The pass-through of central bank lending to banks:
Evidence from the ECB’s TLTRO Programme

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Abstract

This paper models the tradeoffs and relative efficiency of direct central bank lending to banks and quantitatively analyzes the most significant such policy, the ECB’s Targeted Long Term Refinancing Operations Programme (TLTRO). I construct a banking model with bank market power in deposits and lending in which banks borrow funds with TLTROs and choose to adjust deposits, liquid asset holdings, and loans. The pass-through of the policy to bank lending is increasing in the degree of competition in loan markets and the marginal value of liquid assets to banks and decreasing in the degree of competition in deposit markets. Intuitively, these factors affect the elasticity of loan supply and demand and the degree of pass-through is similar to tax incidence where higher elasticities translate into larger shifts in quantities. In a partial equilibrium calibration, banks allocate 27% of TLTRO funds to loans, 32% to liquid assets, and 41% to substitute for deposits, which is consistent with empirical evidence. TLTROs are more effective when banks hold few liquid assets, and TLTRO is three times stronger than QE at increasing loan supply. A DSGE extension implies that an increase in TLTROs of 10% of outstanding loans triggers a 3.2% expansion in loans, increases GDP by 3.4% and inflation by 2%, and is equivalent to a 54 basis points cut to the policy rate.
1 Introduction

Direct lending from central banks to private banks at cheap interest rates has been widely implemented after the Great Recession with the aim of stimulating lending to the private sector. For example, the European Central Bank has provided unprecedented amounts of funds to private banks at cheap interest rates and long horizons with its Targeted Long Term Refinancing Operations (TLTROs), the second most important unconventional monetary policy program in terms of central bank balance sheet expansion after Quantitative Easing (QE). While QE has been extensively studied both empirically and theoretically, the impact of TLTROs and other similar policies has been evaluated mainly through empirical research.

The quantitative pass-through of TLTROs to the real economy (how many € of TLTROs generate new loans) depends on how banks substitute other sources of funding such as deposits and debt, or allocate these funds to new loans, government bonds, and excess reserves. If the objective of the central bank is to stimulate the supply of new loans, any other use of TLTROs represents a leakage of the policy which is not directed to the real economy. This paper provides a simple theoretical framework to evaluate to what extent banks use the new funding to expand loan supply, discusses the economics governing the pass-through of TLTROs to loans, evaluates the determinants of TLTROs’ effectiveness, and allows a comparison between TLTROs and QE.

I develop a banking model with bank market power in deposits and lending where banks borrow funds with TLTROs and choose how much of new central bank funding substitutes other sources of funding such as deposits, is stored as excess reserves, is used to purchase government bonds, or increases the supply of new loans. Liquid assets like government bonds and reserves provide a benefit because they insure against liquidity risk. As TLTROs are cheaper than other sources of funding, banks substitute part of their deposit base with the new funds obtained by the central bank. The increase in liquid assets and the decrease in deposit funding determine a leakage of the policy, as not all the funds are used to generate new loans. Quantitatively, the leakage is decreasing in the degree of competition in the loan market and the marginal liquidity benefit provided by liquid asset holdings and increasing in the competitiveness in the deposit markets. As for tax incidence, higher elasticities translate into stronger variation in quantities after the implementation of the policy. In the calibrated partial equilibrium model, banks use 27% of TLTRO funds to increase lending, 32% to increase liquid asset holdings and 41% to substitute deposits. In addition, TLTROs are more effective when banks hold few liquid assets, and are three times stronger than QE in increasing loan supply. I then extend the model to a dynamic stochastic general equilibrium setup where an increase in TLTRO funding by 10% of outstanding loans raises aggregate real loans by 3.2%, GDP by 3.4%, inflation by 2%, and is equivalent to a 54 basis points cut to the policy rate.

In the model, banks have market power in the loan and deposit markets, access TLTROs up to a maximum level set by the central bank, hold government bonds and reserves, and pay a cost for...
insuring against liquidity risk which is decreasing in liquid assets (bonds and reserves). The policy instrument controlled by the central bank is the maximum level of funds (borrowing allowance) that banks can access via TLTROs. As these operations represent a very cheap source of funding, banks choose to use the entire amount of available funds.\footnote{\textsuperscript{1}} When the central bank expands the maximum borrowing allowance, banks gain access to new central bank funding, increase their demand for liquid assets in order to reduce liquidity costs, expand loan supply, and diminish other more expensive sources of external funding. Bank market power in the loan and deposit markets determines a downward sloping demand curve for loans and an upward sloping supply curve for deposits. The existence of convex liquidity costs implies a downward sloping demand curve for liquid assets. An expansion in TLTROs shifts the loan supply curve and the demand curve for liquid assets. When a bank increases its funding via TLTROs, it cuts the lending rate in order to expand loan supply. Optimally, the decline in the lending rate is associated with a cut in the deposit rate, which leads to an outflow of deposits (deposit channel). Moreover, the decline in the marginal benefit of lending induces an increase in demand for government bonds and reserves (liquidity channel). Overall, the decline in deposit funding and the increase in liquid assets reduce the expansion in lending determined by the policy. Quantitatively, the relative contribution of these channels depends on the loan demand elasticity, the deposit supply elasticity, and the demand elasticity for bonds and reserves. These factors affect the elasticity of loan supply and demand and the pass-through is similar to tax incidence where higher elasticities are associated to larger movements in quantities. The leakage of the policy is weaker when the loan market is competitive, the deposit market uncompetitive, and the bank demand elasticity for liquid assets is low. In this case, loan supply can be increased by a smaller change in the lending rate, while the negative adjustment in deposits and the positive adjustment in liquid assets are smaller due to their steeper deposit supply and liquid asset demand schedules.

To be consistent with empirical micro-estimates in the TLTRO literature, I calibrate parameters in the liquidity cost function, loan demand and deposit supply elasticities to match estimates on the impact of TLTROs on lending, government bonds and reserves. In the calibrated partial equilibrium model, banks use 27% of TLTRO funds to increase lending, 7% to buy government bonds, 25% to hold excess reserves and 41% to substitute other funding (deposits in the model). The growth rate of loans is consistent, by construction, to empirical estimates reported in Barbiero et al. (2021) and Altavilla et al. (2023). The implied pass-through for 1tn€ of policy shock is larger than the 18.5% and the 13% estimated respectively by Andrade et al. (2019) and Carpinelli and Crosignani (2017) for VLTROs.\footnote{\textsuperscript{2}}

\footnotetext[1]{The maximum borrowing allowance has been increased by the ECB various times to increment the expansionary effect of the policy. For this reason, the maximum borrowing allowance is the policy tool controlled by the central bank in this model.}

\footnotetext[2]{Various papers (Crosignani et al. (2020), Jasova et al. (2018), Carpinelli and Crosignani (2017)) documented that banks used large parts of VLTROs to purchase government bonds, while this channel was way weaker with TLTROs (Benetton and Fantino (2021), Altavilla et al. (2020), Laine (2021), De Haan et al. (2019)). As a consequence, the model calibrated to match the impact of TLTROs implies a stronger pass-through to loans and a weaker pass-through to government bonds compared to}
The model also predicts that the impact of TLTROs on loans is stronger for banks operating in more competitive markets or holding a low share of liquid assets.\textsuperscript{3}

I use this framework to evaluate the impact on lending of other policies that expand central bank balance sheets such as QE. When the central bank purchases 1€ of government bonds in the market, bond yields are compressed and banks find it profitable to substitute their holdings of government bonds with other assets such as loans. The elasticity of bank demand for government bonds and loan demand elasticity pin down how many € of new loans are created by banks. The partial equilibrium model predicts that 9% of QE injections result in an increase in loan supply, compared to 27% for TLTROs. The reason is that the change in government bond yields generated by 1€ of QE triggers a 0.35€ reduction of government bond holdings by private banks.\textsuperscript{4} The implication is a smaller liquidity injection for banks with QE compared to TLTROs, where the entire central bank intervention is channelled through the banking sector. It is important to stress that QE was implemented not only to stimulate bank lending and more broadly this policy affects the real economy through different channels and agents. Nevertheless, it is possible to derive the direct effect of central bank asset purchases on loans in this banking model.

Finally, I extend the partial equilibrium banking setting to a dynamic stochastic general equilibrium model to evaluate the aggregate impact of TLTROs on lending and real activity. In this richer setup, I introduce dynamics to the model, exogenous aggregate reserves which are controlled by the central bank, and endogenize bank equity, loan demand and deposit supply via optimal decisions by other agents. The general equilibrium model allows an evaluation of the aggregate impact of TLTROs and QE on economic activity and a discussion of monetary policies at the zero lower bound after recessionary shocks or liquidity shocks. A TLTRO shock of 10% of outstanding loans (around 1tn€ in steady state) increases aggregate loans by 3.2%, GDP by 3.4% and inflation by 2%. A QE shock of the same amount increases loans by 2.2%, GDP by 2.4% and inflation by 1.4%. Even when the policy rate is at the zero lower bound, TLTROs can mitigate negative shocks as much as a cut in the policy rate by 54 basis points. Moreover, if banks are hit by adverse liquidity shocks, TLTROs can provide funding even if the central bank does not want to cut the policy rate.\textsuperscript{5}

\textbf{Institutional setting.} In June 2014 the ECB announced a series of Targeted Long Term Refinancing Operations (TLTROs) aimed at improving bank lending to the Euro Area non-financial private sector. European banks were allowed to borrow from the ECB at favourable rates up to a certain share of the total amount of their eligible private loans, defined as loans to the Euro Area non-financial private sector, excluding loans to households for house purchase. Favourable interest rate conditions were subject to the literature on VLTROs.

\textsuperscript{3} This result is consistent with García-Posada and Marchetti (2016), Andrade et al. (2019), Benetton and Fantino (2021), Andreeva and García-Posada (2020), Boeckx et al. (2020).

\textsuperscript{4} See Koijen et al. (2021).

\textsuperscript{5} This is a relevant scenario when the central bank is hiking interest rates to fight inflation and banks face sudden outflows of funding.
fulfillment by banks of a specific benchmark in terms of lending growth (for this reason these operations were named "Targeted"). The maturity was set to four years and TLTROs were conducted at quarterly frequency in various rounds. Additional operations with different characteristics in terms of interest rate and borrowing allowance have been announced also in March 2016 (TLTRO-II), June 2019 (TLTRO-III) and April 2020 (PELTRO). Before the introduction of TLTROs, banks were allowed to borrow funds from the ECB through LTROs. The main difference between the two programs was related to the shorter maturity of LTROs and the absence of a benchmark in terms of lending growth. In this paper I abstract from the maturity extension and the targeted nature of TLTROs and focus on their role as a cheap source of funding compared to other liabilities. This assumption allows to discuss the leakage of the policy through bonds, reserves and deposits, while keeping the model simple.\textsuperscript{6} Figure 1 shows the evolution of the asset side of the ECB balance sheet. The red area corresponds to securities held for monetary policy purposes (QE) and the blue area relates to liquidity operations such as LTROs and TLTROs. In terms of the quantitative impact on the ECB balance sheet, TLTROs became the second most important unconventional monetary policy tool after QE. Figure 2 shows the share of LTROs and TLTROs to eligible outstanding private loans. Blue bars represent the injections of refinancing operations to European banks, while the red line shows the outstanding stock. Between 2014 and 2019, the stock TLTROs reached a level of almost 15% of outstanding eligible loans in the Euro Area, with a peak of 35% during the pandemic.

Figure 1: ECB balance sheet (Assets)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{ECB balance sheet (Assets)}
\end{figure}

Notes: This figure reports the asset side of the balance sheet of the European Central Bank in millions of Euro. The red area refers to QE, while the blue area refers to LTROs and TLTROs. The remaining area correspond to other items in the balance sheet such as gold, claims to non-Euro Area residents denominated in foreign currency and other assets.

\textsuperscript{6} For a presentation of TLTROs as a discount window with extended maturity see Cahn et al. (2017). Da Silva et al. (2021) discuss the targeted nature of TLTROs.
Related Literature. There exists a literature which estimates the impact of LTROs and TLTROs on lending growth using aggregate and bank-level data for the Euro Area or for specific countries. A first set of studies is related to the 3-year LTROs implemented in 2011 and 2012 (known also as VLTROs). Darracq-Paries and De Santis (2015), Balfoussia and Gibson (2016), Casiraghi et al. (2016), García-Posada and Marchetti (2016), and Carpinelli and Crosignani (2017) find expansionary impacts of the policy on industrial production (around 5.7%) and loans (between 2 and 2.9 percentage points), especially for banks holding a low level of sovereign bonds and in less concentrated markets. Jasova et al. (2018) find that every 100€ of VLTROs translate in 2.5€ of new loans and in an increase in securities, Andrade et al. (2019) estimate an expansion in loans by 18.5€ with a stronger impact for banks having a low level of liquid assets, and Carpinelli and Crosignani (2017) find that banks more exposed to funding dry-up increased credit by 13€ and purchased 44€ of government bonds, while banks less exposed to the dry-up purchased 83€ in government bonds. Crosignani et al. (2020) estimate an overall increase of 5.4% in government bond holdings in Portugal.

A related branch of the literature studies the effect of TLTROs. Benetton and Fantino (2018, 2021) estimate an overall impact of TLTRO-I on loan growth of 4% after one year, and find that Italian banks participating in TLTROs increased lending by 17% (with a stronger effect in more competitive markets) and reduced the fraction of government bond holdings in their balance sheet compared to non-participating institutions. Andreeva and García-Posada (2020), Bats and Hudepohl (2019), Afonso and Sousa-Leite (2020), Da Silva et al. (2021), and Laine (2021) provide evidence that TLTROs increased lending (between 9 and 16 percentage points), especially in very competitive markets, and did not signif-
icantly impact government bond holdings. Altavilla et al. (2023) estimate that the April 2020 recalibration of TLTRO-III increased lending by 1.4 percentage points per year. Boeckx et al. (2020) show that the impact is stronger for banks which are small, illiquid, more reliant on wholesale funding, and less capitalized. De Haan et al. (2019) compare VLTROs and TLTROs and find that VLTROs incentivized carry trade by banks, while TLTROs were associated with an increase in liquid asset share in the balance sheet. Barbiero et al. (2021) provide a meta-analysis of estimated impacts of TLTROs on annual lending growth where the median is around 0.95% for 1tn€ policy shock. They also show that the share of TLTRO funds stored by banks in their own Eurosystem accounts ranges between 20% and 50% 20 days after the TLTRO settlement. Overall, estimates in the literature suggest that VLTROs and TLTROs had a positive effect on lending growth, especially in more competitive markets and for illiquid banks. The impact tends to be weaker for VLTROs as a large part of the funds was used to purchase government bonds, whereas TLTROs have been more successful in stimulating loan supply as the diversion of funds to liquid assets was smaller.

Da Silva et al. (2021) develop a partial equilibrium banking model to study TLTROs, taking into account the existence of eligible and ineligible loans, and how the interest rate on TLTROs is determined depending on how banks increase lending relative to the benchmark. Cahn et al. (2017) model LTROs as a discount window with longer maturity in a DSGE model with a frictional banking sector and find that without LTROs output would have been 2.5% lower over 2009. Mouabbi and Sahuc (2019) quantify the impact of TLTROs and other unconventional monetary policies in a general equilibrium model where the policies are modelled as a change in the shadow interest rate. Van der Kwaak (2017) analyzes LTROs in a DSGE model with balance-sheet-constrained banks facing a portfolio choice between loans and government bonds and find that the effect of the policy is neutral. In this paper, I instead abstract from the maturity structure and the existence of eligible and ineligible loans, and focus on the allocation of TLTRO funds in a banking model where banks have market power and own liquid assets to decrease liquidity risk.

This paper is also related to research on how negative rates are transmitted to the economy through banks. In particular, Ulate (2021) presents a banking model where banks intermediate the transmission of monetary policy, have some monopoly power in the loan and deposit markets, and are subject to a lower bound on deposit rates. He shows how negative rates can at the same time stimulate loan supply and depress bank profitability thereby muting their expansionary effect. In this paper, rather than focusing on the lower bound on deposit rates, I study how banks intermediate the transmission of monetary policy through negative interest rates.

7. In a previous version of the paper (Altavilla et al. (2020)) they show that the impact on the growth rate of government bond holdings is higher than the effect on loans after one year, and becomes insignificant after two years.
8. They find that in absence of unconventional measures, Euro Area GDP growth would have been on average 1.09% below its actual level over the period 2014-2017.
9. The reason is that banks use government bonds as collateral to obtain central bank funding and therefore they initially substitute loans with government bonds.
of TLTROs in a similar monopolistically competitive setting where I add liquidity costs and an active management of government bond holdings and reserves. Additionally, I incorporate the banking sector into a general equilibrium model in a similar way. Abadi et al. (2022) develop a frictional banking model to discuss the determinants of the reversal interest rate, which is the rate at which accommodative monetary policy becomes contractionary for lending. Similarly, Eggertsson et al. (2019) construct a banking model where negative policy rates become progressively less efficient or even contractionary. With this literature I share the study of unconventional monetary policy in a monopolistically competitive banking setup which is embedded in a general equilibrium New Keynesian model (Gertler and Karadi (2011)).

The paper is organized as follows. Section 2 presents the partial equilibrium banking model, its calibration and the main results. Section 3 extends the partial equilibrium model to a dynamic stochastic general equilibrium setting. Section 4 concludes.

2 A banking model with TLTRO and liquidity

In this section, I develop the monopolistically competitive banking model and discuss the intuition behind the transmission mechanism of TLTROs to loans. The setup is static and in partial equilibrium. The model is then calibrated using European data and is integrated into a dynamic general equilibrium model in Section 3.

2.1 Model

There exists a continuum of \( j = [0, 1] \) banks endowed with a given amount of equity \( F_j \) which is exogenous. The liabilities side of the bank balance sheet includes equity \( F_j \), deposits \( D_j \), and TLTRO funding \( O_j \). On the asset side, banks issue loans \( L_j \), hold central bank reserves \( R_j \), and purchase government bonds \( A_j \). Banks have some monopoly power both in the loan market and in the deposit market. Market power could be attributed to switching costs, menu costs, or asymmetric information. It follows that banks face a downward-sloping loan demand curve and an upward-sloping deposit supply curve.

Banks choose the interest rate on loans \( i^L_j \), the quantity of loans, the deposit rate \( i^D_j \), the amount of deposits, the amount of TLTRO funding \( O_j \), and the quantity of government bonds \( A_j \) and reserves \( R_j \). They take as given the interest rate on reserves \( i^R \), the interest rate on government bonds \( i \), and the interest rate on TLTROs \( i^o \). Table 1 provides an overview of the balance sheet of bank \( j \).

Note that the central bank directly controls three variables: the maximum amount of central bank funding to banks \( \tilde{O} \), the interest rate on central bank funding \( i^o \), and the policy rate \( i^R \). Therefore, as
Table 1: Balance sheet of bank $j$

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans: $L_j (i^L_j)$</td>
<td>Deposits: $D_j (i^D_j)$</td>
</tr>
<tr>
<td>Bonds: $A_j (i)$</td>
<td>Central bank funding: $O_j (i^o)$</td>
</tr>
<tr>
<td>Reserves: $R_j (i^R_j)$</td>
<td>Equity: $F_j$</td>
</tr>
</tbody>
</table>

Notes: This table reports the balance sheet of bank $j$. For each balance sheet item, the corresponding rate of return is reported in round brackets.

in the ECB’s institutional setup of TLTROs, private banks can borrow funds from the central bank at a given interest rate $i^o$, up to a maximum borrowing allowance $O_j$. This way of modelling TLTROs captures the fact that this policy is essentially a constrained source of cheap funding for banks, and allows a discussion of the quantitative pass-through rather than the interest rate pass-through of the policy.

Banks pay a cost for liquidity risk that is decreasing in government bonds and reserves, which are assumed to be liquid assets. Intuitively, private banks need enough liquid assets for precautionary and regulatory reasons to cover possible outflows of funds. Microfoundations for liquidity costs are provided by Freixas and Rochet (2008): with some probability banks face an outflow of funds which, if big enough, leads banks to pay a penalty for the liquidity shortage. It follows that liquidity costs are a decreasing convex function of liquid assets. For simplicity and tractability, in this paper I assume a reduced form convex liquidity cost function (similar to Eggertsson et al. (2019)). Government bonds and reserves contribute to the reduction in liquidity costs. For this reason, holding liquid assets provides and additional benefit on top of the rate of return on those assets. The liquidity cost function $C(A_j, R_j, F_j)$ is assumed to be convex and decreasing in the shares of government bonds and reserves to equity, such that $C_A < 0$, $C_R < 0$, $C AA > 0$, $C RR > 0$.

The bank maximises profits subject to the balance sheet constraint, the demand for loans, the supply of deposits and the central bank funding constraint:

$$
\max_{i^L_j, L_j, i^D_j, D_j, O_j, A_j, R_j} \left( (1 + i^L_j) L_j + (1 + i) A_j + (1 + i^R_j) R_j \right) \\
- (1 + i^D_j) D_j - (1 + i^o) O_j - C(A_j, R_j, F_j)
$$

10. In principle, the intuition would be preserved by assuming the existence of a single liquid asset as what matters is the existence of a liquidity cost function. The distinction between bonds and reserves is introduced because a TLTRO injection determines a corresponding increase in reserves for the aggregate banking sector, something that becomes relevant in the general equilibrium extension. The assumption that bonds and reserves might not be perfect substitutes is also due to the fact that the two assets provide different liquidity benefits as they have different maturities and government bonds can be used by banks as collateral in repos. This is consistent with their different response to TLTROs estimated in empirical studies.
Balance sheet constraint: \[ L_j + A_j + R_j = D_j + O_j + F_j \]

Loan demand: \[ L_j = \left( \frac{1 + i^L_j}{1 + i_L} \right)^{-\varepsilon^L} L, \quad \varepsilon^L > 1 \]

Deposit supply: \[ D_j = \left( \frac{1 + i^D_j}{1 + i^D} \right)^{-\varepsilon^D} D, \quad \varepsilon^D < -1 \]

Central bank funding constraint: \[ O_j \leq \bar{O}. \]

Note that banks take as given aggregate loans \( L \), aggregate deposits \( D \), the aggregate lending and deposit rates \( i^L \) and \( i^D \), their own equity \( F_j \), the policy rate \( i^R \), the government bond yield \( i^o \) and the TLTRO interest rate \( i^o \).

First order conditions yield the following equilibrium equations:

\[ 1 + i^L_j = \mu^L \left( 1 + i^R - C_R \right) \quad (1) \]
\[ 1 + i^D_j = \mu^D \left( 1 + i^R - C_R \right) \quad (2) \]
\[ 1 + i^R - C_R = 1 + i - C_A \quad (3) \]
\[ 1 + i^R - C_R = 1 + i^o + \delta \quad (4) \]
\[ \delta (O_j - \bar{O}) = 0, \quad \delta \geq 0 \quad (5) \]

where \( \delta \) is the lagrange multiplier on the central bank funding constraint, and \( \mu^L \) and \( \mu^D \) are respectively the markup and markdown on the marginal cost to set the lending rate and the deposit rate.

Equations (1) and (2) show that the bank optimally sets the lending rate and the deposit rate respectively as a markup and a markdown over the marginal cost. The marginal cost is given by the gross interest rate on reserves net of the marginal liquidity benefit of holding reserves. In a model without liquidity costs, the marginal cost would be only equal to the interest rate on reserves. In this model, the marginal cost of issuing loans takes into account the fact that holding reserves provides a liquidity benefit, with an implied optimal lending rate which is higher than in a model without liquidity costs (as \( C_R < 0 \)).

Equation (3) imposes that the bank should be indifferent between holding reserves and government bonds. As the policy rate \( i^R \) is historically smaller than the government bond yield \( i \), liquidity costs guarantee the existence of an interior solution which depends on \( C_R \) and \( C_A \) as long as they are functions of reserves and government bonds. As an example, holding the policy rate fixed, a decline in the government bond yield implies a decrease in \( C_A \) and/or an increase in \( C_R \). By assumption, \( C_{AA} > 0 \) and...
\( C_{RR} > 0 \) thus implying a decrease in government bond holdings and an increase in demand for reserves.

Equation (4), results from the first order condition with respect to TLTRO funding \( O_j \). The intuition is that if the cost of borrowing funds via TLTROs is low enough \((i^o \leq i^R - C_R)\), banks are going to use TLTRO funding up to the borrowing limit \( \bar{O} \). The shadow price of the central bank funding constraint is captured by the lagrange multiplier \( \delta \geq 0 \). If, instead, the borrowing cost of TLTROs is too high \((i^o > i^R - C_R)\), banks use other sources of funding and do not use TLTROs. Since the introduction of TLTROs in 2014\(^{11}\), \( i^o \) has always been equal or lower than the interest rate on reserves \( i^R \). Given that by assumption \( C_R > 0 \), it follows that the condition \( i^o < i^R - C_R \) has always been true. For this reason, this paper focuses on the case where participating banks decide to use all TLTRO funding made available from the central bank (making the central bank funding constraint binding). An additional reason supporting this assumption is that the central bank has control both on the interest rate on TLTROs \( i^o \) and the maximum borrowing constraint \( \bar{O} \). In principle, then, the central bank could always choose \( \bar{O} \) and set the TLTRO interest rate low enough to convince banks to borrow the entire amount of TLTRO funds available. Moreover, most of the bidding banks especially from stressed European countries have been borrowing a very large share of their maximum borrowing allowance (around 95% in some auctions).

The equilibrium conditions can be combined into equations (6).

\[
\begin{align*}
\frac{1 + i^L_j}{\mu^L_j} &= (1 + i) - C_A = (1 + i^R) - C_R = \frac{1 + i^D_j}{\mu^D_j} = (1 + i^o) + \delta
\end{align*}
\]

(6)

The marginal benefit of lending must be equal to the marginal benefit of holding government bonds, the marginal benefit of holding reserves, the marginal cost of issuing deposits and the marginal cost of TLTRO funding. Then, any variation in the policy rate \( i^R \), the government bond yield \( i \) or the TLTRO borrowing allowance \( \bar{O} \) (which affects \( \delta \)) impacts the endogenous variables in the model.

2.2 The pass-through to loans of an expansion in TLTRO

In order to understand how an increase in TLTRO funds results in new loans, let’s first focus on loan supply when banks have access to TLTROs as depicted in the first panel of Figure 3. For the time being, assume the absence of liquidity costs and liquid assets and focus on the deposit funding channel. Given that TLTROs are the cheapest source of funding, banks fund the first part of their loans using central bank funding. The first portion of the loan supply schedule is flat because the interest rate on TLTROs (marginal cost for the bank) is constant. After reaching the maximum TLTRO borrowing allowance,

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\(11\) Before the introduction of TLTROs, banks had access to LTROs which were characterized by shorter maturities and an absence of a benchmark in terms of lending growth.
banks fund the remaining part of their loans using deposits. This portion of the loan supply schedule is upward-sloping because banks face an upward-sloping deposit supply curve. The slope depends on the degree of competition in the deposit market which is captured by the elasticity $\varepsilon^D$. In other words, in order to gain more deposits, banks need to offer higher deposit rates.

The second panel of Figure 3 shows the impact of an increase in the TLTRO borrowing allowance $\bar{O}$ on bank loan supply. The green line represents loan supply after the TLTRO shock. A larger share of loans can now be funded using TLTROs at a constant marginal cost which is equal to $i_o$, and the loan supply schedule shifts to the right. Thus, an increase in TLTROs represents a loan supply shock, precisely because it provides a larger amount of funding at a low marginal cost.

The impact of a TLTRO shock on loans in equilibrium depends both on loan demand and loan supply. The slope of loan supply schedule is a function of competition in the lending market which is captured by the elasticity $\varepsilon^L$. If the loan market is perfectly competitive, the elasticity is infinite and the loan supply curve is flat. This is the case where the entire amount of new TLTRO funds gets channelled into new loans. The left panel of Figure 4 shows this case where the quantitative pass-through of TLTRO to loans is full. I define full pass-through when $1\%$ of new TLTRO funds increases lending by $1\%$. The reason is that with a flat loan demand curve, banks can increase lending without changing the lending rate. In absence of a change in the marginal benefit of lending, also the other equilibrium conditions are unaffected and there is no change in deposit funding or other assets. This is a limiting case which suggests that stronger competition in the lending market increases the pass-through to loans of TLTROs. This property of the model is also consistent with empirical findings in Andreeva and García-Posada (2020) and Benetton and Fantino (2021) who find that the policy is stronger in areas where the banking
Figure 4: Pass-through of TLTRO shock to loans

Notes: This figure shows the impact of a TLTRO shock on equilibrium loans. The left panel presents the case of a perfectly competitive market for loans where the demand elasticity is infinite. In this case the pass-through of the policy is full because there is no need to change the lending rate and the deposit rate to expand lending. The right panel presents the case of partial pass-through where the loan demand schedule is downward-sloping. The expansion of lending is associated with a decline in the deposit rate. In this case there is leakage due to an outflow of deposits determined by the decline in the deposit rate. The amount of funding available to the bank is then partially reduced as banks substitute part of their deposit funding with TLTROs.

Deposit channel. The right panel of Figure 4 presents the relevant case, where banks face a downward-sloping loan demand curve due to monopolistic competition in the lending market and $\varepsilon^L < \infty$. After an increase in TLTRO funds banks expand their loan supply, but in equilibrium the increase in loans can only be sustained by a decrease in the lending rate. The decline in the marginal benefit of lending has to be matched by a decrease in the marginal cost of deposits via a reduction in the deposit rate. Due to the upward-sloping shape of the deposit supply curve, the result is a decrease in deposit funding. Overall, the increase in loans after 1€ TLTRO shock is going to be less than 1€ because banks use part of the new funds to substitute from deposit funding. In this sense, there is leakage from the policy objective of expanding loan supply. The intuition is analogous to tax/transfer incidence because the impact of new funds is going to be relatively stronger on the more elastic items of the balance sheet. If loan demand is very elastic relative to deposit supply, the adjustment in loans is going to be stronger and deposit substitution is going to be smaller, thus implying less leakage. Graphically, loan demand elasticity pins down the slope of the loan demand curve, whereas deposit supply elasticity affects the slope of the loan supply curve. The implication is a strong pass-through of the policy to the quantity of new loans when loan demand is very elastic. In fact, in equilibrium the lending rate and the deposit rate are changing by the same amount and the resulting variation in quantities depends on the relative elasticities of loans and deposits. To sum up, compared to the case of full pass-through of TLTROs to loans, now there is some leakage in the policy because part of the funds are used to substitute deposit funding (deposit channel) and only a share of new TLTRO funds are going to generate new loans.\textsuperscript{12}

\textsuperscript{12} This model abstracts from other sources of external funding such as bank debt. Altavilla et al. (2020) find that TLTROs...
Liquidity channel. So far, we have abstracted from liquid assets (reserves and government bonds) and liquidity costs in order to focus on the deposit channel. The left panel of Figure 5 shows how the existence of a liquidity channel affects the loan supply curve. The first implication is that the flat portion of the supply curve is smaller because part of TLTRO funds are used to buy liquid assets, thereby leaving less funds to supply loans. The second implication is that the upward-sloping portion of the supply curve is flatter because, as the lending rate varies, funds can be reallocated between liquid assets and loans. Now the slope depends also on the second derivatives of the liquidity cost function $C_{AA}$ and $C_{RR}$ which pin down bank demand elasticity for government bonds and reserves.

The right panel of Figure 5 presents the impact of a TLTRO shock on loans in the model with liquidity costs and compares it with the model that abstracts from liquidity. Now the horizontal shift in the loan supply curve is smaller because part of the funds are used to purchase government bonds or held as excess reserves. The resulting equilibrium increase in loans is then smaller compared to the previous case, leading to a bigger leakage in the policy. Equation (6) helps in understanding the mechanism. An increase in TLTRO funding reduces the shadow cost of the central bank funding constraint $\delta$ and the marginal cost of funding. Banks cut the lending rate in order to increase loans, but at the same time also the marginal benefit of holding government bonds and reserves has to fall in equilibrium. Holding the policy rate $i^R$ and the government bond yield $i$ fixed, the marginal benefit of holding liquid assets declines as the bank expands its holdings of government bonds and reserves. Intuitively, banks optimally use part of the new TLTRO funds to cover liquidity costs. The quantitative response depends on bank demand elasticity for government bonds and reserves which is a function of the shape of the liquidity cost function. Again, the intuition is similar to tax/transfer incidence: if demand for government bonds and reserves is very elastic, a larger part of TLTRO funds is going to be used to purchase liquid assets and a smaller share ends up in new loans. Overall, when the central bank provides 1€ of new TLTROs, a part of that € is going to substitute deposit funding (deposit channel), a part is going to be used to increase holdings of bonds and reserves (liquidity channel), and the remaining part ends up in new loans. The deposit channel and the liquidity channel determine a leakage in the policy and the pass-through of TLTROs to loans is partial.

Total pass-through. The pass-through of TLTROs to loans can be also evaluated analytically as $\frac{\partial L_j}{\partial \bar{O}}$. Differentiating the balance sheet constraint together with the first order conditions gives the following result:

$$\frac{\partial L_j}{\partial \bar{O}} = \left[ 1 + \frac{\varepsilon D_j}{\varepsilon L_j} D_j + \frac{1}{\varepsilon L_j} L_j \left( 1 + i^R - C_R \right) \frac{C_{AA} C_{RR} - C_{AR}^2}{(C_{AA} + C_{RR} - 2C_{AR})} \right]^{-1}$$

(7)

The blue term is related to the deposit channel and the red term is related to the liquidity channel. It tends to reduce banks’ debt suggesting an important source of funding substitution. Adding debt in the liability side of banks would be a natural extension to the model, but the same intuition would be preserved.
Notes: The left panel shows how the introduction of liquid assets and liquidity costs changes the shape of the loan supply schedule. The flat portion is shorter because part of TLTRO funds are used to purchase liquid assets, while the slope of the second portion becomes flatter as variations in the lending rate induce reallocation from/to liquid assets. The right panel shows how a TLTRO shock is subject to a bigger leakage because part of the funds are now used to purchase liquid assets and the horizontal shift in the curve is smaller.

absence of those two terms (e.g. when $\varepsilon^L = \infty$) the pass-through is full as $\frac{\partial L_j}{\partial \bar{O}} = 1$. In this case, each $\varepsilon$ of additional TLTROs ends up in new loans, as shown graphically in Figure 4.

In general, the deposit channel (blue term) is always positive as $\varepsilon^D < -1$. The sign of the liquidity channel (red term) is positive if $C_{AA} > C_{AR}$ and $C_{RR} > C_{AR}$, a condition which is going to be satisfied in the calibration presented in this paper. The fact that both terms are positive implies a partial pass-through as $\frac{\partial L_j}{\partial \bar{O}} < 1$. This is the case presented in Figure 5. The blue term captures the fact that banks substitute part of the deposit funding with TLTROs depending on elasticities $\varepsilon^L$ and $\varepsilon^D$. Higher competition in lending market (high $\varepsilon^L$) determines lower deposit substitution and stronger increase in loans. Similarly, stronger competition in deposit market (high $|\varepsilon^D|$) implies stronger deposit substitution and weaker increase in loans.

The liquidity channel arises because banks allocate part of new resources into government bonds and reserves to reduce liquidity costs. When demand elasticity for bonds and reserves is low, a smaller share of new funds is diverted to liquid assets and the increase in loans is stronger (red term is smaller). The reason is that demand elasticity for bonds and reserves is inversely related to $(C_{AA} - C_{AR})$ and $(C_{RR} - C_{AR})$.

2.3 Introducing Quantitative Easing

The existence of government securities allows a discussion of the direct effects on loans of Quantitative Easing policy via the banking sector. The stated aim of TLTROs was to stimulate bank lending to the real economy, while QE was introduced to affect the economy more broadly and via different channels (households, firms, financial markets). The focus of this section is the impact of QE via the bank lending

\begin{itemize}
\item[(13)] The opposite is implausible because it would imply bonds and reserves moving in opposite directions after a TLTRO shock, which is not what is observed in the data.
\end{itemize}
channel.

Assume that the central bank buys $A^{QE}$ assets and these purchases affect the interest rate on government bonds $i$. In the model presented so far, QE can be modelled as a change in the interest rate $i$ engineered by the central bank $\frac{\partial i}{\partial A^{QE}}$. This specification makes it possible to compare the two policies in quantitative terms, where TLTROs appear as a change in $\bar{O}$ and QE as a change in $\bar{A}$. Therefore, the loan impact of QE can be derived as:

$$\frac{\partial L_j}{\partial A^{QE}} = \frac{\partial L_j}{\partial i} \bigg|_{\text{endogenous}} \ast \frac{\partial i}{\partial A^{QE}} \bigg|_{\text{exogenous}}$$

The derivative of loans with respect to the interest rate $i$ can be derived analytically from the model whereas the second term on the right hand side of the equation corresponds instead to the yield impact of QE and it is not modelled in this framework. The yield impact of QE in Europe has been estimated in the literature for various maturities and horizons (see Eser et al. (2019) and Altavilla et al. (2021)) and is assumed to be equal to the constant parameter $\gamma^a \equiv \frac{\partial i}{\partial A^{QE}}$.

### 2.4 Parametrized Liquidity Cost Function

The reason for holding liquid assets comes from the fact that banks pay a cost for liquidity risk $C(A_j, R_j, F_j)$ (see Eggertsson et al. (2019), Freixas and Rochet (2008)). Intuitively, private banks need enough liquid assets to cover possible outflows of short-term funding for precautionary and regulatory reasons. Therefore, holding government securities and reserves provides two benefits to the bank: the return on the asset and the decline in liquidity costs. For simplicity, I assume a reduced form liquidity cost function. Freixas and Rochet (2008) provide microfoundations for a liquidity cost function which is decreasing in liquid assets and convex.

Equation (8) presents the functional form of the liquidity cost function in this model:

$$C(A_j, R_j, F_j) = \kappa \left( \frac{\alpha A_j^\rho + (1 - \alpha) R_j^\rho}{F_j^{1/\rho}} \right)^{-\gamma} F_j$$

As long as $\kappa > 0$, $\gamma > 0$, $\rho < 1$, and $0 < \alpha < 1$, liquidity costs are decreasing and convex in the share of liquid assets to equity.

There are two important assumptions underlying this functional form. The first is that liquidity costs are a function of the share of liquid assets to equity. This assumption preserves the property that liquidity costs are decreasing in bonds and reserves, and can be microfounded by assuming that banks face the risk of an outflow of funds which is equal to a constant share of their equity (as a proxy
of their balance sheet). The second important assumption in equation (8) is that government bonds and reserves (liquid assets) are combined via a CES aggregator. In principle, the economic intuition presented so far does not require the existence of two separate liquid assets, and the liquidity channel could be driven only by substitution towards government bond holdings. However, introducing central bank reserves is important in this setup as the central bank settles TLTROs by crediting a corresponding amount of reserves in private banks’ accounts. Then, single banks can always redistribute reserves to other banks, but the aggregate amount of reserves is exogenously set by the central bank. Therefore, the existence of reserves becomes important in the general equilibrium model. Another key assumption is the type of aggregation of liquid assets in the liquidity cost function. In principle, the aggregation could be linear as in Eggertsson et al. (2019). However, government bonds and reserves are different in terms of liquidity, maturity, riskiness, and ability to provide collateral in operations with the central bank and other banks. Moreover, perfect substitutability would imply that the elasticity of banks’ demand functions for government bonds and reserves would be the same and their response after a TLTRO shock would be equal. This result would be in conflict with empirical estimates of the response of government bond holdings and reserves after a TLTRO shock. The constant returns to scale CES aggregator ensures enough flexibility to calibrate different responses of government bonds and reserves after a shock, nests the case of perfect substitutability, and preserves properties in terms of shares that are useful in the calibration of the general equilibrium model.

Overall, the liquidity cost function presented in equation (8) is decreasing in liquid assets \( C_A < 0, C_R < 0 \) and convex \( C_{AA} > 0, C_{RR} > 0 \).

### 2.5 Additional modifications to the quantitative model

Two modifications are added to the model to make calibration more consistent with observed data on banks and interest rates. First, the asset side of banks’ balance sheet includes other net assets \( S_j \) which are exogenous. They are constructed as \( S_j = F_j + D_j + O_j - L_j - A_j - R_j \). This feature does not alter the dynamics of the model and is introduced to equate assets and liabilities for banks and at the same time use observed data for loans, bonds, reserves, deposits, TLTROs and capital.

The second modification is the introduction of an exogenous benefit of issuing deposits \( \theta^D \) (as in Ulate (2021) and Abadi et al. (2022)), and an exogenous cost of holding government bonds \( \theta^{gp} \). The benefit of issuing deposits can be seen as an additional fee charged to depositors or alternatively a benefit to the bank of having a large and stable deposit base. This modification allows a decoupling of the policy

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14. Eggertsson et al. (2019) do not provide an explicit form for their liquidity cost function, but assume that it is an increasing function of external funds. Abadi et al. (2022) assume that bank’s liquid assets must exceed a constant fraction of deposits issuance. My choice of using the ratio of liquid assets to equity makes sure that liquidity costs depend on the ratio of liquid assets in bank’s balance sheet (which is proxied by equity), rather than their absolute level. Moreover, the fact that equity is exogenous in the model ensures that the calibration of the cost function depends only on movements in assets. Additionally, the choice of using shares has the benefit of making the general equilibrium model more tractable.
rate $i^R$, the marginal liquidity cost $C_R$ and deposit supply elasticity $\varepsilon^D$ in the calibration. The cost of holding government bonds captures the different riskiness of this type of asset compared to reserves and makes sure that the spread in the rate of return of the two assets is not only explained by their distinct contribution to the liquidity cost function. Also these additional features do not alter the dynamics of the model.

Given these two modifications, the balance sheet constraint becomes:

$$L_j + A_j + R_j + S_j = D_j + O_j + F_j$$  \hspace{1cm} (9)

and equilibrium conditions for the deposit rate and for government bond holdings become:

$$1 + i^D_j = \mu^D (1 + i^R - C_R + \theta^D)$$  \hspace{1cm} (10)

$$1 + i^R - C_R = 1 + i - C_A - \theta^{ip}$$  \hspace{1cm} (11)

### 2.6 Calibration

The model is calibrated using monthly Euro Area data for the aggregate banking sector from 2014 to 2019 obtained from ECB SDW database.\(^{15}\) There are nine parameters to be calibrated: loan demand elasticity $\varepsilon^L$, deposit supply elasticity $\varepsilon^D$, the parameters of the liquidity cost function $\kappa$, $\alpha$, $\gamma$, $\rho$, the exogenous cost/benefits of issuing deposits and holding bonds $\theta^D$ and $\theta^{ip}$, and the yield impact of QE $\gamma^a$. Deposit supply elasticity is calibrated as $\varepsilon^D = -275$ at the quarterly frequency as in Abadi et al. (2022) (which is also very close to Ulate (2021) who sets the parameter equal to $-268$). Both papers calibrate this parameter to target historical averages of the deposit rate spread. The reason to calibrate deposit supply elasticity from the literature is twofold: first, the sample covers a period where deposit rates have been very sticky compared to other interest rates as they got very close to the zero lower bound; second, it would be difficult to decouple loan demand elasticity and the benefit of issuing deposits $\theta^D$ in Equation (10) using available data as both parameters are pinned down by the historical spread between the deposit rate and the policy rate. The yield impact of QE for 1tn€ central bank purchases is calibrated using empirical results from Altavilla et al. (2021). They estimate an impact on the 10-year European yields of -36 basis points for a 1tn€ envelope, such that $\gamma^a = -0.36\%$ annualized.

The remaining seven parameters are calibrated to match historical averages for equations (1), (10) and (11) and target elasticities obtained from estimates in the empirical literature on TLTROs and QE. The first empirical target is the response of loan growth after 1tn€ TLTRO shock. Altavilla et al. (2023)

\(^{15}\) Data sources and transformations are reported in Appendix A.

\(^{16}\) Calibrating $\varepsilon^D$ using these data would imply an implausibly large level of competition in the deposit market which is at odds with other parametrizations in the literature.
estimate that the $1.3\text{tn}\,\text{€}$ June uptake of TLTRO-III had an aggregate impact on loan growth 1.4 percentage points over a year. Rescaling the impact to $1\text{tn}\,\text{€}$ and assuming that it lasts for three years returns an overall impact of 3.2 percentage points in loan growth. Barbiero et al. (2021) present a meta-analysis of the estimated impact of TLTROs on loan growth after one year where the median is around 0.95 percentage points after a $1\text{tn}\,\text{€}$ TLTRO injection. Again, assuming that the impact lasts for three years, the overall impact is 2.8 percentage points, which is the figure I target in the calibration.

The second target is the share of TLTRO funds stored in banks’ own reserves accounts with the central bank. This is estimated by Barbiero et al. (2021) to be around 30% after 20 days from TLTRO settlement. Therefore, I target $\frac{\partial B}{\partial O}$ to be equal to 0.30.

The third target is the growth rate of government bond holdings after a TLTRO shock. Various papers such as Altavilla et al. (2020), Carpinelli and Crosignani (2017), Crosignani et al. (2020), Jasova et al. (2018) find that banks using TLTRO funding increase substantially their holdings of government bonds. Altavilla et al. (2020) estimate that the response in the growth rate of bonds after one year is larger than the response in loans by a factor of around 1.33. For this reason I target the growth rate of government bond holdings to be equal to 1.33 times the growth rate of loans, which returns 3.7 percentage points overall.

The last target is related to the share of bonds that the central bank purchases from private banks with Quantitative Easing. This target helps to pin down the response of government bond holdings to a Quantitative Easing shock. Koijen et al. (2021) estimate a demand system for government bonds and show how the portfolio of various investor sectors has rebalanced from 2015 to 2017, when the ECB started its program of asset purchases. The ECB bought a total of $1.33\text{tn}\,\text{€}$ of government bonds (net of new issuances by governments) between 2015 and 2017 where $0.47\text{tn}\,\text{€}$ were purchased from private banks. This implies that banks sold 35% of overall government bonds purchased by the central bank. Thus, in the calibration I target $\frac{\partial A}{\partial A^{QE}} = -0.35$.

Calibrated parameters appear in most of the targeted equations and enter non-linearly. For this reason, I choose parameters by numerically minimizing the squared deviation of model equations from targets.\(^{19}\)

The parameters that minimize the objective function are presented in Table 2. Table 3 shows how the various targeted equations are matched by the calibration. The average lending rate is very close to the historical average, while average deposit rate and average spread between policy rate and government bond yield are almost perfectly matched. The growth rate of loans and bonds after a $1\text{tn}\,\text{€}$ TLTRO

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17. Altavilla et al. (2020) estimate the response of loans to TLTROs up to two years and show that the response in loans is monotonically increasing over the horizon. However, the maturity horizon of TLTROs was four years, suggesting that it might be possible that loans keep growing for a longer period of time. In the calibration, to calculate the overall impact of a TLTRO shock, I conservatively assume that loans keep growing at the same rate for three years.

18. Confidence intervals range between 50% and 20%.

19. A more detailed presentation of the calibration exercise is presented in Appendix B.
Table 2: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa )</td>
<td>0.015</td>
<td>Liquidity cost function param.</td>
<td>Calibration</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.17</td>
<td>Liquidity cost function param.</td>
<td>Calibration</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.86</td>
<td>Liquidity cost function param.</td>
<td>Calibration</td>
</tr>
<tr>
<td>( \rho )</td>
<td>-0.02</td>
<td>Liquidity cost function param.</td>
<td>Calibration</td>
</tr>
<tr>
<td>( \varepsilon^L )</td>
<td>169</td>
<td>Loan demand elasticity (quarterly)</td>
<td>Calibration</td>
</tr>
<tr>
<td>( \varepsilon^D )</td>
<td>-275</td>
<td>Deposit supply elasticity (quarterly)</td>
<td>Abadi et al. (2022)</td>
</tr>
<tr>
<td>( \theta^D )</td>
<td>0.007</td>
<td>Benefit of issuing deposits (quarterly)</td>
<td>Calibration</td>
</tr>
<tr>
<td>( \theta^{ip} )</td>
<td>0.007</td>
<td>Cost of holding government bonds (quarterly)</td>
<td>Calibration</td>
</tr>
<tr>
<td>( \gamma^a )</td>
<td>-0.36</td>
<td>Yield impact of 1tn QE (ann. %)</td>
<td>Altavilla et al. (2021)</td>
</tr>
</tbody>
</table>

Notes: This table reports the parametrization of the partial equilibrium model.

Table 3: Targets

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{i}_L )</td>
<td>2.44</td>
<td>2.36</td>
<td>Historical Average 14-19</td>
</tr>
<tr>
<td>( \bar{i}_D )</td>
<td>1.47</td>
<td>1.46</td>
<td>Historical Average 14-19</td>
</tr>
<tr>
<td>( \bar{i}_R - \bar{i} )</td>
<td>-1.57</td>
<td>-1.56</td>
<td>Historical Average 14-19</td>
</tr>
<tr>
<td>( \partial L/\partial O )</td>
<td>2.8</td>
<td>2.8</td>
<td>Barbiero et al. (2021)</td>
</tr>
<tr>
<td>( \partial R/\partial O )</td>
<td>0.30</td>
<td>0.25</td>
<td>Barbiero et al. (2021)</td>
</tr>
<tr>
<td>( \partial A/\partial O )</td>
<td>3.7</td>
<td>3.7</td>
<td>Altavilla et al. (2020)</td>
</tr>
<tr>
<td>( \partial A/\partial A^{QE} )</td>
<td>-0.35</td>
<td>-0.35</td>
<td>Kojien et al. (2021)</td>
</tr>
</tbody>
</table>

Notes: This table reports the targeted historical averages and elasticities in the partial equilibrium model with their respective sources. \( \partial L/\partial O \) and \( \partial A/\partial A^{QE} \) are expressed in percentage points.

shock and the change in government bonds after 1tn\( \text{€} \) QE shock are the same as the target. The share of TLRTOS that is kept by banks as reserves is 25%, which is lower than the target, but still within the confidence bands reported in Barbiero et al. (2021). Appendix C shows how the calibration changes with alternative target elasticities.

2.7 Results from the partial equilibrium model

1tn\( \text{€} \) TLRTOS increase loans by 0.27tn\( \text{€} \). The calibrated partial equilibrium model can be used to assess the impact of a TLRTO shock. Figure 6 presents the response of bank’s balance sheet variables to 1tn\( \text{€} \) positive TLRTO shock. Loans increase by 0.27tn\( \text{€} \), banks buy 0.07tn\( \text{€} \) of government bonds, increase reserves by 0.25tn\( \text{€} \) and substitute other funding (in this case deposits) by 0.41tn\( \text{€} \). Overall, the pass-through of TLRTOS to loans is reduced by 0.41tn\( \text{€} \) due to the deposit channel and by 0.32tn\( \text{€} \) due to the liquidity channel. The implication is that large part of the funds is used either to substitute other sources of external funding or to increase holdings of liquid assets. These properties of the model are a direct outcome of the calibration which targets the growth rates of loans, government bonds, and the share of reserves held by banks after a TLRTO shock. To compute the change in \( \text{€} \) rather than the growth rate, the model is shocked assuming that all bank balance sheet items are at their historical
Figure 6: Impact of a 1tn€ TLTRO shock

Notes: This figure reports the change in loans, government bonds, reserves and deposits after a 1tn€ increase in TLTROs. All variables are reported in tn€.

average between 2014 and 2019. The quantitative impact presented depends on the relative elasticities of loan demand, deposit supply and liquid assets demand.

**Pass-through is stronger when liquidity is scarce.** An interesting implication of the model is that the pass-through of TLTROs to loans is stronger when banks hold few liquid assets. Figure 7 presents this result. The upper panel in Figure 7 shows how loan growth can vary between 0.5% and 4% depending on the initial holdings of liquid assets, where the baseline growth using historical averages is 2.8%. This result comes from the convexity of the demand functions for bonds and reserves, implying that low initial liquidity is associated with lower elasticities in the demand for liquid assets and a smaller increase their quantity. As a consequence, a larger share of TLTRO funds is directed to new loans. This is an interesting implication of the model which is consistent with empirical results presented in Andrade et al. (2019), García-Posada and Marchetti (2016), and Boeckx et al. (2020). The lower panel shows how loan growth after 1tn€ TLTRO shock varies substantially depending on the initial holdings of liquid assets (measured as a share of equity). When banks hold very few government bonds or reserves, the change in the marginal benefit of additional liquidity is bigger. In other words, when liquidity is scarce, receiving TLTRO funds can improve liquidity costs with a smaller adjustment in liquid assets, thereby leaving more resources to expand lending.

**TLTRO 3 times more effective than QE.** The partial equilibrium banking model can also be used to compare the impact on lending of TLTROs and QE shocks of the same size. These two unconventional monetary policies are both quantitative, in the sense that they involve expansions of the central bank balance sheet which can be measured in trillions of €. Figure 8 shows how private banks react to an increase in TLTRO funding and an increase in central bank’s asset purchases of the same size. 1tn€
Figure 7: Impact of TLTROs on loans as a function of liquid assets

Notes: The upper panel reports the growth rate of loans after 1tn€ TLTRO shock in three cases: when banks have low levels of initial liquidity, when banks have high levels of initial liquidity, and when initial levels of liquidity are at the historical average. The lower panel reports the growth rate of loans after 1tn€ TLTRO shock (vertical axis) as a function of initial holdings of government bonds and reserves (horizontal axes). Loan growth is reported in percentage terms, while government bonds and reserves are reported as a share of bank equity.

Expansion in TLTROs increases lending by 0.27tn€, while 1tn€ QE purchases determine an increase in loans by 0.09tn€. It is important to stress that the two policies affect banks’ balance sheets in different ways: while the entire amount of TLTROs represent new funding to banks, Quantitative Easing induces substitution between government bonds and loans. The implication is that the increase in lending from QE depends on the amount of government bonds which is sold by private banks due to the compression in bond yields engineered by central bank purchases. In this model, then the impact depends on government bond bank demand elasticity. An increase in QE by 1tn€ reduces government bond yields by 36bp (annualized) which is associated to a drop in government bond holdings by 0.35tn€. Part of these 0.35tn€ are then used to expand lending by 0.09tn€ and increase reserves by 0.12tn€. The drop in interest rates determines also a reduction in the deposit rate which compresses deposits by 0.14tn€.

Overall, the differential impact on lending of TLTROs and QE comes from the fact that banks respond to 1tn€ QE by cutting their holding of government bonds by 0.35tn€, whereas 1tn€ TLTRO shock is entirely absorbed by banks as new funding. Notice that the results on QE presented here are only related to the direct effect of the policy through the banking sector and do not take into account how QE affects the real economy more broadly through different channels and agents.

Table 4 presents the pass-through of TLTROs and QE for different calibrations of the model. The pass-through of TLTROs to loans ranges between 0.25tn€ and 0.38tn€, while the impact of QE is between 0.03tn€ and 0.16tn€.20

20. More details on the different calibrations and on other variables are presented in Appendix D.
3 TLTRO in a general equilibrium model

The static partial equilibrium model outlined in Section 2 provides useful intuitions on the mechanism governing the pass-through of TLTROs to loans and other banks’ balance sheet items. However, due to its static and partial equilibrium nature, it is not sufficient to assess the impact of unconventional monetary policies on output and inflation. In this Section, I extend the model in two dimensions: introducing dynamics and closing the model in a general equilibrium setting with other agents in order to make loan demand and deposit supply endogenous.

The general equilibrium model features endogenous loan demand from firms and deposit supply from households which were exogenous in the partial equilibrium setting. As a consequence, a TLTRO shock which expands loan supply and increases economic activity may have positive feedback effects on loan demand and deposit supply. In addition, although reserves represent a choice variable from the perspective of a single bank, aggregate reserves are exogenously set by the central bank. The general equilibrium model then features an exogenous increase in reserves after an expansion of TLTRO funding from banks. The same is true for QE shocks, where the amount of government bonds sold by banks has to be matched by a corresponding increase in central bank reserves.
Bank equity is now endogenous and evolves over time because a share of profits is retained by banks to build up capital. The implication is that as banks get bigger and increase their capital, they need to raise more liquid assets in order to cover liquidity costs. Moreover, the introduction of dynamics makes policy shocks persistent (rather than permanent) and forward looking banks take into account the expected persistence of the shock when making their optimal choices.

Overall, the model presented in this Section can shed light on the impact of TLTROs and QE on loans, GDP and inflation taking into account the evolution of bank capital, exogenous central bank reserves and feedback from loan demand and deposit supply. Moreover, this model allows for a discussion of the central bank policy reaction to negative shocks with conventional and unconventional monetary policy instruments when the policy rate is at the zero lower bound. Finally, the model can be used to study how TLTROs constitute a useful policy tool when banks are hit by adverse liquidity shocks but the central bank does not want to reduce the policy rate.

3.1 Model Setup

The model builds on Gertler and Karadi (2011) and Ulate (2021) where the banking sector is an extension of the one presented in the partial equilibrium section of this paper. Gertler and Karadi (2011) develop a DSGE model with financial intermediaries facing balance sheet constraints and study the effect of unconventional monetary policy. Ulate (2021) builds on their model and modifies the banking sector to study the effect of negative interest rates on bank lending behavior in a setup where financial intermediaries have market power in the loan and deposit markets, and are subject to a lower bound on the deposit rate. In this paper, what’s different from previous models are the banking sector and the policy tools available to the central bank. Banks issue loans, demand deposits, hold government bonds and reserves, are subject to liquidity costs and have to option to fund with TLTROs. The central bank controls reserves, the policy rate via a Taylor Rule, and can decide to engage in TLTROs or Quantitative Easing. The government consumes goods, sets lump-sum taxes and issues government bonds. The rest of the general equilibrium model shares a common setup with Gertler and Karadi (2011) and Ulate (2021). There are five agents: households, intermediate goods producers, capital producers, retailers, and banks. Figure 9 presents an overview of all the agents in the model and their main characteristics. Retailers are subject to Calvo price rigidities and aggregate intermediate output demanded from firms. Intermediate goods firms demand capital and labor. They rent capital from capital producers using loans obtained from private banks, and labor from households. Capital producers are subject to investment adjustment costs and supply capital at a price which is not fixed at one. Households consume the retail good, supply labor to intermediate firms and supply deposits to private banks.
3.2 Agents

3.2.1 Households

The economy is populated by a continuum of households of measure 1 who consume, save by supplying deposits to banks and holding money, and supply labor to intermediate goods firms. Households maximize expected discounted lifetime utility as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \varphi_t \left( \frac{(C_t - hC_{t-1})^{1-\sigma} - \sigma}{1-\sigma} - \frac{\lambda N_t^{1+\frac{1}{\eta}}}{1 + \frac{1}{\eta}} \right)$$

(12)

21. A natural extension of the model would be to allow households to hold government bonds. This assumption would introduce an additional channel which would make QE even more stimulative, as households would sell government bonds to the central bank after a QE shock. However, the extension would not significantly affect the impact of TLTROs and for this reason it is not presented.
where $\beta$ is the discount factor, $\varphi_t$ is a shock to the discount factor, $h$ governs habit persistence$^{22}$, $\sigma$ is the inverse of the intertemporal elasticity of substitution, $\chi$ is the scale parameter associated to labor disutility, and $\eta$ is the Frisch elasticity of labor supply. Common assumptions are $0 < \beta < 1$, $0 < h < 1$, and $\sigma, \chi, \eta > 0$. Households can save either by supplying deposits $D_t$ at the gross interest rate $1 + i_t^D$ or hold money $M_t$. They supply labor $N_t$ and receive the nominal wage $W_t$. They consume the final good $C_t$ at the price $P_t$, receive nominal profits $\Pi_t$ from the ownership of all firms, and pay lump sum taxes $T_t$ to the government. The budget constraint is then given by:

$$P_tC_t + D_t + M_t = W_tN_t - T_t + (1 + i_t^D)D_{t-1} + \Pi_t + M_{t-1}$$ (13)

Optimality conditions are:

$$\chi N_t^{\frac{1}{\eta}} = \phi_t W_t$$ (14)

$$\phi_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \left[ \frac{\varphi_{t+1}}{\varphi_t} \right] (C_{t+1} - hC_t)^{-\sigma}$$ (15)

$$1 = \beta E_t \left[ \frac{\varphi_{t+1}}{\phi_t} \frac{\varphi_t}{\varphi_{t+1}} (1 + i_t^D) \frac{1}{1 + \pi_{t+1}} \right]$$ (16)

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the net inflation rate.

### 3.2.2 Retail firms

Retail firms use intermediate inputs demanded from intermediate goods firms to produce differentiated varieties of a retail good $Y_t(s)$. Varieties are then aggregated into a final good via a CES aggregator.

$$Y_t = \left( \int_0^1 Y_t(s) \frac{s^{-1}}{1-s} ds \right)^{\frac{1}{1-\varepsilon}}$$ (17)

Demand for retail goods and the price index are:

$$Y_t(s) = \left( \frac{P_t(s)}{P_t} \right)^{-\varepsilon} Y_t$$ (18)

$$P_t = \left[ \int_0^\infty P_t(s)^{1-\varepsilon} ds \right]^{\frac{1}{1-\varepsilon}}$$ (19)

where $\varepsilon$ is the elasticity of demand across differentiated retail goods.

Retail firms are subject to price frictions, as they can reset their price with probability $1 - \gamma_p$ (Calvo...
(1983)). They solve:

$$\max_{P_t^*} E_t \sum_{\tau=0}^{\infty} \beta^\tau (\gamma P_t^*)^\tau \Lambda_{t,t+\tau} \frac{P_t}{P_{t+\tau}} Y_{t+\tau} (s) \left[ P_t^* (s) - P_{t+\tau}^m \right]$$

subject to demand for $Y_t(s)$. $\Lambda_{t,t+\tau}$ is the stochastic discount factor and $P_t^m$ is the price of the intermediate good. Optimality conditions are standard and they are reported in the Appendix.

### 3.2.3 Intermediate goods firms

Intermediate goods firms produce intermediate inputs using labor and capital. In order to obtain capital $K_t$, they need to borrow from banks. The timing is as follows: at the end of $t - 1$ firms borrow $K_t$ from the bank and use the capital stock to produce goods in period $t$. After production takes place, firms return the capital stock to the bank. The production function for intermediate goods is then:

$$Y_t^m = Z_t (\xi_t K_t)^{\alpha_k} N_t^{1-\alpha_k}$$

where $Z_t$ is total factor productivity, $\xi_t$ is a capital efficiency shock, and $0 < \alpha_k < 1$ is the capital share parameter in the Cobb-Douglas production function. Firms then choose labor and capital as follows:

$$\max_{N_t, K_t} P_t^m Y_t^m - W_t N_t - Z_t K_t$$

where $P_t^m$ is the price of the intermediate good, and $Z_t K_t$ is the dividend paid by banks for each unit of capital. Optimality conditions are:

$$(1 - \alpha_k) P_t^m Y_t^m N_t = W_t$$

$$\alpha_k P_t^m Y_t^m K_t = Z_t K_t$$

Note that the two conditions imply that intermediate firms make zero profits. As in Gertler and Karadi (2011) and Ulate (2021), banks are residual claimants of intermediate goods firms, as banks receive all residual stochastic returns to banks. Therefore, the return for banks of lending a unit of capital is:

$$1 + \delta_t = \frac{Q_{t+1} \xi_{t+1} (1 - \delta)}{Q_t} + P_{t+1}^m \alpha_k \frac{Y_{t+1}^m}{K_{t+1}}$$

where $Q_t$ is the nominal price of capital, and $\delta$ is the capital depreciation rate.
3.2.4 Capital producers

Capital producers generate new capital subject to investment adjustment costs. The law of motion of capital is:

\[ K_{t+1} = (1 - \delta)\xi_t K_t + I_t \]  

(26)

where \( I_t \) is investment. The nominal price of capital is \( Q_t \). Capital producers choose the real price of capital \( \frac{Q_t}{P_t} \) by maximizing the following:

\[
\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,\tau} \left[ \left( \frac{Q_{\tau}}{P_{\tau}} - 1 \right) - f \left( \frac{I_{\tau}}{I_{\tau-1}} \right) \right] \]  

(27)

Investment adjustment costs satisfy \( f(1) = f'(1) = 0 \) and \( f''(1) > 0 \) as in Christiano et al. (2005), Gertler and Karadi (2011), Ulate (2021). The optimality condition which pins down the price of capital is standard and is reported in the Appendix.

3.2.5 Banks

The setup for the banking sector is similar to the one presented in Section 2, with three main modifications that are introduced to add dynamics, and make the model more realistic.

First, bank capital is now endogenous as banks can now retain part of their profits to build up equity. Each period, banks retain a constant fraction of profits \( \omega \) and return the rest as dividends to the owner. Moreover, banks pay each period a constant fraction \( \varsigma \) of their net worth as managerial cost. These assumptions make bank equity an endogenous variable which varies over time as in Equation (28), and is potentially affected by policy shocks.

Second, banks receive a stochastic return from firms, rather than a deterministic interest rate. The return on loans is stochastic because banks charge firms a fraction of their total return on capital. The implication is that the lending rate is determined in \( t + 1 \) and contains expectations, as the return on capital can be affected by unexpected shocks occurring between \( t \) and \( t + 1 \). This assumption is helpful to make shocks to capital efficiency (recessions) relevant to banks and bank equity.

Finally, in the general equilibrium setting aggregate reserves are exogenous. Single banks can always choose the amount of reserves they hold by exchanging them with other banks, but the aggregate amount of reserves in the economy is set by the central bank. This is relevant in the analysis of unconventional monetary policy as policy interventions such as TLTROs and Quantitative Easing determine an exogenous increase in central bank reserves. Therefore, each \( \欧元 \) supplied via TLTROs or purchased in government bonds by the central bank is associated to an equivalent increase in reserves.

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24. Eberly et al. (2012) show that this specification of investment adjustment costs is consistent with firm-level data.
As in Ulate (2021) define total profits, net of managerial costs and adjusted for inflation as $X_{j,t+1}$:

$$X_{j,t+1} = i_t^R F_{j,t} + (i_{j,t+1}^D - i_t^R) L_{j,t} + (i_{j,t} - \theta_D^P - i_t^R) A_{j,t} - i_t^R S_{j,t} - (i_{j,t}^D - \theta_{D}^P - i_t^R) D_{j,t} - (i_t^P - i_t^R) O_{j,t} - C(A_{j,t}, R_{j,t}, F_{j,t}) - (1 - \varsigma) F_{j,t-1} \pi_{t+1}$$

The assumption is that banks pay a fraction $\omega$ of $X_{j,t+1}$ as dividends and keep the remaining share:

$$DIV_{j,t+1} = (1 - \omega) X_{j,t+1}$$

As a consequence, real bank equity evolves as follows:

$$\frac{F_{j,t+1}}{P_{t+1}} = \frac{F_{j,t}}{P_{t}} (1 - \varsigma) + \omega \frac{X_{j,t+1}}{P_{t+1}}$$

(28)

Banks maximize the present discounted value of dividends:

$$\max E_t \sum_{s=0}^{\infty} \beta^{s+1} \Lambda_{t,t+s+1} DIV_{j,t+s+1}$$

The first order conditions for all the variables except for the lending rate are Equations (10), (11), (4) and (5). The return on loans is instead now stochastic and has the following equilibrium condition:

$$E_t (1 + i_{t+1}^L) = \mu_L (1 + i_t^R - C_R(A_{j,t}, R_{j,t}, F_{j,t}))$$

(29)

3.2.6 Government and central bank

The central bank has three monetary policy instruments to stabilize the economy: the policy rate $i_t^R$, TLTROs $O_t$, and Quantitative easing $A_t^{QE}$. The policy rate is determined by a Taylor Rule subject to a lower bound which is equal to the steady state level of the policy rate:

$$i_t^R = \max \left( (1 - \rho_i) \left( i_t^R + \psi_i \pi_t + \psi_y \frac{Y_t - \bar{Y}}{\bar{Y}} \right) + \rho_i i_{t-1}^R + \epsilon_m^p, \bar{i}_t^R \right)$$

(30)

where $\rho_i \in (0, 1)$ is the smoothing parameter of the Taylor Rule, $\bar{i}_t^R$ and $\bar{Y}$ are steady state policy rate and output, $\psi_i$ and $\psi_y$ are both positive and govern the importance of inflation and output gap in the reaction function of the central bank, and $\epsilon_m^p$ is the exogenous monetary policy shock.

TLTROs and QE both follow autoregressive processes. The government bond yield $i_t$ is determined by three components: the policy rate $i_t^R$, an exogenous premium associated to government bonds $\theta_i^p$.

25. The lower bound is not set to zero because the steady state level of the policy rate is already negative. Setting the lower bound below the steady state level of the policy rate would not affect the results presented in this paper.
(which captures the spread with the risk-free rate), and the QE component.

\[ i_t = i_t^R + \theta_i^t + \gamma^a A_t^{QE} \]  

(31)

where \( \gamma^a < 0 \) is the yield impact of QE. Notice that in absence of QE, the government bond yield is given by the sum of the policy rate and an exogenous spread. When the central bank purchases government bonds, the government bond yield falls and the magnitude depends on \( \gamma^a \) which is calibrated using estimates from Altavilla et al. (2021).

The central bank controls also aggregate reserves. Whenever the monetary authority increases the amount of TLTROs available to banks, reserves increase by the same amount. Similarly, government bonds purchased by the central bank from private banks determine an increase in aggregate reserves. Thus, the equation governing the amount of reserves in the economy is the following:

\[ R_t = \bar{R} + (O_t - \bar{O}) + I_{QE} \cdot \bar{A} - A_t \]  

(32)

where \( I_{QE} \) is an indicator function which is equal to 1 in case of a QE shock and 0 otherwise. Notice that in this model the only agent holding government bonds are banks. Therefore, to close the model, the change in reserves has to be equal to the amount of government bonds sold by private banks to the central bank, which is an endogenous choice and depends on the impact of QE on government bond yields.

Finally, the government budget constraint and the central bank balance sheet are aggregated as follows:

\[ A_t + R_t - O_t = (1 + i_{t-1})A_{t-1} + (1 + i_t^R)R_{t-1} - (1 + i_{t-1}^o)O_{t-1} + G_t - T_t \]  

(33)

where \( G_t \) is government spending and follows an AR(1) process, and \( T_t \) are lump sum taxes.

### 3.2.7 Resource constraints and shocks

Assuming a symmetric equilibrium across all banks, we can drop the \( j \) subscript for all the variables and work with aggregate variables. The resource constraint is given by

\[ Y_t = C_t + I_t + G_t + f \left( \frac{I_t}{I_{t-1}} \right) I_t + \theta_i^t \frac{A_{t-1}}{P_t} - \theta_i^o \frac{D_{t-1}}{P_t} \]  

(34)

\[ + \frac{F_{t-1}}{P_t} + \frac{f(A_{t-1}, R_{t-1}, F_{t-1})}{P_t} \]

Aggregate loans issued by banks need to be equal to the value of capital:

\[ L_t = Q_t K_{t+1} \]  

(35)
All the shocks in the model follow autoregressive processes except for the monetary shock.

3.3 Calibration of the general equilibrium model

The calibration of the general equilibrium model follows three principles: (i) calibrate the banking sector with parameters obtained in the partial equilibrium model, (ii) match ratios and interest rates in steady state using historical averages, and (iii) use standard parameters from the literature for the remaining blocks. The model is calibrated at the quarterly frequency.

Parameters for the banking sector are the same as in Section 2. The reason for calibrating the liquidity cost function and the loan demand and deposit supply elasticities as in the static setup is that those parameters do not involve intertemporal relations and for this reason they should not be affected by the dynamic setup of the general equilibrium model. In fact, loan demand and deposit supply elasticities pin down the markup and markdown over the marginal cost in the equations for the lending rate and the deposit rate, while the liquidity cost function determines the bank demand elasticity for liquid assets. Therefore, loan demand elasticity \( \varepsilon^L \) is equal to 169 at the quarterly frequency, and deposit demand elasticity \( \varepsilon^D \) is set equal to -275. This parametrization implies a markup \( \mu^L \) for the lending rate of 1.006 and a markdown \( \mu^D \) for the deposit rate of 0.996. The liquidity cost function is the same as in the partial equilibrium model: the scale parameter \( \kappa \) is set to 0.015, the elasticity parameter \( \gamma \) is 0.17, the share of government bonds in the CES aggregator \( \alpha \) is 0.86 and the substitution parameter \( \rho \) is -0.002. The exogenous benefit of issuing deposits and cost of holding government bonds \( \theta^D \) and \( \theta^{ip} \) are both equal to 0.007. The fraction of profits staying in banks \( \omega \) is equal to 12% as in Ulate (2021) and the share of equity paid as managerial costs \( \varsigma \) is pinned down using steady state relations and is equal to 0.22%.

The impact of 1tn\( £ \) QE on 10-year government bond yields is taken from Altavilla et al. (2021) and is set to -36 basis points annualized. The discount rate \( \beta \) is 0.9945 as a result of the steady state deposit rate and household Euler Equation.

The share to equity of various banking balance sheet variables is calibrated in steady state using historical averages between 2014 and 2019. The loan-to-equity share is equal to 3.95, the share of government bonds is 0.68, the share of deposits is 3.65, the share of reserves is 0.50, and the share of TLTROs is 0.24. Historical averages are used also to calibrate interest rates in steady state: the lending rate is 2.44% annualized, the deposit rate is 1.47%, the policy rate is -0.32%, and the 10-year government bond yield is 1.25%. The steady state amount of government bonds purchased via Quantitative Easing is set to 1.17tn\( £ \) and pins down the government bond premium \( \theta^i \) in steady state which is equal to 0.005.

TLTROs follow an AR(1) process where \( \rho_o \) is the autoregressive coefficient, and \( \varepsilon^o_t \) is a TLTRO shock. AN AR(2) process is assumed for Quantitative easing, where \( \rho_{qe,1} \) and \( \rho_{qe,2} \) are the autoregressive coefficients, and \( \varepsilon^{qe}_t \) is an exogenous QE shock. The rationale for assuming an AR(2) process
for QE is that this kind of policy is implemented over time after the announcement rather than being conducted only in one period. Expressing the two equations in logs, we have:

\[ o_t = (1 - \rho_o)\bar{o} + \rho_o o_{t-1} + \epsilon_t^o \quad (36) \]

\[ o_{t}^{qe} = (1 - \rho_{qe,1} - \rho_{qe,2})\bar{a}_{t}^{qe} + \rho_{qe,1} a_{t-1}^{qe} + \rho_{qe,2} a_{t-2}^{qe} + \epsilon_t^{qe} \quad (37) \]

The autoregressive coefficient for a TLTRO shock \( \rho_o \) is estimated using an AR(1) regression for the sample 1999-2022 and is equal to 0.97. Similarly, the autoregressive coefficients for the QE shock \( \rho_{qe,1} \) and \( \rho_{qe,2} \) are estimated with an AR(2) regression in the sample 2015-2022 and are respectively equal to 1.84 and -0.86.

The remaining parameters are taken from the literature and are reported in Table 5 with their corresponding sources.

### 3.4 Results from the general equilibrium model

#### 3.4.1 The impact of TLTROs

The impact of a TLTRO shock can be now evaluated through the lens of this rich dynamic model. Given that the shock is expansionary, solving the model with traditional methods would trigger the contractionary response from the Taylor Rule, which is not the ideal setting to evaluate the impact of a policy which is usually implemented when the policy rate is fixed. For this reason, in order to hold the policy rate constant after the shock, the model is solved by bounding above the policy rate to its steady state level with occasionally binding constraints (Guerrieri and Iacoviello (2015)).

Figure 10 shows the impulse response of some variables to a TLTRO shock equivalent to 10% of outstanding loans in steady state. As in the partial equilibrium model, banks use the new resources to cut the lending rate and increase lending. Part of TLTRO funding is also used to purchase government bonds which grow substantially. The expansion in reserves is mechanical in the general equilibrium model and is not reported. The increment in the supply of loans is associated to an increase in the capital stock used by intermediate goods firms in their production which triggers a rise in investment. As a consequence of the increase in the production of intermediate goods, also aggregate output, consumption and inflation are impacted positively by the shock. Quantitatively, the peak response of loans is 3.2% growth, output raises by 3.4% after 3 quarters and inflation increases by 2% in annualized terms. For each € of TLTROs, banks increase lending by 0.32€, government bond holdings by 0.27€, and capital by 0.04€. As already mentioned, reserves in the banking sector increase by the same amount of the TLTRO injection. What’s different compared to the partial equilibrium model is the growth in deposits which is equivalent to 0.55€. The reason is that the increase in lending triggers an expansion in production. As households
Table 5: General equilibrium parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Target or source</th>
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<tbody>
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<td></td>
<td></td>
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<tr>
<td>$\kappa$</td>
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<td>Calibration</td>
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<td>$\gamma$</td>
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<td>Liquidity cost exp param</td>
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<td>Liquidity cost ela sub</td>
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<td>Cost of holding bonds</td>
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<td>$\omega$</td>
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<td>Smoothing param. Taylor Rule</td>
<td>Ulate (2021)</td>
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<td>Yield impact of 1tn QE (ann. %)</td>
<td>Altavilla et al. (2021)</td>
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<td>Gertler and Karadi (2011)</td>
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<td>AR(1) estimation 99-22</td>
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<td>AR(2) coeff. QE shock</td>
<td>AR(2) estimation 15-22</td>
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Notes: This Table present the calibration of the general equilibrium model. Parameters in bold are taken from the static partial equilibrium model.
get richer, they save part of the additional resources into deposits. This general equilibrium effect was not captured in the partial equilibrium model.

Another interesting result of the model is the response of the economy to a capital efficiency shock. This kind of shock has been used by Gertler and Karadi (2011) and Ulate (2021) to model the Great Recession. I study three cases to evaluate the ability of the central bank to provide accommodation with conventional and unconventional policy tools after negative shocks to the economy. First, I do not constrain the policy rate ("No ZLB") and solve the model with traditional perturbation methods. This is a standard setup where the policy rate can respond to negative shocks via the Taylor Rule. Then, I constrain the interest rate on reserves to be bounded downwards by its steady state level ("ZLB"). In this case, the monetary authority cannot stimulate the economy in a recession. Finally, I allow the central bank to respond to the shock by expanding the availability of TLTROs even if the policy rate is still bounded below ("ZLB+TLTRO"). These last two cases are solved using occasionally binding constraints where the policy rate is bounded below by its steady state level. Figure 11 presents results for this exercise. Capital efficiency falls by 1%, with a persistence parameter of $\rho_\xi = 0.8$. In absence of any policy response (black line), output falls by around 2.4% and loans decline by 3.8%. Similarly, also inflation, consumption and investment decline substantially. When the policy rate is unconstrained (blue dotted line), monetary policy mutes the peak drop in output to 1.7% and in loans to 3% on impact. The red dashed line presents the case where the policy rate is constrained, but the monetary authority implements TLTROs to stimulate the economy with a shock equal to 10% of outstanding loans. Notably, the accommodation provided by TLTROs is very close to the "No ZLB" case, suggesting that this type of unconventional monetary policy instrument is strong enough to replace standard interest rate policies when the policy rate is at the lower bound. The model predicts that a TLTRO shock equivalent to 10% of outstanding loans in steady state has an impact on output as effective as a 54 basis points drop in the policy rate.

3.4.2 TLTROs vs QE

A key feature of the model is the ability to compare the impact of a TLTRO shock and a QE shock of the same initial magnitude. Figure 12 presents the impulse response of various variables to both shocks of comparable size. Consistent with results from the partial equilibrium model, TLTROs provide a larger stimulus to the economy. The intuition is similar to the one presented in partial equilibrium: the entire amount of TLTROs is injected to the banking sector, whereas only a part of central bank purchases of government bonds is sold by private banks. For this reason, the amount of stimulus to the economy through the banking sector is bigger with TLTROs. The peak impacts on GDP and loans are respectively 3.4% and 3.2% for TLTROs, and 2.4% and 2.2% for QE. Government bond holdings by banks move in
Figure 10: IRFs to a TLTRO shock of 10% of outstanding loans

Notes: This figure shows the impulse responses after a positive TLTRO shock of 10% of outstanding loans, when the policy rate is fixed at steady state. All variables are in percentage deviations from steady state, except for inflation and the loan rate which are reported in annualized percentage deviations from steady state.

Figure 11: IRFs to a Capital Efficiency shock

Notes: This figure shows the impulse responses after a 1% decrease in capital efficiency $\xi_t$. The black line reports the response when the policy rate is fixed at the lower bound, the blue dotted line reports the response when the policy rate is unconstrained, and the red dashed line reports the response when the policy rate is constrained, but the central bank increases TLTROs by 10% of outstanding loans. All variables are in percentage deviations from steady state, except for inflation and the loan rate which are reported in annualized percentage deviations from steady state.
Figure 12: IRFs to a TLTRO shock and a QE shock - baseline

Notes: This figure shows the impulse responses after a TLTRO shock of 10% of outstanding loans (1tn€ in steady state) (blue line), and a QE shock of 1tn€ (red line) when the policy rate is fixed at steady state. The TLTRO shock follows an AR(1) specification. The QE shock is 1tn€ in the first period and peaks to 3.97tn€ after 8 quarters due to the AR(2) specification. All variables are in percentage deviations from steady state, except for inflation and the loan rate which are reported in annualized percentage deviations from steady state.

opposite directions as expected: TLTROs induce banks to purchase government bonds to use part of the new funding to cut liquidity costs, whereas QE determines a decline in government bond holdings as their yield is compressed by the central bank intervention. Compared to the partial equilibrium results, the gap between the impact of TLTROs and QE on loans is smaller. This outcome comes mainly from the different persistence of the two shocks. TLTROs are modelled as a very persistent AR(1) process, whereas QE is modelled as an AR(2) process with the first autoregressive coefficient larger than 1 and the second coefficient negative. The implication is that after an initial QE shock of 1tn€, agents expect an expansion in asset purchases by the central bank in the future, which peak to 3.97tn€. This feature of QE, where central banks purchase gradually government bonds over time rather than in a single period, is consistent with the historical experience. Alternatively, an initial QE shock of 0.25tn€ peaks to 1tn€ after eight quarters and returns an impact on output and loans equal to 0.61% and 0.55%.

For a better quantitative comparison, Figure 13 presents a comparison of a TLTRO shock and a QE shock of the same size where both shocks follow the same AR(1) process. In this case, QE increases GDP by 0.85% and loans by 0.82%, thus preserving the wide gap between the effectiveness of the two policies already discussed for the partial equilibrium model.

3.4.3 A shock to liquidity costs for banks

This model is also well suited to study the impact of adverse liquidity shocks to banks and how TLTROs can be used as a policy tool to counteract their negative effects on the banking sector. I model liquidity
shocks by adding an exogenous shifter $\xi_t^{liq}$ to the liquidity cost function:

$$C(A_t, R_t, F_t) = \xi_t^{liq} \kappa \left( \frac{[\alpha A_t^\rho + (1 - \alpha) R_t^\rho]^{1/\rho}}{F_t} \right)^{-\gamma} F_t$$

An exogenous increase in $\xi_t^{liq}$ raises liquidity costs all else equal. In order to keep liquidity costs constant, then, banks would need to increase their holdings of liquid assets and reduce lending. Intuitively, an increase in $\xi_t^{liq}$ can be seen as an higher probability of an outflow of funds from the bank. The blue dotted line in Figure 14 shows the impact of a 10% increase in liquidity costs with a persistence of 0.95.

Loans fall by 0.2% and investment declines by 1.2%. The negative impact on the economy leads to an outflow of deposits of 0.2% and a decline in bank capital. Banks respond to the shock also by increasing their holdings of government bonds in order to counteract the liquidity shock. Suppose now that the central bank does not want to reduce the policy rate to help banks facing the shock. This can be true for example when the central bank is tightening the monetary policy stance and does not want to signal an easing of monetary conditions. In this situation, the central bank can always provide funds to banks with TLTROs without changing the path of the policy rate. The red dashed line displays the response to a liquidity shock when the central bank increases TLTRO funding available to banks. Banks use the central bank funding to purchase government bonds to adjust liquidity costs. The outcome is that and the decline in loans, investment, and bank capital is reverted. Moreover, also the outflow of deposits is avoided, as the contractionary effects on the economy are muted. Overall, this exercise shows how...
Figure 14: IRFs to a liquidity shock

Notes: This figure shows the impulse responses after a 10% positive shock to the liquidity cost shifter $\xi^{liq}$ when the policy rate is constrained. The blue dotted line shows the response after the shock without TLTROs. The red dashed line shows the response after the shock when the central bank expands TLTROs by 10% of outstanding private loans.

TLTROs can be used as an effective policy tool in order to counteract liquidity shock to banks even when the central bank is increasing the policy rate.

4 Conclusions

The impact of TLTROs on economic activity crucially depends on how private banks decide to use the new funds received from the central bank. Empirical research has established that TLTROs had expansionary effects on lending and output, but also that banks used those funds to purchase government bonds, store excess reserves and substitute other sources of funding. In this paper, I develop a theoretical framework to think about the mechanism behind the pass-through of this form of unconventional monetary policy to the real economy, taking into account the different uses that private banks make when receiving central bank loans. Banks face monopolistic competition both in the loan and deposit markets, hold government bonds and reserves, have access to central bank funding, and pay a cost for insuring against liquidity risk which is decreasing in their holdings of liquid assets. When a bank receives TLTROs, it increases loan supply, reduces loan demand to substitute with cheaper funding, and expands its holdings of government bonds and reserves in order to reduce liquidity costs. The magnitude of these responses depends on the relative elasticities of loan demand, deposit supply and bank demand for liquid assets. The intuition is analogous to tax/transfer incidence: the more elastic markets are going to display stronger variation in quantities compared to the least elastic ones.

I calibrate the model to be consistent with the empirical research on TLTROs which makes use
of bank-level data. In this calibrated partial equilibrium model, banks choose to use 27% of TLTRO funds to increase lending, 7% to purchase government bonds, 25% to hold excess reserves and 41% to substitute other funding. An interesting implication is that TLTROs are more effective when banks have few liquid assets because the marginal liquidity benefit is larger when liquidity is scarce.

I then use this framework to evaluate the impact on lending of Quantitative Easing, the other main unconventional monetary policy which has been implemented through an expansion of central bank balance sheet. When the central bank purchases 1€ of government bonds in the market, bond yields are compressed and banks substitute their holdings of government bonds with other assets. The elasticity of bank demand for government bonds and loan demand elasticity pin down how many € of new loans will be created by banks. The model predicts that TLTROs are three times stronger than Quantitative Easing in increasing loan supply. The reason is that QE generates a smaller liquidity injection to the bank balance sheet as the change in government bond yields generated by 1€ QE determines a 0.35€ reduction of government bond holdings by private banks, whereas with TLTROs the entire amount of central bank intervention represents fresh funding to the banking sector.

The model is then extended to a dynamic stochastic general equilibrium setup, where an increase in TLTRO funding by 10% of outstanding loans determines an expansion in aggregate real loans by 3.2%, an increase in GDP by 3.4% and in annualized inflation by 2%. The general equilibrium model is also suitable to study the impact of TLTROs when the policy rate is constrained by a lower bound and the economy is hit by adverse shocks. The model shows that TLTROs are an effective substitute of interest rate policy to counteract recessionary shocks: an increase in TLTROs equivalent to 10% of outstanding loans triggers the same response in output as a cut in the policy rate by 54 basis points.
References


A Data Sources and transformations

List of data sources used in the paper.

**LTROs and TLTROs (Stock):** *Longer-term refinancing operations - Eurosystem*. Source: ECB SDW. Key: ILM.W.U2.C.A050200.U2.EUR.

**LTROs and TLTROs (operations):** *Longer-term refinancing operations*. Source: ECB website.²⁶

**QE:** *Securities held for monetary policy purposes*. Source: ECB SDW. Key: ILM.W.U2.C.A070100.U2.EUR.


**Loans to households:** *Loans vis-a-vis euro area households reported by MFIs excl. ESCB in the euro area (stocks)*. Source: ECB SDW. Key: BSI.M.U2.N.A.A20.A.1.U2.2250.Z01.E.

**Loans to nonfinancial corporations:** *Loans vis-a-vis euro area NFCs reported by MFIs excl. ESCB in the euro area (stocks)*. Source: ECB SDW. Key: BSI.M.U2.N.A.A20.A.1.U2.2240.Z01.E.

**Loans for house purchases:** *Lending for house purchase vis-a-vis euro area households reported by MFIs excl. ESCB in the euro area (stocks)*. Source: ECB SDW. Key: BSI.M.U2.N.A.A22.A.1.U2.2250.Z01.E.

**Eligible private loans:** Loans to households - Loans for house purchases + Loans to nonfinancial corporations.

**Loans:** Loans to households + Loans to nonfinancial corporations.

**Government Bonds:** *Holdings of Debt securities issued by euro area General Government reported by MFIs excl. ESCB in the euro area (stocks)*. Source: ECB SDW. Key: BSI.M.U2.N.A.A30.A.1.U2.2100.Z01.E.

**Reserves:** *Loans vis-a-vis the Eurosystem reported by MFIs excl. ESCB in the euro area (stocks)*. Source: ECB SDW. Key: BSI.M.U2.N.A.A20.A.1.U2.1100.Z01.E.

**Deposits from households:** *Deposit liabilities vis-a-vis euro area households reported by MFIs excl. ESCB in the euro area (stocks)*. Source: ECB SDW. Key: BSI.M.U2.N.A.L20.A.1.U2.2250.Z01.E.

**Deposits from nonfinancial corporations:** *Deposit liabilities vis-a-vis euro area NFCs reported by MFIs excl. ESCB in the euro area (stocks)*. Source: ECB SDW. Key: BSI.M.U2.N.A.L20.A.1.U2.2240.Z01.E.

**Deposits:** Deposits from households + Deposits from nonfinancial corporations.

**Capital:** *Capital and reserves reported by MFIs excl. ESCB in the euro area (stocks)*. Source: ECB SDW. Key: BSI.M.U2.N.A.L60.X.1.Z5.0000.Z01.E.

**Lending Rate:** *Bank interest rates - loans to corporations (outstanding amounts) - euro area*. Source: ECB SDW. Key: MIR.M.U2.B.A20.A.R.A.2240.EUR.O.

**Deposit rate:** *Bank interest rates - deposits from households with an agreed maturity (on outstanding amounts) - euro area*. Source: ECB SDW. Key: MIR.M.U2.B.L22.A.R.A.2250.EUR.O.

**Policy Rate:** Deposit Facility Rate. Source: ECB website.\(^{27}\)

**Government Bond Yield:** Euro area 10-year Government Benchmark bond yield - Yield. Source: ECB SDW. Key: FM.M.U2.EUR.4F.BB.U2_10Y.YLD.

### B Partial equilibrium model calibration

I have seven objective equations (three equilibrium conditions and four targets from empirical estimates):

\[
g_1(\theta) = 100 \times ((1 + \bar{i}^L) - \mu^L (1 + \bar{i}^R - C^R(\bar{A}, \bar{R}, \bar{F})))
\]

\[
g_2(\theta) = 100 \times ((1 + \bar{i}^D) - \mu^D (1 + \bar{i}^R - C^R(\bar{A}, \bar{R}, \bar{F}) + \theta^D))
\]

\[
g_3(\theta) = 100 \times (\bar{i}^R - \bar{i} - C^R(\bar{A}, \bar{R}, \bar{F}) + C^A(\bar{A}, \bar{R}, \bar{F}) - \theta^\nu)
\]

\[
g_4(\theta) = 100 \times \left(0.028 - \frac{\partial L}{\partial \theta} \frac{\partial O}{O} \right)
\]

\[
g_5(\theta) = 0.3 - \frac{\partial R}{\partial \theta} \frac{\partial O}{O}
\]

\[
g_6(\theta) = 100 \times \left(0.037 - \frac{\partial A}{\partial \theta} \frac{\partial A}{O} \right)
\]

\[
g_7(\theta) = -0.35 - \frac{\partial A}{\partial A^{QE}}
\]

where the upper bar refers to the historical average for the variable.

Define:

\[
g(\theta) = [g_1(\theta); g_2(\theta); g_3(\theta); g_4(\theta); g_5(\theta); g_6(\theta); g_7(\theta)]
\]

The objective function to be minimized is then \(g(\theta)'g(\theta)\).

### C Parameter sensitivity to targets

Table 6 presents calibration results using alternative targets (in bold). Loan growth after a TLTRO shock of 1tn€ is assumed to be between 2.5% and 3.9% as estimated in Altavilla et al. (2020) where the baseline is 2.8%. The growth rate of government bonds is between 1.3% and 5.2%, thereby covering the wide uncertainty related to this elasticity.\(^{28}\) The share of TLTRO funds stored in banks’ own reserves accounts was estimated by Barbiero et al. (2021) to be between 20% and 50%. Finally, in this exercise


\(^{28}\) 1.3% is obtained by halving the lower estimate of loan growth (2.5%), while 5.2% is calculated by multiplying the upper estimate for loan growth (5.2%) by 1.33 which is the coefficient used in the baseline exercise.
Table 6: Sensitivity of the calibrated parameters to the targets

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<td>2.8</td>
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Notes: This table reports the results of the calibration exercise with different assumptions on the targets. Targets are reported in bold when different from the baseline. Each column represents a different calibration exercise. Parameter $\varepsilon^D$ is always assumed to be equal to -275 and $\gamma$ to be equal to -0.36%. $\frac{\partial L}{\partial O}$ and $\frac{\partial A}{\partial O}$ are expressed in percentage points.

The share parameter $\alpha$ in the CES aggregator of the cost function tends to be quite stable, ranging between 0.70 and 0.95. The lowest levels are associated to a strong impact of TLTROs or QE on bond growth, or to a weak impact of TLTROs on reserves. Loan demand elasticity $\varepsilon^L$ takes values between 151 and 205, and mainly depends on the targeted loan growth after a TLTRO shock. The benefit of issuing deposits $\theta^D$ and the cost of holding bonds $\theta^p$ all range between 0.004 and 0.008. The elasticity parameter in the cost function $\gamma$ ranges between 0.11 and 3.82, where the highest values are associated to very negative values of $\rho$ (which is the parameter governing the elasticity of substitution between bonds and reserves in the CES aggregator and ranges between -3.75 and 0.45). These parameters tend to get bigger in absolute value when the target change in reserves after TLTROs and the target change in bonds after QE become very different. Finally, the scale parameter $\kappa$ adjusts according to the magnitude of the other parameters.

D Robustness of partial equilibrium results

Table 7 presents the impact of 1tn€ TLTRO and QE shocks in the model calibrated with different combinations of the parameters that have been presented in Table 6. The pass-through of TLTROs to loans ranges between 0.25tn€ and 0.38tn€, while leakage from government bonds ranges between 0.03tn€ and 0.09tn€. The change in reserves takes values between 0.06tn€ and 0.36tn€ while most of the calibrations return values above 0.18tn€. Leakage from deposits varies between -0.36tn€ and -
0.49tn€. Regarding QE, the impact on loans is between 0.03tn€ and 0.16tn€ with an associated change in government bond holdings ranging from -0.28tn€ and -0.53tn€. The maximum decline in deposits is -0.20tn€ while some calibrations return a figure of -0.06tn€.

Table 7: Impact of 1tn€ TLTRO and QE shocks - Robustness

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<th>Deposits</th>
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<td>TLTRO</td>
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<tr>
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<td>0.27</td>
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<td>0.03</td>
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</tr>
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</table>

Notes: This table reports the impact of 1tn€ TLTRO and QE shocks on loans, government bonds, reserves and deposits for various alternative combinations of the parameters outlined in Table 6. All the changes are expressed in tn€.