QUANTITY COMMITMENTS IN MULTIUNIT AUCTIONS: EVIDENCE FROM CREDIT EVENT AUCTIONS

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ABSTRACT. Credit Default Swaps (CDS) are financial derivative products that insure bond investors against firm-default. In a credit event, CDS payouts are determined in a two-stage auction. In addition to the standard information rents in multiunit auctions, learning across rounds, initial quantity commitments, and heterogeneous positions in CDS impact auction outcomes. This paper develops and estimates a structural model of bidding in these auctions and uses it to quantify the role of strategic bidding. I consider counterfactual changes to the auction format, including a double auction design with step function bidding, which reduces shading, increasing the insurance coverage.

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1. Introduction

U.S. firms issue nearly $2 trillion in corporate debt each year. The large institutional investors who purchase this debt will often hedge against default risk by buying insurance using Credit Default Swaps (CDS). CDS are derivative contracts which provide insurance against a credit event (e.g. bankruptcy, restructuring) on some set of obligations. In addition to hedging default risk, CDS also allow investors to speculate. This insurance market has a gross notional volume outstanding of roughly $10 trillion.

When a credit event occurs, issuers of CDS have to make payments to the buyers of protection. The target payment is the difference between the par value on a bond and its post credit event value. The focus of this paper is in the determination of this value. When CDS contracts were first introduced, settlement involved a physical transfer of bonds from buyers of insurance to sellers, and an insurance payment from sellers to buyers equal to the par value of the bond. This arrangement, however, is complicated by the fact that the market may have more CDS contracts than bonds. Buyers of insurance do not need to own the bond. This means physical settlement can lead to a short squeeze where the few investors who own the bond can charge a very high price to investors needing to source it in order to realize the insurance payout. As a result, in 2005 the participants in this market agreed to instead settle these contracts using a cash payment, the value of which is determined by holding a two-stage auction for bonds.

At each auction, bonds are bought and sold by large investment banks. The current auction format involves a nonstandard two-stage design. In a first stage, the auctioneer accepts initial quantity commitments, i.e. buy and sell commitments that are enforced in the second stage. This first stage determines whether there is an excess demand for bonds or an excess supply. In the second stage the auctioneer uses a multi-unit auction to determine the market-clearing price. This price ($p$ per dollar of face value), is paid for all bonds traded in the auction, and a default payment of $1 - p$ is paid by CDS sellers to CDS buyers. Since investors are active in the auction and hold CDS contracts, they may have incentives to distort auction prices and therefore contract payoffs. This is a concern because the CDS market has a few large players who could be in a position to influence prices. Consistent with the existence of strategic bidding, bond prices in the auctions

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1 A similar incentive arises in other settings where prices are used both for exchange and in calculating a benchmark. For example, see Zhang (2022) for a theoretical analysis of spot markets for derivatives.

2 Some non-dealer firms desire more direct participation in the auctions, see Rutledge (2009). A lawsuit, New Mexico State Investment Council. v. Bank of America et al. (2022), alleges that the dealers influenced both the auction design and clients’ acceptance of the auction process.
are usually a few cents on the dollar below their secondary market counterparts (Coudert and Gex (2010), Gupta and Sundaram (2012)). Because the auction price determines insurance payouts, differences from the efficient price can reduce the insurance provided.

This paper makes three contributions. First, I develop a novel structural model of bidding in the two stage multi-unit CDS auctions that I use along with unique dataset on bidding to identify and estimate the distribution of bidders’ private values for post-default bonds simultaneously with their insurance positions. Second, I use the model estimates to quantify the distortions from information rents under the current auction design and explore the channels causing these distortions. Third, I evaluate the improvements that could be achieved by changing the auction format.

Unlike in standard multiunit auction settings where the econometrician is only interested in learning a bidder’s private value from their bids, we need to identify both bidder private values and their CDS positions in order to quantify distortions and evaluate alternatives. I extend identification arguments from multi-unit auctions by using restrictions on the shape of the marginal value curve (i.e., bounded and weakly decreasing in quantities) to jointly bound the set of CDS positions and marginal values for every bidder.

Model estimates allow me to document the size of the distortions due to information rents and to quantify several strategic channels. It is important to quantify these channels as theoretical work focusing on different channels has proposed different solutions. My focus is on the double auction proposed by Du and Zhu (2017) to avoid excluding participants. The authors consider constraints on behavior across rounds in an independent private value setting where the dealers have no price impact and hold zero average CDS positions.

I evaluate whether a counterfactual change to a double auction, where bidders simultaneously submit supply and demand orders, can reduce the pricing distortions. Relative to the current format, the double auction eliminates initial commitments, increases the uncertainty about opponents’ demands, and removes constraints on the direction of eligible

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1Chernov et al. (2013) use an environment with perfect information and common values to highlight the role of short sale constraints and constraints prohibiting some participants from holding defaulted bonds. They recommend a change to pro-rata rationing to eliminate underpricing equilibria of the common value auction (as in Kremer and Nyborg (2004)). I perform several tests that suggest that the first round price quote captures most of the common value information. Peivandi (2015) focuses on bilateral settlement pre-auction and shows that the only mechanisms that deliver unbiased prices are equivalent to a mechanism with a posted price. The posted price does not incorporate bidders’ private information, and may reduce insurance provision.

2In settings with imperfect competition the double auction is not fully efficient because bidder’s strategically account for the price impact of their bids, e.g. in Kyle (1989), Vives (2011), Ausubel et al. (2014).
bids. The distortions from these constraints have been established theoretically in Du and Zhu (2017). However, their result that a double auction would improve price discovery relies on assumptions, rejected by the data, that bidders have zero-average CDS positions and that their marginal values decrease at a common rate. Because dealers are estimated to be mostly net buyers of CDS, the incentives for buyers and sellers to distort their bids do not cancel out in a double auction. In addition, in my model the constraints have an ambiguous effect on prices, because they increase bidders’ price impacts and bid shading. Therefore, whether the double auction improves outcomes is an empirical question.

To evaluate the effect of hypothetical changes to the auction rules, I apply a new computational approach to calculate the equilibrium strategies of bidders under counterfactual auction scenarios. Direct computation of equilibrium in multi-unit auction settings has been elusive. This has limited the counterfactuals considered to exercises that provide an upper bound on the benefits of eliminating bid shading (e.g., Hortaçsu and McAdams (2010), Kastl (2011)). The main computational challenge is that bidders’ strategy functions are high dimensional and complex. This implies that both Euler-based approaches, and approaches based on parametrizations of the strategy functions (e.g., Armantier et al. (2008)), cannot be applied. Instead, my method begins by guessing a data-generating process (DGP) for equilibrium bids and adjusts the DGP until the distribution of values that rationalizes those bids, given the rules of the game, matches the true distribution of values. The approach leverages the fact that inverting a bid to recover values is much easier than solving for the optimal bid given values. In a companion paper, Richert (2021), I provide general results and simulations to evaluate the performance of this approach.

The distortions that I estimate arising from the auction design are economically meaningful. Strategic bidding leads to an under-pricing bias of 2.2 cents on the dollar and induces variance in contract payouts, reducing insurance coverage from CDS to 94-96% of full insurance. I show that a counterfactual change to a double auction could decrease the bias by 67%, decrease the standard deviation of auction risk by 70%, and cover 98-99% of the risk. To provide context on how the additional risk may impact real outcomes, I scale the estimated impact of CDS contracts on firm value from Danis and Gamba (2018), and find the double auction could increase firm value by 0.07-0.11%.

Section 2 presents details of the CDS auction institution and introduces the data, 3 introduces the model, 4 discusses identification, 5 presents the estimation 6 considers the results and section 7 examines the counterfactual experiments.
CDS are financial derivatives which provide insurance against a pre-determined set of credit events occurring on a pre-specified set of bonds.\footnote{For a survey of the literature on CDS markets, contract terms, and pricing, see Augustin et al. (2014).} These contracts initially used physical settlement, akin to basing settlement on scrapage value in other insurance markets. In physical settlement, the insurance buyer delivers the bond to the insurance seller and in return receives the par value of the bond. This leaves the buyer with the full initial value, and the insurer can claim any bond recoveries. However, in addition to bond owners, CDS contracts may be purchased by speculators that do not own the underlying bond. These so-called Naked CDS contracts allow speculators to use CDS to bet on firm creditworthiness. The presence of speculators adds liquidity to the market, but also means that the volume of CDS is often many times the outstanding volume of bonds, and so physical settlement of all contracts would require the bonds to be recycled through the market. This could produce a short squeeze, preventing physical settlement from providing fair insurance for naked buyers (Gupta and Sundaram (2015)). Physical settlement also produces an inefficient allocation of bonds when some CDS buyers have a higher value of holding the bond through the recovery process than sellers.

These issues were anticipated in the lead up to the default of Delphi in 2005, where there was $25B of CDS contracts written on $2B of bonds. To address the problems with physical settlement, the dealer-banks decided to settle contracts in cash at a price determined in an auction for the underlying bonds. A two-stage auction design was proposed to allow participants to replicate the outcomes of physical settlement. In the first stage, dealers submit physical settlement requests and in the second a uniform price multi-unit auction is held to clear the market. The dealers were not attempting to design the optimal mechanism; the objective was to replicate physical settlement specified in the outstanding contracts. To receive the same payments and transfers as under physical settlement, a dealer requests to buy/sell in the first stage as many bonds as they would have transferred physically. Following 2009, the auctions were written into all CDS contracts.\footnote{There have been two major changes in this market since the first auctions in 2006: the big bang and small bang protocol. The main effect of these rules (effective 2009) were to tie the CDS contract payouts to auction prices.}

There were 209 credit events between 2006 and the Fall of 2019. Of these, 84 were loan credit default swaps (LCDS) and 125 CDS. LCDS are similar to CDS contracts but have loans rather than bonds as the underlying reference obligation. There were 8 auctions which did not proceed to the second stage, and in 16 cases auctions were not held,
The following table presents auction-level summary statistics. Price is the final market clearing price from the auction. IMM is the initial market midpoint calculated using bidders’ first round price quotes. The NOI is the excess supply or demand from summing over each bidder’s quantity commitments. Probability to buy takes a value 1 if the auction results in excess supply (accepts demand bids at stage two). There are 185 auctions in the sample. In total there are 1998 eligible bonds across all auctions. Maturity in years. FRN % denotes the share of the eligible bonds that are floating rate notes (coupon payments linked to a benchmark rate, usually LIBOR).

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Sd</th>
<th>[P10,P90]</th>
<th></th>
<th>Mean</th>
<th>Sd</th>
<th>[P10,P90]</th>
</tr>
</thead>
<tbody>
<tr>
<td>N Dealers</td>
<td>11.06</td>
<td>2.54</td>
<td>[8,14]</td>
<td>N Bonds</td>
<td>10.80</td>
<td>27.77</td>
<td>[1,298]</td>
</tr>
<tr>
<td>Price ($.01)</td>
<td>43.01</td>
<td>32.46</td>
<td>[4.0,88.5]</td>
<td>Max maturity</td>
<td>10.46</td>
<td>11.94</td>
<td>[0,50]</td>
</tr>
<tr>
<td>IMM</td>
<td>43.41</td>
<td>32.02</td>
<td>[4.8,88.8]</td>
<td>Min maturity</td>
<td>2.98</td>
<td>4.92</td>
<td>[0,50]</td>
</tr>
<tr>
<td></td>
<td>NOI</td>
<td>($millions)</td>
<td>95.14</td>
<td>167.84</td>
<td>[2,234]</td>
<td>Max coupon %</td>
<td>5.5</td>
</tr>
<tr>
<td>Probability to buy</td>
<td>0.632</td>
<td></td>
<td></td>
<td>Min coupon %</td>
<td>2.7</td>
<td>3.3</td>
<td>[0,11.8]</td>
</tr>
<tr>
<td>Share FRN %</td>
<td>33.99</td>
<td>41.8</td>
<td>[0,1]</td>
<td>Share FRN %</td>
<td>33.99</td>
<td>41.8</td>
<td>[0,1]</td>
</tr>
</tbody>
</table>

resulting in a sample of 185 auctions.\(^7\) I collect data from creditfixings.com, (administered by Creditex and MARKIT) on all bids made in credit event auctions, and obtain the lists of eligible bonds from the determinations committee. The eligible debt includes corporate and sovereign bonds, syndicated loans, commercial mortgage backed securities (CMBS) and mortgage backed securities (MBS).\(^8\) Summary statistics are displayed in Table 1. The auctions have an average of 11 participants, usually the nine global dealers and two largest regional dealers. Dealers also aggregate client orders, if any. I identify unobserved client orders by imposing structure on bidding—discussed in Section 4.2 and Appendix I.

The recovery price determined in the auction averages 43.41 cents to the dollar. That is, the auction price for the bond is 43.41 percent of the par value of the bond; there is, however, substantial variation across auctions. The IMM, which is an average of price quotes given by dealers at the first stage of the auction, is fairly similar to this final price. NOI (net open interest) denotes the volume of excess supply or demand for bonds resulting from the first stage. In total, 355 bidders submit requests to buy in the first stage of one of the 185 auctions, 535 submit requests to sell, and 1167 submit zero quantity bids in the first stage. Around 63 percent of the auctions result in excess supply.

From the determination that a credit event has occurred to the final payout, the auction process is administered jointly by a committee whose members are selected based on

\(^7\)In the design of the auctions the ISDA determined a set of situations where an auction is not required to be held. This occurs if for certain maturity buckets there are no deliverable obligations in the bucket that is not shared with a shorter-dated bucket, or if the determinations committee decides an auction on that bucket is not warranted due to limited notional volume of transactions within the bucket.

\(^8\)Results are similar if I exclude the 5 auctions that involved sovereign bonds from the analysis.
their global notional volumes. These large dealers are obliged to participate in most auctions—failure to participate could threaten their eligibility to participate in future auctions. The committee selects the set of deliverable obligations for the auction. In cases where the issuer has debt of multiple maturities or risk levels separate auctions may be held. This means that the bonds that can be delivered in a particular auction are systematically homogeneous. For each eligible bond, I obtain volume and trait information from Bloomberg and obtain loan information from DealScan. For 56 of the auctions, the bonds are covered by TRACE and so I also obtain trading data for these bonds around the auction date. Summary statistics for the deliverable obligations in each auction are provided in Table 1. On average, 11 bonds are eligible for submission into the auction. The bond characteristics vary substantially across auctions. The within-auction variation is summarized in Table OS.2. Only a fraction of the eligible bonds are exchanged at the auction. Dealers should anticipate which bonds are cheapest to deliver and expect these to be submitted first. All dealers should expect the same bonds to be sold, and the estimated values will represent bidders’ values for that set of bonds.

CDS contracts are traded over-the-counter and disaggregated trading data are not available. Prior to 2010, reporting requirements for these transactions were limited. After 2010, information on all standardized and confirmed CDS transactions involving U.S. entities was reported to the DTCC. This data is available to regulators through the DTCC’s Trade Information Warehouse. Paulos et al. (2019) uses this data from 2014-2017 for the subset of dealers regulated by the Federal Reserve. They show that dealers are typically net buyers of protection in the auction. Given these data are regulatory, market participants do not observe trade data and do not know each others positions.

2.1. Evidence of market power. Figure 1 plots the average transaction price in the secondary market (and the auction price on day 0). The V-shaped pricing pattern is consistent with the findings in existing papers. Coudert and Gex (2010), and Gupta and Sundaram (2012), for example, document that the auction price tends to be well below both the pre- and post-auction trading prices.

The Trade Reporting and Compliance Engine is the FINRA database for the mandatory reporting of over-the-counter transactions in eligible fixed income securities. Following the financial crisis, some CDS moved to central clearing—mostly index CDS and more liquid companies, C.f. Slive et al. (2012), and has not affected the single names on which credit events occurred. This result is in contrast to Eisel et al. (2022), which finds that dealers are net sellers of CDS. Dealers may hold different positions in firms which are likely to default or may adjust positions pre-auction.
Although this price gap is consistent with the presence of market power it could also come from other sources. For example, dealers may take on, at a discount, larger bond positions around the auction, with clients selling both CDS and bonds to dealers. There could also be risk in the bond price as auction outcomes may reveal information about the bond value. This relationship is highlighted in Table A.1 of the appendix. While the auction price has no explanatory power for post-auction bond prices, the post-auction prices are correlated with the IMM, suggesting that some information relevant for the secondary market may be revealed during the first-stage of the auction.\footnote{The concentration of the quantity of bonds won by the largest winner does not explain the price gap around the auction date, suggesting the gap may not be due to differences in expectations of dealers’ actions through the recovery process.}

2.2. Current auction format. The auction begins with a stage where bidders (i) submit initial quantities that they want to commit to settle at the final auction price and (ii) price quotes, at a quantity and maximum spread set by the determination committee depending on the liquidity of the defaulted assets. Following the first stage, the auctioneer adds up all the quantity commitments and announces this along with the average price quote. They then hold a uniform price multiunit auction to clear the excess supply or demand. I illustrate the process with an example auction.

2.2.1. Initial Quantity. For an example of the initial quantities see Table 2. Quantities to buy and sell are summed across dealers to determine the Net Open Interest (NOI). In the example presented in Table 2 there is a NOI of $47.397M. The auction is therefore in excess supply. The bidders are required to make initial quantity submissions that are in the same direction as their net CDS position, restricting the transfers to those that would
This table presents the initial round quantities from the auction for Parker Drilling Co. An offer is a commitment to supply bonds. A bid is a commitment to buy bonds. There are $507M of eligible bonds in this auction.

<table>
<thead>
<tr>
<th>ID</th>
<th>Dealer</th>
<th>Bid/offer</th>
<th>Size ($M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Barclays Bank PLC</td>
<td>Offer</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>BNP Paribas SA</td>
<td>Offer</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Credit Suisse</td>
<td>Offer</td>
<td>7.953</td>
</tr>
<tr>
<td>4</td>
<td>Deutsche Bank</td>
<td>Offer</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Goldman Sachs International</td>
<td>Bid</td>
<td>6.53</td>
</tr>
<tr>
<td>6</td>
<td>J.P. Morgan Securities LLC.</td>
<td>Offer</td>
<td>26.974</td>
</tr>
<tr>
<td>7</td>
<td>Merrill Lynch, Pierce, Fenner &amp; Smith Inc.</td>
<td>Offer</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>Morgan Stanley &amp; Co. LLC</td>
<td>Offer</td>
<td>9.0</td>
</tr>
<tr>
<td>9</td>
<td>Societe Generale</td>
<td>Offer</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Total for Auction (Net Open Interest)</td>
<td>Offer</td>
<td>47.397</td>
</tr>
</tbody>
</table>

occur under physical settlement. Therefore a bank that owns more CDS contracts than it has sold can only submit requests to sell.

To understand the incentives of dealers in this stage, consider a dealer that is a net buyer of insurance. This dealer wants the auction to establish a low price for the bonds, which will result in a larger payout on their CDS position. The dealer receives a cash settlement of $(1 - p^\text{auc})$ on CDS they own, and receives $p^\text{auc}$ for bonds sold at the auction. By supplying additional units in the first stage, the dealer reduces their final exposure to the auction price.\textsuperscript{13} The dealer also considers what opponents will learn from the NOI.\textsuperscript{14}

2.2.2. Initial Quotes. The first stage also includes a simultaneous submission of bid and offer price quotes. The bid-offer spread and the quote size are fixed before each auction. The quotes are used to set a price floor (ceiling) in the auction when bidders are selling (buying) and are carried forward into the auction as part of the second stage bid. Prior to the second stage, this average quote is announced. The price caps set a limit such that a dealer with a net CDS position larger than the total quantity being auctioned does not have an incentive to push the price to 100 or zero. To calculate this cap from the quotes, the auctioneer discards crossing/touching markets (where a buy price is above a sell price),

\textsuperscript{13}These features are related to those of sequential markets in Treasury, or electricity market settings as in Allaz and Vila\textsuperscript{1993}, and Ito and Reguant\textsuperscript{2016}. Unlike those settings, the initial round is settled at the final auction price (rather than a separate price) and the cost of the first round comes from being unable to adjust due to the second stage constraints.

\textsuperscript{14}Figure OS.3 shows how expected price varies with the NOI.
and takes the ‘best half’: the highest half of the remaining bids and lowest half of the offers and calculates the average— called the Initial Market Midpoint (IMM). To set the cap, a pre-determined spread is added to the average if it is an auction to buy or subtracted if it is to sell. These quotes are carried over into the auction in the direction matching the NOI, at an auction-specific quantity, set in advance by the auctioneer. Any bids that are off-market are carried over at the IMM. Finally, the bids are used to determine fines for off-market bids. If an offer to buy is above an offer to sell, this indicates a trade-able market, and the off-market party is fined based on the size of this difference multiplied by the fixed quote size. These fines, in addition to only the average quote being reported, limits the ability to use a quote to signal relative to the LIBOR case (Bonaldi (2017)).

2.2.3. Uniform Price Auction. After the initial submissions, the initial market midpoint and the size and direction of the open interest are announced. The market is then given between 30 minutes and two hours to incorporate this information. Next, a uniform price auction is held to clear the excess quantity. The bids submitted in the example auction are plotted in panel A of Figure 2. These bids are then summed to calculate a demand (or supply) curve and the point where it intersects the total quantity to be sold determines the clearing price, as shown in panel B of Figure 2.

When bidding in this stage, the dealer chooses a demand curve to submit. Uncertainty about opponents’ values leads participants to bid strategically. When deciding on the bid, each bidder considers the distribution of residual supply curves. Knowledge of the NOI provides each bidder with information on the location of the aggregate supply curve and because the NOI results from the initial quotes it also informs them about the opponents’ signals. The final price is paid for all bonds acquired in the auction, for the initial quantity commitment and for all CDS contracts. The CDS positions affect dealers
incentives to shade by changing the effective number of units on which they pay the final price. The first-round submissions allow bidders to decrease their exposure to the auction price, which reduces the heterogeneity across bidders. Finally, the uni-directional second round may constrain some participants from expressing their demand.

2.2.4. Role of Directional Constraints. In the current auction format each round limits the quantities that can be submitted. In the first stage a buyer (seller) of insurance can submit only orders to sell (buy) bonds. In the second stage if there is excess demand (supply) only orders to buy (sell) are accepted. As highlighted by Du and Zhu (2017) this means that some bidders do not have a chance to participate. However, when participants have market power, it also impacts their bid by changing their expected price impact.

To illustrate this force consider an auction with 2 bidders. Suppose that bidder A has a high value for the bonds and is a large owner of insurance contracts. The other bidder, B, has a low value for bonds and zero insurance position. Efficient trade should transfer bonds from B to A. The left panel of Figure 4 illustrates the willingness to pay of bidder A, along with the willingness to supply of bidder B. In the first stage of the auction, bidder A can commit to supply bonds while bidder B can neither supply nor demand bonds. Suppose that in the first stage A commits to supplying $y_A$. This results in an excess supply of bonds $NOI = y_A$, so the auctioneer only accepts demand orders in the second stage. This means that bidder B is excluded from supplying, and the residual demand faced by A is given by B’s demand (the negative supply at low prices) and is vertical from the price where B makes no purchases, as illustrated in the right panel. This puts upward pressure on prices with the new clearing price the y-intercept of $v^A_d$.

However, there is a second effect from the bidders’ strategic responses. When there are no constraints, if bidder A shades their demand by requesting a slightly smaller quantity, the clearing price moves by a small amount, along B’s demand curve. In the presence of constraints, however, Bidder A faces a residual supply curve which is much steeper (vertical until the level where $v^B$ becomes willing to demand bonds). This means that if A requests to buy a slightly smaller quantity than their initial commitment, there would be a large drop in prices (from the intercept of $v^A$ to the intercept of $v^B$). By shading, Bidder A obtains a large increase in profits: they obtain almost exactly the same bond position and receive a large increase in payments for the cash settlement of her insurance position at the lower price. Despite the fact that the constraints exclude a supplier (putting upward pressure on prices), the strategic responses cause the price to fall, due to the large increase in the price impact of Bidder A.
2.3. Distinguishing Between Common and Independent Private Values. Most empirical work on auctions requires the economist to make a modelling assumption on the information structure of the game. For tractability, given the complicated dynamic, multi-unit setting, this choice is limited to either the independent private values framework (IPV) or one based on common values (CV). In the CDS context there are factors which could lead both of these assumptions to be reasonable, and in theoretical work both have been used (Du and Zhu [2017] and Chernov et al. [2013], respectively). Specifically, IPV may be reasonable if the price quotes effectively aggregate the common value information held by different dealers and the remaining variation in values was driven by bidders’ expectations of their own customer order flows, their value of liquidity, their expertise in managing the complicated legal process of restructuring/liquidation, or their cost of holding bonds through the recovery process. On the other hand, common values is a reasonable assumption if there is a liquid resale market where these inventory/management costs are negligible. In Appendix B I empirically test for the presence of several correlations which are predicted if common values are important in the strategic bidding decisions. Although there is likely to be some aspect of both private and common values in this setting, the results of these tests do not provide evidence of the Winner’s Curse or of important within-auction correlations in bidder values. This is consistent with second-round bids being largely driven by idiosyncratic values.

3. Model

3.1. Players and Endowments. The participants in the auction game are a set of dealers who are eligible to bid in the CDS auctions, $I_d \subseteq I$, the complete set of owners and sellers of CDS, and owners of the underlying bonds. On the auction date, each dealer, $i$ is endowed with a CDS position $n_i$ and bonds $B_i$. If $n_i \geq 0$, the dealer is a net buyer of
protection while, if \( n_i \leq 0 \), the dealer is a net seller. Since these are derivative contracts, there is someone on each side of the position and \( \sum_i n_i = 0 \). Note that this aggregation condition holds over the entire set of market participants, not only the subset of dealers who bid in the auction. Both the quantities \( B \) and \( n \) are denominated in hundreds of millions of dollars outstanding, so a bond payoff if no credit event occurs is \( 100B_i \) million. The final auction price is expressed as cents on the dollar.

3.2. Information. Before making their choices, bidders receive independent draws of a vector \( m_i = (s_i, n_i) \) from \( F_m \), where \( s_i \) is a vector of private signals, and \( n_i \) is the one-dimensional position in CDS contracts. The vector \( m \) is drawn from an absolutely continuous joint distribution with no holes and no mass points. Bidders know the distribution \( F_m \) but not the draws of their opponents. Let \( y_i \) denote the initial quantity commitment of dealer \( i \) to purchase or sell bonds and let \( v_i(q - y_i, s_i) \) denote the marginal value for the \( q^{th} \) unit of a bond purchased at auction. I assume that these functions are bounded, weakly increasing in each component of \( s_i \), and decreasing in \( q \).

In addition to the vector of private value-relevant private information, bidders receive a signal of the expected recovery value \( \eta_i = R + \xi_i \), and \( \xi_i \sim F_{\xi} \). These draws are IID across bidders and the bidders know the distribution \( F_{\eta} \). I assume that this recovery value affects the bidders marginal value through a level shift: \( R + v_i(q, s) \). Both \( R \) and \( \xi \) are independent of the private signals and positions. Finally, all bidders face a dealer-specific cost of submitting a bid in the first round, \( c_{\kappa i} \sim \kappa \), and after the first stage, a (complexity) cost of submitting each step in the second stage, \( c_i \sim \tau \).

Prior to the first-round bidding, each bidder receives orders from clients to submit physical settlement requests on their behalf. These orders are aggregated with the dealers’

---

15While the bond or CDS positions of a dealer may reflect their private information, it seems reasonable that they are not perfectly correlated. Due to, for example, frictions in OTC markets, c.f. Duffie et al. (2005), Hugonnier et al. (2019), Li and Schürhoff (2019) and Di Maggio et al. (2017).

16This rules out that a bidders own position is informative of their opponents positions. This would be an important concern if, for example, one bidder owned most of the outstanding bonds and could therefore infer that their opponents held minor positions. In this setting the bonds owned by dealers usually make up a small share of the total outstanding, with large volumes of bonds owned by outside investors.

17This is indistinguishable from a model where bidder preferences depend on an initial position of bonds \( B_i \) which is also part of their private information (ie. \( m = (s_i, n_i, B_i) \)), in which case \( v_i(B_i + q - y_i, s_i) = v_i(q - y_i, s_i, B_i) \), as long as the auction does not cause a change in \( i \)'s post-auction value of owning \( B_i \). Without data on bond positions, the roles of signals and bonds in determining the marginal value cannot be separately identified, so I treat \( v(q, \cdot) \) as the structural primitive and as a consequence simplify notation by writing \( v_i(q - y_i, s_i) \) throughout.

18The choice of an additive \( R \) is motivated by the fact that the variance of bidders’ values does not appear to shift with the level of \( R \). Evidence for this is presented in Online Appendix E.2.
own physical settlement request when reported to the auctioneer. This can cause reported requests to be in excess of, or in an opposing direction, to the dealers’ insurance positions. I assume that these orders arrive independently of all dealer’s private information, and each dealer receives a unique draw \((y^c_i)\) from the distribution of these shocks \(\Upsilon\).

3.3. Actions and Timing. The bidders start with a quantity of bonds, a net position of CDS contracts, some private signal indicating their private benefit from finishing the auction with \(q\) units, a signal about the expected recovery value \(\eta\), an order to submit on behalf of their clients for physical settlement, and a cost of submitting a first round bid.

Given their signals and position, dealers choose an initial round quantity \((y_i)\) to commit to purchase/sell at the auction stage and a price quote that is used to determine the IMM. The restrictions on participation from the auction rules mean that if \(n_i \leq 0\), then \(y_i \in [n_i, 0]\) while if \(n_i \geq 0\), \(y_i \in [0, n_i]\). Bidders choose \(y_i\) in the set: \(\mathcal{Y}_i \equiv \{y_i | y_i \in [\min(n_i, 0), \max(0, \min(B_i, n_i))]\}\). The choice \(y_i\) is discrete with increments of the minimum deliverable bond denomination. The bidder submits a total quantity order, which is the sum of own and customer commitments \(y^o_i \equiv y_i + y^c_i\). This may differ in sign from the \(y_i\), which is constrained by \(n_i\). The total commitments are \(NOI \equiv \sum_{i \in I} y^o_i\).

After the first stage, bidders learn the open interest, and the initial market midpoint \(\Omega = (NOI, IMM)\). In addition, they know their own contribution to the NOI, \(y^o_i\), and so they can deduce that the total submissions of opponents were \(NOI - y^o_i\). In the second stage, bidders choose an action from the restricted set of strategies denoted by \(\gamma(p|m_i, \Omega, y^o_i)\). This function describes the quantity \(\gamma\) allocated to bidder \(i\) at price \(p\).

The strategies \(\gamma_i\) for each player lie in the set of possible actions \(A_i\):

\[
A_i = \{(b_i, q_i, K_i) : dim(b_i) = dim(q_i) = K_i \in \{0, 1, 2, ..\bar{K}\}, b_{ik} \in \{0, 0.125, 0.25, ..., 100\}, q_{ik} \in [\min(0, NOI), \max(0, NOI)], b_{ik} \geq b_{ik+1}, q_{ik} \leq q_{ik+1}\}.
\]

3.4. Initial Market Price Quote. I take the initial round price quote as a reflection of the value-relevant information common to all bidders. This assumption simplifies the model and is reasonable in this setting because: (i) only the average quote is reported before the second stage, (ii) each bidder has limited ability to manipulate the averages, and (iii) any attempt to engage in manipulation is likely to result in fines and exclusion of the quote from the calculation of the average.

**Assumption 1.** \(p^{IMM}\) is a monotone function of \(R\) and after the first round results are announced aggregates all the information in individuals’ signals about the common value.
Assumption 1 is key to allowing us to characterize the equilibrium behavior in the second-stage bidding game. It plays a helpful role in the empirical analysis by removing the common information components from valuations and provides an auction-specific measure of the bidders perceived values to help control for across-auction heterogeneity. Assumption 1 also imposes that after learning the $p^{IMM}$, expectations do not depend on the initial private signal, $\xi$, that informed bidders’ first round quotes. This is a reasonable approximation of the optimal updating behavior because the $IMM$ aggregates information from all participants so is likely to be more informative than $\eta$.\footnote{Appendix C shows that for calibrated parameters, bidders’ expectations after learning the $IMM$ have low variance, and Table A.2 shows first round quotes are not strongly correlated with second round bids.}

The bidders problem trades off: by decreasing their quote, a bidder decreases the IMM, which decreases the expected price floor or ceiling and may signal to opponents a lower expected resale value for the bond, leading to lower opposing bids. However, by lowering their quote, dealers (i) increase the chance that their quote is not in the average, (ii) reduce their bid that will be carried over into the second stage, and (iii) increase the chance that they receive a fine. I argue that the costs from the fine disciplines the distortions so that although the $IMM$ may be a biased, it is monotonic in the common value component $R$.

3.5. Stage 2: Auction Payouts. In stage 2, bidders submit either a supply or demand curve as appropriate to clear the open interest announced after the first stage. Because this submission occurs after learning the $NOI$, which is a function of opponents’ choices made given their private information, the players’ expected distribution of opponents’ signals in this stage will depend on the first stage strategies. In addition, the distribution of opponents signals that each player expects will differ due to their knowledge of their own contribution to the $NOI$. This distribution can be written as:

$$F_{m|\Omega,y^0_i} = \int_{[m,\bar{m}] \times [y^c,\bar{y}^c] \times j \neq i} 1(NOI - y^0_i = \sum_{j \neq i} y_j(m_j, \eta, y^c_j) + y^c_j) \prod_{j \neq i} f(m_j) \Upsilon(y^c_j) dmdy^c.$$ 

Assume that these beliefs leave positive mass on every $m \in [m, \bar{m}]$, are absolutely continuous, and have no holes and no mass points. I will show that these properties are satisfied such that beliefs are consistent with Bayesian updating given the equilibrium strategies. Therefore, these strategies and beliefs are a Perfect Bayesian equilibrium.

The bidder chooses the strategy in $\gamma_i$ in order to maximize the expected auction profits. Let the distribution of opponents’ signals given the information in $\Omega$ be denoted by $L$.

$$\Pi^A(m_i, y^0_i, L, \Omega) = \max_{\gamma \in (\cdot | m_i, y^0_i, \Omega)} \int_m \int_q \Pi(m_i, b, q) dH(q, b | m, L, \gamma(m, y^0_i, \Omega)) dL(m | y^0_i, \Omega) - \sum_{k=1}^{K_i} c_{ik}.$$
The bidder’s profits in the auction have three components: (i) the cash settlement on their existing CDS positions—paid at the auction clearing price, (ii) the auction payments—made for the quantity bought in the auction plus the commitment from the first round, and (iii) the benefit from the bonds bought/sold in the auction. I rewrite the problem as:

$$
\max_{\{b_k,q_k\}_{k=1}^{K_i}} \sum_{k=1}^{K_i} \int_{b_{k+1}}^{b_k} \left[ (100 - p)(n_i) \right] \text{cash settlement} \\
+ \left[ R + v(q_k - y_i, s_i) \right] q_k - \left[ p(q_k - y_i) \right] f(p) \text{Benefit from final bonds} \\
\text{quantity}\ 
$$

where $c_{ik}$ is the cost of submitting each step and defined in section 3.2. For demand bids, optimality of the chosen bid implies the set of first order conditions (FOC) given in Equation 1:\textsuperscript{20}

When a tie occurs the quantity is split pro-rata. In any PBNE, for almost every $s_i$, every step $k$ in the $K_i$ step function must satisfy the following equation:\textsuperscript{21}

$$
Pr(b_k > P^c > b_{k+1} | y_i^c, \Omega) \left[ R + v(q_k - y_i, s_i) - E_{M-a|\alpha_i}(P^c | b_{ik} > P^c > b_{ik+1}; y_i^c, \Omega) \right] \\
Pr(b_k = P^c \land Tie) E[\left[ R + v(q(S, \gamma(S)) - y_i, s_i) - b_k \right] \frac{dQ^c}{dq_k} | P^c = b_k \land Tie] \\
Pr(b_{k+1} = P^c \land Tie) E[\left[ R + v(q(M, \gamma(M)) - y_i, s_i) - b_{k+1} \right] \frac{dQ^c}{dq_k} | P^c = b_k \land Tie] \\
= (q_k + n_i - y_i) \frac{\partial E[p^c; b_k > p^c > b_{k+1}; y_i^c, \Omega]}{\partial q_k}.
$$

Simplifying to remove ties and collecting $\alpha_i = y_i^c, \Omega$:

$$
Pr(b_k > p^c > b_{k+1} | \alpha_i) \left[ R + v(q_{ik}, s_i) - E_{M-a|\alpha_i}(P | b_{ik} > p^c > b_{ik+1}; \alpha_i) \right] \\
= (q_k + n_i - y_i) \frac{\partial E[p^c; b_k > p^c > b_{k+1}; \alpha_i]}{\partial q_k}. \tag{2}
$$

A similar argument can be applied to bids to supply bonds, leading to the equation:

$$
Pr(b_{ik-1} > p^c > b_{ik} | \alpha_i) \left[ -R - v(q_{ik}, s_i) + E_{M-a|\alpha_i}(P | b_{ik-1} > p^c > b_{ik}; \alpha_i) \right] \\
= (q_k + n_i - y_i) \frac{\partial E[p^c; b_{ik-1} > p^c > b_{ik}; \alpha_i]}{\partial q_k}. \tag{3}
$$

\textsuperscript{20}This is constructed from perturbations in the quantity $q_k$. This FOC does not account for any bounds on the price. In reality an auction to buy (sell) has a price ceiling (floor) of 2 * spread above (below) $p^IMM$. This may lead to corner solutions where a dealer purchases (sells) all the quantity at the price cap (floor). Whether this is a concern in practice depends on the support of $(n - y)$. This does not occur often in the observed bidding data and so I ignore this case in the discussion.

\textsuperscript{21}This result is derived in Kastl (2011) for the standard multiunit auction. Once I apply the conditioning described above, my model is a special case of this game.
To simplify expressions, in the following sections I focus on the case of excess supply. All the expressions are easily adapted to the case of excess demand.

Equation 1 is similar to the FOC derived in Kastl (2011), with the important additions of the price impact from cash settlement and initial quantity commitments. The LHS of the equation represents the marginal cost of quantity shading—the difference between the marginal utility and the expected price; while the RHS represents the marginal benefit of quantity shading from the savings on the inframarginal units. There are two important differences relative to Kastl (2011): (i) bidders learn about the expected level of competition and the total supply based on the NOI and so the expectations condition on this outcome, (ii) the CDS position, less quantity commitments, influences the importance of the price savings from quantity shading. For buyers of CDS it increases the number of units for which they pay the price, making them more sensitive to the price changes that they may cause. This causes CDS buyers to bid lower prices to buy (or makes them willing to supply more at lower prices)\(^{22}\)

3.6. First Stage Quantity. The first stage quantity choice involves many strategic considerations. First, it changes the bidders exposure to the auction clearing price. Second, it changes the total quantity for sale in the auction (altering the distribution of marginal values to clear the market). Finally, it affects the expected level of competition for \(i\)'s opponents due to its impact on the announced quantity. The bidder chooses a quantity of bonds from \(y_i \in Y_i\) in order to maximize their expected profits from the auction:

\[
\max_{y_i \in Y_i} E[\Pi^A(m_i, y_i + y_i^c, \Omega, L)|m_i, \eta_i].
\]

**Assumption 2.** \(y_i^c\), the set of customer order shocks, is independent of the dealers own position \(n_i\) and has full support on the set of possible NOI.

**Assumption 3.** Each dealer draws a cost \(\kappa\) of submitting a nonzero \(y_i\). The support, \(Supp(\kappa)\), includes costs that satisfies the following. \(\exists \delta_n > 0\) such that \(\forall n_i \in [-\delta_n, \delta_n]\) there exists an open set of signals \(\tilde{s}_i\), for which \(\forall y_i \in Y_i\), \(\exists \Delta > 0\) such that \(\max_{\delta \in \{\delta|\delta|\leq |\Delta|\}} \Pi(y_i + \delta) - \Pi(y_i) \leq \kappa_i \in Supp(\kappa)\).

The assumption is required to guarantee that for some positive mass of signals (with net CDS positions sufficiently close to zero), it is optimal for them to choose \(y_i = 0\) for

\(^{22}\)The existence of equilibrium is discussed in Online Appendix D.1. The existence of equilibrium in multiunit uniform price auctions with restricted strategy sets is an open question, however in the standard setting Kastl (2012) proves the existence of an epsilon equilibrium. That result does not apply to the CDS setting. I follow Kastl (2011) and impose a fine discrete grid of price levels. This is the case in practice as bidders can only express their prices to the nearest 1/8th of a cent.
any customer order shock that they receive.\footnote{For example, the assumption is satisfied if the cost distribution has an unbounded support.}

**Proposition 1.** $\forall NOI \in [\overline{NOI}, \overline{NOI}]$, the probability density function $f_{NOI}(NOI) > 0$ and is continuous.

**Proof.** By assumption, the cost of submitting is greater than the largest jump in profits between two neighbouring choices of $y$. This implies that there is some positive mass (some interval in $n$ near $n=0$) of signals whose optimal first round choice is $y_i = 0$. These bidders pass through shocks from client orders directly in their initial orders. This means that the density of NOI is continuous, with full support. \hfill $\Box$

The presence of directly submitted customer orders means that it is never possible for a bidder to distinguish which part of the observed NOI arose from submissions by the bidders and which part from customers. This means a bidder cannot rule out any vector $m$ from the observed NOI. Updating consistent with Bayes rule then implies that the distribution of private information $F(s_0,n_0,s_1,n_1,...,s_N,n_N|\Omega, y_o_i)$ satisfies the assumptions made in Section 3.5, therefore an equilibrium exists in the second stage with these beliefs. These shocks also mean that changes in a bidder’s first-stage commitment only shifts the mean of the conditional distribution of NOI. The first stage is therefore an incomplete information game, with continuous payoffs and so there exists an equilibrium.

### 4. Identification

The objective is to identify the joint distribution of marginal value curves and CDS positions, the distribution of step submission costs and the distribution of customer order shocks. I argue that all of these distributions are set-identified\footnote{I do not separately identify the signals and bond position in the function $v()$. Doing so would require additional structure. As there is a secondary market for bonds, there are not meaningful constraints for bond positions: i.e., a dealer could sell more bonds than owned.} Although additional restrictions on the shape of $v()$ can greatly simplify the identification discussion, in this section I provide intuition for how the data restrict the sets of possible distributions without the use of functional-form restrictions.

The main identification argument uses a GPV-type approach (Guerre et al. (2000)) to estimate the bidders’ marginal values for additional units that rationalize each observed bid. Unlike in GPV, or the standard multiunit auction case, where the unobservable value can be written as a function of observables, the credit event auction includes both unobservable values and CDS positions. For any CDS position, there is a unique unobserved
value that rationalizes the observed bids. Imposing that marginal values are monotone decreasing eliminates all the CDS positions which imply nonmonotonic marginal value curves. This leaves us with a set of CDS positions and marginal value functions that may be consistent with the behavior of each bidder.

4.1. Marginal Value and CDS positions. To begin, I show that a curve

\[ \tilde{v}(q) = v(q, s_i, B_i) + (n - y) \frac{\partial \mathbb{E} [P; b_k > p > b_{k+1} \mid \alpha_i]}{\partial q} \frac{1}{\mathbb{P}(b_k > p > b_{k+1} \mid \alpha_i)} \]

is identified at the subset of quantities where steps are submitted. As in Kast [2011], the terms \( \mathbb{P}(b_k > p > b_{k+1} \mid \alpha_i) \), \( E_{m_{-i} \mid \alpha_i}(P|b_{ik} > p > b_{ik+1}, \alpha_i) \), and \( \frac{\partial \mathbb{E} [P; b_k > p > b_{k+1} \mid \alpha_i]}{\partial q} \) are directly identified from observed bidding data. Rearranging equation 3 gives the newly defined curve \( \tilde{v}(q) \) as a function of identified objects.

\[ \tilde{v}(q) = v(q, s) + R - (n_i - y_i) \left[ \frac{\partial \mathbb{E} [P; b_k > p > b_{k+1} \mid \Omega, y_i]}{\partial q} \frac{1}{\mathbb{P}(b_k > p > b_{k+1} \mid \Omega, y_i)} \right] \]

\[ = E_{m_{-i} \mid \Omega, y_i}(P|b_{ik} > p > b_{ik+1}, \Omega, y_i) + (q_k) \left[ \frac{\partial \mathbb{E} [P; b_k > p > b_{k+1} \mid \Omega, y_i]}{\partial q} \frac{1}{\mathbb{P}(b_k > p > b_{k+1} \mid \Omega, y_i)} \right] \]

Given knowledge of the curve \( \tilde{v}(q) \) as well as the ratio of the price impact \( \frac{\partial \mathbb{E} [P; b_k > p > b_{k+1} \mid \alpha_i]}{\partial q} \) to the probability of clearing \( \mathbb{P}(b_k > p > b_{k+1} \mid \Omega, y_i) \), and the monotonicity and boundedness (assumed in the structure of the model) of the \( v(q, s_i) \) allows us to bound the \( v(q) \) and the possible \( n - y \) simultaneously. That is: for \( q_k > q_{k-1} \) it must be that \( v(q_{k-1}) \geq v(q_k) \). If \( \frac{\partial \mathbb{E} P}{\partial q} \) is not monotone across the set of \( q_k \) where the curve \( \tilde{v}(q) \) is observed, this provides an upper and lower bound. Intuitively, \( n - y \) must be such that the observed changes in \( \tilde{v} \) can be rationalized with \( \frac{\partial \mathbb{E} P}{\partial q} \) and a bounded, monotone decreasing function.

As an example, Figure 5 draws in black an observed bid curve defined by the set of steps. The dashed lines denote the observed ratio of price impact to win probability multiplied by different factors \((n - y)\). The round dots above the observed bids denote the \( \tilde{v}(q) \), calculated from the observed price impact, probability and expected clearing price. The triangular dots show the marginal value curve associated with a particular level of \((n - y)\). That is, the triangles are defined so that the sum of the triangle and dashed line give the round dots \( \tilde{v}(q) \). In the left panel the implied marginal value curve is not monotone decreasing. This allows us to conclude that the \((n - y)\) factor is too large and cannot be part of the identified set. The right panel illustrates two possible \((n - y)\). The dashed curves (in indigo and green) illustrate the ratio of price impact to win probability multiplied by each of these factors, respectively. The green triangles show
The black lines denote observed submitted bids. The dashed lines denote the observed ratio of price impact to win probability multiplied by different factors \((n-y)\). The round dots denote \(\tilde{v}(q)\). The triangular dots show the implied marginal value curve; the sum of the triangle and dashed line give the round dots \((\tilde{v}(q))\). In the left panel the \((n-y)\) factor is too large and the implied marginal value curve is not monotone decreasing. The right panel illustrates two possible marginal \((n-y)\) factors. The dashed curves (in indigo and green) illustrate the ratio of price impact to win probability multiplied by each of these factors. The green triangles show the implied marginal value curve associated with the green dashed curve and the indigo triangles the marginal value curve implied by the indigo dashed curve.

The bounds on \(\tilde{v}\) also help restrict the set of \((n-y)\) that are consistent with the observed bids. For example, if \(\tilde{v}(q_s) \leq 0\), the fact that \(v(q_s) \geq 0\) implies \((n-y) \frac{\partial E}{\partial q} \leq \tilde{v}(q_s)\) and for the upper bound, that \(100 + (n-y) \frac{\partial E}{\partial q} \geq \tilde{v}(q_s)\). This set of restrictions is quite informative, as many bids contain more than one step, which share the same \((n-y)\).

The information content of this identification argument depends on the observed differences in the value curve and price impact of shading across quantity levels, which all share the same \((n-y)\) within a given bidder. Take two quantity levels \(q_1 < q_2\) at which bidder \(i\) submitted bids.

\[
\tilde{v}(q_1) - \tilde{v}(q_2) = v(q_1, s_i) - v(q_2, s_i) + (n-y) \left( \frac{\partial E[P; b_1 \geq p \geq b_{1+1}|\alpha_i]}{\partial q_1} - \frac{\partial E[P; b_2 \geq p \geq b_{2+1}|\alpha_i]}{\partial q_2} \right).
\]

The LHS of this equation is observed, as is the term inside the final set of brackets. By monotonicity of the marginal value curve, the difference \(v(q_1, s_i) - v(q_2, s_i) \geq 0\) is known. If the difference in \((\frac{\partial E[P; b_1 \geq p \geq b_{1+1}|\alpha_i]}{\partial q_1} - \frac{\partial E[P; b_2 \geq p \geq b_{2+1}|\alpha_i]}{\partial q_2}) \geq 0\) then this provides an upper bound for \(n-y\), while if it is negative it provides a lower bound. Let's consider two pairs of points, with one pair providing an upper and the other pair the lower bound. The true
value is \((n - y) = \frac{\tilde{v}(q_1) - \tilde{v}(q_2) - (v(q_1, s_i) - v(q_2, s_i))}{\partial E[P_{b_1 \geq p \geq b_1+1} | \alpha_i]}\), which can be decomposed into the observed (difference in \(\tilde{v}\)) and the unobserved but bounded (difference in \(v(q)\)).

4.2. Entry costs and Client orders. I construct bounds on the distribution of client orders \((y^c)\) by leveraging the direction constraints on physical settlement requests. Since \(y^c\) are independent of the original position, this allows me to identify the distribution of client order shocks. To begin, take the bidders with \((n_i - y_i) > 0\) (and therefore \(y_i \geq 0\)). For these dealers, \(y^c \leq y^o\). This means that the distribution of \(y^o\) on this subset of bidders gives a valid upper bound on the client-order shock distribution: \(Pr(y^o \leq y | (n_i - y_i) > 0) \geq Pr(y^c < y | (n_i - y_i) > 0)\). Similarly, for dealers with \((n_i - y_i) < 0\) (and therefore \(y_i \leq 0\)), it must be the case that \(y^c \geq y^o\). This means that the distribution of \(y^o\) on this subset of bidders is a lower bound for the distribution of customer shocks. Because the \(y^c\) shocks are independent of \(n_i\), the only sample-selection comes from the mass of bidders where the sign of \(n_i\) cannot be inferred. This occurs if: (i) the bidder chooses to submit their entire position \(n_i = y_i\), (ii) \(0 \in [(n_i - y_i), (n_i - y_i)]\). This mass is observed and so by adding it to the upper bound from the selected sample we obtain an upper bound on the distribution of client shocks. The bounds on \(n_i + y^c_i = (n_i - y_i) + y^o_i\) and the distribution of \(y^c\) provide bounds on the distribution of \(n\), conditional on each curve \(v(q, s_i)\).

The distribution of costs for submitting an additional step \(\iota\) can be bounded from above by calculating the maximum profit difference a bidder could achieve by adding an additional step, and from below by comparing the true profit to expected profit with one less step. I do not consider identification of \(\kappa\) as it plays no role in the counterfactuals.

The discussion so far showed that the \(m_i\) are identified conditional on choosing a non-zero number of steps. However, this leaves a problem of selection on observables; which can be corrected. First note that for every signal there is a positive probability of submitting at least one step, as variation in the NOI reverses the set of bidders most likely to be excluded. Further, at each signal vector \(m_i\), I can calculate the expected change in profits from a one step bid. We can compare these differences to the distribution of costs, which is already identified, to calculate the probability of submitting zero steps at each \(m_i\).

5. Estimation

Despite being non-parametrically set-identified, a fully nonparametric estimation would require far more data than are currently available. Therefore, I impose some parametric restrictions to reduce the dimensionality of the problem. I perform tests supporting many
of these assumptions in Appendix 6.4. Online Appendix F presents an illustration of the nonparametric bounds in the data, as well as the full set of nonparametric estimates for the bidders’ insurance positions. There are several important challenges for estimation of this model: (i) the model is dynamic, (ii) dealers have both private information and private positions, and (iii) it is common to submit only a smaller number of steps.

To begin, assume that bidders’ marginal valuation curves are linear. That is, the marginal value can be represented by: (i) a value for the first unit of bonds acquired in the auction, and (ii) a rate at which the marginal benefit from additional units declines.

**Assumption 4.** The dimension of the private signal is 2 and the form of the marginal value is linear

\[ v(q, s) = s_1 - s_2 q. \]

This implies that for all bidders who place more than three steps, the linear restriction is over-identified and therefore these cases can be used for testing. In Online Appendix E.6 I show that the R-square from the linear fit is high and that the addition of a quadratic term does not result in a large change in either the R-square or model estimates.

**Assumption 5.** \( p^{IMM} \) is a sufficient statistic for the observed (across auction) variation in bond traits \( Z \) and these traits only impact \( R \), not the joint distribution of \( s_1, s_2, n \).

When estimating the distribution of opposing bids that a bidder expects to face, we need to condition on \( \Omega, y_i^q \) and the observed characteristics of the set of bonds eligible for submission to the auction. The observed IMM picks up a large amount of the across auction heterogeneity, including differences due to the observable bond traits and those that are observable to bidders but not the econometrician. I test whether bond traits and volumes affect values beyond their role in determining the IMM. Results are reported in Online Appendix E.5. These variables have no explanatory power beyond the IMM and therefore I treat the IMM as a sufficient statistic for the observable differences in \( Z \). This greatly reduces the dimensionality of the estimation problem and as a result improves power.

I parameterize the distribution of \( s_1, s_2 \) and \( n \) using 4-, 4- and 6-parameter cubic B-splines, respectively, to describe the quantile functions of the marginal distributions and impose that the correlation structure is given by a Gaussian Copula. I parameterize the distribution of entry costs as Normal and estimate the mean and variance and the distribution of the customer order shock as mean zero Normal, and estimate the variance.

In the previous section I showed that the model is non-parametrically set identified. I now discuss why the model that I estimate with these additional restrictions is point
identified. First, for every bidder that submits three or more steps we learn a unique \((n - y), s_1, s_2\). For any combination of \((n - y), s_1, s_2\) we also know the difference in profits from using \(K = 1, 2, 3, \ldots\) steps. By comparing the probability of submitting \(K\) steps when the difference in expected profits are some fixed level, we can identify the probability of a submission cost exceeding/not exceeding that level. Since the submission cost distribution goes from \([0, \infty)\) and the change in expected profits are weakly positive, then for any draw of \((n - y), s_1, s_2\) the bidder will sometimes submit three or more steps and so the probability of that vector is known. For each bidder we also observe the \(y^o\). As in the previous section, we can construct bounds on this distribution using restrictions on the eligible submissions. This does not guarantee a unique \(\sigma_{y^c}\). However, if we assume that the distribution of \(y_i\) has a compact support, then given the normality of the errors \(y^c\), results from Bertrand et al. (2019) insure identification given \(y^o\). To identify the common values, I assume \(R = b_p(p^\text{IMM}), \ b_p(0) = 0, b_p(100) = 100, \ p^\text{IMM}, \ E[s_1|p^\text{IMM} = 0]\) and \(E[s_1|p^\text{IMM} = 100]\) are observed, \(s\) is independent of \(p^\text{IMM}\), so \(b_p(p^\text{IMM}) = E[s_1|p^\text{IMM} = 0] - E[s_1|p^\text{IMM} = 100]\).

The estimation contains three distinct steps. In the first, I use a weighted resampling estimator developed in the literature on multi-unit auctions to estimate \(Pr(b_k > p > b_{k+1}|\Omega, y^o)\), \(E_{m-i|\Omega, y^o}(P|b_{ik} > p > b_{ik+1}, \Omega, y^o)\) and \(\frac{\partial E[P|b_k > p > b_{k+1}|\alpha_i]}{\partial q_k}\), where weights are used to control for selection on observables as well as other behavioral responses of bidders to these observables. In the second step I estimate functions which approximate the differences in profits for a given bidder of bidding using 0, 1, 2, or 3 steps. In this way I can control for selection. Conceptually, this calculation could be made inside of the final step, however nesting this calculation is not computationally feasible. In the final step, I combine the estimates of these components, with the restrictions from the FOCs, to form a set of moment conditions which allow for the parameters of the joint distribution of \(s_1, s_2\) and \(n\), and the parameters of the entry costs and customer order shock distribution to be jointly estimated. The next sections discuss each of these components.

5.1. Stage 1: Resampling. All the terms in the bidder’s FOC are functions of three terms: (i) \(Pr(b_k > p > b_{k+1}|\Omega, y^o)\)—the probability of being allocated quantity \(q_k\) associated with price bid \(b_k\), (ii) \(E_{m-i|\Omega, y^o}(P|b_{ik} > p > b_{ik+1}, \Omega, y^o)\)—the expected clearing price conditional on winning \(q_k\), and (iii) \(\frac{\partial E[P|b_k > p > b_{k+1}|\alpha_i]}{\partial q_k}\)—the price impact of increasing \(q_k\).

In the first stage I construct estimates of these terms, following Hortaçsu and McAdams (2010), Kastl (2011) and Clark et al. (2021). To handle shifts in bids due to observable dif-

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25For example, when a bidder submits two steps, the FOC provides a set of possible \(m_i\), but some points in that region are very unlikely to bid only 2 steps and this must be accounted for in the aggregation.
ferences across auctions. Hortaçsu and McAdams (2010) propose a conditioning approach weighting by the traits in the resampling process used to approximate these terms. With this approach, weights are used to control for both selection and behavioral responses to observables $\Omega, y^o$. The challenge in this setting is that $\Omega$ includes $p^{IMM}, NOI, y_i$, so that the kernel weights must reflect the similarity of the information set faced by individual bidders. To do this I use the logic that bidders with similar information sets should expect similar opposing bids. I use kernel weights so that opponents of bidders with similar $NOI$ and similar $y_i$ are most likely to be included in the simulated residual supply curves.

The resampling scheme should put the most weight on an opponent showing up that looks like the opponents of a bidder with a particular information set. For example, if bidder 1 in auction 1 and bidder 3 in auction 15 have the same information sets, they should expect to face opposing bids from the same distribution of opponents’ bids. To evaluate this in a tractable way, begin by finding the bidder with the most similar information set to bidder $i$, in each other auction. For each of these most similar bidders measure the difference between their information sets, and, using this distance, define an auction-level weight that will be applied to all the opponents of that most similar bidder, while giving zero weight to resampling the single most similar bidder. This gives weights:

$$w_{Aj} = \begin{cases} \frac{\sum l \in A K\left(\frac{\alpha_l - \alpha_i}{bw}\right)}{\sum l \in A K\left(\frac{\alpha_l - \alpha_i}{bw}\right)} / \mathcal{I}_{dj} & l^* \neq j \\ 0 & l^* = j \end{cases}$$

(4)

Nothing in the information set is estimated; these components are all observed. Implementing this in practice requires resampling from the quantity and price shares, which helps avoid extreme draws. This normalization has no effect asymptotically, because as the bandwidth shrinks, samples are drawn from auctions with identical $p^{IMM}, NOI$.

5.2. Stage 1b: Selection. In estimating the second stage of the model, it is important to incorporate the bidders who submit less than three steps, despite the fact that the signals and private position $[s_1, s_2, n]$ that rationalizes their observed bid cannot be uniquely pinned down. For bidders that use less than three steps, there are three unknown values to estimate but less than three observed points. Rather than a unique vector of private information, therefore, the restrictions from the FOCs give us a set of signals and positions that could be consistent with the observed bid.

Each bidder decides how many steps to use in their bid function by comparing the expected profits from including an additional step to the cost of submitting a bid with that step. Because the differences in expected profits depend on bidders’ private information,
some \([s_1, s_2, n]\) are more likely to result in submissions with a given number of steps. The probability that a type \([s_1, s_2, n]\) submits \(K\) steps, can be calculated by comparing the expected benefit to this type of bidder of including an additional step to the cost distribution of the individual-specific random cost of submitting an additional step.

To incorporate the bidders who submit less than three steps, I integrate over the set of possible values consistent with the observed bid while re-weighting each bidder type in the integral to account for the probability that a bidder of that type would submit \(K_i\) steps. Given the practical difficulty of computing expected differences in profits from submitting an alternative number of steps for each bidder, I specify these differences as:

\[
\Pi(3, (s_1, s_2, n - y), \Omega, Z) - \Pi(2, (s_1, s_2, n - y), \Omega, Z) = h_3((s_1, s_2, n - y), Z, \Omega, \beta) + u,
\]

\[
\Pi(2, (s_1, s_2, n - y), \Omega, Z) - \Pi(1, (s_1, s_2, n - y), \Omega, Z) = h_2((s_1, s_2, n - y), Z, \Omega, \beta) + u,
\]

where \(h_k\) is a second order complete polynomial in \(n, s_1, s_2, 1(\text{NOI} > 0), \text{IMM}\). I then compute estimates of these equations by calculating the optimal bids with 1, 2, and 3 steps for 1000 random draws of possible signal vectors, uniformly sampled between the bounds of the signals \([\underline{s}_1, \bar{s}_1]x[\underline{s}_2, \bar{s}_2]x[\underline{n}, \bar{n}]\), where the bounds are estimated using the set of bidders who submitted more than three steps in the data (and hence for whom the signal vector is known). I assign each signal vector to an auction with traits chosen from a randomly selected auction and compute the profit differences on that sample.\(^{26}\)

5.3. Stage 2: Aggregation. In the first stage I obtained consistent estimates of the coefficients in a linear system that bidders’ bids must satisfy. Depending on the number of steps submitted this system might be over, exactly, or under-identified. I then solve the optimal set of parameters using simulated method of moments where simulation allows for integration over the multiple solutions that satisfy the system of equations for \(K_i = 1, 2\).

The linear system formed within a bidder by their set of \(K_i\) optimality conditions gives a system of equations, where each step satisfies:

\[
s_1 - s_2q - (n_i - y_i)\left[\frac{\partial E[P, b_k > p > b_{k+1} | \Omega, y_i^\alpha]}{\partial q_k}Pr(b_k > p > b_{k+1} | \Omega, y_i^\alpha)\right] = \frac{\partial E[P, b_k > p > b_{k+1} | \Omega, y_i^\alpha]}{Pr(b_k > p > b_{k+1} | \Omega, y_i^\alpha)}.
\]

\(^{26}\) I truncate the change in expected profits (the dependent variable) at $500M to improve the fit at levels where the cost shock plays an important role (small profit differences). The fitted model still implies probabilities of submitting an extra step very close to 1 for truncated bidders.
For each bidder, I calculate an estimate of the private positions \([s_1; s_2; (n-y)]_i\). Collecting the terms above and rewriting this in matrix form gives
\[
\hat{A}_i[s_1; s_2; (n-y)]_i = \hat{d}_i.
\]
For all bidders the objects \(A, d\) are measured with error. For bidders with fewer than three steps, I simulate \((n-y)\) and so all the finite sample errors occur in the dependent variable. However, when three or more steps are submitted, the term which multiplies \((n-y)\) is a regressor with measurement error. I adopt a minimum distance shrinkage approach to correct for these errors.\(^{27}\)

I then solve the following set of moment conditions simultaneously. The model is basically a random effects model, with selection and censoring, and where the explanatory variables contain measurement error. Standard errors are calculated using the bootstrap, where resampling is done at the auction level and a bootstrap draw, is held fixed throughout the first stage resampling estimator, the selection estimation and the second stage.\(^{28}\)

For each of the marginal distributions for levels of \(\alpha\) at each decile, where \(m_j\) denotes the jth element of \(m_i\), \(x_j(\alpha)\) is the inverse of the marginal distribution \(F(x_j|\theta) = \alpha\), and \(M_i\) is the set of \(m, y_i\) for which \(\hat{A}m = \hat{d}\) giving
\[
E[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} 1(m_j \leq x_j(\alpha))(m, y^0 - y^c) \in M_i) Pr(K|\Delta \Psi(m_i, y_i), \theta) h(y^c; \theta)f_0(m; \theta)dmdy^c - \alpha = 0, \tag{5}
\]
and a moment condition for the covariance
\[
E[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (m - \mu_o)(m - \mu_m) 1((m, y^0 - y^c) \in M_i) Pr(K|\Delta \Psi(m_i, y_i), \theta) h(y^c; \theta)f_0(m; \theta)dmdy^c - \theta_\mu] = 0. \tag{6}
\]
To pin down the distribution of \(y^c\), I leverage the restrictions on \(y_i\) that each bidder can submit. These restrictions together with the observed \(y^o\) imply a set of possible submissions \(\mathcal{Y}_y\). When combined with a \(y^c\), each \(y_i \in \mathcal{Y}_y\) is associated with some \(y^c_i\), and it must be the case that when these sets are aggregated across bidders the implied probability of being below some point \(\hat{y}\) lines up with the probability in the \(y^c\) distribution:
\[
E[\int 1(y^c \leq x_j(\alpha))(y^o - y^c \in \mathcal{Y}_y)h(y^c; \theta)dy^c - h(y^c; \theta)] = 0.
\]

\(^{27}\)The resulting bias is given by \([s_1; s_2; n-y] = [s_1; s_2; n-y] + (A^TA)^{-1}A^Tc_2(-n-y-q).\) To evaluate this bias, I calculate measurement error \((c_2)\) by bootstrap resampling of the first stage, and apply a correction by solving that equation. Note that asymptotically this \(c_2\) vanishes and so even without the correction the estimates are consistent.

\(^{28}\)The bias-correction factor for bidders who submit 3 or more steps is held fixed across replications. It is estimated using 1000 bootstrap replications of the first stage, and it would be computationally infeasible to correct this on each sample. Additional uncertainty from this term is likely to play only a small role.
I also assume that \( y_i \) has a compact support given by the minimum and maximum holdings reported by Paulos et al. (2019) and verify that the estimated \( y_i \) are inside this support.

Finally, to pin down the parameters of the \( c_i \) distribution, I use the observed probability of submitting \( K \) steps, along with the observed differences in the profit functions to construct moments:

\[
E[\Phi(\Delta \Pi_{32}, \hat{\mu}, \hat{\sigma}) - 1(K_i = 3)] = 0, \tag{7}
\]
\[
E[(1 - \Phi(\Delta \Pi_{32}, \hat{\mu}, \hat{\sigma}))\Phi(\Delta \Pi_{21}, \hat{\mu}, \hat{\sigma}) - 1(K_i = 2)] = 0, \tag{8}
\]
\[
E[(1 - \Phi(\Delta \Pi_{21}, \hat{\mu}, \hat{\sigma}))\Phi(\Delta \Pi_{10}, \hat{\mu}, \hat{\sigma}) - 1(K_i = 1)] = 0, \tag{9}
\]
\[
E[(1 - \Phi(\Delta \Pi_{10}, \hat{\mu}, \hat{\sigma})) - 1(K_i = 0)] = 0, \tag{10}
\]

for three, two, one, and the zero steps respectively, where \( \Delta \Pi_{jk} \) denotes \( \Pi(j, m_i) - \Pi(k, m_i) \), integrated over the possible vectors \((m_i, y_i)\) with parameters \( \theta \) as in the previous conditions (E.g., Equation 5).

6. Results

6.1. SMM Estimates. The estimated parameters are presented in Table 3. The distribution of CDS positions is presented in tens of millions of dollars. The signal distribution is in terms of cents over or under the common value component. The signal distribution is truncated at an auction-specific minimum to insure bidders have non-negative values. The marginal distributions implied by these estimates are plotted in Figure OS.4. The distribution of CDS positions \((n)\) implied by the estimation is fairly close to the distribution reported in Paulos et al. (2019). The estimated correlation between \( s_1 \) and \( n \) is negative. This is consistent with the incentive of bidders to hold too many CDS in order to avoid being constrained during the credit event auction process, as discussed in Du and Zhu (2017). It is also consistent with bidders with low bond values buying more insurance.

6.2. Expected Surplus. In order to contextualize the estimates I compare the expected surplus and expected change in price that would result if bidders used truthful bidding, i.e. if bidders reported directly their implied value functions. This comparison removes the incentive to bias the price from the CDS contract position as well as from the competitive effects from information rents. The results integrate over possible draws of the individual private information using 1000 simulated draws of potential bidders. The results of this calculation show that, on average, the prices are lowered by a median of 2.07 cents on the dollar, or mean of 2.20 cents on the dollar, as a result of market power in the auction.
Table 3. Value Distribution Parameters

This table presents the coefficients for the spline quantile functions for the three marginal distributions and correlations from estimating equations 5 and 6. Standard errors in parentheses.

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Slope</th>
<th>CDS position</th>
<th>IMM-bias</th>
<th>Other Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7.361</td>
<td>0.0173</td>
<td>-18.911</td>
<td>1.341</td>
<td>Entry cost mean</td>
</tr>
<tr>
<td>(1.829)</td>
<td>(0.070)</td>
<td>(2.439)</td>
<td>(0.270)</td>
<td>(1.688)</td>
</tr>
<tr>
<td>-0.785</td>
<td>0.1094</td>
<td>-12.826</td>
<td>2.895</td>
<td>Entry cost Std</td>
</tr>
<tr>
<td>(1.643)</td>
<td>(0.440)</td>
<td>(2.018)</td>
<td>(0.348)</td>
<td>(3.610)</td>
</tr>
<tr>
<td>-0.262</td>
<td>2.815</td>
<td>0.804</td>
<td>4.331</td>
<td>Client Shock Std</td>
</tr>
<tr>
<td>(1.574)</td>
<td>(0.425)</td>
<td>(0.632)</td>
<td>(0.250)</td>
<td>(1.479)</td>
</tr>
<tr>
<td>9.026</td>
<td>2.624</td>
<td></td>
<td></td>
<td>Correlation: $s_1, s_2$</td>
</tr>
<tr>
<td>(1.800)</td>
<td>(0.898)</td>
<td></td>
<td></td>
<td>-0.197</td>
</tr>
<tr>
<td>20.179</td>
<td>10.826</td>
<td></td>
<td></td>
<td>Correlation: $s_1, n$</td>
</tr>
<tr>
<td>(0.674)</td>
<td>(2.43)</td>
<td></td>
<td></td>
<td>-0.466</td>
</tr>
<tr>
<td></td>
<td>19.677</td>
<td></td>
<td></td>
<td>Correlation: $s_2, n$</td>
</tr>
<tr>
<td></td>
<td>(1.525)</td>
<td></td>
<td></td>
<td>0.565</td>
</tr>
</tbody>
</table>

These results are similar to the gaps between the auction price and secondary market prices described in Figure 1. Working with the estimates of my structural model I can evaluate the shading in a broader sample (not limited to those with trade reporting requirements to TRACE), and using a more direct measure of bidders’ willingness to pay.

6.3. **First-stage behavior.** Analytic solutions for the first-stage optimal strategies are unavailable and numerical solution of these strategies would require calculating the expected profits in stage 2 for every own submission and set of opponents’ submissions in round 1 conditional on the vector of private information. Instead of solving these strategies I simply present the pattern of choices observed. I examine the correlations using the estimated private information together with the raw data on first stage submissions. Even post-estimation we do not pin down the private information for a particular individual and so this calculation is done by integrating over the set of possible draws in $\mathcal{M}_i$.

The regression estimates show that the size of the initial submission is positively correlated with the initial value for bonds but has limited movement with the positions $n$ and the slopes $s_2$. The expectation of the common value component relative to the opponents expectations also seems to play a key role: bidders with high signals about this component submit substantially smaller physical settlement requests (sell fewer bonds).

6.4. **Evaluating Assumptions.** In setting up the model I made four important assumptions. First, I assumed that the dealer was able to jointly optimize the entire set of bids
Table 4. First Stage Submissions Results

This table presents the correlation of the private information with the choice of y. Standard errors in parentheses (not accounting for estimation error in \(n, s_1, s_2\)).

<table>
<thead>
<tr>
<th></th>
<th>(y_i)</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)</td>
<td>-0.0012</td>
<td>(0.0149)</td>
</tr>
<tr>
<td>(s_1)</td>
<td>0.512</td>
<td>(0.231)</td>
</tr>
<tr>
<td>(s_2)</td>
<td>-2.836</td>
<td>(1.896)</td>
</tr>
<tr>
<td>(\eta - R)</td>
<td>-57.485</td>
<td>(10.315)</td>
</tr>
<tr>
<td>constant</td>
<td>8.151</td>
<td>(1.474)</td>
</tr>
</tbody>
</table>

which they submit. This could be a problem if many of the steps are submitted on behalf of customers, reflecting orders that the dealer received and which they decided to pass through directly as part of their bid. Online Appendix I estimates bounds on the share of bids submitted by dealers and customers, and shows that estimates of the dealers’ insurance positions are similar when accounting for customer orders.

Second, I assumed that bidders truthfully report their initial price quotes. I evaluate this assumption in Online Appendix E.4. First, I compare the expected price change a bidder could achieve by manipulating their IMM quote with the size of a fine and show that the fine is much larger than the expected benefit of a small price change. Second, I examine the correlation between a bidder’s own quote and their quantity submission: if a bidder is using the quote to manipulate the outcome these should be positively correlated. Instead, I find a small negative correlation. Third, I assumed that conditioning on the IMM is sufficient to capture all the relevant across-auction heterogeneity in the bonds. Online Appendix E.5 presents a set of regressions showing that bond traits have no explanatory power for bids conditional on the IMM and open interest. Finally, I assumed marginal values were linear. To test this assumption I first show the R-square from the within-bidder fit for bidders with more than 3 steps is high, and then show that the estimated positions and value curves are highly correlated with the estimates from re-estimating the model with the inclusion of a quadratic term. Results are presented in Online Appendix E.6.

6.5. Decomposition. In this section I perform a decomposition to understand the role of the various strategic channels that produce the observed bidding behavior. To do this, I present a partial equilibrium exercise which eliminates various strategic impacts, and allows each bidder to re-optimize their bid holding fixed the behavior of their opponents.
The current two-stage format results in three main features. The first is learning based on the NOI. Learning from the NOI can be decomposed into two different parts: learning about the total supply that is offered, and learning about opponents’ private information, resulting from the fact that the NOI is constructed from their endogenous quantity commitments. The second are the uni-directional constraints. If the second stage were a double auction, some bidders might like to submit bids supplying the good and some demanding it in the relevant price range. The current format, however, restricts bidders’ possible orders, resulting in the exclusion of some bidders who are unable to express their preferences. This exclusion changes the expected price impact, and hence the desired shading of the remaining bidders. The third feature is the position-reduction effect. When a bidder commits to $y_i$ in the first round, it effectively reduces the number of its insurance contracts which are settled at the final price.

I consider three separate experiments, for each I focus on changes in the price bid for 10 percent of the total quantity offered. In the first, I ask how each bidder $i$ would change their response to the existing bid distribution if they were unable to condition their expectations of the residual supply curves they face on the NOI. This means $i$ has no information about the total quantity of bonds available to them, nor are they able to refine their expectations about the competing bids they will face. To calculate the bidders’ unconditional expectations, I simulate residual supply curves where both opposing bids and excess supply or demand from the first stage are drawn randomly. When subjected to this uncertainty, the bidders decrease their bids by an average of 1.925 cents, suggesting that announcement has a pro-competitive effect. In the second experiment I calculate how bidder $i$ would respond if they knew the quantity being sold but were unable to condition on this when forming expectations about the set of competing bids they are likely to face. Relative to the first exercise, this decreases the uncertainty about the location of the residual supply curves. Bidders respond by increasing their bids by an average of 0.671 cents. In the final exercise I examine the effect of position reductions by replacing $(n - y)$ by $n$ (set $y = 0$) and recalculating the bidder’s optimal bid. The average bid falls by 0.106 cents per dollar, but includes bidders that submit zero first stage requests and hence make no change to their bids (57% of the sample). The median change is 0 while the 5th percentile is -1.225 and 95th percentile 2.104. The decomposition does not allow

\footnote{Results for 50 and 90 percent are similar in all cases except the experiment eliminating the NOI announcement, where the changes are smaller, suggesting submitted bid curves become less steep.}

\footnote{This does not capture the effect of learning from the endogeneity of the quantity for sale. That effect would account for opposing bidders’ responses to the quantity level, when the quantity was not informative about the signals but only capacity. Therefore, it cannot be calculated within a partial equilibrium setup.}
us to analyze the role of constraints, which operate through the first stage commitments.

7. Counterfactual

The first counterfactual I consider is a change from the current two-stage auction to a double auction format. The double auction asks bidders to submit a step-function bid of quantities to buy or sell and prices at which they would be willing to trade. This eliminates the direction restriction and the first stage from the current design. Without the direction restriction, the first stage quotes would never be binding so the key change is the acceptance of both supply and demand orders in the auction. A major challenge for the CDS auction mechanism is that the final clearing price establishes both the CDS cash settlement amounts and serves as a price for the exchange of bonds. Because dealers tend to be net owners of CDS, the cash settlement feature provides dealers with a coordinated incentive when strategically bidding. As a second counterfactual I maintain the double auction design but impose restrictions on participation based on bidders’ CDS positions.

An important property in establishing the theoretical result that the double auction improves on the current format, is the requirement that the CDS positions of the participants are net zero (Du and Zhu (2017)). The estimation results (and the raw data explored in Paulos et al. (2019)) suggest that at the time of the auction, dealers are net buyers of CDS. This introduces a price bias, as it incentivizes more shading on the demand side of the market than the supply side—which will tend to push prices down. In the rest of this section I examine the auction outcomes allowing for step-function bidding, nonzero average positions, and private draws for the slopes of marginal values.

The computation of equilibrium in multiunit auction models with step-function bidding has so far been an intractable problem. The challenge arises as equilibrium bid strategies map high-dimensional values $v(q)$ into high-dimensional sets of $K_i$ price-quantity pairs. These strategies may be highly nonlinear and little is known about their properties. This makes standard methods for computing these functions infeasible. I develop a method to compute the equilibrium in these settings. In Richert (2021) I provide a set of simulations to demonstrate the performance of the method. Section 7.1 describes the method. Section 7.2 provides details of the implementation. Results are presented in Section 7.3.

7.1. Counterfactual Solution Method. I propose to numerically solve for the equilibrium distribution of bids taking as given the distribution of values estimated from the data and the set of equations characterizing equilibrium behavior. To numerically solve for this distribution, I search for the set of bid-distributions for which the distribution
of types (e.g., private values) that rationalizes these bids in a Bayes-Nash equilibrium matches the known primitive distribution of types. The search proceeds in four steps: (i) guess a bid distribution, (ii) use the model equilibrium constraints to map the bids to values, (iii) check whether the implied distribution of values is the same as the known value distribution, (iv) if not: update the guess of the bid distribution and repeat steps (i)-(iii). This novel procedure can be formalized as the solution to a problem that is very similar (and in some cases equivalent) to indirect inference, Gourieroux et al. (1993).

Let the system of equilibrium equations (given by the FOC) be given by $D(b(q), G_B, \gamma) = [s_1, s_2, n]$. Define an auxiliary model, used to describe the value distribution, as follows:

$$\arg\min_{\alpha_s} \int K(\frac{\alpha_s - a}{h})dF_l(a) - L_{l,s}$$

for each element $\alpha_s \in \alpha$, where $L_{l,s}$ is the $s^{th}$ grid point in $L_l = (0.01, 0.02, ... 0.99)$ and $l \in \{s_1, s_2, n\}$. $K$ is a kernel function, $h$ a fixed bandwidth, $F_l$ denotes the marginal distribution of the $l^{th}$ dimension of private information. I solve for the set $\Gamma \equiv \{\gamma | Q(\gamma) = 0\}$ where $\gamma$ denotes a parameterization of the bid distribution $G_B$, and $Q$ is a criterion function $Q(\gamma) = \sum_s (\alpha_s(\gamma) - \alpha_0)^2$, measuring the distance between the parameters of the auxiliary model, $\alpha_0$, which obtain the best fit when $F_l$ is the true value distribution and those which obtain the best fit $\alpha(\gamma)$ when $F_l$ is the distribution implied by $D$ at $\gamma$.

The solution method does not solve directly for the equilibrium but for a set which contains the equilibrium. Despite this loss of information, the method has several advantages. First, solving the optimal bid to respond to any distribution of opposing bids is computationally intensive. My approach avoids preforming this calculation in the fixed-point search and instead uses the necessary conditions, which can be computed quickly.

Rather than parameterize the bid strategy function (as in Armantier et al. (2008)), which maps marginal value curves into bids, I parameterize the $2K$-dimensional joint distribution of bids. By parameterizing the bid distribution, the inversion mapping can be computed without reference to $F(v)$ and the entire set of counterfactuals consistent with a set-identified model can be computed quickly, a fact I leverage in Online Appendix J.

The proposed solution expresses errors in the distance between the true and implied distribution of values rather than in violations of FOCs. While Armantier et al. (2008) solves for the optimal constrained strategy to respond to the expected constrained be-

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31To see why, suppose that $\gamma_l$ describes an equilibrium distribution of actions for the true signal distribution $F_0$. By construction, $F_0(\hat{v}) = F(\hat{v})$ and so $\alpha_0 = \alpha$. This means $Q(\gamma) = 0$ and $\gamma_l \in \Gamma$. 
behavior of opponents, my approach adjusts the bid distribution towards the unconstrained best-response to the constrained behavior of opponents. This makes the results easier to interpret: at a potential solution, the values one would estimate from simulated bidding data cannot be distinguished from the true values. When strategies are quite restricted this may also lead my solution to provide a better approximation. Finally, I use a criterion function based on an auxiliary model to compare the implied and true value distributions. This allows high-dimensional value distributions to be compared on a lower dimensional set of traits, with the auxiliary model possibly misspecified. The auxiliary model can also smooth over potential discontinuities caused by: (i) jumps or undefined values of $D(b(q))$, (ii) any mass points or regions with zero probability in the distribution of $v$, which may cause the solution under other criterion functions to fail to converge.\footnote{Jumps or undefined $D$ may occur at bids that have zero probability in equilibrium. Mass points may arise in the general model but are ruled out by the empirical assumptions on $v$.}

7.2. Solution Details. In this section I discuss the choice of parametrization and criterion function in the counterfactual double auction game. These details are not required to understand the results presented in Section 7.3.

Let $G_{B,Q|K}(b_{i1}, \ldots, b_{ik_i}, q_{i1}, \ldots, q_{ik_i} | k_i) \pi_K(k_i)$ denote the joint distribution of prices, quantities, and steps, where $\pi_K$ is the distribution of steps. I restrict the strategy space to $\bar{K} = 8$. The bid distribution is described using sixteen parameters. Given this parametrization, for any value of $\gamma$, I can simulate the distribution of residual supply curves and then back out the implied private value distributions using the system of FOCs.

I parameterize the bid distribution by describing the distribution of quantity levels and price increments. I use a simulated set of 1000 bidders. For each bidder, I draw $K_i$ increment pairs, where $K_i$ is sampled uniformly on the support $[0, 1, 2, 3, \ldots, 8]$. For each bidder I then draw a set of $(e_k, f_k)$, which describe the price change and quantity level from a baseline at each of the $K_i$ steps. With these in hand we have $b_k = \sum_{k'=1}^{k-1} e_{k-k'} + \bar{\gamma}_p$ and $q_k = \sum_{k'=1}^{k} f_k + \bar{\gamma}_q$, where $\bar{\gamma}_p$ and $\bar{\gamma}_q$ are parameters that determine price and quantity level shifts that apply to all bidders. I allow the $(e_k, q_k, q_{k-1})$ to be correlated. I parametrize the marginal distributions of $e_1$ and $f_1$ using 4-parameter cubic B-splines, $G_{E_1}(:, \gamma_e)$ and $G_{F_1}(:, \gamma_q)$, characterized by parameter vectors $\gamma_{e_1}$ and $\gamma_{f_1}$ while the marginal distribution of $G_{F_k}(:, \gamma_f)$ for $k \in (2, ..\bar{K})$ as a beta distribution with parameter vector $\gamma_f$ and $G_{E_k}(:, \gamma_e)$ as a beta distribution with parameter vector $\gamma_e$ for $k \in (2, ..\bar{K})$. I model the correlation structure as a Gaussian copula $C[\cdot, \cdot; \gamma_c]$, where $\gamma_{c_2}$ is a $2 \times 2$ correlation matrix.
with elements \( \rho_{eq} \) and \( \gamma_{c3} \) is a $3 \times 3$ correlation matrix \( \rho_q, \rho_{eq} \), and the third correlation is \( \rho_q, \rho_{eq} \) which gives conditional independence between \( e_k \) and \( q_{k-1} \) given \( q_k \):

\[
G_{E,Q|K}(e_1, \ldots, e_5, f_1, \ldots, f_5|K_i) = \mathcal{C} \left[ G_{E_1}(e_1; \gamma_e), G_{F_1}(f_1; \gamma_f); \gamma_{c2} \right] \times \prod_{k=2}^{K_i} \mathcal{C}_3 \left[ G_{E_k}(e_k; \gamma_e), G_{F_k}(f_k; \gamma_q), G_{F_{k-1}}(f_k-1; \gamma_q); \gamma_{c3} \right].
\]

Finally, to account for the fact that the probability of \( K_i \) steps is not uniform, for each simulated bidder I calculate a weight that reflects the probability of appearing with \( K_i \)-steps. To specify this I assume that the probability of putting each additional step is Poisson, with parameter \( \gamma_n \). For notational convenience I collect all the relevant parameters into a single vector \( \gamma = [\bar{\gamma}_q, \bar{\gamma}_p, \gamma_e, \gamma_{c1}, \gamma_{c2}, \gamma_{c3}, \gamma_n] \).

For the criterion function I match the distance between the CDFs of the marginal distributions at a grid of points \( L_l \) for \( l \in \{s_1, s_2, n\} \) defined by \( F_i^{-1}(\alpha) = L_l \) for \( \alpha = (0.01, 0.02, 0.03, \ldots, 0.99) \) and fit the element-wise squared distance between the off-diagonal elements in the matrices of estimated correlations. I base the calculation of these bid distributions off the set of bidders who submit three or more steps. For each bidder I calculate the selection probabilities by fitting a function \( h_p(s_1, s_2, n, K) \) using the observed probabilities of \( s_1, s_2, n \) under \( K = 3, 4, \ldots, \bar{K} \) and then extrapolating this for \( K = 1, 2 \). Finally, the system of FOC may be ill-conditioned for some simulated bids under some parameters of the bid distribution, resulting in large jumps of the criterion function. To improve this, I integrate over a grid of CDS positions \( n \) and at each grid point solve the best \( s_1, s_2 \) and assigning a relative likelihood to each \( n \) by assuming the errors at each of the grid points are normally distributed. As the grid gets dense in \( n \), and the error variance (which plays a role akin to a bandwidth in a kernel) goes to zero, this is equivalent to the direct solution procedure.

I perform the search following procedure 1 in Chen et al. (2018) which constructs confidence sets for an identified set using an adaptive sequential monte-carlo routine on the criterion function. The algorithm discards draws which are relatively unlikely and duplicates those which are, then mutates the draws via a MCMC step.33

7.3. Results. Two benchmarks provide a useful baseline for comparison to the counterfactual results. First, the counterfactual of truthful bidding in these auctions. This would be the result if there were no information rents and no strategic bid shading. Second, the

33I use two blocks of parameters $B=40$, and set tuning parameters as in Chen et al. (2018).
When computing the counterfactual equilibrium I fix the common value quote at its median 32.375. I expect similar shading across different levels of this conditioning variable. To predict the amount of shading in the current format, I calculate the gap from the IMM to the auction price. Across auctions the IMM is on average 0.3993 cents above the final auction price and the size of this gap is independent of the IMM level. This implies that for \( R = 32.375 \), auction prices in the current format are 31.98 while under truthful bidding they are \( 32.375 + (2.2 - 0.3993) = 34.18 \).

The main policy counterfactual is a change to a double auction format. The results of this exercise suggest that the double auction could increase the price in the auctions to between 33.44 and 33.47 from 31.98 today and could decrease the standard deviation of prices around the expected outcome to 1.23 cents from 3.37 cents in the current format. Consistent with these results, the average slopes of the residual supply curves in the double auction are 58% below those in the data, reducing bidders’ price impacts.

The counterfactual change to a double auction reduces the risk faced by investors in two ways. First, it directly reduces the auction outcome risk. Outcome risk is generated by the fact that the bias in any given auction is unpredictable and can be measured using \( \text{Var}(p_{\text{auc}} - E[p_{\text{auc}}|R]) \). The current auction format has a standard deviation in these outcomes of 5.56 cents/dollar (or 3.37 when outliers are omitted). The counterfactual double auction reduces this substantially, to 1.23 cents/dollar. The second source of risk is the risk generated by the price bias. Plots illustrating the role of this bias are provided in Online Appendix H. Because the bias is a fixed cents/dollar rather than a percentage of the

\[ \text{It is not possible to solve the equilibrium of the current auction format and so I compare outcomes to the data. In the non-parametric case, the outcome in the data would be equivalent to the model equilibrium. The main parametric restriction is the linear form of values; to show that this does not drive results I calculate the optimal bid for each bidder who submitted three or more steps (allowing me to pin down their } s_1, s_2, (n - y) \text{ imposing the linear form. I then compare the calculated optimal bid to their observed bid. The resulting bids are similar: in 95 percent of cases the change in expected clearing price conditional on the bid made is less than } 1\text{e-10} \text{ and so it seems unlikely that this drives the results.} \]
final recovery price, and because the recovery amount is unknown before the credit event, investors cannot adjust their holdings to offset the pricing bias. If investors adjusted their positions for the expected level of recoveries, they would be underpaid when recoveries are low and overpaid when high. In the current format this risk has a standard deviation of 1.17 cents/dollar, which is reduced under the counterfactual format to 0.39 cents/dollar.

A major challenge for the CDS auction mechanism is that the final clearing price jointly determines: (i) the CDS cash settlement amounts, and (ii) the price for bonds exchanged. Because dealers tend to hold net positions on the same side of the market, the cash settlement feature provides them with a coordinated incentive to manipulate their bids. As a second counterfactual I consider a change where a limit is set such that bidders with either buy or sell side insurance positions above the limit are not allowed to participate in the auction. This means some participants are unable to express a desire to purchase or sell bonds, but these excluded participants are those with the largest incentives to manipulate prices. Despite excluding the participants with the largest positions, the average holdings of participants are similar to those in the current format. Relative to the double auction, participants face reduced competition, increasing the price impact of each participant, resulting in a larger bias in prices and increased uncertainty.

To evaluate the efficiency of the auction I compare the expected gains in surplus of bidders under the current and double auction designs to the gains in surplus they would obtain from their final bond positions if bonds were assigned to the most efficient holders. The current design achieves only 36% of the possible gains in surplus from reallocating bonds. The double auction improves on this somewhat, achieving 39% of the possible surplus. However, both the current and double auction designs achieve inefficient allocations, as in both cases the allocations are influenced by the CDS positions of participants which are irrelevant under the efficient benchmark. This increase in surplus to bidders is offset by their increased average payments on their insurance positions under the double auction. On average, in the double auction bidders receive $100,500-126,000 less than under the current auction design for the settlement of their insurance positions, while the efficiency improvement increases their surplus by an average of $37,000-90,876, suggesting that the dealers would lose a small amount of surplus from the change in auction design. In appendix J I show the double auction still provides an improvement when dealers can adjust their CDS or bond positions in response to the change in auction format.

The extra risk in these contracts has real economic impacts, and represents an important loss of welfare from a contract with full insurance. The reduced ability for investors to
insure themselves could increase the costs of holding bonds, and could reduce the gains to firms of having CDS written on their debt. To provide a rough sense of the magnitude of the welfare impact, I scale up an estimate of the impact of the current contracts on firm value. The scaling makes the strong assumption that insurance of 95% of the risk leads to 95% of the increase in firm value that would be observed under full coverage. The percentage of the total risk that could be insured under the current contracts is 94-96%, while the double auction would provide 98-99% coverage. Danis and Gamba (2018) estimate that the current contracts cause an increase in firm value of 2.9% when they are introduced on a firm, increasing firms investment and leverage. This would suggest that replacing the current auction rules with a double auction would increase firm value by 2.97-3.01% instead of 2.9%, or an additional increase in firm value of 0.07-0.11% for firms on which CDS is written. Oehmke and Zawadowski (2016) show that firms on which CDS is written have average assets of 51.2 billion, and the median firm has assets of 10.6 billion. Therefore, the gains in value represent increases of 35.8-56.3 billion for the average firm and 7.4-11.7 billion for the median firm.

8. Conclusion

I develop and estimate a structural model describing bidding behavior in credit event auctions. The current auctions have two stages with bidders providing initial quantity commitments and then market-clearing using a uniform price auction. To model these auctions, I extend models of bidding in multiunit auctions to handle initial positions. I then show how bidding data can be used to identify both the private values and CDS positions of dealers without placing parametric restrictions on the shape of dealers marginal value functions. Given this, I estimate the private information from bidding behavior and use the estimates to perform a decomposition exercise of the importance of a set of strategic channels. Finally, I directly solve for the counterfactual equilibrium in the multiunit auction to study the outcome of a change to a double auction.

I find that the current design results in substantial market power for dealers, and as a result, holders of CDS contracts are exposed to risk. This risk is large, with current contracts providing only 94-96% of the coverage from complete insurance. By changing the auction mechanism to a double auction, I find the risk induced by price bias could

35This estimate may be best thought of as an upper bound, some earlier work including Ashcraft and Santos (2009) fail to find evidence of an impact from CDS on bond markets though other recent work including Bilan and Gündüz (2022) also find evidence of a positive link.

36Because the default probability is unknown I compute the risk for a grid of probabilities from 2% to 90% and report the maximum and minimum.
be reduced by 67% and the risk from the variance in auction outcomes by 70%. This increases the effective insurance provided by CDS contracts to 98-99% of complete insurance. Given the important role of CDS markets, this increased ability to hedge risk could have substantial advantages for firms. A rough calculation suggests a possible gain of 0.07-0.11% in firm value from this change in auction rules.

References


**Appendix A. Post-Auction Price Impact**

For the 56 auctions where I observe trade-level data, I check if the auction price has any predictive power for post-auction prices after conditioning on the information available to bidders when submitting their round-two bids. Results are presented in Table A.1. I do not find any evidence of a correlation with bond prices 1, 5, or 30 days post-auction.

**Appendix B. Evidence of Independent Private Values**

In this section I empirically test for the presence of several correlations which are predicted if common values play an important role in the second stage bidding decisions but which do not occur under IPV. Results can be seen in Table A.2. I first perform the test...
This table presents results from a regression predicting the post-auction price using the IMM price and the auction price. Results suggest that the final price is independent of the auction price. This is consistent with no information being revealed about the common value of the bond in that price. The price after each number of days is calculated as a volume weighted average and the sample is the set of auctions for which bond prices are available in TRACE. Prices are cleaned following Dick-Nielsen (2009). The securities missing price information include MBS, CMBS, and syndicated loans. Similar results are obtained when additional controls for bidding behavior are included. Regressions have 56 observations and include a constant term. Standard errors in parentheses.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Price after 30 Days</th>
<th>Price after 5 Days</th>
<th>Price after 1 Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMM price</td>
<td>1.739</td>
<td>1.513</td>
<td>0.525</td>
</tr>
<tr>
<td></td>
<td>(0.624)</td>
<td>(0.493)</td>
<td>(0.302)</td>
</tr>
<tr>
<td>Auction price</td>
<td>-0.771</td>
<td>-0.477</td>
<td>0.477</td>
</tr>
<tr>
<td></td>
<td>(0.592)</td>
<td>(0.467)</td>
<td>(0.286)</td>
</tr>
</tbody>
</table>

proposed by Gupta and Sundaram (2015), which uses the variance of initial round quotes as a proxy for uncertainty. When uncertainty is high, bid shading in the second round should increase under common values due to the Winner’s Curse. I find no significant correlation between these measures. I then focus on the ‘independent’ piece of the assumption and formally test this using a procedure proposed by Hickman et al. (2021). For each bidder I regress their bid on the mean opposing bid. I find that the average opposing bid does not predict each bidder’s bid. The test suggests that unobserved auction heterogeneity does not play an important role. In addition, Appendix A reports results which provide no evidence that auction outcomes impact post-auction prices. Finally, I show that the bidders’ own beliefs relative to the IMM level have no explanatory power for their second stage bids, suggesting that bidding across rounds reflects different information.
Table A.2. Testing for the Winner’s Curse

The first panel reports results from regressing the average slope of a bidder’s stage two bid, against a proxy for the winners curse. The presence of the winner’s curse suggests steeper stage 2 bids. The regression controls for the bidders first stage quantity submission, the auction NOI, the N-steps submitted and a constant. The second panel reports results from regressing each bidders average stage 2 bid, against a measure of their initial beliefs about value (before announcement of the IMM). The regression controls for the bidders first stage quantity submission, the auction NOI, the N-steps submitted, the max q bid on, IMM and a constant. There are 289 and 830 observations. The third panel reports results from regressing the average bid for each bidder in stage 2 against the average bid by opposing bidders, controlling for factors that would explain across auction variation in the bid level in an IPV setting. A non-zero coefficient on the mean opposing bid would lead us to reject the null hypothesis of IPV. The regression includes a constant and controls for IMM and NOI and following Hickman et al. (2021) adopts a cubic polynomial in N and the average opposing bids to control for variation in shading resulting from optimal bidding in an IPV model. Standard errors in parentheses.

<table>
<thead>
<tr>
<th>Slope of bids</th>
<th>Mean bid (NOI&lt;0)</th>
<th>Mean bid (NOI&gt;0)</th>
<th>Mean bid Opp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMM vari.</td>
<td>0.000 (0.004)</td>
<td>-0.643 (0.444)</td>
<td>0.335 (0.227)</td>
</tr>
<tr>
<td></td>
<td>$IMM_i - IMM$</td>
<td></td>
<td>0.929 (0.886)</td>
</tr>
</tbody>
</table>
Appendix C. Appendix for online publication to accompany
“Quantity Commitments in Multiunit Auctions: Evidence from Credit Event
Auctions”
by Eric Richert

Appendix D. Additional Proofs

D.1. Equilibrium Existence. To understand the role of the restricted strategy sets it is useful to compare the results to the unrestricted case from Wilson (1979) in the IPV case. Using calculus of variations gives
\[ v(q, s) = b - (q + n - y) \frac{H_q(b, q | s)}{H_b(b, q | s)} \]
where \( H \) represents the probability that the residual supply is less than or equal to the quantity \( q \) at price \( b \). Using this together with Proposition 4 from Kastl (2012) implies that as \( K \) goes to infinity, any restricted equilibrium approaches this solution and these empirical FOC are valid for inference conditional on an equilibrium existing.

Although the existence of an equilibrium in the uniform price auction with restricted strategy sets is an open question, in the standard setting Kastl (2012) proves the existence of an epsilon equilibrium. This argument cannot be applied to the credit event auction setting, because the proof makes use of the separability between the benefit of winning and the price paid, to argue that if a bidder is unrestricted in number of steps they will not bid above their value. This separability property does not apply in credit event auctions, as a bidder may be better off bidding above their value in order to impact the clearing price of their existing CDS position. To guarantee that an equilibrium exists, in the uniform price multiunit auction game, I follow the suggestion of Kastl (2011) and impose that there exists a fine discrete grid of price levels. This is the case in practice, as bidders can only express their prices to the nearest 1/8th of a cent. In this case, Kastl (2011) argues that the FOC for the quantity choice are still valid, and an equilibrium is guaranteed to exist (at least in mixed strategies) as it is a finite game.

Appendix E. Additional Tables

E.1. Additional Tables and Figures. Table OS.1 presents some additional statistics describing the bidding behavior of different auction participants. The sample is loosely divided into participants that are involved regularly (the 9-10 global dealers) and the less frequent regional participants whose participation varies with the frequency of defaults in a location. The table shows that there is considerable variation within participant in the direction of their initial quantity commitments, suggesting that while the dealers are most often holders of insurance, they are net sellers in some cases. It also appears to be quite common that bidders submit only 1 step in the second stage. These bidders do not actively bid in the second stage as 1 step is carried-over from the initial price quotes.

Figure OS.1 plots the maximum and minimum quantity of bonds purchased at an auction by each bidder. By construction, the total bought and sold must sum to zero in

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1The proposition requires randomness in the quantity being sold which is announced in this game. However, the carried over amounts from the first stage price quotes effectively lead to a random (predetermined and non-strategically linked) residual quantity at any price level.
Table OS.1. Auction Participation

The following table presents summary statistics for participation of the bidders. Each number is a count of the number of auctions in which the bidder participated, submitted a positive or negative first round quantity commitment or used each number of steps in their second stage bids.

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Participated</th>
<th>$y_i &gt; 0$</th>
<th>$y_i &lt; 0$</th>
<th>1 step</th>
<th>2 steps</th>
<th>3 steps</th>
<th>4 steps</th>
<th>5+ steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>7</td>
<td>2</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>20</td>
<td>102</td>
<td>22</td>
<td>13</td>
<td>50</td>
<td>22</td>
<td>6</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>21</td>
<td>96</td>
<td>19</td>
<td>11</td>
<td>83</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>180</td>
<td>43</td>
<td>42</td>
<td>72</td>
<td>31</td>
<td>16</td>
<td>13</td>
<td>40</td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>128</td>
<td>32</td>
<td>27</td>
<td>67</td>
<td>22</td>
<td>7</td>
<td>4</td>
<td>24</td>
</tr>
</tbody>
</table>

every auction. Auctions where a big quantity of bonds was bought/sold are more likely to appear in this figure. Most bidders appear to both buy and sell in the auctions. The purchase of a large quantity by a single bidder appears slightly more common that the sale of a large quantity by a single bidder. In Panel B of the figure, there is no obvious time trend which may have been a concern if some of the dealers were known to have poor financial health during some periods of the sample.

Figure OS.2 shows the price realized in auctions depending on which type of credit event caused the auction. Even across types of credit events, there is a large amount of heterogeneity in the remaining value of the firms. However in regressions, there is no

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2In a similar way that healthy banks bidding for failed banks might themselves be constrained, c.f. Granja et al. (2017)
**Figure OS.1. Purchases**

Panel A of the figure plots the max and minimum quantity of bonds purchased at an auction by each bidder. Panel B plots the purchased quantity of each dealer over time.

![Graph showing max and min quantity of bonds purchased by each bidder, and purchased quantity of each dealer over time.]

**Figure OS.2. Event Types: Prices**

The figure plots the price realized in auctions depending on which type of credit event caused the auction. Even across types of credit events, there is a large amount of heterogeneity in the remaining value of the firms.

![Graph showing price realized in auctions for different types of credit events.]

Evidence that the type of event is correlated with the gaps between the pre-auction price, auction price, and post-auction resale prices.

Figure OS.3 looks at the impact of the total quantity submitted by opponents in the first stage and the expected price in the auctions. This highlights the intuitive relationship that a small quantity to be cleared results in more competitive bidding, leading to a high price. However, as the quantity that needs to be cleared increases, in the second stage bidders can shade their bids more, and the expected price falls.

The within-auction variation is summarized in Table OS.2. The table shows summary traits of the bonds, including volume, duration, convexity and conversion factor. Duration and convexity measure the exposure of the bond to interest rate risk. Duration has substantial variability within auction, which is heavily influenced by the fact that the set
Figure OS.3. Expected Price Quantity Others
Nonparameteric smoothed estimates of the expected price as the \( NOI_i = NOI - (\text{dealer j’s commitment}) \) varies. The expected price is calculated as a fraction of the price cap, and expectations are taken by simulating residual supply curves which imposes the assumption that bids are conditionally independent in the second stage given the NOI submission of opponents and the bidders own submission.

![Figure OS.3](image)

Figure OS.4. Marginal Distribution: CDS Positions
The left panel plots the estimated distribution of CDS positions \((n_i)\). The right panel plots the estimated distribution of effective intercept. The plots show kernel smoothed densities from 10000 simulated draws from the distributions implied by the quantile functions with parameters in Table 3.

![Figure OS.4](image)

of eligible bonds often contains some share of floating rate notes (FRN) and some share of long-term coupon bonds. Convexity is much more similar within auctions than across. The volume of individual issues varies substantially within auction.

Figure OS.4 plots the estimated marginal distributions of the slope, and intercept of bidders’ private values for the bonds and their insurance positions.

E.2. Multiplicative Form. If the common value component of the bond entered multiplicatively with bidders’ own private values in the model then the dispersion of private information would be increasing in the level of R. This would mean that for auctions with small R the dispersion in private values matters little, while in auctions with a big R this plays a central role. If this was true we would expect to see the level of information rents, and the gap with the pre-auction price information growing in R. Figure OS.5 plots the
Table OS.2. Bond Measures

Volume ($B$), duration convexity and conversion factor as calculated for each bond in the eligible set that can be submitted to the auction. Each variable $x_{ja}$ for auction $j$ admissible bond $a$ has between variable $x_j$ and within $x_{ja} - x_j + x$, where $x$ is the global mean. While the "within" reported minimum eg. for volume is negative, this does not indicate negative volume of any issuance but refers to the deviation from each auctions average issuance size and naturally, some of those deviations must be negative. Across 185 auctions there are a total of 1,998 eligible bonds.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume</td>
<td>overall</td>
<td>7.24</td>
<td>85.80</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>between</td>
<td>9.55</td>
<td>0</td>
<td>127</td>
</tr>
<tr>
<td></td>
<td>within</td>
<td>81.60</td>
<td>-108</td>
<td>3380</td>
</tr>
<tr>
<td>duration</td>
<td>overall</td>
<td>3.27</td>
<td>10.81</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>between</td>
<td>9.71</td>
<td>0.87</td>
<td>103</td>
</tr>
<tr>
<td></td>
<td>within</td>
<td>9.50</td>
<td>-31.34</td>
<td>123</td>
</tr>
<tr>
<td>convexity</td>
<td>overall</td>
<td>0.86</td>
<td>1.49</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>between</td>
<td>1.33</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>within</td>
<td>1.21</td>
<td>-3.62</td>
<td>10.38</td>
</tr>
</tbody>
</table>

Figure OS.5. Auction price vs NOI

The figure plots the price realized in auctions against the IMM quoted.

E.3. Collusion Test. Although the setting features repeated interaction of a small set of participants, it would likely to be difficult to sustain collusion as (i) violation would be...
difficult to detect and (ii) there are usually one or two regional players in each auction who do not frequently participate. Detection is difficult in this setting because bidders may receive orders which they place on behalf of their customers. This means that when deviating from the prescribed collusive behavior, bidders could simply claim to be placing the bid on behalf of a customer. Since this cannot be verified by the other participants this makes detection and deciding when to punish more complicated, making collusion more difficult to sustain. In addition to this, the set of bidders that wants to push the price up and the set that wants to collude to push the price down (i.e. the sets of individuals that all benefit from working together), varies across auctions. Finally, if prices were pushed substantially in one direction bidders with large insurance positions on the opposite side of the market would have a strong incentive to deviate from the agreement. These challenges, together with the reduced form evidence, suggest that collusion in the second stage bidding game would be quite difficult.

The resale opportunity present in the bond market allows for an additional test of the null hypothesis of no collusion. In a model of collusion we would expect bidding behavior similar to that described in Laksa et al. (2018). If the data was generated by collusive bidding, then bidders’ implied values that rationalize observed bids in a competitive bidding model would be well below the true values. If we then saw bidders willing to buy bonds immediately after the auction at higher resale prices this might suggest a violation of the competitive bidding model. In the data, the median value implied is 0.96 cents below the IMM at the expected clearing quantity, however the 65th percentile is the IMM and the 78th is the average markup for the clearing price. At the 90th percentile the value is 5 cents above the IMM and at the 99th it is 45 cents above the IMM. These results seem to be broadly consistent with the observed post-auction behavior and not suggestive of collusion.

E.4. IMM manipulation. The average change in price that dealers can expect by manipulating their IMM quote is 0.02 cents. This is small as if you quote a number that is different from others your quote is dropped and since only half of the quotes are used and the average is rounded to the nearest 1/8th of a cent increment after averaging, it is difficult to influence this calculation with a unilateral deviation. At the 95th percentile of expected benefits when integrating over the estimated distribution of possible n and using the distribution of clearing prices in the data, this gives an increase of 4,198 dollars of surplus. The mean cost from quoting off-market is 24,000 so a bidder that is optimizing should be more worried about that effect and quote their best guess of initial price.

Given the incentives to profit from the insurance positions, first-stage quotes should be negatively correlated with the bidders’ insurance positions, i.e. a bidder who is a net buyer of CDS should quote lower prices in the first round. Table OS.5 presents results from regressing the level of individual price quotes on an indicator which takes the value 1 if the nonparameterically estimated lower bound is greater than zero (column 1) and upper bound is greater than zero (column 2). I use the nonparametric estimate as these are available for all bidders rather than only those submitting at least 3 steps. While the coefficient has a negative sign, it is not statistically significant in either specification.
TABLE OS.3. Position and price quotes

The following table presents results from regressing the level of individual price quotes on an indicator which takes the value 1 if the nonparametrically estimated lower bound is greater than zero (column 1) and upper bound is greater than zero (column 2). I use the nonparametric estimate as these are available for all bidders rather than only those submitting at least 3 steps. In all cases I control for the baseline expected recovery value using the final IMM. Standard errors in parentheses.

<table>
<thead>
<tr>
<th>Variable</th>
<th>IMM Submission</th>
<th>IMM Submission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auction IMM</td>
<td>0.9927</td>
<td>0.9929</td>
</tr>
<tr>
<td></td>
<td>(0.0014)</td>
<td>(0.0014)</td>
</tr>
<tr>
<td>CDS buyer</td>
<td>-0.1346</td>
<td>-0.1202</td>
</tr>
<tr>
<td></td>
<td>(0.094)</td>
<td>(0.1151)</td>
</tr>
<tr>
<td>Constant</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

TABLE OS.4. Position and price quotes

The following table presents results from regressing the level of individual price quotes-IMM on an indicator which takes the value 1 if the noi submission is positive. Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>IMM Submission</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS buyer</td>
<td>-0.1296</td>
</tr>
<tr>
<td></td>
<td>(0.235)</td>
</tr>
<tr>
<td>Constant</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table OS.4 presents an alternative specification, comparing bidders who commit to sell bonds (and so must be buyers of insurance according to the auction rules). Again the coefficient is negative but not statistically significantly different from zero.

E.5. Sufficiency of IMM. Given the strong relationships documented between outcomes and the IMM price, I proposed that the IMM should be considered a sufficient statistic for the auction level heterogeneity. In this section I show that while there is some evidence that bond traits influence the IMM amounts, there is no evidence that they influence residual bids beyond this point. Therefore, conditioning on the IMM should be sufficient to capture the auction specific differences in bonds.

Finally, I check the relationship of the traits of the deliverable bonds to the auction outcomes. I check this relationship both at the bidder-level, regressing the residualized bids on the bond traits in Table OS.5 and at the auction level in Table OS.6. This shows that the bond traits have some power in explaining the IMM quote that a bidder provides but no power to explain their residualized bid after conditioning on the IMM. The regression at the auction level finds no statistically significant effect of the bond traits. Given these results, I do not include bond traits in the main estimation. These results indicate that once I condition on the initial market price they have no explanatory power.
### Table OS.5. Bond Traits: Bidder Level

Residualized bids from nonparametric regression on IMM and NOI. Bonds useful in determining IMM submission but not in bids conditional on IMM common signal and NOI. Standard errors in parentheses.

<table>
<thead>
<tr>
<th>Variable</th>
<th>IMM Submission</th>
<th>Residualized bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>-4.888</td>
<td>-0.0177</td>
</tr>
<tr>
<td></td>
<td>(2.437)</td>
<td>(0.0318)</td>
</tr>
<tr>
<td>Conversion</td>
<td>5.284</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(3.394)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Convexity</td>
<td>0.531</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>(0.266)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>Volume</td>
<td>0.0002</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0325)</td>
</tr>
<tr>
<td>Auction NOI</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Constant</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>N</td>
<td>1965</td>
<td>1965</td>
</tr>
</tbody>
</table>

### Table OS.6. Bond Traits: Auction Level


<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>Auction Price</th>
<th>Auction IMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>duration</td>
<td>-0.643</td>
<td>-6.901</td>
</tr>
<tr>
<td></td>
<td>(1.495)</td>
<td>(8.161)</td>
</tr>
<tr>
<td>conversion</td>
<td>0.692</td>
<td>3.751</td>
</tr>
<tr>
<td></td>
<td>(1.985)</td>
<td>(10.85)</td>
</tr>
<tr>
<td>convexity</td>
<td>0.0538</td>
<td>0.587</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
<td>(0.910)</td>
</tr>
<tr>
<td>volume</td>
<td>3.25e-06</td>
<td>0.000154</td>
</tr>
<tr>
<td></td>
<td>(1.82e-05)</td>
<td>(9.91e-05)</td>
</tr>
<tr>
<td>NOI</td>
<td>-0.00718</td>
<td>-0.0389</td>
</tr>
<tr>
<td></td>
<td>(0.00242)</td>
<td>(0.0129)</td>
</tr>
<tr>
<td>IMM</td>
<td>1.004</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0571)</td>
<td></td>
</tr>
<tr>
<td>IMM²</td>
<td>-5.19e-05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000568)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.599</td>
<td>50.05</td>
</tr>
<tr>
<td></td>
<td>(1.643)</td>
<td>(6.773)</td>
</tr>
<tr>
<td>Observations</td>
<td>178</td>
<td>178</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.971</td>
<td>0.068</td>
</tr>
</tbody>
</table>
Figure OS.6. Sample Bounds from Monotonicity

The black lines denote observed submitted bids. The round dots denote $\tilde{v}(q)$. The triangular dots show the implied marginal value curve with a $(n-y)$ factor of -50 and +50. In the left panel the implied insurance position at -50 is a feasible curve while at 50 monotonicity is violated. In the right panel both -50 and 50 can be rejected as the implied value curves violate monotonicity. This Figure is the sample analog of two randomly selected bidders.

Section G discusses in detail the incentives involved in the IMM submission decision and considers a calibration exercise to examine the information revealed through the IMM announcement.

E.6. Linearity Test. I test the linearity assumption in two ways. First, by using the overidentifying restriction from the subsample of bidders that submit 3 or more bids. In that sample, the median R squared is 0.98 and the mean 0.87. As a second test I estimate the model with a quadratic specification for marginal values. The median change in the estimated CDS position is 0.015 million and even at the 75th percentile the change is only 1.5 million. The estimated positions under the two sets of CDS positions $n$ are also strongly positively correlated. Because of this, I maintain the linear restriction for the primary specification.

Appendix F. Nonparametric Estimation of Insurance Positions

Figure OS.6 presents the analog of Figure 5 for two randomly selected bidders in the sample. In the left panel, the marginal values implied by zero insurance position and by net holdings of -50 million are consistent with the monotonicity restriction, while an insurance position of 50 leads to a violation of monotonicity and can be rejected as part of the identified set for this bidder. In the left panel both insurance positions of -50 million and 50 million lead to violations of monotonicity and can be rejected.

Enforcing bidder by bidder the boundedness and monotonicity restrictions gives a set of non-parameteric bounds. The left panel of Figure OS.7 plots the lower bound for each bidder in the sample. The right panel of the figure shows the width (or informativeness) of the bounds for each bidder. In many cases, these nonparametric restrictions provide very informative bounds (with width close to zero). This suggests that these restrictions contribute substantial information to the estimation results.
**Figure OS.7. Sample Bounds from Monotonicity**

The left panel plots the nonparametric estimate of the lower bound of the bidders’ insurance position. The right panel plots the width of the bound (upper bound-lower bound).

**Appendix G. Stage 1 Price Quotes**

The bidders choice of first stage price quotes is a complex strategic decision. These quotes serve many roles in the auction: (i) the quoted price is carried over as a bid for a fixed quantity (usually 2 million dollars) of bonds in the second stage, (ii) the quotes are aggregated by taking an average excluding the outliers which is announced to all participants between rounds, (iii) the average plus 2 times the spread determines a price cap or floor which stands for bids submitted in the second stage auction, and (iv) the quotes determine a set of fines for bidders who submit off-market quotes.

These many roles mean that bidders’ may have incentives to strategically report their quotes from a number of different sources and their strategies are likely to be complex. The data contain some information that indicates the importance of the different channels. The carried over bids are sometimes relevant for clearing the auction, for example most auctions have carried over amounts of 2 million and an average of 11 participants implying 22 million of carried over bids. 74 auctions have a total excess supply/demand less than 22 million and so these bids may play an important role in the final price determination. The price cap binds for only 3 percent of the bids made in the second stage auctions, but in 16 percent of the auctions it plays an important role in determining the price. Fines are given 169 times in the data and have an average level of 32000 dollars.

Assumption [1] imposes that once bidders know the IMM their own private information on the common value component of bond values is no longer relevant (ie. the bidders mostly agree on the common component of the recovery value). This assumption is critical for the tractability of the empirical exercise. Although bidders may have many reasons to manipulate their price quote, their chosen quote is likely to be correlated with their own signal, which influences their expectations about what opposing quotes they will face. If bidders make reports that are correlated with their own signal, the IMM will aggregate the signals from across many bidders. For bidders trying to learn about the mean the IMM which combines information from many draws is likely to be much more informative than the individual bidders’ single information. In addition, the complex
formulas for the calculation of the IMM make it difficult for a bidder to calculate the role of their own quote in establishing the final value, reducing the value of relying both on their quote and the announced IMM.

To get a better understanding of the updating process I perform a simulation exercise to understand what bidders learn with parameters calibrated to match the quotes in the data. I impose that each bidder learns a signal about the true common value, and also has draws some private benefit from misreporting their value either up or down. This private benefit draw is a reduced form way to capture the complex benefits that a bidder may receive from manipulating the price quote. For example the benefit may come from the change in profitability from the bidders carried over bids, or the change in expected profits from their influence on the price cap or floor.

For each bidder I calculate their expected impact on the IMM from submitting different price quotes along a grid of possible submissions ranging from 10 cents on the dollar below the true value to 10 cents on the dollar above the true value. In calculating the expected impact I assume that bidders expect to face quotes drawn from the empirical distribution of quotes submitted into auctions with similar post-auction prices to the common value signal $R + \eta$ that the bidder received. I then calculate for each simulated information set, the quote on the grid that maximizes the bidders’ expected surplus given by the private marginal benefit of manipulation multiplied by the price impact of their quote less any fines. I will assume that the distribution of private signals and private benefits are both normal and the private benefit is independent of the private signal about the common value for bonds.

Given this structure there are three critical parameters which are unknown, that will determine the amount of information revealed by the IMM announcement. First, the precision of the initial signals about this component, which is governed by the parameter $\sigma_\eta$. Second, the mean and variance of the distribution of private benefits from manipulation. I set a coarse grid in these three parameters and for each grid point solve the choice of initial quotes as described above for 100 randomly drawn private benefits and initial signals. I then compare the distribution of the implied optimal quotes to the distribution of quotes submitted in the data and choose the parameters which minimize the difference between these two distributions. This gives the key inputs to the updating process: a signal variance, a quote variance and a correlation between the signal a bidder receives and their submitted quote.

Given these parameters I can simulate signals $\eta$. For each signal I can calculate the optimal submission for that bidder and after repeating for each bidder at a simulated auction, can obtain a resulting IMM. Using this I can calculate the expectations of each bidder $E[R|\eta]$ and $E[R|\eta, p_\text{IMM}]$ assuming they update according to bayes rule and know all relevant distributions, and that the underlying distribution of R is exactly equal to the post-auction resale price distribution in the data.

Two quantities play an important role in the outcomes. The variance across bidders (within-auction) of $E[R|\eta, p_\text{IMM}]$ which indicates the remaining role of the signals $\eta$ and the ratio of this variance to $E[R|\eta]$ which indicates how much the bidders learned. The
participating bidders appear to have a fairly precise knowledge of the common value component with initial expectations having an expected variance of 0.6 cents. Once the IMM is announced this disagreement drops dramatically and bidders almost completely agree with each other. The remaining variance under the calibrated parameters is 0.002 cents, which is roughly .3 percent of the variance in initial expectations. In experimenting with the parameter values it appears that even under quite small correlations between quotes and initial signals the IMM quoting mechanism results in expectations that are far less variable across participants. The small variance in the expectation across different initial signals after learning the initial market quote suggests that heterogeneity in bidders’ expectations of the common value post announcement are not likely to play an important role.

While collusion in stage 2 bids may be difficult to sustain it is possible that bidders instead collude on their first stage quotes. However, even under collusion it seems likely that the optimal quote level depends on the initial signals received by bidders and so the level chosen is likely to be highly informative to bidders of opponents signals, reducing their reliance on their own initial signal. For example, when a bidder has a high $\eta$, they are willing to buy bonds at higher prices. Making a low quote would decrease the price cap making it more likely that they are constrained, allowing them to purchase fewer bonds at the attractive price. They would therefore want to bargain for a slightly higher quote and the final IMM would reflect this information weighted against the other collusive participants. Therefore it seems likely that even under a collusive regime, the IMM level would substantially reduce the reliance of bidders on their initial signals of the common value. Any bias due to collusion in these first stage quotes is captured in estimation by the function $R(IMM)$, which is parameterized as a cubic B-spline.

**Appendix H. Risk Calculation**

Figures [OS.8](#) and [OS.9](#) illustrate the risk induced by the fact that there is a constant level of expected bias while recovery values are uncertain before the auction. Results are shown for the level of bias under the current auction format and under the counterfactual double auction design.

**Appendix I. Customer Orders**

The model presented in Section 3 treats all submitted bids as if they were made by the dealer. That is, the dealers are assumed to have some value for acquiring the bonds, which may be driven by the ability to sell the bond to a client post-auction, but the dealer makes the strategic decision about the set of steps to submit in a bid. The same assumption is made in the long literature on the estimation of Treasury Auctions, where small clients place orders with dealers that are not directly observed. In this section, I calculate conservative bounds on dealer and customer participation rates. I then consider a selection model that suggests that client orders are not driving the results.
There is indirect evidence that clients do sometimes dictate orders to their dealer. For example, we sometimes observe bids for different quantities at the same price, or bids for more quantity than the total available supply. The first of these occurs in roughly 15 percent of bids and the second occurs for roughly 10 percent of dealers. While the first may be due to bidders’ internal accounting practices, reporting different steps to account for different bonds offered the second is difficult to rationalize within a dealer. These suggest lower bounds on the rate of customer participation but may not positively identify all customer orders.

To estimate a conservative lower bound on dealer participation, I use the insurance positions of dealers reported in [Paulos et al. (2019)] and assume that dealers have positions drawn from this distribution and zero value for every bond they purchase. I then lay out a grid of possible entry/bid formation costs running from zero up to 50 million dollars and calculate the set of positions that would find participation in the auction profitable for the only gain of increasing insurance profits. This provides a lower bound for the probability that a dealer wants to participate on their own behalf of 14 percent. With
Figure OS.10. Nonparameteric Bounds: With Customers

The figure plots the nonparameteric estimates of the distribution of insurance positions for bidders that submitted three or more steps as part of their bid curve. The second set of curves compares the distribution estimated when we explicitly account for the probability that some of the dealers’ steps may have been submitted to them by a client.

The same set of entry costs, an upper bound, from assuming values of 100 for every bond purchased, implies a participation rate of 62 percent. Note, even in the data, over 40 percent of the dealers do not submit additional second stage bids and so a participation rate of 62 percent actually exceeds the rate observed in the data.

To understand the effect of the possible incorrect attribution of bids to dealers on the structural estimates I consider a selection model. First, I assume that customers submit only orders using a single step. This is consistent with evidence in Treasury auctions, c.f. [Kastl (2011), Hortaçsu and McAdams (2010)] and may be because they are smaller, less sophisticated or less accustomed to bidding. I leverage the fact that the customer order will be observed whenever the dealer is bidding for the full quantity on offer and the customer makes a bid for any positive quantity at a price lower than the minimum price from the dealers’ own bid. This allows me to obtain an estimate of the likelihood that a given step in the data would be positively identified as having been made by a customer. This can be combined with an estimate of the probability that such a bid was made at all, to obtain an estimate of the probability that any step was submitted by a customer. Once this probability is known for every step, then when estimating the values, I can draw many possible assignments: where an assignment is a list of the steps $k$ from a given dealer that were submitted by the dealer and the steps $k'$ submitted by customers. For each assignment I can re-estimate the implied marginal values and insurance positions. To describe the effect, I re-estimate the nonparameteric bounds on insurance positions, as this should be the part of estimation most affected by the assumption. The results are plotted in Figure [OS.10]. The estimated bounds look quite similar to the original bounds, and so I conclude that the selection effect from customer orders is not likely playing an important role in the model estimates.
In the results so far I have assumed that the joint distribution of $s_1, s_2, n$ was a primitive and would remain fixed in the counterfactuals. This assumption seems reasonable given the CDS and bond positions are taken on prior to the default event occurring. Therefore, they are likely to be much more reflective of market-making and trading activities by the dealers, their costs of holding bonds and CDS, and their perceptions about the probabilities of default than the expected auction outcomes. However, we may be worried for example that a reduction in the expected surplus at the auction from holding CDS leads dealers to hold a smaller initial position. In this section I discuss the most plausible ways that the joint distribution might be affected by the change in incentives to hold various positions in the counterfactual auction formats. I then develop a set of changes to the value distribution that are plausible and use this to compute a set of bounds for counterfactual equilibria for any joint distribution in the set.

There are two possible changes that one may worry could occur that would affect the joint distribution. The first, are changes to the CDS position caused by shifts in the benefit of holding a particular CDS position for a given marginal value curve and given pre-auction benefit of holding CDS. The second, are changes to the bonds bought/sold before the auction which could shift bidders along the marginal value curve (ie. lead to bidding behavior according to $v(q) = s_1 - s_2 \Delta B_i - s_2 q$).

First consider changes in the CDS position. These changes may play an important role through the constraints they impose on a bidders’ set of feasible actions. For example, these constraints may prohibit a bidder from obtaining their desired final position in bonds. This incentive is discussed at length in Du and Zhu (2017) and they show that under the current auction format the desire to be unconstrained leads bidders with intermediate levels of pre-auction benefit from holding CDS on both the buy and sell side to hold slightly larger positions. The lack of constraints in the double auction should eliminate this expansion. In the double auction, bidders also no longer have the option of a physical settlement round. Given the concentration of buyers/sellers I still expect the double auction to achieve a downward bias in general on the price, which could provide an incentive for buyers to increase their positions and sellers to decrease their positions (such that $n$, rather than $n - \gamma$ is subject to the price bias). These shifts in the distribution will increase price biases in the CDS auctions and so the baseline results may overstate the possible improvement. Because it is likely that most of the position is determined by factors unrelated to the auction, I consider as a reasonable set of bounds, perturbations that allow for an increase of up to $+10$ percent of each CDS buying bidders existing CDS position and a decrease of 10 percent on the positions of seller dealers.

Given the expected price pressures from cash settlement, bidders expect the bonds traded in the auction to do so at a discount to the market price of bonds in both the current and double auction format. In the baseline change to a double auction there

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3There is also a limited amount of trading (and limited liquidity) that takes place in the lead up (and during) the auction. For the trace-eligible sub-sample of auctions the median trade volume on the auction day is $6.5M$ of bonds.
is a slight reduction in the level of the discount for bonds purchased in the auction. This would suggest that bonds purchased in the auction are relatively less attractive and may lead high value bidders to purchase additional bonds before the auction date. This change in positions is expected to lead to less aggressive bidding and lower prices, so the main double auction results may only be an upper bound on the possible set. The bond market is quite illiquid and especially so following default (as documented in Feldhütter et al. (2016)) and so large adjustments of positions will generally be extremely costly. Therefore I examine robustness of the results to a shift in the intercept distribution that is consistent with a shift along the value curve equivalent to a maximum purchase of $1 Million of bonds by high (above median) value bidders prior to the auction.

The results of this exercise suggest the final price after adjustments in position will be in the interval 33.34-33.97 and the standard deviation of outcomes relative to the expected price is 0.55-1.23. This means that once the position changes are accounted for, the double auction continues to improve on the current format.

REFERENCES


