What are the sources of boom-bust cycles?*

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Abstract

Why do some expansions end up in recessions while others do not? We argue that the answer lies in the distinction between fundamental-driven and expectation-driven expansions. We find that technology shocks generate either expansions or recessions, whereas shocks to expectations generate boom-bust type of dynamics. We show that a Real Business Cycle model with endogenous borrowing limits rationalizes boom-bust phenomena in response to self-fulfilling shifts in expectations unrelated to current and future technology. Credit market amplification plays a central role during expectation-driven expansions, while only mildly shapes the economic dynamics induced by technology shocks. Crucially, both types of expansions are characterized by an increase in credit growth, suggesting that policies aimed to limit credit expansions might not be optimal.

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1 Introduction

The types of forces that drive business cycles are a debated subject. A view maintains that expansions and recessions arise from independent positive and negative persistent changes in some exogenous forces. In contrast, there is the view according to which business cycle fluctuations are due to forces that are internal to the economy and that endogenously favor recurrent periods of boom followed by a bust. Notwithstanding the different implications that these two perspectives have on the autocovariance patterns of the data, evidence in favor of either view is hard to find. We argue that the reason underlying the absence of conclusive evidence stems from the coexistence of fundamental and non-fundamental shocks that differently shape the dynamics of key economic aggregates.

In particular, while positive transitory shocks to total factor productivity (TFP) generate prolonged expansions after which the economy slowly reverts back to its long-run trend, “pure” sentiment shocks – shocks to expectations orthogonal to TFP and to the rational expectations of future TFP – lead to boom-bust type of dynamics. The main contribution of this paper is twofold. Motivated by reduced form evidence of regular economic cycles and the inability of TFP shocks to induce oscillatory dynamics, we show that a real business cycle (RBC) model with financial frictions can give rise to boom-bust type of responses to self-fulfilling shifts of expectations regarding the solvency of market borrowers, and simultaneously matches the economic response to a transitory TFP shock. Second, we provide novel empirical evidence that sentiment shocks generate boom-bust dynamics consistent with the predictions of the model.

The paper is divided into three parts. In the first part, we document the existence of periodic motions in U.S. macroeconomic and financial variables that repeat themselves in a regular cycle. We do so by showing that the spectral densities of these variables display a peak at periodicities of around 8 to 10 years. Importantly, the hump in the spectral density appears to be more pronounced among the set of financial variables, suggesting that boom-bust phenomena originate from the financial sector of the economy. We then look at the dynamics induced by a temporary shock to utilization-adjusted TFP and find that TFP is unable explain the observed periodic behavior. Specifically, a positive TFP shock leads to a temporary expansion that is not systematically followed by a recession. By examining the
conditional spectral density implied by a TFP shock, we conclude that these shocks are not responsible for the unconditional spectral properties of the data.

Motivated by the presence of a spectral peak common across both financial and macroeconomic aggregates, in the second part of the paper, we build an otherwise standard RBC with financial frictions. The model matches the empirical response of a TFP shock, while it generates boom-bust fluctuations in response to shifts of expectations on firms’ creditworthiness. The model features an endogenous debt limit where firms’ borrowing capacity is limited by their expected equity value. There is a default-deterring amount of loan that households lend to firms. This generates a feedback loop between firms’ market value and households’ income: an increase in firms’ market value reduces borrowers’ incentives to default allowing firms to borrow more and produce more. Higher production increases households’ income and their willingness to save, which feeds back into higher market value of the firms.

We find that the model displays self-fulfilling equilibria together with endogenous boom-bust cycles for plausible parameters governing the tightness of the credit constraint. Boom-bust phenomena are triggered by changes in agents’ expectations on borrowers’ ability to repay. A positive shift of expectations relaxes the financial constraint thereby leading to larger production, investment, and consumption. During expectation-driven expansion, firms’ profitability declines. However, firms’ market value rises due to the increase in households’ income which stimulates the demand for firms’ assets. As firms’ debt expands, their ability to invest and distribute dividends decreases, ultimately generating a reversal. During the contraction, financial constraints tighten, production decreases leading to lower households’ income. The contraction ends up in a recession as firms fail to internalize the negative effects of their production decisions on future households’ demand for firms’ assets.

Conversely, when the economy is hit by a technology shock, the financial amplification mechanism is considerably more muted. The reason is because a TFP shock increases firms’ market value but also raises managers’ incentives to divert funds due to larger revenues. As a result, TFP-fueled expansions are not characterized by excessive growth of firms’ debt relative to their profitability. In fact, a positive TFP shock increases firms’ profits therefore preventing the expansion to turn into a recession.

Importantly, the fact that credit increases during both types of expansions but only
sentiment-driven expansions sow the seeds of a subsequent bust, casts doubts on using credit growth as a predictor of a crisis. Despite its simplicity, our model suggests that policy makers who aim at reducing firms’ leverage during an expansion should be cautious in not to limit those credit expansions associated with efficient investment opportunities.

In the third part of the paper, we estimate sentiment shocks and test the predictions of the model. Specifically, we construct an indicator that summarizes the revisions of expectations on the future economic outlook using quarterly data on expectations from the Survey of Professional Forecasters and the Survey of Consumers. We use the indicator to identify exogenous shifts in expectations that are uncorrelated with past, present and future realizations of TFP. We further control for a number of leads and lags of shocks to expectations on TFP in order to isolate shifts in expectations that are pure sentiment from those originating from beliefs on future TFP. Using local projections, we find that sentiment shocks generate significant boom-bust dynamics in all the aggregate variables that we examine, and explain up to 40% of real GDP at business cycle frequencies, consistent with the findings of Angeletos et al. (2018) and Chahrour and Ulbricht (2017). When compared to the TFP shock, sentiment shocks appear to generate only slightly larger fluctuations in non-financial corporate debt but opposite impact responses of the labor wedge. We test the performance of the model in two ways. First we find that the model is able to reproduce the empirical impulse responses to both sentiment and TFP shocks. Second, we compute the theoretical spectral density implied by the two shocks and find that they are in line with their empirical counterparts.

Related literature. This paper lies in the intersection between the strand of finance literature that focuses on credit cycles and the broad macroeconomic literature that aims at understanding the sources of business cycles.

That the financial system is prone to generate economic instability through endogenous credit booms traces back at least to Minsky (1975) and Kindleberger (1978). More recently, the idea that an increase in credit associated with a decrease in borrowing costs can be a powerful predictor of future economic crisis has been empirically tested and verified using both macro and micro level data. For example, Schularick and Taylor (2012) and Jordà et al. (2013), using data on 14 developed countries from 1870 to 2008, demon-
strate that rapid credit expansions forecast declines in real activity.\footnote{Other examples include \textcite{Laeven2013}, \textcite{DemirgucKunt1998}, \textcite{Gourinchas2012}, \textcite{Claessens2011}, \textcite{Reinhart2009a}, \textcite{Borio2009}, \textcite{Gourinchas2001}, \textcite{Kaminsky1999}, \textcite{Hardy2001} and \textcite{Goldfajn2006}.} Using data on the credit quality of corporate debt issuers, \textcite{Greenwood2013} find that high share of risky loans tends to forecast low corporate bond returns. \textcite{Krishnamurthy2017} show that crisis are preceded by a period of high credit to GDP growth and leverage, and low spread and risk premium. We complement this literature by providing conditional evidence on the link between a credit boom and the ensuing recession in that we show that positive sentiment shocks - unlike TFP shocks - are systematically followed by a recession. Our evidence on sentiment shocks relates to \textcite{Lopez-Salido2017} who focus on credit market sentiment identified using credit spreads and find that high credit market sentiments are a predictor of future negative output growth. We complement their analysis by showing that sentiment shocks not only predict a negative output growth but also prolonged periods during which the level of output is below trend.

We relate to the literature that aims at rationalizing boom-bust phenomena. A subset of this literature builds model of chaos and limit cycles. \textcite{Boldrin1990} survey the literature and conduct formal analyses on the conditions under which limit cycles can emerge. In a recent paper, \textcite{Beaudry2019} revisit the reduced form evidence on the spectral densities of a series of economic variables and build a model of limit cycles where small exogenous shocks give rise to perpetual economic cycles. While our model can also exhibit limit cycles for regions of the parameter space that imply a sufficiently tight financial constraint, our aim is rather to rationalize the conditional boom-bust evidence from the empirical analyses which favor transitory oscillatory dynamics. A close paper to ours is \textcite{Gorton2014} who distinguish between “good” and “bad” credit booms depending whether or not they end up in a crisis. They find that shocks in the trend of productivity are associated to “good” credit booms, whereas “bad” booms are typically associated with a decline in productivity. We differ from them in at least three aspects. First, we look at cycles at short and medium-run frequencies while their focus is on booms that last ten years on average. Second, their model generate asymmetric cycles where a boom suddenly and dramatically ends in a crisis, whereas our model displays smoother dynamics. Third, we emphasize that the shocks responsible for boom-bust episodes are
orthogonal to movements of TFP. In Boissay et al. (2016) the increase in households’ savings triggers a recession because a decline in the real rate enhances adverse selection problems. In our model the increase in savings brings about a recession because it reflects an increase in firm’s debt which tightens financial markets.

Furthermore, we relate to the class of models that generate self-fulfilling rational expectations equilibria due to credit market amplification. Examples of this class are Benhabib and Wang (2013), Benhabib and Wen (2004), Liu and Wang (2014), and Azariadis et al. (2015). While their emphasis is on a single shock, our model is built to capture the important different responses to fundamental and sunspot shocks.

Lastly, our theoretical framework shares some similarities with models of stock market bubbles as in Miao and Wang (2018), in that, debt limits depend upon firms’ market value and sentiment shocks can be interpreted as bubbles. However, models of stock market bubbles formalize the burst of a bubble as an exogenous event. In contrast, in our model sentiment shocks rationalize both the formation of a bubble and its subsequent burst.

2 Motivation

In a recent article, Beaudry et al. (forthcoming) provide evidence in favour of U.S. business cycles being characterized by cyclical forces. In particular, they show that the spectral densities of a number of economic aggregates exhibit a common local peak at periodicities of 32 to 50 quarters. The spectral density is a useful diagnostic tool of cyclicality for two reasons. First, a peak in the spectral density signals the presence of oscillatory dynamics in the autocovariance function of the data. Second, it answers the question on whether the cyclical pattern is a property of the business cycle or it stems from lower frequency forces unrelated to business cycles.

Motivated by their findings, we are interested in understanding the sources of such oscillatory behaviour. To this end, this section shows results from two distinct exercises. First, we look at the unconditional spectral properties of a broad set of quarterly macroeconomic and financial variables and find results that confirm the findings of Beaudry et al. (forthcoming). Second, we ask whether technology shocks account for these empirical regularities.

The notion of cyclicality that we use is analogous to Beaudry et al. (forthcoming), that is a series is cyclical if its autocovariance function displays oscillations.
Figure 1 reports the spectral density of a series of macroeconomic and financial variables. Variables are detrended using a band pass filter that removes fluctuations with periodicities longer than 100 quarters. Our preferred specification uses data from 1981:Q1 to 2018:Q2 as it is consistent with the data sample we use for the estimation of impulse responses. However, results are robust to using different data lengths. With the exception of utilization-adjusted TFP, all variables exhibit a peak in the spectral density in the interval between 32 and 50 quarters. There are not notable qualitative differences in the shape of the spectral density across the variables, suggesting the presence of an underlying mechanism responsible for the cyclical behaviours rather than idiosyncrasies in the variables examined.

Importantly, the financial variables exhibit a more pronounced peak relative to the macroeconomic variables. Results seem broadly consistent with the idea that the cyclical features of the data originate from shocks propagating through the financial sector, whereas shocks that primarily hit the real sector of the economy generate less oscillatory dynamics. To validate this conjecture, we look at the economic responses to a TFP shock. Specifically, we use quarterly utilization-adjusted TFP available on Fernald’s website and identify technology shocks as the innovation of detrended TFP after regressing it on its own lags, lags of the first principal component of a large dataset of aggregate economic variables and news shocks estimated following Barsky and Sims (2011). We estimate impulse responses using the method of local projections proposed by Jordà (2005). Specifically, we estimate the $h$-th coefficient of the impulse response function by regressing each variable at time $t + h$ on the shock at time $t$.

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3 The spectral density is computed using the Schuster’s periodogram.
4 Because filtering the series could induce a spurious hump in the spectral density, we check that results are robust to various detrending techniques and frequency bands.
5 Beaudry et al. (forthcoming) do not find a peak in the spectral density of real GDP. The difference is in the Hamming window used to compute the unconditional spectral density. The authors claim that the overall presence of cyclicality combined with absence of a peak in real GDP is due to the behaviour of technology. This would be true if technology shocks did not imply cyclical dynamics while other shocks do, which is the point of this paper.
6 Results are robust to different detrending techniques, additional controls, and different number of lags and principal components. See Appendix B for results and additional details.
7 Details on local projections are in the Appendix C.
Figure 1: Unconditional spectral density of quarterly and seasonally adjusted U.S. macroeconomic and financial variables from 1981 to 2018. TFP is utilization-adjusted total factor productivity. GDP is real gross domestic product. Investment is real consumption of durables plus real gross private domestic investment. Hours is hours of all persons in nonfarm business sector. Change in debt is the flow of nonfinancial business debt securities and loans. Credit is total credit for private nonfinancial sector. Financial Conditions Index is an index of financial condition provided by Chicago Fed. BAA T-Bill Spread is the difference between the yield of BAA corporate bonds and the treasury bill at 10-year horizon. All variables are detrended using Band-Pass filter excluding periodicities above 100 quarters.

suggests to insert a large number of lags in the VAR estimation undermining its feasibility.

The top panel of Figure 2 shows the impulse responses of real GDP, investment and the change in nonfinancial corporate debt as a fraction of GDP, to a positive transitory technology shock. An unanticipated improvement of TFP leads to a hump-shaped response of real GDP and investment, aggregate debt rises during the initial build-up and decreases while the economy returns to its long run trend. To verify whether these impulse responses can account for the spectral properties of the data, we compute the spectral densities implied by the estimated coefficients of the moving averages. The bottom panel of Figure 2 shows that the spectral densities of real GDP and investment conditional to a TFP shock are monotonically increasing over business cycle periodicities. This poses a challenge to TFP-based explanations of boom-bust cycles. In next subsection we provide a formal test for the presence of a local peak in the spectral density implied by a structural shock.
Figure 2: Impulse responses (top panel) and conditional spectral densities (bottom panel) implied by a technology shock. A technology shock is estimated as the residual in utilization-adjusted TFP controlling for 6 lags of TFP, first principal component of a large dataset and news shocks estimated via Barsky and Sims (2011). Endogenous variables are the same described in Figure 1 and are stationarized using Band-Pass filter excluding periodicities above 100 quarters. Impulse responses are estimated via local projections and confidence intervals are computed the block-bootstrap technique suggested by Kilian and Kim (2011). Conditional spectral densities are estimated from the truncated moving average implied by the impulse responses.

2.1 Conditional test for a local peak

The lack of a local peak in the spectral density of output, investment, and TFP observed in Figure 2 suggests that technology shocks cannot account for spectral properties of the data shown in Figure 1. To make the point, we construct a test for the presence of a significant local peak in the spectral density conditional to a structural shock. The test procedure echoes Canova (1996) and Reiter and Woitek (1999) who design a test for the presence of a peak for the unconditional spectral density. Details of our procedure are presented in the Appendix E. The idea is to test if the shape of the conditional spectral density around a particular frequency range is not statistically different from the spectral density implied by an autoregressive process of order one. More specifically, define $D_1$ the average estimated spectral density over a range around 34 quarters, and $D_2$ the average estimated spectral density over a range around 45 quarters. The test statistic is the ratio $D \equiv D_1/D_2$. A
value of $D$ bigger than one indicates the spectral density is decreasing in the range 34 to 45 quarters. The spectral density associated to an AR(1) process, in contrast, is monotonically increasing in the periodicity. Therefore we test the null hypothesis $H_0 : D = D^*$ where $D^*$ is the value implied by an AR(1) with persistent parameter estimated from the data, against the alternative $H_1 : D > D^*$. Results for the technology-implied spectral density are reported in Table 1. We fail to reject the null hypothesis of absence of a local peak for GDP, investment, and TFP.

Taken together our reduced form and conditional evidence points at the presence of oscillatory properties of the data that do not appear to be captured by movements in TFP. In the next section we build a model that helps us rationalizing the findings and propose “pure” sentiment shock - defined as shifts in expectations unrelated to fundamental - as a natural candidate to explain the spectral properties of the data. In section 4 we construct novel empirical evidence in favor of this hypothesis and show that the model can reproduce the responses to sentiment and technology shocks together with the unconditional spectral densities of the data.

3 A model of conditional cycles

To capture the cyclical feature of the data, we build an otherwise standard RBC model with an endogenous debt limit. The key feature of the model is a credit market amplification channel which originates from a positive feedback loop between firms’ market value and households’ income. An increase in the market value of firms relaxes the debt limit thereby increasing production. The resulting increase in households’ income further drives up firm’s value. The model exhibits stationary sunspot equilibria for plausible parametrizations. Crucially, the model generates boom-bust dynamics in response to a sunspot shock but not to a technology shock. The reason is because the credit market amplification channel is key in propagating expectation shocks while it has a small contribution in shaping the dynamics in response to technology shocks. In particular, an improvement of technology has an ambiguous effect on the borrowing limit: on the one hand, firms’ market value rises thereby relaxing the borrowing constraint; on the other hand, it increases the amount of revenues that firms’ can abscond with. The net effect is a moderate reaction of credit markets during technology-driven expansions.
Crucially, the model stands in stark contrast to the class of models of self-fulfilling business cycle that provide microfoundations to the aggregate increasing returns to scale economy described in Benhabib and Farmer (1994). Amplification in the form of increasing returns would strongly influence the propagation of technology shocks, thus, while these models can generate endogenous oscillatory dynamics, they cannot simultaneously account for the empirical evidence on technology shocks.

For expositional reasons, we present first a benchmark model featuring intertemporal debt as the only state variable. In the next section we identify sentiment shocks in the data and augment the model with capital and external consumption habit to match empirical responses. We further validate model’s performance by showing that it does a good job in matching the spectral properties of the data.

3.1 Firm sector
There is a continuum $i \in [0, 1]$ of firms with a gross revenue function $F(z_t, k_t, n_t) = z_t k_t^{\theta} n_t^{1 - \theta}$. The variable $z_t$ is the stochastic level of productivity common to all firms, $n_t$ is the labor input, $k_t$ is the capital input which we assume to be constant and equal to one for now. Firms borrow inter period from households. We assume that debt $b_t$ carries a tax advantage such that given the interest rate $r_t$, the effective gross interest rate for the firm is $R_t = 1 + r_t (1 - \tau)$ where $\tau$ is the tax benefit. Thus, firms are effectively more impatient than households so that if financial markets are not too tight the equilibrium stock of debt will be positive. In addition to the intertemporal debt, firms raise funds with an intraperiod loan, $l_t$, to finance working capital. Because revenues are realized at the end of the period working capital is required to cover the intraperiod cash flow mismatch. The loan $l_t$ is paid at the end of the period with no interest.

The timing of the events is the same as in Jermann and Quadrini (2012). Shocks realize at the beginning of the period. Firms start with an outstanding debt equal to $b_t$ and choose labor $n_t$, the new intertemporal debt $b_{t+1}$ and distribute dividends $d_t$. Since payments are

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8 Examples of this class are Benhabib and Wang (2013) and Liu and Wang (2014).
9 The assumption of two types of debt is made for analytical convenience. In particular the intratemporal debt can be replaced with cash that firms carry from the previous period. Cash would then be used to finance working capital and pay part of dividends.
made before producing, the intraperiod loan is
\[ l_t = w_t n_t + \phi(d_t) + b_t - b_{t+1}/R_t, \]
where \( \phi(d_t) = d_t + \kappa(d_t - \bar{d})^2 \) includes a convex adjustment cost of dividends which captures documented evidence of preferences for dividend smoothing (Lintner, 1956). The end of period firm’s budget constraint is
\[ b_{t+1}/R_t + F(z_t, n_t) = w_t n_t + \phi(d_t) + b_t. \] (1)

It follows that firm’s revenues are equal to the intraperiod loan, that is \( l_t = F(z_t, n_t) \).

**Incentive constraint.** When production is completed, firms decide whether or not repay the intraperiod loan they owe to the household. Consistent with recent evidence on the procyclicality of unsecured debt (see Azariadis et al., 2015), we assume that contract enforcement is imperfect so that firms have incentives to default. If a firm defaults, it can be caught with probability \( \gamma \), in which case the firm would be liquidated and perpetually excluded from future access to credit. If a firm is not caught, it continues to retain access to credit in future periods. In either case, firm will abscond with its end of period revenues.\(^{10}\)

Specifically, firm defaults if
\[ y_t + (1 - \gamma)E_t m_{t,t+1} V_{t+1} + \gamma E_t m_{t,t+1} \max\{0, \psi \bar{k} - b_{t+1}/R_t - y_t/m_{t,t+1}\} > E_t m_{t,t+1} V_{t+1}, \]
where \( y_t \equiv F(z_t, n_t) \), \( m_{t,t+1} \) is the households’ stochastic discount factor, \( V_{t+1} \) is the firm’s future value defined as the net present value of future dividends, and the parameter \( \psi \) is positive and strictly smaller than one in order to capture the partial irreversibility of capital.

Because there is no intra-period uncertainty, there exist default-deterring credit limits. Assuming that the liquidation value of a defaulting firm is smaller than firm’s liabilities, the incentive constraint can be simplified to
\[ \gamma E_t m_{t,t+1} V_{t+1} \geq y_t. \] (2)

\(^{10}\)Assuming that in the case of being caught a firm would also lose its revenues does not quantitatively alter our results.
Because firm’s value depends negatively on its outstanding debt, the incentive constraint will always bind in equilibrium.

The problem of the individual firm can be written recursively as

$$V_t = \max_{d_t, m_t, b_{t+1}} \left\{ d_t + E_t \left[ m_{t,t+1}V_{t+1} \right] \right\}$$

subject to (1) and (2).

Firm’s first order conditions are

$$\left(1 + \mu_t \gamma \right) R_t E_t \left[ m_{t,t+1} \phi'(d_t) \phi'(d_{t+1}) \right] = 1$$

$$\left(1 - \theta \right) \frac{y_t}{n_t} = \frac{w_t}{1 - \mu_t \phi'(d_t)}$$

where $\mu_t$ is the Lagrange multiplier associated to the incentive constraint. Equation (4) is the first order condition of new intertemporal debt $b_{t+1}$. It states that the marginal cost of debt increases with the tightness of the credit limit $\mu_t$ and the effective firm’s discount factor which is the household’s discount factor times the expected decrease in the adjustment cost of dividends. From the first order condition of labor input (5), looser borrowing constraint increases labor demand and allows firms to borrow more intra-period. The resulting increase in labor income and dividends increases households’ asset demands further relaxing the borrowing constraint.

Furthermore, looser credit constraints also increase the intertemporal loan. To see this, combine the budget constraint of the firms with the optimality condition for labor:

$$\frac{b_{t+1}/R_t - b_t}{y_t} = \frac{\phi(d_t)}{y_t} + (1 - \theta)(1 - \mu_t \phi'(d_t)).$$

As credit market relaxes, that is $\mu_t$ decreases, for a given dividend to output ratio, the intertemporal debt rises.

### 3.2 Households sector and general equilibrium

There is a continuum of homogeneous utility-maximizer households. Households are the owners of firms. They hold equity shares and noncontingent bonds issued by firms. House-
holds’ instantaneous utility function is

\[ U(c_t, n_t) = \frac{c_t^{1-\omega} - 1}{1 - \omega} + \alpha \log(1 - n_t). \]

The household’s budget constraint is

\[ c_t + s_{t+1}p_t + \frac{b_{t+1}}{1 + r_t} = w_t n_t + b_t + s_t(d_t + p_t) - T_t \tag{6} \]

where \( s_t \) is the equity shares and \( p_t \) is the market price of shares. The government finances the tax benefits to firms through lump-sum taxes equal to

\[ T_t = B_{t+1} / [1 + r_t(1 - \tau)] - B_t / (1 + r_t). \]

The first order conditions with respect to \( n_t, b_{t+1}, \) and \( s_t \) are

\[ w_t = -\frac{U_n(c_t, n_t)}{U_c(c_t, n_t)} \tag{7} \]

\[ U_c(c_t, n_t) = \beta(1 + r_t)E_t U_c(c_{t+1}, n_{t+1}) \tag{8} \]

\[ p_t = \beta E_t \left\{ \frac{U_c(c_{t+1}, n_{t+1})}{U_c(c_t, n_t)} (d_{t+1} + p_{t+1}) \right\} \tag{9} \]

Given the aggregate states \( s \), that are productivity \( z \) and aggregate bonds \( B \) we can define the general equilibrium as follows:

**Definition:** A recursive competitive equilibrium is defined as a set of functions for (i) households’ policies \( c^h(s, b), n^h(s, b) \) and \( b^h(s, b) \); (ii) firms’ policies \( d(s, b), n(s, b), \) and \( b(s, b) \); (iii) firms’ value \( V(s, b) \); (iv) aggregate prices \( w(s), r(s), \) and \( m(s', s) \); (v) law of motion for the aggregate states \( s' = \psi(s) \). Such that: (i) household’s policies satisfy conditions (7) and (8); (ii) firm’s policies are optimal and \( V(s, b) \) satisfies the Bellman’s equation (3); (iii) the wage and the interest rate clear the labor and bond markets; (iv) the law of motion \( \psi(s) \) is consistent with individual decisions and stochastic processes for productivity.

### 3.3 Inspecting the mechanism

The key externality in the model is that households do not take into account the effects of their savings decisions on the financial constraint. Likewise, firms only partly internalize the effects of their production decisions on their market value. In particular, they understand that a higher level of debt reduces their market value by limiting their ability to
distribute dividends, but they do not internalize the effects of their decisions on variations of their market value due to changes in the present and future stochastic discount factor. This generates a positive feedback loop between firms’ market value and households’ income. Absent of adjustment cost of dividends, credit market amplification depends upon the elasticity of firms’ production to the households’ stochastic discount factor. This elasticity is equal to

\[
\frac{\partial \log(y_t)}{\partial \log(m_{t,t+1})} = \frac{\beta\tau}{\gamma(1-\mu)(1-\tau+\tau\beta)^2} \left[ \frac{(1-n)(1-\theta)}{(\omega-1)(1-n)(1-\theta)+1} \right] \equiv \xi,
\]

where \(\mu = \tau(1-\beta)/\gamma(1-\tau+\tau\beta)\).

If credit market frictions are severe, that is the probability of being excluded from financial market \(\gamma\) is low or the tax advantage on debt \(\tau\) is high, firms are more responsive to changes in their continuation value reflected by changes in the stochastic discount factor. Sufficiently high values of \(\xi\) give rise to self-fulfilling equilibria. Assume lenders expect higher firms’ value; this relaxes the financial constraint and implies an increase in the credit supply. Thus, production and households’ labor income increase which raise firms’ market value through an increase in the stochastic discount factor \(m_{t,t+1}\) validating the initial shift in expectations.

Formally, take a first order approximation around the steady state, aggregate output can be expressed as

\[
\hat{y}_t = \frac{\omega \xi}{\omega \xi - 1} E_t \hat{y}_{t+1} - \frac{1}{\zeta(\omega \xi - 1)} \hat{z}_t
\]

where \(\zeta \equiv (\omega - 1)(1-n)(1-\theta) + 1\).

When \(\omega \xi > 1/2\), current aggregate output is a convex function of future output which is sufficient to generate indeterminacy.

Note that the impact of technology shocks on aggregate output is ambiguous. By increasing end of period revenues, a positive technology shock raises firm’s incentives to divert funds thereby increasing the right-end-side of the incentive constraint in eq. (2). Whether firm’s market value increases more than firm’s revenue depends upon firm’s willingness to distribute dividends. We find that for plausible parametrizations, the Lagrange multiplier \(\mu_t\) increases in response to a positive technology shock.

A current loosening of financial constraints leads firms to borrow more and hinge upon
their ability to borrow in the future. In fact, firm’s value depends upon the amount of intertemporal debt $b_{t+1}$ which in turn depends positively upon the outstanding debt $b_t$ at the beginning of the period. The resulting dynamic substitutatibility between current and future production allows for the possibility of boom-bust dynamics. The following proposition lists the necessary conditions under which boom-bust fluctuations may obtain in response to perturbations from the economy’s steady state.

**Proposition 1**  *Boom-bust phenomena obtain only if*

i. *The equilibrium is indeterminate.*

ii. *Adjustment costs are non zero, that is $\kappa > 0$.*

*Proof is relegated in Appendix F.*

Condition (i) states that if the credit market amplification channel is strong enough, so that indeterminacy obtains, then the economy can also be subject to oscillatory dynamics.\(^{11}\) The intuition is that after an initial expansion, firms have accumulated large amount of debt which limits their ability to borrow and produce. As firms decrease production they do not internalize the adverse effects on their market value. The stronger are the effects of this externality the larger is the drop in current production. The reason why adjustment cost of dividends is necessary to obtain cycles is more subtle. Besides the static amplification mechanism described above, the model displays dynamic substitutability between current and future production generated by movements in firms’ net worth. An increase in new debt brings about higher current production but it decreases future firms’ net worth which negatively affects the subsequent level of production. Absent dividend adjustment costs, firms with high level of outstanding debt would finance production by decreasing the amount of distributed dividends, therefore limiting the impact that changes of net worth on their production decisions, thus preventing the large accumulation of debt after the expansion to generate a recession.

### 3.4 Parametrization and theoretical impulse responses

The sunspot shock is defined as an i.i.d. expectation error of firm’s value that is not correlated with fundamentals

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\(^{11}\) This property is not specific to the environment described here. Gu et al. (2013) discuss the link between indeterminacy and cycles in the context of financial frictions of different forms.
\[ V_t - E_{t-1}V_t = u_t \]

where \( u_t = \epsilon_{s,t} + \psi_z \epsilon_{z,t} \).

\( \epsilon_{s,t} \) and \( \epsilon_{z,t} \) are respectively the sunspot shock and the technology shock.\(^{12}\) The natural logarithm of technology is assumed to follow an AR(1) process as

\[ \hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_{z,t}. \]

We calibrate the model to a quarterly frequency consistent with the frequency of the data. We set \( \beta \) in order to match a 3\% annual interest yield on bonds. Following Jermann and Quadrini (2012) \( \tau \) and \( \theta \) are set equal to 0.35 and 0.36, respectively. The inverse of households’ intertemporal elasticity of substitution \( \omega \) is set to 1.211 while the degree of adjustment cost to dividends, \( \kappa \), is 2.3 so as to generate oscillatory dynamics. The parameter \( \rho_z \) governs the persistence of the technology process and is set equal to 0.93 consistent with the law of motion of detrended TFP estimated in the data. We assume the expectation error \( u_t \) and the technology shock to be uncorrelated, so that \( \psi_z \) is equal to zero.\(^{13}\)

Figure 3 shows the theoretical impulse responses of the model to a sunspot shock and to a technology shock.

In response to the sunspot shock the economy experiences an initial boom characterized by an increase output, consumption and hours. The associated increase in debt has two effects. On the one hand, it reflects an increase in households’ savings which increases the supply of credit generating a decrease in the real rate and an increase in firms’ market value. On the other hand, larger outstanding debt hinders firms’ ability to pay current and future dividends which deteriorates their market value. Which of these two forces prevails depends upon the level of firms’ profitability. As production increases firms’ profitability decreases so that firms’ market value decreases, the financial constraint tightens

\(^{12}\) Note that inserting the sunspot on output would not alter our results. It is easy to show that

\[ V_t - E_{t-1}V_t = \omega(Y_t - E_{t-1}Y_t). \]

\(^{13}\) Note that \( \psi_z \) equal zero implies a zero-impact response of output and firm’s value after a technology shock. While this is an implausible restriction that will be relaxed in the quantitative exercise, it allow us to generate more hump-shaped dynamics in response to a technology shock.
Figure 3: Theroretical impulse responses to a technology shock (blue dotted line) and to sunspot shock (red dashed line). Outstanding debt is $b_{t+1}$ Change in debt to output is defined as $\Delta b_{t+1}/y_t$. Operating profits are $y_t - w_t n_t$. Technology shock is normalized to have a unitary impact effect on technology while sunspot shock is normalized to have a unitary effect on output after one year.

and output starts declining. During the contraction phase, households are less willing to lend which results in an increase in the real rate, a decrease in firm’s value and a further tightening of the financial market. This negative vicious circle reinforces as households’ savings decline, ultimately bringing about a recession. Importantly, even though agents know about the incoming recession their actions magnifies the decline in output.

A positive technology shock generates hump-shaped dynamics in all the main macroeconomic variables. By increasing incentives to divert funds, a positive technology shock tightens the financial constraint which dampens the impact response of output. Importantly, the response of debt and output is comparable to the ones after a sunspot shock, suggesting that looking at measures of firms’ indebtedness such as the debt to GDP ratio may not be the best predictor of a crisis.

Crucially, sentiment-driven fluctuations arise also in an economy where fundamentals, that is technology, preferences, or government policies, do not change and this is common knowledge. This distinguish them from noise shocks arising from ex post erroneous beliefs.
on future changes of technology. Bearing this distinction in mind, in the next section, we estimate expectation shocks unrelated to fundamentals and to rational expectations of fundamentals. We find that these shocks generate boom-bust dynamics consistent with the quantitative prediction of an extended version of the model.

4 Identifying sunspot shocks using survey data

In this section we estimate the sunspot shock as a “pure” sentiment shock, that is a shock that reflects a change in expectations disconnected from changes in expectations on future TFP and realizations of TFP. To this end, we use quarterly one-year-ahead expectations on a number of key macroeconomic variables formed by both professional forecasters and households. We proceed in three steps.

First, we construct an indicator $\hat{S}_t$ that summarizes revisions in the expected economic outlook using quarterly revisions by professional forecasters in the expected one year growth of real GDP, nominal GDP, and industrial production, together with the Michigan survey of expected change of business conditions in one year. The indicator is constructed taking the first principal component from these series from 1981:Q1 to 2018:Q2.

Second, we regress the constructed indicator $\hat{S}_t$ on a battery of controls in order to capture variations in expectations that are “extrinsic”, that is, exogenous to fundamentals and to changes in expectations on future fundamentals. Formally, let the process of detrended TFP be represented by the following news representation

$$
\log(TFP)_t = A(L) \log(TFP)_{t-1} + \varepsilon^z_t + \sum_{k=1}^{\infty} \varepsilon^k_{t-k}
$$

where $\varepsilon^k_{t-k}$ is a news shock on TFP k-period ahead which is part of time $t$ agents’ information set, and $\varepsilon^z_t$ is the surprise shock of technology. Let $S^K_t$ be the indicator that summarizes revision of agents expectations on the economic activity $K$-period ahead. We assume that these revisions depend upon current technology shocks, expectations on future technology, and sentiment shocks. Specifically,

$$
S^K_t = \lambda_0 \log(TFP) + \sum_{k=1}^{K} \alpha_k \varepsilon^k_t + \varepsilon^s_t
$$

where expectations on future technology are a linear combination of news upon technol-
ogy up to $K$ horizons. Hence, in order to identify sentiment shocks one needs to cleanse changes in expectations, proxied by $\hat{S}_t$, from the realized level of TFP and expectations about future TFP up to the horizon $K$. In other words, we want the estimated sentiment shock to satisfy two conditions: (i) the estimated sentiment shock must be uncorrelated with future TFP realizations; (ii) the sentiment shock has to be uncorrelated with noise shocks, defined as ex-post wrong beliefs on future TFP.\(^{14}\)

We proxy expectations on future TFP with TFP news shocks identified as in Barsky and Sims (2011). However, this controlling set may no be large enough to satisfy the two conditions above. To overcome this issue we add two additional set of controls. First, we control for future realizations of TFP so as to guarantee that the estimated shock has no impact on future TFP. Second, as shown by Chahrour and Jurado (2018), one can recover noise shocks by adding future news and realizations of TFP to the econometrician’s information set. Thus, we further control for future realizations of the identified news shock. Specifically, sentiment shocks are estimated from the following equation:

$$\hat{\varepsilon}_t^s = \hat{S}_t - \sum_{k=0}^{K} \hat{\lambda}_k TFP_{t+k} + \sum_{k=0}^{K} \hat{\alpha}_k \varepsilon_t^{BS} - X_t \hat{\beta}$$

where $\varepsilon_t^{BS}$ is the news shock estimated using the procedure in Barsky and Sims (2011), and $X_t$ is a vector of additional control variables, including past realizations of TFP and news, other shocks to fundamentals such as monetary policy and fiscal shocks, and past values of the first two principal components from a large data set of U.S. aggregate variables. Interestingly, even after controlling for virtually all available sources of fundamental fluctuations, estimated sentiment shocks explain approximately half of the changes in the expectation indicator $\hat{S}_t$.

In the last step, we estimate the impulse response to a sentiment shock using Local Projections as in Jordà (2005). Specifically, for each variable of interest $Y$, we run the following equation:

\(^{14}\) As shown by Beaudry and Portier (2004) noise shocks in the form of ex-post wrong beliefs on future TFP can give rise to Pigouvian cycles and therefore are a competing candidate to the explanation of the reduced form evidence presented in Section 1. However, we find that controlling for this particular type of beliefs has small quantitative changes on the variance explained by the sentiment shock, suggesting that noise shocks play only a minor role in shaping expectations.
A series of regressions

\[ Y_{t+h} = \theta^h \hat{\varepsilon}_t + \sum_{j=1}^J \left[ \delta_j \hat{\varepsilon}_{t-j} + \lambda_j Y_{t-j} + PC_{t-j} \Gamma_j \right] + \nu_{t+h} \quad \text{for } h = 0, 1, \ldots, H \]  

(11)

where \( \theta^h \) is the response of \( Y \) to a sentiment shock after \( h \) periods, and \( PC \) is a vector including the first two principal component from a set of U.S. aggregate variables. We use four lags, that is \( J = 4 \), in the baseline specification.

**Figure 4:** Impulse responses (top panel) and conditional spectral densities (bottom panel) implied by a sentiment shock. A sentiment shock is estimated as the residual of the principal component of a series of SPF forecast revisions after controlling for past information, current and future TFP. Endogenous variables are the same described in Figure 1 and are detrended using Band-Pass filter which excludes periodicities above 100 quarters. Impulse responses are estimated via local projections and bootstrap is estimated with the same procedure suggested by Kilian and Kim (2011). Conditional spectral densities are estimated from the truncated moving average implied by the related impulse responses.

Figure 4 shows the responses to a sentiment shock of real GDP, real Investment, and the change of non-financial corporate debt divided by real GDP. In response to a sentiment shock, real GDP, investment and debt flow exhibit significant oscillatory dynamics. In particular after a positive sentiment shock, the economy enters in an expansion followed

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15 The shock is normalized to have an unit impact on GDP after four quarters.
by a recession after about two years. Importantly, the conditional spectral density to a sentiment shock implies a peak associated to periodicities of 8 to 10 years in line with the reduced form evidence presented earlier. Table 1 reports the p-values for the test of a local peak in the spectral density implied by sentiment shocks. The null hypothesis of absence of a local peak is rejected for all variables, with the obvious exception of TFP. In addition, from a variance decomposition exercise, we find that sentiment shocks explain up to 40% of real GDP at business cycle frequencies.

<table>
<thead>
<tr>
<th>Sentiment Shock</th>
<th>GDP</th>
<th>Investment</th>
<th>Change in Debt / GDP</th>
<th>TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sentiment Shock</td>
<td>3.64%</td>
<td>4.82%</td>
<td>2.24%</td>
<td>28.4%</td>
</tr>
<tr>
<td>Tech Shock</td>
<td>28.52%</td>
<td>5.54%</td>
<td>0.1%</td>
<td>89.84%</td>
</tr>
</tbody>
</table>

Table 1: P-values for the test of a local peak in the spectral density implied by sentiment shock (first row) and technology shock (second row).

4.1 Robustness checks

In this section we show that the results in Figure 4 are robust to different detrending techniques, additional controls, and the expectation variables used to construct the indicator $S_t$. Given that our endogenous variables are non-stationary, in the baseline specification we detrend the variables using a Band-Pass filter which excludes periodicities above 100 quarters. In order to argue that the oscillatory dynamics implied by a sentiment shock is not specific to the detrending technique, in Figure 5 we show robustness checks where endogenous variables are detrended using $(i)$ first differences (and the cumulated), $(ii)$ linear time trend, $(iii)$ quadratic time trend, and $(iv)$ Hodrick-Prescott filter. Results are in line with the baseline specification and most of the estimates lie between the confidence intervals of the main specification.
Figure 5: Impulse responses (top panel) and conditional spectral densities (bottom panel) implied by a sentiment shock. Point Estimates are from the baseline specification presented in Figure 4. RC 1 is the first robustness check where variables are differentiated before employ the LP estimator and then cumulated in the plot above. RC 2 and RC 3 are the second and the third robustness check where variables are linearly and quadratically detrended, respectively. RC 4 is the last robustness check where variables are HP-filtered.

Figure 6 reports results for four additional variations of the baseline specification. First, we increase the number of lags and the number of principal components in the regression equation of the sentiment shock. Second, we control for the present and the past of other shocks to fundamentals such as oil shocks, fiscal shocks, military spending news shocks and monetary policy shocks. Third, we check whether results are sensitive to the choice of the indicator for the revisions of expectations. Specifically, we use only revisions on one-year-ahead output growth from the SPF and find results that are not significantly different from the baseline. Finally, we check that results are robust to the number of lags and principal components used in the LP.
Figure 6: Impulse responses and conditional spectral densities implied by a technology shock. Point Estimates is the baseline specification presented in Figure 2. RC 1 and RC 2 are the first and the second robustness check where variables are linearly and quadratically detrended, respectively. RC 3 is the third robustness check where we add more controls when we estimate a technology shock. RC 4 is the last robustness check where we use different number of lags and principal component when we estimate LP impulse responses.

5 Model with capital and external consumption habit

In this section we augment the model with variable capital. In addition, we allow for the possibility of investment-adjustment costs and external consumption habit. The relevant parameters are estimated via impulse response matching. The equilibrium equations of the extended model are:

\[ w_t U_c(c_t, c_{t-1}, n_t) = -U_n(c_t, c_{t-1}, n_t) \]  \hspace{1cm} (12)

\[ \beta E_t[m_{t,t+1}(R_t - \tau)] = 1 - \tau \]  \hspace{1cm} (13)

\[ w_t n_t + b_t - \frac{b_{t+1}}{R_t} + d_t = c_t \]  \hspace{1cm} (14)

\[ [1 - \mu_t \phi'(d_t)] F_n(z_t, k_t, n_t) = w_t \]  \hspace{1cm} (15)
\[ k_{t+1} = (1 - \delta) k_t + \left[ \frac{s_1}{1 - \nu} \left( \frac{i_t}{k_t} \right)^{1-\nu} + s_2 \right] k_t \] (16)

\[ E_t \left\{ m_{t,t+1} \frac{\phi'(d_t)}{\phi'(d_{t+1})} (1 + \mu_t \gamma) \left[ (1 - \phi'(d_{t+1}) \mu_{t+1}) F_k(z_{t+1}, k_{t+1}, n_{t+1}) + 
    \frac{1}{s_1} (\frac{i_{t+1}}{k_{t+1}}) (1 - \delta + \frac{s_1 \nu}{1 - \nu} (\frac{i_{t+1}}{k_{t+1}})^{1-\nu} + s_2) \right] \right\} = 1 \] (17)

\[ (1 + \mu_t \gamma) E_t \left[ m_t \frac{\phi'(d_t)}{\phi'(d_{t+1})} R_t \right] = 1 \] (18)

\[ y_t = w_t m_t - b_t + \frac{b_{t+1}}{R_t} i_t = \phi_t(d_t) \] (19)

\[ \gamma E_t [m_{t,t+1} V_{t+1}] = y_t \] (20)

where \( y_t = F(z_t, k_t, n_t) = z_t k_t^\theta n_t^{1-\theta} \) and \( \phi(d_t) = d_t + \kappa(d_t - d^{ss})^2 \). Moreover, the stochastic discount factor is \( m_{t,t+1} = \beta(U_{c,t+1}/U_{c,t}) \) and value of the firm is defined as \( v_t = d_t + E_t[m_{t} v_{t+1}] \). Finally, \( U_c(c_t, c_{t-1}, n_t) = (c_t - \omega c_{t-1})^{-\alpha} \) and \( U_n(c_t, c_{t-1}, n_t) = -\alpha(1 - n_t)^{-\omega_2} \).

5.1 Calibration and impulse response matching

Following Christiano et al. (2005) we divide the model parameters in two different groups. The first group is calibrated while the remaining parameters are estimated via impulse response matching. We calibrate the model to a quarterly frequency. The discount factor \( \beta \) is set to 0.9926 to match an annual 3% yield of bonds in steady state. Capital’s share of income \( \theta \) is set to 0.36. We assume a quarterly depreciation rate on capital of 0.025. Tax shield \( \tau \) is set equal to 0.35 following Jermann and Quadrini (2012). The parameter \( \alpha \) is chosen such that the steady state value of \( n \) is equal to one third. The parameter \( \omega_2 \) is set equal to one to imply a Frisch labor supply elasticity equal to 2. Moreover, \( \varsigma_1 \) and \( \varsigma_2 \) are set such that in the steady state the depreciation rate is equal to \( \delta \) and the steady state Tobin’s \( q \) is equal to one. Furthermore, \( \psi_z \) - which governs the response of firm’s value to a technology shock - is set in order to match the empirical impact response of technology to output.

The second group includes the vector of parameters \( \Sigma = (\omega, \iota, \gamma, \nu, \kappa, \rho_z) \). These param-
eters are set to minimize the distance between the empirical and model-implied impulse responses. In particular, we chose $\Sigma$ that minimizes the following objective

$$J = \min_{\Sigma} [\hat{\Psi} - \Psi(\Sigma)]'V^{-1}[\hat{\Psi} - \Psi(\Sigma)]$$

where $\hat{\Psi}$ denotes the empirical impulse responses of GDP, Consumption, hours worked and TFP to both technology and sentiment shocks, $\Sigma$ is the vector of estimated parameters, and $\Psi(\Sigma)$ is the model-implied counterpart of $\hat{\Psi}$. $V$ is a diagonal matrix which gives different weights to the target estimates. Table 2 reports the parameter values of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Disutility of labor</td>
<td>8.785</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>CRRA labor</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9926</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tax shield</td>
<td>0.35</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Capital share</td>
<td>0.36</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation</td>
<td>0.025</td>
</tr>
<tr>
<td>$\varsigma_1$</td>
<td>Capital adj. cost (1)</td>
<td>$\delta^\nu$</td>
</tr>
<tr>
<td>$\varsigma_2$</td>
<td>Capital adj. cost (2)</td>
<td>$\delta - \delta/(1 - \nu)$</td>
</tr>
<tr>
<td>$\psi_z$</td>
<td>Technology on $V_t$</td>
<td>0.29</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Technology persistence</td>
<td>0.93</td>
</tr>
<tr>
<td>$\omega$</td>
<td>CRRA consumption</td>
<td>1.3219</td>
</tr>
<tr>
<td>$\iota$</td>
<td>Consumption habit</td>
<td>0.699</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Capital adj. cost (3)</td>
<td>0.59154</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Dividend adj. cost</td>
<td>0.44606</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Incentive parameter</td>
<td>0.094009</td>
</tr>
</tbody>
</table>

Table 2: Model’s parameter values.

### 5.2 Model performance

Figures 7 and 8 plot the theoretical impulse response of the model against their empirical counterparts. The model does a good job in reproducing the empirical impulses to both shocks. In particular, we estimate the model consistent measure of labor wedge and find that the responses are in line with the predictions of the model.
Figure 7: Empirical and model-implied impulse responses to a sentiment (sunspot) shock. All the responses are percentage deviation from the steady state except for change in debt to GDP ratio which is express in levels. Empirical impulse responses are estimated using the same procedure presented in Figure 4. Labor wedge is estimated following Zhang (2018).

Figure 8: Empirical and model-implied impulse responses to a technology shock. All the responses are percentage deviation from the steady state except for change in debt to GDP ratio which is express in levels. Empirical impulse responses are estimated using the same procedure presented in Figure 2. Labor wedge is estimated following Zhang (2018).
Finally, Figure 9 shows the empirical conditional spectral densities against their model counterpart. The theoretical spectral densities implied by the model are within the range of the confidence bands of the empirical ones.

**Figure 9**: Empirical and model-implied conditional spectral densities. Endogenous empirical and theoretical variables are the same described in Figures 1 and 3. Conditional spectral densities are estimated from the truncated moving average implied by the related impulse responses.
References


HARDY, D. AND C. PAZARBAŞIOĞLU (1998): “Leading indicators of banking crises: was Asia different?”


A Unconditional Spectral Density

Figure 10: Unconditional spectral density of quarterly and seasonally adjusted U.S. macroeconomic and financial variables from 1981 to 2018. TFP is utilization-adjusted total factor productivity. GDP is real gross domestic product. Investment is real consumption of durables plus real gross private domestic investment. Hours is hours of all persons in nonfarm business sector. Change in debt is the flow of nonfinancial business debt securities and loans. Credit is total credit for private nonfinancial sector. Financial Conditions Index is an index of financial condition provided by Chicago Fed. BAA T-Bill Spread is the difference between the yield of BAA corporate bonds and the treasury bill at 10-year horizon. All variables are stationarized using Band-Pass filter excluding periodicities above 100 quarters. Confidence intervals are computed following the procedure described in Beaudry et al. (forthcoming).

B Robustness Checks to Technology Shocks

Figure 11 reports impulse responses together with conditional spectral densities implied by a technology shock for the baseline specification presented in Figure 2 and a series of robustness checks. In particular, RC 1 and RC 2 are the first and the second robustness check where variables are linearly and quadratically detrended, respectively. RC 3 is the third robustness check where TFP is controlled using 8 lags of TFP, the first 2 principal components and news shocks. RC 4 is the last robustness check where we use different number of lags and principal component when we estimate LP impulse responses.
Figure 11: Impulse responses and conditional spectral densities implied by a technology shock. Point Estimates is the baseline specification presented in Figure 2. RC 1 and RC 2 are the first and the second robustness check where variables are linearly and quadratically detrended, respectively. RC 3 is the third robustness check where we add more controls when we estimate a technology shock. RC 4 is the last robustness check where we use different number of lags and principal component when we estimate LP impulse responses.

C Local Projections

To estimate LP impulse responses we follow standard techniques as firstly introduced by Jordà (2005). Given the stationary series $y_t$ and shock $\varepsilon_t$, impulse responses can be estimated as follows,

$$y_{t+h} = \theta_h \varepsilon_t + \sum_{j=1}^{J} \left[ \delta_j \varepsilon_{t-j} + \lambda_j y_{t-j} + \gamma_j x_{t-j} \right] + \nu_{t+h} \quad \text{for } h = 0, 1, \ldots, H$$

(21)

where $\theta_h$ represents response of $y_t$ to shock $\varepsilon_t$ at horizon $h$ and $x_t$ are additional controls which in our estimation represent principal components from a large dataset of macroeconomic variables.

C.1 Inference

Following Kilian and Kim (2011) we estimate confidence interval using the block bootstrap procedure. As emphasized by Kilian and Kim (2011), we opt for this approach because the
error term in the local projections regressions is most likely serially correlated. The LP impulse response estimator for horizon $h$ depends on the tuple,

$$T_h = [y_{t+h} \ \ \varepsilon_t \ \ \varepsilon_{t-1} \ \ \ldots \ \varepsilon_{t-j} \ \ y_{t-1} \ \ \ldots \ \ y_{t-1}]$$ (22)

To preserve the correlation in the data, build the set of all $T_h$ tuples for $h = 0, 1, \ldots, H$. For each tuple $T_h$, employ the following procedure:

1. Define $g = T - l + 1$ overlapping blocks of $T_h$ of length $l$.\(^\text{16}\)
2. Draw with replacement from the blocks to form a new tuple $T_h^b$ of length $T$.
3. Estimate $\theta_h^b$ from $T_h^b$ using LP estimator.

### D Variance Decomposition

Variance decomposition is estimated following Gorodnichenko and Lee (2017). In particular, we define the population share of variance explained by the future innovations in $\varepsilon_t$ to the total variations in the unpredictable component of $y_{t+h}$ as,

$$v_h = \frac{\sigma_{\varepsilon}^2 \sum_{i=0}^{h} \theta_i}{\text{Var}(f_{t+h|t-1})}$$ (23)

where $\text{Var}(\varepsilon_t) = \sigma_{\varepsilon}^2$ and $\theta_i$ are LP estimators. Moreover $f_{t+h|t-1}$ can be estimated from the following regression,

$$y_{t+h} = \sum_{j=1}^{J} \delta_j \varepsilon_{t-j} + \sum_{i=1}^{I} \lambda_i y_{t-i} + \sum_{q=1}^{Q} \gamma_q x_{t-q} + f_{t+h|t-1}$$ (24)

where $x_{t-q}$ represents a vector of additional controls.

Since the estimator $v_h$ does not guarantee estimates to be between 0 and 1, we use the

\(^{16}\) Notice that $l = (T - I - J + 2)^{\frac{1}{2}}$ is defined following Berkowitz, Birgean and Kilian (1999). Results are not sensitive to alternative choices of $l$. 

following estimator,\(^17\)

\[
\tilde{v}_h = \frac{\sigma^2 \sum_{i=0}^{h} \theta_i}{\sigma^2 \sum_{i=0}^{h} \theta_i + \text{Var}(\nu_{t+h} - \sum_{i=0}^{h-1} \theta_i x_{t+h-i})}
\]

(25)

where \(\nu_{t+h}\) is coming from the LP regression,

\[
y_{t+h} = \theta_h \varepsilon_t + \sum_{j=1}^{J} \delta_j \varepsilon_{t-j} + \sum_{i=1}^{I} \lambda_i y_{t-i} + \nu_{t+h}.
\]

(26)

### D.1 Inference

To estimate confidence intervals for \(\tilde{v}_h\), we directly use the non-parametric confidence intervals estimated for \(\theta_i\). In particular, use simulated \(\theta^b_i\) to estimate,

\[
\tilde{v}^b_h = \frac{\sigma^2 \sum_{i=0}^{h} \theta^b_i}{\sigma^2 \sum_{i=0}^{h} \theta^b_i + \text{Var}(\nu_{t+h} - \sum_{i=0}^{h-1} \theta^b_i x_{t+h-i})}
\]

(27)

and select confidence intervals.

### E Conditional Spectral Density and Cyclicalility Test

Consider the case where stationary variable \(y_t\) is explained by two shocks: \(\varepsilon_{1,t}\) and \(\varepsilon_{2,t}\). In this case, \(y_t\) can be represented with the following infinite moving average,

\[
y_t = \sum_{h=0}^{\infty} \theta_{1,h} \varepsilon_{1,t-h} + \sum_{h=0}^{\infty} \theta_{2,h} \varepsilon_{2,t-h}
\]

(28)

Since the estimated impulse responses cannot cover an infinite number of lags consider the truncate moving average,

\[
y_t \approx \sum_{h=0}^{H} \theta_{1,h} \varepsilon_{1,t-h} + \sum_{h=0}^{H} \theta_{2,h} \varepsilon_{2,t-h}
\]

(29)

\(^{17}\)See Local Projections Based Method by Gorodnichenko and Lee (2017). In particular, we refer to Equation (9’) at page 5.
Since we are interested in the conditional cyclicality implied by the two shocks, we focus on the conditional moving average,

\[ y_{k,t} \approx \sum_{h=0}^{H} \theta_{k,h} \varepsilon_{k,t-h} \quad \text{for } k = 1, 2. \]  

(30)

where \( y_{k,t} \) represents the realized value of \( y_t \) only conditional on shock \( \varepsilon_{k,t} \) for \( k = 1, 2 \).

Conditional spectral densities are parametrically estimated by taking the Fourier transform of the estimated truncated moving average. Estimators are,

\[ s_k(\omega) \approx \left[ \sum_{h=0}^{H} \theta_{k,h} e^{ih\omega} \right] \sigma_k^2 \left[ \sum_{h=0}^{H} \theta_{k,h} e^{-ih\omega} \right] \quad \text{for } k = 1, 2. \]  

(31)

where \( \omega \in (0, \pi] \) represents frequencies, \( i = \sqrt{-1} \), \( \theta_{k,h} \) is the LP estimator, and \( \sigma_k^2 \) is a standard estimator for \( Var(\varepsilon_{k,t}) \).  

E.1 Inference

Similarly to what we have done for the variance decomposition, to estimate confidence intervals for \( s_k(\omega) \), we directly use the non-parametric confidence intervals estimated for \( \theta_h \). In particular, use simulated \( \theta^b_h \) to estimate,

\[ s^b_k(\omega) \approx \left[ \sum_{h=0}^{H} \theta^b_{k,h} e^{ih\omega} \right] \sigma_k^2 \left[ \sum_{h=0}^{H} \theta^b_{k,h} e^{-ih\omega} \right] \quad \text{for } k = 1, 2. \]  

(32)

and select confidence intervals.

E.2 Test

1. Filter each variable you want to test using a Band-Pass filter which excludes frequencies below 2 and above 100.

2. Estimate the autoregressive parameter \( \rho_y \) implied by this stationary variable using standard regression techniques.

3. Simulate - for each variable \( y \) - \( B (\geq 2000) \) AR(1) processes with persistence param-

\[ ^{18} \text{Notice that for estimating } s_k(\omega) \text{ we need to build a grid for } \omega \in (0, \pi]. \text{ Although results are not sensitive to different grid size, in our main results grid is 0.001 in order to guarantee a precise estimate to ten-year frequencies.} \]
eter $\rho_y$ fed with normally distributed random disturbances.$^{19}$

4. For each simulated series estimate its disturbances, impulse response coefficients with LP estimator $\theta_h$ and conditional spectral density via $s_k(\omega)$ where $k$ is the estimated innovation from each simulated $AR(1)$ process.

5. Following Canova (1998) and Beaudry et al. (forthcoming) we test if the estimated conditional spectral densities for shocks $\varepsilon_t$ ($\hat{s}_\varepsilon(\omega)$) are indistinguishable from the ones derived from the simulated $AR(1)$ process ($\hat{s}_a(\omega)$).

- Notice that $H_0: \hat{D}_\varepsilon = \hat{D}_a$ and $H_1: \hat{D}_\varepsilon > \hat{D}_a$
- $\hat{D}_k = \hat{s}_k(\omega_1)/\hat{s}_k(\omega_2)$
- $\omega_1 \in (\pi/40, \pi/28)$ and $\omega_1 \in (\pi/72, \pi/48)$

6. Test is estimated non-parametrically.

- Define $\hat{D}_k^b = \hat{s}_k^b(\omega_1)/\hat{s}_k^b(\omega_2)$ as the simulation of $\hat{D}_k$ from $\hat{s}_k^b$.
- Estimate, for each $b$, $\hat{\zeta}^b = \hat{D}_\varepsilon^b - \hat{D}_a^b$ as the difference between the simulation for $\hat{D}_\varepsilon^b$ and $\hat{D}_a^b$.
- P-value is the number of $\hat{\zeta}^b > 0$ over the total number of simulations $B$.

F Proof of Proposition 1

Cyclical dynamics obtain if at least two eigenvalues of the reduced form system of the model are complex and conjugate. Under determinacy this is not possible because there would be two eigenvalues, one stable and the other one unstable. Indeterminacy is characterized by a system with two stable eigenvalues, possibly complex and conjugate. The loglinearized deterministic version of the model can be written as,

$$
\begin{pmatrix}
2\kappa d & \frac{\tau \beta \omega}{1 - \tau + \tau \beta} \\
1 - \beta & \beta - \omega
\end{pmatrix}
\begin{pmatrix}
\hat{d}_{t+1} \\
\hat{y}_{t+1}
\end{pmatrix}
= 
\begin{pmatrix}
\frac{2\kappa d}{1 + \mu \gamma} & M \\
0 & 1 - \omega
\end{pmatrix}
\begin{pmatrix}
\hat{d}_t \\
\hat{y}_t
\end{pmatrix}
$$

(33)

$^{19}$This simulated series has the same length of the data used in the empirical section. Since our sample start slightly after 1980 then we have about 150 observations.
where

\[ M \equiv \frac{\tau_\beta \omega}{1 - \tau + \tau_\beta} - \gamma \frac{1 - \mu}{1 + \gamma \mu} \left( \omega - 1 + \frac{1}{(1 - \theta)(1 - n)} \right) \]  \hspace{1cm} (34)

Notice that when \( \kappa \) is equal to zero then the reduced-form of the system is independent of \( \hat{d}_t \) implies that one eigenvalue is equal to zero ruling out the possibility to have two complex and conjugate eigenvalues.