Unequal Assignments to Public Schools and the Limits of School Choice*

Mariana Laverde†

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Abstract

This paper studies the limits of school choice policies in the presence of residential sorting. Using data from the Boston Public Schools choice system, I show that white pre-kindergarteners are assigned to higher-achieving schools than minority students, and that cross-race school achievement gaps under choice are no lower than would be generated by a neighborhood assignment rule. To understand why choice-based assignments do not reduce gaps in school achievement, I use rich data on applicants' rank-order choices to estimate preferences over schools, and consider a series of counterfactual assignments. I find that between 60% and 70% of the gap in achievement at the schools assigned to black and hispanic students relative to those assigned to white students is explained by travel costs to high-performing schools. Differences in preferences for schools explain about 30% of the gap, while algorithm rules have no significant effect. Importantly, if black and hispanic parents faced the average travel costs of white parents, the improvement in school achievement for minority students would be coupled with school assignments that are on average preferred to these students.

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†University of Chicago Harris School of Public Policy. Email: mlaverde@uchicago.edu

1 Introduction

Since the late 1980s, many cities across the United States have adopted centralized school choice systems.[1] These systems allow families a choice among public schools, as opposed to neighborhood assignments where school districts assign students to schools based on proximity to residences. Since typically lower-achievement schools are in low-income areas populated by racial and ethnic minorities, neighborhood assignments replicate residential segregation and sustain educational inequality across racial and income groups. By decoupling residences and schools, choice systems are believed to create opportunity for desegregation and equal access to educational quality. As Boston Public Schools' superintendent wrote in the proposal for the 1988 choice plan: "My overall goal is to create a student assignment plan that provides all Boston students with high-quality desegregated education" (Boston Desegregation Project 1988).

This paper asks how effectively choice systems reduces cross-racial gaps in access to quality education relative to a geographic assignment, and why. Using assignments data from Boston Public Schools (BPS), I begin by showing that under Boston's choice system white pre-kindergartners are assigned to schools with higher achievement than black and hispanic students. Moreover, average achievement of the schools assigned to white, black, and hispanic students is the same as that generated under a neighborhood system where students are assigned based on proximity to schools.[2] School choice assignments are identical to neighborhood assignments for about 20% of students. The remaining students are on average assigned to schools with marginally higher achievement, with the highest gains concentrated in the white population.[3] As a consequence, choice does not translate into more access to high-performing schools for hispanic and black families, nor does it result in reductions in the achievement gap at the assigned schools compared to white students.

An effective policy response to the above depends on why the effects of choice are limited. I argue

[1] According to the non-profit *Education Commission of the States*, 47 states plus the District of Columbia have passed laws to allow or mandate a version of school choice. School districts that have implemented open enrollment include New York, Boston, Cambridge, Charlotte, and New Haven

[2] I generate a neighborhood assignment matching students to schools in proximity order while taking into account school capacities. Specifically, I run a DA algorithm where preferences and priorities are fully determined by distance.

[3] I use information of students assigned to schools in the first round of applications. Since there are more seats than students assigned in the first round, average improvements for all students are possible.

cross-race differences in choice-based assignments may stem from (i) differences in travel costs, (ii) differences in preference for schools, or from (iii) assignment rules that generate different probabilities of assignment conditional on parents' preferences. To distinguish between these channels, I combine detailed application data from BPS with a structural model of school demand to estimate racial-specific preferences for schools. I use the estimated parameters to generate counterfactual exercises that quantify the contribution of various mechanisms to the observed school assignments. In each exercise, I estimate the reduction in the gap in average school achievement between the schools assigned to white students and black or hispanic students.

The mechanisms that explain the differential assignments of white, black and hispanic students need to stem from either differences in the demand of high-achieving schools, or from assignment rules that generate different probabilities of assignment conditional on parents' preferences. In my model, parents' demand for schools is determined by two main components. First, the distance between the school and the families' residence, and second, the value parents place in all other school characteristics. The former determines the travel cost to a school. The latter is the location-independent value of a school; that is, the attractiveness of a school that is independent of distance to the students' residence.

Assignment rules also depend on the residential location of students. There are two ways in which this happens. First, school districts typically prioritize students for assignment based on proximity to schools. This means that students that live closer to high-achieving schools are more likely to get assigned to these schools.[4] Second, school districts can restrict the menu of schools parents can apply to based on closeness to a students' residence.[5] These rules may reduce the probability that black and hispanic students get assigned to high-achieving schools if these schools are sufficiently far away.

To disentangle the three mechanisms, I use detailed data on all first-round applicants to a seat in pre-kindergarten in BPS between 2010 and 2012. The data includes the rank-order list of

[4] Dur et al. (2018) show that having a proximity priority under the precedence order used in Boston, does not importantly increase the fraction of students admitted with this priority. This means that living in the walk-zone of a schools does not have a big impact on the probability of being assigned to that school.

[5] Restricting school menus based on geography is not very common across school districts. BPS has had this type of restrictions since the early 1990s.

schools submitted to BPS and the residential location and race of each applicant. I first estimate group-specific preference parameters from a random utility model using the rankings submitted by parents to BPS[6]. In Boston, parents submit rank-order lists of schools to the school district, who then uses these rankings and a set of school priorities to generate assignments using the student-proposing Deferred Acceptance (DA) algorithm (Gale and Shapley 1962). The structural demand model allows me to separately identify parents' assessment of travel costs, and the valuation of school attractiveness net of this cost. Using these parameters I simulate counterfactual rankings and assignments, after changing the location-independent preferences of schools, the travel costs, or the assignment rules. Boston is a good setting to estimate the demand for schools since the DA does not reward strategic play, and the district does not impose limits on the length of the rankings submitted. Mechanisms that do not meet these criteria may not generate truthful reports (Abdulkadiroglu et al. 2005, Haeringer and Klijn 2009, Calsamiglia et al. 2010)

In a first counterfactual, I simulate submitted rankings and assignments for black and hispanic students after a change in residential location. Specifically, new locations for black and hispanic students are randomly drawn from white students' residences. In each new location, I use the estimated parameters to generate rankings and subsequently assignments using the DA algorithm. I change the residential location of one student at a time, to make sure I can sustain the assumption that schools and hence preferences are unchanged. This counterfactual parallels the Moving to Opportunity (MTO) experiment that relocated families from high-poverty neighborhoods to low-poverty communities in the late 1990's.[7] Results from the counterfactual I propose show first-order implications of a change in residential location within a city.

Changing the residential location of a student is a bundled treatment. Students that are relocated face different travel costs to high-achieving schools, while assignment rules that are location-specific impact the probability of assignment to schools with higher achievement. To disentangle between the effect of assignment rules and travel costs, in a second counterfactual I independently vary assignment rules. Specifically, I first generate assignments assuming that there are no restrictions over choice menus, and later consider the case where proximity priorities are eliminated.

[6] Similar model are estimated in Abdulkadiroglu et al. (2017) and Pathak and Shi (2013)

[7] Papers that study the impacts of this experiment include Ludwig et al. (2013), Chetty et al. (2016), Katz et al. (2001), Kling et al. (2007), Clampet-Lundquist and Massey (2008)

Finally, I simulate assignments under a change in the location-independent preferences for schools. In these counterfactuals, I generate assignments where black and hispanic students take white students' preference parameters, while the original location of each student's residence is unchanged. Results from this counterfactual highlight how preferences for location-independent school characteristics impact the observed gap. Differences across races in these preferences may capture trade-offs made by parents between demographic and academic school characteristics, as well as any other dimensions of preference heterogeneity.

I find that after a change in residential location the gap in achievement at the schools assigned to treated students and white students reduced by about 60% to 70% relative to the original gap. A change in the location-independent preferences of schools explains 30% of the original gap. Finally, eliminating proximity priorities and choice menu restrictions does not have any effect. This suggests that the impact of a residential location change is fully explained by changes in travel costs to high-achieving schools.

The salience of travel costs on the resulting school assignments has important policy implications. It suggests that school choice alone does not always mitigate the undesirable effects of residential sorting and that there may be gains from coordinating the efforts of school and housing authorities. Increasing investment in schools close to constrained students, while guaranteeing housing affordability can increase access to quality education and possibly reduce school segregation if less constrained students react to these investments. Alternatively, policies that incentivize residential desegregation can lead to more equity in schooling.

This paper contributes to the empirical literature that studies the impact of heterogeneity in ranking behavior on the results from school choice mechanisms (Hastings et al. 2009, Borghans et al. 2015, Glazerman and Dotter 2017, Oosterbeek et al. 2019, Burgess et al. 2015). Related to my findings, Hastings et al. (2009) find that black families in Charlotte trade-off high school performance with a low fraction of same-race peers. The authors show that this trade-off hinders the competitive pressures that are believed to deliver system-wide school improvements under choice. My work highlights the trade-off between distance and performance. I find that this trade-off undermines the equity goal of the policy, and I show it has sizable consequences relative to the effect of preference heterogeneity. My analysis complements evidence from Glazerman and Dotter (2017) by generating estimates of the contribution of several channels to the observed heterogeneity. This literature is under the umbrella of a broader set of papers that study theoretically and empirically the implications of school choice[8] on sorting and stratification (Epple and Romano 1998, Hsieh and Urquiola 2006, Altonji et al. 2015, MacLeod and Urquiola 2015).

Moreover, this paper shows that choice systems alone may not be sufficient to create opportunity for residents of *low-opportunity neighborhoods*. Growing up in these areas has an important impact on adult earnings and education (Chetty and Hendren 2018), and some of these effects are related to access to school quality (Laliberte 2018). This paper shows that guaranteeing access to school quality for these populations requires not only including high-quality choices in their menus but having quality choices close to home. As a result, choice systems would benefit from parallel policies that aimed to reduce residential segregation. An example of such policies includes those proposed by Bergman et al. (2019).

Finally, my analysis adds to a recent series of studies leveraging preference data from centralized school assignments to study school demand (Hastings et al. 2009, Borghans et al. 2015, Abdulkadiroglu et al. 2017, Abdulkadiroğlu et al. 2017, Glazerman and Dotter 2017, Kapor et al. 2018, Agarwal and Somaini 2018, Luflade 2018). Some of these papers study parents' demand under mechanisms that provide incentives to misrepresent preferences, while others study the welfare and fairness associated with assignment rules that give parents incentives to strategize relative to strategy-proof mechanisms. I build on previous work by using data from a strategy-proof mechanism with no restriction on list length, to rationalize differences in assignments across racial and ethnic groups.

My analysis focuses on studying differences in average achievement at the schools assigned to white, black, and hispanic students. Average achievement is a bundled measure of the academic ability of the students a school enrolls, and of the capacity of a school to generate improvements in student outcomes; that is the effectiveness of a school. In this paper, I am not able to speak of differences in effectiveness as opposed to peer composition, and how gaps in achievement map onto these. Nevertheless, schools that enroll high-achieving peers have been found to be more effective (Abdulkadiroglu et al. 2017). This suggests that higher achievement may be correlated with school

[8] In these set of papers, school choice is broadly defined to include vouchers and other forms of choice.

effectiveness.

The rest of the paper is organized as follows. Section 2 discusses the institutional context and the data restrictions,and it also summarizes the main observed differences in application behavior and assignments across races. Section 3 presents reduced form evidence on the mechanisms. Section 4 presents the model used to recover demand parameters, discusses the assumptions and analyzes the results. Section 5 describes the methodology and assumptions made to run counterfactual exercises and the results. I conclude in Section 6.

2 Elementary School Choice in Boston

2.1 The Assignment Mechanism

Parents of students entering pre-kindergarten in Boston rank schools from a set of eligible schools determined by the students' residential location. During the study period, Boston was divided into three zones shown in Figure 2a. There are about 24 schools that offer a pre-kindergarten program in each zone, and each school has multiple programs. Most schools have at least one general education program and some have programs for English language learners.[9] Students are eligible for any general education program in their residence zone, plus any within a mile from their home. There are also a handful of city-wide schools that can accept applications from all over the city. I refer to the set of schools a parent can rank as the parents' choice-menu.[10] Importantly, parents can rank as many schools as they want to.

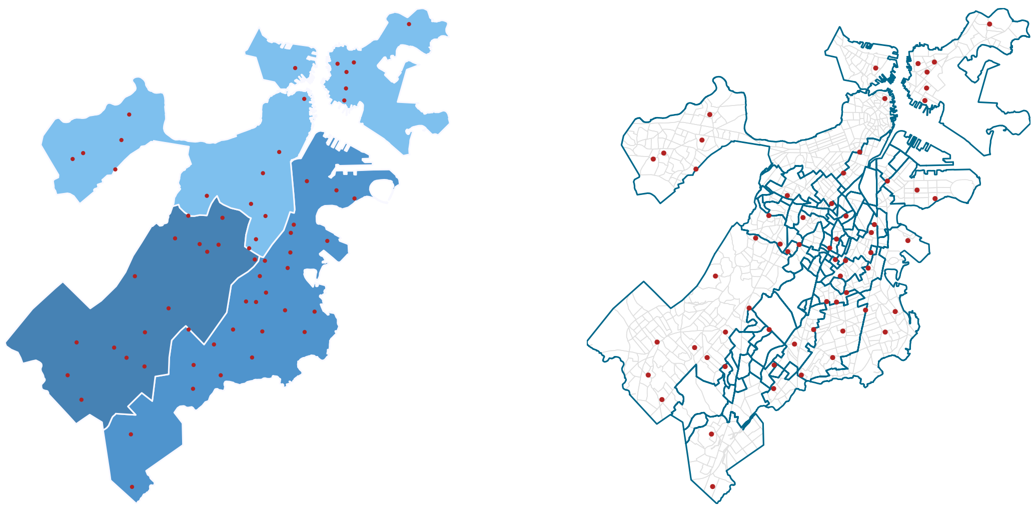
Students are prioritized for admission at each school using a priority structure determined by the school district that is common across schools. Under this priority, students having a sibling at a school are prioritized over students that do not have a sibling. Also, students that live within a mile

[9] There are also programs for substantially-separate special education students but these students undergo an assignment process that does not follow the choice assignment

[10] Eligibility criteria for applying to English language programs includes not being a native English speaker and score below a threshold in a BPS administered language test. There are also geographic restrictions similar to those of general education programs. Pathak and Shi (2013) discusses how geographic eligibility restrictions may not be binding for language programs. Given this I assume, as Pathak and Shi (2013) do, that English language learner students can apply to any program across the city

Figure 1: North, West and East Zones and Choice Menus

(a) Zones (b) Students with the same Choice Menu



Note: Figure built using data from Boston Public Schools. Red points are schools with a pre-kindergarten program in 2010.

of a school -usually called the walk-zone- have priority over students that live farther. Specifically, the first priority is given to students that both have a sibling and live in the walk-zone. The second priority is for students who have a sibling, and third priority for those that live in the walk-zone. In each group, ties are broken with a random number that guarantees priorities generate a strict ordering of students.

The assignment mechanism is a version of Gale and Shapley's (1962) student-proposing DA (Abdulkadiroglu et al. 2005; Pathak and Sonmez 2008). This algorithm uses ranks submitted by parents and the described priorities to generate an assignment as follows:

- *Step 1:* Only first choices are considered. Applicants are sorted in priority order and applicants in excess of capacity are rejected. Those who are not rejected are provisionally accepted.

- *Step k:* For students rejected in step $k-1$, their next preferred option is considered. Each school ranks the set of provisionally admitted students jointly with those being considered in this step by priority order. The program provisionally admits those with the highest priority and rejects students in excess of capacity. The algorithm stops when every rank list has been exhausted or if there are no rejections.

Under the DA, parents have no incentive to misrepresent their true preferences (Dubins and Freedman 1981, Roth 1982). Incentives to strategize may generate rankings that respond not only to preferences but also to beliefs about admission chances. Moreover, restrictions to the length of submitted rankings may not generate truthful reports (Haeringer and Klijn 2009, Calsamiglia et al. 2010). Boston is a good setting to study parents' reports since it uses a strategy-proof mechanism with no restrictions to rank lengths.

The assignment generated by the DA is stable. This means that there are no bilateral trades that would make students better-off, and would respect priorities: If a student wants to transfer from school j to schools j', it must be true that the student has a lower priority at j' than all students assigned to that school (Gale and Shapley 1962, Abdulkadiroglu et al. 2005).

2.2 Data

I use data from BPS that covers the universe of first-round applicants to pre-kindergarten programs for the years 2010 to 2012. Over 80% of students are assigned in the first round (Pathak and Shi 2017). Data includes submitted ranks, the assigned school, the position of the assigned school in the submitted ranking, and the priority that generated the assignment. I also observe the school that each student enrolled in, as well as a set of demographic characteristics that includes the residential location[11], race, language spoken at home, and whether the students is an English language learner.[12]

I use yearly data on school characteristics from the Massachusetts Department of Education (DOE). This includes data on the racial makeup of each school, the fraction of low-income students enrolled in Kindergarten[13]; and test results from the Massachusetts Comprehensive Assessment System (MCAS), specifically the fraction of students scoring *advanced or proficient* in math and English

[11] I observe the geocode of residence. Geocodes are a partition of the city in 868 polygons of average area of 0.1 sq. miles

[12] I remove from my sample students with an invalid geocode (2% of the sample)

[13] Income status is measured by the DOE

tests by grade.[14]

Using the location of each school and the geocode of residence of each student, I measure the distance to each school as the linear distance between the geocode's centroid and the school. I recreate the algorithm used by BPS to generate walk-zone priority status for each student-school pair: student i is in the walk-zone of school j if a one-mile radius from school j intersects the geocode of residence of i. Similarly, I define the choice-menu of each student using data on the zone in which each school and geocode lies.

Table 1: Descriptive Statistics: Applicants

	All	*Black*	*Hispanic*	*White*
Applicants	6,358	21.0%	46.4%	21.8%
Applications				
Size of Choice-Menu	24.5	26.0	24.7	22.9
Length of Submitted List	5.4	5.8	5.4	5.1
Assignments				
Rank of Assigned School	2.0	2.1	2.0	1.9
Distance to Assigned School	1.2	1.3	1.3	0.9
% Unassigned Students	30.0	24.0	30.0	34.0

Ideally, I would have the sibling priority status of every student at every school. Nevertheless, I only observe the sibling priority status of student i at school j, if i was assigned to j with this priority. Throughout the analysis, I assume that all students that are not assigned with a sibling priority do not have a sibling priority at any school, and that students assigned with a sibling priority at j do not have a sibling priority at other schools. An analysis of rankings and assignments reveals that for most of the schools this is a good assumption. In most schools, students with a sibling priority were not rejected. Only for a handful of schools, we can't rule out this happened. This means that in the set of schools that each student finds acceptable the assumption is likely to be correct. If students have a sibling priority in multiple schools, I would only be able to account for the priority

[14]This test is administered to students starting in 3rd grade

at the school ranked higher.

The sample has $6,358$ applicants to pre-kindergarten between 2010 and 2012. Close to half of the applicants to pre-kindergarten in Boston are hispanic, while black and white students are around one-fifth of the sample each.[15] Choice-menus have on average 25 schools, and students rank on average 5 options. Black students submit longer lists while white students submit shorter lists. Pre-kindergarten attendance is not mandatory, in consequence, there are applicants who won't be assigned to any school. About 30% of the students that apply in the first round are unassigned. Unassigned students can re-apply in a subsequent round. Out of all unassigned students, about 80% do not enroll in any public school.

The set of pre-kindergarten applicants is not representative of Boston population. While less than one-fifth of Boston residents are Hispanic, almost half of applicants to pre-kindergarten are Hispanic. On the contrary, white students seem to apply to public pre-kindergarten programs less often. They represent half of Boston population, but only about 20% of applicants. African-Americans make up around 20% of Boston residents and pre-kindergarten public school applicants.

Between 2010 and 2012, there were a total of 67 public schools that offered a pre-kindergarten program. Not all schools were in all years. The schools are far from being homogeneous in terms of demographics and achievement. The average share of 3rd grade students scoring *advanced or proficient* in math is 44%. There is high variance across schools, the school with the lowest achievement had 2% of students scoring *advanced or proficient* in math while for the highest-performing school the fraction was close to 90%. On average schools have 32% of black students and 15% of white students. Since both white and black students represent about 20% of all applicants, this means that there are several schools with a high concentration of black students, while there are several schools with a low fraction of white students. Each school has on average 70% of low-income students, and the school with the lowest fraction of low-income students has 8%.

[15] Asian students are around 7% of the applicants in my sample. Due to the small sample size, I do not estimate preferences for this population, nor I analyze their assignments.

Table 2: Descriptive Statistics: Schools

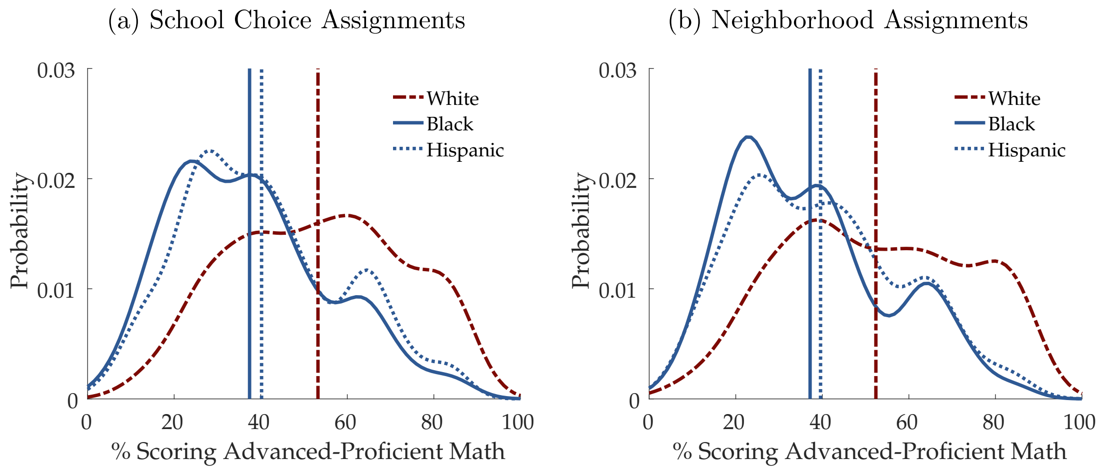
	Mean	*St. Dev.*	*Min*	*Max*
Capacity	31.3	16.5	6.0	108.0
Achievement				
% Scoring Advanced-Proficient Math	44.0	19.2	2.0	86.0
% Scoring Advanced-Proficient English	38.4	15.9	10.0	86.0
Demographics				
% Black Students	32.4	19.3	2.1	79.7
% White Students	14.5	14.8	0.0	65.8
% Hispanic Students	43.8	19.0	14.3	90.8
% Low-Income Students in Kindergarten	69.9	18.5	8.3	96.3
Observations		189 (68 distinct schools)		

Note: I do not observe achievement data for all schools in all years. There are a total of 17 missing observations (school-year pairs) of schools that do not offer third grade or for which data is restricted due to a small set of test takers.

3 Reduced form Evidence: The importance of Geography and Preferences

Between 2010 and 2012, white, black and hispanic pre-kindergarteners in Boston were assigned to schools that had different levels of achievement and demographic composition. I measure achievement at school j for assignments in the school-year t, as the fraction of 3rd-grade students scoring *advanced or proficient* at the math MCAS tests in $t-1$. Equivalently, the demographic characteristics of a school will be measured one year prior to the assignments. These measures approximate the status of schools before parents submit their applications. While white students were assigned to schools where more than half of students scored *advanced or proficient*, these measures were close to 40% for black and hispanic students (Figure 3a). In terms of demographics, white students were assigned to schools with near 60% of low-income kindergarteners, for hispanic and black students the percentage is closer to 80%.

Figure 3: Distribution of School Achievement under School Choice and Neighborhood Assignments



Note: Distribution of school achievement for students assigned to general education programs between 2010 and 2012, and counterfactual distributions built if these students were assigned to the school closest to their homes.

Surprisingly, these statistics barely change if these students were instead assigned to their neighborhood schools. Using the location of each student assigned to pre-kindergarten during this period, and the location and capacity of each school, I generate an alternative allocation that resembles a neighborhood assignment.[16] Figure 3b shows the distribution and average achievement at the schools where white, black and hispanic students would have been assigned under this alternative scenario. The plots not only look very similar, but for each group, I cannot reject the null hypothesis that the average achievement is equal across assignments.[17]

Around 20% of students are assigned to the same school under the neighborhood and choice assignments. This number is larger for white students and smaller for black students. The students not assigned to their neighborhood school under choice are assigned to schools that are on average 1 mile farther than their neighborhood schools, and have around 1 pp higher achievement. The average black student travels 1.3 miles more and is assigned to a school with 0.3 pp higher achievement. Under choice, white students have the largest gain in achievement for every extra

[16] I generate this assignment running a DA where preferences and priorities are determined by proximity: students prefer schools closer to home, and schools prioritize students that live closer to schools.

[17] I cannot reject the null hypothesis that the means are equal using a t-test where I allow for differences in the variance across groups. Two tail p-values are 0.3, 0.6 and 0.3, for white, black and hispanic students, respectively

mile traveled. Black students seems to be selecting into schools with lower poverty rates under choice, while hispanic students are assigned to schools with fewer black students (Table 3).

Table 3: Descriptive Statistics: Neighborhood Assignments and DA Assignments

	All	White	*Black*	*Hispanic*
% of Students Assigned to the Same School	23.3	31.1	17.6	21.7
Students Assigned to a Different School				
Achievement - Neighborhood	40.9	50.8	36.3	39.1
Achievement - DA	42.2	52.1	36.6	39.8
Distance - Neighborhood	0.5	0.5	0.4	0.4
Distance - DA	1.6	1.3	1.7	1.7
% Low-Income in Kindergarten - Neighborhood	71.9	59.1	77.8	74.5
% Low-Income in Kindergarten - DA	70.7	57.3	75.7	74.3
% Black Students - Neighborhood	34.3	24.4	43.3	34.2
% Black Students - DA	32.6	23.5	42.5	31.1
% White Students - Neighborhood	13.4	24.3	7.3	11.9
% White Students - DA	14.7	26.4	8.5	12.7

This exercise suggests that giving parents a choice does not translate into more equitable access to high-achieving schools. This can be explained by cross-race differences in ranking behavior, or by assignment rules that favor access to high-achieving school to some students over others.

Differences in ranking behavior can stem from differences in preferences for school attributes that are independent of school location, or from different costs expressed in distance traveled to high-achieving schools. Since schools are bundles of characteristics, when parents decide what schools to rank they need to make concessions across these. If parents have a different valuation for any single school characteristic, the resulting rankings across races may look very different. For instance, Hastings et al. (2009) argue that black students in Charlotte rank high-achieving schools less often, because these students face a trade-off between higher achievement and a low fraction of same-race peers. On the other hand, if black and hispanic students had to travel farther to high-achieving schools, we would likely see fewer of these schools high in their rankings. These explanations, although observationally similar have different policy implications.

Eligibility restrictions and walk-zone priorities are assignment rules that have the potential to generate inequities in the likelihood of assignment to high-achieving schools. If choice-menus of black and hispanic families have on average lower-achieving schools, or if white parents are more likely to have a walk-zone priority at high-achieving schools, white students will be more likely assigned to high-achieving schools conditional on the preferences reported.

In this paper I estimate the contribution of three mechanisms to the cross-race gap in school achievement. First, the contribution of cross-race differences in preferences for school attributes that are independent of schools location. Second, the contribution of cross-race differences in distance to high-achieving schools and finally, the contribution of assignment rules.

Figure 4 sheds lights on these mechanisms. While average achievement in choice-menus is similar across groups, once I account for proximity I find that schools in the walk-zone of white students' residences have higher achievement than those in the walk-zone of black and hispanic students' homes. Moreover, the schools in the walk-zone of white students' residences have on average higher achievement than the rest of schools in their choice-menu, the opposite is true for black and hispanic students. This means that geography may be an important explanation, whether it factors through walk-zone priorities of differences in access costs.

Schools ranked first by white students tend to have higher achievement than first choices of hispanic and black students. Differences in revealed preferences may stem from different valuations for school characteristics, or from different costs of accessing schools, expressed in the distance travelled. Table 4 shows evidence that longer distances between minority students and high-achieving schools may be an important part of the explanation[18].

4 Estimating Parent Preferences

In this section, I present the model and assumptions used to recover parents' preferences for schools. At the end of the section I discuss the estimated parameters.

[18] Related to this Walters (2018) finds suggestive evidence that in Boston charter middle schools tends to locate in lower-achieving areas of the city

Figure 4: Average Achievement in Choice-Menus,
Walk-Zones, First Choices and Assignments

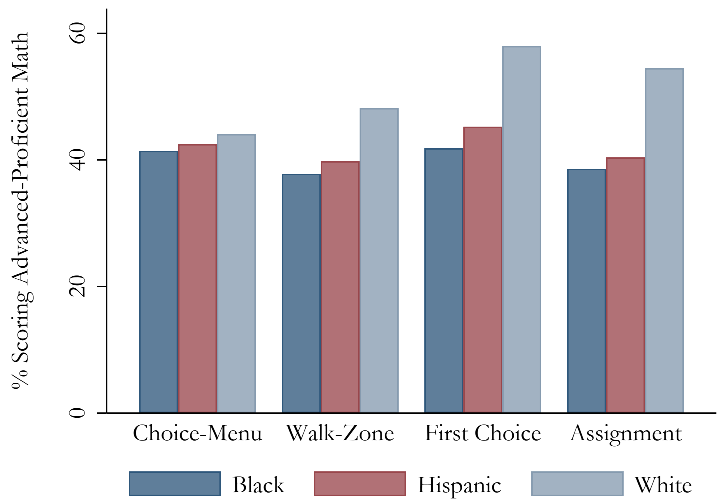


Table 4: Relation Between Distance to Schools and School Achievement

	School Achievement		
	White	Black	Hispanic
Distance	-1.39^{***}	1.60^{***}	0.46^{***}
	(0.07)	(0.07)	(0.05)
Observations	27,905	22,325	52,569

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: Each column shows a regression between school achievement and distance. Each observation is a pair student-school for schools in the choice-menu of every student. Standard errors in parenthesis.

4.1 Model and Identification

I model preferences using a random utility model where $i \in \mathcal{I}$ index students and $j \in \mathcal{J}$ index schools. Each student belongs to a group $r \in \{White, Black, Hispanic\}$, and $\mathcal{I}^r$ is the set of students in group r. These categories are exclusive as they are in the data. I refer to r as denoting race, following the convention used by BPS. The indirect utility of student i of race r from attending school j is:

$$U_{ij} = \delta^r_{jt} + \beta^r D_{ij} + X'_{ij}\gamma^r + \epsilon_{ij} \tag{1}$$

where δ^r_{jt} summarizes the racial-specific attractiveness of school j in year t that is independent of school location. D_{ij} is the linear distance from i's residence to j, measured in miles. The parameter β^r summarizes parents' preferences for proximity. Notice that each student i is observed only once in my sample, in consequence, each i is associated with a single t and a single r. I do not include these subscripts in all the variables and parameters in 1 for simplicity. The matrix X_{ij} includes three indicator variables, that capture within race individual heterogeneity. The first is an indicator whether student i has a sibling at school j, the second for whether i is a language learner and j offers a language program, and the third for whether j has a language program specializing in i's first language. ϵ_{ij} captures idiosyncratic tastes for schools. This is observed by the student but not by the econometrician. I assume ϵ_{ij} is iid. *T1EV*, with a scale parameter σ^r that I allow to vary across races.

Truth-telling. I assume that submitted rankings are truthful. This means that parents rank all acceptable schools in true preferences order. A school is acceptable if it has a higher value than the outside option. This assumption is motivated by the algorithm's incentive compatibility and the property that there are no restrictions on the number of schools parents can rank. Having restrictions over the length of submitted lists, even under strategy-proof assignments, can generate reports that are not truthful (Haeringer and Klijn 2009, Calsamiglia et al. 2010, Luflade 2018). Boston's choice system is one of some that satisfies both properties. This makes it a good setting to estimate the proposed model.

Truth-telling can be violated if admission outcomes are largely predictable. Under this setting, parents may misrepresent preferences by not ranking schools that are desirable but are perceived to have low admission probabilities. This is more likely to be a worry in settings where priorities for admission and their distribution across applicants are known before submitting applications, and historical cutoffs are observable to applicants (Fack et al. 2019).

Consistent with this assumption, if $R_i = (R_{i1}, \cdots, R_{il_i})$ is the rank-ordered list submitted by i and

$\mathcal{J}_i$ is the choice-menu of i then,

$$R_{i1} = \underset{j \in \mathcal{J}_i}{\arg\max} \quad U_{ij} \tag{2}$$

$$R_{ik} = \underset{j \in \mathcal{J}_i \setminus \{R_{im}: m<k\}}{\arg\max} \quad U_{ij} \tag{3}$$

Moreover, if U_{i0} is the utility of the outside option then,

$$U_{ij} > U_{i0} \quad \forall \quad j \in R_i \tag{4}$$

$$U_{i0} > U_{ij} \quad \forall \quad j \in \mathcal{J}_i \setminus R_i \tag{5}$$

The utility U_{i0} represents the expected utility of the alternatives if unassigned after the first round.

Identification. As is common in logit models, the parameters are identified modulo the scale parameter of the idiosyncratic shock, σ^r. Moreover, I normalize the utility of the outside option to zero, and in consequence the school mean-effects are estimated as deviations with respect to the outside option (Train 2009). Specifically, I estimate

$$\left(\frac{\delta^r_{jt} - \delta^r_{0t}}{\sigma^r}, \quad \frac{\beta^r}{\sigma^r}, \quad \frac{\gamma^r}{\sigma^r} \right) \qquad \text{for all } j, r, t \tag{6}$$

where δ^r_{0t} is the mean utility of the outside option.

Identification of the distance parameters relies on the assumption that ϵ_{ij} is independent of D_{ij} conditional on school j's fixed effect and X_{ij}. This means that families may sort into neighborhoods according to average tastes for observable and unobservable school characteristics and those in X_{ij}. The assumption will be violated if families sort according to idiosyncratic tastes ϵ_{ij}. In this case, the distance parameter may be biased downward showing that students care for distance more than they really do.

Two distinct sources of variation identify school mean utilities and preferences for proximity. Rankings of students who are equidistant to any two pairs of schools generate the variation used to identify school attractiveness. Students ranking schools farther over schools closer is the variation used to identify the preferences for proximity.

Estimation. I estimate utility parameters by maximum likelihood, using all the first-round rankings submitted to BPS between 2010 and 2012. Details about the likelihood function are shown in Appendix B.

4.2 Parameter Estimates

Table A.1 summarizes $(\hat{\beta}^r, \hat{\gamma}^r)$ for all races. Negative signs for the distance parameters mean that on average parents value proximity. The magnitude of these parameters is similar to that obtained by Pathak and Shi 2013 who carry out a similar analysis for a sample that overlaps mine.[19] Positive values of the parameters in γ^r mean that parents value that siblings go to the same school. Also, language programs are valued by language learner student parents.

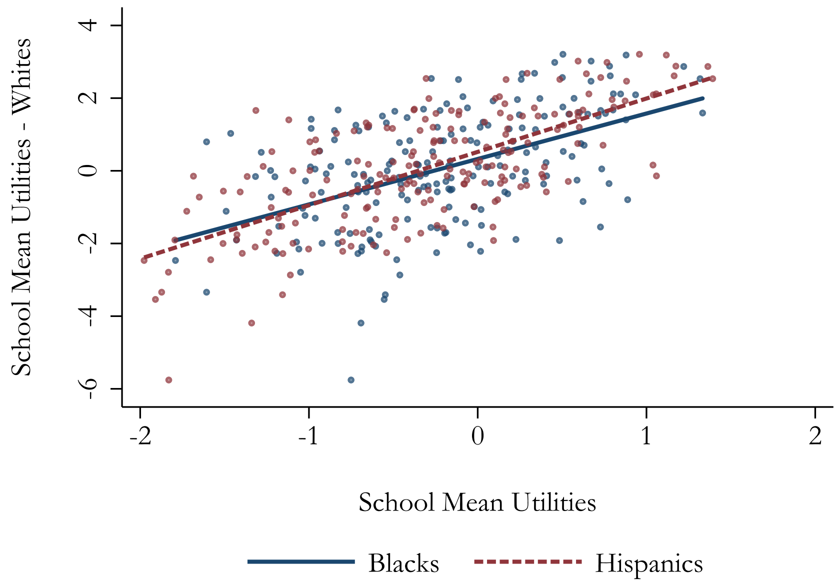
School mean utilities summarize the overall attractiveness of a school that is independent of distance. These parameters and their first moments cannot be directly compared across races, but the order generated does provide a way to assess commonalities in the valuation of schools across races. Figure 5 shows a positive correlation between the coefficients of white parents and parents of other groups. This suggests, there are underlying school characteristics that all groups value. Hispanic families parameters have a higher correlation with white families parameters than black families do.

Table A.6, shows that higher-achieving schools tend to have higher mean utilities. School demographics are related to shool mean utilities also. Schools with higher fractions of white students and a lower fraction of low-income students are more attractive.

Preference parameters cannot be directly compared across races. Doing so requires assuming some relation between the scale parameters σ_r. A way to asses how do preferences for proximity compare across groups without additional assumptions is to use the parameters of the model to simulate rankings, and evaluate how rankings change when the distance to a school is marginally increased. Results from this simulation are determined not only by the estimated preferences for distance, but also by the distribution of school mean utilities and other preference parameters, and the discrete

[19] The authors do not carry-out a race-specific model estimation. A weighted average of the race-specific parameters is similar to the values they obtain.

Figure 5: Correlation of School Mean Utilities δ_{jt}^{r}



Note: Scatter plot of school mean effects of black and hispanic students with white students' school effects. The correlation between hispanic and white students' parameters is 0.69, between black and white students parameters is 0.5 and between hispanic and black is 0.71.

nature of rankings.

Concretely, I generate a series of rankings from the parameters of my model and random realizations of the idiosyncratic taste shock. I compare simulations after increasing D_{ij} for all i by 0.1 miles to simulations generated with the original distances. Table A.8 shows the average number of positions a school would gain after running the exercise for every school in the sample. I find little evidence of cross-race differences in ranking elasticities with respect to distance. On average, a school is ranked 0.15 positions lower after the proposed distance change.

Fit. To evaluate the fit of the model I use the estimated parameters to generate rankings over schools for each student. Under the assumption that families will rank every acceptable school, the parameters of the model generate the ordering and the length of the lists. I use these simulated rankings to run the DA algorithm, and I compare the assignment generated with the simulated rankings to the assignment obtained with the rankings submitted by families to BPS. I find that the parameters closely predict the distribution of achievement at the schools where white, black and hispanic students are assigned to in 2011, as well as the distance to the assigned school for these groups (Figure A.4). For this year the simulated lists generated have an average length of 5.6 schools, while the average length of the rankings submitted to BPS is 4.7 in that year.

5 Counterfactual Assignments

In this section, I describe the counterfactual exercises, discuss the assumptions, and analyze the results. I consider three counterfactuals. First I study how assignments of black and hispanic students change if they lived in locations drawn from white students' residences. In a second counterfactual, I analyze how assignments of black and hispanic students would change if their preference parameters were equal to the preference parameters of white students. Now, the residential location determines (1) a students' choice menu, (2) the schools where a student has a proximity priority, and (3) the joint distribution of distance and school mean utilities. In consequence, results from the counterfactual where locations change includes the effect of these three channels. To disentangle between these, I run two additional counterfactuals where I estimate the effects of eliminating choice-menu boundaries and walk-zone priorities.

For each counterfactual exercise, I generate counterfactual rankings using the estimated preference parameters and random realizations of the idiosyncratic taste shock. The parameters of the model identify the order in which a truthful family would rank schools under alternative settings. Moreover, the model endogenizes the length of the list, under the assumption that families rank all acceptable schools.

5.1 Changing the location of a student

I described how first choices of white, black and hispanic parents are different, but also how location can affect submitted preferences by changing the availability of choices close to home. To disentangle the effect of location-independent preferences for schools from the bundle of effects that come with location, a researcher would ideally want to observe how assignments of black and hispanic students look if they lived in a different location, holding all else equal.

The framework I've built allows me to answer that question. Using the preference parameters described in section 4.2 I simulate submitted rankings after a location change. With these rankings, I generate a counterfactual assignment that I compare to the assignment generated in the original setting. Specifically, I simulate rankings and assignments in a counterfactual where an alternative residential location of one student -black or hispanic- is randomly drawn from the distribution of residences of white students. I relocate one student at a time and analyze the assignment assuming that school characteristics are unchanged. Moving a larger number of students may not be consistent with the assumption that schools are unchanged. The effects of relocation that I estimate are partial equilibrium effects, and need to be interpreted as such.

In practice, I proceed as follows:

1. Generate assignments under the original setting:
   (a) Take all applicants and schools from 2011. Using the parameters $(\tilde{\delta^r_{jt}}, \tilde{\beta^r}, \tilde{\gamma^r})$, and a realization[20] of $\epsilon = (\epsilon_{01}, \cdots, \epsilon_{0\mathcal{J}}, \epsilon_{11}, \cdots, \epsilon_{\mathcal{I}\mathcal{J}})$ generate rank-order lists, R_i for all i. The length of the submitted list is determined by the position of the outside option in the ranking: only schools preferred to the outside option are ranked
   (b) Using the ranking profile $R = (R_1, \cdots, R_{\mathcal{I}})$, generate an assignment running the DA
   (c) Repeat for m realizations of ϵ
2. Generate counterfactual assignments:
   (a) Generate random locations by pairing each black or hispanic student, i_b, in the 2011 sample, with k white students, i_w, in that year's sample[21]. Each pair (i_b, i_w) represents a location change for i_b. In the counterfactual, i_b will take i_w's choice-menu, walk-zone priorities, sibling priority, and distance to schools will be updated accordingly
   (b) Consider one pair (i_b, i_w). Under the counterfactual, all students will keep their location except for i_b
   (c) Consider only the same pair (i_b, i_w). For each realization of ϵ used in 1. generate rank-order lists assuming that each list has the same length of the ranking originally submitted

[20] Each ϵ_{ij} is drawn independently from a *T1EV* distribution with scale parameter 1.

[21] This is done generating random draws of white students with replacement

to BPS. Notice that for each realization of ϵ, the lists of all untreated students under the original and counterfactual will be equal

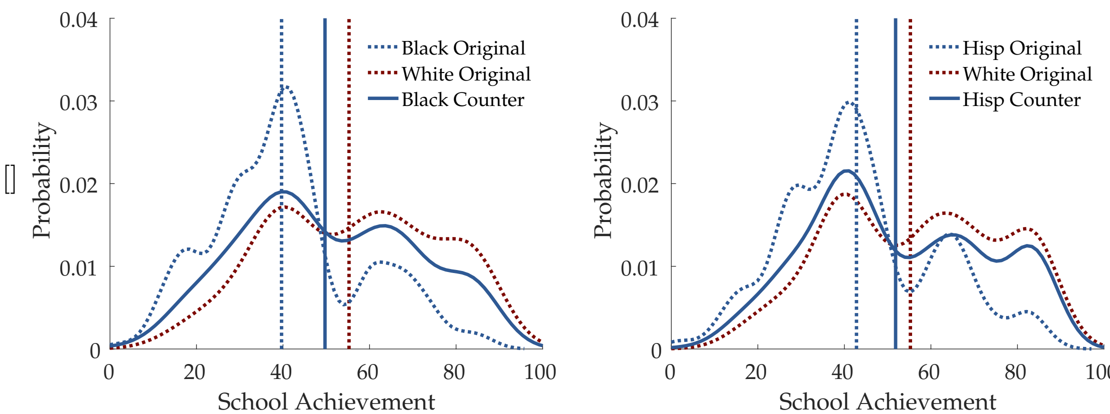
(d) For each profile R generate an assignment running the DA

(e) Repeat for all pairs (i_b, i_w)

For each simulated counterfactual assignment I recover the school assigned to the treated student and the characteristics of that school. After pooling together the assignments of all treated students, I compare the average achievement at the schools assigned under the counterfactual to the average achievement they would have gotten under the original setting. Finally, I evaluate the resulting gap under the counterfactual and the change relative to the original gap.

Figure 6: Change location of a student: Achievement at the Assigned School

(a) Change location of a black student (b) Change location of a hispanic student



Note: Distribution of achievement in schools assigned to black and hispanic students under a counterfactual assignment where they are randomly assigned to a new residence drawn from the distribution of whites' residences. This is compared to the distribution for black and white students in their original location.

In Figure 6, I compare the assignments of the pooled set of treated students who's locations changed, to the assignments of white and treated students under their original location. After changing the location of a black student, achievement at the assigned school increased from 41% to 51%. For hispanic students achievement increased from 43% to 52%. Recall that under the original setting, white students are assigned to schools with average achievement of 56%. This means that a change in location reduces the gap between black and white students by 63%, and by 69% between hispanic and white students.

5.2 Changing preference parameters

To study the contribution of differences in the location-independent value of schools, and other preferences, I run counterfactual assignments where black and hispanic students rank schools according to white students' preference parameters. In this counterfactual, residential locations and school locations are unchanged. For consistency with the previous counterfactual, I change the preference parameters of one student at a time.

To generate these assignments I proceed as follows:

1. Generate assignments under the original setting:
   (a) Take all applicants and schools from 2011. Using the parameters $(\tilde{\delta^r_{jt}}, \tilde{\beta^r}, \tilde{\gamma^r})$, and a realization[22] of $\epsilon = (\epsilon_{01}, \cdots, \epsilon_{0\mathcal{J}}, \epsilon_{11}, \cdots, \epsilon_{\mathcal{I}\mathcal{J}})$ generate rank-order lists, R_i for all i. The length of the submitted list is determined by the position of the outside option in the ranking: only schools preferred to the outside option are ranked
   (b) Using the ranking profile $R = (R_1, \cdots, R_{\mathcal{I}})$, generate an assignment running the DA
   (c) Repeat for m realizations of ϵ
2. Generate counterfactual assignments:
   (a) Take a student $i_b \in \mathcal{I}^b$. This will be the treated student
   (b) Replace the values of the parameters $(\tilde{\delta^b_{jt}}, \tilde{\beta^b}, \tilde{\gamma^b})$ for $(\tilde{\delta^w_{jt}}, \tilde{\beta^w}, \tilde{\gamma^w})$ only for i_b
   (c) For each realization of ϵ used in 1. generate rank-order lists for all students assuming that each list has the same length of the ranking originally submitted to BPS. Notice that each realization of ϵ, the lists of all untreated students under the original and counterfactual will be equal
   (d) For each profile R generate an assignment running the DA
   (e) Repeat for all $i_b \in \mathcal{I}^b$ and $i_h \in \mathcal{I}^h$

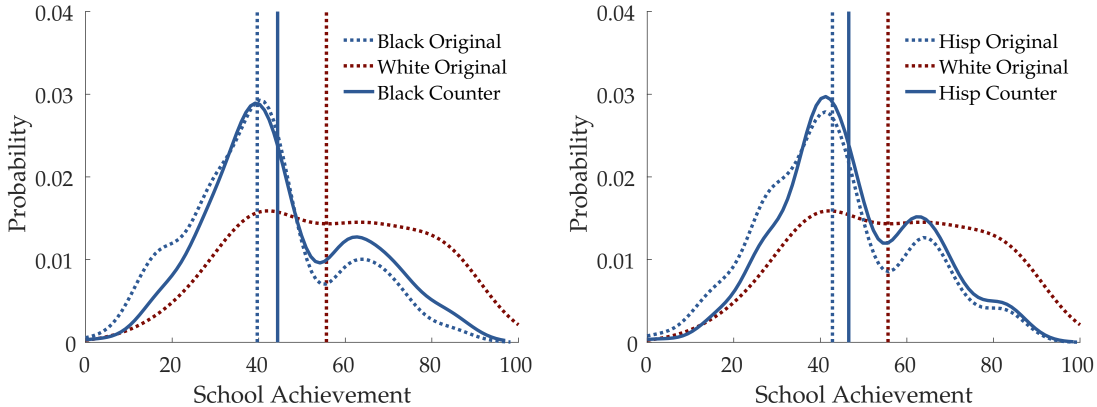
Achievement at the assigned schools increases for black and hispanic students after a change in preference parameters. For black students, achievement increased from 41% to 46%. For hispanic

[22] Each ϵ_{ij} is drawn independently from a *Gumbel* distribution with scale parameter 1.

students, achievement goes from 43% to 47%. This means that a change in preferences reduces the gap between black and white students, and hispanic and white students by 30%.

Figure 7: Change preferences of a student: Achievement at the Assigned School

(a) Change preferences of a black student (b) Change preferences of a hispanic student



Note: Distribution of achievement in schools assigned to black and hispanic students under a counterfactual assignment where these students have the preference parameters of white students. This is compared to the original distribution of school achievement for black, hispanic and white students.

5.3 Eliminate Choice Menus and Walk-zone Priorities

The effects of a change in the location include the effects of a change in the choice-menu, the walk-zone priorities, and the joint distribution of school effects and distance. To disentangle between the effects of each of these, I run two additional counterfactuals. In the first counterfactual I eliminate choice-menu boundaries, this means that I allow students to rank schools from all over the city. In the second counterfactual, I eliminate walk-zone priorities. Under this counterfactual rank lists won't change, differences in the assignments are generated only by differences in priorities.

Specifically, I proceed as follows:

1. Generate assignments under the original setting:

   (a) Take all applicants and schools from 2011. Using the parameters $(\tilde{\delta}^r_{jt}, \tilde{\beta}^r, \tilde{\gamma}^r)$, and a realization[23] of $\epsilon = (\epsilon_{01}, \cdots, \epsilon_{0\mathcal{J}}, \epsilon_{11}, \cdots, \epsilon_{\mathcal{I}\mathcal{J}})$ generate rank-order lists, R_i for all i. The

[23] Each ϵ_{ij} is drawn independently from a *Gumbel* distribution with scale parameter 1.

length of the submitted list is determined by the position of the outside option in the ranking: only schools preferred to the outside option are ranked

(b) Using the ranking profile $R = (R_1, \cdots, R_{\mathcal{I}})$, generate an assignment running the DA

(c) Repeat for m realizations of ϵ

2. Generate counterfactual assignments:

(a) For each realization of ϵ in 1. generate rank-order lists for all students, assuming that there are no restrictions to choice-menus and that each list has the same length as the ranking originally submitted to BPS

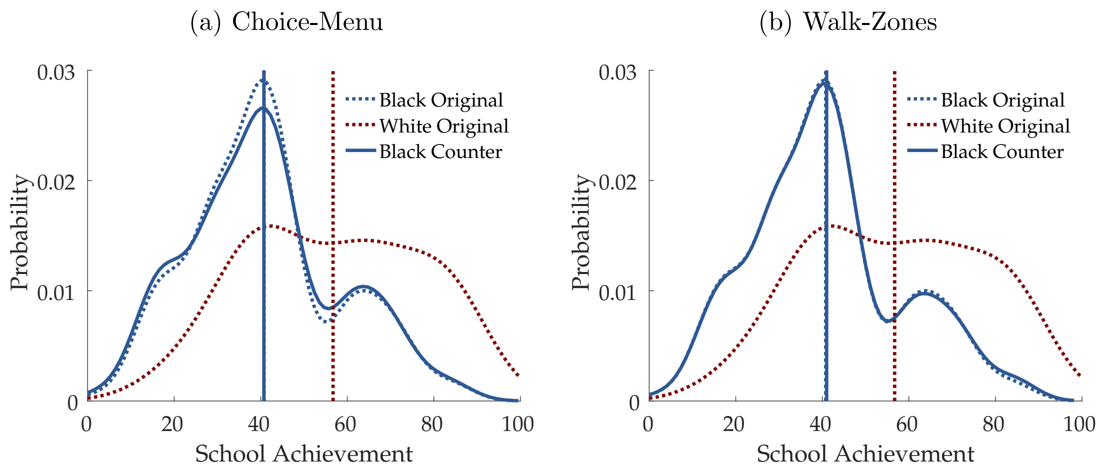
(b) For each profile R generate an assignment running the DA

When limits to choice-menus are eliminated, achievement at the schools assigned to white, black and hispanic students does not change. For each realization of ϵ, I compare the average achievement at the school assigned to students of each race in the original and counterfactual settings. I find that only in one out of 100 realizations of ϵ, I can reject the null-hypothesis that the average achievement is different for black and hispanic students across the counterfactual and original settings. For white students, I can reject the null-hypothesis in 8 of 100 realizations of ϵ.

If the walk-zone priority was eliminated, less would change. For each realization of ϵ, I compare the average achievement at the school assigned to students of each race in the original and counterfactual settings. Out of 100 realizations of ϵ, I find that in any can reject the null-hypothesis that the average achievement is different for black, hispanic or white students across the counterfactual and original settings

Figure 8 shows the mean density and average of achievement at the schools assigned to white, black and hispanic students. The plots show how, allowing parents to rank schools around the city without changing families' location, changes very little the quality of the assignments. Similar results are obtained if the proximity priority was eliminated. This means that the location effect estimated earlier is the effect of a reconfiguration of the distance to schools.

Figure 8: Eliminate Choice-Menu Restrictions and Walk-Zone Priorities: Achievement at the Assigned School



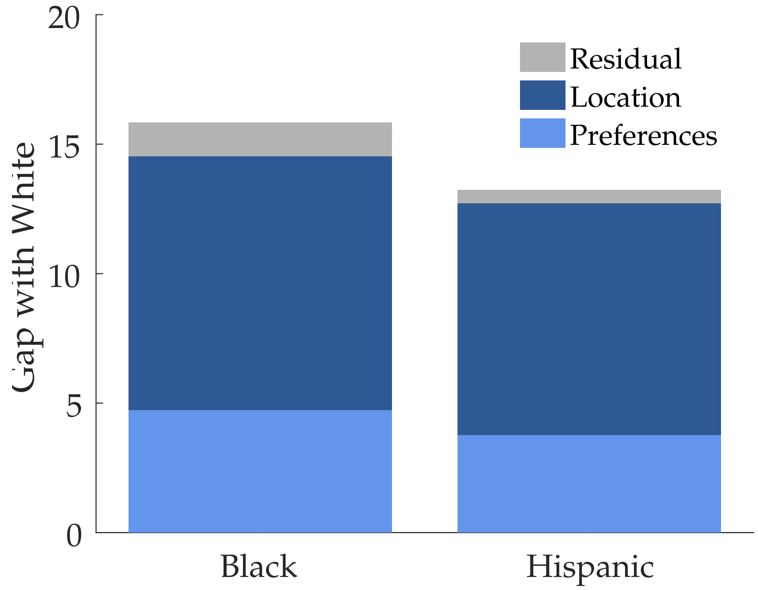
5.4 Summary

Figure summarize the reductions in the gap in achievement at the schools assigned to black, hispanic, and White students. White students are assigned to schools that have 15.8 pp higher achievement than schools assigned to black students, and 13.2 pp higher achievement than schools assigned to hispanic students. Location is the main contributor to this gap, 63% for black students and 69% for hispanic students. The effect of location is mediated by a change in the access to schools close to home, as opposed to rules of the algorithm that are location specific. Differences in the location-independent value of schools explain around 30% of the gap.

5.5 Change in the School-Match After a Location Change

Using the model parameters I can assess whether black and hispanic students are assigned to schools with higher value after a location change. To do this I compare, for each treated student, the location-independent value of the school assigned under the original setting and the counterfactual. Let $\mu(i) \in \mathcal{J}$ be the school assigned to i under the original setting and $\tilde{\mu}(i) \in \mathcal{J}$ be the school assigned to i under the counterfactual. If N^r is the number of students of race r, then the following is the average change in school value for students of race r expressed in miles

Figure 9: Decomposition of the gap



$$\frac{1}{N^r}\sum_{i\in\mathcal{I}^r}\frac{\delta^r_{\tilde{\mu}(i)}-\delta^r_{\mu(i)}}{\beta^r}.$$

I find that after a location change, Black and hispanic students are matched to schools that are on average better school matches. The average change in school value for black and hispanic students is equivalent to reducing students' travel distance by 0.4 miles. After the change in location, black students are assigned to schools that are on average 0.14 miles farther. Hispanic students, on the other hand, are assigned to schools that are 0.01 miles farther from home relative to the original assignments.

6 Conclusion

Choice-based assignments are intended to increase equity and foster diversity by offering students the option to sort into their preferred schools, and by weakening the link between residences and schools. I show, in Boston, cross-racial gaps in access to quality are no lower under choice relative to a neighborhood assignment. Then I show that the main contributor to this gap are cross-race differences in travel costs to high-achieving schools. Location-independent school preferences account for a smaller share of the gap. Importantly, assignment rules that factor geographies in the algorithm have no significant effect.

The salience of travel costs under choice-based assignments shows a first-order channel of why neighborhoods matter. This suggests geography plays a crucial role in the effectiveness of educational policy. Thus, coordinated efforts of housing and school authorities to increase investment in schools close to constrained students, while guaranteeing housing affordability, can increase access to quality and possibly reduce school segregation. Alternatively, policies that incentivize residential desegregation can lead to more equity.

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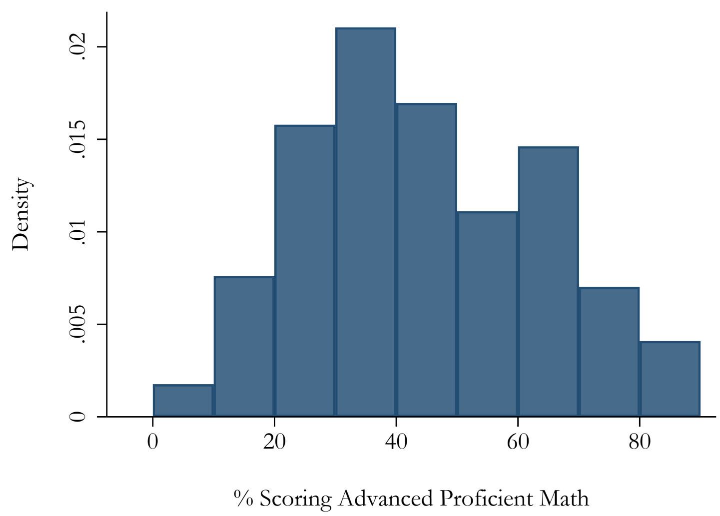
Appendix

A Supplementary Tables and Figures

A.1 Descriptive Statistics

Figure A.1: Histogram of School Achievement



Note: Histogram of school achievement measured as the fraction of 3rd grade students scoring advanced of proficient in the math MCAS tests.

A.2 Preference Estimates

Table A.1: Estimates of Preferences for Distance, Sibling and Language Programs

	White	Black	Hispanic
Distance	-0.93	-0.44	-0.55
	(0.01)	(0.01)	(0.01)
Sibling	4.44	4.21	4.48
	(0.16)	(0.10)	(0.08)
ELL match	0.49	0.06	0.28
	(0.10)	(0.06)	(0.03)
ELL language match	1.29	0.07	0.68
	(0.42)	(0.50)	(0.04)

Note: Standard errors computed with the inverse of the Hessian in parenthesis

Table A.2: School Mean Utilities

		2010			2011			2012	
School Code	White	Black	Hispanic	White	Black	Hispanic	White	Black	Hispanic
1	2.570	0.501	0.73	2.530	1.32	1.39	2.120	0.803	1.06
	(0.12)	(0.16)	(0.10)	(0.11)	(0.13)	(0.09)	(0.10)	(0.15)	(0.10)
2	2.983	0.456	0.766	2.880	0.781	1.174	2.465	0.671	0.881
	(0.12)	(0.16)	(0.10)	(0.11)	(0.16)	(0.10)	(0.10)	(0.15)	(0.11)
3		-1.277	-1.416	-1.304	-0.962	-1.180	-1.96	-1.073	-1.255
		(0.17)	(0.18)	(0.60)	(0.20)	(0.16)	(0.78)	(0.16)	(0.15)
4	-0.163	-1.201	0.142	0.108	-0.991	0.049	-0.324	-0.652	0.041
	(0.17)	(0.14)	(0.06)	(0.17)	(0.16)	(0.07)	(0.15)	(0.11)	(0.07)
5	-1.888	0.226	-0.415						
	(0.70)	(0.09)	(0.10)						
6	-1.906	-1.429	-1.794	-3.410	-0.547	-1.156	-2.000	-0.827	-1.509
	(0.56)	(0.18)	(0.18)	(1.43)	(0.16)	(0.15)	(0.53)	(0.13)	(0.17)
7	1.541	0.246	-0.112	1.590	1.332	0.134	1.255	0.061	0.508
	(0.29)	(0.28)	(0.13)	(0.23)	(0.26)	(0.12)	(0.22)	(0.34)	(0.10)
8	-4.189	-0.691	-1.339	-0.632	0.177	-0.750	-0.997	-0.169	-0.651
	(2.28)	(0.15)	(0.18)	(0.53)	(0.13)	(0.14)	(0.45)	(0.12)	(0.13)
9	0.243	-0.766	-0.500	0.273	-0.162	0.012	0.224	-0.298	-0.227
	(0.15)	(0.19)	(0.12)	(0.15)	(0.18)	(0.11)	(0.12)	(0.17)	(0.12)
10	0.178	-0.524	-0.647	0.575	0.178	0.152	0.105	0.060	0.161
	(0.15)	(0.19)	(0.12)	(0.13)	(0.17)	(0.11)	(0.12)	(0.16)	(0.11)
11	1.198	-0.238	-0.264	1.198	0.014	0.601	0.771	-0.004	0.297
	(0.12)	(0.17)	(0.11)	(0.11)	(0.18)	(0.10)	(0.10)	(0.16)	(0.11)
12	-2.791	-1.049	-1.832	-0.253	-0.633	-0.894	-1.696	-0.829	-1.237
	(1.10)	(0.16)	(0.18)	(0.39)	(0.17)	(0.13)	(0.55)	(0.13)	(0.14)
13	-1.229	-1.023	-0.789	-0.366	-0.455	-0.078	-1.065	0.019	-0.199
	(0.46)	(0.21)	(0.15)	(0.36)	(0.20)	(0.11)	(0.36)	(0.13)	(0.11)
14	-0.558	-0.713	-1.507	-1.437	-0.073	-1.080	-0.665	-0.450	-0.978
	(0.28)	(0.15)	(0.18)	(0.47)	(0.16)	(0.16)	(0.26)	(0.15)	(0.14)
15	-0.533	-0.350	-1.138	-0.364	0.390	-0.419	-0.415	-0.293	-0.600
	(0.38)	(0.15)	(0.17)	(0.33)	(0.15)	(0.13)	(0.33)	(0.15)	(0.12)
16							-1.009	-0.779	-1.223
							(0.26)	(0.12)	(0.12)
17	-1.725	-0.756	-0.678	-0.507	-0.152	-0.710	-2.253	-0.243	-0.539
	(0.64)	(0.14)	(0.10)	(0.45)	(0.14)	(0.11)	(0.68)	(0.11)	(0.10)
18	0.545	-0.936	-0.939	0.827	-0.218	-0.225	0.182	-0.359	-0.364
	(0.24)	(0.16)	(0.13)	(0.23)	(0.15)	(0.10)	(0.21)	(0.12)	(0.11)
19	-5.757	-0.750	-1.830	-1.112	-1.490	-1.724	-2.095	-0.686	-1.318
	(4.75)	(0.15)	(0.19)	(0.53)	(0.25)	(0.19)	(0.68)	(0.14)	(0.15)
20	0.654	0.342	0.053	1.559	0.652	0.455	1.135	0.342	0.472
	(0.30)	(0.21)	(0.16)	(0.25)	(0.24)	(0.14)	(0.24)	(0.23)	(0.13)

Note: Standard error estimated with the inverse of the Hessian in parenthesis

Table A.3: School Mean Utilities Continued

		2010			2011			2012	
School Code	White	Black	Hispanic	White	Black	Hispanic	White	Black	Hispanic
21	1.638	-0.491	-0.110	1.994	0.570	0.105	1.154	-0.086	0.176
	(0.28)	(0.30)	(0.19)	(0.28)	(0.26)	(0.18)	(0.26)	(0.28)	(0.17)
22	1.153	0.186	-0.159	1.507	0.713	0.357	0.750	0.612	0.592
	(0.28)	(0.25)	(0.20)	(0.30)	(0.25)	(0.16)	(0.26)	(0.23)	(0.14)
23	3.206	0.505	0.958	2.870	1.220	1.363	2.091	0.935	1.040
	(0.33)	(0.15)	(0.12)	(0.38)	(0.13)	(0.10)	(0.32)	(0.12)	(0.11)
24	-0.495	-0.171	-0.451	0.533	0.506	0.197	-0.262	0.282	0.160
	(0.37)	(0.15)	(0.15)	(0.38)	(0.13)	(0.11)	(0.34)	(0.13)	(0.12)
25	1.696	-0.321	-0.375	1.502	0.395	0.595	1.338	0.031	0.040
	(0.13)	(0.15)	(0.12)	(0.12)	(0.14)	(0.09)	(0.10)	(0.13)	(0.10)
26	0.827	-0.202	-0.137	0.500	0.289	0.181	-0.347	0.076	0.134
	(0.17)	(0.16)	(0.11)	(0.18)	(0.16)	(0.11)	(0.19)	(0.14)	(0.11)
27	-2.153	-1.059	-1.002	-2.215	0.016	-0.799	-1.697	-0.188	-0.721
	(0.50)	(0.19)	(0.13)	(0.59)	(0.17)	(0.12)	(0.35)	(0.14)	(0.11)
28	-2.467	-1.790	-1.976	-1.760	-0.584	-1.426	-1.527	-1.178	-1.161
	(0.68)	(0.21)	(0.20)	(0.57)	(0.16)	(0.17)	(0.37)	(0.17)	(0.15)
29	-3.340	-1.606	-1.872	-2.210	-0.688	-1.201	-2.278	-0.710	-1.154
	(0.90)	(0.16)	(0.14)	(0.59)	(0.14)	(0.11)	(0.45)	(0.11)	(0.10)
30	1.166	-0.986	-0.504	1.204	-0.646	-0.654	1.674	-0.827	-0.378
	(0.22)	(0.23)	(0.12)	(0.20)	(0.26)	(0.13)	(0.16)	(0.24)	(0.11)
31							-0.582	-0.660	-0.591
							(0.20)	(0.18)	(0.14)
32	1.507	-0.564	0.265	2.041	-0.087	0.129	1.732	-0.060	0.335
	(0.14)	(0.18)	(0.10)	(0.13)	(0.19)	(0.10)	(0.11)	(0.15)	(0.10)
33	-0.724	-1.223	-1.649	-0.147	0.207	-1.684	0.528	-0.429	-1.219
	(0.36)	(0.37)	(0.19)	(0.24)	(0.28)	(0.20)	(0.17)	(0.29)	(0.16)
34							1.60	0.60	0.59
							(0.12)	(0.14)	(0.09)
35	-0.582	0.160	-0.354	0.051	0.735	0.228	-0.844	0.624	0.103
	(0.36)	(0.11)	(0.10)	(0.32)	(0.11)	(0.09)	(0.35)	(0.09)	(0.09)
36	1.029	-1.463	-0.836	1.464	-0.418	0.191	1.319	-0.541	-0.661
	(0.14)	(0.28)	(0.15)	(0.12)	(0.22)	(0.12)	(0.10)	(0.20)	(0.16)
37	-0.095	-0.676	0.379	-0.292	-0.188	0.536	-0.580	-0.239	0.406
	(0.29)	(0.31)	(0.10)	(0.24)	(0.36)	(0.09)	(0.26)	(0.27)	(0.09)
38	2.671	0.263	0.644	2.613	0.785	1.164	1.923	0.091	0.873
	(0.12)	(0.16)	(0.10)	(0.11)	(0.15)	(0.10)	(0.10)	(0.17)	(0.10)
39	-2.271	-1.190	-1.395		-0.535	-1.095	-2.866	-0.461	-1.110
	(0.80)	(0.16)	(0.13)		(0.16)	(0.12)	(0.89)	(0.11)	(0.11)
40	-0.581	-0.707	-1.407	0.415	0.118	-0.391	0.139	-0.171	-0.407
	(0.34)	(0.15)	(0.15)	(0.27)	(0.13)	(0.10)	(0.22)	(0.11)	(0.10)

Note: Standard error estimated with the inverse of the Hessian in parenthesis

Table A.4: School Mean Utilities Continued

	2010			2011			2012		
School Code	White	Black	Hispanic	White	Black	Hispanic	White	Black	Hispanic
41	0.047	-0.963	-0.164	-0.190	-0.442	-0.545	-0.279	-1.020	-0.404
	(0.31)	(0.34)	(0.11)	(0.24)	(0.35)	(0.11)	(0.23)	(0.36)	(0.10)
42	0.852	-0.848	-0.960	1.256	-0.721	-0.645	1.110	-0.848	-0.341
	(0.16)	(0.19)	(0.15)	(0.15)	(0.23)	(0.14)	(0.12)	(0.19)	(0.13)
43	1.660	-0.222	-1.314	1.401	0.877	-1.119	1.988	0.368	-0.289
	(0.19)	(0.22)	(0.18)	(0.16)	(0.20)	(0.17)	(0.14)	(0.20)	(0.11)
44	-0.156	-1.316	-1.365	-0.082	-0.897	-0.339	-0.150	-0.340	-0.674
	(0.28)	(0.21)	(0.17)	(0.29)	(0.21)	(0.12)	(0.23)	(0.13)	(0.13)
45	0.796	-1.606	-0.994	1.420	-0.986	-0.310	1.099	-0.703	-0.732
	(0.22)	(0.22)	(0.13)	(0.20)	(0.21)	(0.11)	(0.17)	(0.14)	(0.12)
46	2.684	0.337	0.293	3.185	0.879	1.117	3.018	0.672	0.595
	(0.19)	(0.12)	(0.09)	(0.18)	(0.12)	(0.08)	(0.15)	(0.10)	(0.09)
47	-1.918	0.484	0.093	-0.795	0.888	0.118	-1.546	0.727	0.090
	(0.82)	(0.11)	(0.10)	(0.55)	(0.11)	(0.10)	(0.60)	(0.09)	(0.10)
48	-0.140	0.408	1.059	0.703	0.630	0.742	0.158	-0.351	1.038
	(0.28)	(0.20)	(0.09)	(0.19)	(0.25)	(0.09)	(0.20)	(0.28)	(0.09)
49	-1.950	-0.630	-0.801	-1.341	-0.130	-0.485	-1.332	-0.415	-0.604
	(0.44)	(0.11)	(0.09)	(0.37)	(0.11)	(0.08)	(0.29)	(0.10)	(0.08)
50	-1.032	-1.036	-0.660	-0.141	-0.027	-0.180	-0.196	-0.157	-0.497
	(0.40)	(0.17)	(0.11)	(0.33)	(0.15)	(0.10)	(0.25)	(0.12)	(0.10)
51	0.508	-1.282	-0.966	0.829	-0.402	-0.258	0.836	-0.041	-0.075
	(0.23)	(0.21)	(0.14)	(0.25)	(0.18)	(0.12)	(0.18)	(0.12)	(0.10)
52	-0.081	-0.307	-0.053	0.107	0.004	0.395	-0.178	-0.466	0.389
	(0.30)	(0.29)	(0.11)	(0.24)	(0.35)	(0.09)	(0.24)	(0.32)	(0.09)
53	-0.875	-0.601	-0.317	-0.438	-0.319	0.255	-0.402	-0.711	0.290
	(0.40)	(0.30)	(0.12)	(0.27)	(0.38)	(0.10)	(0.25)	(0.34)	(0.09)
54	-0.100	-0.731	-0.579	-0.357	-0.243	-0.082	-0.800	-0.416	-0.279
	(0.15)	(0.19)	(0.11)	(0.17)	(0.19)	(0.10)	(0.15)	(0.18)	(0.11)
55							0.01	-1.227	-0.994
							(0.13)	(0.23)	(0.16)
56	-1.994	-0.922	-1.094	-2.037	-0.042	-0.617	-1.615	-0.641	-0.800
	(0.74)	(0.19)	(0.17)	(0.69)	(0.20)	(0.15)	(0.47)	(0.19)	(0.15)
57				-3.538	-0.554	-1.909	-0.591	-0.927	-1.120
				(1.26)	(0.18)	(0.23)	(0.23)	(0.16)	(0.15)
58	0.121	-0.870	-1.271	0.526	-0.267	-0.380	0.270	-0.509	-0.712
	(0.27)	(0.24)	(0.23)	(0.27)	(0.23)	(0.16)	(0.21)	(0.19)	(0.18)
59	2.541	-0.274	-0.305	2.515	0.257	0.486	2.512	-0.113	0.168
	(0.21)	(0.19)	(0.16)	(0.21)	(0.19)	(0.12)	(0.16)	(0.17)	(0.13)
60	0.227	0.405	-0.008	0.395	0.558	-0.260	-0.256	0.588	-0.064
	(0.29)	(0.09)	(0.09)	(0.30)	(0.11)	(0.10)	(0.31)	(0.09)	(0.09)

Note: Standard error estimated with the inverse of the Hessian in parenthesis

Table A.5: School Mean Utilities Continued

		2010			2011			2012	
School Code	White	Black	Hispanic	White	Black	Hispanic	White	Black	Hispanic
61	-1.716	-0.195	-0.337	-0.030	0.282	-0.087	0.141	0.132	0.085
	(0.60)	(0.25)	(0.18)	(0.41)	(0.27)	(0.16)	(0.28)	(0.23)	(0.15)
62	1.673	0.802	0.623	2.114	0.846	0.658	1.960	0.235	0.796
	(0.25)	(0.18)	(0.14)	(0.23)	(0.21)	(0.13)	(0.20)	(0.23)	(0.12)
63	0.776	-0.463	-0.261	0.812	0.209	0.205	1.001	-0.157	0.166
	(0.23)	(0.19)	(0.14)	(0.22)	(0.18)	(0.12)	(0.18)	(0.16)	(0.12)
64	-0.531	-0.161	-0.292	-0.351	0.778	-0.377	-1.575	-0.240	-0.205
	(0.33)	(0.16)	(0.11)	(0.26)	(0.16)	(0.11)	(0.42)	(0.20)	(0.10)
65	1.181	0.263	-0.537	1.578	0.757	-0.247	1.661	0.728	-0.156
	(0.21)	(0.16)	(0.15)	(0.16)	(0.17)	(0.13)	(0.14)	(0.16)	(0.12)
66	1.845	-0.183	0.160	1.756	0.504	0.806	1.560	0.540	0.544
	(0.12)	(0.16)	(0.10)	(0.12)	(0.15)	(0.09)	(0.10)	(0.13)	(0.10)
67	-1.899	-0.637	-1.428	-1.501	0.022	-0.240	-0.987	-0.019	-0.022
	(0.74)	(0.17)	(0.21)	(0.55)	(0.18)	(0.13)	(0.43)	(0.16)	(0.11)
68	-2.446	-0.479	-1.582	-1.559	-0.277	-0.493	-2.075	-0.604	-0.750
	(0.94)	(0.13)	(0.18)	(0.66)	(0.14)	(0.12)	(0.59)	(0.14)	(0.13)

Note: Standard error estimated with the inverse of the Hessian in parenthesis

Table A.6: Relation Between School Mean Utilities and School Characteristics - Individual Regressions

	Standardized School Mean Effects - δ_j^r		
	White	Black	Hispanic
	(1)	(2)	(3)
% Scored Advanced-Proficient Math	0.027***	0.017***	0.018***
	(0.003)	(0.004)	(0.004)
% Scored Advanced-Proficient English	0.037***	0.018***	0.022***
	(0.004)	(0.005)	(0.004)
% of White Students	0.052***	0.021***	0.035***
	(0.003)	(0.005)	(0.004)
% of Black Students	−0.027***	−0.010**	−0.028***
	(0.003)	(0.004)	(0.003)
% of Hispanic Students	−0.008*	−0.005	0.009*
	(0.004)	(0.004)	(0.004)
% Low-Income Students in Kindergarten	−0.020***	−0.007*	−0.013***
	(0.003)	(0.003)	(0.003)

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: Each coefficient is from an independent regression where the dependent variable is the standardized δ_j^r. Standard errors in parenthesis.

Table A.7: Relation Between School Mean Utilities and School Characteristics - Pooled Regressions

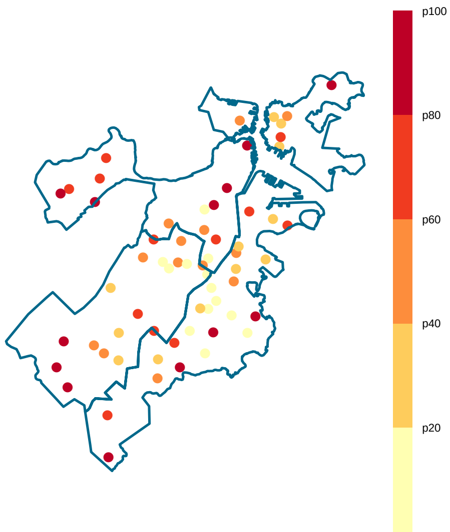
	Standardized School Mean Effects - δ_j^r		
	White	Black	Hispanic
	(1)	(2)	(3)
% Scored Advanced-Proficient Math	0.005	0.012	0.007
	(0.004)	(0.006)	(0.005)
% Scored Advanced-Proficient English	0.006	−0.004	0.0001
	(0.006)	(0.009)	(0.007)
% of White Students	0.011	0.005	0.027**
	(0.007)	(0.011)	(0.009)
% of Black Students	−0.032***	−0.012	−0.010
	(0.006)	(0.009)	(0.007)
% of Hispanic Students	−0.020***	−0.011	0.009
	(0.005)	(0.007)	(0.006)
% Low-Income Students in Kindergarten	−0.007*	−0.001	−0.001
	(0.003)	(0.005)	(0.004)
Observations	169	170	170
R^2	0.671	0.166	0.411

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

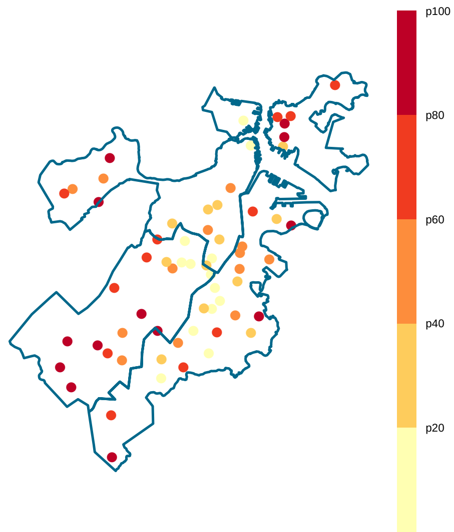
Note: Coefficient from regression between the standardized δ_j^r and school characteristics. Standard errors in parenthesis.

Figure A.2: Location of 2011 Schools by Deciles of Mean Utility

(a) Black Students



(b) Hispanic Students



(c) White Students

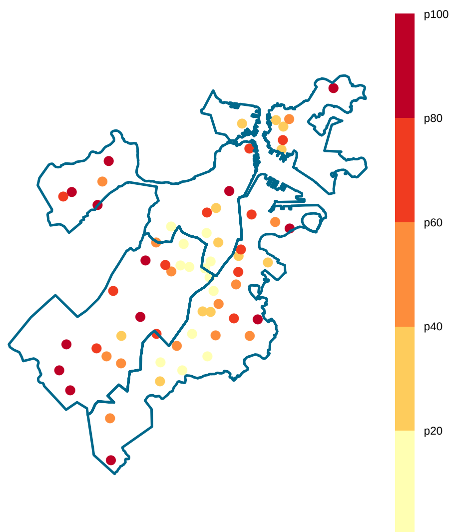
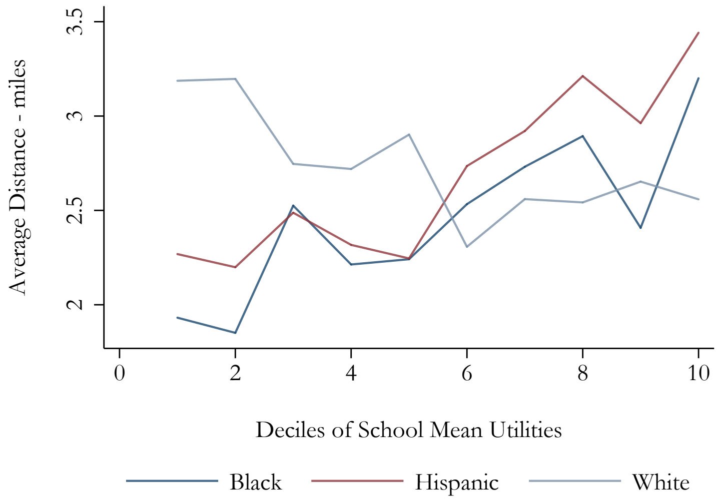


Figure A.3: Distance and School Mean Utility



Note: Average distance between students and schools by deciles of school mean utility δ^r_{jt}

Table A.8: Simulated change in positions after an extra 0.1 miles

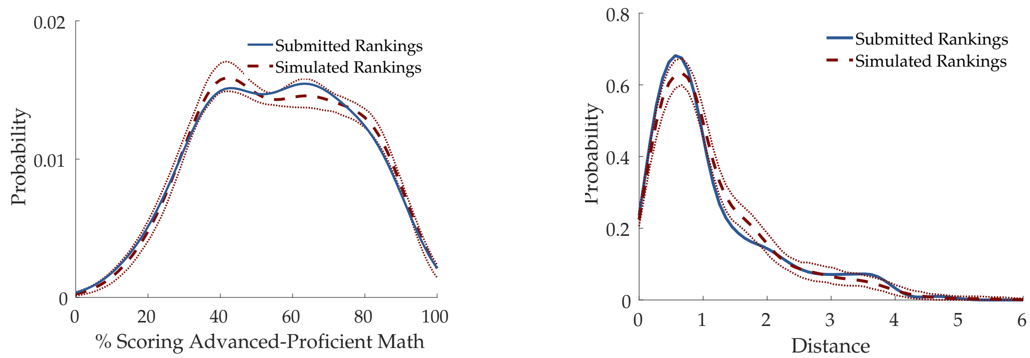
	Change in position		
	White	Black	Hispanic
	(1)	(2)	(3)
Distance (Δ 0.1 miles)	0.149	0.149	0.147

Note: Average number of positions gained by a school after an increase in travelled distance of 0.1 miles. Simulations generated using the estimated preference parameters and random realizations of ϵ

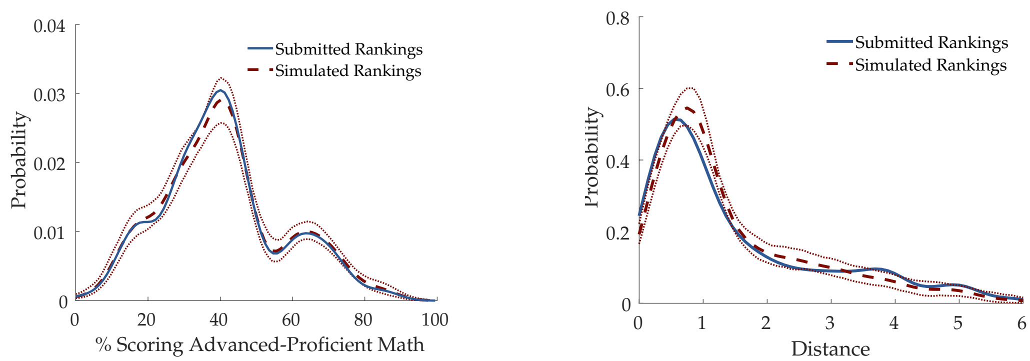
A.3 Model Fit

Figure A.4: Fit of Estimated Preference Parameters: Achievement and Distance to Assigned School

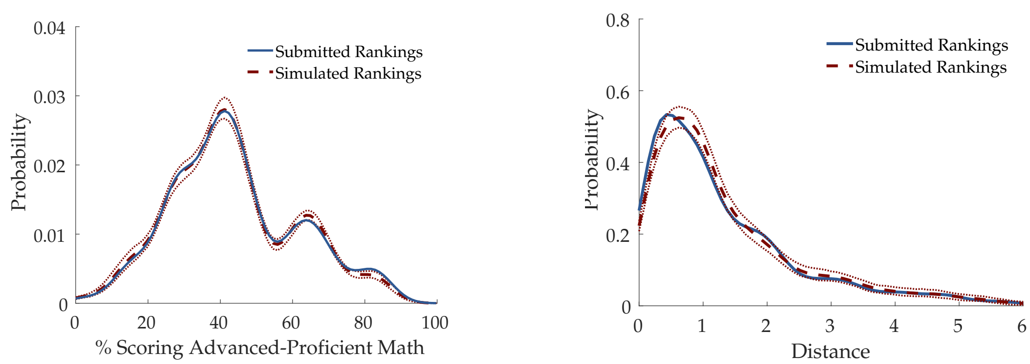
(a) White



(b) Black



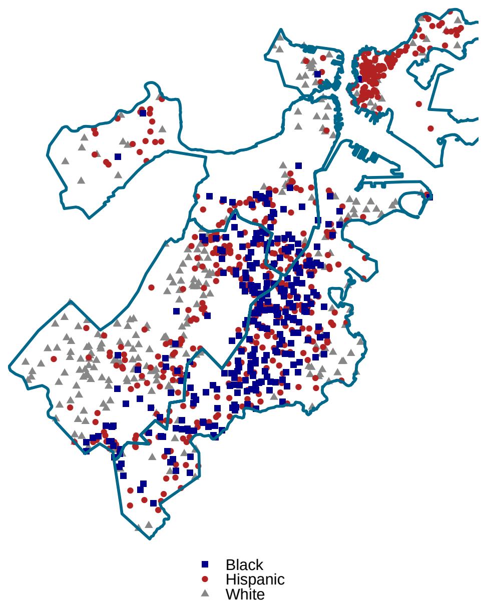
(c) Hispanic



Note: Submitted rankings distributions are obtained from running the DA using the rankings submitted by parents to BPS. Simulated rankings distributions are obtained from rankings generated using demand parameters and 100 random realizations of ϵ. I plot the piece-wise median density, and the 5% and 95% densities.

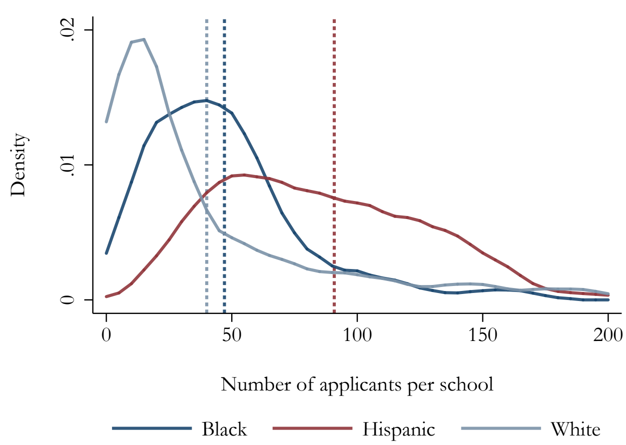
A.4 Distribution of Students in Space

Figure A.5: Spatial Distribution of Applicants by Race



Note: Each point represents 10 students from the 2010-2012 pooled data, randomly located at the census tract level.

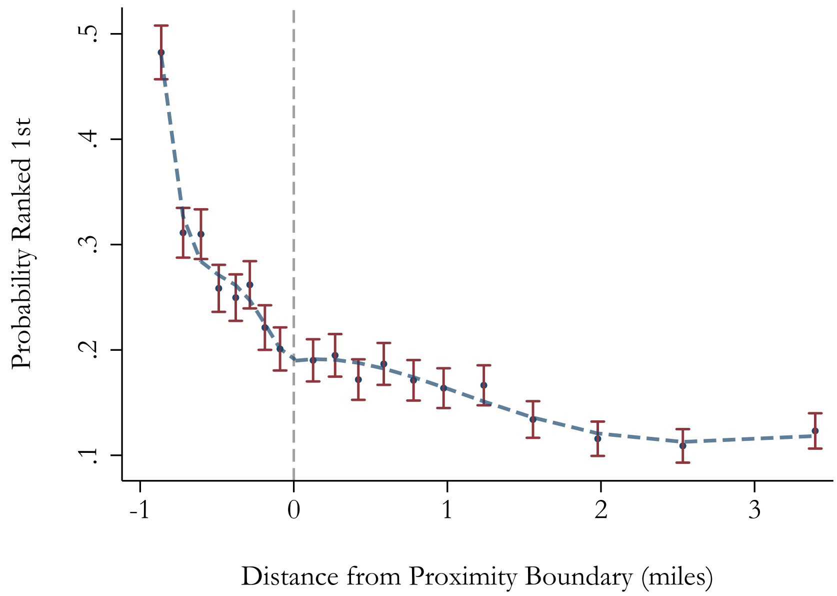
Figure A.6: Spatial Distribution of Applicants by Race



Note: Distribution of number of students from each race that applied to each school

A.5 No Sorting in Boundaries

Figure A.7: Probability of Ranking a School First by Distance to the Proximity Boundary



Note: Probability of ranking a school first as a function of the distance to the boundary of the proximity priority.

A.6 Counterfactual Assignments

Figure A.8: Eliminate Choice-Menu Restrictions and Walk-Zone Priorities: Achievement at the Assigned School

(a) Choice-Menu (b) Walk-Zones

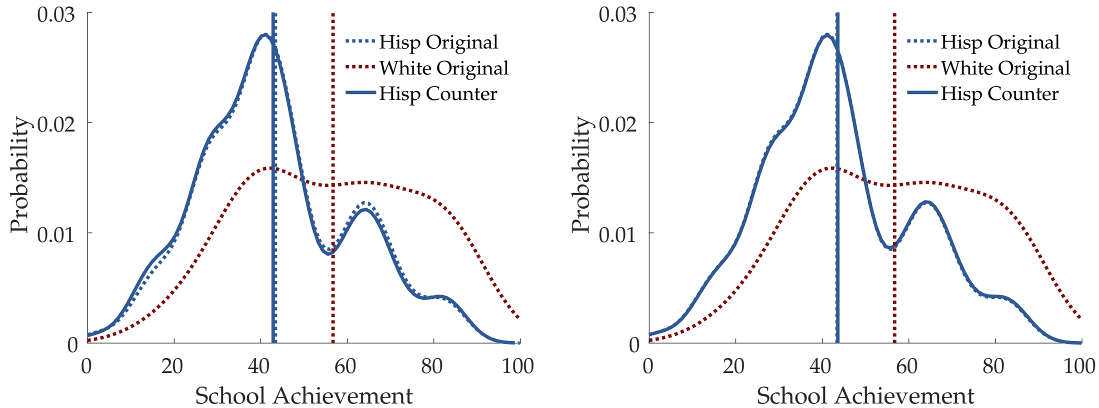


Figure A.9: Location Change: Distance to Assigned School

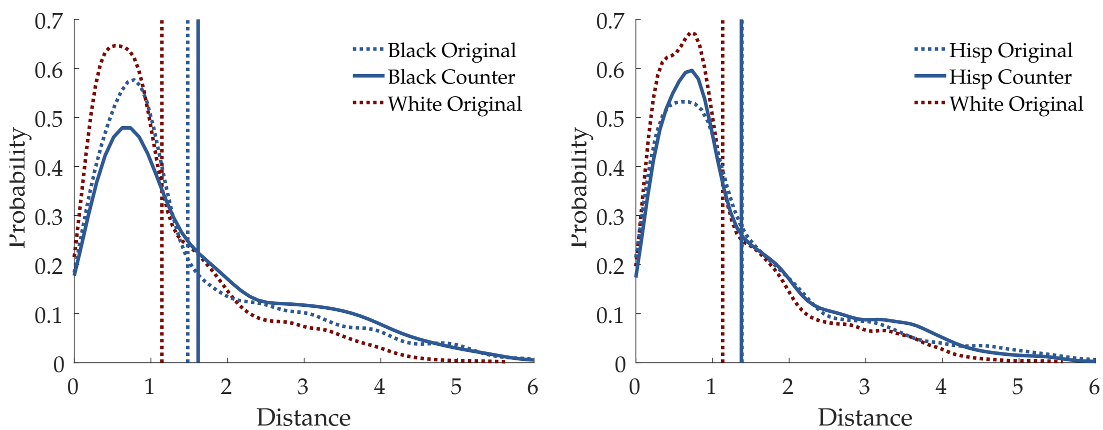
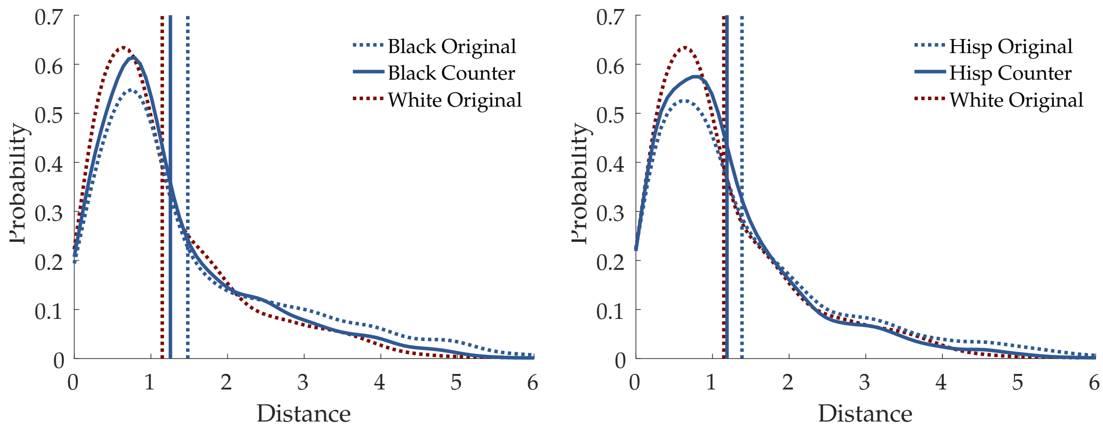


Figure A.10: Preference Change: Distance to Assigned School



B Maximum Likelihood Function

Let $R_i = (R_{i1}, \cdots, R_{il_i})$ be teh rank-order list submitted by i. The likelihood of R_i is

$$\mathcal{L}(R_i) = \left[\prod_{k=1}^{l_i} \frac{\exp(U_{iR_{ik}})}{1 + \sum_{j \in \mathcal{J}_i \setminus \{R_{im}: m<k\}} \exp(U_{iR_{ij}})} \right] \left[\frac{1}{\sum_{j \in \mathcal{J}_i \setminus \{R_{im}: m<l_i\}} \exp(U_{iR_{ij}})} \right] \tag{7}$$

I find the values of δ^r, β^r, γ^r that maximize

$$\Pr(R_1, \cdots, R_{\mathcal{I}}) = \prod_{i \in \mathcal{I}} \mathcal{L}(R_i) \tag{8}$$