market power, the less exchange rate is responsive

[25]The dynamics of total demand are the results of the underlying compositional forces. Both competitive and monopolistic investors' demands, b_t^C and $b_t^{S,i}$, drop when the excess return falls. However, when market power increases, the reaction of $b_t^{S,i}$ is smaller, conditional to the same change in excess return. The smaller response of total demand for larger concentration is then explained by the fact that, as λ increases, more weight is given to the demand of non-competitive traders.

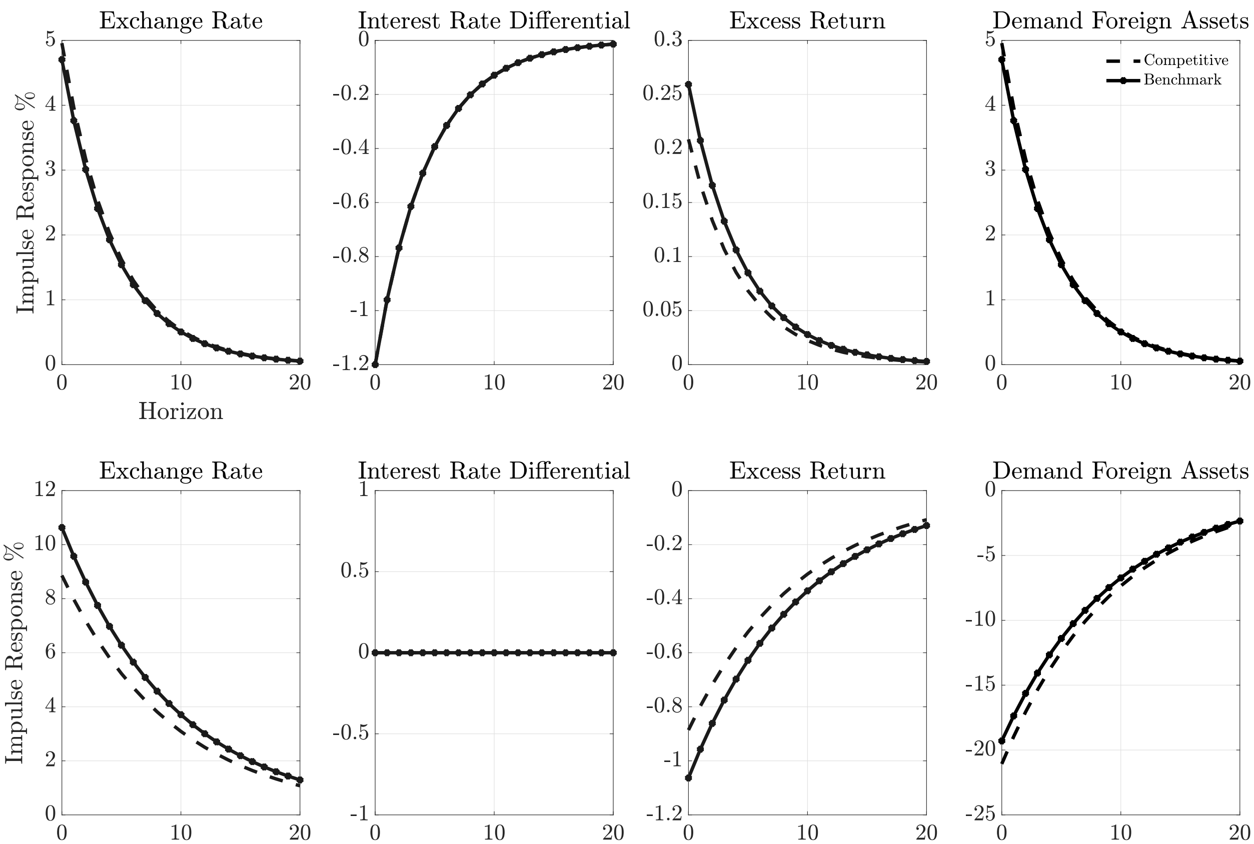

Figure 2: Impulse response to one standard deviation shock in fundamental (first row) and noise shock (second row).

to fundamentals shocks. Total demand is illustrative of the mechanism. Investors increase their holdings of foreign assets by less when they are able to exert market power, due to the presence of price impact, for a given shock to fundamentals. Consequently, a smaller impact on total demand dampens the effects on exchange rate. The amplified reaction of the excess return is again driven by the fact that the monopolist investor is, *de facto*, more risk averse and requires larger risk premia to absorb foreign assets.

Notice that the result in Proposition 1 can be generalized in terms of excess return dynamics.

Remark 1. *The presence of non-competitive traders dampens (amplifies) the effects on exchange rate when the impact on the excess return on foreign assets has the same (opposite) sign as the impact on the exchange rate.*

Conditional and unconditional variance. Our model allows us to study the implication of market power on both conditional and unconditional volatility of the exchange rate.

We simulate the model and estimate the conditional volatility in response to exogenous shock and how it evolves for different levels of market power.[26]

Proposition 2. *The volatility of exchange rate explained by noise shocks is monotonically increasing in market power.*

The left panel of figure 3 show that, conditional to exogenous shocks, the volatility of the exchange rate due to noise are increasing in market power. Under our benchmark parametrization, the variance explained by noise is $\approx$ 84% of the total variance at the impact. Assuming fully competitive markets, the noise component explain only $\approx$ 76% of the total variance in exchange rate. The reason is a direct consequence of the behavior of exchange rate informativeness. Less competitive markets decrease the information content of exchange rate, amplifying the response of the exchange rate to noise shock. Thus, the overall variance due to the noise component is increasing in market power. The conditional variance at longer horizon is increasingly explained by noise because the noise process is more persistent.[27]

The unconditional variance of the exchange rate can be derived from the law motion in equation 1:

$$\text{Var}(s) = \frac{\mu^2}{(1-\mu\rho_u)^2}\left[\frac{1}{1-\mu^2} + \frac{\rho_u^2}{1-\rho_u^2}\right]\sigma_u^2 + \frac{(1-\mu)^2}{(1-\mu\rho_x)^2 b^2}\left[\frac{1}{1-\mu^2} + \frac{\rho_x^2}{1-\rho_x^2}\right]\sigma_x^2$$

As expected, the unconditional volatility is a combination of the variances of both fundamental and noise shocks.

Result 1. *Given standard parametrizations, the unconditional variance of the exchange rate is monotonically increasing in market power.*

The right panel of figure 3 shows result 1. The rational relies again on the dynamics of exchange rate informativeness. An increase in market power decreases the information content

[26] The number of simulations is 3000 and, for each iteration, the model runs for 1000 periods with 4000 burn-in.

[27] Panel C in figure 2 in Bacchetta and Van Wincoop (2006) shows that the exchange rate change variance explained by noise is higher when information is dispersed and declines at longer horizons. Investors' heterogeneity in market power increases the share of variance explained by noise as information dispersion. However, differently from their framework, the persistence of our fundamental shock is lower, resulting in opposite long-run predictions.

of s_t, attributing relatively more weight to one of the two processes, the noise shock. Since the volatility of the noise process is higher, an higher weight to the noise component increases the unconditional volatility of the exchange rate. Appendix C shows that, theoretically, the effect of an increase in market power is not monotonic. There exists a threshold value in the ratio between the variance of the fundamental and noise shocks such that if the ratio is higher, the volatility is monotonically increasing in market power. Given our parametrization, the noise process is sufficiently more volatile than fundamentals and the condition is always satisfied, implying a monotonic relationship. The robustness of this result ultimately depends on the values of few others parameters like the persistence of the two processes and b. Our calibration is very conservative because other reasonable parametrizations (with higher ρ_x, lower ρ_u or b) would all strengthen presence of monotonicity.[28]

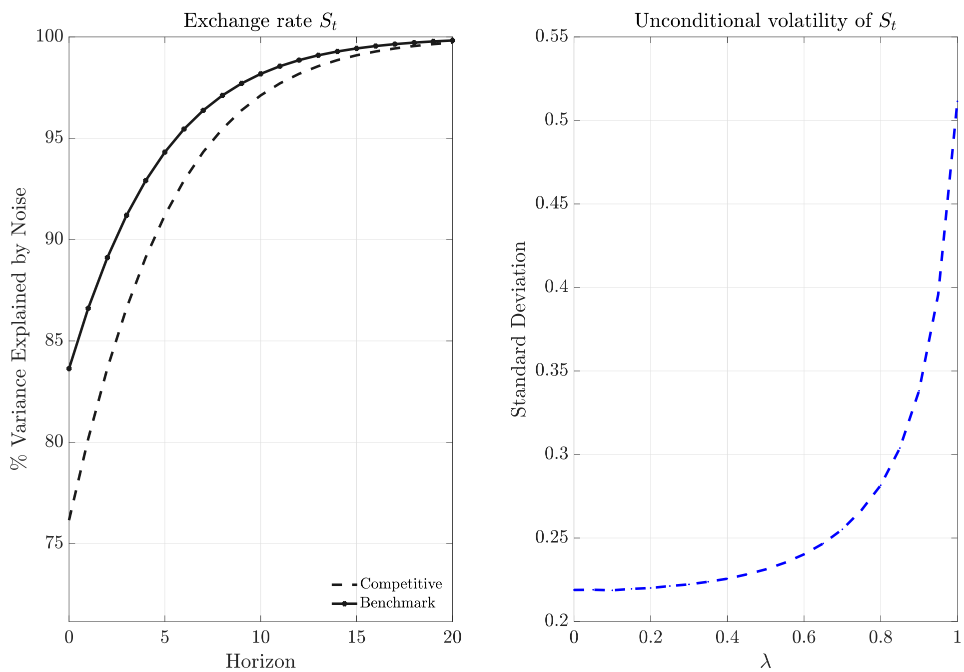

Figure 3: Conditional (left panel) and unconditional (right panel) volatility of the exchange rate.

Excess return predictability. A major puzzle in the literature is to explain why currencies tend to appreciate when interest rates are high. We show that the model is able

[28] Appendix C shows that noise shocks should be at least 45% less volatile than in our calibration in order not to have a monotonic relationship between market power and unconditional variance. σ_x should be an order of magnitute lower for higher ρ_x, lower ρ_u or b, implying unreasonable values considering that it is common belief that fundamentals are less volatile than noise.

to account for the forward premium puzzle and the implication of an increase in market power on excess return predictability.

Define the k-period ahead excess return $q_{t+k} = s_{t+k+1} - s_{t+k} - (i_{t+k} - i^*_{t+k})$ and consider the following regression:

$$q_{t+k} = \alpha + \beta_k(i_t - i^*_t) + \epsilon_{t+k} \tag{2}$$

Proposition 3. *The Fama predictibility coefficient, β_1, is negative and decreasing in market power or, in other words, the excess return is more predictable as market power increases.*

Proof. See Appendix C. ∎

The right panel of figure 4 shows the numerical relationship between the excess return predictability coefficient β_1 as a function of market power (in this case λ). The coefficient is negative for all values of λ and its magnitude is monotonically increasing in market power. To understand that, re-write the excess return from equation 1 as:

$$E_t q_{t+1} = \frac{\Phi}{B}\left(Be^{s_t} - X_t\right) \tag{3}$$

where the right-hand side represents the deviation from UIP which can interpret as the risk premium required by investors for holding a foreign asset. The risk premium depends on two components: the net supply of foreign assets and the size of non-competitive investors captured by Φ, which is increasing in λ or decreasing in N. Our model predicts that an increase in market power increases the risk premium on holding a foreign asset: less competitive markets mean lower market risk appetite because investors' positions are less elastic and, thus, an higher risk premium to absorb the net supply foreign assets. Notice that our model predicts systematic deviations from UIP, even in the presence of fully competitive markets. When markets are competitive, a risk premium is required to clear the market, absorbing the net supply $Be^{s_t} - X_t$. Changes in market power modify the premium required to absorb the imbalance in the supply of assets.

While UIP implies that the Fama coefficient is zero, empirical evidence typically finds a negative number. Our model predicts that β_1 is given by:

$$\beta_1 = -(1-\mu)\frac{1}{1-\mu\rho_u} < 0$$

The negative covariance between the expected risk premium on a foreign asset and the interest rate differential is increasing in market power. A shock in interest rate differential moves the excess return and thus the risk premium. An higher level of λ means that investors' positions are less elastic which, in turn, requires larger movements in the risk premium to make investors willing to take larger positions. Therefore, the excess return reacts more to changes in fundamentals, making it more predictable.[29] This is consistent with the variance decomposition of the excess return shown in the bottom left panel of figure 3: higher shares of the variance in excess return are explained by fundamental when market concentration increases.

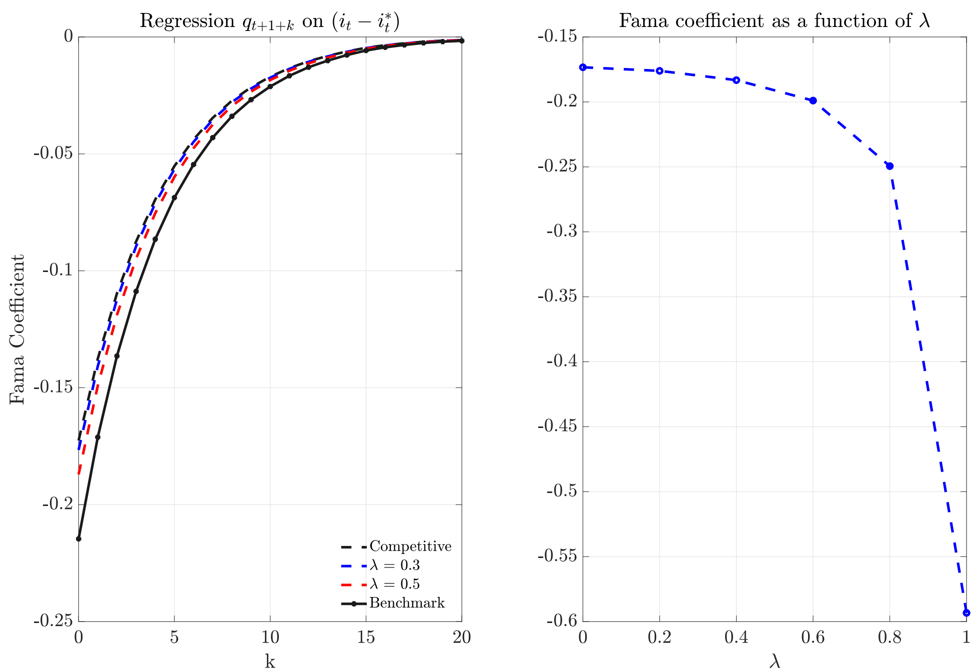

Figure 4: β_k (left) and excess return predictability (right).

Appendix C generalizes the result in Proposition 3 showing that β_k is monotonically increasing in k and approaching zero for $k = \infty$, as shown in the left panel of figure 4. This is not consistent with the predictability reversal puzzle documented in Bacchetta and Van Wincoop (2010) and Engel (2016), which refers to the fact that there is a reversal in the sign of expected excess returns at longer horizons. We are not surprised of this because market power works through an amplification mechanism that does not entail any adjustment

[29]Interestingly, β_1 is equal to zero if the supply of asset is constant when denominated in domestic currency, that is, B is not multiplied by e^{s_t}. In this particular case, the excess return depends only on the noise component X_t, which is orthogonal to fundamental shocks. Therefore, β_1 is equal to zero even if there are systematic deviations in UIP. In other words, risk premium is still positive (UIP does not hold) but it is not predictable ($\beta_1 = 0$).

friction.[30]

Exchange rate disconnect. A main concern in the literature is the poor explanatory power of standard theories of nominal exchange rate. Meese and Rogoff (1983) first documented that exchange rates are disconnected from fundamentals, at least in the short run. We provide a rationale for this puzzle levereging on investors' heterogeneity, in the spirit of Bacchetta and Van Wincoop (2006), and focusing on the role of financial markets, as in Gabaix and Maggiori (2015).[31] The standard measure used to evaluate the puzzle is the R-squared of the following regression:

$$s_{t+1+k} - s_t = \alpha + \beta_k (i_t - i_t^{\star}) + \varepsilon_{t+k+1} \tag{4}$$

Figure 5 shows the R-squared at different horizons (k up to 20) and for different levels of market power. Independently of the level of market concentration, the model predicts that the puzzle is less acute for long-run exchange rate movements, consistently with the literature. A competitive market predicts that current fundamentals explains from 4% to 12%

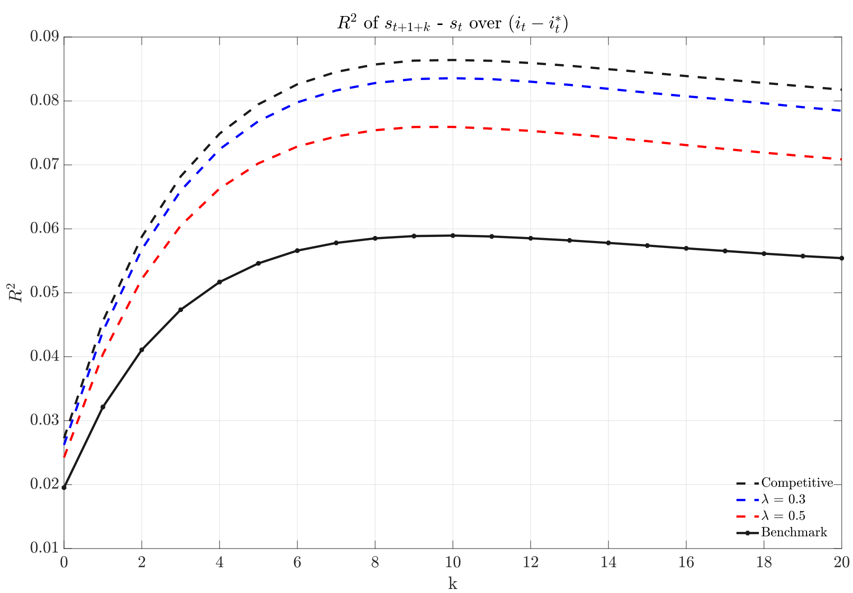

Figure 5: Exchange rate disconnect at different horizons.

[30] Bacchetta and Van Wincoop (2010) and Bacchetta and Van Wincoop (2019) impose infrequent portfolio ajdustment frictions to match the reversal and other puzzles. Our model is consistent with their frictionless case (or "actively managed portfolio" case), see figure 3 in Bacchetta and Van Wincoop (2010).

[31] Differently form the latter, which introduces financial variables to predict exchange rates, we study the underlying market structure to explain the poor performance of standard macro variables.

of the fluctuations in the exchange rate, depending on the horizon. Instead, our benchmark calibration implies that current fundamentals explain from 2 to $\approx 5\%$ of the fluctuations in the exchange rate. Less competitive financial markets are able to rationalize a low explanatory power because noise shocks are amplified by the presence of non-competitive investors. Exchange rate informativeness is decreasing in market power and, thus, exchange rate fluctuations are overwhelmingly explained by the noise component.

4 Testable prediction

We use the disaggregate information provided by the New York Fed Biannual FXC Report about our measure of market power for each currency pair. In particular, the measure of market power we are using refers to the share of total transactions intermediated by the top first quintile of dealers, that is, λ. We consider the same set of currencies used to calibrate the model, excluding Euro, British Pound and Japanese Yen as the small open economy assumption could not hold for those countries.[32] We use the cross section of λs available from the April 2019 NY Fed Report to test a set of predictions obtained from our model. We look at four main testable predictions: (i) unconditional variance of the currency; (ii) excess return predictability; (iii) exchange rate disconnect; (iv) trading volume-concentration relationship. The results are reported in figure 6.

Our model predicts that the (unconditional) exchange rate volatility is increasing in market concentration (result 1). The top-left panel in figure 6 confirms that an increase in market concentration is strongly associated with an increase in currency volatility.

The theory delivers clear testable implications about the excess predictability and the exchange rate disconnect puzzles. The top-right panel and bottom-left in figure 6 and **??** show the cross section relationship between the fama coefficient β_1 and the R^2 obtained from the exchange rate disconnect regression, respectively. Both correlation coefficients are negative, supporting that currency pairs associated with less competitive markets are more disconnnect from fundamentals and exhibit a more predictable excess return.

Finally, a very general prediction of our model is that less competitive markets discourage participation. Standard market microstructure textbooks show that frictions like market power can decrease the volume of trading (see Foucault et al. (2013)). This happens also in

[32]We consider the same time period, from 1993 to 2019, at daily frequency. Implications are tested at daily frequency as we believe that the influence of the underlying market structure on the exchange rate shows at higher frequency.

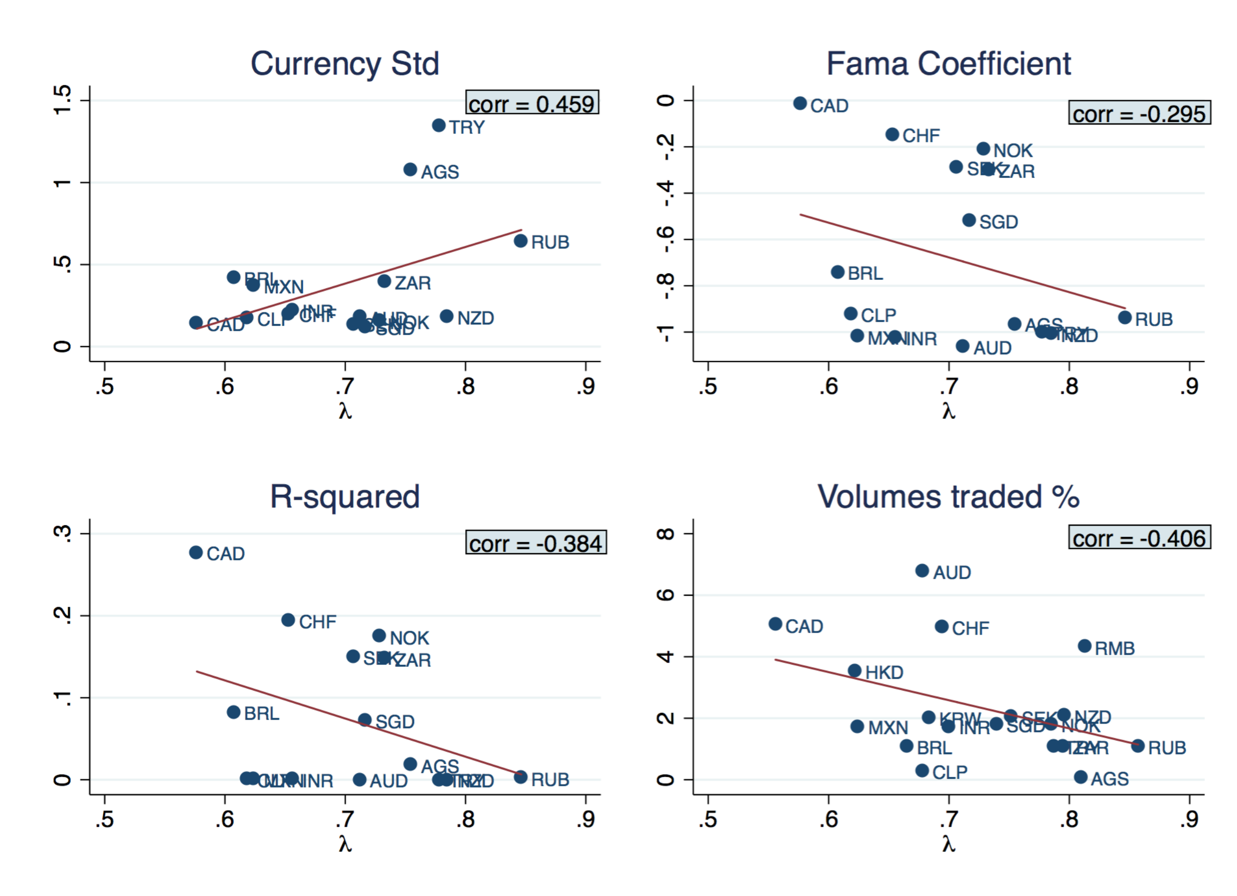

Figure 6: Testable predictions: market power correlation

our setting as shown by the optimal portfolio allocation, $b_t^{S,i}$, which decreases for higher levels of market power. Therefore, we expect to see that currencies with higher concentration are also the ones with lower volumes traded in the market. We consider the percentage share of trading volume for each currency pair over the total trading volume in the NY OTC Forex market. Consistently with our model, the bottom-right panel in figure 6 shows that currencies that are relatively traded less are also those where a larger share of transactions is intermediated by top dealers.

A main concern is that the unconditional correlations shown in figure 6 capture spurious patterns between our measure of market concentration and the outcome variables due to market thinness. Markets that are characterized by a small number of investors (i.e., thin markets) could present low trading volume, high volatility and low liquidity. To mitigate potential endogeneity bias, we estimate the effect of market power on each outcome variable controlling for an exogenous measure of market thinness. We borrow from the international

	Sd	Volumes	Fama Coeff	R2
λ	2.971	-11.62	-1.028	-0.311
	(1.346)	(5.840)	(1.568)	(0.356)
Distance	-0.0276	0.110	-0.0176	-0.00576
	(0.0240)	(0.118)	(0.0279)	(0.00634)
constant	-1.443	9.788	0.203	0.345
	(0.864)	(3.955)	(1.006)	(0.228)
N	15	17	15	15

Standard errors in parentheses

Table 1: OLS regression: $y_i = \alpha + \beta\lambda_i + \gamma \text{distance}_i + \epsilon_i$ where y_i is the outcome variables (sd, volumes, fama coefficients and r-square).

trade literature in using distance (from the US) as a proxy for market size. We report the coefficients in Table 1. Even controlling for market thinness, our predictions are empirically supported. The effect of market concentration on each outcome variable has the correct sign, in line with our theory and with the correlations previously reported.

5 Welfare and Policy Implications

Since we explicitly model preferences of all market participants, our model can study the welfare implications of a change in the underlying market organization. Notice that the tractability of our model comes at the expense of a comprehensive analysis of the welfare effects, as discussed in the policy implication section.

Welfare. Mean-variance preferences imply that the indirect utility of investor j is quadratic its portfolio positions b^j and ultimately depends on the variance of the excess return q_{t+1}.[33]

Let define the excess return $q_{t+1} = E_t s_{t+1} - s_t - \Delta i_t$. The expected welfare for a competitive trader is given by:

$$E(u^C) = \omega + E\left(\frac{q_{t+1}^2}{2\rho\sigma_t^2}\right)$$

while the expected welfare for a strategic investor i is:

$$E(u^{S,i}) = \omega + E\left(\frac{q_{t+1}^2}{2\rho\sigma_t^2}\right)\frac{2\alpha_i - 1}{\alpha_i^2} \qquad \alpha_i = 1 + \frac{\lambda_i}{1 + B\rho\sigma_t^2 - \lambda}$$

[33] Precisely on the squared of the excess return, since q_{t+1} has zero mean.

The presence of imperfectly competitive markets unambiguously increases competitive traders' welfare. The less competitive the market, the lower the average risk appetite of market participants, the higher the excess return. Higher q_{t+1} benefits competitive traders. Strategic investors' welfare can be written as the product of two terms. One is the average welfare of a competitive trader, which is decreasing in the level of competition. The other is a term that depends on the underlying market structure (λ_i λ and N). The latter adjusts investor's welfare accounting for the presence of price impact and higher perceived risk aversion. For the same level of excess return, strategic investors have a lower utility. Notice that the second term in increasing in the level of competition, implying an overall non monotonic relationship between $E(u^{S,i})$ and market power. Strategic investors' welfare decreases when markets are highly non competitive because, even if the excess return increases, the price impact is so high that the excess return is discounted more, as in Kacperczyk et al. (2018). The left panel of figure 7 shows the hump shaped dynamics of the welfare of a strategic investor when competition decreases.[34]

Given individual welfare, we define the average aggregate welfare (in a symmetric oligopoly):

$$E(u) = \sum_i \lambda_i E(u^{S,i}) + (1-\lambda)E(u^C) = \omega + E\left(\frac{q_{t+1}^2}{2\rho\sigma_t^2}\right)\left[1 - \lambda\left(\frac{\alpha-1}{\alpha}\right)^2\right]$$

. where $\alpha = 1 + \lambda N^{-1}(1-\lambda+B\rho\sigma_t^2)^{-1}$.[35]

We use our framework to explore the welfare consequences of the consolidation and rise in concentration experienced in the last three decades in the foreign exchange rate markets, as extensively documented by the BIS Triennial Survey of Foreign Exchange Markets.[36]

Focusing on the US foreign exchange rate market, the number of dealers accounting for (at least) 75% of total volume in 1990 was around 20.[37] Today, the same share of total transactions is intermediated by just four banks, as we calibrated N from the NY Fed FX report (2019). The right panel of figure shows that aggregate and individual welfare has increased since the 90s. Aggregate welfare has increased by 31% and competitive investors

[34] In this case we fix a certain value for λ and change N. The same dynamics qualitatively hold in the opposite case.

[35] See Appendix B for the derivation of all equations.

[36] These trends are not particularly surprising given that all players in the FX are banks and the consolidation forces that the whole financial sector experienced in those years, as reported by Corbae and D'Erasmo (2020) and others.

[37] BIS Triennial Survey of Foreign Exchange Markets, 1990.

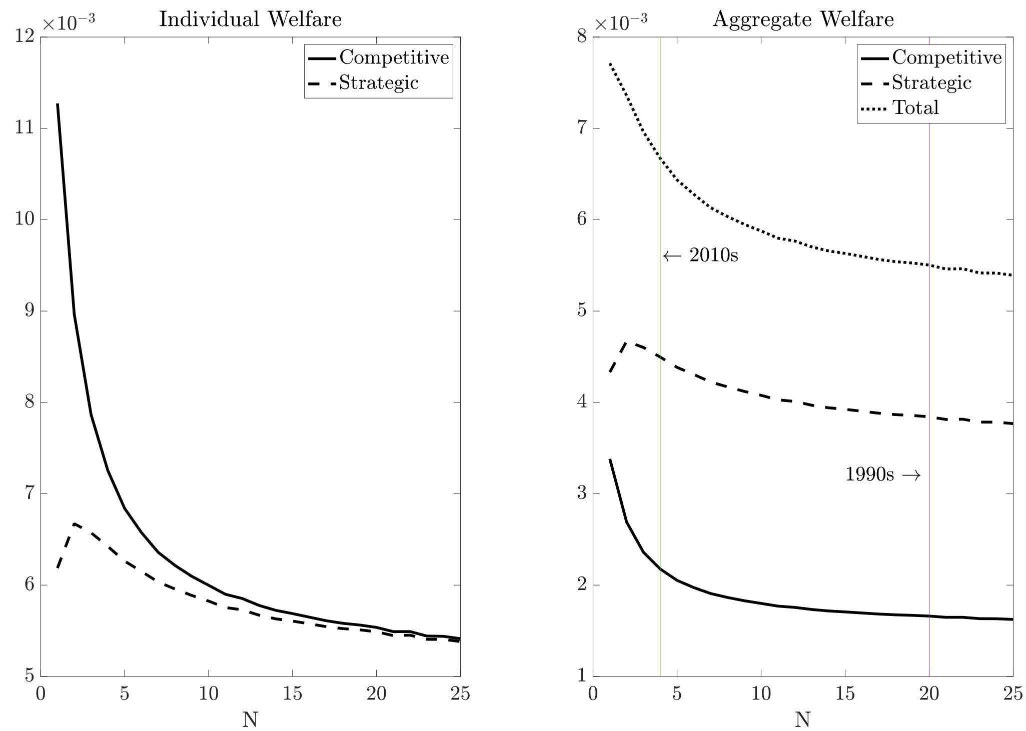

Figure 7: Individual welfare (left) and welfare dynamics over time (right).

has benefited relatively more (competitive investors' welfare share over aggregate welfare has increased from 30% to 33%).

Policy implications. Is further consolidation in the FX market desirable? Through the lens of our model, aggregate welfare could decrease depending on whether the increase in market power takes the form of a fall in the number of strategic investors ($N \downarrow$) or a rise in the size of non-competitive traders ($\lambda \uparrow$). In the former case, aggregate welfare would increase, as shown in the right panel of figure 7. However, distributional concerns would arise, as strategic investors would be worsen off. In the latter case, it can be shown that aggregate welfare could decrease because an increase in λ would give more weight to strategic investors' welfare, which is declining for extremely high levels of market power. Aggregate welfare could become hump shaped when λ increases. Therefore, given our parametrization, consolidation in the FX market has already expressed all the potential benefits in the last thirty years.

More generally, it is hard to draw unambiguous policy implications also because our measure of welfare refers only to investors, disregarding that other important channels could affect the economy. In particular, a less competitive FX market increases exchange rate volatility, which could have welfare consequences through price adjustment, consumption and FDI volatility, uncertainty, and other forces highlighted by the literature (see Singleton (1987) and subsequent). Our stylized model does not account for all these additional GE forces because they are outside of the scope of this paper.

6 Conclusion

The foreign exchange market is opaque and highly concentrated, reasons due to which perfectly competitive financial markets are unrealistic. We have explored the implications of the presence of non-competitive traders for exchange rate dynamics. We have shown that market concentration dampens or amplifies the response of the exchange rate, depending on the shock, and increases exchange rate volatility. Moreover, it can help rationalizing two puzzles related to the relationship between exchange rates and interest rates, namely excess return predictability and exchange rate disconnect. Welfare analysis shows that investors' welfare has increased by 30% in the last three decades due to the consolidation dynamics in the financial sector.

The analysis has focused on the role of non-competitive markets in the determination of exchange rate dynamics, which was underexplored. The model is stylized in order to derive basic insights and analytical results. At the same time, it delivers a set of predictions that are confirmed by a cross currency analysis, proving the relevance of our theory. Our model is tractable and can be generalized to address richer settings. Future works should be directed to a much rigorous documentation of the effects of market power on the dynamics of exchange rate, the interaction between non-competitive markets and more complex information structures, a more accurate welfare analysis in order to derive sound policy implications.

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Appendix A

Mapping market power from data to model

Data sources on the foreign exchange market are hardly available or comprehensive, reflecting the opaque and decentralized structure that characterized the market. Since 1990, the Bank of International Settlements collects and publishes information on turnover, instruments used, market participants etc..., providing one of few sources of data at global level. The BIS Triennial Surveys provide a clear picture of the high concentration in the foreign exchange market, both geographically and within market.

The Triennial Survey complements more frequent regional surveys conducted by national foreign exchange committees like the New York Fed Biannual FXC Report, which provides similar information at higher frequency (biannual) since 2005 for the US market (see figure 8).

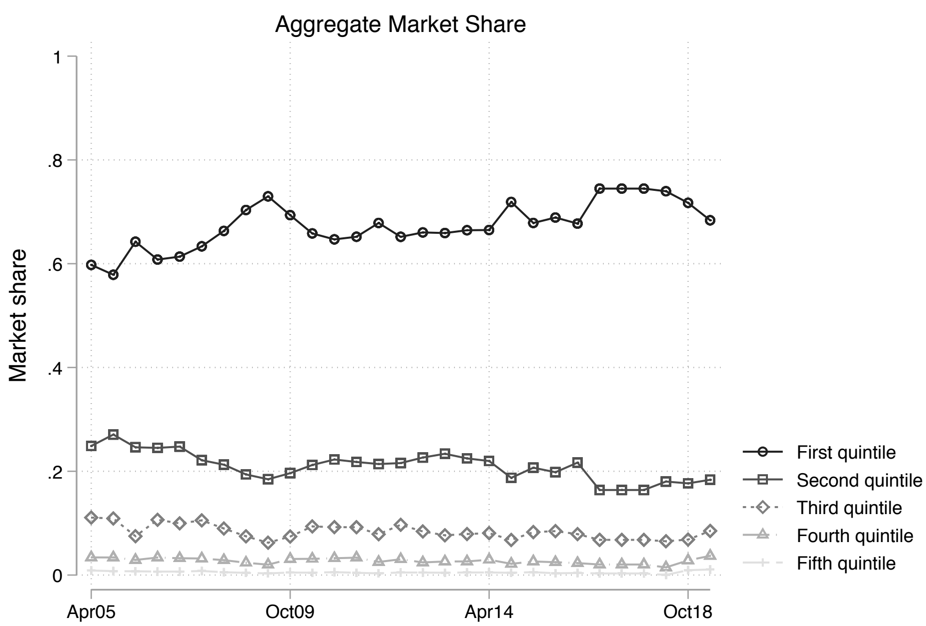

Figure 8: Market concentration in the NY OTC Forex market (Source: NY Fed Biannual FXC report).

We choose to calibrate our model focusing on the US market because i) we believe it closely reflects the overall global dynamics in the foreign exchange rate market since it represents the second market worldwide and ii) data are more granular and allow cross-sectional analysis. In our model we use the market share of the top dealers to calibrate the size of the non-competitive segment, λ. From figure 8, a reasonable value is 70%, which is the average share of the first quintile of dealers. The number of large investors that have strategic behavior in the foreign market, N, is calibrated to the number of players falling into the first quintile of the distribution. The average number of top traders over the time horizon considered is $4-5$.

Importantly, the NY Fed also provides information for each currency pairs (USD againt other currencies). In particular, for each currency pair, the report provides information on the market share for each quintile of the distribution. In other words, we have a cross-currency measure of λ. The number of institutions in the first quintile does not change across currency (N is constant). This variation allows us to validate the implications of the model cross currencies because it reflects variation in the level of market power.

When using foreign exchange rate data, it is important to keep in mind that FX is a decentralized market based on a OTC structure in which dealers play a pivotal role. The data available generally refer to the market-making desks of large financial players which have to report their currency activities (frequently called "reporting dealers"). On the other hand, our model is phrased in terms of investors and this could cause some inconsistency in mapping the data into the model. We argue that concentration in dealership reflects (to some extent) market power among investors because i) dealers often take substantial open positions and undertake their own investment activities, as highlighted by Lyons (1997) or Lyons et al. (2001); ii) large investors can have the endogenous incentives to enter in OTC markets and play the role of intermediaries (see Atkeson et al. (2015) and Dugast et al. (2019)); iii) the very same top dealers in the Forex market are also among the largest global investment banks (Citigroup, Barclays, JP Morgan, UBS, HSBC, Deutsche Bank, etc...).[38]

Finally, we evaluate the relevance of our proxy for market power, testing a standard theoretical relationship between market power and liquidity. An extensive literature in market microstructure associates the presence of non-competitive traders to higher bid-ask spreads, which can be considered as a markup measure.[39] We collect the daily bid-ask spread from Bloomberg for a set of 15 currencies for the 2019 calendar year.[40] Figure 9 shows the ex-

[38]In the FX in 2019, among the top 50 global dealers, only six were non-bank institutions (Euromoney, 2019).

[39]For instance, standard argument in a market microstructure textbook is that opaqueness and information asymmetry can lessen competition among dealers and thus reduce market liquidity, increasing the bid-ask spread (Foucault et al. (2013), Chapter 8).

[40]Refer to footnote 17 for details on the set of curriencies considered.

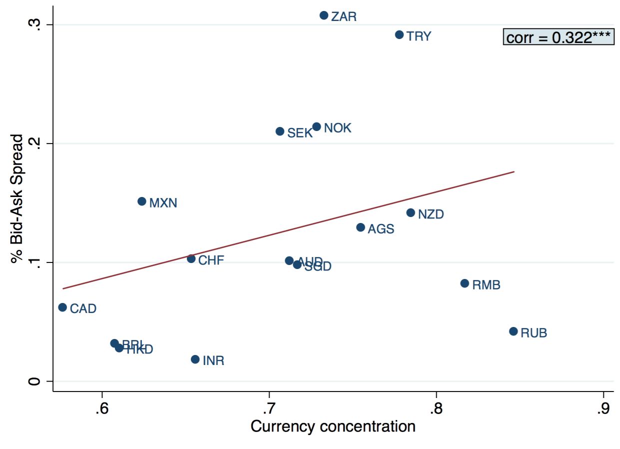

Figure 9: Cross-currency relationship between λ and bid-ask spread.

istence of a positive relationship between our measure of market power and the average daily bid-ask spread on the cross section of currencies. Therefore, figure 9 is empirically supporting our conjecture that concentration in the dealership market can be considerd as a (approximate) measure of the size of non-competitive traders in the market.

Appendix B

Derivation Demand Functions

Each investor j solves the following problem:

$$\max_{b_t^j} E_t(w_{t+1}^j) - \frac{\rho}{2} Var_t(w_{t+1}^j)$$

$$\text{s.t. } w_{t+1}^j = (\omega - b_t^j)i_t + (i_t^\star + s_{t+1} - s_t)b_t^j$$

Taking the derivative of the objective function w.r.t. b_t^j, we find:

$$b_t^j = \frac{E_t(s_{t+1}) - s_t + i_t^\star - i_t}{\rho Var_t(s_{t+1}) + \frac{\partial s_t}{\partial b_t^j}}$$

for each investor $j \in [0, 1]$. Only a fraction $1 - \lambda$ of investors are atomic; λ is divided into N non-competitive segmenst of size λ_i (with $i \in \{1, N\}$). Therefore:

$$b_t^C = \frac{E_t(s_{t+1}) - s_t + i_t^\star - i_t}{\rho Var_t(s_{t+1})}$$

$$b_t^{S,i} = \frac{E_t(s_{t+1}) - s_t + i_t^\star - i_t}{\rho Var_t(s_{t+1}) + \frac{\partial s_t}{\partial b_t^{S,i}}}$$

where the $\frac{\partial s_t}{\partial b_t^j}$ is 0 for the competitive investors and is positive for the non-competitive.

Derivation Price Impact

Consider the market clearing condition:

$$(1-\lambda)b_t^C + \sum_i^N \lambda_i b_t^{S,i} + (x_t + \bar{x})\bar{W} = B(1+s_t)$$

where

$$b_t^C = \frac{E_t(s_{t+1}) - s_t + i_t^\star - i_t}{\rho Var_t(s_{t+1})}$$

and

$$b_t^{S,i} = \frac{E_t(s_{t+1}) - s_t + i_t^\star - i_t}{\rho Var_t(s_{t+1}) + \frac{\partial s_t}{\partial b_t^{S,i}}}$$

From standard Implicit function theorem, we can write:

$$(1-\lambda)\frac{\partial b_t^C}{\partial s_t}\frac{\partial s_t}{\partial b_t^{S,i}} + \lambda_i = B\frac{\partial s_t}{\partial b_t^{S,i}}$$

Thus:

$$\frac{\partial s_t}{\partial b_t^{S,i}} = \frac{\lambda_i}{B - (1-\lambda)\frac{\partial b_t^C}{\partial s_t}} \qquad \text{with } \frac{\partial b_t^C}{\partial s_t} \equiv -\frac{1}{\rho Var_t(s_{t+1})}$$

Therefore:

$$\frac{\partial s_t}{\partial b_t^{S,i}} = \frac{\lambda_i \rho Var_t(s_{t+1})}{B\rho Var_t(s_{t+1}) + (1-\lambda)} \equiv \frac{1}{N}\frac{\lambda\rho\sigma_t^2}{B\rho\sigma_t^2 + (1-\lambda)} > 0$$

where the last equality holds in case of a symmetric oligopoly. The price impact is positive for $\forall(B, \lambda, N, \lambda_i, \rho, \sigma)$.

Welfare

We can derive the expected welfare of each investor plugging the demand schedule into the budget constrain and the level of wealth into investors' preferences.

Consider a competitive investor with demand b_t^C defined previously. End-life wealth w_t^C is therefore:

$$w_t^C = \omega + q_{t+1}\frac{s_{t+1} - s_t + i_t^\star - i_t}{\rho\sigma_t^2}$$

where $q_{t+1} = E_t(s_{t+1}) - s_t + i_t^\star - i_t$.

Given mean-variance prefernces and assuming rational expectations, utility of an atomic investors is:

$$\begin{aligned}
u_t^C =& E_t(w_t^C) - \frac{\rho}{2}Var_t(w_t^C)\\
&\omega + E_t\left(\frac{s_{t+1} - s_t + i_t^\star - i_t}{\rho\sigma_t^2}q_{t+1}\right) - \frac{\rho}{2}Var_t\left(\frac{s_{t+1} - s_t + i_t^\star - i_t}{\rho\sigma_t^2}q_{t+1}\right)\\
&\omega + \frac{q_{t+1}^2}{\rho\sigma_t^2} + E_t(\varepsilon_q) - \frac{\rho}{2}\frac{q_{t+1}^2}{(\rho\sigma_t^2)^2}Var_t(s_{t+1} - s_t + i_t^\star - i_t)\\
&\omega + \frac{q_{t+1}^2}{2\rho\sigma_t^2}
\end{aligned}$$

where $E_t(\varepsilon_q) = 0$ under rational expectation and $Var(s_{t+1} - s_t + i_t^\star - i_t) = \sigma_t^2$ by the definition.[41]

Therefore the expected (average) welfare for an atomistic investor is simply the unconditional expectation of u_t^C, which depends ultimatly on the variance of the excess return.

Similarly, we can derive the welfare for a strategic investor.

$$\begin{aligned}
u_t^{S,i} =& E_t(w_t^{S,i}) - \frac{\rho}{2}Var_t(w_t^{S,i}) \qquad \text{where } w_t^{S,i} = \omega + q_{t+1}\frac{s_{t+1} - s_t + i_t^\star - i_t}{\rho\sigma_t^2 + \frac{\partial s_t}{\partial b_t^{S,i}}}\\
&\omega + \frac{q_{t+1}^2}{\alpha_i\rho\sigma_t^2} - \frac{\rho}{2}\frac{q_{t+1}^2}{(\alpha_i\rho\sigma_t^2)^2}\sigma_t^2 \qquad \text{where } \alpha_i = 1 + \frac{\lambda_i}{B\rho\sigma_t^2 + 1 - \lambda}\\
&\omega + \frac{q_{t+1}^2}{\rho\sigma_t^2}\left(\frac{1}{\alpha_i} - \frac{1}{2\alpha_i^2}\right)\\
&\omega + \frac{q_{t+1}^2}{\rho\sigma_t^2}\left(\frac{2\alpha_i - 1}{\alpha_i^2}\right)
\end{aligned}$$

Finally, (average) total welfare is the aggregation of individual welfare weighted by the

[41] Under rational expectation $s_{t+1} = E_t s_t + \varepsilon_q$.

size of each segment, as follow:

$$
\begin{aligned}
E(u) =&(1-\lambda)E(u^C) + \sum_i^N \lambda_i E(u_t^{S,i}) \\
&\omega + \left(\frac{q_{t+1}^2}{\rho\sigma_t^2}\right)\left[1-\lambda+\sum_i^N \lambda_i \left(\frac{2\alpha_i - 1}{\alpha_i^2}\right)\right] \\
&\omega + \left(\frac{q_{t+1}^2}{\rho\sigma_t^2}\right)\left[1-\lambda \left(\frac{\alpha - 1}{\alpha}\right)^2\right]
\end{aligned}
$$

where the last equation holds in case of a symmetric oligopoly (with $\alpha = 1 + \frac{\lambda}{N(1-\lambda+B\rho\sigma^2)}$).

Appendix C

Lemma 1

In international portfolio choice models, the value of the supply of foreign assets in domestic currency (indirectly) depends on the value of the exchange rate when foreign assets are denominated in foreign currency. Differently from standard models of strategic trading (as Kyle (1989)), non-competitive traders internalize not only their price effect on the quantity demanded but also on the quantity (value) supplied. Compared to closed economy models or cases in which foreign assets are denominated in domestic currency, the presence of this "supply effect" implies a weakly lower price impact.

Let pi^F and pi^D be the price impact on a foreign and a domestic asset, respectively.

$$
pi^F \equiv \frac{\partial s_t}{\partial b_t^{S,i}} = \frac{\lambda_i \rho \sigma_t^2}{B\rho\sigma_t^2 + (1-\lambda)} \qquad pi^D \equiv \frac{\partial p_t}{\partial b_t^{S,i}} = \frac{\lambda_i \rho \sigma_t^2}{(1-\lambda)}
$$

where p_t is the price of the domestic asset. It is easy to show that $pi^F \leq pi^D \quad \forall (B, \rho, \sigma_t^2, \lambda_i, \lambda)$. The intuition is fairly simple. The increase in the price of a currency (foreign currency appreciates) rises the nominal value of the supply of foreign assets when denominated in domestic currency. The overall effect of tradings on the exchange rate is lower due to the presence of a revaluation effect.

Proposition 1

According to proposition 1, an increase in market power amplifies (dampens) the response of the exchange rate to noise (fundamental) shock.

Proof. Consider the law motion of the exchange rate, equation 1. s_t can be rewritten as a forward looking sum of fundamentals and noises as follow:

$$s_t = -\mu \sum_{k=0}^{\infty} \mu^k \left(\Delta i_{t+k}\right) + \frac{1-\mu}{b} \sum_{k=0}^{\infty} \mu^k \left(x_{t+k}\right)$$

Therefore, the impulse response to a unit shock in noise and fundamental that raises the exchange rate at impact are:

$$\text{IRF}\ (s_{t+j}, j=0) = \begin{cases} \dfrac{\mu}{1-\mu\rho_u}, & \text{for } \varepsilon_u = -1 \\ \\ \dfrac{(1-\mu)}{(1-\mu\rho_x)b}, & \text{for } \varepsilon_x = 1 \end{cases}$$

Taking the derivative w.r.t. μ, we find:

$$\frac{\partial \text{IRF}(s_{t+j}, j=0)}{\partial \mu} = \begin{cases} \dfrac{1}{(1-\mu\rho_u)^2} > 0 \\ \\ -\dfrac{(1-\rho_x)}{(1-\mu\rho_x)^2 b^2} < 0 \end{cases}$$

Since μ is decreasing (increasing) function of λ (N), the response of the exchange rate to a unit shock in fundamental is dampened while noise shock are amplified as λ increases (N decreases). ■

Result 1

According to result 1, the unconditional volatility in exchange rate is non-monotonic in market power.

Proof. Consider the law of motion of the exchange rate, equation 1, substituting the process for fundamental and noise in s_t we write:

$$s_t = -\mu \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \mu^k \rho^j \varepsilon^u_{t+k-j} + \frac{1-\mu}{b} \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \mu^k \rho_x^j \varepsilon^x_{t+k-j}$$

After some algebra, s_t can be written as summation of its backward and forward components:

$$s_t = -\frac{\mu}{1-\mu\rho_u} \left[\sum_{k=0}^{\infty} \mu^k \varepsilon^u_{t+k} + \sum_{k=1}^{\infty} \rho_u^k \varepsilon^u_{t-k} \right] + \frac{1-\mu}{b(1-\mu\rho_u)} \left[\sum_{k=0}^{\infty} \mu^k \varepsilon^x_{t+k} + \sum_{k=1}^{\infty} \rho_x^k \varepsilon^x_{t-k} \right]$$

and derive the unconditional variance of the exchange rate:

$$\mathrm{Var}(s) = \frac{\mu^2 \sigma_u^2}{(1-\mu\rho_u)^2} \left[\frac{1}{1-\mu^2} + \frac{\rho_u^2}{1-\rho_u^2} \right] + \frac{(1-\mu)^2 \sigma_x^2}{(1-\mu\rho_x)^2 b^2} \left[\frac{1}{1-\mu^2} + \frac{\rho_x^2}{1-\rho_x^2} \right]$$

which tells us that the unconditional volatility of the exchange rate is a combination of the variances of fundamental and noise shocks.

Taking the derivative of $\mathrm{Var}(s)$ w.r.t. μ, we find:

$$\begin{aligned} \frac{\partial \mathrm{Var}(s)}{\partial \mu} = & \frac{\mu \sigma_u^2}{(1-\mu\rho_u)^3} \left[\frac{1}{1-\mu^2} + \frac{\rho_u^2}{1-\rho_u^2} \right] + \frac{\mu^3 \sigma_u^2}{(1-\mu\rho_u)^2 (1-\mu^2)^2} - \\ & \frac{(1-\mu)(1-\rho_x)\sigma_x^2}{(1-\mu\rho_x)^3 b^2} \left[\frac{1}{1-\mu^2} + \frac{\rho_x^2}{1-\rho_x^2} \right] + \frac{\mu(1-\mu)^2 \sigma_x^2}{(1-\mu\rho_x)^2 (1-\mu^2)^2 b^2} \end{aligned}$$

Therefore, the unconditional volatility of the exchange rate is increasing in λ iff:

$$\begin{aligned} & \frac{(1+\mu\rho_x)\sigma_x^2}{(1-\mu\rho_x)^2 (1+\mu)(1+\rho_x) b^2} - \frac{\mu \sigma_x^2}{(1-\mu\rho_x)^2 (1+\mu)^2 b^2} > \\ & \frac{\mu \sigma_u^2}{(1-\mu\rho_u)^2} \frac{(1+\mu\rho_u)}{(1-\mu^2)(1-\rho_u^2)} + \frac{\mu^3 \sigma_u^2}{(1-\mu\rho_u)^2 (1-\mu^2)^2} \end{aligned}$$

which can finally re-written as the condition:

$$\frac{\mathrm{Var}(x)}{\mathrm{Var}(\Delta i)} \frac{1}{b^2} > \left[\frac{(1+\mu^2\rho_x)(1-\rho_x)}{\mu(1+\mu\rho_u)(1-\mu^2) + \mu^3(1-\rho_u^2)} \frac{(1-\mu\rho_u)^2 (1-\mu)^2}{(1-\mu\rho_x)^2} \right]^{-1}$$

which intuitively happens whenever the variance of the noise shock is sufficiently high.

■

We believe that the non monotonic case is not relevant given our parametrization or other reasonable/standard values of the calibration. The validity of result 1 depends upon the fact that the noise process is more volatile and more persistent than the fundamental one. Let define $\underline{\sigma_x}$ as the minimum value of the volatility of the noise process such that the relationship between market power and exchange rate variance is not monotone anymore. In our benchmark case, $\underline{\sigma_x} \approx 0.03$, 65% lower than our benchmark value of σ_x. Figure 10 shows $\underline{\sigma_x}$ for different combinations of N and λ.

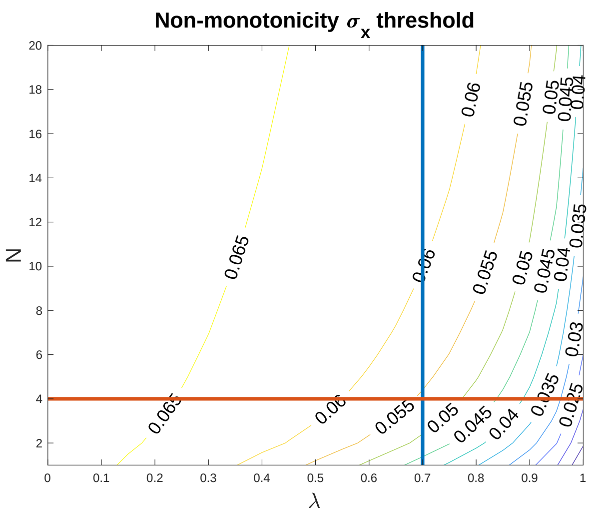

Figure 10: $\underline{\sigma_x}$ for different combinations of N and λ.

For other values of λ or N, $\underline{\sigma_x}$ is at least 50% lower than the implied value of σ_x by figure 1. In a competitive market, $\sigma_x \approx 0.12$ and monotonicity does not arise if $\underline{\sigma_x} < 0.066$. In highly non-competitive markets, $\sigma_x \approx 0.06$ and monotonicity does not arise if $\underline{\sigma_x} < 0.015$.

Moreover, notice that the threshold value depends on ρ_x, ρ_u and b. Result 1 is robust because our calibration is particularly conservative. Only more persistent noise processes or less persistent fundamental processes would be consistent with standard calibrations; similarly, only higher values of home bias (lower b) would be acceptable. Higher values of ρ_x, lower values of ρ_u and lower b all decrease the threshold, relaxing the condition for monotonicity.

Proposition 3

According to proposition 3, excess returns are more predictable as market power increases.

Proof. Consider the law motion of the exchange rate, equation 1:

$$s_t = \mu \left(E_t \left(s_{t+1} \right) + i_t^\star - i_t \right) + (1-\mu) \frac{\bar{x}}{b} + (1-\mu) \frac{1}{b} x_t$$

where only the first term depends on fundamentals.

The j-period change in currency price can be obtained from the law motion for the exchange rate as follows:

$$\Delta s_{t+j} = -\mu \sum_{k=0}^{\infty} \mu^k \left(\Delta i_{t+j+k} - \Delta i_{t+k} \right)$$

With Δs_{t+j} in hand, we can calculate:

$$\begin{aligned}
\beta_1 = \frac{\mathrm{Cov}\left(\Delta s_{t+1} - \Delta i_t ; \Delta i_t\right)}{\mathrm{Var}(\Delta i_t)} &= \left[\mathrm{Cov} \left(-\mu \sum_{k=0}^{\infty} \mu^k \left(\Delta i_{t+k+1} - \Delta i_{t+k} \right) ; \Delta i_t \right) - \mathrm{Var}\left(\Delta i_t\right) \right] / \mathrm{Var}(\Delta i_t) \\
&= \left[-\mu \sum_{k=0}^{\infty} \mu^k \mathrm{Cov}\left(\Delta i_{t+k+1} - \Delta i_{t+k} ; \Delta i_t \right) - \mathrm{Var}\left(\Delta i_t\right) \right] / \mathrm{Var}(\Delta i_t) \\
&= \left[-\mu \sum_{k=0}^{\infty} \mu^k \rho_u^k (\rho_u - 1) \mathrm{Var}(\Delta i_t) - \mathrm{Var}(\Delta i_t) \right] / \mathrm{Var}(\Delta i_t) \\
&= -(1-\mu) \frac{1}{1 - \mu \rho_u} < 0
\end{aligned}$$

which is negative for each value of μ and increasing (decreasing) in μ (in market power). ■

Notice that predictability reversal does not arise in our model, differently from Bacchetta and Van Wincoop (2010) and Engel (2016). Formally define the j-period ahead excess return $q_{t+j} = s_{t+j+1} - s_{t+j} - (i_{t+j} - i^*_{t+j})$ and consider the following regression:

$$q_{t+j} = \alpha + \beta_j (i_t - i_t^*) + \epsilon_{t+j} \tag{5}$$

It can be shown from the fact that the regression coefficient β_j is monotonically decreasing to zero for j that goes to infinity.

Consider $q_{t+j} = \Delta s_{t+j} - \Delta i_{t+j-1}$ where Δs_{t+j} is defined as above. We can write:

$$\begin{aligned}
\beta_j =& \frac{\text{Cov}(q_{t+j}, \Delta i_t)}{\text{Var}(\Delta i_t)} \\
& \frac{1}{\text{Var}(\Delta i_t 9} \left(\text{Cov}(\Delta s_{t+j}, \Delta i_t) - \text{Cov}(\Delta i_{t+j-1}, \Delta i_t)\right) \\
& \frac{1}{\text{Var}(\Delta i_t)} \left[\text{Cov}\left(-\mu \sum_{k=0}^{\infty} \mu^k \left(\Delta i_{t+k+j} - \Delta i_{t+k+j-1}\right); \Delta i_t\right) - \text{Cov}\left(\Delta i_{t+j-1}, \Delta i_t\right)\right] \\
& \frac{1}{\text{Var}(\Delta i_t)} \left[\left(-\mu \sum_{k=0}^{\infty} \mu^k \text{Cov}\left(\Delta i_{t+k+j} - \Delta i_{t+k+j-1}\right); \Delta i_t\right) - \text{Cov}\left(\Delta i_{t+j-1}, \Delta i_t\right)\right] \\
& - \mu \sum_{k=0}^{\infty} \mu^k (\rho_u^{k+j} - \rho_u^{k+j-1}) - \rho_u^{j-1} \\
& - \mu \rho_u^{j-1} (\rho_u - 1) \frac{1}{1 - \mu \rho_u} - \rho^{j-1} = -\rho^{j-1} \frac{1 - \mu}{1 - \mu \rho_u} \leq 0
\end{aligned}$$

Moreover, notice that $\frac{\partial \beta_j}{\partial j} = -(j-1)\rho_u^{j-1}\left(\frac{1-\mu}{1-\mu\rho_u}\right) < 0$. Therefore, for $j \to \infty$, the coefficient $\beta_j \to 0$ monotonically, excluding any reversal. This is not surprise since, differently from Bacchetta and Van Wincoop (2010) and Engel (2016), our framework does not entail any adjustment friction.