

Credibility of Monetary Policy with Fiscal Conditionality

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Abstract

Shall a central bank strike back when the treasury tries to break its independence? We study an environment where only the central bank has a commitment technology to announce future policy. We contrast the implications of two monetary rules: one where the central bank follows a standard rule, e.g. inflation targeting, against one where monetary decisions lean against fiscal influence. We show that designing explicit fiscal conditionality not only improve economic outcomes, but also can enhance the overall credibility of a central bank. In particular, these relative credibility gains are higher when public debt is high and nominal, and lower when debt is real with long maturity.

Keywords: *Public debt, fiscal-monetary policy, time-consistency, credibility and limited commitment.*

JEL classification: E02, E52, E58, E61, E62

Over the past decades, central banks have been granted independence, with the objective to shield monetary policymakers from political influence and reduce inflationary bias. Under this institutional arrangement, politicians remain in charge of taxes, debt and deficit, while central bank's mandates are tailored around different concepts of price stability. This policy assignment is challenged by mounting political pressures and government interference with central banks.¹ Electoral concerns or imperatives of public debt management are increasingly weighting on monetary policy credibility. In this context, we ask whether a central bank should explicitly lean against fiscal pressures to preserve its independence. Our answer is positive, for two reasons. Providing explicit incentives to the fiscal authority not only improves economic outcomes, but also can enhance the overall credibility of a central bank. In particular, the analysis shows that relative credibility gains are the highest at higher level of nominal public debt and when debt is real with short maturity.

Surprisingly, several components of existing monetary frameworks already include provisions contingent on fiscal decisions, especially in the Eurozone. Multiple supporting schemes of the European Central Bank

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¹Binder (2018) documents political pressure faced by 118 central banks from 2010 to 2018. Pressure is widespread: 39% of the central banks report at least one event of pressure. In an average quarter, there are reports of political pressure on over 5% of the central banks. In over 90% of cases, the political pressure is for easier monetary policy.

(ECB) are conditional on the pursuit of fiscal programs, with the implicit objective to contain the spending bias of treasuries. These conditions are present in Emergency Liquidity Assistance (ELA) programs to banks or in the Outright Monetary Transactions (OMT) of sovereign bonds.² The refinancing framework of the ECB includes similar clauses, where an investment grade rating of public debt is a pre-requisite to participate in refinancing operations.³

The presence of fiscal contingencies in monetary interventions is controversial as it might per se compromise the mandate of an independent central bank and threaten the credibility of monetary policy makers. For instance, Orphanides (2013) argues *overburdening monetary policy may eventually diminish and compromise the independence and credibility of the central bank, thereby reducing its effectiveness to preserve price stability*. We take this concern seriously and study formally whether monetary policy with fiscal conditionality is effective to discipline treasuries without compromising the pursuit of price stability.

To this end, we analyze a non cooperative game between a monetary and fiscal authority, where only the central bank has a commitment technology. At a *constitutional* stage, the central bank discloses its operational framework and announces a monetary rule, which specifies policy decisions for every possible state of the economy. Then, at each point in time, the fiscal authority moves first and implements a policy decision. Given this fiscal choice, the monetary authority faces the following alternative: either to set policy according to the announced rule, or to renounce the promise and implement a sequentially optimal policy decision. Importantly, deviating from the rule is costly to the central bank. The magnitude of the cost captures the degree of commitment of the monetary authority.⁴

Under this timing, the fiscal authority might have an incentive to influence the central bank to renounce its rule and reap the short-term benefits of policy discretion. The credibility of a monetary rule is then defined as the minimum degree of commitment that contains the fiscal incentives to challenge the monetary pledge, hence eliminates all monetary incentives to renounce the rule.

We contrast the welfare and credibility effects of two monetary policy rules. Following a *standard rule*, the central bank commits to set policy without responding to fiscal decisions.⁵ An alternative *strategic rule* instead prescribes to set monetary policy explicitly conditional on fiscal decisions. We focus on strategic rules that rely on off-equilibrium threats designed to discourage fiscal deviations from a desired equilibrium path. This way the central bank can influence the treasury and induce an equilibrium outcome that coincides with the one that would prevail if both authorities had full commitment. By design, *strategic rules* dominate *standard rules* from a welfare perspective, but what is the effect of added conditionality on credibility? To address this question, we study the equilibrium implications of this game in two different yet related

²Conditional monetary assistance is explicit for the OMT program: "A necessary condition for Outright Monetary Transactions is strict and effective conditionality attached to an appropriate European Financial Stability Facility/European Stability Mechanism (EFSE/ESM) program." The argument against the OMT program, brought to the German Constitutional Court in 2014, stressed precisely that conditional support to treasuries was beyond the mandate of the ECB, because it was in conflict with the prohibition of monetary financing of member states. See Siekmann and Wieland (2014). ELA for Greek banks turned out to be a critical element in the bargaining game between European institutions and the newly elected Greek government in 2015.

³For details on the collateral framework policies of the European Central Bank, see Claeys and Goncalves Raposo (2018).

⁴The extreme cases where the cost is zero or infinite correspond respectively to no commitment or full commitment.

⁵The rule is *standard* in the sense that it refers to a conventional policy assignment: the central bank controls inflation and the treasury controls taxes, deficit and debt.

frameworks.⁶

First, we present the institutional environment and the implications of different monetary rules in an extension of the linear-quadratic environment of Barro and Gordon (1983). The policy game is static in the sense that each policy maker plays only once. Absent a commitment technology, both monetary and fiscal authorities are confronted with time-inconsistency problems that give rise to an *inflation bias* and, by analogy, a *fiscal bias*. In this context, a *standard* rule prescribes the central bank to deliver unconditionally an inflation target. A *strategic* rule is designed to both eliminate the *fiscal bias* and achieve the inflation target. Under these two regimes, the credibility of a monetary rule decreases when the relative gains of renouncing the rule are large. Further, contrasting credibility between rules provides our central result: the strategic monetary rule requires a lower degree of commitment precisely when renouncing the unconditional inflation target is more tempting. In other terms, strategic rules enhance the credibility of the central bank to deliver its inflation target when the monetary authority is exposed the most to the possibility of successful fiscal influence.⁷

Extending this analysis, we study the implications of differences in objective functions of policy makers. We consider first the case of an inflation conservative central banker, as in Rogoff (1985). As is well understood, the credibility of an unconditional standard rule is improved by the appointment of a conservative central banker. In contrast, the credibility of strategic monetary rules is at stake when the central banker is too conservative. Indeed, a conservative central banker perceives the threats embedded in strategic rules as more costly. Second, we examine a self-interested treasury that puts more weight on short-term stimulation of output out of political economy consideration. Here, the credibility of strategic monetary rules is not sensitive to fiscal self-interest. More generally, the following insight applies: a strategic rule is designed using the objective function of the fiscal authority to provide incentives, while the credibility is evaluated using central bank's objective.

To account explicitly for the role of public debt in monetary-fiscal interactions, we fit the policy game into a dynamic cash-credit economy as in Lucas and Stokey (1983). The cash-credit economy brings together concerns for the conduct of monetary and fiscal policy under lack of commitment. The monetary authority is tempted to generate unexpected inflation to inflate away outstanding debt. The fiscal authority is tempted to set policy to manipulate interest rates and the price of newly-issued debt. As before, we contrast equilibrium outcomes under two classes of monetary rules: one where the central bank commits to a *standard* constant money growth rate, and one where the central bank designs a *strategic* rule, with the objective to eliminate the sequential incentives of the fiscal authority to manipulate interest rates.

The analysis yields the following results. First, a higher level of outstanding debt increases the degree of commitment required to implement either monetary rule. Second, the nature of public debt is critical for the design of monetary rules and associated credibility. If debt is nominal, then simple constant money growth rate rules eliminate fiscal incentives to manipulate interest rates: indeed, fiscal incentives to manipulate

⁶Independently of the nature of the monetary rule, evaluating credibility requires to assess on and off equilibrium paths for both monetary and fiscal authorities.

⁷Within the model, central bank's credibility is at stake when monetary deviations from the inflation target are relatively not costly and generate large effects on output.

the price of newly issued debt is offset by a reevaluation of outstanding debt.⁸ Still, pursuing such an unconditional rule requires a higher degree of credibility than following a strategic rule, where monetary policy is set explicitly contingent on fiscal choices. Further, when public debt returns are indexed to variations in the price level, the degree of commitment required to support the strategic monetary rule is lower. Indeed, as established in the linear-quadratic framework, the credibility cut-off depends on the relative benefits of renouncing the rule: under nominal debt, a central bank defaulting on its promise can achieve larger welfare gains by inflating outstanding liabilities, in contrast to the case where debt is real.

We also investigate the influence of debt maturity on the credibility of strategic monetary rules. As discussed in Debortoli, Nunes, and Yared (2018), long term real debt mitigates fiscal incentives to manipulate interest rates.⁹ In our environment, this lowers the incentives of the fiscal authority to challenge the monetary rule, which in turn decreases the credibility cut-off of strategic rules. This effect is absent when debt is nominal, since monetary incentives to deviate from a rule depend on outstanding debt, and fiscal incentives to challenge the monetary authority are constrained by associated reevaluation of outstanding debt.

Finally, we show that a central bank can improve the credibility of its promises by deviating along the equilibrium path from its optimal policy target: when the central bank commits to an inflation target higher than optimal, then the relative benefits to renouncing the rule are lower, improving the credibility of the original promise.

Related literature. This paper investigates whether a central bank can overcome the Tinbergen rule, namely whether it can achieve several targets with a limited set of instruments. In particular, our analysis assesses *the risk (...) that pursuing multiple objectives simultaneously brings the central bank into the realm of politics. This can compromise its independence and risk losing sight of price stability.* [Orphanides (2013)] In contrast, we show precisely that taking into account the risk of fiscal dominance improves the credibility of an independent central bank to deliver price stability.

In particular, we evaluate strategic monetary-fiscal interactions in an environment where both policy institutions face credibility constraints. Our cash credit economy brings together the incentives of the fiscal authority to manipulate the real interest rate (as in Lucas and Stokey (1983), Debortoli and Nunes (2013) and Debortoli, Nunes, and Yared (2017)) and the inflation bias of monetary policy. The inflation bias is driven by the benefits to inflate nominal claims and collect resources from seignorage, as studied by Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008).

Our analysis develops a non cooperative game between the central bank and the treasury, as in Dixit and Lambertini (2003). Our institutional set up features asymmetric commitment, in the sense that only the central bank has the ability to commit and respond to fiscal policy.¹⁰ This approach is followed by Gnocchi (2013) and Gnocchi and Lambertini (2016): fiscal contingent monetary strategies clearly welfare dominate

⁸This offsetting effect is absent when debt is real/indexed.

⁹In the case where debt is real and maturity long, our environment contains the results from Debortoli, Nunes, and Yared (2018) for fiscal policy and Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008) for monetary policy: optimal fiscal-monetary policy is time consistent.

¹⁰Formally, both the central bank and the treasuries are benevolent, but due to asymmetric commitment, they don't have the same objective function. We also discuss how relaxing the assumption of benevolent policy makers influence our results, considering monetary conservatism, as in Adam and Billi (2008) and Niemann (2011a), or fiscal short-termism, as discussed in Alesina and Passalacqua (2016).

restricted ones.¹¹

Our interest in evaluating credibility across policy regimes is novel though. Several approaches have been proposed to model partial commitment. Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2013) consider exogenous stochastic commitment shocks, while Clymo and Lanteri (2018) suggest that policy makers have a commitment technology limited in time. In our framework, policy plans can be revised endogenously against a cost. The magnitude of this cost is related to the degree of commitment of the central bank.¹² This way, we can characterize the *credibility* of monetary pledges, as the minimum degree of commitment that implements a rule in equilibrium. By contrasting credibility across monetary rules, we propose a novel criterion to guide policy recommendations.

Finally, the game-theoretic implications of *strategic monetary rule* are similar to Bassetto (2005), Atkeson, Chari, and Kehoe (2010) or Camous and Cooper (2019), where off equilibrium policies influence equilibrium outcomes. These studies develop this idea in environments plagued by multiple equilibria with the objective to implement a unique and superior equilibrium outcome.¹³ In contrast, we are interested in the effectiveness of this class of monetary interventions to eliminate the time inconsistency of optimal fiscal policy and the influence on the credibility of a central bank to deliver price stability.

Plan. The rest of the paper is organized as follows. Section 1 presents the institutional environment in a linear-quadratic framework, exposes the construction of monetary strategies and discusses the main results. Section 2 then embeds the policy game in a dynamic cash credit economy and analyzes the influence of public debt on the design of strategic monetary rules and associated credibility. Section 3 concludes. All proofs are relegated to an appendix.

1 Linear-Quadratic Framework

This section develops ideas in a linear-quadratic framework, in the tradition of Barro and Gordon (1983).¹⁴ After presenting the economic and policy environment, we pursue two objectives: first, study the construction of *strategic* monetary rules designed to lean against fiscal pressures, and second, contrast the credibility of these rules with a *standard* framework of strict inflation targeting.

¹¹The analysis in Gnocchi (2013) is conducted in a New Keynesian environment without debt, with a focus on stabilization vs. provision of public good. Our cash-credit economy with debt allows us to study a different problem, namely the capacity of the central bank to eliminate the incentives of the treasury to manipulate the real interest rate.

¹²In Section 1.5, we discuss how the modeling of our commitment technology relates to *reputation* or *trigger-type* environment.

¹³For instance, Bassetto (2005) highlights the shortcomings of standard Ramsey analyses for fiscal policy. Atkeson, Chari, and Kehoe (2010) study *sophisticated monetary policy* to implement a unique equilibrium outcome. Finally, Camous and Cooper (2019) study how monetary strategies can eliminate self-fulfilling debt crises.

¹⁴Variants of this environment to study monetary-fiscal interactions have been developed by Dixit and Lambertini (2003). Also by Dixit (2000) in the context of a monetary union.

1.1 Economic Environment

A fiscal authority chooses a tax instrument τ , while a central bank chooses inflation π . A “Phillips curve” captures the influence of policy decisions (τ, π) on output y :

$$y = \tau - \tau^e + \alpha(\pi - \pi^e), \quad (1)$$

where $\alpha > 0$ is the relative efficiency of monetary policy to stimulate the economy.¹⁵ As usual, expansionary policy choices (τ, π) can stimulate output beyond its natural level - here normalized to $y^n = 0$, only if unanticipated by private agents’ expectations (τ^e, π^e) .

Economic outcomes are ranked according to the following social loss function, shared by both monetary and fiscal authorities:¹⁶

$$\mathcal{L}(e, \tau, \pi) = \frac{1}{2} [(\tau - \tau^*)^2 + \lambda(\pi - \pi^*)^2 + \gamma(y - y^*)^2], \quad (2)$$

where $\lambda > 0$ captures the cost of monetary deviations from an optimal target π^* relative to fiscal deviations from τ^* , $\gamma > 0$ stands for the temptation to stimulate output y towards a *first best* level $y^* > 0$.¹⁷

Private agents form rational expectations, therefore in equilibrium they perfectly anticipate the conduct of public policy:

$$\tau^e = \tau \quad \pi^e = \pi. \quad (3)$$

Given the linear structure of the economy, one can write private agents’ expectations as:

$$e = \tau^e + \alpha\pi^e. \quad (4)$$

1.2 Cooperative Optimal Policy

We consider first benchmark equilibrium outcomes when the central bank and the treasury cooperate over the choice of policy instruments with the same degree of commitment. This is equivalent to studying the joint choice of (τ, π) by a consolidated government.

When the government operates under an infinite degree of commitment, it internalizes how policy choices (τ^c, π^c) influence private agents expectations e^c .¹⁸ Accordingly, it understands that output cannot be stim-

¹⁵Efficiency α has an intuitive interpretation in the dynamic economy with public debt presented in Section 2: higher α corresponds to the case of nominal debt, where monetary policy can inflate the real value of debt and lower total tax burden, while lower α corresponds to real debt.

¹⁶We discuss this assumption in Section 1.5 and consider interactions of policy makers with different ranking of economic outcomes.

¹⁷The normalized natural level of output $y^n = 0 < y^*$ is inefficiently low due to the presence of distortionary taxation for instance. This gives policy makers a motive to use policy tools to stimulate output. τ^* could stand either for an optimal provision of a public good, or supporting a redistributive policy program.

¹⁸Formally, under *commitment*, the government moves first and decides upon (τ^c, π^c) , then private agents form expectations, and finally the government implements its pre-announced policy. See Appendix A.1 for a formal treatment.

ulated beyond the natural level, and chooses policy (τ^*, π^*) to minimize the loss function (2):

$$\tau^c = \tau^* \quad \pi^c = \pi^* \quad e^c = \tau^* + \alpha\pi^* \quad y^c = 0. \quad (5)$$

This equilibrium outcome is not sensitive to preference parameters (λ, γ) .

When the government lacks commitment, it makes policy decisions sequentially, i.e. after private agents form expectations. Under this regime of *discretion*, the government is tempted to stimulate output towards y^* . In equilibrium though, private agents anticipate the conduct of public policy, and output is not stimulated:

$$\tau^d = \tau^* + \gamma y^* \quad \pi^d = \pi^* + \frac{\alpha}{\lambda} \gamma y^* \quad e^d = \tau^* + \gamma y^* + \alpha \left(\pi^* + \frac{\alpha}{\lambda} \gamma y^* \right) \quad y^d = 0. \quad (6)$$

Under discretion, policy choices are characterized by a *fiscal bias* and a *monetary bias* that make the social loss higher than under commitment. The monetary bias under discretion is increasing in α - the relative efficiency of monetary policy to stimulate output, and decreasing in λ - the relative cost of monetary deviation from the target π^* .

1.3 Non-cooperative Policy Game with Asymmetric Commitment

We consider now a non-cooperative game between monetary and fiscal authorities, where policy institutions have different degrees of commitment. This game allows to characterize the credibility of different monetary rules.

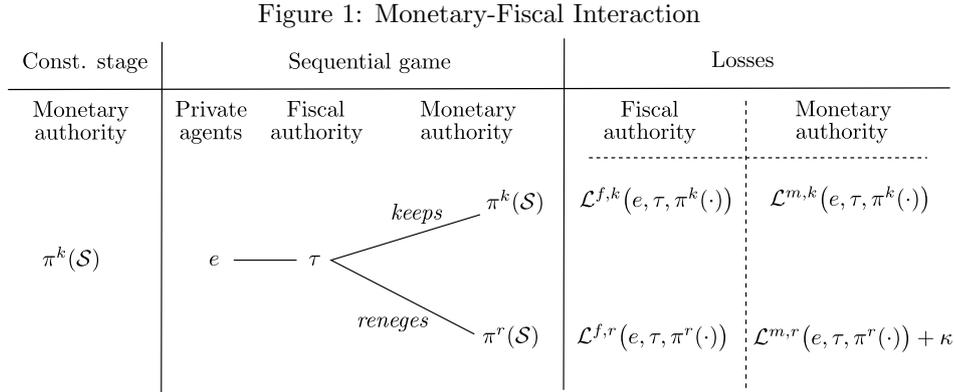
Timing and decisions. Initially, at a *constitutional stage*, the central bank announces a *policy rule* $\pi^k(\mathcal{S})$. The rule prescribes the choice of π as a function of the state $\mathcal{S} = (e, \tau)$, which consists of private agents expectations e and fiscal decisions τ . Then, the following sequence of actions takes place:

- i. Private agents form expectations $e = (\tau^e, \pi^e)$.
- ii. The fiscal authority sets τ .
- iii. Given $\mathcal{S} = (e, \tau)$, the central bank,
 - either *keeps* its promise and follows its policy rule $\pi^k(\mathcal{S})$,
 - or *renege*s, incurs a cost $\kappa \geq 0$, and implements $\pi^r(\mathcal{S})$ different from $\pi^k(\mathcal{S})$.

Policy objectives. Both monetary and fiscal authorities rank economic outcomes using the loss function $\mathcal{L}(\cdot)$.¹⁹ To keep track of the sequential nature of the game, we index the loss function with the identity of the policymaker and the decision of the central bank to *keep* or *renege* on its promised policy rule. For instance, $\mathcal{L}^{f,k}(\cdot)$ is the loss as evaluated by the fiscal authority conditional on the central bank keeping its promise, and $\mathcal{L}^{m,r}(\cdot)$ is the loss as evaluated by the central bank conditional on renegeing.

¹⁹Despite sharing the same loss function, monetary and fiscal authorities do not have the exact same objective function, since only the central bank incurs the cost κ in case it renegees its promise.

Figure 1 graphically summarizes the interactions of monetary and fiscal authorities described above.



This figure displays the sequence of choices and associated loss to each authority.

Fiscal influence. The set-up with asymmetric and limited commitment provides scope for the fiscal authority to influence the central bank to renounce its promised rule. As Stackelberg leader, the treasury can strategically choose τ that makes the central bank renege on $\pi^k(\mathcal{S})$ and implement $\pi^r(\mathcal{S})$. In the latter case, the central bank implements the sequential optimal policy: $\pi^r(\mathcal{S}) = \operatorname{argmin}_{\pi} \mathcal{L}(e, \tau, \pi)$. Note that the central bank enjoys a first mover advantage at the constitutional stage and can possibly embed fiscal contingencies in its rule $\pi^k(\mathcal{S})$ to contain the incentives of the fiscal authority to challenge its rule.

Degree of Monetary Commitment. The extent to which the central bank resists the temptation to renounce and re-optimize depends on its degree of commitment captured by the cost κ .²⁰ If $\kappa = 0$, then no monetary rule can be credibly implemented and the equilibrium outcome coincides with (6), when a consolidated government chooses policy under discretion. If κ is arbitrarily large, then any monetary rule can be credibly implemented. We characterize credibility of a generic monetary rule allowing for intermediate values of $\kappa \in (0, \infty)$.

1.3.1 Credibility of the Central Bank

To characterize the credibility of a monetary rule we proceed in two steps. First, describe fiscal choices conditional on whether the central bank keeps or reneges on a generic rule $\pi^k(\mathcal{S})$. Second, derive conditions on the degree of commitment κ such that along the equilibrium path the central bank follows its rule $\pi^k(\mathcal{S})$. Accordingly, the characterization of credibility requires to study fiscal and monetary decisions on and off equilibrium paths.

If the fiscal authority anticipates that the central bank keeps its promise and follows its rule $\pi^k(\mathcal{S})$, it chooses $\tilde{\tau}$ defined as:

$$\tilde{\tau} = \operatorname{argmin}_{\tau} \mathcal{L}^{f,k}(e, \tau, \pi^k(\mathcal{S})). \quad (7)$$

²⁰In Section 1.5, we discuss a formal characterization of κ as a reputation cost. A similar modeling approach for limited commitment is followed by Farhi, Sleet, Werning, and Yeltekin (2012).

Along this equilibrium path, private agents' expectations satisfy $\tilde{e} = \tilde{\tau} + \alpha\pi^k(\tilde{e}, \tilde{\tau})$. We proceed under the assumption that expectations of private agents remain anchored on this equilibrium path.²¹

The choice $\tilde{\tau}$ could be dominated from the perspective of the fiscal authority if the central bank could be induced to renege on its rule $\pi^k(\mathcal{S})$ and implement $\pi^r(\mathcal{S})$. The set of all such *profitable fiscal deviations* – that yield a lower loss to the treasury – is defined as follows:

$$\mathbb{T}(\tilde{e}) = \{\tau \mid \mathcal{L}^{f,r}(\tilde{e}, \tau, \pi^r(\tilde{e}, \tau)) \leq \mathcal{L}^{f,k}(\tilde{e}, \tilde{\tau}, \pi^k(\tilde{e}, \tilde{\tau}))\}. \quad (8)$$

This set reflects the incentives of the fiscal authority to challenge the monetary policy rule.

Turning to the central bank, the monetary policy rule $\pi^k(\mathcal{S})$ is credible if the degree of commitment κ is high enough to deter the temptation of deviation.

Definition 1. A monetary rule $\pi^k(\mathcal{S})$ is credible if for all profitable fiscal deviations $\tau \in \mathbb{T}(\tilde{e})$:

$$\mathcal{L}^{m,k}(\tilde{e}, \tau, \pi^k(\tilde{e}, \tau)) \leq \mathcal{L}^{m,r}(\tilde{e}, \tau, \pi^r(\tilde{e}, \tau)) + \kappa,$$

where $\pi^r(\mathcal{S}) = \operatorname{argmin}_{\pi} \mathcal{L}(e, \tau, \pi)$.

Furthermore, we define a *credibility cut-off* as the minimum degree of commitment κ under which the monetary policy rule is credible.

Definition 2. Let $\bar{\kappa}$ be the credibility cut-off of a monetary rule $\pi^k(\mathcal{S})$, defined as:

$$\bar{\kappa} = \max_{\tau \in \mathbb{T}(\tilde{e})} \mathcal{L}^{m,k}(\tilde{e}, \tau, \pi^k(\tilde{e}, \tau)) - \mathcal{L}^{m,r}(\tilde{e}, \tau, \pi^r(\tilde{e}, \tau)).$$

This definition makes clear that the credibility of a monetary rule relies on monetary-fiscal interactions: $\bar{\kappa}$ is the minimum cost that eliminates the incentives of the monetary authority to renounce its policy rule for every profitable fiscal deviation $\tau \in \mathbb{T}(\tilde{e})$. In other terms, a monetary rule is credible if and only if $\kappa \geq \bar{\kappa}$. A rule associated with a high credibility cut-off $\bar{\kappa}$ requires a high degree of commitment to be defended against strategic fiscal decisions, and accordingly be implemented in equilibrium.

1.4 Policy Game Analysis

The policy game is simple yet it has all the necessary features required to study the key questions of interest. Can a central bank design a rule to lean against fiscal influence and *de facto* share commitment with the fiscal authority? Is such monetary rule credible? We approach these questions by analyzing and contrasting how the policy game plays out under two alternative monetary rules:

- (1.) a *standard* rule, that prescribes the central bank to set inflation on target π^* unconditionally,
- (2.) a *strategic* rule, that prescribes the central bank to set policy contingent on the conduct of fiscal policy with the objective to both reach its policy target π^* and induce the fiscal authority to implement τ^* .

²¹In Appendix A.4.5 we discuss a simple version of the Taylor-like principle that ensures that self-fulfilling expectations and multiple equilibria do not arise.

In particular, we compare the credibility cut-offs associated with these two rules. The objective is to understand whether adopting a *strategic* monetary rule could compromise the core objective of the central bank to reach the policy target π^* .

1.4.1 Standard Monetary Rule

The standard rule of setting inflation on target unconditionally is defined formally as follows:

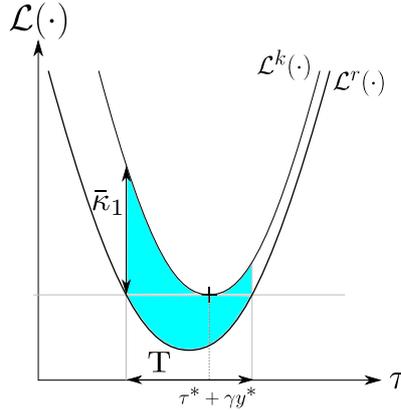
$$\forall \mathcal{S}, \pi^k(\mathcal{S}) = \pi^*. \quad (9)$$

If credible, this policy rule results in the following equilibrium outcome:²²

$$\tau_1 = \tau^* + \gamma y^*, \quad \pi_1 = \pi^*, \quad e_1 = \tau^* + \gamma y^* + \alpha \pi^*, \quad y_1 = 0. \quad (10)$$

The central bank naturally implements its policy target π^* , while the fiscal decision is still characterized by the *fiscal bias*. Expectations reflect policy choices and output is not stimulated beyond its natural level.

Figure 2: Credibility Cut-off under Standard Rule



This figure illustrates the characterization of credibility cut-off $\bar{\kappa}_1$, when the central bank follows a standard inflation target rule. Given the sequential nature of the game, credibility is evaluated for a set $T(e_1)$ of fiscal choices τ , represented by the blue area. The credibility cut-off $\bar{\kappa}_1$ is then given by the maximum distance within that set between $\mathcal{L}^k(\cdot)$ and $\mathcal{L}^r(\cdot)$, corresponding respectively to the cases where the central bank *keeps* its promise (9) or *renege*s and implements $\pi^r(\mathcal{S})$.

Recall that the central bank might be tempted to renounce the promised rule and the fiscal authority has incentives to influence this decision. Sustaining equilibrium (10) requires a high enough degree of commitment κ . The following proposition characterizes the credibility cut-off.

Proposition 1. *The credibility cut-off for the standard monetary policy rule (9) is*

$$\bar{\kappa}_1 = \frac{(y^*)^2}{2}(\gamma - \eta)(1 + \gamma) \left(\frac{\sqrt{1 + \gamma} + \sqrt{\gamma - \eta}}{1 + \eta} \right)^2,$$

²²See formal derivation in Appendix A.2.

where $\eta \equiv \frac{\lambda\gamma}{\lambda+\gamma\alpha^2}$. In addition:

$$\frac{d\bar{\kappa}_1}{d\lambda} < 0 \quad \text{and} \quad \frac{d\bar{\kappa}_1}{d\alpha} > 0.$$

Proof. See Appendix A.2 ■

The credibility cut-off $\bar{\kappa}_1$ is the minimum degree of commitment that discourages the central bank from renegeing on the standard rule, at all off equilibrium paths within the set of profitable fiscal deviations $\mathbb{T}(e_1)$. In particular, $\bar{\kappa}_1$ is determined by the off equilibrium path where the fiscal authority implements the most contractionary policy $\tau_l = \min\{\mathbb{T}(e_1)\}$, followed by an expansionary offsetting monetary action $\pi^r(e_1, \tau_l)$. This is illustrated in Figure 2.

The dependence of the credibility cut-off $\bar{\kappa}_1$ on parameters λ and α makes clear how the credibility of the standard rule is intertwined with the commitment problem of the central bank. Consider an off equilibrium path where the central bank renounces its rule and reoptimizes. The lower is the cost of inflation λ , the more inflation would the central bank generate to stimulate output.²³ The higher is α , the more effective is monetary policy at stimulating output. Either way, the relative gains from renouncing the rule increases, which makes the credibility cut-off higher.

1.4.2 Strategic Monetary Rule

We now consider *strategic* monetary rules, which set monetary policy contingent on fiscal decisions. The construction of these rules rests on two components. First, the central bank picks the desired equilibrium outcome, which coincides with the one under cooperation and full commitment:

$$\tau_2 = \tau^*, \quad \pi_2 = \pi^*, \quad e_2 = \tau^* + \alpha\pi^*, \quad y_2 = 0. \quad (11)$$

Second, these rules are strategic because the central bank constructs off-equilibrium threats to discourage fiscal deviations from (11). Formally,

$$\mathcal{L}^{f,k}(e_2, \tau, \pi^k(e_2, \tau)) \geq \mathcal{L}^{f,k}(e_2, \tau^*, \pi^*), \quad \forall \tau \quad (12)$$

The existence of this class of rules is straightforward, given the capacity of the central bank to set arbitrary π driving the loss $\mathcal{L}^{f,k}(\cdot)$ arbitrarily high for all τ .

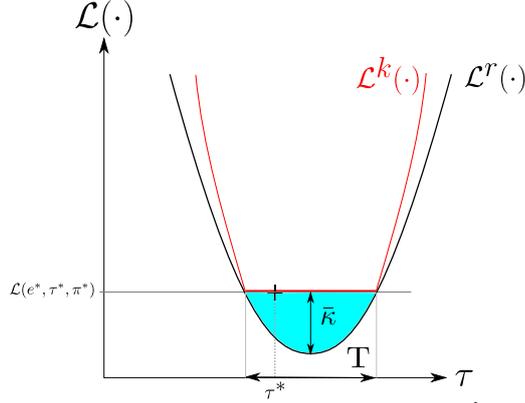
We further refine this condition to minimize the associated credibility cut-off. To this end, we consider rules with threats that are just enough to discipline incentives of the fiscal authority along relevant off equilibrium paths:

$$\mathcal{L}^{f,k}(e_2, \tau, \pi^k(e_2, \tau)) = \mathcal{L}^{f,k}(e_2, \tau^*, \pi^*), \quad \forall \tau \in \mathbb{T}(e_2), \quad (13)$$

A strategic rule that satisfies this additional restriction prescribes to adjust monetary policy in response to profitable fiscal deviations $\mathbb{T}(e_2)$, so as to maintain the social loss at the level reached at the desired

²³This is explicit in the outcome when consolidated government chooses policy under discretion; see (6).

Figure 3: Credibility Cut-off under Strategic Rule



This figure represents the construction of a strategic monetary rule $\pi^k(\mathcal{S})$ aimed to implement the equilibrium outcome (11), with a minimum degree of commitment. To induce the fiscal authority to implement τ^* , the central bank designs its rule so that the loss function for all τ is higher evaluated at $(e_2, \tau, \pi^k(\mathcal{S}))$ than at the desirable equilibrium outcome (e_2, τ^*, π^*) . To minimize the commitment intensity required to support this rule, monetary interventions offset the welfare implications of fiscal deviations, within the relevant set $T(e_2)$, in blue on the figure.

equilibrium.²⁴ The construction of such rule is illustrated in Figure 3. The credibility cut-off is pinned down by the off equilibrium path where policy choices (τ, π) coincide with the discretionary choice under cooperation.

Proposition 2. *The credibility cut-off of a strategic monetary policy rule that satisfies (11) and (13) is:*

$$\bar{\kappa}_2 = \frac{\gamma(y^*)^2}{2} \frac{\gamma(\lambda + \alpha^2)}{\lambda + \gamma\alpha^2 + \lambda\gamma}.$$

In addition, we have:

$$\frac{d\bar{\kappa}_2}{d\lambda} < 0 \quad \text{and} \quad \frac{d\bar{\kappa}_2}{d\alpha} > 0.$$

Proof. See Appendix A.3 ■

The dependence of the credibility cut-off $\bar{\kappa}_2$ on parameters λ and α reflects the intuition provided in Section 1.4.1: the degree of commitment required to follow $\pi^k(\mathcal{S})$ is directly influenced by the relative gains to renouncing the rule and implement $\pi^r(\mathcal{S})$.

Using Propositions 1 and 2, we contrast credibility cut-offs and derive conditions under which *strategic* rules require a lower degree of commitment than *standard* rules.

Proposition 3. *The credibility cut-off $\bar{\kappa}_2$ associated with strategic monetary rules is lower than the credibility cut-off $\bar{\kappa}_1$ associated with the standard rule, for relatively low values of λ and high values of α . Formally,*

- for every $\lambda > 0$, there is a threshold $\bar{\alpha} > 0$ such that for all $\alpha > \bar{\alpha}$, $\bar{\kappa}_1 > \bar{\kappa}_2$.

- for every $\alpha > 0$, there is threshold $\bar{\lambda} > 0$ such that for all $\lambda < \bar{\lambda}$, $\bar{\kappa}_1 > \bar{\kappa}_2$.

Proof. See Appendix A.3 ■

²⁴For all $\tau \notin T(\cdot)$, the monetary rule can specify any $\pi \geq \pi^r(\mathcal{S})$, as the fiscal authority will not consider such a policy path to challenge the strategy of the central bank.

Contrasting credibility cut-off across rules provides our central result: the strategic rule requires a lower degree of commitment precisely when renouncing a rule is most tempting. In particular, what makes the standard inflation target relatively easier to sustain is that the equilibrium implications of the fiscal bias lower the relative gains to *renounce* the inflation target. What makes the strategic rule relatively easier to sustain is that it is designed precisely to minimize the welfare costs of *keeping* the rule for all possible fiscal deviations. The credibility benefit of following the strategic rule dominates precisely when the relative gains to renouncing any of these rules are high.

In summary, the central bank can design strategic rules that rely on off-equilibrium threats to discipline the treasury and improve economic outcomes. Importantly strategic rules require less commitment intensity precisely when the central bank is most exposed to possible successful fiscal influence.

1.5 Extensions

The linear-quadratic framework allows to investigate an additional set of questions. In Appendix A.4, we present formal treatment for the following elements.

Credibility and aggregate shocks. In the linear quadratic framework, time inconsistency and stabilization policies are orthogonal: both monetary and fiscal policy biases do not depend on the realization of an aggregate shock to the Phillips curve. Accordingly, the credibility of monetary rules is not sensitive to the realization of aggregate shock. In turn, in environments where incentives to renounce a rule would depend on the realization of an aggregate shock, policy biases and credibility would be state contingent.²⁵

Credibility and policy objectives. The environment presented previously accommodates easily the study of monetary-fiscal interactions when policy makers do not share identical policy objective functions. In particular, we consider two common adjustments of policy objectives: *monetary conservatism* and *self-interest* of fiscal authority.

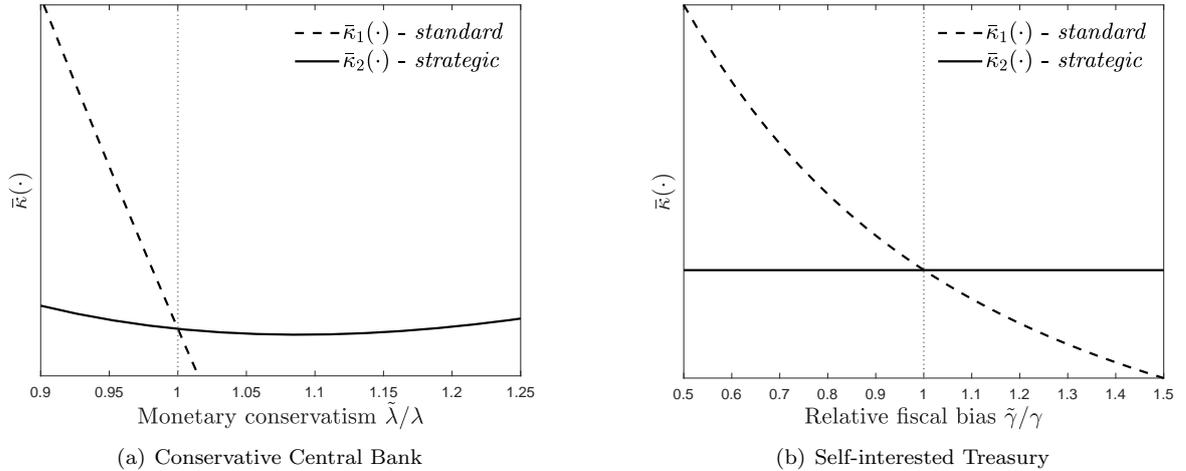
Monetary conservatism refers to the appointment of a monetary policy maker that puts a higher relative welfare weight on inflation stabilization.²⁶ As is well understood, monetary conservatism decreases the credibility required to implement the unconditional inflation target - the standard rule, since upon renouncing the inflation target, a more conservative central banker would implement a lower monetary stimulus, yielding lower welfare gains. This dimension is also present under the strategic rule. However, under the strategic rule, stronger conservatism also makes off-equilibrium threats more costly to the central bank. The latter effect dominates at higher levels of conservatism. Overall, as illustrated in the left panel Figure 4, only moderate level of monetary conservatism relax the commitment intensity required to sustain a strategic rule.

Treasury self-interest refers to higher temptation of the fiscal authority to stimulate output, captured by

²⁵The correlation between aggregate shock and credibility cut-off depends on the particular ingredients of such a model. In the dynamic cash credit economy with public debt discussed in Section 2, a negative income shocks would increase the burden of outstanding debt, raising the credibility cut-off.

²⁶The benefits of appointing an inflation conservative central banker have been discussed first by Rogoff (1985). Recent contributions, including Adam and Billi (2008) and Niemann (2011b), study the interactions of monetary conservatism with fiscal policy.

Figure 4: Credibility Cut-offs under Different Policy Objectives



This figure compares credibility cut-offs $\bar{\kappa}$ for each monetary rule: *standard* and *strategic*. Panel (a) reports credibility for different level of central bank conservatism. Panel (b) compares the credibility across regimes for different degrees of fiscal bias. (Numerical values are set so that $\bar{\kappa}_1 = \bar{\kappa}_2$ when authorities have the similar benevolent preferences: $\lambda = 3/2$, $\gamma = 1/2$, $\alpha = 0.952$.)

a higher weight that the treasury attaches to deviations of output from the first-best level.²⁷ As illustrated in the right panel of figure 4, the credibility cut-off of the strategic monetary rule does not depend on the strength of fiscal self-interest. The strategic monetary rule does change with fiscal self-interest to provide appropriate incentives to the treasury, but its credibility, evaluated with the preferences of the monetary authority, is not influenced by the intensity of the fiscal temptation to stimulate output.²⁸

Credibility and inflation target. We have considered so far strategic rules where on equilibrium the central bank delivers the optimal inflation level π^* . But if the commitment intensity falls short of the required credibility cut-off, could the central bank adjust its equilibrium target to relax the shortage of credibility? We consider strategic rules designed to eliminate the fiscal bias, as in Section 1.4.2, but which deliver on equilibrium an inflation rate $\pi^s > \pi^*$.²⁹

This adjustment of the monetary target improves the credibility of the strategic rule. The intuition is straightforward: by targeting an inflation rate higher than π^* , the central bank reduces the relative gains from renouncing the rule, which in turn decreases the degree of commitment required to support this strategy in equilibrium.³⁰

²⁷The terminology fiscal *self-interest* is borrowed from Yared (2019). Political frictions can fuel this type of fiscal behavior, incl. electoral constraints or budgetary processes.

²⁸In contrast, fiscal self-interest reduces the commitment intensity required to deliver an unconditional inflation target, because the *fiscal bias* in equilibrium under this regime reduces the relative gains for the monetary authority to renounce its rule.

²⁹The optimal policy mix (π, τ) under restricted commitment would require a more comprehensive analysis of policy trade-offs. We do not pursue this route, as we are not interested in optimal constrained equilibria per se. We rather anchor our discussion in the design of the monetary policy frameworks.

³⁰A similar result is derived in the dynamic cash credit economy, see Section 2.5.3.

Credibility and reputation. Reputation — or trigger-type equilibria — is a common construction to support commitment: private agents' expectations carry the history of policy decisions, so that a policy authority deviating from a rule would suffer reputational costs over subsequent periods.³¹ We discuss in Appendix A.4.4 how our concepts of *commitment intensity* and *credibility cut-off* relate to these constructions within a repeated version of the game presented.

Importantly, reputational equilibria emphasize strategic interactions between private agents and policy makers, without adding economic insight to our discussion of monetary-fiscal interactions. Given the intuitive interpretation of our reduced form reputation cost κ , we choose to abstract from these elements.

2 Dynamic Cash-Credit Economy

The institutional environment developed in Section 1 is now introduced in a dynamic cash-credit economy as in Lucas and Stokey (1987). The objective is to study how public debt influences credibility of *standard* and *strategic* monetary rules. With public debt, the monetary-fiscal game is properly dynamic: credibility is evaluated against the level of newly issued debt, both on and off equilibrium paths.

2.1 Economic Environment

Time is discrete and each period is indexed with $t \geq 0$. The economy is populated by a representative agent and a government. The resource constraint of the economy is

$$c_t + d_t + g = 1 - l_t, \quad (14)$$

where c_t is private consumption of a *credit good*, d_t is private consumption of a *cash good*, $g > 0$ is exogenous and constant public consumption, l_t is leisure, while production is linear in labor $y_t = 1 - l_t$.

Private agents. A representative household enjoys utility from private consumption and leisure:

$$\sum_{t=0}^{\infty} \beta^t U(c_t, d_t, l_t) = \sum_{t=0}^{\infty} \beta^t (\alpha \log c_t + (1 - \alpha) \log d_t + \gamma l_t), \quad (15)$$

where $\beta \in (0, 1)$ is the time discount factor.³²

The household supplies labor to competitive firms that produce both cash and credit goods. She earns a real wage equal to the (unitary) marginal product of labor and is taxed at a linear rate τ_t . At a price q_t , she can buy (sell) nominal risk-free government bonds, B_t^h . The nominal flow budget constraint in period t reads:

$$P_t c_t + P_t d_t + q_t B_t^h + M_t^h = P_t (1 - \tau_t) (1 - l_t) + B_{t-1}^h + M_{t-1}^h, \quad (16)$$

where P_t is the price level, and M_t^h is the stock of money, carried over from period t into next period.

³¹Various versions of this construction are discussed in Barro and Gordon (1983), Stokey (1989) or Chari and Kehoe (1990).

³²Under this utility specification, nominal debt only, and not temptations to raise seignorage revenue, weighs on the credibility of monetary policy. See Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008).

The purchase of cash good d_t is subject to a cash-in-advance constraint, where beginning-of-period stock of money M_{t-1}^h sets a cap on expenses:³³

$$P_t d_t \leq M_{t-1}^h. \quad (17)$$

Finally, exogenous debt limits are in place to prevent Ponzi schemes but they do not bind in equilibrium.

Government. The government consists of a fiscal and a monetary authority. The treasury controls the tax rate τ_t on labor income and the supply of government bonds B_t . The central bank controls the growth rate of money supply:

$$\sigma_t = M_t/M_{t-1} - 1. \quad (18)$$

Every period, policies satisfy the budget constraint of the government, which in nominal terms reads:

$$q_t B_t + M_t + P_t \tau_t (1 - l_t) = P_t g + B_{t-1} + M_{t-1}. \quad (19)$$

Initial outstanding debt, $B_{-1} = B_{-1}^h$, and stock of money, $M_{-1} = M_{-1}^h$, are exogenous and nonnegative.

Real Debt. In the analysis, we contrast the nominal bond economy with one where government bonds are indexed to inflation. These bonds, labelled b_t , are effectively a promise to deliver real payoffs. The budget constraint of the government with inflation-indexed bonds reads:

$$q_t P_{t+1} b_t + M_t + P_t \tau_t (1 - l_t) = P_t g + P_t b_{t-1} + M_{t-1}. \quad (20)$$

Naturally, in that case, the budget constraint of the household is adjusted similarly.³⁴

Competitive equilibrium. Our analysis considers competitive equilibria that arise in this economy.

Definition 3. *A competitive equilibrium in an economy with nominal debt consists of a price system $\{P_t, q_t\}_{t=0}^\infty$, a private sector allocation $\{c_t, d_t, l_t, M_t^h, B_t^h\}_{t=0}^\infty$, and a government policy $\{M_t, B_t, \tau_t\}_{t=0}^\infty$ s.t.:*

- *Given initial asset positions $\{M_{-1}^h, B_{-1}^h\}$ as well as price system and policy, the allocation solves the maximization program of the representative household (15) subject to the sequence of household budget constraints (16), the cash-in-advance constraints (17), and exogenous debt limits.*
- *Given initial liabilities $\{M_{-1}, B_{-1}\}$ as well as allocation and price system, the policy satisfies the sequence of government budget constraints (19).*
- *All markets clear, hence at all times $M_t^h = M_t$, $B_t^h = B_t$, and the resource constraint (14) holds.*

Naturally, there are multiple competitive equilibria, indexed by different government policies, and this creates scope for the analysis of policy choice. The following expressions characterize household choice given

³³The timing is induced by a segmented market assumption, as in Svensson (1985), Nicolini (1998) or Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008): the market for cash good opens before asset market.

³⁴The exposition focuses on the economy with nominal bonds. We only highlight differences with real bonds when relevant, otherwise only notations need to be adjusted. In addition, we study how debt maturity influences our results in Section 2.5.

government policy:³⁵

$$U_{l,t} = U_{c,t}(1 - \tau_t) \quad \Leftrightarrow \quad (1 - \tau_t) = \frac{\gamma}{\alpha} c_t, \quad (21)$$

$$U_{c,t} = \beta \frac{P_t}{P_{t+1}} U_{d,t+1} \quad \Leftrightarrow \quad P_{t+1} d_{t+1} = \frac{\beta(1 - \alpha)}{\gamma} P_t (1 - \tau_t), \quad (22)$$

$$U_{c,t} = \beta \frac{1}{q_t} \frac{P_t}{P_{t+1}} U_{c,t+1} \quad \Leftrightarrow \quad P_{t+1} c_{t+1} = \beta \frac{1}{q_t} P_t c_t. \quad (23)$$

These equations highlight how public policy influences household choices. A positive tax rate drives a wedge on the consumption-leisure choice as described by (21). Equation (22) maps current consumption of credit good c_t with next period consumption of cash good d_{t+1} : the wedge is driven by variations in the price level $\frac{P_t}{P_{t+1}}$, which is the real return to holding money M_t . Equation (23) is a standard Euler equation, where the intertemporal allocation in credit good is driven by the inverse real interest rate \tilde{q}_t :

$$\tilde{q}_t \equiv q_t \frac{P_{t+1}}{P_t} = \beta \frac{U_{c,t+1}}{U_{c,t}}. \quad (24)$$

Finally, inequality $(1 - \alpha)c_t \geq \alpha d_t$ is a complementary slackness condition due to the cash-in-advance constraint (17). This constraint imposes an upper bound on nominal interest rate, $q_t \leq 1$, which in turn imposes the following restriction on the money growth rate: $(1 + \sigma_t) \geq \beta$.³⁶

A convenient way to characterize competitive equilibria is to substitute away prices and allocation using equilibrium conditions to obtain an implementability constraint in terms of the sequence of policy instruments.³⁷

Lemma 1. *A sequence of tax rates and money growth rates, $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$, supports the competitive equilibrium when government debt is nominal if and only if the following constraint is satisfied for all $t \geq 0$ given z_{-1} :*

$$\beta \left[\frac{(1 - \alpha)\beta}{1 + \sigma_{t+1}} \right] z_t - \alpha(1 - \tau_t) - (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} + \Phi = \left[\frac{(1 - \alpha)\beta}{1 + \sigma_t} \right] z_{t-1}, \quad (25)$$

where $\Phi \equiv (\beta(1 - \alpha) + \alpha - \gamma g)$, $z_t \equiv B_t/M_t$, and Ponzi-schemes are ruled out by the exogenous debt limits.

Proof. See Appendix B.1 ■

In (25), z_t is the bond-to-money ratio, the relevant state variable to measure outstanding government debt. The analogous constraint for the economy with real debt is:

$$\beta \left[\frac{\gamma}{(1 - \tau_{t+1})} \right] b_t - \alpha(1 - \tau_t) - (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} + \Phi = \left[\frac{\gamma}{(1 - \tau_t)} \right] b_{t-1}. \quad (26)$$

³⁵The optimality conditions (21) – (23) remain the same if government debt is real instead of nominal. Hence if nominal and real bond economies are characterized by different equilibria, this is driven by government decisions and its effect on household choices, and not directly by a change in household choices.

³⁶When $q_t = 1$, we assume households keep the minimum amount of money required to purchase goods, hence the cash-in-advance constraint always holds with equality.

³⁷Following Lucas and Stokey (1983), the optimal policy literature often uses the *primal approach*, whereby one substitutes away prices and policy instruments using the equilibrium conditions to obtain an implementability constraint in terms of the allocation. We are reversing this approach to keep the focus on the interaction of policy makers with different policy instruments.

Unless the distinction is of essence, we use the following general way of writing constraints (25) and (26):

$$0 = f(s_t, s_{t-1}, \sigma_{t+1}, \sigma_t, \tau_{t+1}, \tau_t), \quad (27)$$

where $s_t \in \{z_t, b_t\}$ refers to the state variable of the nominal or real debt economy.

We further assess welfare via an indirect flow utility function.

Lemma 2. *In the competitive equilibrium induced by policy $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$, the flow utility of the representative household is given by the following indirect utility function:*

$$U(\tau_t, \sigma_t) = \alpha [\log(1 - \tau_t) - (1 - \tau_t)] + (1 - \alpha) \left[\log \left(\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right) - \beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right]. \quad (28)$$

Proof. See Appendix B.1 ■

Illustrative calibration. The analytical results are illustrated with numerical simulations of the model. We use a standard calibration for an annualized model that matches some key statistics and long-run ratios, as reported in Table 1.

Table 1: Parameter values

Parameter	Description	Value
β	Discount factor	0.96
γ	Preference leisure weight	5
α	Preference credit good weight	0.5
g	Public spending	0.05

Parameters are set to target moments of the first best allocation - which is the solution to the maximization of (15) subject to the sequence of resource constraints (14). The implied moments are $g/(c+d) = 0.25$, the fraction of time devoted to leisure $l = 0.75$ and an equal consumption of cash and credit good.

Correspondence across economies. Importantly, to contrast allocations and policy outcomes across economies with nominal z_{-1} or indexed debt b_{-1} , we use a correspondence introduced in Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008): debt levels are comparable as long as present value of primary policy surpluses are equalized. Using intertemporal versions of (25) and (26), this correspondence reads:³⁸

$$\frac{(1 - \alpha)\beta}{1 + \sigma_0} z_{-1} = \frac{\gamma}{1 - \tau_0} b_{-1}, \quad (29)$$

where σ_0 is $t = 0$ money printing rate under nominal debt and τ_0 the $t = 0$ tax rate under real debt. Hence, the principle of correspondence requires to equalize policy surpluses in a comparable institutional environment.

³⁸See Appendix B.2 for explicit intertemporal implementability conditions.

2.2 Cooperative Optimal Policy

As benchmark, we study first the case where monetary and fiscal authorities cooperate over the choice of policy instruments. Policy choices are in effect under the control of a single government entity. We contrast choices with and without commitment to highlight the properties of the optimal policy plan, and the incentives to deviate from it.

Cooperation with commitment. Consider a government with a commitment technology to implement a dynamic policy plan $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$ defined at $t = 0$. This commonly studied *Ramsey plan* is useful to understand whether the associated policy is time-consistent, and when it is not, where the sequential incentives to deviate stem from.³⁹

Proposition 4. *Given $s_{-1} > 0$, the Ramsey plan has the following characteristics:*

- *“timeless optimal allocation”*: for all $t \geq 1$, constant path of consumption and leisure. Similarly, tax rates and public liability s_{t-1} , are constant over time. The money printing rate follows the Friedman rule: $\sigma_t = \beta - 1$.
- *“incentives to deviate”*: at $t = 0$, allocation and policy choices differ from the “timeless” levels. In particular, $\tau_0 < \tau_1$ and $\sigma_0 > \sigma_1$. Debt dynamics satisfies:
 - for all $z_{-1} > 0$, newly issued nominal debt z_0 satisfies: $z_0 < z_{-1}$.
 - in the case of real debt $b_{-1} > 0$, there is $\hat{b} > 0$ s.t. $b_0 < b_{-1}$ if and only if $b_{-1} > \hat{b}$.

Proof. See Appendix B.2 ■

Figure 5 represents graphically the dynamic path of allocations, policy decisions and prices in such a Ramsey equilibrium. Both when debt is nominal and real, the allocation for $t \geq 1$ reflects the tax-smoothing structure of the model: consumption bundle $\{c_t, d_t, l_t\}_{t=0}^{\infty}$, taxes and the bond-to-money ratio (or real debt level) are constant over time. Finally, the Friedman rule applies: the nominal interest rate is one, i.e. $q_t = 1$, and the stock of money decreases at a constant rate $\sigma_t = \beta - 1$.⁴⁰ We refer to this stationary economic outcome as the “timeless” optimal allocation and policy decisions.⁴¹ In the rest of the analysis, the *timeless optimal allocation* is the normative benchmark against which different policy environments are evaluated. Given outstanding liabilities s_{-1} , note $W^{ta}(\cdot)$ the associated lifetime welfare of the representative household:

$$W^{ta}(s_{-1}) = \frac{1}{1-\beta} U(\tau^*(s_{-1}), \sigma^*), \quad (30)$$

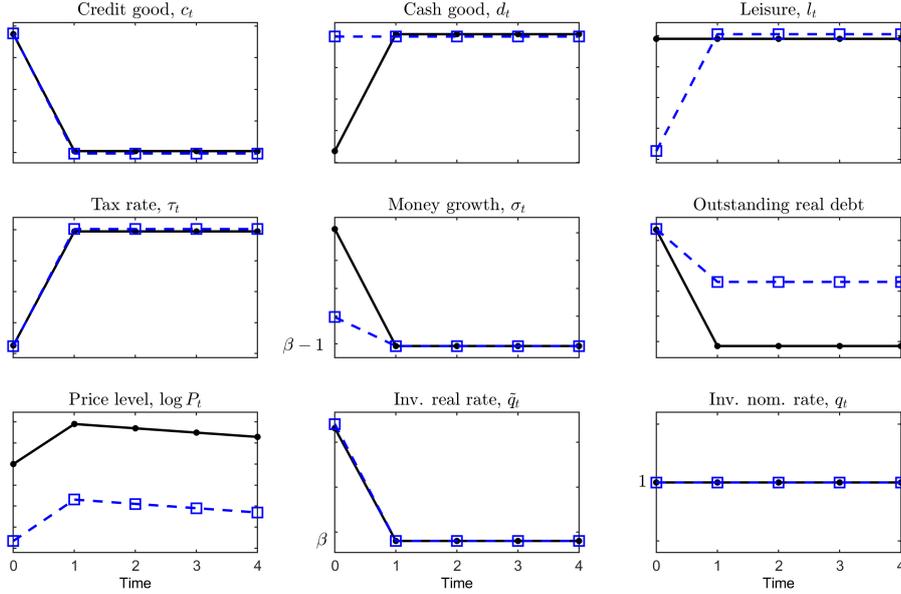
where $\tau^*(s_{-1})$ solves the implementability condition (27) under stationary level of debt $s = s_{-1}$ and Friedman deflation $\sigma^* = \beta - 1$.

³⁹ “If the continuation plan of a Ramsey plan is not a Ramsey plan, then the Ramsey plan is time-inconsistent”, Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008).

⁴⁰ The intuition for the Friedman rule is that the government should compensate the household for the friction induced by the cash-in-advance constraint, i.e. for the utility cost of time β .

⁴¹ The “timeless perspective” of optimal policy plans is an equilibrium concept suggested by Woodford (1999): optimal policy and allocations should be characterized ignoring initial conditions, as if they were derived in the distant past.

Figure 5: Ramsey equilibrium: joint government with commitment



This figure represents allocations, policy instruments and prices when policy is conducted under co-operation and commitment. The **black** line refers to the case of nominal debt, the **blue** line to real debt. The “timeless” optimal allocation from $t = 1$ on reflects the tax-smoothing structure of the model, while policy choices at $t = 0$ highlights the incentives to deviate from this stationary policy plan. Nominal and real debt economies are made comparable using the correspondence (29).

The allocation at $t = 0$ differs when $B_{-1} > 0$ or $b_{-1} > 0$: public debt induces policy makers to deviate from a pre-announced policy plan. These incentives are multiple, as the model captures both monetary and fiscal policy. The money printing rate deviates from the Friedman rate to inflate away the real value of nominal debt.⁴² Further, fiscal policy engineers a tax cut $\tau_0 < \tau_1$, to influence the real interest rate $1/\bar{q}_0$ and increase the market value of newly issued debt.⁴³

Given this characterization, the welfare under the Ramsey plan is simply :

$$W^{rp}(s_{-1}) = \max_{\tau, \sigma, s} U(\tau, \sigma) + \beta W^{ta}(s). \quad (31)$$

Cooperation without commitment. When the government does not have a commitment technology to credibly implement a dynamic policy plan, then arises a dynamic game across policy makers tempted to reap short term welfare gain with systematic *deviations*. Figure 6 represents the dynamic path in a Markov-perfect equilibrium.⁴⁴ At each point in time, the government wants to achieve some welfare gain with strategic money printing or tax policy. But it anticipates future policy makers behave similarly. As the incentives to *deviate* increase with the outstanding level of debt, debt left to future policy makers decreases

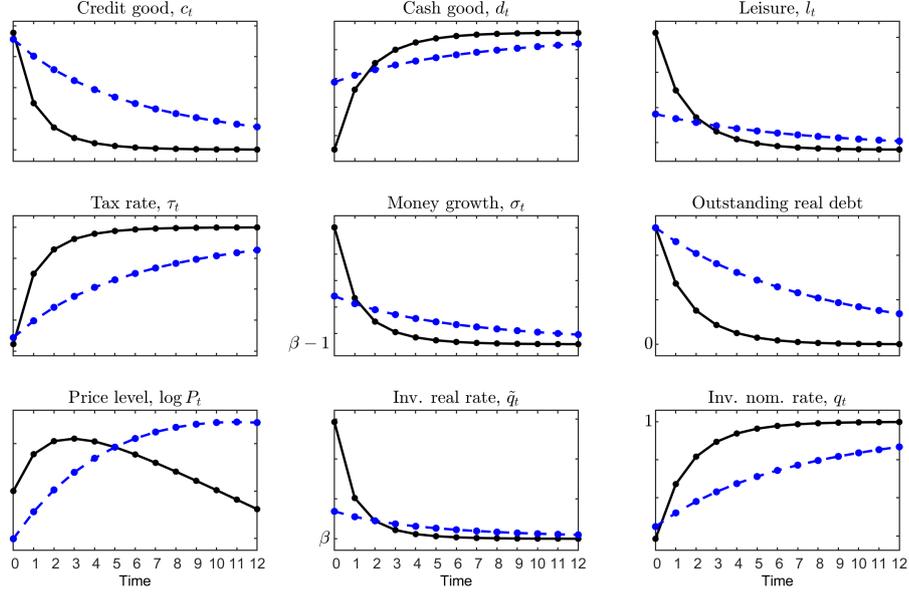
⁴²As mentioned, the log utility assumption ensures that variation in the elasticity of money demand over time is not a source of time inconsistency per se, see Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008).

⁴³The fiscal incentives to manipulate the real interest rate has been studied in Debortoli and Nunes (2013) and Debortoli, Nunes, and Yared (2017). In particular, the higher willingness of households to buy public debt when $\tau_0 < \tau_1$ and $c_0 > c_1$ is driven by intertemporal substitution effects.

⁴⁴Appendix B.3 defines formally the equilibrium concept, characterizes the long-run level of debt and details the numerical solution. As discussed in the Appendix, we report one specific equilibrium, as is common in the literature, see for instance Debortoli and Nunes (2013).

gradually, up to the point where there is no incentives to deviate from the stationary optimal policy plan, i.e., when $B_t = 0$ or $b_t = 0$.

Figure 6: Markov equilibrium: joint government without commitment



This figure represents the dynamic allocation induced by cooperative policy makers without commitment. The **black** line refers to the case of nominal debt, the **blue** line to real debt. In both cases, the sequential game induces a gradual reduction in the level of debt, down to $z_t = 0$ or $b_t = 0$, i.e. to the point where policy makers no longer have the incentives to deviate from the optimal stationary policy plan. Nominal and real debt economies are made comparable using the correspondence (29).

2.3 Non-cooperative Policy Game with Asymmetric Commitment

The institutional set-up with asymmetric commitment from Section 1.3 is now applied to the dynamic cash-credit good economy to contrast credibility for different monetary rules. We consider the following dynamic game where monetary and fiscal policy makers act strategically. Our focus being on the interactions across policy authorities, we treat households as non strategic.⁴⁵

Timing and decisions. At a constitutional stage before $t = 0$, the central bank announces a monetary rule ρ^{Mk} for setting money growth rates σ_t at all times in the future.⁴⁶ After the constitutional stage, the game unfolds dynamically. In every period $t \geq 0$, the fiscal authority moves first and chooses the tax rate τ_t . The central bank moves second and chooses the money growth rate σ_t . The central bank can *keep* the promise and follow the pre-announced rule ρ^{Mk} or *renege* on the promise and deviate from the rule. Given policy choices, household choose consumption, leisure and savings. The resulting dynamic transition

⁴⁵The definition of the game in Atkeson, Chari, and Kehoe (2010) requires that for all histories, the continuation outcome constitute a competitive equilibrium. This is not a concern in our environment, where the two players of the game operates within the set of competitive equilibria, as defined by the implementability constraint (27).

⁴⁶Superscript M in ρ^{Mk} stands for *monetary* authority and as made clear below, k indicates that the central bank *keeps* its promise when it decides to follow this rule along the game.

of government debt is represented by a function ϱ^s such that $s_t = \varrho^s(s_{t-1}, \tau_t, \sigma_t)$.⁴⁷

Policy objectives and commitment technology. Policy makers are benevolent but differ in their commitment technology. Every period, the fiscal authority sets the tax rate so as to maximize the discounted utility of household. The central bank then implements its rule ϱ^{Mk} unless it finds it profitable to renege its promise, namely if it improves the utility of the representative household starting from this period net of an institutional loss $\kappa \geq 0$. This cost $\kappa \geq 0$ born by the central bank reflects the commitment technology of the monetary authority.⁴⁸

Policy rules and strategies. We study monetary rules ϱ^{Mk} that set the money growth rate as a function of outstanding liabilities s_{t-1} , and the tax rate τ_t set by the fiscal authority. The sequential actions of all players involved in the game are described by stationary Markovian strategies. First, the fiscal authority sets the tax rate as a function of outstanding liabilities, $\tau_t = \varrho^F(s_{t-1})$. Second, the decision of the central bank of whether to follow the pre-announced rule is described by an indicator function, $\mathbb{I}^r(s_{t-1}, \tau_t)$, equal to one when the central bank reneges on the promise and zero otherwise. When reneging, the choice of the money growth rate by the central bank is described by a policy strategy $\varrho^{Mr}(s_{t-1}, \tau_t)$. Formally, an equilibrium of this game is as follows.

Definition 4. *Given a monetary rule ϱ^{Mk} , a Markov-perfect equilibrium of the policy game consists of a transition function ϱ^s , policy strategies $(\varrho^{Mr}, \mathbb{I}^r)$ and ϱ^F , and value functions $V^M(\cdot), V^F(\cdot)$ such that:*

(i) *Given $(\varrho^s, \varrho^{Mr}, \mathbb{I}^r)$, the fiscal policy ϱ^F and value function V^F solve the following Bellman equation:*

$$V^F(s_{t-1}) = \max_{\tau_t} U(\tau_t, \sigma_t) + \beta V^F(\varrho^s(s_{t-1}, \tau_t, \sigma_t)),$$

where

$$\sigma_t = [1 - \mathbb{I}^r(s_{t-1}, \tau_t)]\varrho^{Mk}(s_{t-1}, \tau_t) + \mathbb{I}^r(s_{t-1}, \tau_t)\varrho^{Mr}(s_{t-1}, \tau_t).$$

(ii) *Given (ϱ^s, ϱ^F) , the monetary policies $(\varrho^{Mr}, \mathbb{I}^r)$ and value function V^M solve the following Bellman equation:*

$$V^M(s_{t-1}, \tau_t) = \max_{\mathbb{I}_t^r \in \{0,1\}} [1 - \mathbb{I}_t^r]V^{Mk}(s_{t-1}, \tau_t) + \mathbb{I}_t^r [V^{Mr}(s_{t-1}, \tau_t) - \kappa],$$

where the value functions V^{Mk} and V^{Mr} , corresponding respectively to the central bank keeping or reneging on its rule ϱ^{Mk} :

$$V^{Mk}(s_{t-1}, \tau_t) = U(\tau_t, \varrho^{Mk}(s_{t-1}, \tau_t)) + \beta V^M \left(\underbrace{\varrho^s(s_{t-1}, \tau_t, \varrho^{Mk}(s_{t-1}, \tau_t))}_{s_t}, \underbrace{\varrho^F(\varrho^s(s_{t-1}, \tau_t, \varrho^{Mk}(s_{t-1}, \tau_t)))}_{\tau_{t+1}} \right),$$

$$V^{Mr}(s_{t-1}, \tau_t) = \max_{\sigma_t} U(\tau_t, \sigma_t) + \beta V^M(\varrho^s(s_{t-1}, \tau_t, \sigma_t), \varrho^F(\varrho^s(s_{t-1}, \tau_t, \sigma_t))).$$

⁴⁷The transition function captures the conditions associated to the competitive equilibrium Definition 3 and characterized in Lemma 1.

⁴⁸The polar cases where $\kappa = +\infty$ and $\kappa = 0$ correspond respectively to full monetary commitment and no monetary commitment.

(iii) Given $(\varrho^F, \varrho^{Mr}, \mathbb{I}^r)$, the transition function ϱ^s induces a competitive equilibrium for any $(s_{t-1}, \tau_t, \sigma_t)$:

$$0 = f(s_t, s_{t-1}, \sigma_{t+1}, \sigma_t, \varrho^F(s_t), \tau_t),$$

where

$$\sigma_{t+1} = \mathbb{I}^r(s_t, \varrho^F(s_t))\varrho^{Mr}(s_t, \varrho^F(s_t)) + [1 - \mathbb{I}^r(s_t, \varrho^F(s_t))]\varrho^{Mk}(s_t, \varrho^F(s_t)).$$

The definition highlights the sequential nature of policy moves within periods.⁴⁹ The fiscal authority is a Stackelberg leader, and can thus challenge the pre-announced monetary rule ϱ^{Mk} . On the other hand, the central bank retains a first mover advantage at the constitutional stage, with the possibility to design threats in reaction to fiscal deviations from a desirable policy plan.

2.3.1 Credibility of the Central Bank

An equilibrium path of this game is the outcome of non-cooperative actions by policy makers. Both are interested in minimizing tax distortions and intertemporal losses, but only the central bank is endowed with a commitment technology, parametrized by the cost κ of renouncing a promise. Intuitively, a monetary rule ϱ^{Mk} announced at the constitutional stage is credible if the central bank keeps the promise along the equilibrium path.

Definition 5. Let s_{-1} be initial government liability and $\{\tilde{\sigma}_t, \tilde{\tau}_t, \tilde{s}_t\}_{t=0}^{\infty}$ an equilibrium outcome of the policy game given a monetary rule ϱ^{Mk} . The rule ϱ^{Mk} is credible given s_{-1} if $\tilde{\sigma}_t = \varrho^{Mk}(\tilde{s}_{t-1}, \tilde{\tau}_t) \forall t \geq 0$.

To characterize the commitment intensity required to ensure the credibility of a monetary rule, we specify the sequential incentives of fiscal and monetary authorities on and off equilibrium path.

Start with the fiscal authority and let $V^{Fk}(s_{t-1}, \tau_t)$ be the value to the fiscal authority of choosing a tax rate τ_t , conditional on the monetary policy *keeps* its promise to follow ϱ^{Mk} :

$$V^{Fk}(s_{t-1}, \tau_t) = U(\tau_t, \varrho^{Mk}(s_{t-1}, \tau_t)) + \beta V^F\left(\varrho^s(s_{t-1}, \tau_t, \varrho^{Mk}(s_{t-1}, \tau_t))\right). \quad (32)$$

Similarly, let $V^{Fr}(s_{t-1}, \tau_t)$ be the value function to the fiscal authority when setting a tax rate τ_t conditional on monetary policy *renouncing* the pre-announced rule:

$$V^{Fr}(s_{t-1}, \tau_t) = U(\tau_t, \varrho^{Mr}(s_{t-1}, \tau_t)) + \beta V^F\left(\varrho^s(s_{t-1}, \tau_t, \varrho^{Mr}(s_{t-1}, \tau_t))\right) \quad (33)$$

Consider an equilibrium path $\{\tilde{\sigma}_t, \tilde{\tau}_t, \tilde{s}_t\}_{t=0}^{\infty}$ under a credible monetary rule. Along this path, the choices of the fiscal authority satisfy:

$$\tilde{\tau}_t = \operatorname{argmax}_{\tau_t} V^{Fk}(\tilde{s}_{t-1}, \tau_t) \quad \text{and} \quad V^F(\tilde{s}_{t-1}) = V^{Fk}(\tilde{s}_{t-1}, \tilde{\tau}_t), \quad (34)$$

where the second expression captures the monetary decision to follow its rule along the equilibrium path.

⁴⁹The equilibrium definition makes the intraperiod leadership structure of policy interactions explicit, given differences in state variables for each authority.

At \tilde{s}_{t-1} , the fiscal authority would deviate from $\tilde{\tau}_t$ and set a tax rate $\tau_t \neq \tilde{\tau}_t$ if it were to lead to a welfare improvement, conditional on the central bank renouncing its promise. Define accordingly $\mathbb{T}(\tilde{s}_{t-1})$, the set of *profitable fiscal deviations* at \tilde{s}_{t-1} :

$$\mathbb{T}(\tilde{s}_{t-1}) = \{\tau_t \mid V^{Fr}(\tilde{s}_{t-1}, \tau_t) \geq V^F(\tilde{s}_{t-1})\}. \quad (35)$$

Consider now the central bank: its incentives to *renounce* its promise at $(\tilde{s}_{t-1}, \tau_t)$ for all $\tau_t \in \mathbb{T}(\tilde{s}_{t-1})$ are evaluated against the value to *keep* its promise and follow the rule. The central bank follows the rule at \tilde{s}_{t-1} when

$$\kappa \geq \Delta(\tilde{s}_{t-1}) = \max_{\tau_t \in \mathbb{T}(\tilde{s}_{t-1})} \{V^{Mr}(\tilde{s}_{t-1}, \tau_t) - V^{Mk}(\tilde{s}_{t-1}, \tau_t)\}. \quad (36)$$

In words, the rule is implemented at \tilde{s}_{t-1} against any fiscal decisions if the commitment intensity is high enough to eliminate all monetary incentives to renounce the rule, and in particular when it would yield the highest welfare gain to the monetary authority.⁵⁰

We define the credibility cut-off $\bar{\kappa}(s_{-1})$ as the minimum commitment intensity that supports credible implementation of the monetary policy rule.

Definition 6. *Given initial liabilities s_{-1} , the credibility cut-off of the monetary rule ϱ^{Mk} is defined as*

$$\bar{\kappa}(s_{-1}) = \min\{\kappa \mid \kappa \geq \Delta(\tilde{s}_{t-1}) \forall t \geq 0\}. \quad (37)$$

Note that evaluation of credibility involves off equilibrium paths, hence $\Delta = \max_{\{\tilde{s}_{t-1}\}_{t=1}^{\infty}} \Delta(\tilde{s}_{t-1})$ might be a function of the degree of commitment κ . If it is the case, provided the dependence is monotone, the credibility cut-off $\bar{\kappa}$ is a fixed point of $\Delta(\kappa)$. Otherwise, the credibility cut-off is simply equal to Δ .

2.4 Standard and Strategic Monetary Rules

We contrast the policy implications of two classes of monetary rules, *standard* and *strategic*, and compare their credibility cut-offs.

2.4.1 Standard Monetary Rule

Recall that the optimal *timeless* policy plan characterized in Proposition 4 prescribes the central bank to set a constant negative money growth rate $\sigma^* = \beta - 1$. We study the effect of a similar monetary rule in the context of the policy game. At the constitutional stage the central bank announces a *standard* rule, independent of economic conditions, and in particular of fiscal policy decisions:

$$\varrho^{Mk}(s_{t-1}, \tau_t) = \sigma \geq \beta - 1, \quad \forall s_{t-1}, \tau_t. \quad (38)$$

⁵⁰This characterization of credibility makes clear that a credible rule eliminates all the fiscal incentives to challenge the rule and the monetary incentives to renounce the rule.

Proposition 5. *Let κ be arbitrarily large, and the central bank commits to a standard rule of type (38). If debt is nominal, then the policy game results in a stationary allocation and choice of policy instruments: for all $t \geq 0$, $z_t = z_{-1}$.*

Proof. See Appendix B.4 ■

This proposition has stark implications for the conduct of public policy under asymmetric commitment: under the same conditions, if the central bank commits unconditionally to the money growth rate prescribed by the Friedman rule $\sigma = \beta - 1$, then the implied allocation coincides with the optimal *timeless* one characterized in Proposition 4.

Corollary 1. *Let κ be arbitrarily large and debt be nominal. If $\sigma = \beta - 1$, then the induced allocation and policy choices coincide with the timeless optimal allocation.*

Proof. See Appendix B.4 ■

Only when debt is nominal, and if the central bank is endowed with a commitment technology that eliminates deviation from its rule, then the fiscal authority has no longer the incentive to manipulate the interest rate and influence the price of newly issued bonds. Indeed, variations in the tax rate generate an offsetting revaluation of outstanding nominal liabilities. This effect does not operate when debt is real.

Further, when the central bank follows the constant deflation rate prescribed by the Friedman rule, the induced equilibrium coincides with the timeless allocation. Standard monetary rule with nominal debt extends commitment across the government. Still, this rule might require a high degree of commitment to be credible.

2.4.2 Strategic Monetary Rule

We now consider a class of monetary rules designed to induce the timeless optimal allocation with the additional objective to minimize the associated degree of commitment. Following the construction presented in Section 1.4.2, these *strategic rules* are built to eliminate the incentives of the fiscal authority to challenge the monetary rule. They rely on off-equilibrium threats to maintain fiscal incentives to follow the appropriate policy in equilibrium.

The strategic rule ϱ^{Mk} is constructed to satisfy two key properties. First, the central bank selects the timeless allocation as the desired equilibrium and sets monetary policy to support it. In particular, let $\tau^*(s_{t-1})$ be the tax rate required to sustain the *timeless* allocation and keep $s_t = s_{t-1}$ at any $t \geq 0$. In turn, the monetary policy rule is such that:

$$(p.1) \text{ if } \tau_t = \tau^*(s_{t-1}), \text{ then } \varrho^{Mk}(s_{t-1}, \tau_t) = \beta - 1.$$

If both fiscal and monetary policy play according to this path, then the timeless allocation is the induced equilibrium outcome. The values to both policy authorities is then $W^{ta}(s_{-1})$, as defined in (30).

Second, the central bank designs threats, to discourage fiscal deviations from the desired equilibrium. Importantly, the credibility of a monetary rule is intertwined with the magnitude of these threats: the

stronger the threat, the larger the degree of commitment required to implement the rule. Accordingly, we restrict our attention to strategic rules that minimize the degree of commitment required to implement them.⁵¹ The central bank calibrates its off-equilibrium reaction to offset the incentives of the fiscal authority to deviate from the equilibrium path:

(p.2) if $\tau_t \in \mathbb{T}(s_{t-1})$ and $\tau_t \neq \tau^*(s_{t-1})$, then $\varrho^{Mk}(s_{t-1}, \tau_t) = \sigma$ such that:

$$V^{Fk}(s_{t-1}, \tau_t) = W^{ta}(s_{t-1}), \quad (39)$$

where $\mathbb{T}(s_{t-1})$ is the set of profitable fiscal deviations, which takes the following form under the strategic rule:

$$\mathbb{T}(s_{t-1}) = \{\tau \mid V^{Fr}(s_{t-1}, \tau) \geq V^{Fk}(s_{t-1}, \tau^*(s_{t-1})) = W^{ta}(s_{t-1})\}. \quad (40)$$

Given s_{-1} , a strategic rule is credible if the degree of commitment of the central bank is sufficient to eliminate the most profitable fiscal deviation within $\mathbb{T}(s_{-1})$.⁵²

Credibility under nominal debt. The following proposition characterizes the credibility cut-off required to implement such strategic rules when debt is nominal.

Proposition 6. *Given initial liabilities $z_{-1} > 0$, there is a strategic monetary rule ϱ^{Mk} which implements the timeless allocation as an outcome of the game, if and only if the commitment intensity κ of the central bank satisfies:*

$$\kappa \geq \bar{\kappa}_2(z_{-1}) = \underbrace{\max_{\tau, \sigma, z} U(\tau, \sigma) + \beta W^{ta}(z) - W^{ta}(z_{-1})}_{=W^{rp}(z_{-1})}, \quad (41)$$

with $\frac{d\bar{\kappa}_2(z_{-1})}{dz_{-1}} > 0$.

Proof. See Appendix B.5. ■

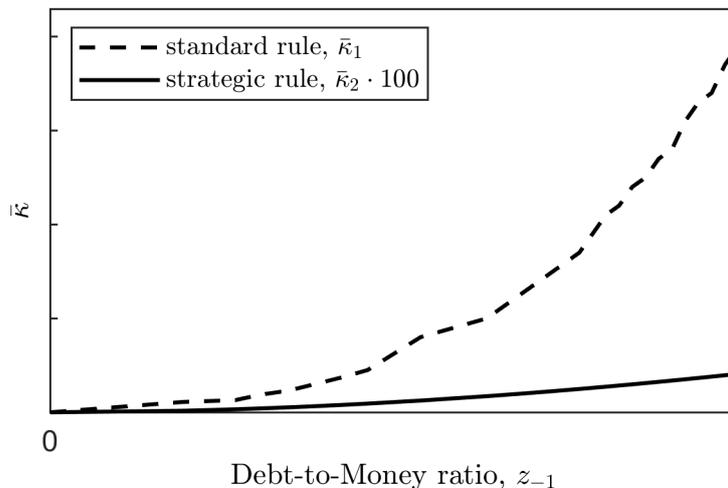
The credibility of the strategic rule that implements the *timeless allocation* with welfare $W^{ta}(s_{-1})$ is evaluated against the most profitable fiscal deviation, followed by the central bank reneging on its rule. When debt is nominal, this sequential policy path coincides with the cooperative choice under commitment—the *Ramsey plan*—associated with welfare $W^{rp}(z_{-1})$, see Proposition 4.

This simple characterization arises for two reasons. First the level of debt z issued under the Ramsey plan is lower than outstanding debt z_{-1} , and second the credibility cut-off is increasing in outstanding debt. If monetary threats are credible to eliminate today the incentives of the fiscal authority to deviate from $\tau^*(z_{-1})$, then the monetary authority would as well deter (off equilibrium) fiscal incentives to deviate from

⁵¹In particular, as shown in Appendix B.5, the existence of these strategies does not rely on the central bank imposing arbitrarily large inflation to the economy.

⁵²For $\tau \notin \mathbb{T}(s_{-1})$, the central bank rule can specify arbitrary σ , since the payoff to the fiscal authority is strictly lower than to abide by the stationary allocation for any σ .

Figure 7: Credibility Cut-Offs - Nominal Debt



This figure represents credibility cut-offs for monetary rules under nominal debt. It contrasts the commitment intensity required to sustain a *standard* rule of unconditional money growth rate $\sigma = \beta - 1$ rule with a *strategic* rule. The different scales reflect the higher credibility requirement of the standard rule.

$\tau^*(z)$ tomorrow for all $z \leq z_{-1}$. Overall, the strategic rule is credible only if the commitment intensity of the central bank can eliminate the temptation to reoptimize and implement the Ramsey plan.

Figure 7 illustrates Propositions 5 and 6, i.e. the credibility challenge associated to each type of monetary rules under nominal debt.⁵³ Given a level of debt z_{-1} , if commitment intensity κ is high enough to implement either rule in equilibrium, then the equilibrium path is the timeless allocation. Obviously, the credibility cut-off under the standard rule is higher at any level of debt than under the strategic rule, precisely designed to minimize credibility.

Note that the credibility of monetary rules is directly linked to the level of debt z_{-1} . Indeed, the larger the level of outstanding liabilities, the higher the relative gains to renounce the rule. In particular, in the extreme case where $z_{-1} = 0$, there is no policy incentives to manipulate the interest rate or generate inflation beyond Friedman deflation $\sigma = \beta - 1$, hence $\bar{\kappa}_1(0) = \bar{\kappa}_2(0) = 0$.

Credibility under real debt. The credibility of a monetary rule is primarily influenced by the relative gain to renouncing it. When debt is real, there is no possibility to inflate outstanding debt, so that the relative gains to renouncing the rule are lower. This explains why the credibility cut-off of strategic rules is lower when debt is indexed rather than nominal, as illustrated in Figure 8. Formally, under indexed debt, a similar expression to (41) provides an upper bound on the credibility cut-off $\bar{\kappa}_2(b_{-1})$:⁵⁴

$$\bar{\kappa}_2(b_{-1}) \leq W^{rp}(b_{-1}) - W^{ta}(b_{-1}). \quad (42)$$

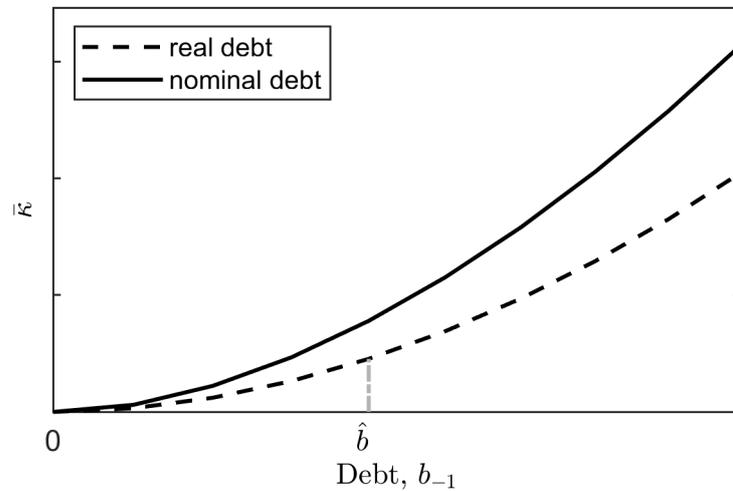
⁵³The characterization of the credibility cut-off for *standard* rules $\bar{\kappa}_1(z_{-1})$ is numerical, since the off equilibrium paths at which credibility is evaluated are associated with spells of central bank playing *reneges* in future periods. Appendix B.7 provides elements related to the solution of the Markov game and characterization of $\bar{\kappa}_1(z_{-1})$.

⁵⁴Appendix B.5 derives formally this expression.

The inequality illustrates the dynamic nature of this game. It is binding if and only if $b_{-1} \geq \hat{b}$, where \hat{b} is the cut-off level of debt characterized in Proposition 4. For higher level of outstanding debt $b_{-1} \geq \hat{b}$, newly issued debt under a Ramsey plan satisfies $b \leq b_{-1}$. If the commitment intensity of the central bank is high enough to eliminate fiscal deviation today at b_{-1} , it is enough to implement (off equilibrium) the timeless allocation tomorrow at lower levels b . Accordingly, the inequality (42) is binding.

For lower levels of outstanding debt $b_{-1} \leq \hat{b}$, a Ramsey planner issues a higher level of debt $b > b_{-1}$: the continuation utility associated with any deviation (τ, σ) is lower than $W^{rp}(b_{-1})$, since it might not be possible to sustain (off equilibrium) the timeless allocation tomorrow at higher level of debt b . The most profitable deviation within $T(b_{-1})$ yields a lower relative gain, which reduces the commitment intensity required to defend the strategic rule against fiscal deviations today. The inequality (42) is strict.

Figure 8: Credibility of Strategic Rules: Nominal vs. Real Debt



This figure contrasts the commitment intensity required to sustain the strategic rule under nominal and real debt. \hat{b} corresponds to the level of debt such that the dashed line is an upper bound on $\bar{\kappa}_2(b_{-1})$ for $b_{-1} \leq \hat{b}$. Nominal and real debt economies are made comparable using the correspondence (29).

2.5 Extensions

In this section, we investigate an additional set of questions. First, we study the sensitivity of credibility cut-offs to the maturity of public debt. Next we characterize how the economy behaves when the degree of monetary commitment falls short of the credibility cut-off. We finally study how deviations from the Friedman rule can alleviate an insufficient degree of monetary commitment.

2.5.1 Debt Maturity and Strategic Rule

Maturity structure is known to be an important factor that affects discretionary incentives of policy-makers. Lucas and Stokey (1983) show that a portfolio of government securities with a properly structured maturity profile can eliminate the incentives of the treasury to manipulate interest rates.⁵⁵

⁵⁵See Debortoli, Nunes, and Yared (2018) for an updated discussion.

To analyze how debt maturity alters monetary-fiscal interactions and the credibility of monetary rules, we introduce long-term government debt as in Woodford (2001). Let government bonds be perpetual with payoffs decaying at the exponential rate $\rho \in [0, 1]$. The *maturity* of long-term debt is increasing in ρ .⁵⁶ With nominal debt of maturity ρ , the household budget constraint becomes:⁵⁷

$$P_t c_t + P_t d_t + q_t B_t^h + M_t^h = P_t(1 - \tau_t)(1 - l_t) + (1 + \rho q_t)B_{t-1}^h + M_{t-1}^h, \quad (43)$$

First, consider cooperative optimal policy under full commitment. Figure 9 presents the associated dynamic equilibrium paths for different values of debt maturity ρ . The economy eventually converges to a stationary allocation supported by the constant money growth rate set according to the Friedman rule and a constant tax rate. The longer debt maturity, the longer the transition path to this *timeless allocation*. Indeed the fiscal interest rate manipulation needs to be implemented over several periods to match the profile of outstanding debt. In short, debt maturity matters.

Consider now strategic rules designed to implement a stationary *timeless* allocation. In this environment, the credibility cut-off (41) for $\rho \geq 0$ satisfies

$$\bar{\kappa}_2(z_{-1}, \rho) = \max_{\tau, \sigma, z} U(\tau, \sigma) + \beta W^{ta}(z) - W^{ta}(z_{-1}), \quad (44)$$

where the reoptimization program is constrained by the following implementability condition, sensitive to maturity ρ :

$$\beta(1 - \alpha) \frac{z}{1 - \rho} - \alpha(1 - \tau) - (1 - \alpha)\beta \frac{(1 - \tau)}{(1 + \sigma)} + \Phi = \frac{(1 - \alpha)\beta}{1 + \sigma} \frac{z_{-1}}{1 - \rho}. \quad (45)$$

(44) holds with equality since newly issued nominal debt under the deviation satisfies $z \leq z_{-1}$.⁵⁸ The most profitable deviation against which credibility is evaluated is similar the case with the one-period bond economy (41).

Key is whether maturity influences credibility (44). We show that the credibility of strategic monetary rules is sensitive to the maturity of outstanding debt only if debt is real. Formally,

Proposition 7. *If debt is nominal, credibility of strategic monetary rule is not sensitive to debt maturity. In contrast, credibility is sensitive to debt maturity if debt is real: in the limit case $\rho = 1$, then the timeless allocation is self-enforcing / time consistent.*

Proof. See Appendix B.6. ■

The intuition for these results again highlights monetary-fiscal interactions. Consider first the case of real debt, illustrated in figure 10: the longer debt maturity, the lower the credibility cut-off at all levels of debt.⁵⁹ Indeed, longer debt maturity mitigates the interest rate manipulation motive of the treasury, which

⁵⁶Maturity is measured in terms of duration, which is formally defined as the weighted average term to maturity of bond payoffs. The steady-state duration of nominal debt is $x = (1 - \beta\rho/(1 + \sigma))^{-1}$. With real debt, duration is $x = (1 - \beta\rho)^{-1}$.

⁵⁷All other elements of the model are adjusted similarly. Appendix B.2 details the environment.

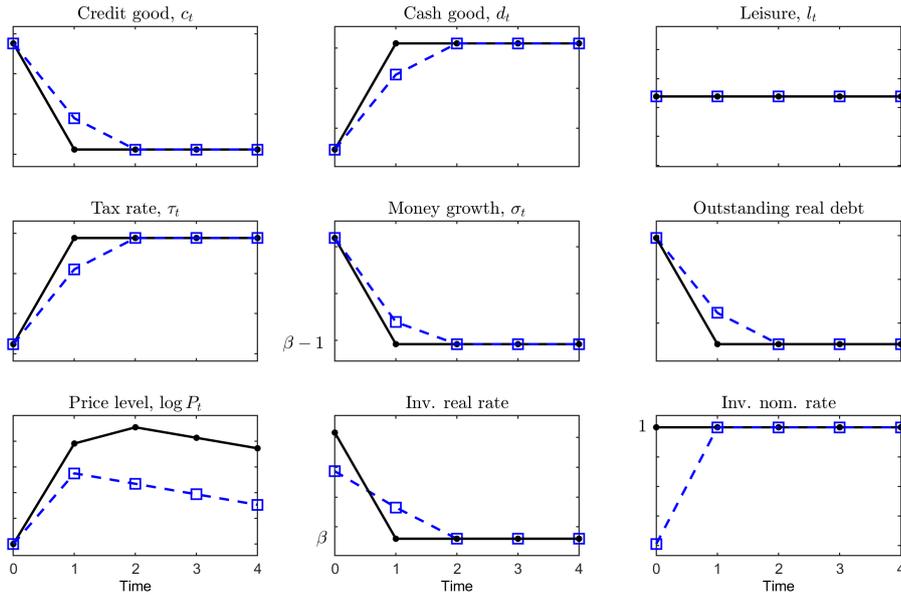
⁵⁸This is not necessarily the case under real debt, in which case the expression equivalent to (45) is a strict inequality. This is another illustration of the dynamic properties of the game.

⁵⁹We verify numerically that newly issued debt under the most profitable deviation is lower than outstanding debt and indicate cut-offs where (44) is binding or not.

lowers its incentives to challenge the monetary rule. In the limit case of consol bonds, $\rho = 1$, the interest rate manipulation motive of the treasury is mute and the central bank has no incentives to generate strategic inflation.⁶⁰

When debt is nominal, maturity does not matter for two reasons. First, from the central bank's perspective, the incentives to inflate away outstanding debt depends on its stock, not its maturity.⁶¹ Second, as was the case in Proposition 5, when debt is nominal and the central bank credibly anchors the equilibrium path, then the fiscal temptation to manipulate interest rate backfires with a revaluation of outstanding debt, independently of debt maturity.

Figure 9: Ramsey equilibrium under different debt maturities



This figure represents time allocation, policy instruments and prices when policy is conducted under joint commitment with nominal debt. The **black** line refers to a 1-year duration ($\rho = 0$), the **blue** line to a 3-years duration ($\rho = 0.67$). Economies with different debt maturities are made comparable using the correspondence discussed in Section 2.1.

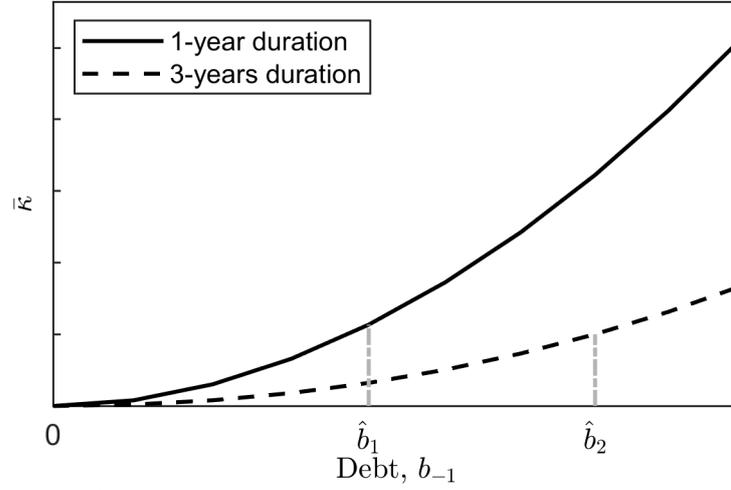
2.5.2 Equilibrium Path under Lack of Credibility.

Consider an initial debt position z_{-1} such that the degree of commitment falls short of the credibility cut-off for a strategic rule: $\kappa < \bar{\kappa}_2(z_{-1})$. Figure 11 presents the equilibrium path in such a scenario. Early policy choices are characterized by a fiscal tax cut that induces the central bank to renounce its rule and generate inflation. Outstanding debt decreases up to a level z such that the strategic rule is credible $\kappa \geq \bar{\kappa}_2(z)$.

⁶⁰Again, the unitary intertemporal elasticity of substitution in (15) eliminates the incentives to collect seignorage. See Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008).

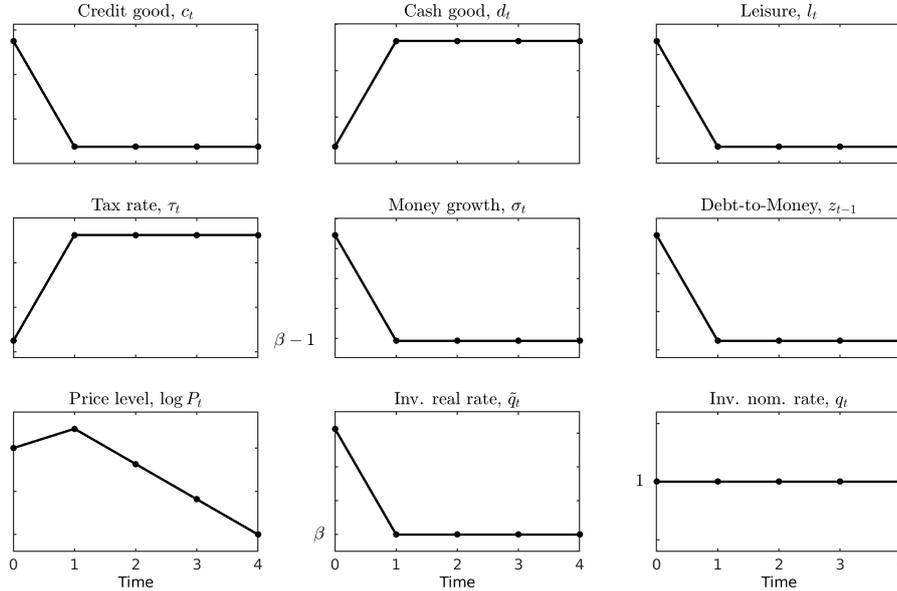
⁶¹For a given level of debt z_{-1} , an increase in maturity ρ is associated with higher payoffs. This effect is taken into account in the correspondence used to compare economies with different debt maturities.

Figure 10: Credibility of Strategic Rules for different debt maturities



This figure contrasts the commitment intensity required to sustain the strategic rule under real debt and different degrees of debt maturity $\rho \in \{0, 0.52\}$. \hat{b} correspond to level of debt such that the line is an upper bound on $\bar{\kappa}_2(b_{-1}, \rho)$ for $b_{-1} \leq \hat{b}$

Figure 11: Dynamic path under lack of credibility



This figure represents the dynamic path of policy choices and allocation when initial outstanding debt z_{-1} higher than the credibility cut-off a central bank's would need to implement a strategic rule: $\kappa < \bar{\kappa}_2(z_{-1})$. At $t = 0$, the fiscal authority implements a tax cut to manipulate the real interest rate; it induces the central bank to renounce its rule and inflate outstanding debt. Newly issued debt decreases up to a level z such that: $\kappa \geq \bar{\kappa}_2(z)$. From $t = 1$ on, the rule is credible, the allocation is stationary.

2.5.3 Equilibrium Money Growth Rate and Credibility of Strategic Rule.

Can the central bank adjust its policy framework when the degree of commitment κ is lower than the credibility cut-off $\bar{\kappa}_2(z_{-1})$ required to implement a strategic rule?

Consider the central bank designs a strategic rule as presented in Section 2.4.2, with an equilibrium money growth rate $\tilde{\sigma} \geq \beta - 1$.⁶² By Proposition 5, if the rule is credible, then the equilibrium allocation is stationary. Note $\tilde{\tau}(z_{-1}, \tilde{\sigma})$ the tax rate associated to this stationary allocation. The welfare to both authorities is then:

$$W^s(z_{-1}, \tilde{\sigma}) = \frac{\beta}{1 - \beta} U(\tilde{\tau}(z_{-1}, \tilde{\sigma}), \tilde{\sigma}), \quad (46)$$

where the superscript s stands for *stationary*.⁶³ The fiscal authority considers the following set of deviations from $\tilde{\tau}(z_{t-1}, \tilde{\sigma})$ at z_{t-1} :

$$T(z_{t-1}, \tilde{\sigma}) = \{\tau \mid V^{Fr}(z_{t-1}, \tau) \geq W^s(z_{-1}, \tilde{\sigma})\}. \quad (47)$$

The strategic rule is defined as:

(p.1) if $\tau_t = \tilde{\tau}(z_{t-1}, \tilde{\sigma})$, then $\varrho^{Mk}(s_{t-1}, \tau_t) = \tilde{\sigma}$,

(p.2) if $\tau_t \in T(z_{t-1}, \tilde{\sigma})$ and $\tau_t \neq \tilde{\tau}(z_{t-1}, \tilde{\sigma})$, then $\varrho^{Mk}(z_{t-1}, \tau_t) = \sigma$ such that:

$$V^{Fk}(z_{t-1}, \tau_t) = W^s(z_{t-1}, \tilde{\sigma}). \quad (48)$$

Figure 12 represents the credibility cut-offs $\bar{\kappa}_2(z_{-1}, \tilde{\sigma})$ required to implement strategic rules for different equilibrium money growth rate $\tilde{\sigma} \geq \beta - 1$.⁶⁴ At high level of debt, the degree of commitment required to enforce a stationary allocation is decreasing in the equilibrium money printing rate. This builds upon the intuition that at higher equilibrium money printing rate $\tilde{\sigma} > \beta - 1$, the relative gains to the monetary authority to renouncing to the rule are lower. Accordingly, the credibility of the rule is higher.⁶⁵

3 Conclusions

Shall a central bank lean against fiscal influence? Does it compromise its capacity to ensure price stability? Our analysis of monetary and fiscal interactions under asymmetric commitment has contrasted two types of monetary rules. The key difference across these rules is whether the central bank explicitly reacts to fiscal decisions.

We show that *strategic* rules, which are designed to eliminate the incentives of fiscal authorities (i) to deviate from an optimal policy plan and (ii) to induce the central bank to renounce its rule have several

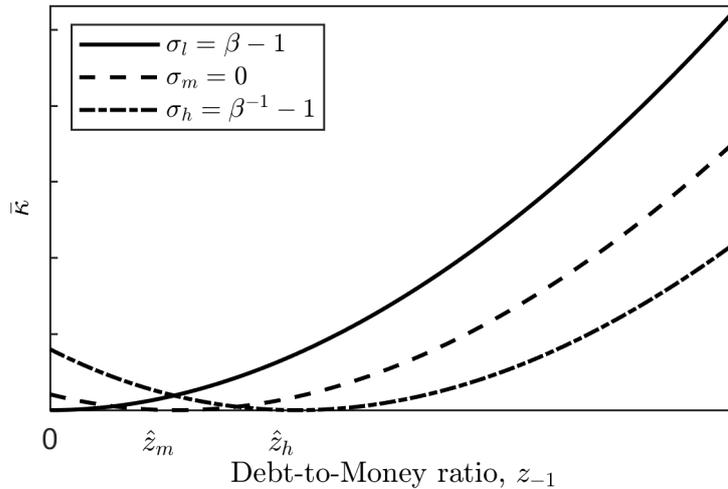
⁶²This analysis is similar to the discussion on the inflation target in Section 1.5 for the linear-quadratic game.

⁶³In particular, $W^s(z_{-1}, \beta - 1) = W^{ta}(z_{-1})$.

⁶⁴Appendix B.7 provides elements related to the numerical characterization of $\bar{\kappa}_2(z_{-1}, \tilde{\sigma})$.

⁶⁵In contrast, at lower levels of debt, deviating from the Friedman rate of money growth increases required credibility, since the relative gains to renege the promise are low and the permanent welfare cost on equilibrium higher.

Figure 12: Credibility of Strategic Rules: Equilibrium Money Printing Rates



This figure compares credibility cut-offs $\bar{\kappa}_2(z_{-1}, \bar{\sigma})$ for strategic rules under different levels of equilibrium money growth rates $\bar{\sigma} \geq \beta - 1$. At higher levels of debt, an increase in the equilibrium money growth rate reduces the relative gains to renouncing the rule, which lowers the commitment intensity required to sustain a stationary allocation. \hat{z}_k for $k \in \{m, h\}$ correspond to cut-off levels of outstanding debt z_{-1} such that the dashed line are actually an upper bound on $\bar{\kappa}_2(z_{-1}, \bar{\sigma}_k)$ if $z_{-1} < \hat{z}_k$.

benefits. First, the central bank can rely on its commitment technology to stir the economy toward better economic outcomes. Second, these strategic rules do not necessarily require a higher degree of commitment to be implemented. Quite surprisingly, these rules are more credible when the threats of fiscal dominance are the highest, that is to say when the fiscal authority is able to exert pressure on the central bank to reoptimize and reap short term gains from policy discretion.

In this context, public debt is a central determinant of credibility. We unveil an interesting insight. When debt is nominal, unconditional money growth rate policies curb the time inconsistency problem of fiscal policy, but they require significant degree of commitment to be supported in equilibrium. In contrast, when debt is real, monetary interventions need to rely on off equilibrium threats to eliminate fiscal deviations from the dynamic policy plan, and the associated commitment intensity is lower.

Our economy with a representative agent abstracts from the distributive consequences of monetary and fiscal interactions. Introducing heterogeneous agents would be interesting per se to study the implications of fiscal political economy and institutional independence on monetary-fiscal interactions.

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Appendix - For Online Publication Only

A Linear-Quadratic Framework

A.1 Policy choices under cooperation - section 1.2

Under cooperation, a single benevolent authority decides on policy instruments. Under commitment, the government sets (τ, π) before private agents form expectations $e = \tau^e + \alpha\pi^e$. The objective is to minimize the welfare loss (2), subject to the Phillips curve (1) and private agents expectations (3). Substituting the constraints into the loss function yields:

$$\min_{\tau, \pi} \frac{1}{2} \left[(\tau - \tau^*)^2 + \lambda(\pi - \pi^*)^2 + \gamma(0 - y^*)^2 \right]. \quad (49)$$

The first order conditions naturally leads to $\tau^c = \tau^*$ and $\pi^c = \pi^*$.

When the government does not have a commitment technology, it takes policy decisions after private agents formed expectations $e = \tau^e + \alpha\pi^e$. The objective is to minimize the welfare loss (2), subject to the Phillips curve (1), given e :

$$\min_{\tau, \pi} \frac{1}{2} \left[(\tau - \tau^*)^2 + \lambda(\pi - \pi^*)^2 + \gamma(\tau - \tau^e + \alpha(\pi - \pi^e) - y^*)^2 \right] \quad (50)$$

The first order conditions give policy reactions to private agents expectations:

$$\tau - \tau^* + \gamma(\tau - \tau^e + \alpha(\pi - \pi^e) - y^*) = 0 \quad (51)$$

$$\lambda(\pi - \pi^*) + \gamma\alpha(\tau - \tau^e + \alpha(\pi - \pi^e) - y^*) = 0 \quad (52)$$

In equilibrium, private agents hold rational expectations (3), which yields $\tau^d = \tau^* + \gamma y^*$ and $\pi^d = \pi^* + \frac{\gamma\alpha}{\lambda} y^*$.

A.2 Standard monetary rule - section 1.4.1

Equilibrium outcome. Consider the case where the central bank degree of commitment is unrestricted: $\pi_1 = \pi^e = \pi^*$. The fiscal authority solves (7):

$$\tilde{\tau} = \operatorname{argmin}_{\tau} \mathcal{L}(e, \tau, \pi^*), \quad (53)$$

subject to (1). The first order condition yields:

$$\tau - \tau^* + \gamma(\tau - \tau^e - y^*) = 0. \quad (54)$$

In equilibrium, $\tau^e = \tau$, which gives $\tau_1 = \tau^* + \gamma y^*$ and $e_1 = e^* + \gamma y^*$. The loss in equilibrium is then:

$$\mathcal{L}(e_1, \tau^* + \gamma y^*, \pi^*) = \frac{\gamma(1 + \gamma)}{2} (y^*)^2. \quad (55)$$

Proposition 1. The objective is to characterize the credibility cut-off required to eliminate all off equilibrium paths where, given a fiscal choice $\tau \neq \tau_1$, the central bank renounces π^* and implements $\pi^r(\mathcal{S})$. Given $e_1 = e^* + \gamma y^*$, we characterize $\pi^r(\cdot)$, the set of possible fiscal deviations and then derive the degree of commitment required for the central bank not to renounce π^* within this set.

Sequential monetary reoptimization. Given (e, τ) and conditional on *renege*, the central bank implements $\pi^r(e, \tau) = \operatorname{argmin}_{\pi} \mathcal{L}^{m,r}(e, \tau, \pi)$. Simple computations lead to:

$$\pi^r(\mathcal{S}) = \frac{\lambda \pi^* + \gamma \alpha (y^* + e - \tau)}{\lambda + \gamma \alpha^2}. \quad (56)$$

Set of profitable fiscal deviations. Using (8) evaluated at (10):

$$\mathbb{T}(e_1) = \{\tau \mid \mathcal{L}^{f,r}(e_1, \tau, \pi^r(\mathcal{S})) \leq \mathcal{L}^{f,k}(e_1, \tau_1, \pi^*)\}, \quad (57)$$

where $\pi^r(\cdot)$ is given by (56) evaluated at $e_1 = e^* + \gamma y^*$:

$$\pi^r(e_1, \tau) = \pi^* + \frac{\gamma \alpha}{\lambda + \gamma \alpha^2} ((1 + \gamma) y^* + \tau^* - \tau). \quad (58)$$

With a quadratic the loss function $\mathcal{L}(\cdot)$, we get $\mathbb{T}(e_1) = [\tau_l, \tau_h]$, where τ_x are the solutions to:

$$\mathcal{L}(e_1, \tau, \pi^r(e_1, \tau)) = \frac{\gamma(1 + \gamma)}{2} (y^*)^2. \quad (59)$$

Rewrite this equation as:

$$(\tau - \tau^*)^2 + \frac{\gamma \lambda}{\lambda + \gamma \alpha^2} (\tau - \tau^* - (1 + \gamma) y^*)^2 = \gamma(1 + \gamma) (y^*)^2, \quad (60)$$

and derive

$$\tau_x - \tau^* = y^* \frac{\eta(1 + \gamma) \pm \sqrt{(1 + \gamma)(\gamma - \eta)}}{1 + \eta}, \quad (61)$$

where $\eta = \frac{\gamma \lambda}{\lambda + \gamma \alpha^2}$.

Credibility cut-off. Given $e_1 = e^* + \gamma y^*$ and applying Definition 2, the standard rule is credible if and only if:

$$\kappa \geq \max_{\tau \in \mathbb{T}(e_1)} \bar{\kappa}(\tau) = \mathcal{L}^{m,k}(e_1, \tau, \pi^*) - \mathcal{L}^{m,r}(e_1, \tau, \pi^r(e_1, \tau)). \quad (62)$$

Note that $\bar{\kappa}(\tau)$ is a second order positive polynomial, so its highest value is reached for either τ_l or τ_h . As $\bar{\kappa}(\tau)$ is minimum for $\tau = \tau^* + (1 + \gamma) y^*$ and that $[\tau_l, \tau_h]$ is centered around $\tau = \tau^* + (1 + \gamma) y^* \frac{\gamma \lambda}{\lambda + \gamma \alpha^2 + \gamma \lambda}$,

with $\frac{\gamma\lambda}{\lambda+\gamma\alpha^2+\gamma\lambda} < 1$, we get $\bar{\kappa}(\tau_l) > \bar{\kappa}(\tau_h)$. Accordingly,

$$\bar{\kappa}_1 = \bar{\kappa}(\tau_l) = \mathcal{L}^k(e_1, \tau_l, \pi^*) - \mathcal{L}^r(e_1, \tau_l, \pi^d(e_1, \tau_l)). \quad (63)$$

Computations lead to:

$$\bar{\kappa}_1 = \frac{(y^*)^2}{2}(\gamma - \eta)(1 + \gamma) \left(\frac{\sqrt{1 + \gamma} + \sqrt{\gamma - \eta}}{1 + \eta} \right)^2. \quad (64)$$

Comparative statics. From (64), derive:

$$\frac{d\bar{\kappa}_1}{d\eta} = -\frac{(y^*)^2}{2}(1 + \gamma) \left[\frac{\sqrt{1 + \gamma} + \sqrt{\gamma - \eta}}{1 + \eta} \right] \left[\frac{\sqrt{1 + \gamma} + \sqrt{\gamma - \eta}}{1 + \eta} + 2 \frac{\gamma - \eta}{(1 + \eta)^2} \left(\frac{1}{2} \frac{1 + \eta}{\sqrt{\gamma - \eta}} + \sqrt{1 + \gamma} + \sqrt{\gamma - \eta} \right) \right], \quad (65)$$

which is unambiguously negative. Together with

$$\frac{d\eta}{d\lambda} = \frac{(\gamma\alpha)^2}{(\lambda + \gamma\alpha^2)^2} > 0 \quad \frac{d\eta}{d\alpha} = -\frac{2\alpha\gamma^2\lambda}{(\lambda + \gamma\alpha^2)^2} < 0, \quad (66)$$

we get

$$\frac{d\bar{\kappa}_1}{d\lambda} < 0 \quad \frac{d\bar{\kappa}_1}{d\alpha} > 0. \quad (67)$$

A.3 Strategic rule - section 1.4.2

Credibility cut-off. As explained, the credibility cut-off satisfies:

$$\bar{\kappa}_2 = \mathcal{L}^{m,k}(e_2, \tau^*, \pi^*) - \min_{\tau, \pi} \mathcal{L}^{m,r}(e_2, \tau, \pi), \quad (68)$$

with $e_2 = e^* = \tau^* + \alpha\pi^*$. We get $\mathcal{L}^k(e_2, \tau^*, \pi^*) = \frac{\gamma(y^*)^2}{2}$.

Consider the optimization program $\min_{\tau, \pi} \mathcal{L}^r(e_2, \tau, \pi)$. The first order conditions are:

$$(\tau - \tau^*) + \gamma(\tau - \tau^* + \alpha(\pi - \pi^*) - y^*) = 0, \quad (69)$$

$$\lambda(\pi - \pi^*) + \gamma\alpha(\tau - \tau^* + \alpha(\pi - \pi^*) - y^*) = 0. \quad (70)$$

Solving this system with unknown variables $\tau - \tau^*$ and $\pi - \pi^*$ leads to the following solution:

$$\tau - \tau^* = \frac{\gamma\lambda y^*}{\lambda + \gamma\alpha^2 + \lambda\gamma} \quad \text{and} \quad \pi - \pi^* = \frac{\gamma\alpha y^*}{\lambda + \gamma\alpha^2 + \lambda\gamma}. \quad (71)$$

Evaluating the loss function at this policy outcome:

$$\mathcal{L}^r(\cdot) = \frac{1}{2} \frac{\lambda\gamma(y^*)^2}{\lambda + \gamma\alpha^2 + \lambda\gamma}. \quad (72)$$

Altogether, we get:

$$\bar{\kappa}_2 = \frac{\gamma(y^*)^2}{2} \frac{\gamma(\lambda + \alpha^2)}{\lambda + \gamma\alpha^2 + \lambda\gamma}. \quad (73)$$

Comparative statics. From (73), derive:

$$\frac{d\bar{\kappa}_2}{d\lambda} = -\frac{\gamma(y^*)^2}{2} \frac{\gamma\alpha^2}{(\lambda + \gamma\alpha^2 + \lambda\gamma)^2} < 0 \quad ; \quad \frac{d\bar{\kappa}_2}{d\alpha} = \frac{\gamma(y^*)^2}{2} \frac{2\alpha\lambda\gamma}{(\lambda + \gamma\alpha^2 + \lambda\gamma)^2} > 0. \quad (74)$$

Proposition 3. We want to derive conditions on parameters (α, λ) , such that $\frac{\bar{\kappa}_1}{\bar{\kappa}_2} > 1$, where $\bar{\kappa}_1$ is given by (64) and $\bar{\kappa}_2$ in (73). Note $f(\alpha, \lambda) = \frac{\bar{\kappa}_1}{\bar{\kappa}_2}$ and derive:

$$f(\alpha, \lambda) = \frac{\alpha^2(1 + \gamma)(\lambda + \gamma\alpha^2)}{(\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2)} (\sqrt{1 + \gamma} + \sqrt{\gamma - \eta})^2, \quad (75)$$

with $\eta = \frac{\gamma\lambda}{\lambda + \gamma\alpha^2}$. Expanding the quadratic factor:

$$(\sqrt{1 + \gamma} + \sqrt{\gamma - \eta})^2 = \frac{\lambda + \gamma\lambda + \gamma\alpha^2}{\lambda + \gamma\alpha^2} + \frac{2(\gamma\alpha)^2}{\lambda + \gamma\alpha^2} + 2\gamma\alpha\sqrt{\frac{1 + \gamma}{\lambda + \gamma\alpha^2}}, \quad (76)$$

and get:

$$f(\alpha, \lambda) = \frac{\alpha^2(1 + \gamma)}{\lambda + \alpha^2} + \frac{2\gamma^2(1 + \gamma)\alpha^4}{(\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2)} + \frac{2\gamma(1 + \gamma)^{\frac{3}{2}}\alpha^3\sqrt{\lambda + \gamma\alpha^2}}{(\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2)}. \quad (77)$$

We immediately get:

$$f(0, \lambda) = 0 \quad \lim_{\alpha \rightarrow +\infty} f(\alpha, \lambda) > 1 + \gamma \quad (78)$$

$$f(\alpha, 0) > 1 + \gamma \quad \lim_{\lambda \rightarrow +\infty} f(\alpha, \lambda) = 0 \quad (79)$$

Note $g(\alpha, \lambda) = \frac{\alpha^2}{\lambda + \alpha^2}$. This function is increasing in α and decreasing in λ . Note $h(\alpha, \lambda) = \frac{\alpha^4}{(\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2)}$. This function is decreasing in λ . Further,

$$\frac{dh(\cdot)}{d\alpha} = \frac{\frac{dN}{d\alpha}D - \frac{dD}{d\alpha}N}{D^2} \quad (80)$$

with

$$N = \alpha^4 \quad \frac{dN}{d\alpha} = 4\alpha^3 \quad (81)$$

$$D = (\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2) \quad \frac{dD}{d\alpha} = 2\alpha(\lambda + 2\gamma\alpha^2 + 2\lambda\gamma) \quad (82)$$

$\frac{dh(\cdot)}{d\alpha}$ has the sign of H :

$$H = \frac{1}{2\alpha^3} \left(\frac{dN}{d\alpha} D - \frac{dD}{d\alpha} N \right) \quad (83)$$

$$= 2(\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2) - \alpha^2(\lambda + 2\gamma\alpha^2 + 2\lambda\gamma) \quad (84)$$

$$= 2\lambda^2 + \lambda\alpha^2 + \lambda^2\gamma + 2\lambda\gamma\alpha^2 > 0 \quad (85)$$

Note $k(\alpha, \lambda) = \frac{\alpha^2 \sqrt{\lambda + \gamma\alpha^2}}{(\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2)}$. Deriving the monotonicity properties:

$$\frac{dk(\cdot)}{d\alpha} = \frac{\frac{dN}{d\alpha} D - \frac{dD}{d\alpha} N}{D^2} \quad (86)$$

with

$$N = \alpha^3(\lambda + \gamma\alpha^2)^{\frac{1}{2}} \quad \frac{dN}{d\alpha} = \alpha^2(\lambda + \gamma\alpha^2)^{-\frac{1}{2}}(3\lambda + 3\gamma\alpha^2) \quad (87)$$

$$D = (\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2) \quad \frac{dD}{d\alpha} = 2\alpha(\lambda + 2\gamma\lambda + 2\gamma\alpha^2) \quad (88)$$

$\frac{dk(\cdot)}{d\alpha}$ has the sign of K :

$$K = \frac{(\lambda + \gamma\alpha^2)^{-\frac{1}{2}}}{\alpha^2} \left(\frac{dN}{d\alpha} D - \frac{dD}{d\alpha} N \right) \quad (89)$$

$$= (3\lambda + 4\gamma\alpha^2)(\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2) - 2\alpha^2(2\gamma\lambda + 2\gamma\alpha^2 + \lambda)(\lambda + \gamma\alpha^2) \quad (90)$$

$$= \lambda^2\alpha^2 + \gamma\lambda\alpha^4 + 4\lambda\gamma^2\alpha^4 + \lambda^2(3\lambda + 4\gamma\alpha^2) + 3\lambda^3\gamma + 6\lambda^2\gamma\alpha^2 > 0 \quad (91)$$

Overall:

$$\frac{df(\cdot)}{d\alpha} = (1 + \gamma)\frac{dg(\cdot)}{d\alpha} + 2\gamma^2(1 + \gamma)\frac{dh(\cdot)}{d\alpha} + 2\gamma(1 + \gamma)^{\frac{3}{2}}\frac{dk(\cdot)}{d\alpha} > 0, \quad (92)$$

which together with (78) gives:

$$\forall \lambda > 0 \exists \bar{\alpha} > 0 \text{ s.t. } \forall \alpha > \bar{\alpha}, \bar{\kappa}_1 > \bar{\kappa}_2. \quad (93)$$

Similarly,

$$\frac{dk(\cdot)}{d\lambda} = \frac{\frac{dN}{d\lambda} D - \frac{dD}{d\lambda} N}{D^2} \quad (94)$$

with

$$N = \alpha^3(\lambda + \gamma\alpha^2)^{\frac{1}{2}} \quad \frac{dN}{d\lambda} = \frac{\alpha^3}{2}(\lambda + \gamma\alpha^2)^{-\frac{1}{2}} \quad (95)$$

$$D = (\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2) \quad \frac{dD}{d\lambda} = \alpha^2 + 2(\lambda + \lambda\gamma + \gamma\alpha^2). \quad (96)$$

$\frac{dk(\cdot)}{d\lambda}$ has the sign of K :

$$K = \frac{2(\lambda + \gamma\alpha^2)^{-\frac{1}{2}}}{\alpha^3} \left(\frac{dN}{d\alpha} D - \frac{dD}{d\alpha} N \right) \quad (97)$$

$$= (\lambda + \lambda\gamma + \gamma\alpha^2)(\lambda + \alpha^2) - 2(\alpha^2 + 2(\lambda + \lambda\gamma + \gamma\alpha^2))(\lambda + \gamma\alpha^2) \quad (98)$$

$$= -(3\lambda^2 + \lambda\alpha^2 + 3\lambda^2\gamma + 6\lambda\gamma\alpha^2 + \gamma\alpha^4 + 4\lambda\gamma^2\alpha^2 + 4\gamma^2\alpha^4) < 0. \quad (99)$$

Overall:

$$\frac{df(\cdot)}{d\lambda} = (1 + \gamma) \frac{dg(\cdot)}{d\lambda} + 2\gamma^2(1 + \gamma) \frac{dh(\cdot)}{d\lambda} + 2\gamma(1 + \gamma)^{\frac{3}{2}} \frac{dk(\cdot)}{d\lambda} > 0, \quad (100)$$

which together with (79) gives:

$$\forall \alpha > 0 \exists \bar{\lambda} > 0 \text{ s.t. } \forall \lambda < \bar{\lambda}, \bar{\kappa}_1 > \bar{\kappa}_2. \quad (101)$$

A.4 Extensions

The linear-quadratic framework allows to investigate an additional set of questions. This section provides detailed exposition for the elements presented in Section 1.5.

A.4.1 Credibility and aggregate shocks

The presence of shocks ϵ to the Phillips curve generates a stabilization objective for policymakers, with state contingent policy choices. These shocks might influence the decision of the central bank to keep a promise $\pi^k(\mathcal{S})$ or renounce and reoptimize: is credibility state contingent?

Formally, output is influenced by the realization of a shock ϵ , whose CDF is noted $F(\epsilon)$:

$$y = \tau - \tau^e + \alpha(\pi - \pi^e) + \epsilon \quad (102)$$

The optimal policy under commitment solves:

$$\min_{\{\tau_\epsilon, \pi_\epsilon\}, e} \frac{1}{2} \int_{\epsilon} (\tau_\epsilon - \tau^*)^2 + \lambda(\pi_\epsilon - \pi^*)^2 + \gamma(y_\epsilon - y^*)^2 dF(\epsilon) \quad (103)$$

subject to the Phillips curve and private agent's rational expectations:

$$e = \int_{\epsilon} \tau_\epsilon + \alpha\pi_\epsilon dF(\epsilon) \quad (104)$$

The solution to this program yields:

$$\tau_\epsilon^c = \tau^* - \frac{\gamma\lambda}{\lambda + \gamma\alpha^2 + \lambda\gamma} \epsilon \quad ; \quad \pi_\epsilon^c = \pi^* - \frac{\gamma\alpha}{\lambda + \gamma\alpha^2 + \lambda\gamma} \epsilon \quad ; \quad e = \pi^* \quad (105)$$

Under a strategic rule, the central bank reacts to deviation τ from τ_ϵ^c given ϵ . Formally, set $\pi^k(\mathcal{S})$ such

that:

$$\mathcal{L}^{f,k}(e^*, \epsilon, \tau, \pi^k(\cdot)) = \mathcal{L}^{f,k}(e^*, \epsilon, \tau_\epsilon^c, \pi_\epsilon^c) \quad (106)$$

Given a realization of ϵ , credibility of the strategic rule is $\bar{\kappa}_2(\epsilon)$:

$$\bar{\kappa}_2(\epsilon) = \mathcal{L}^{m,k}(\pi^*, \epsilon, \tau_\epsilon^c, \pi_\epsilon^c) - \min_{\tau} \mathcal{L}^{m,r}(\pi^*, \epsilon, \tau, \pi^d(\mathcal{S})) \quad (107)$$

The solution to the reoptimization path yields:

$$\tau_\epsilon^d = \tau^* + \frac{\gamma\lambda(y^* - \epsilon)}{\lambda + \gamma\alpha^2 + \lambda\gamma} \quad ; \quad \pi_\epsilon^d = \pi^* + \frac{\gamma\alpha(y^* - \epsilon)}{\lambda + \gamma\alpha^2 + \lambda\gamma} \quad (108)$$

Rewrite $\bar{\kappa}_2(\epsilon)$ as

$$\bar{\kappa}_2(\epsilon) = \frac{1}{2} [(\tau_\epsilon^c - \tau^*)^2 + \lambda(\pi_\epsilon^c - \pi^*)^2 + \gamma(y_\epsilon^c - y^*)^2] - \frac{1}{2} [(\tau_\epsilon^d - \tau^*)^2 + \lambda(\pi_\epsilon^d - \pi^*)^2 + \gamma(y_\epsilon^d - y^*)^2] \quad (109)$$

and substitute to verify that $\frac{d\bar{\kappa}_2(\epsilon)}{d\epsilon} = 0$.

A.4.2 Credibility and policy-makers objective

Propositions 1, 2 and 3 are derived under the assumption that monetary and fiscal authorities, despite differences in the degree of commitment, make decisions using identical objective functions. We relax this assumption and consider two common adjustments of policy objectives: monetary *inflation conservatism* and *self-interest* of fiscal authority.

The analysis accommodates different objective functions in a straightforward way. Changes of monetary policy objective affect the credibility cut-off directly as it is based on losses as evaluated by the central bank:

$$\bar{\kappa} = \max_{\tau \in \mathbb{T}(\tilde{e})} \mathcal{L}^{m,k}(\tilde{e}, \tau, \pi^k(\tilde{e}, \tau)) - \mathcal{L}^{m,r}(\tilde{e}, \tau, \pi^r(\tilde{e}, \tau)), \quad (110)$$

where $\pi^r(\mathcal{S}) = \operatorname{argmin}_{\pi} \mathcal{L}^{m,r}(e, \tau, \pi)$, and $\mathcal{L}^m(\cdot)$ may reflect an objective function different from (2).

Changes of fiscal objective function affect directly the set of profitable fiscal deviations as it is based on the losses evaluated by the treasury:

$$\mathbb{T}(\tilde{e}) = \{ \tau \mid \mathcal{L}^{f,r}(\tilde{e}, \tau, \pi^r(\tilde{e}, \tau)) \leq \mathcal{L}^{f,k}(\tilde{e}, \tilde{\tau}, \pi^k(\tilde{e}, \tilde{\tau})) \}, \quad (111)$$

where $\mathcal{L}^f(\cdot)$ may reflect an objective function different from (2).

Monetary Conservatism. Conservatism is reflected in a higher weight that the central bank attaches to deviations of inflation from target π^* : $\tilde{\lambda} > \lambda$. The objective function of the fiscal authority is 2. We consider the effects of monetary conservatism on credibility cut-offs for both standard and strategic rules. A graphical illustration is presented on the left panel of Figure 4.

Under a standard rule, when the central bank is inflation conservative: $\tilde{\lambda} > \lambda$, the equilibrium outcome is (10). The set of profitable fiscal deviations reads:

$$\mathbb{T}(e_1) = \{\tau \text{ s.t. } \mathcal{L}^{f,r}(e_1, \tau, \pi^r(\cdot)) \leq \mathcal{L}^{f,k}(e_1, \tau_1, \pi^*)\}. \quad (112)$$

The credibility cut-off is then:

$$\bar{\kappa}_1 = \max_{\tau \in \mathbb{T}(e_1)} \mathcal{L}^{m,k}(e_1, \tau, \pi^*) - \mathcal{L}^{m,r}(e_1, \tau, \pi^r(\cdot)). \quad (113)$$

A similar adjustment applies to derive the credibility cut-off under strategic rule.

Under the standard rule, stronger conservatism unambiguously reduces the degree of commitment required to implement the monetary target π^* . This reduction of $\bar{\kappa}_1$ is driven by the decline in relative gains from renouncing the inflation target: indeed, a more conservative central banker renouncing the rule would implement a lower monetary stimulus, yielding lower welfare gains. This effect is well understood in the literature and is also present under the strategic rule. However, under the strategic rule, stronger conservatism also makes off-equilibrium threats more costly to the central bank. The latter effect dominates at higher levels of conservatism, which makes the credibility cut-off $\bar{\kappa}_2$ eventually increase as $\tilde{\lambda}$ goes up.

Fiscal Self-interest. Consider implications of self-interest that increases the short-term temptation of the fiscal authority to stimulate the economy. It is reflected in a higher weight that the treasury attaches to deviations of output from the first-best level, $\tilde{\gamma} > \gamma$. The monetary policy objective is kept as in the baseline analysis. The right panel of Figure 4 provides a graphical illustration of the results.

Under a standard rule, the equilibrium outcome if the inflation target is credible is:

$$\tau_1 = \tau^* + \tilde{\gamma}y^* \quad \pi_1 = \pi^* \quad e_1 = \tau^* + \tilde{\gamma}y^* + \alpha\pi^* \quad y_1 = 0, \quad (114)$$

where $\tilde{\gamma}$ is the preference parameter of the fiscal authority. The set of profitable fiscal deviations is:

$$\mathbb{T}(e_1) = \{\tau \text{ s.t. } \mathcal{L}^{f,r}(e_1, \tau, \pi^r(\cdot)) \leq \mathcal{L}^{f,k}(e_1, \tau_1, \pi^*)\}. \quad (115)$$

The credibility cut-off is then:

$$\bar{\kappa}_1 = \max_{\tau \in \mathbb{T}(e_1)} \mathcal{L}^{m,k}(e_1, \tau, \pi^*) - \mathcal{L}^{m,r}(e_1, \tau, \pi^r(\cdot)). \quad (116)$$

A similar adjustment applies to derive the credibility cut-off under strategic rule.

The key result is that the credibility cut-off of the strategic monetary rule does not depend on the strength of fiscal self-interest. The strategic monetary rule $\pi^k(\mathcal{S})$ does change with $\tilde{\gamma}$ to adjust for the changing incentives of the treasury, as is the set of profitable fiscal deviations. However, the strategic rule is designed in such a way that the off-equilibrium fiscal deviation that pins down the credibility cut-off remains unaffected by $\tilde{\gamma}$. In contrast, under the standard rule the pivotal off-equilibrium path varies with the level

of fiscal self-interest.⁶⁶ Hence, the credibility cut-off corresponding to the standard rule is sensitive to fiscal self-interest $\tilde{\gamma}$.

A.4.3 Credibility and inflation target

Consider credibility of a strategic rule where the central bank targets an inflation rate different $\pi^s > \pi^*$, with the following desired equilibrium.

$$\tau_2 = \tau^* \quad \pi_2 = \pi^s > \pi^* \quad e_2 = \tau^* + \alpha\pi^s \quad y_2 = 0. \quad (117)$$

Following the exposition in Section 1.4.2, such a rule is designed to provide incentives to the fiscal authority:

$$\forall \tau \in \mathbb{T}(e_2), \mathcal{L}^{f,k}(e_2, \tau, \pi^k(e_2, \tau)) = \mathcal{L}^{f,k}(e_2, \tau^*, \pi^s), \quad (118)$$

where $\mathbb{T}(e_2)$ is defined as in (8), adjusted for the equilibrium inflation target π^s . The equilibrium monetary target $\pi^s > \pi^*$ generates a systematic welfare loss, but lowers the commitment intensity $\bar{\kappa}_2(\pi^s)$ required to eliminate the fiscal bias and implement (117). Formally,

$$\left. \frac{d\bar{\kappa}_2(\cdot)}{d\pi^s} \right|_{\pi^s=\pi^*} < 0. \quad (119)$$

The intuition is straightforward: by targeting an inflation rate higher than π^* , the central bank reduces the relative gains from renouncing $\pi^k(\mathcal{S})$, which in turn decreases the degree of commitment required to support this strategy in equilibrium.

The formal derivation of (119) follows Appendix A.3 with $e_2 = \tau^* + \alpha\pi^s$ and $\pi^s > \pi^*$:

$$\bar{\kappa}_2(\pi^s) = \mathcal{L}^{m,k}(e_2, \tau^*, \pi^s) - \min_{\tau, \pi} \mathcal{L}^{m,r}(e_2, \tau, \pi), \quad (120)$$

where

$$\mathcal{L}^{m,k}(e_2, \tau^*, \pi^s) = \frac{1}{2} [\lambda(\pi^s - \pi^*)^2 + \gamma(y^*)^2] \quad (121)$$

Consider the optimization program $\min_{\tau, \pi} \mathcal{L}^{m,r}(e_2, \tau, \pi)$. The first order conditions:

$$\tau - \tau^* + \gamma(\tau + \alpha\pi - e_2 - y^*) = 0 \quad (122)$$

$$\lambda(\pi - \pi^*) + \gamma\alpha(\tau + \alpha\pi - e_2 - y^*) = 0 \quad (123)$$

Get $\tau - \tau^* = \frac{\lambda}{\alpha}(\pi - \pi^*)$ and then derive:

$$(\lambda + \lambda\gamma + \gamma\alpha^2)\pi = \lambda(1 + \gamma)\pi^* + \gamma\alpha^2\pi^s + \gamma\alpha y^* \quad (124)$$

⁶⁶In particular, an increase in $\tilde{\gamma}$ yields a higher fiscal bias. For the central bank then, the relative gains to renouncing the rule and stimulate output are lower, hence the downward slopping shape of $\bar{\kappa}_1(\cdot)$.

which rewrites:

$$\pi - \pi^* = \frac{\gamma\alpha(\alpha(\pi^s - \pi^*) + y^*)}{\lambda + \lambda\gamma + \gamma\alpha^2} \quad (125)$$

so that the loss function under *renege* is:

$$\mathcal{L}^r(\cdot) = \frac{\gamma\lambda}{2} \frac{(\alpha(\pi^s - \pi^*) + y^*)^2}{\lambda + \lambda\gamma + \gamma\alpha^2} \quad (126)$$

and

$$\bar{\kappa}_2(\pi^s) = \frac{1}{2} \left[\lambda(\pi^s - \pi^*)^2 + \gamma(y^*)^2 - \frac{\gamma\lambda}{\lambda + \lambda\gamma + \gamma\alpha^2} (\alpha(\pi^s - \pi^*) + y^*)^2 \right] \quad (127)$$

Finally:

$$\left. \frac{d\bar{\kappa}_2(\pi^s)}{d\pi^s} \right|_{\pi^s=\pi^*} = -\frac{\gamma\lambda y^*}{\lambda + \lambda\gamma + \gamma\alpha^2} < 0. \quad (128)$$

A.4.4 Credibility and reputation

Reputation—or trigger-type equilibria—is a common construction to support commitment.⁶⁷ Consider an infinite repetition of the game described before. The objective of the central bank is to implement (11) by following a strategic rule $\pi^k(\mathcal{S})$ of the type derived in Section 1.4.2, without the exogenous cost κ in case of renouncement to the rule. Credibility is derived from the long run consequences of renouncing the rule, namely the infinite repetition of the discretionary equilibrium outcome (6).

Let $T(e^*)$ be the set of profitable deviations, as in (8). The central bank follows $\pi^k(e, \tau)$ given by (11) and (13) for all $\tau \in T(e^*)$ if $e = e^*$. Otherwise, if $e \neq e^*$ or $\tau \notin T(e^*)$, it implements $\pi^r(e, \tau)$. In turn, the process for private households expectations keep track of the history of central bank's decisions:

$$\begin{aligned} e = e^* & \quad \text{if} \quad \pi_{-1} = \pi^k(e_{-1}, \tau_{-1}) \\ e = e^d & \quad \text{if} \quad \pi_{-1} = \pi^r(e_{-1}, \tau_{-1}) \end{aligned} \quad (129)$$

In other terms, if the central bank implemented $\pi^r(\mathcal{S})$ once in the past, private agent expectations carry this information.⁶⁸ The strategic rule $\pi^k(\mathcal{S})$ implements the equilibrium outcome (11) if and only if:

$$\forall \tau \in T(e^*) \quad \underbrace{\Delta(\tau) = \mathcal{L}(e^*, \tau, \pi^k(\mathcal{S})) - \mathcal{L}(e^*, \tau, \pi^r(\mathcal{S}))}_{\text{Short Term Gain}} - \underbrace{\frac{\beta}{1-\beta} [\mathcal{L}(e^d, \tau^d, \pi^d) - \mathcal{L}(e^*, \tau^*, \pi^*)]}_{\text{Long Term Cost}} \leq 0, \quad (130)$$

where β is a discount factor. Under this construction, deviating from the rule yields a short term gain, which is evaluated against the cost of an infinite repetition of the discretionary equilibrium outcome (6). This long term cost component maps precisely into κ , the cost incurred by the central bank when breaking a promise.

⁶⁷Various versions of this construction are discussed in Barro and Gordon (1983), Stokey (1989) or Chari and Kehoe (1990).

⁶⁸Finite forms of punishment are also possible.

It must be high enough to deter the short run incentives to renounce the rule, i.e. it must be higher than the cut-off value $\bar{\kappa}$, associated to the short term gain in (130).

A.4.5 Private agents expectations

As mentionned in Section 1.3, our evaluation of credibility focuses on fiscal-monetary interactions. In other terms, private agents' expectations are assumed anchored to the equilibrium path. As such, variations in private agents' expectations cannot threaten the implementation of a strategy.

In this section, we highlight that a simple additional term in the central bank rule can overrule any concerns related to private agents' expectations. Consider a simplified game between a central bank and private agents. First, the central bank commits to π^* . Then private agents form expectations π^e and finally the central bank:

- either implements π^* , output is $y = \pi^* - \pi^e$ and the loss to the economy is

$$\mathcal{L}(\cdot) = \frac{1}{2} [(\pi - \pi^*)^2 + \gamma(y - y^*)^2], \quad (131)$$

- or renounces its promise, pays a welfare cost κ and implements

$$\pi^r = \operatorname{argmin}_{\pi} \frac{1}{2} [(\pi - \pi^*)^2 + \gamma(\pi - \pi^e - y^*)^2]. \quad (132)$$

There are two possible equilibrium paths, related to the degree of commitment κ :

- if private agents form $\pi^e = \pi^*$, then the central bank implements $\pi = \pi^*$ if and only if $\kappa \geq \bar{\kappa}$.
- if private agents form $\pi^e = \pi^d = \pi^* + \gamma y^*$, then the central bank renounces π^* and implements π^d if and only if $\kappa \leq \bar{\kappa}$.

Importantly, $\bar{\kappa} \leq \bar{\bar{\kappa}}$, which means that for intermediate values of $\kappa \in [\bar{\kappa}, \bar{\bar{\kappa}}]$, the credibility of the monetary target π^* depends on self-fulfilling variations in private agents' expectations. To avoid this strategic uncertainty, the central bank needs to modify its rule as follow:

$$\pi^s(\pi^e) = \pi^* + \alpha(\pi^e - \pi^*) \quad \text{with } \alpha \in (0, 1) \quad (133)$$

Under this strategy, if $\kappa \leq \bar{\kappa}$, then the only equilibrium path is one where $\pi^e = \pi^*$.

B Cash-Credit Economy

B.1 Policy representation of a competitive equilibrium

Proof of Lemma 1. Start by proving necessity. First, combine the resource constraint (14), binding cash-in-advance constraint and market clearing conditions to rewrite the household's budget constraint (16) in real terms, after dividing it by M_{t-1} :

$$z_t(1 + \sigma_t)q_t + (1 + \sigma_t) - \left(\frac{1 - \tau_t}{d_t}\right)g + \tau_t\left(1 + \frac{c_t}{d_t}\right) - 1 = z_{t-1},$$

where $z_t \equiv B_t/M_t$. Second, using household's optimality conditions (21) to (23):

$$\beta \left[\frac{(1 - \alpha)\beta}{1 + \sigma_{t+1}} \right] z_t - \alpha(1 - \tau_t) - (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} + \Phi = \left[\frac{(1 - \alpha)\beta}{1 + \sigma_t} \right] z_{t-1},$$

where $\Phi \equiv (\beta(1 - \alpha) + \alpha - \gamma g)$.

To prove sufficiency, consider a sequence $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$ of policy instruments that satisfies implementability constraints (25) and let $\{B_t, M_t\}_{t=0}^{\infty}$ be the associated paths of government nominal liabilities. We derive a sequence of quantities and prices that satisfy Definition (3). Let $B_t^h = B_t$ and $M_t^h = M_t$ for all $t \geq 0$, cash good consumption sequence $\{d_t\}_{t=0}^{\infty}$ satisfies

$$d_t = \frac{\beta(1 - \alpha)}{\gamma} \frac{(1 - \tau_t)}{(1 + \sigma_t)}, \quad (134)$$

credit good consumption $\{c_t\}_{t=0}^{\infty}$ and leisure $\{l_t\}_{t=0}^{\infty}$ be given by (21) and (14). Also, let bond prices $\{q_t\}_{t=0}^{\infty}$ be given by $q_t = \beta/(1 + \sigma_{t+1})$. With this construction, (22) and (23) are satisfied and the sequence of implementability constraints (25) implies that (16) and (19) are satisfied. Hence, all conditions of a competitive equilibrium are met by these sequences.

Similar elements allow to derive (26) for real debt.

Proof of Lemma 2. Use the resource constraint (14) to substitute leisure into the utility function:

$$U(c_t, d_t) = \alpha \log(c_t) - \gamma c_t + (1 - \alpha) \log(d_t) - \gamma d_t,$$

where constant terms independent of policy are discarded, without loss of generality. Next, substitute c_t and d_t with policy instrument choices τ_t and σ_t with (21) and (134):

$$U(\tau_t, \sigma_t) = \alpha [\log(1 - \tau_t) - (1 - \tau_t)] + (1 - \alpha) \left[\log \left(\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right) - \beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right].$$

B.2 Ramsey equilibrium: proof of Proposition 4.

Nominal debt. The Ramsey policy problem determines a whole sequence of policy instruments to maximize households welfare subject to the implementability constraints, given $z_{-1} > 0$:

$$\begin{aligned} \max_{\{\tau_t, \sigma_t\}} & \sum_{t=0}^{\infty} \beta^t \left\{ \alpha [\log(1 - \tau_t) - (1 - \tau_t)] + (1 - \alpha) \left[\log \left(\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right) - \beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right] \right\} \\ \text{subject to} & \sum_{t=0}^{\infty} \beta^t \left\{ \Phi - \alpha(1 - \tau_t) - (1 - \alpha) \beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right\} = \left[\frac{(1 - \alpha)\beta}{(1 + \sigma_0)} \right] z_{-1}, \end{aligned} \quad (135)$$

where the intertemporal implementability constraint (135) is derived by forward substitution of (25) in the absence of Ponzi schemes. The first-order conditions read as follows:

$$1 = (1 + \lambda) \left[\alpha(1 - \tau_t) + (1 - \alpha) \beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right], \quad \forall t \geq 0, \quad (136)$$

$$1 = (1 + \lambda) \beta \frac{(1 - \tau_t)}{(1 + \sigma_t)}, \quad \forall t \geq 1, \quad (137)$$

$$1 = (1 + \lambda) \beta \frac{(1 - \tau_0)}{(1 + \sigma_0)} + \lambda \frac{\beta}{(1 + \sigma_0)} z_{-1}, \quad (138)$$

where $\lambda \geq 0$ is the Lagrange multiplier attached to (135). (136) and (137) imply $\tau_t = \bar{\tau} \equiv \lambda/(1 + \lambda)$ and $\sigma_t = \bar{\sigma} \equiv \beta - 1$ for all $t \geq 1$. Using (21), (134) and (14), it is straightforward to get that consumption of both goods, leisure and debt-to-money ratio are constant for all $t \geq 1$, with

$$z_t = \bar{z} \equiv \frac{1}{(1 - \alpha)(1 - \beta)} \left[\Phi - \frac{1}{1 + \lambda} \right], \quad t \geq 0. \quad (139)$$

Contrasting optimality conditions for $t = 0$ and $t \geq 1$ yields $\tau_0 < \bar{\tau}$ and $\sigma_0 > \bar{\sigma}$. Finally, substituting (136) into (135) gives $\bar{z} = \frac{\beta}{1 + \sigma_0} z_{-1}$, i.e. if $z_{-1} > 0$ then $\bar{z} < z_{-1}$.

Real debt. Under real debt, the right hand side of the intertemporal implementability condition (135) is $\frac{\gamma}{1 - \tau_0} b_{-1}$. Let $\lambda > 0$ be the associated Lagrange multiplier. (136) holds for all $t \geq 1$, (137) holds for all $t \geq 0$. Replace (138) with:

$$1 = (1 + \lambda) \left[\alpha(1 - \tau_0) + (1 - \alpha) \frac{\beta(1 - \tau_0)}{1 + \sigma_0} \right] - \lambda \frac{\gamma}{(1 - \tau_0)} b_{-1}. \quad (140)$$

Stationary real debt level reads:

$$b_t = \bar{b} \equiv \frac{1}{\gamma(1 - \beta)(1 + \lambda)} \left[\Phi - \frac{1}{1 + \lambda} \right], \quad t \geq 0. \quad (141)$$

Use (136) and (140) into the intertemporal implementability condition and get:

$$\bar{b} = \underbrace{\left[\frac{1 + 2\lambda}{1 + \lambda} \right]}_{>1} \underbrace{\left[\frac{\beta}{1 + \sigma_0} \right]}_{<1} b_{-1}. \quad (142)$$

To characterize the relation between \bar{b} and b_{-1} , let's consider non negative level of debt such that $\bar{b} = b_{-1}$ from (142).⁶⁹ Obviously, $b_{-1} = 0$ is a fixed point, associated with Lagrange multiplier $\lambda = 1/\Phi - 1$. Next, we show there is unique $b^p > 0$ such that if $b_{-1} = b^p$, then $\bar{b} = b^p$. Equation (142) implies :

$$\left[\frac{1 + 2\lambda^p}{1 + \lambda^p} \right] \left[\frac{\beta}{1 + \sigma_0} \right] = 1. \quad (143)$$

Using (143) and (137), rewrite (140) to characterize b^p using the associated Lagrange multiplier λ^p :

$$b^p = \frac{\alpha (1 + 2\lambda^p)}{\gamma (1 + \lambda^p)^3} > 0, \quad (144)$$

which together with the definition of \bar{b} (141) implies that the Lagrange multiplier λ^p is a solution to

$$(1 + \lambda)[\Phi(1 + \lambda) - 1] - \alpha(1 - \beta)(1 + 2\lambda) = 0. \quad (145)$$

This expression is quadratic in λ with one negative and one positive root. The positive root is an admissible solution, which implies existence of a unique positive debt fixed point $b^p > 0$. Further, $\lambda^p > 1/\Phi - 1$ from (141).

Finally, we need to show that $\bar{b} > b_{-1} > 0$ if and only if $0 < b_{-1} < b^p$. Following the steps (142) to (144), one gets that $\bar{b} > b_{-1}$ for all $b_{-1} > 0$ if and only if $b_{-1} < \frac{\alpha (1+2\lambda)}{\gamma (1+\lambda)^3}$. As λ is increasing in b_{-1} and $\frac{\alpha (1+2\lambda)}{\gamma (1+\lambda)^3}$ is decreasing in λ , we get that $\bar{b} > b_{-1}$ for lower values of b_{-1} , i.e. for $b_{-1} \in (0, b^p)$.

B.3 Joint government, without commitment

Definition. Consider the economy with nominal debt. A Markov Perfect equilibrium is a triplet of functions $\{\tau(z_{-1}), \sigma(z_{-1}), z(z_{-1})\}$ and value function $V(z_{-1})$ that solves:

$$V(z_{t-1}) = \max_{\tau_t, \sigma_t, z_t} \left\{ \alpha [\log(1 - \tau_t) - (1 - \tau_t)] + (1 - \alpha) \left[\log \left(\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right) - \beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right] + \beta V(z_t) \right\},$$

subject to

$$\beta \left[\frac{(1 - \alpha)\beta}{1 + \sigma(z_t)} \right] z_t - \alpha(1 - \tau_t) - (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} + \Phi = \left[\frac{(1 - \alpha)\beta}{1 + \sigma_t} \right] z_{t-1}. \quad (146)$$

A differentiable Markov-Perfect equilibrium satisfies (146) and the following optimality conditions:

$$1 = (1 + \theta_t) \left[\alpha(1 - \tau_t) + (1 - \alpha)\beta \frac{(1 - \tau_t)}{1 + \sigma_t} \right], \quad (147)$$

$$1 = (1 + \theta_t) \beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} + \theta_t \frac{\beta}{1 + \sigma_t} z_{t-1}, \quad (148)$$

$$0 = \theta_{t+1} - \theta_t \left[1 - \frac{z_t}{(1 + \sigma_{t+1})} \frac{d\sigma(z_t)}{dz_t} \right], \quad (149)$$

where $\theta_t \geq 0$ is the Lagrange multiplier attached to (146).

⁶⁹There is also a negative debt fixed point such that $\lambda = 0$ and the implied allocation coincides with the first-best.

Long run convergence. In steady state, equation (149):

$$0 = \bar{\theta} \frac{\bar{z}}{(1 + \bar{\sigma})} \frac{d\sigma(\bar{z})}{dz}, \quad (150)$$

which indicates the possible existence of different steady states. Steady state properties and transition dynamics depend on the derivative $d\sigma(z_t)/dz_t$.⁷⁰ Our numerical simulations display an equilibrium which converges to a zero debt steady state, $\bar{z} = 0$.

Real debt. Constraint (146) is replaced by (26), state variable is b_{t-1} instead of z_{t-1} , and the optimality conditions (147)–(149) for a differentiable equilibrium:

$$1 = (1 + \theta_t) \left[\alpha(1 - \tau_t) + (1 - \alpha)\beta \frac{(1 - \tau_t)}{1 + \sigma_t} \right] - \theta_t \frac{\gamma}{(1 - \tau_t)} b_{t-1}, \quad (151)$$

$$1 = (1 + \theta_t) \beta \frac{(1 - \tau_t)}{(1 + \sigma_t)}, \quad (152)$$

$$0 = \theta_{t+1} - \theta_t \left[1 + \frac{b_t}{(1 - \tau_{t+1})} \frac{d\tau(b_t)}{db_t} \right]. \quad (153)$$

In steady state, equation (153):

$$0 = \bar{\theta} \frac{\bar{b}}{(1 - \bar{\tau})} \frac{d\tau(\bar{b})}{db}.$$

As with nominal debt, our numerical simulations feature the steady state with zero debt, $\bar{b} = 0$.

Numerical solution. The model is solved using value function iteration to search for a fixed point of the value function and associated decision rules.

1. Define a grid over s and guess equilibrium functions V^0, τ^0, σ^0 .
2. For every grid point s_{t-1} , solve the optimization problem of the government

$$\max_{\tau_t, \sigma_t, s_t} \left\{ \alpha \left[\log(1 - \tau_t) - (1 - \tau_t) \right] + (1 - \alpha) \left[\log \left(\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right) - \beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right] + \beta V^0(s_t) \right\},$$

subject to

$$0 = f(s_t, s_{t-1}, \sigma^0(s_t), \sigma_t, \tau^0(s_t), \tau_t).$$

Call the decision rules that solve this problem (τ^1, σ^1, s^1) and let

$$\begin{aligned} V^1(s_{t-1}) &= \alpha \left[\log(1 - \tau^1(s_{t-1})) - (1 - \tau^1(s_{t-1})) \right] \\ &\quad + (1 - \alpha) \left[\log \left(\beta \frac{(1 - \tau^1(s_{t-1}))}{(1 + \sigma^1(s_{t-1}))} \right) - \beta \frac{(1 - \tau^1(s_{t-1}))}{(1 + \sigma^1(s_{t-1}))} \right] + \beta V^0(s^1(s_{t-1})). \end{aligned}$$

3. Check convergence of the decision rules and the value function. If convergence is above a desired threshold, set $V^0 = V^1, \tau^0 = \tau^1, \sigma^0 = \sigma^1$ and return to step 2.

⁷⁰This feature echoes the analysis of fiscal policy under discretion with real debt in Debortoli and Nunes (2013).

The algorithm relies on interpolating the value function and the decision rules off the grid points. The results presented in the paper are based on the implementation of the algorithm in Matlab. Equilibrium functions are interpolated using the cubic splines routine from the CompEcon toolbox; see Miranda and Fackler (2002). The optimization problem of the government is solved using IPOPT, an open source nonlinear optimization solver, in the OPTI toolbox; see Currie and Wilson (2012). Computation speed is improved by parallelizing the step of solving the optimization problem on the grid.

B.4 Nominal debt, standard rule

Proof of Proposition 5. The monetary authority with commitment follows a constant money growth rate rule: $\sigma_t = \sigma \geq \beta - 1$. The Markov-Perfect equilibrium of the game is associated with a solution of the following dynamic programming problem of the fiscal authority:

$$V^F(z_{t-1}) = \max_{\tau_t, z_t} \left\{ \alpha [\log(1 - \tau_t) - (1 - \tau_t)] + (1 - \alpha) \left[\log \left(\beta \frac{(1 - \tau_t)}{(1 + \sigma)} \right) - \beta \frac{(1 - \tau_t)}{(1 + \sigma)} \right] + \beta V^F(z_t) \right\},$$

subject to

$$\beta \left[\frac{(1 - \alpha)\beta}{1 + \sigma} \right] z_t - \alpha(1 - \tau_t) - (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma)} + \Phi = \left[\frac{(1 - \alpha)\beta}{1 + \sigma} \right] z_{t-1}. \quad (154)$$

The first-order envelope conditions result in the following expressions for all $t \geq 0$:

$$1 = (1 + \theta_t) \left[\alpha(1 - \tau_t) + (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma)} \right], \quad (155)$$

$$0 = \theta_{t+1} - \theta_t, \quad (156)$$

where $\theta_t \geq 0$ is the Lagrange multiplier associated with (154). (156) implies $\theta_t = \bar{\theta}$ for all $t \geq 0$. Hence, starting from $t = 0$, $\tau_t = \bar{\tau}$ as can be seen from (155) and the equilibrium allocation is constant as per characterization from Lemma 1. Importantly, using the implementability constraint (154) we get that the debt-to-money ratio is equal to its initial outstanding value, i.e., $\bar{z} = z_{-1}$.

Proof of Corollary 1. When $\sigma = \beta - 1$, conditions (155) and (156) imply $\tau_t = \bar{\tau} \equiv \bar{\theta}/(1 + \bar{\theta})$. The Lagrange multiplier $\bar{\theta} \geq 0$, in turn, is implicitly determined by the following equation derived from (154):

$$z_{-1} = \frac{1}{(1 - \alpha)(1 - \beta)} \left[\Phi - \frac{1}{1 + \bar{\theta}} \right]. \quad (157)$$

This characterization coincides with the “timeless allocation” induced by the choice of a joint government under commitment for $t \geq 1$, as described in Proposition 5. In particular, comparing (157) and (139) shows that $\bar{\theta} = \lambda$ when z_{-1} is equal to the long-run debt-to-money ratio in a Ramsey equilibrium. The corresponding tax rate and money growth rates also match. Therefore, the equilibrium of the game described in Definition (4) coincides with the stationary “timeless” allocation of a Ramsey plan with stationary level of debt $\bar{z} = z_{-1}$.

Real debt. Constraint (154) is replaced by (26), the state variable is b_{t-1} instead of z_{t-1} , and the optimality conditions (155)–(156):

$$1 = (1 + \theta_t) \left[\alpha(1 - \tau_t) + (1 - \alpha)\beta \frac{(1 - \tau_t)}{1 + \sigma_t} \right] - \theta_t \frac{\gamma}{(1 - \tau_t)} b_{t-1}, \quad (158)$$

$$0 = \theta_{t+1} - \theta_t \left[1 + \frac{b_t}{(1 - \tau_{t+1})} \frac{d\tau(b_t)}{db_t} \right]. \quad (159)$$

In contrast to the nominal debt case, the implied equilibrium allocation is not stationary whenever $b_{-1} > 0$.⁷¹

B.5 Strategic rule - Proposition 6

Monetary rule ϱ^{Mk} . For all $s_{t-1} \geq 0$,

- if $\tau_t = \tau^*(s_{t-1})$, then $\varrho^{Mk}(s_{t-1}, \tau_t) = \beta - 1$,
- if $\tau_t \in \mathbb{T}(s_{t-1})$ and $\tau_t \neq \tau^*(s_{t-1})$, then $\varrho^{Mk}(s_{t-1}, \tau_t) = \sigma$ such that:

$$V^{Fk}(s_{t-1}, \tau_t) = W^{ta}(s_{t-1}), \quad (160)$$

where

$$\mathbb{T}(s_{t-1}) = \left\{ \tau \mid V^{Fr}(s_{t-1}, \tau) \geq V^{Fk}(s_{t-1}, \tau^*(s_{t-1})) = W^{ta}(s_{t-1}) \right\}. \quad (161)$$

Equilibrium path if credible. Given $s_{-1} > 0$, if credible, the strategic monetary rule is designed to implement on equilibrium the timeless allocation, i.e. the solution to the game induces the following dynamic path of policy choices:

$$\tau_t = \tau^*(s_{-1}) \quad \sigma_t = \beta - 1 \quad s_t = s_{-1}, \quad \forall t \geq 0, \quad (162)$$

where $\tau^*(s_{-1})$ solves a stationary (25), or (26) if debt is real. In that case, the welfare to both monetary and fiscal authorities is $W^{ta}(s_{-1})$, given by (30).

Existence of monetary threats. To prove existence, consider the case of nominal debt and the following sequential policy decision: the fiscal authority chooses $\tau \in \mathbb{T}(s_{-1})$, then the central bank can:

- implement the equilibrium money growth rate $\sigma^* = \beta - 1$, in which case the welfare to the fiscal authority is lower than at the desired equilibrium path by virtue of Proposition 5:

$$\forall \tau, V^F(\tau, \sigma^*, z_{-1}) \leq W^{ta}(\tau^*, \sigma^*, z_{-1}),$$

⁷¹While this characterization is similar to that of the jointly optimal policy under discretion, the lack of adjustment in money growth rate changes the derivative $d\tau(b_t)/db_t$.

- or it can optimize and implement $\sigma^r(\tau, z_{-1}) \geq \sigma^*$, in which case by definition of $T(z_{-1})$:⁷²

$$\forall \tau \in T(z_{-1}), V^F(\tau, \sigma^r(\cdot), z_{-1}) \geq W^{ta}(\tau^*, \sigma^*, z_{-1}).$$

By continuity, there is $\sigma \in [\sigma^*, \sigma^d]$ s.t.:

$$\forall \tau \in T(z_{-1}), V^{Fk}(\tau, \sigma, z_{-1}) = W^{ta}(\tau^*, \sigma^*, z_{-1})$$

This expression implicitly defines $\varrho^{Mk}(z_{-1}, \tau)$ for $\tau \in T(z_{-1})$.

Characterization of credibility. This section follows Section 2.3.1 to characterize the commitment intensity required to implement the strategic monetary rule given s_{-1} . By construction, the welfare to the monetary authority (on and off equilibrium) conditional on the central bank *keeps* its rule

$$V^{Mk}(s_{t-1}, \tau_t) = W^{ta}(s_{t-1}), \quad (163)$$

and the welfare (off equilibrium) conditional on central bank *renege*s its rule

$$V^{Mr}(s_{t-1}, \tau_t) = \max_{\sigma, s} U(\tau, \sigma) + \beta V^M(s, \varrho^F(\cdot)). \quad (164)$$

The rule is credible at (s_{t-1}, τ_t) if and only if

$$\kappa \geq \Delta(s_{t-1}, \tau_t) = V^{Mr}(s_{t-1}, \tau_t) - V^{Mk}(s_{t-1}, \tau_t) = V^{Mr}(s_{t-1}, \tau_t) - W^{ta}(s_{t-1}), \quad (165)$$

and at s_{t-1} if and only if

$$\kappa \geq \max_{\tau_t \in T(s_{t-1})} \Delta(s_{t-1}, \tau_t) \quad (166)$$

In particular

$$\max_{\tau_t} V^{Mr}(s_{t-1}, \tau_t) = \max_{\tau_t} \max_{\sigma_t, s_t} U(\tau_t, \sigma_t) + \beta V^M(s_t, \varrho^F(\cdot)) \quad (167)$$

$$\leq W^{ra}(s_t) = \max_{\tau_t, \sigma_t, s_t} U(\tau_t, \sigma_t) + \beta W^{ta}(s_t) \quad (168)$$

where the inequality comes from the fact that the highest welfare that can be reached given s_{t-1} is induced by a Ramsey plan, see Proposition 4.

We form and then verify the following conjecture: if ϱ^{Mk} is credible at $s_{-1} \geq 0$, then it is credible at $0 \leq s'_{-1} \leq s_{-1}$. Under the conjecture, and if the continuation debt level s_t of a Ramsey plan given s_{t-1}

⁷²Formally, $\sigma^r(\cdot) = \operatorname{argmax}_{\sigma} V^M(\cdot)$.

satisfies $s_t \leq s_{t-1}$,

$$\max_{\tau_t} V^{Mr}(s_{t-1}, \tau_t) = W^{ra}(s_{t-1}). \quad (169)$$

Indeed, (i) $\tau^{ra}(s_{t-1})$ the “initial” Ramsey deviation belongs to $T(s_{t-1})$. (ii) given $\tau^{ra}(s_{t-1})$, the maximum welfare to the monetary authority is reached if the central bank reoptimizes, implements $\sigma^{ra}(s_{t-1})$ and induces $s_t = s^{ra}(s_{t-1})$ with continuation welfare $W^{ta}(s_t)$. (iii) if the rule is credible today, then the timeless allocation is credible at s_t if $s_t < s_{t-1}$, in which case the continuation welfare in (167) is $V^M(s_t, \varrho^F(s_t)) = W^{ta}(s_t)$ since under these conditions $\varrho^F(s_t) = \tau^{ta}(s_t)$. Overall, the strategic rule is credible at s_{t-1} if and only if:

$$\kappa \geq \tilde{\kappa}(s_{-1}) = W^{ra}(s_{t-1}) - W^{ta}(s_{t-1}), \quad (170)$$

and the continuation level of debt under the Ramsey plan is $s_t \leq s_{t-1}$. The credibility cut-off writes then:

$$\bar{\kappa}_2(s_{t-1}) = \tilde{\kappa}(s_{-1}) = W^{ra}(s_{t-1}) - W^{ta}(s_{t-1}). \quad (171)$$

If the continuation level of debt under a Ramsey plan is $s_t > s_{t-1}$, as in (42), then the inequality (168) is strict since the timeless allocation (off equilibrium tomorrow) under s_t might not be induced by the central bank given its commitment intensity κ . In that case:

$$\bar{\kappa}_2(s_{t-1}) < \tilde{\kappa}(s_{-1}) = W^{ra}(s_{t-1}) - W^{ta}(s_{t-1}). \quad (172)$$

Verify the conjecture. Consider the following program:

$$W(z_{-1}) = \max_{\tau, \sigma, z} U(\tau, \sigma) + \beta W^{ta}(z) \quad (173)$$

subject to the implementability constraint (25) and possibly an additional constraint $z = z_{-1}$. Note $\lambda > 0$ and $\mu > 0$ the respective Lagrange multipliers. With both constraints, $W(z_{-1}) = W^{ta}(z_{-1})$ and with the implementability condition only, $W(z_{-1}) = W^{ra}(z_{-1})$. Then:

$$\frac{d\bar{\kappa}_2(z_{-1})}{dz_{-1}} = \frac{dW^{ra}(z_{-1})}{dz_{-1}} - \frac{dW^{ta}(z_{-1})}{dz_{-1}}, \quad (174)$$

$$= -\lambda^{ra} \frac{(1-\alpha)\beta}{1+\sigma^{ra}} - (-\lambda^{ta}(1-\alpha) - \mu^{ta}), \quad (175)$$

where the second equality comes from the envelope conditions of each program. Reorganizing,

$$\frac{d\bar{\kappa}_2(z_{-1})}{dz_{-1}} = (1-\alpha) \left[\lambda^{ta} - \lambda^{ra} \frac{\beta}{1+\sigma^{ra}} \right] + \mu^{ta} \quad (176)$$

Since $\lambda^{ta} \geq \lambda^{ra}$ and $\sigma^{ra} > \beta - 1$, one gets $\frac{d\bar{\kappa}_2(z_{-1})}{dz_{-1}} \geq 0$.

When debt is real, a similar argument holds for the upper bound $\tilde{\kappa}(b_{-1})$:

$$\frac{d\tilde{\kappa}(b_{-1})}{db_{-1}} = \gamma \left[\frac{\lambda^{ta}}{1 - \tau^{ta}} - \frac{\lambda^{ra}}{1 - \tau^{ra}} \right] + \mu^{ta} \geq 0, \quad (177)$$

since $\tau^{ta} \leq \tau^{ra}$ and $\lambda^{ta} \geq \lambda^{ra}$.

B.6 Long-Term Government Debt

Economic environment. We consider first the case of nominal debt. Let B_t be a perpetual nominal bond with payoffs decaying at the exponential rate $\rho \in [0, 1]$, as, e.g., in Woodford (2001). The consolidated government budget constraint (19) becomes:

$$q_t B_t + M_t + P_t \tau_t (1 - l_t) = P_t g + (1 + \rho q_t) B_{t-1} + M_{t-1}. \quad (178)$$

Adjusting the household budget constraint (16) accordingly results in the following Euler equation (23):

$$U_{c,t} = \beta \underbrace{\frac{(1 + \rho q_{t+1})}{q_t} \frac{P_t}{P_{t+1}}}_{\text{one-period real rate}} U_{c,t+1}, \quad (179)$$

Now consider the case with government debt indexed to inflation. Let b_t be a perpetual real bond with payoffs decaying at the exponential rate $\rho \in [0, 1]$. The consolidated government budget constraint (20) becomes:

$$q_t P_{t+1} b_t + M_t + P_t \tau_t (1 - l_t) = P_t g + (P_t + \rho q_t P_{t+1}) b_{t-1} + M_{t-1}. \quad (180)$$

Euler equation with indexed debt:

$$U_{c,t} = \beta \frac{(P_{t+1} + \rho q_{t+1} P_{t+2})}{q_t P_{t+1}} \frac{P_t}{P_{t+1}} U_{c,t+1} \quad (181)$$

The maturity of long-term debt is conveniently characterized by the Macaulay duration. In particular, we compute the weighted average term to maturity of bond payoffs. The steady-state duration of nominal debt is equal to $x = (1 - \beta\rho/(1 + \sigma))^{-1}$. With real debt, it becomes $x = (1 - \beta\rho)^{-1}$. Note that one-period debt is a special case with $\rho = 0$. More generally, the higher is ρ the longer is the duration of debt.

Competitive equilibrium. Consider the nominal-debt case first. The implementability constraint (25) reads:

$$\beta \left[\frac{(1 - \alpha)\beta}{1 + \sigma_{t+1}} \right] (1 + \rho q_{t+1}) z_t - \alpha(1 - \tau_t) - (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} + \Phi = \left[\frac{(1 - \alpha)\beta}{1 + \sigma_t} \right] (1 + \rho q_t) z_{t-1}. \quad (182)$$

with the following additional recursive expression:

$$q_t = \beta \frac{(1 + \rho q_{t+1})}{(1 + \sigma_{t+1})}. \quad (183)$$

Real debt. Implementability:

$$\beta \left[\frac{\gamma(P_{t+1} + \rho q_{t+1} P_{t+2})}{(1 - \tau_{t+1}) P_{t+1}} \right] b_t - \alpha(1 - \tau_t) - (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} + \Phi = \left[\frac{\gamma(P_t + \rho q_t P_{t+1})}{(1 - \tau_t) P_t} \right] b_{t-1}, \quad (184)$$

with the following additional recursive expression:

$$q_t = \beta \frac{(1 - \tau_t)}{(1 - \tau_{t+1})} \frac{P_t}{P_{t+1}} \frac{(P_{t+1} + \rho q_{t+1} P_{t+2})}{P_{t+1}}. \quad (185)$$

Ramsey equilibrium with nominal debt. Iterating forward (182) and (183), we get the intertemporal implementability condition:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \Phi - \alpha(1 - \tau_t) - (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right\} = (1 - \alpha)\beta z_{-1} \sum_{t=0}^{\infty} \left\{ (\rho\beta)^t \prod_{k=0}^t \frac{1}{1 + \sigma_k} \right\} \quad (186)$$

The Ramsey policy problem is defined as in (135) with (186) as a constraint. The first-order conditions are

$$1 = (1 + \lambda) \left[\alpha(1 - \tau_t) + (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right], \quad \forall t \geq 0, \quad (187)$$

$$1 = (1 + \lambda)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} + \theta_t(1 + \sigma_t) - \frac{\rho^{t-1}}{\beta} \left[(1 + \lambda)\beta \frac{(1 - \tau_{t-1})}{(1 + \sigma_{t-1})} - 1 + \lambda\beta z_{-1} \prod_{k=0}^{t-1} \frac{1}{1 + \sigma_k} + \theta_{t-1}(1 + \sigma_{t-1}) \right], \quad \forall t \geq 1, \quad (188)$$

$$1 = (1 + \lambda)\beta \frac{(1 - \tau_0)}{(1 + \sigma_0)} + \beta z_{-1} \sum_{t=0}^{\infty} \left\{ (\rho\beta)^t \prod_{k=0}^t \frac{1}{1 + \sigma_k} \right\} + \theta_0(1 + \sigma_0), \quad (189)$$

and the implementability constraint can be rewritten as

$$\frac{1}{(1 - \alpha)(1 - \beta)} \left[\Phi - \frac{1}{1 + \lambda} \right] = \beta z_{-1} \sum_{t=0}^{\infty} \left\{ (\rho\beta)^t \prod_{k=0}^t \frac{1}{1 + \sigma_k} \right\}. \quad (190)$$

The solution algorithm works as follows. First, guess λ and generate recursively the corresponding sequence $\{\tau_t, \sigma_t\}_{t=0}^{\infty}$ using (187)–(189). Second, check if implementability (190) is satisfied. Then, adjust λ and repeat if needed.

Ramsey equilibrium with real debt. TBA we get the intertemporal implementability condition:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \Phi - \alpha(1 - \tau_t) - (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right\} = \gamma b_{-1} \sum_{t=0}^{\infty} \left\{ \frac{(\rho\beta)^t}{1 - \tau_t} \right\} \quad (191)$$

The Ramsey problem first-order conditions are

$$1 = (1 + \lambda) \left[\alpha(1 - \tau_t) + (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} \right] - \lambda \frac{\gamma \rho^t}{1 - \tau_t} b_{-1}, \quad \forall t \geq 0, \quad (192)$$

$$1 = (1 + \lambda)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} + \theta_t(1 + \sigma_t), \quad \forall t \geq 0, \quad (193)$$

Correspondence. Economies with different debt maturities $\rho_1 < \rho_2$ are made comparable using the correspondence discussed in Section 2.1. Formally, economies with different maturity profile ρ_i and ρ_j have comparable discounted policy surpluses if outstanding initial debts $s_{-1,i}$ and $s_{-1,j}$ satisfies:

$$\beta z_{-1,i} \sum_{t=0}^{\infty} \left\{ (\rho_i \beta)^t \prod_{k=0}^t \frac{1}{1 + \sigma_{k,i}} \right\} = \beta z_{-1,j} \sum_{t=0}^{\infty} \left\{ (\rho_j \beta)^t \prod_{k=0}^t \frac{1}{1 + \sigma_{k,j}} \right\} \quad \text{when debt is nominal,} \quad (194)$$

Proposition 7. Consider first economies with nominal debt and different debt maturity $\rho_2 \neq \rho_1$. The principle of correspondence requires that these economies are comparable for level of debt $z_{-1,1}$ and $z_{-1,2}$ that satisfy

$$\frac{z_{-1,1}}{1 - \rho_1} = \frac{z_{-1,2}}{1 - \rho_2} \quad (195)$$

The credibility cut-off satisfies (44) and the implementability condition to evaluate each term is (45). Using (195), one can easily note that both $\max_{\tau, \sigma, z} U(\tau, \sigma) + \beta W^{ta}(z)$ and $W^{ta}(z)$ are scalable by maturity under the correspondence, so that:

$$\bar{\kappa}_2(z_{-1,1}, \rho_1) = \bar{\kappa}_2(z_{-1,2}, \rho_2)$$

In other terms, when debt is nominal, maturity does not influence the credibility of strategic monetary rules.

Consider now real debt. The principle of correspondence requires to compare economies with different debt maturities at the stationary allocation. The implementability condition (184) reads:

$$\beta \left[\frac{\gamma(1 + \rho \tilde{q}_{t+1})}{(1 - \tau_{t+1})} \right] b_t - \alpha(1 - \tau_t) - (1 - \alpha)\beta \frac{(1 - \tau_t)}{(1 + \sigma_t)} + \Phi = \left[\frac{\gamma(1 + \rho \tilde{q}_t)}{(1 - \tau_t)} \right] b_{t-1}, \quad (196)$$

Economies with real debt and different debt maturity $\rho_1 \neq \rho_2$ are comparable as long as:

$$\frac{b_{-1,1}}{1 - \tau(b_{-1,1})} \frac{1}{1 - \beta \rho_1} = \frac{b_{-1,2}}{1 - \tau(b_{-1,2})} \frac{1}{1 - \beta \rho_2}, \quad (197)$$

where $\tau(b_{-1,i})$ solves (196) at $b_{-1,i} = b_i$ and $\tilde{q} = \frac{\beta}{1 - \beta \rho}$ given by the stationary recursive condition (185). Naturally, under the principle of correspondence, $W^{ta}(b)$ is scalable by maturity. Let's focus on the deviation, where the continuation allocation is a stationary allocation:

$$\max_{\tau, \sigma, b} U(\tau, \sigma) + \beta W^{ta}(b)$$

subject to

$$\beta \frac{\gamma}{1 - \tau(b)} \frac{b}{1 - \beta\rho} - \alpha(1 - \tau) - (1 - \alpha)\beta \frac{1 - \tau}{1 + \sigma} + \Phi = \frac{\gamma}{1 - \tau} b_{-1}(1 + \rho\tilde{q}), \quad (198)$$

where $\tilde{q} = \beta \frac{1 - \tau}{1 - \tau(b)} \frac{1}{1 - \beta\rho}$. Combining these expressions make it explicit that the deviation is not sensitive to debt maturity if and only if $\tau = \tau(b)$, namely if there is no deviation from the stationary allocation.

B.7 Annex on Numerical Computations

This section provides elements regarding numerical simulations reported in Section ??, and in particular (36), i.e.

$$\bar{\kappa}_i(s_{-1}) = \max_{\tau_0 \in \mathbb{T}(s_{-1})} V^{Mr}(s_{-1}, \tau^0) - V^{Mk}(s_{-1}, \tau^0) \quad (199)$$

Standard (unconditional rule) with nominal debt. The central bank commits unconditionally to $\sigma = \beta - 1$, what is the credibility cut-off given z_{-1} ? The numerical solution reported in Figure 7 panel (a) derives $\bar{\kappa}_1(z_{-1})$ as follows. Given κ , solve the game using the value function iteration algorithm described previously. Then compute $\Delta(z_{-1}) = \max_{\tau_0 \in \mathbb{T}} V^{Mr}(\tau_0, z_{-1}) - V^{Mk}(\tau_0, z_{-1})$. Then infer \bar{z}_{-1} s.t. $\kappa = \Delta(\bar{z}_{-1})$, and get $\bar{\kappa}_1(\bar{z}_{-1}) = \kappa$.

Strategic monetary rule with nominal debt. The central bank commits to a strategic monetary rule, but target an equilibrium money printing rate $\tilde{\sigma} \geq \beta - 1$. The derivation of associated credibility cut-off $\bar{\kappa}_2(z_{-1}, \tilde{\sigma})$ goes as follows. By proposition 5, if the rule is credible, then the implemented allocation is stationary $(\tilde{\tau}, \tilde{\sigma}, z_{-1})$ and the associated welfare reads

$$W^{st}(z_{-1}, \tilde{\sigma}) = \frac{\beta}{1 - \beta} U(\tilde{\tau}, \tilde{\sigma}), \quad (200)$$

where the superscript *st* stands for stationary. In particular, $W^{st}(z_{-1}, \beta - 1) = W^{ta}(z_{-1})$. Let $\tilde{\Delta}(z_{-1}, \tau_0, \tilde{\sigma})$ be the monetary temptation wedge at (z_{-1}, τ_0) . It satisfies:

$$\tilde{\Delta}(z_{-1}, \tau_0, \tilde{\sigma}) \leq \tilde{\kappa}(z_{-1}, \tilde{\sigma}) = \max_{\tau_0, \sigma, z} U(\tau_0, \sigma) + \beta W^{st}(z, \tilde{\sigma}) - W^{st}(z_{-1}, \tilde{\sigma}) \quad (201)$$

As in (B.5), if $z \leq z_{-1}$, then $\bar{\kappa}_2(z_{-1}, \tilde{\sigma}) = \tilde{\kappa}(z_{-1}, \tilde{\sigma})$. If $z \geq z_{-1}$, then $\bar{\kappa}_2(z_{-1}, \tilde{\sigma}) \leq \tilde{\kappa}(z_{-1}, \tilde{\sigma})$, since

$$\max_{\tau_0, \sigma, z} U(\tau_0, \sigma) + \beta V^M(z, \tilde{\sigma}) \leq \max_{\tau_0, \sigma, z} U(\tau_0, \sigma) + \beta W^s(z, \tilde{\sigma}). \quad (202)$$