Dissecting Business Cycles

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Abstract

This paper introduces a novel identification strategy to examine the relative role of aggregate supply and demand shocks in driving business cycles. It dissects real GDP fluctuations into long-run and short-run components, identifying them as long-run supply and short-run demand shocks based on conditional correlations of macro variables. Both shocks contribute significantly to business cycle volatility and drive the co-movements of relevant macroeconomic variables. Additionally, the study identifies a second category of long-run supply shocks that do not impact business cycles, revealing substantial normative and policy implications for benchmark DSGE models estimated in a full information setting. By employing theoretical insights and estimation of DSGE models by matching the dynamic causal effects of the identified business cycle shocks, the paper advocates for parameter estimation in a limited information setting.

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1. Introduction

The inquiry into the nature of fluctuations in real gross domestic product (GDP) revolves around the distinction between transitory and persistent disturbances. A central question pertains to the characterization of recessions: do they entail temporary deviations of GDP from its trend, or do they predominantly signify shifts in the underlying trend itself? The literature on business cycles grapples with this fundamental query, delving into the relative role of real determinants such as productivity in long-term equilibrium versus the short-run fluctuations of demand that steer business cycle dynamics. Discerning whether recessions stem from low-frequency supply or high-frequency demand shocks holds significant implications for formulating effective policy responses by central banks.

The conventional framework of Real Business Cycle (RBC) models and their extensions positions Total Factor Productivity (TFP) shocks as pivotal drivers of labor wedges and consequent business cycle patterns. Nevertheless, a series of empirical investigations of this hypothesis by Gali (1999), Basu et al. (2006), and Angeletos et al. (2020), has contested this hypothesis. They argue against the notion by highlighting the estimated conditional correlations of hours and productivity display either zero or negative associations for short-run and long-run technology shocks. This discordance has prompted a reevaluation of demand shocks as plausible contributors to business cycle dynamics, particularly in light of their compatibility with a subset of New Keynesian models. Empirical findings from studies by Blanchard and Quah (1989), Angeletos et al. (2020) and Benhima and Poilly (2021) have further reinforced the case for demand-driven business cycles.

This paper aims to use a novel extension of structural vector autoregression (SVAR) methodology to review further and synthesize the existing empirical literature concerning the respective roles of aggregate supply and demand shocks in driving business cycles. The applications of structural vector autoregression (SVAR) identification in related studies primarily focused on identifying dynamic causal effects of an average aggregate long-run productivity shock and assumed the existence of only one category of long-run productivity shocks that may or may not drive business cycles. In contrast to previous literature this paper employs an empirical approach that allows for the consideration of two categories of long-run supply shocks: one may drive business cycles and one may not. Specifically, it dissects business cycle fluctuations into their constituent long-run (low-frequency) and short-run (high-frequency) com-
ponents. This empirical strategy builds on the contributions of Uhlig (2003) and Angeletos et al. (2020), aimed at identifying a pair of orthogonal shocks that effectively account for the maximal business cycle volatility observed in real GDP. However, this paper introduces an additional constraint: one of these shocks is precluded from explaining low-frequency fluctuations in either real GDP, real consumption, or utilization-adjusted Total Factor Productivity (TFP). In essence, this methodological evolution goes beyond mere observation of reduced-form responses to business cycle shocks or long-run productivity shocks; instead, it offers insights into the question of whether there exists a subset of long-run shocks that explains significant business cycle volatility and the concurrent co-movement patterns of macroeconomic variables.

The classification of the two orthogonal shocks, identified as business cycle (BC) supply and demand shocks, relies on examining the impulse response functions (IRFs) of the remaining variables within the vector autoregression (VAR) model. Both of these shocks independently contribute to the co-movement observed in business cycles and account for a substantial portion of the volatility exhibited in output, consumption, investment, hours worked, and labor productivity. The short-run shock, subject to the constraint of not explaining long-run fluctuations, is identified as a ‘BC-demand’ shock, as it leads to significant procyclical fluctuations in real GDP, inflation, and federal funds rates. Nevertheless, the unrestricted shock also manifests long-run impacts on the key macroeconomic quantity variables, along with TFP and negative conditional correlation in inflation and the federal funds rate. This long-run shock is identified as a ‘BC-supply’ shock, characterized by long-run IRFs of output, consumption, investment, TFP, and labor productivity, and giving rise to countercyclical inflation.

Drawing upon the long-run volatility of macro variables explained by the aforementioned ‘BC-supply’ shock, this study also provides evidence of a significant portion of long-run productivity shocks that do not exert an influence on business cycles. This poses a challenge for parameter estimation in medium-scale Dynamic Stochastic General Equilibrium (DSGE) models when estimated under full information. Central banks often employ various versions of these models to analyze the role of economic frictions and for policy analysis. I formally argue that in the presence of cross-frequency restrictions within DSGE models and a notable fraction of long-run non-business cycle shocks in the data, parameter estimates become biased, with consequential implications for these models’ business cycle outcomes.
To delve into the normative and policy implications of models estimated under full information, this study utilizes the benchmark medium-scale DSGE model proposed by Smets and Wouters (2007) originally estimated using Bayesian likelihood estimation and estimates it using impulse response matching methodology akin to Christiano et al. (2005). This model encompasses a range of nominal and real frictions, along with seven structural shocks. Its inclusion serves two primary objectives. Firstly, it offers a more comprehensive structural interpretation of the ‘demand’ shock identified in our empirical analysis, as the model can accommodate shocks that generate both standard business-cycle dynamics and deviations from those patterns. Notably, the model’s setup underscores the adaptability and applicability of our proposed identification strategy and results across various Vector Autoregression (VAR) specifications, given that Smets and Wouters (2007) employs a seven-variable VAR of observables without incorporating TFP as an observable.

Secondly, conventional frictionless RBC models typically predict that long-run TFP fluctuations have an expansionary effect, while other macroeconomic models, which account for sticky prices and imperfect information, suggest the opposite in short run. These alternative models propose that technological advancements may initially lead to short-run declines in employment or hours worked due to price rigidities but result in long-run increases when prices can adjust. This dynamic leads to a positive conditional correlation between the output gap and inflation, even in response to long-run TFP shocks, resulting in divine coincidence akin to demand-driven business cycles in new Keynesian framework.

However, when the Smets and Wouters (2007) model is estimated using impulse response matching, it reveals a policy tradeoff for monetary authorities due to a negative comovement between inflation and the output gap. This contrast in results challenges the normative and policy implications compared to the same model estimated under full information setting, highlighting the sensitivity of policy recommendations to the choice of estimation methodology.

This paper makes contribution to two distinct strands of literature within the domain of business cycles. The first strand, rooted in the SVAR framework, exemplified by the works of Blanchard and Quah (1989), Shapiro and Watson (1998), and Angeletos et al. (2020), revolves around quantifying the relative impact of aggregate supply and demand shocks on the fluctuations inherent in business cycles. However, the analyses conducted in these aforementioned studies, alongside those pursued by Gali (1999), Basu et al. (2006), Barsky et al. (2014), Barsky and Sims (2011), Neville
et al. (2014a), Kurmann and Sims (2021), Benhima and Poilly (2021), collectively challenge the conventional notion of long-run TFP fluctuations as the primary driver of business cycles. Instead, these investigations posit demand shocks as the pivotal drivers behind business cycle dynamics.

In contrast, this study deviates from this perspective and aligns itself with the SVAR evidence presented by Beaudry and Portier (2006), Chahrour and Jurado (2018) & Chahrour et al. (2023). These works advocate for long-run TFP-driven business cycles while not explicitly addressing the relative influence of different shocks. Building upon this foundation, the current paper advances the comprehension of the role played by long-run TFP shocks in shaping business cycles. It achieves this by employing the benchmark VAR model rooted in the framework of Angeletos et al. (2020), which thoroughly dissects business cycle fluctuations into their distinct long-run supply and short-run demand components. This approach highlights the intricate dynamics underpinning business cycles, thus accentuating the significant contributions of both long-run TFP shocks and demand shocks in driving patterns of co-movement.

The second strand of literature to which this paper contributes is the literature on the identification of macroeconomic equations through structural shocks. Notable works in this domain include Rotemberg and Woodford (1997) and Christiano et al. (2005), which estimate DSGE models by matching impulse response functions. More recently, Barnichon and Mesters (2020) introduced a new approach involving regressions in impulse response space, and Lewis and Mertens (2022) presented an improved approach. This paper extends this literature by advocating for the use of conditional variation in identified business cycle shocks to discipline structural model parameters, as opposed to the Bayesian likelihood approach using unconditional moments.

The paper is organized as follows. Section 2 delves into the data and outlines the methodology employed to dissect business cycle fluctuations and identify business-cycle (BC) supply and demand shocks. In Section 3, the primary empirical findings are presented. Moving on to Section 5, I estimate the parameters of the medium-scale Dynamic Stochastic General Equilibrium (DSGE) model of Smets and Wouters (2007) required to match the identified dynamic causal effects of business cycle shocks. This section also undertakes a comparative analysis of the model, estimated using conditional moments, and the Bayesian likelihood estimation of the original estimated model. Finally, Section 6 concludes.
2. Data and Method

The data utilized in the main analysis of this study includes quarterly observations of ten macroeconomic variables. These variables include the unemployment rate (u), the real per capita levels of GDP (Y), hours worked (h), investment (I), consumption (C), labor productivity in the nonfarm business sector (Y/h), the level of utilization-adjusted total factor productivity (TFP), the labor share (wh/Y), the inflation rate (π), as measured by the rate of change in the GDP deflator, and the nominal interest rate (R), as measured by the federal funds rate. The sample for this study begins in 1955:I, the earliest date of availability for the federal funds rate, and ends in 2019:IV.

The Vector Autoregression (VAR) model employed in this study takes the form:

$$A(L)Y_t = \mu_t,$$

Where $Y_t$ is a vector of $n$ macroeconomic variables under examination, $A(L)$ is a matrix polynomial represented by the sum of $A_\tau L^\tau$, with $A(0) = A_0 = I$, and $l$ is the number of lags included in the VAR. The vector of residuals, $\mu_t$, follows the assumption of $E(\mu_t\mu_t') = \Sigma$ for a positive definite matrix $\Sigma$. The large size of the VAR necessitated the use of Bayesian methods and a Minnesota prior for estimation. The baseline specification employed 2 lags, as suggested by standard Bayesian criteria.

The method is based on the assumption that there exists a linear relationship between the residuals, denoted by $\mu_t$, and a set of mutually orthogonal shocks, represented by $\varepsilon_t$. Mathematically, this relationship can be represented by the equation $\mu_t = C\varepsilon_t$, where $C$ is an invertible $n \times n$ matrix.

Another key assumption in the analysis is that the orthogonal shocks, $\varepsilon_t$, are independently and identically distributed over time. Additionally, we assume that the covariance matrix of these shocks is equal to the identity matrix, $I$. The interpretation of these orthogonal shocks as “structural” shocks, such as exogenous changes in supply or demand, will be based on the impulse response functions (IRFs) of the variables included in the VAR. By examining the dynamic responses of the variables to a shock in a set of variable, we can gain insight into the underlying causes of the fluctuations in the data.

To identify these shocks, the matrix $C$ is decomposed into the Cholesky decomposition of the VAR residuals covariance matrix, $\hat{C}$, and an orthonormal matrix, $Q$. This leads to the relationship $\varepsilon_t = C^{-1}\mu_t = Q^t\hat{C}^{-1}\mu_t$, where each column of $Q$ corresponds...
to a shock in $\varepsilon_t$.

However, simply satisfying $QQ' = I$ and $CC' = \Sigma$ does not suffice for identifying the underlying shocks. To do so, we impose additional restrictions on $Q$ based on the requirement that it contains the maximal share of all the information in the data about the volatility of a specific variable in a specific frequency band. This approach is different from the typical SVAR exercises in the literature which employ exclusion or sign restrictions motivated by specific theories.

The Wold representation of the VAR model is given by the following equation:

$$Y_t = B(L)\mu_t,$$

Where $B(L)$ is an infinite matrix polynomial, and $\mu_t$ represents the residuals. We then substitute $\mu_t = \tilde{C}Q\varepsilon_t$, where $\tilde{C}$ is the Cholesky decomposition of the VAR residuals covariance matrix, and $Q$ is an orthonormal matrix, leading to the following representation:

$$Y_t = D(L)Q\varepsilon_t = \Theta(L)\varepsilon_t,$$

Where $D(L)$ and $\Theta(L)$ are infinite matrix polynomials, with $D_\tau \equiv B_\tau \tilde{C}$ and $\Theta_\tau \equiv D_\tau Q$ for all $\tau \in 0, 1, 2, \ldots$. The sequence $\{\Theta_\tau\}_{\tau=0}^{\infty}$ represents the impulse response functions (IRFs) of the variables to the structural shocks.

As mentioned, the interpretation of the structural shocks is based on the dynamic responses of the variables to the respective shock. By considering the $(i, j)$ element of the matrix $\Theta_\tau$, one may identify the effect of the $j$th shock on the $i$th variable at horizon $\tau$. This allows to gain insight into the underlying causes of the fluctuations in the data, and identify which shocks are likely to represent exogenous changes in supply or demand.

### 2.1 Identification Strategy

The identification strategy used in this paper builds upon the “max-share” approaches first introduced in the literature by Faust (1998) and Uhlig (2003). These approaches have been subsequently adapted and expanded upon by several other authors, including Barsky and Sims (2011), Kurmann and Otrok (2013), Neville et al. (2014b), Angeletos et al. (2020), Kurmann and Sims (2021) and Charhour et al. (2021) among others. The implementation of this strategy is in the frequency domain, similar to
that used by Angeletos et al. (2020) (ACD henceforth), where the goal is to identify a reduced-form shock that explains the maximum volatility of a targeted variable in a specific frequency band.

In contrast to the ACD approach, this paper adopts a distinct perspective by emphasizing the role of two specific shocks as pivotal drivers of business cycle fluctuations. This standpoint is underpinned by the scree plot illustrated in Figure 1, where the x-axis is limited to 10 in accordance with the maximum number of eigenvalues of a 10-variable VAR. The observed trend in the eigenvalues reveals a convergence to zero from the third eigenvalue onward, while the first two principal components retain significant values. ACD, on the other hand, focuses the first eigenvalue, explaining 80% of business cycle volatility in output, as the main business cycle shock. Drawing from this empirical evidence, the argument advanced in this paper asserts the necessity of two orthogonal business cycle shocks to effectively account for the fluctuations in real GDP per capita across business cycles.

The key novel contribution of this paper lies in the dual employment of targeting and constraining methodologies to identify the two orthogonal business cycle shocks and better explain the volatility of specific variables over a certain frequency band while constraining them to fluctuations of another variable in the same or a different frequency domain.

To illustrate this further, I use a standard data generating process (DGP), where the vector of structural shocks $\epsilon_t$ is decomposed into two distinct categories: business cycle shocks and non-business cycle shocks.

$$
\epsilon_t = [\begin{array}{c}
\epsilon_{sr}^{B,t} \\
\epsilon_{lr}^{B,t} \\
\epsilon_{long-run}^{NB,t} \\
\epsilon_{residual}^{NB,t}
\end{array}]^{'}
$$

The business cycle shocks are represented as $\epsilon_{sr}^{B,t}$ and $\epsilon_{lr}^{B,t}$, while the non-business cycle shocks are represented as $\epsilon_{long-run}^{NB,t}$ and $\epsilon_{residual}^{NB,t}$. The subscript ‘B’ in $\epsilon_{B,t}^{sr}$ and $\epsilon_{B,t}^{lr}$ denotes that these orthogonal shocks are identified by maximizing their contribution to the volatility of the targeted variable, real GDP per capita, over the business cycle frequency band, or frequencies pertaining to a time period of 6-32 quarters.

The superscript ‘sr’ in $\epsilon_{B,t}^{sr}$ denotes a restriction of the shock to explain long-run volatility of real GDP per capita at long-run frequencies, i.e. frequency bands pertaining to time periods of 20-100 years. In other words, $\epsilon_{B,t}^{sr}$ is identified by simultaneously targeting the same variable and frequency bands as $\epsilon_{B,t}^{lr}$, but with the additional restriction of explaining long-run volatility of real GDP per capita. The re-
Results are robust if the restriction is applied to long-run fluctuations of consumption, TFP or labor productivity instead of GDP.

On the other hand, non-business cycle shocks are further classified into two sub-categories: permanent shocks and residual shocks. The permanent shocks, identified as $\epsilon^{\text{long-run}}_{NB,t}$, are orthogonal to the business cycle shocks and lead to persistent changes in real GDP per capita or fluctuations pertaining to frequency bands with time periods above 100 years. On the other hand, the residual shocks, identified as $\epsilon^{\text{residual}}_{NB,t}$, are orthogonal to both the business cycle shocks and the permanent shocks. These residual shocks capture all other non-business cycle shocks that are not captured by the other two categories.

The objective in this paper is to partially identify the business cycle shocks, and this further classification of $\epsilon_t$ allows for a more detailed examination of the results. This classification of structural shocks is useful for highlighting the results of our analysis in comparison to existing literature, such as the works of Blanchard and Quah (1989) and Angeletos et al. (2020). The inclusion of non-business cycle shocks introduces model misspecification, impacting both the SVAR approach and the local identification analysis of linearized DSGE models in full-information contexts. The fifth section of the paper elucidates this phenomenon using a Smets and Wouters (2007) framework.

In the Wold representation from the previous subsection, the variable $Y_t$ can be represented as:
\[ Y_t = D(L)Q\varepsilon_t \]

where \( \varepsilon_t \) is a white noise process and \( Q \) is an orthonormal matrix. The spectral density of a variable \( y_j \) in \( Y_t \) in the frequency band \([f, \bar{f}]\) can be represented by \( D(y_j, f, \bar{f}) \):

\[
D(y_j, f, \bar{f}) = \int_f^{\bar{f}} \left( \mathbf{D}^j (e^{-i\lambda}) \mathbf{D}^j (e^{-i\lambda}) \right) d\lambda
\]

where the sequence \( \{D_r\}_{r=0}^\infty \) represents the Cholesky transformation of the VAR residuals, and \( D^j_r \) represents the jth row of the matrix \( D_r \).

To identify a shock \( \epsilon_{1,t} \), we need to find the column of the orthonormal matrix \( Q \) that represents the shock and explains the maximum volatility of \( y_j \) in the frequency band \([f, \bar{f}]\). This can be represented as:

\[
q_1 \equiv \arg \max_q \int_f^{\bar{f}} \left( \mathbf{D}^j (e^{-i\lambda}) q \mathbf{D}^j (e^{-i\lambda}) q \right) d\lambda \equiv \arg \max_q q' D(y_j, f, \bar{f}) q \text{ s.t. } q' q = 1, \tag{1}
\]

Similarly, if we need to identify a shock \( \epsilon_{2,t} \) that explains the maximum volatility of \( y_j \) in the frequency band \([f, \bar{f}]\), but not the volatility of \( y_k \) in the frequency band \([\omega, \bar{\omega}]\), this can be represented as:

\[
q_2 \equiv \arg \max_q \int_f^{\bar{f}} \left( \mathbf{D}^j (e^{-i\lambda}) q \mathbf{D}^j (e^{-i\lambda}) q \right) d\lambda - \int_{\omega}^{\bar{\omega}} \left( \mathbf{D}^k (e^{-i\omega}) q \mathbf{D}^k (e^{-i\omega}) q \right) d\omega \equiv \arg \max_q q' D(y_j, f, \bar{f}) q - q' D(y_k, \omega, \bar{\omega}) q \text{ s.t. } q' q = 1, \tag{2}
\]

Building on this, the objective is to identify two orthogonal shocks: \( q_{sr}^{\tau B,t} \) and \( q_{lr}^{\tau B,t} \).

These shocks simultaneously should explain the volatility of real GDP per capita at business cycle frequency, but restricting \( q_{sr}^{\tau B,t} \) from explaining the long-run volatility of GDP. The objective function is as follows:
To examine business cycle fluctuations, the framework follows Stock and Watson (1999) where the business cycle frequency band is defined as $[\frac{2\pi}{32}, \frac{2\pi}{6}]$, while the long-run frequency band is specified as $[\frac{2\pi}{400}, \frac{2\pi}{80}]$. The upper bound of the long-run frequency band is based on the findings of Angeletos et al. (2020), while the lower bound of $\frac{2\pi}{400}$ (100 years or 400 quarters) instead of $\approx 0$ is chosen to avoid any potential non-stationarity issues in the estimation process.

Additionally, the optimization procedure is subject to the constraint $q_B^{lr} q_B^{sr} = 0$, ensuring that the long-run and short-run frequency bands are orthonormal. This means that the inner product of the two vectors is zero and each vector has a unit length. The resulting shocks, $q_B^{lr}$ and $q_B^{sr}$, are interpreted as supply and demand shocks, respectively, based on their respective impulse response functions.

### 2.2 Solution Method

The problem of identification, expressed in equation 4, can be represented as,

$$
\max_{X \in \mathbb{R}^n \times p} \mathcal{F}(X), \text{ s.t. } X^T X = I_p
$$

where,

$$
\mathcal{F}(X) = \sum_{i=1}^{k} x_i^T A_i x_i
$$

A maximization of the sum of quadratic forms generated by different matrices.

The objective is to find the set of orthonormal elements in $\mathbb{R}^n$ (where $k \leq n$) that maximizes the functional $\sum_{i=1}^{k} x_i^T A_i x_i$, subject to the constraint $X^T X = I_k$.

This problem is distinct from a standard principal component analysis, where the objective is to find a system of $k$ orthonormal elements in $\mathbb{R}^n$ (where $k \leq n$) that maximize the functional $\sum_{i=1}^{k} x_i^T A x_i$, it is known that the $k$ largest eigenvalues of matrix $A$ and their associated orthonormal eigenvectors are the solution to this optimization problem. These eigenvectors, or principal components, represent the most...
informative directions in the data and capture the maximum amount of variation in
the data.

While in equation 5 the objective is to find the maximum of sum of quadratic forms
generated by different matrices. In most cases it cannot be the sum of the largest
eigenvalues of corresponding matrices, because the eigenvectors corresponding to
the maximal eigenvalues of $A_i$'s are usually not pairwise orthogonal.

In Bolla and Ziermann (1998), the authors prove the existence and uniqueness of
solutions for optimization problems of this nature. The solution is determined using
an adaptive feasible Barzilai-Borwein-like (AFBB) algorithm. The global convergence
of this algorithm is demonstrated in Jiang and Dai (2014), through the use of an
adaptive nonmonotone line search.

3. Empirical Results

This section presents the main empirical findings and discusses a few tentative
lessons for theory.

Figure 2 illustrates the impulse response functions (IRFs) of all variables to the
$q_{B,t}^l$ shock. This shock induces co-movement among key business cycle variables and
plays a substantial role in explaining business cycle volatility, accounting for over
50\% of the volatility in real per capita GDP. Additionally, the TFP impulse response to
the same shock exhibits remarkable persistence, explaining approximately 54\% of its
long-run volatility. Furthermore, the shock is responsible for significant fluctuations
in consumption and investment, both at business-cycle frequencies and in the long
run.

In the context of conditional correlations involving real GDP, TFP, inflation, and
interest rates, the shock has been identified as a long-run TFP shock within a new
Keynesian model. In accordance with a common assumption in the business cycle
literature, long-run TFP changes are considered exogenous. With the presence of
nominal rigidities, a positive TFP shock leads to a decrease in inflation due to a de-
cline in marginal cost, coupled with a decrease in interest rates following a standard
inflation targeting monetary rule.

Figure 3 reports the impulse response functions (IRFs) of all the variables to the
$q_{B,t}^s$ shock. The shock results in co-movement of the key business cycle variables and
explains significant business cycle volatility but not long-run. The impulse responses
are quite significant on impact. No movement or business cycle volatility explained
for TFP.

In contrast to the previous shock, this particular one exhibits no short-run or long-run movements in TFP or labor-productivity IRFs. Furthermore, the positive comovement observed in TFP, consumption, inflation, and interest rates allows us to identify this shock as a demand shock. While the previous supply shock was characterized as a long-run TFP shock, the precise interpretation of this structural shock is model-dependent. In the forthcoming section, I will present arguments, drawing from Smets and Wouters (2007), hereafter referred to as SW07, medium-scale DSGE model, to support the notion that the identified shock represents a risk-premia shock. However, it is essential to acknowledge that the further structural interpretation of the demand shock varies depending on the perspective of the model utilized.
Figure 2: Impulse Response Functions to Supply shock

Impulse Response Functions of all the variables to the identified supply shock. Horizontal axis: time horizon in quarters. Shaded area: 68 percent Highest Posterior Density Interval (HPDI).

Table 1: Supply Shock, Variance Contributions

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$Y$</th>
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<tr>
<td>Short run (6–32 quarters)</td>
<td>32</td>
<td>53.1</td>
<td>29.8</td>
<td>40.6</td>
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<td>[21.2,43.8]</td>
<td>[33.4,70.5]</td>
<td>[21.2,40.2]</td>
<td>[25.7,57.1]</td>
<td>[25.6,39.9]</td>
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<tr>
<td>Long run (80–400 quarters)</td>
<td>35.7</td>
<td>51.8</td>
<td>20.6</td>
<td>47.9</td>
<td>51.4</td>
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<td></td>
<td>[19.1,55.5]</td>
<td>[26.7,72.7]</td>
<td>[4.8,48.2]</td>
<td>[22.4,70.2]</td>
<td>[26.1,71.9]</td>
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<th>$wh/Y$</th>
<th>$\Delta p$</th>
<th>FFR</th>
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<tr>
<td>Short run (6–32 quarters)</td>
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<td>36.6</td>
<td>30.3</td>
<td>19</td>
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<td>[10.7,29.2]</td>
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<tr>
<td>Long run (80–400 quarters)</td>
<td>53.7</td>
<td>54.3</td>
<td>41.9</td>
<td>14.2</td>
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<td></td>
<td>[29.4,71.4]</td>
<td>[29.72.1]</td>
<td>[18,63]</td>
<td>[5.8,29.9]</td>
<td>[6.4,34.7]</td>
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Notes: Variance contributions of the MBC shock at two frequency bands. The first row (Short run) corresponds to the range between 6 and 32 quarters, the second row (Long run) to the range between 80 quarters and $\infty$. The notation used for the variables is the same as that introduced in Section I. 68 percent HPDI in brackets.
Figure 3: Impulse Response Functions to Demand shock

Impulse Response Functions of all the variables to the identified demand shock. Horizontal axis: time horizon in quarters. Shaded area: 68 percent Highest Posterior Density Interval (HPDI).

Table 2: Demand Shock, Variance Contributions

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<td>Short run (6-32 quarters)</td>
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<td>Short run (6-32 quarters)</td>
<td>7.4</td>
<td>22.5</td>
<td>16.5</td>
<td>10.9</td>
<td>50.5</td>
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<tr>
<td></td>
<td>[2.5,15.6]</td>
<td>[12.5,33.9]</td>
<td>[ 7.32.1]</td>
<td>[ 5.6,18.8]</td>
<td>[36.6,60.7]</td>
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<tr>
<td>Long run (80-400 quarters)</td>
<td>0.04</td>
<td>0.04</td>
<td>1.1</td>
<td>3.7</td>
<td>12.9</td>
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<tr>
<td></td>
<td>[ 0, 0.2]</td>
<td>[ 0.01, 0.2]</td>
<td>[ 0.2, 4]</td>
<td>[ 1.1, 9.7]</td>
<td>[ 4.6,26.3]</td>
</tr>
</tbody>
</table>

Notes: Variance contributions of the MBC shock at two frequency bands. The first row (Short run) corresponds to the range between 6 and 32 quarters, the second row (Long run) to the range between 80 quarters and $\infty$. The notation used for the variables is the same as that introduced in Section I. 68 percent HPDI in brackets.
4. Full Information Estimation: Challenges

In this section, the aim is to illustrate how the Bayesian likelihood estimation poses challenges due to the presence of long-run shocks that don’t result in business cycles. This would involve representing the Gaussian log-likelihood function of the state-space models in the frequency domain. The frequency domain provides the decomposition of fluctuations of a variable into fluctuations of different periodicity. This breakdown helps us separate long-term and short-term fluctuations of the variable, which is important for this paper’s purpose.

Let’s start with a canonical representation of a linearized DSGE model:

\[ \Gamma_0 S_t = \Gamma_1 S_{t-1} + \Psi \epsilon_t + \Pi \eta_t \]

Where 1) \( S_t \) is a vector of model variables that include (i) the endogenous variables, (ii) the conditional expectations, (iii) the variables from exogenous processes if they are serially correlated; 2) \( \epsilon_t \) is a vector of exogenous disturbances; 3) \( \eta_t \) is a vector of expectation errors satisfying \( E_{t-1} \eta_t = 0 \) for all \( t \); 4) \( \Gamma_0, \Gamma_1 \) and \( \Pi \) are coefficient matrices; 5) \( \Psi \) a diagonal matrix with standard deviations of the exogenous disturbances.

Assuming the above set of equilibrium conditions that represent optimality conditions have a state-space representation and mapping to a vector of observables \( Y_t \):

\[ S_t = \Theta_1(\theta)S_{t-1} + \Theta_\epsilon(\theta)\Psi(\theta_1)\epsilon_t \]

\[ Y_t = A(L)S_t = A(L)(I - \Theta_1(\theta)L)^{-1}\Theta_\epsilon(\theta)\Psi(\theta_1)\epsilon_t = D(L; \theta)\Theta_\epsilon(\theta)\Psi(\theta_1)\epsilon_t \]

Where 1) \( \theta_1 \) is a vector of standard deviations of exogenous shocks; 2) \( \theta \) is a vector of all deep parameters of the DSGE model except the ones in \( \theta_1 \).

The model implied Spectral Density of variable \( k \) due to shock \( l \) in \( Y_t \) is represented as:

\[ SD(\omega, k; l; \theta, \theta_1) = \frac{1}{2\pi} \mathcal{M}(\omega, y_k, l; \theta)^2 \sigma_l^2, \quad \text{where} \quad \mathcal{M}(\omega, y_k, l; \theta) = D^k(e^{i\omega}; \theta)\Theta_\epsilon(\theta) \]

For the sake of tractability, let’s make a simplifying assumption. Let’s assume the
true data generating process of the variable $y_k$ involves two exogenous shocks $\epsilon_{lB}$ & $\epsilon_{lNB}$. Here, $\epsilon_{lB}$ represents a long-run shock that results in business cycles, while $\epsilon_{lNB}$ represents a long-run that doesn’t cause business cycles.

Since standard DSGE models don’t allow both categories of long-run shocks by having restrictions that allow one to cause business cycles but not the other, the exogenous shock process vector $\epsilon$ in the canonical representation above comprises only one of the structural shocks, which is $\epsilon_{lB}$.

The log-likelihood function of the above state space model in the frequency domain following Harvey (1989) is as follows:

$$\log L(\theta, \theta_1) = -\sum_{j=1}^{T} \left( \log \frac{1}{2\pi} M(\omega_j, y_k, \mathbf{IB}; \theta)^2 \sigma_{\epsilon_{lB}}^2 + \frac{I(\omega_j, y_k)}{\pi} \Theta(\omega_j, y_k, \mathbf{IB}; \theta)^2 \sigma_{\epsilon_{lB}}^2 \right)$$

where, $\omega_j = \frac{2\pi t}{T}$. The likelihood function depends on two arguments: the spectral density of the model $S\mathcal{D}(\omega, k, l; \theta, \theta_1)$ and the periodogram $I_y(\omega_j, k)$ which is the data implied volatility.

$$I(\omega_j, k) = \frac{1}{2\pi} D(y_k, \omega_j, \mathbf{IB}) \sigma_{\epsilon_{lB}}^2 + \frac{1}{2\pi} D(y_k, \omega_j, \mathbf{INB}) \sigma_{\epsilon_{lNB}}^2$$

where as defined in section 2.1, $D(y_k, \omega_j, l) = q_l D^j (e^{-i\omega_j}) D^j (e^{-i\omega_j}) q_l$

Maximising $\log L$ with respect to $\sigma_{\epsilon_{lB}}^2$ gives:

$$\hat{\sigma}_{\epsilon_{lB}}^2(\theta) = \frac{2\pi}{T} \sum_{j=1}^{T} \frac{I(\omega_j, y_k)}{\pi} \Theta(\omega_j, y_k, \mathbf{IB}; \theta)^2$$

Following, Harvey (1989) (pg. 193), the exogenous shock variance may therefore, be concentrated out of the likelihood function, with the result that maximising $\log L$ is equivalent to minimizing

$$S(\theta) = \sum_{j=1}^{t} \frac{I(\omega_j, y_k)}{\pi} \Theta(\omega_j, y_k, \mathbf{IB}; \theta)^2 + \sum_{j=t+1}^{T} \frac{I(\omega_j, y_k)}{\pi} \Theta(\omega_j, y_k, \mathbf{IB}; \theta)^2$$

DSGE models estimated in the time domain are equivalent to fitting the model
over the whole spectral density. These models generate cross-frequency restrictions, the presence of information in the estimation that the model is not intended to explain may affect the estimates.

From a classical result of Kolmogorov, the cross-frequency restrictions on the model with only one long-run shock ($\epsilon_lB$) can be represented with long-run and short-run components as

$$
\sum_{j=1}^{t} \log \mathcal{M}(\omega_j, y_k; \theta)^2 + \sum_{j=t+1}^{T} \log \mathcal{M}(\omega_j, y_k; \theta)^2 = 0
$$

In summary estimation of vector $\theta$ of model parameters is equivalent to minimizing $S(\theta)$ subject to the above constraint.

To understand the bias introduced by the presence of long-run Non-business cycle fluctuations ($D(y_k, \omega_j, \text{INB})$) in the data ($I_y(\omega_j, k)$), let’s assume that the above DSGE model is well specified for business cycle fluctuations.

Suppose $\exists \theta^*$ s.t. $D(y_k, \omega_j, \text{IB}) = |\mathcal{M}(\omega_j, y_k; \theta^*)|^2 \forall \omega_j$

The minimum value of $S(\theta)$ given the cross-frequency constraint is $T \frac{\sigma^2_{\text{IB}}}{2\pi}$. The minimization of $S(\theta)$ is achieved at $\theta^*$ when $D(y_k, \omega_j, \text{INB}) = 0$

$$
S(\theta) = \sum_{j=1}^{t} \frac{1}{2\pi} \frac{D(y_k, \omega_j, \text{IB})\sigma^2_{\text{IB}} + D(y_k, \omega_j, \text{INB})\sigma^2_{\text{INB}}}{\mathcal{M}(\omega_j, y_k; \theta)^2} + \sum_{j=t+1}^{T} \frac{1}{2\pi} \frac{D(y_k, \omega_j, \text{IB})\sigma^2_{\text{IB}} + 0}{\mathcal{M}(\omega_j, y_k; \theta)^2}
$$

$$
S(\theta^*) = \sum_{j=1}^{t} \frac{\sigma^2_{\text{IB}}}{2\pi} \frac{D(y_k, \omega_j, \text{INB})\sigma^2_{\text{INB}}}{|\mathcal{M}(\omega_j, y_k; \theta^*)|^2} + \sum_{j=t+1}^{T} \frac{\sigma^2_{\text{IB}}}{2\pi} \frac{D(y_k, \omega_j, \text{INB})\sigma^2_{\text{INB}}}{|\mathcal{M}(\omega_j, y_k; \theta^*)|^2} = T \frac{\sigma^2_{\text{IB}}}{2\pi} + \sum_{j=1}^{t} \frac{D(y_k, \omega_j, \text{INB})\sigma^2_{\text{INB}}}{|\mathcal{M}(\omega_j, y_k; \theta^*)|^2}
$$
\[ \mathcal{D}(y_k, \omega_j, \text{INB}) \overset{\text{minimize} \ S(\theta)}{\longrightarrow} \sum_{j=1}^{t} \mathcal{M}(\omega_j, y_k, \text{IB}; \theta)^2 \overset{\text{restriction}}{\longrightarrow} \sum_{j=t+1}^{T} \mathcal{M}(\omega_j, y_k, \text{IB}; \theta)^2 \]

\( \theta \) changes such that the model implied long-run volatility increases, resulting in a downward bias on short-run volatility of the model. This argues for estimation in a limited information setting via IRF matching with the business cycle shocks, given the cross-frequency restriction on the DSGE models. The following section showcases the normative and policy implications of the Smets and Wouters (2007) by comparative analysis of the model, estimated using conditional moments, and the Bayesian likelihood estimation of the original estimated model.

This section centers on applying the above identification strategy for comparative analysis with full information estimation using a benchmark medium-scale DSGE model proposed by Smets and Wouters (2007) (SW07). The model serves a two-fold purpose: firstly, to label the demand shock based on both the model and empirical impulse response functions (IRFs); secondly, to examine the normative and policy implications by identifying the model parameters through impulse response matching, a method commonly utilized, among others, in Christiano et al. (2005).

5.1 The Model

Their model includes monopolistic competition in goods and labor markets, sticky prices and wages, partial indexation of prices and wages, investment adjustment costs, habit persistence, and variable capacity utilization. The economy evolves along a balanced growth path, driven by deterministic labor-augmenting technological progress. The endogenous variables in the model, expressed as log-deviations from steady state, are output \( y_t \), consumption \( c_t \), investment \( i_t \), utilized and installed capital \( k^u_t, k_t \), capacity utilization \( \epsilon_t \), rental rate of capital \( r^k_t \), Tobin’s \( q(q_t) \), price and wage markup \( \mu^p_t, \mu^w_t \), inflation rate \( \pi_t \), real wage \( w_t \), total hours worked \( l_t \), and nominal interest rate \( r_t \). The log-linearized equilibrium conditions for these variables are presented in the appendix (7.1). The last equation in the table gives the policy rule followed by the central bank, which sets the nominal interest rate in response to inflation and the deviation of output from its potential level. To determine potential output, defined as the level of output that would prevail without the price and wage mark-up shocks. The business cycle dynamics of the model are driven by seven stationary shocks. Five of them-total factor productivity \( \varepsilon^a_t \), investment-specific technology \( \varepsilon^i_t \), government purchases \( \varepsilon^g_t \), risk premium \( \varepsilon^b_t \) and monetary policy \( \varepsilon^r_t \)-follow AR(1) processes; the remaining two shocks-to wage and price markup \( \varepsilon^p_t \) and \( \varepsilon^w_t \) - follow ARMA(1,1) processes.

5.2 Identification analysis

The model is estimated using data on seven variables: real GDP, real consumption, real investment, real wage, inflation, hours worked and the nominal interest rate. Thus, the vector of observables is given by
For comparative analysis with the dissection strategy suggested in this paper, the SW07 model is estimated with Bayesian likelihood estimation techniques in SW07 using seven key macroeconomic quarterly US time series as observable variables: the log difference of real GDP, real consumption, real investment, and the real wage, log hours worked, the log difference of the GDP deflator, and the federal funds rate. Then, the business cycle shock identification strategy is applied to a VAR with the same seven observables.

The impulse response results, reflecting both long-run and short-run shocks, consistently align with the identification presented in the benchmark VAR model introduced by Angeletos et al. (2020) in the preceding section. The identification process is substantiated by the negative conditional correlation observed between real GDP and inflation, the long-run persistence of the real GDP impulse response as depicted in Figure 4, and the substantial explanatory power exhibited by the macroeconomic variables’ business cycle and long-run volatility, as demonstrated in Table 3. This alignment highlights the identification of the long-run shock being attributed to a supply shock.

Similarly, the identification of the short-run shock as a demand shock is reinforced by the positive conditional correlation observed between real GDP, inflation, and the federal funds rate. These conditional correlations are evident in Figure 4. Moreover, the explanation of the identified model to explain the business-cycle volatility across various macro-variables, highlighted in Table 3, further substantiates the identification of the short-run shock as a demand shock.

Given the array of seven shocks in SW07, there are only two shocks with potential to generate characteristic business-cycle comovement. Within the category of demand shocks that comprises, discount factor shock, risk-premia shock, monetary shock, investment shock, spending shock, and monetary shock it is the risk premium shock helps to explain the comovement of consumption and investment in presence of nominal rigidities. This encourages the further identification of the demand shock as a risk premia shock.

In the upcoming subsection, I delve into the identification of parameters by employing impulse response matching with the same set of observables utilized in SW07. This involves using the previously identified long-run GDP shock as a supply shock.
Figure 4: Smets & Wouter (2007) VAR: Impulse Response Functions

Impulse Response Functions of all the variables to the identified supply shock. Horizontal axis: time horizon in quarters. Shaded area: 68 percent Highest Posterior Density Interval (HPDI).

Table 3: Variance Contributions

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<th>Y</th>
<th>h</th>
<th>I</th>
<th>C</th>
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<td><strong>Supply Shock</strong></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Short run (6–32 quarters)</td>
<td>57.3</td>
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<td>42.9</td>
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<td>[33.9, 76]</td>
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<td>Long run (80–400 quarters)</td>
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<td>69.1</td>
<td>65</td>
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<td>[44.8, 84.4]</td>
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<td>[34.9, 85.3]</td>
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<td><strong>Demand Shock</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short run (6–32 quarters)</td>
<td>42.5</td>
<td>30.7</td>
<td>38.4</td>
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<td>Supply Shock: Short run (6–32 quarters)</td>
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<td>19.78</td>
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<td>Supply Shock: Long run (80–400 quarters)</td>
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<td>65.3</td>
<td>27.7</td>
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<td>[12.9, 51.7]</td>
<td>[33.8, 85.2]</td>
<td>[11, 56]</td>
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<tr>
<td>Demand Shock: Short run (6–32 quarters)</td>
<td>41.2</td>
<td>5.5</td>
<td>8.3</td>
</tr>
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<td></td>
<td>[25.4, 53.6]</td>
<td>[2.3, 12.7]</td>
<td>[3.1, 20.9]</td>
</tr>
</tbody>
</table>

Notes: Variance contributions of the MBC shock at two frequency bands. The first row (Short run) corresponds to the range between 6 and 32 quarters, and the second row (Long run) to the range between 80 quarters and \( \infty \). The notation used for the variables is the same as that introduced in Section I. 68 percent HPDI in brackets.
shock and risk-premia as demand business cycle shocks.

5.3 Estimated Parameters

Consider a model that matches impulse responses of all seven variables to two structural shocks, the identified long-run TFP and a risk-premia shock. Following rank conditions of Iskrev (2010) & Komunjer and Ng (2011), two of the unidentifiable parameters are $\bar{l}, \bar{\pi}$, which affect only the mean of observables; in addition, there is a set of five parameters, namely $\delta, \beta, \Theta, \lambda$ and $\gamma$, any four of which can be identified only if the fifth one is known. Also, the lack of identification corresponding to $\varepsilon_p$ and $\xi_p$ on one hand, and $\varepsilon_w$ and $\xi_w$ on the other. As in SW07, I assume that the curvature parameters $\varepsilon_w, \varepsilon_p$ are known and are both equal to 10. Following SW07, I assume the calibrated values of capital depreciation rate $\delta$, steady state wage markups and fixed cost share. Additionally, the parameters that characterize the stochastic properties of the excluded structural shocks are also unidentifiable. A TFP shock activates the government purchases process, thus identifying $\rho_{ga}$ and $\rho_g$ (see Eq. (21) 7.1) in addition to 21 other parameters. Inclusion of risk premia shocks extends the number of parameters estimated to 23. Table 4 reports the estimated parameter values.

5.4 Comparative Analysis of Estimation Methods

Figure 5 presents impulse response functions (IRFs) to illustrate the effects of different estimation methods on a model’s performance. The blue lines depict the IRFs of the model estimated through IRF matching with identified business cycle shocks (shown in red) in response to a one-standard-deviation Total Factor Productivity (TFP) shock and a risk-premia shock. This estimation method accurately captures the dynamic responses of these identified shocks. Additionally, I include the IRFs of the SW07 model estimated using Bayesian likelihood, represented in black. While the Bayesian estimation aligns well with the empirical IRFs in most aspects, there are disparities in investment and hours worked.

To comprehend the implications of these two estimation methods, this section illustrates the comparative analysis in two parts.

Firstly, Figure 7 focuses on the response to a long-run TFP shock. Here, the two estimation methods lead to significantly different normative and policy implications. In the Bayesian estimated model (in black), inflation and the output gap exhibit a
positive correlation, resembling the divine coincidence often seen in New Keynesian models with demand shocks. In contrast, the model estimated via IRF matching with identified business cycle shocks reveals a negative conditional correlation between the output gap and inflation. This suggests a policy tradeoff for the monetary authority: lowering interest rates to counter falling inflation may lead to a further increase in the output gap.

Secondly, through Figure 8, I argue the two estimations result in different inferences about the underlying internal and external propagation mechanisms of SW07. The mentioned figure has empirical and model IRFs for a risk premia shock. The standard deviation of the risk premia shock ($\sigma_{rp}$) for the Bayesian likelihood estimated is 0.1762. I first replaced it with the IRF matching estimated $\sigma_{rp}$ of 0.0131 which is 17 times smaller than the full information setting value while I kept the rest of the parameters estimated by Bayesian likelihood estimation unchanged. As one may observe in the black IRFs, in the same figure, the risk premia shock becomes insignificant. Next I replace the investment elasticity estimated by Bayesian likelihood (8.0145) with the IRF matching estimated value of 0.0145 and as we may observe risk premia IRFs become significant on impact and persistence. This showcases the key differences in estimation where full information setting argues for a stronger external propagation role in risk premia-driven business cycles while IRF matching argues for a stronger internal propagation through financial frictions.

In summary, the two estimations result in key differences for the same model over the same dataset in terms of normative and policy implications which is crucial given the use of augmented versions of such medium-scale DSGE models at central banks. Based on theoretical insights from the previous section, it argues for limited information estimation to avoid any biased business cycle implications because of non-business cycle information in the full information settings.
Figure 5: Smets & Wouters (2007): Impulse Response Functions

(a) Long-run TFP Shock

(b) Risk Premia Shock
Figure 7: Long-run TFP Shock: Output Gap & Inflation

Figure 8: Risk Premia Shock: Internal vs. External Propagation
6. Conclusion

The paper introduces a novel approach for analyzing GDP fluctuations, uncovering the dynamic causal effects of long-run supply and short-run demand shocks as drivers of business cycles. These shocks collectively account for nearly 99%

Furthermore, the empirical findings reveal the existence of a second category of long-run supply shocks that do not influence GDP's business cycle fluctuations. This, coupled with theoretical insights, emphasizes the parameter bias inherent in medium-scale DSGE models estimated under full information settings. These biased parameters can lead to a downward distortion of the business cycle implications derived from estimated DSGE models.

By estimating medium-scale DSGE models using conditional moments derived from identified business cycle shocks, the paper sheds light on the normative and policy implications of the same model when estimated under full information settings.
References


Basu, Susanto, John Fernald, and Miles Kimball, “Are Technology Improvements Contrac-


Benhima, Kenza and Celine Poilly, “Does Demand Noise Matter? Identification and Implica-


7. Appendix
Table 4: Estimated Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Estimated Value</th>
<th>lb</th>
<th>ub</th>
</tr>
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<tr>
<td>$\rho_{ga}$</td>
<td>Feedback technology on exogenous spending</td>
<td>0.9905</td>
<td>0.0100</td>
<td>2.0000</td>
</tr>
<tr>
<td>$100(\beta^{-1} - 1)$</td>
<td>time preference rate in percent</td>
<td>1.7162</td>
<td>0.0100</td>
<td>2.0000</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>capital share</td>
<td>0.18</td>
<td>0.0100</td>
<td>1.0000</td>
</tr>
<tr>
<td>$\psi$</td>
<td>capacity utilization cost</td>
<td>0.9658</td>
<td>0.0100</td>
<td>1.0000</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>investment adjustment cost</td>
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<td>0</td>
<td>15.0000</td>
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<tr>
<td>$\sigma_c$</td>
<td>risk aversion</td>
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<td>external habit degree</td>
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<td>0.0010</td>
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<td>fixed cost share</td>
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<td>1.0000</td>
<td>3.0000</td>
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<tr>
<td>$l_w$</td>
<td>Indexation to past wages</td>
<td>0.8241</td>
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<td>$\sigma_f$</td>
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<td>0.2500</td>
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<td>1.0000</td>
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<tr>
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<td>Taylor rule output growth feedback</td>
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<td>0.0010</td>
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<td>0.0010</td>
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<td>$\rho$</td>
<td>interest rate persistence</td>
<td>0.7522</td>
<td>0.5000</td>
<td>0.9750</td>
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<td>$\rho_s$</td>
<td>persistence productivity shock</td>
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<td>$\rho_b$</td>
<td>persistence risk premium shock</td>
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<td>persistence spending shock</td>
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<td>$\sigma_b$</td>
<td>Std. risk premium shock</td>
<td>0.0131</td>
<td>0.0100</td>
<td>5.0000</td>
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7.1 Log-linearized equations of the SW07 model
(sticky-price-wage economy)

\[ y_t = c_y c_t + i_y i_t + i^{kss} k_y e_t + \varepsilon_t^q \]
\[ c_t = \frac{\lambda/\gamma}{1 + \lambda/\gamma} c_{t-1} + \frac{1}{1 + \lambda/\gamma} E_t c_{t+1} + \frac{w^{ss}_s s^{ss}_s (\sigma_c - 1)}{\sigma_c (1 + \lambda/\gamma)} (l_t - E_t l_{t+1}) - \frac{1 - \lambda/\gamma}{(1 + \lambda/\gamma) \sigma_c} (c_t - E_t c_{t+1}) - \frac{1 - \lambda/\gamma}{(1 + \lambda/\gamma) \sigma_c} \varepsilon_t^b \]
\[ i_t = \frac{1}{1 + \beta \gamma (1 - \sigma_c)} i_{t-1} + \frac{\beta \gamma (1 - \sigma_c)}{1 + \beta \gamma (1 - \sigma_c)} E_t i_{t+1} + \frac{1}{\Theta \gamma^2 (1 + \beta \gamma (1 - \sigma_c))} q_t + \varepsilon_t^i \]
\[ q_t = \beta (1 - \delta) \gamma^{-\sigma_c} E_t q_{t+1} - \tau_t + E_t \pi_{t+1} + (1 - \beta (1 - \delta) \gamma^{-\sigma_c}) E_t k^{t+1}_t - \varepsilon_t^b \]
\[ y_t = \Theta (\alpha k^a_t + (1 - \alpha) l_t + \varepsilon^a_t) \]
\[ k^i_t = k_{t-1} + \varepsilon_t \]
\[ k_t = (1 - \delta) / \gamma k_{t-1} + (1 - (1 - \delta) / \gamma) i_t + (1 - (1 - \delta) / \gamma) \varphi \gamma^2 (1 + \beta \gamma (1 - \sigma_c)) \varepsilon^i_t \]
\[ \mu^p_t = \alpha (k^a_t - l_t) - w_t + \varepsilon_t^a \]
\[ \pi_t = \frac{\beta \gamma (1 - \sigma_c)}{1 + \beta \gamma (1 - \sigma_c)} E_t \pi_{t+1} + \frac{I_p}{1 + \beta \gamma (1 - \sigma_c) \pi_{t-1}} - \frac{1 - \beta \gamma (1 - \sigma_c) \xi_p}{1 + \beta \gamma (1 - \sigma_c) \pi_{t-1}} (1 - \xi_p) \mu^p_{t} + \varepsilon_t^p \]
\[ r^k_t = l_t + w_t - k_t \]
\[ \mu^w_t = w_t - \sigma l_t - \frac{1}{1 - \lambda / \gamma} (c_t - \lambda / \gamma c_{t-1}) \]
\[ w_t = \frac{\beta \gamma (1 - \sigma_c)}{1 + \beta \gamma (1 - \sigma_c)} (E_t w_{t+1} + E_t \pi_{t+1}) + \frac{1}{1 + \beta \gamma (1 - \sigma_c)} (w_{t-1} + l_w \pi_{t-1}) - \frac{1 + \beta \gamma (1 - \sigma_c) I_w}{1 + \beta \gamma (1 - \sigma_c) \pi_{t-1}} - \frac{1 - \beta \gamma (1 - \sigma_c) \xi_w}{1 + \beta \gamma (1 - \sigma_c) \pi_{t-1} - (1 - \beta \gamma (1 - \sigma_c) \xi_w) (1 - \xi_w) + \mu^w_t + \varepsilon_t^w} \]
\[ r_t = \rho r_{t-1} + (1 - \rho) (r_x \pi_t + r_y (y_t - y^*_t)) + r_{\Delta y} (y_t - y^*_t) - (y_{t-1} - y^*_t)) + \varepsilon_t \]
\[ \varepsilon^a_q = \rho_a \varepsilon^a_{t-1} + \eta^a_t \]
\[ \varepsilon^b_i = \rho_b \varepsilon^b_{t-1} + \eta^b_t \]
\[ \varepsilon^g_q = \rho_g \varepsilon^g_{t-1} + \eta^g_t \]

Note: The model variables are: output \((y_t)\), consumption \((c_t)\), investment \((i_t)\), utilized and installed capital \((k^i_t, k_t)\), capacity utilization \((\varepsilon_t)\), rental rate of capital \((r^k_t)\).
Tobin’s $q (q_t)$, price and wage markup ($\mu^p_t, \mu^w_t$), inflation rate ($\pi_t$), real wage ($w_t$), total hours worked ($l_t$), and nominal interest rate ($r_t$). The shocks are: total factor productivity ($\epsilon^a_t$), investment-specific technology ($\epsilon^i_t$), government purchases ($\epsilon^g_t$), risk premium ($\epsilon^b_t$), monetary policy ($\epsilon^r_t$), wage markup ($\epsilon^w_t$) and price markup ($\epsilon^p_t$).