A Fiscal Theory of Trend Inflation[*]

Francesco Bianchi

Johns Hopkins University

CEPR and NBER

Renato Faccini

Danmarks Nationalbank

Leonardo Melosi

FRB Chicago

CEPR

November 2022

Abstract

We develop a new class of general equilibrium models with partially unfunded debt to propose a fiscal theory of trend inflation. In response to business cycle shocks, the monetary authority controls inflation, and the fiscal authority stabilizes debt. However, the central bank accommodates unfunded fiscal shocks, causing persistent movements in inflation, output, and real interest rates. In an estimated quantitative model, fiscal trend inflation accounts for the bulk of inflation dynamics. As external validation, we show that the model predicts the post-pandemic increase in inflation. Unfunded fiscal shocks sustain the recovery and cause an increase in trend inflation that counteracts deflationary non-policy shocks.

JEL Codes: E50, E62, E30.

Keywords: Unfunded fiscal spending, trend inflation, large public debt, inflation risk, shock-specific rule.

Persistent high inflation is always and everywhere a fiscal phenomenon.

Thomas J. Sargent (2013)

1   Introduction

We build a novel class of general equilibrium models with partially unfunded debt to propose a fiscal theory of trend inflation. In response to business cycle shocks and funded fiscal shocks, the monetary authority controls inflation, and the fiscal authority stabilizes debt. However, the central bank accommodates unfunded fiscal shocks, causing persistent movements in inflation and real interest rates.

We first illustrate the distinction between funded and unfunded fiscal shocks in the context of a Fisherian model. Funded and unfunded shocks coexist in the model. The difference between the two types of shocks is that funded fiscal shocks are backed by future fiscal adjustments, while unfunded fiscal shocks are not. As a result, funded fiscal shocks are irrelevant for inflation, while unfunded fiscal shocks lead to an increase in inflation accommodated by the central bank. We then move to consider the effects of unfunded and funded fiscal shocks in production economies. Absent nominal rigidities, unfunded fiscal shocks cause temporary movements in inflation and do not have real effects, as in the Fisherian model. With nominal rigidities, unfunded fiscal shocks have real effects and cause persistent movements in inflation and real interests rates, leading to a fiscal theory of trend inflation.

We then augment a quantitative New Keynesian (NK) model with unfunded fiscal shocks to assess their importance for US inflation dynamics. The model features all the ingredients that have been proven successful in matching US business cycle dynamics, including a large set of business cycle shocks. With respect to these shocks, monetary policy satisfies the Taylor principle and the fiscal authority is in control of debt stabilization. Thus, in this respect, the model behaves as its counterparts extensively studied in the literature. However, the model also features unfunded fiscal shocks. We model these as shocks to transfers that are not backed by future fiscal adjustments, implying that a share of the overall government debt is unfunded.[1] The central bank accommodates the increase in inflation necessary to stabilize the unfunded amount of debt. As in the textbook version of the new Keynesian model described above, these shocks trigger persistent movements in inflation, output, and real interest rates.

---

[1] We focus on shocks to transfers because historically government purchases (e.g., "G") have been constantly declining as a fraction of GDP since WWII. Thus, government purchases do not seem to represent a problem for fiscal sustainability, while transfers have been increasing over the same period. Our results are robust to allowing both types of spending to be partially unfunded.

The model is endowed with a rich set of shocks, including a persistent shifter to the New Keynesian Phillips Curve (NKPC) that is meant to capture autonomous factors, such as globalization and demographic changes, that can affect inflation in the long run. The model also features hand-to-mouth agents to allow for the possibility that funded fiscal shocks can influence macroeconomic dynamics. Thus, it is an empirical question whether unfunded fiscal shocks play an important role in explaining the data. We show that they do.

A persistent and partially unfunded increase in transfers in the mid-1960s, related to the introduction of the Great Society initiatives, accounts for the persistent rise in inflation during the Great Inflation. Symmetrically, the end of the Great Inflation is explained by a sharp revision in the amount of inflation that the Federal Reserve was going to tolerate to stabilize the portion of unfunded debt. In this respect, the aggressive increase in interest rates implemented by the Federal Reserve Chairman Paul Volcker can be interpreted as a strong signal of this policy change.

Starting from the 1990s, the amount of unfunded spending has been increasing sluggishly, counteracting a deflationary bias due to non-policy shocks, arguably related to demographics and international trade. Finally, large part of the economic rebound at the end of 2020 is attributed to the $2.2 trillion fiscal package introduced in March of the same year to combat the consequences of the pandemic crisis and that we find to be partially unfunded. Interestingly, a shift in the portion of unfunded spending is observed in the third and fourth quarters of 2020, after the Federal Reserve announced its new operating framework which contemplates the possibility of letting inflation overshoot its 2% target after the Pandemic recession.[2] This new monetary policy strategy affected the path of the federal funds rate, which we observe in the estimation.

We then study the *out-of-sample* implications of the model for the post-pandemic period. We proceed in two steps. First, we produce a projection for inflation using only data up to the end of 2020. Out of sample, the model predicts an inflation overshoot for the next four years, with a peak of 4%, and then a gradual return to the 2% target. We show that this scenario depends on the presence of unfunded fiscal shocks in response to the pandemic. Absent these shocks, the model predicts a much weaker recovery and inflation to run persistently below the 2% target. The undershooting of inflation in this counterfactual scenario is driven by the long-lasting non-policy shocks that have been exerting progressively larger deflationary effects on the US economy since the early 2000s.

We then consider the effects of the American Rescue Plan Act (ARPA). We assign part

---

[2]On August 27, 2020, Federal Reserve Chairman Jerome Powell announced the new framework for the first time at the Jackson Hole Economic Symposium as follows: "Following periods when inflation has been running below 2 percent, appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time."

of the increase in spending in the first quarter of 2021 to unfunded transfers based on the historical evidence. We then construct a conditional forecast and show that the model can replicate very closely the inflation figures for 2021. The model now predicts a more robust, overshoot of inflation and a very slow return to the 2% target. This result is obtained based on a conservative scenario about what fraction of the ARPA fiscal shock was unfunded.

Based on these results, we conclude that unfunded spending has played an important role in accounting for inflation dynamics, both historically and in the post-pandemic period. From this point of view, the post-pandemic situation is not necessarily different from the historical experience for the United States. However, two qualifications have to be made. First, in the post-Millennial period fiscal inflation has been counteracting a deflationary bias. The risk of persistent high inflation depends in part on whether this deflationary bias will persist or not. If it were to disappear, perhaps because of a change in the degree of integration of the world economies, the post-Millennial high level of fiscal inflation could become a problem. Second, as of 2022, spending is at an historical maximum. This implies that even small changes in the share of unfunded debt can lead to large swings in inflation.

In the last part of the paper, we conduct a series of additional exercises to highlight the key mechanisms that make unfunded fiscal shocks quantitatively important and to provide additional external validation for our results. First, we trim down the quantitative baseline model to only retain nominal rigidities as in a textbook NK model. We show that when this bare bone NK model is fed with the same unfunded fiscal shocks identified in the data, it delivers a very similar path for inflation, suggesting that the additional features of our baseline model do not drive the results with respect to the empirical relevance of fiscal inflation. These additional features are included to preserve the propagation mechanism of the other shocks and to avoid ruling out a priory other explanations for movements in inflation. Then, we discuss how fiscal trend inflation differs from an exogenous time-varying target. We show that a shock to the target cannot replicate the distinctive features of an unfunded fiscal shock. Finally, we employ sign restrictions to identify unfunded fiscal shocks in a vector autoregression (VAR) and show that the results are in line with our previous analysis.

From the methodological standpoint, this paper develops a new class of models in which policymakers are allowed to react differently to different shocks. This allows us to combine the insights of the Fiscal Theory of Price Level in an otherwise standard TANK model. The paper builds on Bianchi and Melosi (2019) who introduced the concept of shock-specific rules as a way to resolve a conflict between the monetary and fiscal authorities in the presence of a high fiscal burden that the fiscal authority is reluctant or unable to stabilize. In that paper, we apply a shock specific rule to study the macroeconomic effects of introducing an

emergency budget to mitigate a large recession. In this paper, we extend the notion of shock-specific rules to solve general equilibrium models in which monetary and fiscal authorities adopt state-dependent targets. This delivers a fiscal theory of trend inflation that is always at work, not only in response to exceptional events.

This paper is connected to the vast literature on monetary-fiscal policy interaction (Sargent and Wallace 1981; Leeper 1991; Sims 1994; Woodford 1994, 1995, 2001; Cochrane 1998, 2001; Schmitt-Grohe and Uribe 2000, 2002; Bassetto 2002; Reis 2016; Bassetto and Sargent 2021, among many others). Barro (1974) shows that an alternative way to generate non-Ricardian effects is if agents erroneously regard bonds as net-wealth. Aiyagari and Gertler (1985) study the implications of fiscal backing of government bonds for the propagation of shocks. They find that for debt to be irrelevant, the model needs to feature a considerable degree of accommodation with respect to the monetary authority. Leeper and Zhou (2013) find that inflation plays an important role in the optimal marginal financing of fiscal needs in models similar to the one used in our empirical analysis. Hall and Sargent (2011) show that historically most of US debt stabilization has been achieved through a combination of growth, revaluation effects, and low real interest rates. Bianchi and Ilut (2017) estimate a model with regime changes in the monetary/fiscal policy mix and link the high inflation of the 1960s-1970s to a Fiscally-led regime. Bianchi and Melosi (2017) argue that the possibility of a return to such regime can explain the lack of deflation in the aftermath of the Great Recession. In a recent opinion piece, Barro (2022) reaches a similar conclusion to the one proposed in this paper with respect to the post-pandemic inflation: a realignment in prices necessary to stabilize the large pandemic increase in the fiscal burden.

With respect to models with regime changes in the monetary/fiscal policy mix, we move in a new and different direction. Monetary-led and Fiscally-led rules coexist in our model and the policy coordination is shock specific. Shocks to unfunded transfers are dealt with fiscally-led policies. With respect to all other shocks, the monetary authority controls inflation and the fiscal authority is responsible for debt stabilization. As a result, this new modeling approach delivers low-frequency movements in inflation linked to unfunded fiscal shocks, while at the same time preserving the typical propagation of the business cycle shocks employed in New Keynesian models. In this respect, our work is also related to Cochrane (2022) that considers a class of NK models in which pre-existing debt is backed by future surpluses, but in response to unexpected movements in inflation, policymakers are not committed to make fiscal adjustments. The key difference is that Cochrane (2022) assumes that the policy mix is always fiscally-led (Passive Money/Active fiscal), while in our setting a Monetary-led and a Fiscally-led policy mix coexist at the same time. The two papers share the goal of integrating elements of the fiscal theory of price level in a NK framework.

2 Fiscal Trend Inflation

In this section, we introduce a new class of models in which a Monetary-led and a Fiscally-led policy mix coexist at the same time. The propagation of shocks changes depending on the shock specific policy response. This allows us to introduce unfunded fiscal shocks in an otherwise standard model. We illustrate the logic of this new class of models with shock specific rules in the context of a simple Fisherian model (Leeper 1991 and Sims 1994, 2016). Our focus is on fiscal trend inflation, but the method can be applied in other settings in which a researcher is interested in modeling shock specific policy responses. We then move to a production economy and discuss the consequences of introducing nominal rigidities. We show that in a model with nominal rigidities, unfunded fiscal shocks lead to persistent movements in inflation and real effects on output.

2.1 Funded and unfunded fiscal shocks

Stylized model The economy is populated by a continuum of infinitely many households and a government. The representative household has concave and twice continuously differentiable preferences over non-storable consumption goods and is endowed in each period with a constant quantity Y of these goods. The government issues one-period debt B_t to households who can trade them for consumption goods. The representative household chooses consumption and government bonds so as to maximize:

$$\max \ \sum_{t=0}^{\infty} \beta^t U\left(C_t\right),$$

subject to the flow budget constraint $P_t C_t + Q_t B_t + P_t \tau_t = P_t Y + B_{t-1}$, where $\beta < 1$ is the households' discount factor, P_t denotes the price of consumption goods, τ_t denotes real lump-sum net taxes, and $Q_t = 1/R_t$ is the price of the one period government bond B_t, equal to the inverse of the gross interest rate R_t.

The government budget constraint reads $Q_t B_t + P_t \tau_t = B_{t-1}$, where net taxes, τ_t, coincide with the real primary surplus. The fiscal authority follows the following fiscal rule:

$$\tau_t/\tau = \left(b_{t-1}/b\right)^{\gamma} e^{\epsilon_t},$$

where $b_t = B_t/P_t$ denotes real debt, τ and b are the steady-state values for taxes and real debt, respectively, ϵ_t is a shock to lump-sum taxes, and the parameter γ determines how strongly the fiscal authority adjusts primary surpluses to fluctuations in debt.

The central bank behaves according to the following monetary rule:

$$R_t/R = \left(\Pi_t/\Pi\right)^{\phi},$$

where $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate at time t, variables without the time subscript

denote the corresponding steady states, and the parameter ϕ controls the strength with which the central bank reacts to movements of inflation from its target.

Combining the households' Euler equation with the market clearing condition $C_t = Y$ in every period leads to the Fisher equation: $Q_t = \beta \left(\mathbb{E}_t \Pi_{t+1} \right)^{-1}$.

Linearized system of equations We linearize the model equations around the deterministic steady state. Henceforth, hatted variables denote variables in log-deviation from their steady-state values. We obtain the following system of equations:

$$\hat{r}_t = \mathbb{E}_t \hat{\pi}_{t+1} \tag{1}$$

$$\hat{b}_t = \beta^{-1} \hat{b}_{t-1} - \hat{\tau}_t + b(\hat{r}_t - \beta^{-1}\hat{\pi}_t) \tag{2}$$

$$\hat{r}_t = \phi \hat{\pi}_t \tag{3}$$

$$\hat{\tau}_t = \gamma \hat{b}_{t-1} + \epsilon_t. \tag{4}$$

Plugging the monetary rule (3) into the Fisher equation (1) leads to the *monetary block*:

$$\mathbb{E}_t \hat{\pi}_{t+1} = \phi \hat{\pi}_t. \tag{5}$$

Combining the law of motion for real debt (2) with the fiscal rule (4) yields the *fiscal block*

$$\hat{b}_t = \left(\beta^{-1} - \gamma \right) \hat{b}_{t-1} - b \left(\beta^{-1} - \phi \right) \hat{\pi}_t - \epsilon_t. \tag{6}$$

Existence and uniqueness of a solution As shown by Leeper (1991), in this class of models there are two regions of the parameter space that deliver existence and uniqueness of a stationary solution. In the first region, monetary policy is active and responds more than one-to-one to deviations of inflation from its target ($\phi > 1$). The fiscal authority implements the necessary fiscal adjustments to keep debt on a stable path ($\gamma > \beta^{-1} - 1$). Fiscal policy is defined as passive because it passively accommodates the behavior of the monetary authority. We label this policy combination the Monetary-led policy mix. The distinctive feature of the Monetary-led policy mix is that the macroeconomy is completely insulated from the fiscal block and fiscal imbalances are irrelevant for inflation determination in equilibrium (*Monetary and Fiscal Dichotomy*). This is because debt stability is achieved with fiscal adjustments.[3] The first panel of Figure 1 illustrates this point by showing that inflation does not move in response to a negative shock to primary surpluses.

In the second region of the parameter space, labelled Fiscally-led policy mix, the fiscal authority is not committed to implementing the necessary fiscal adjustments. Monetary policy is now passive ($\phi \leq 1$) because it passively accommodates the behavior of the active fiscal authority ($\gamma \leq \beta^{-1} - 1$). Under the Fiscally-led policy mix, the macroeconomy is not

---

[3]In richer models with distortionary taxation and government purchases, fiscal variables affect the macroeconomy, but through a different channel with respect to the one analyzed here.

insulated with respect to fiscal imbalances. In fact, inflation is determined by the need of stabilizing government debt. Consequently, fiscal imbalances affect inflation. The second panel of Figure 1 illustrates this point. Now a negative shock to primary surpluses leads to an increase in inflation. This increase in inflation is fully accommodated by the central bank ($\phi = 0$) and debt stability is preserved.

Shock specific rules and partially unfunded debt We now extend the model to allow the Monetary-led and Fiscally-led policy mixes to coexist. In this new class of models, the dynamics typical of a Monetary-led policy mix coexist with the dynamics typical of a Fiscally-led policy mix. We focus on fiscal shocks, but the logic outlined below applies to all types of shocks that move the fiscal burden of the economy, as illustrated in the richer model considered in our empirical analysis. In what follows, we use the superscript M and F to denote policy parameters that imply a behavior in line with a Monetary-led policy mix and a Fiscally-led policy mix, respectively.

We consider the following fiscal rule:

$$\tau_t/\tau = \left(b_{t-1}/b_{t-1}^F\right)^{\gamma^M} \left(b_{t-1}^F/b\right)^{\gamma^F} e^{\epsilon_t^M + \epsilon_t^F}, \tag{7}$$

where ϵ_t^M and ϵ_t^F denote funded and unfunded fiscal shocks, respectively. With respect to the amount of unfunded debt b_t^F accumulated as a result of the unfunded fiscal shocks, the fiscal authority is not committed to implementing a large enough fiscal adjustment: $\gamma^F < \beta^{-1} - 1$. Instead, the fiscal authority is willing to fully stabilize deviations of debt from its unfunded component: $\gamma^M > \beta^{-1} - 1$. Thus, fiscal policy is passive with respect to the funded component of debt, while it is active with respect to the unfunded component of debt.

The new monetary rule is:

$$R_t/R = (\Pi_t/\Pi_t^F)^{\phi^M}(\Pi_t^F/\Pi)^{\phi^F}. \tag{8}$$

where Π_t^F denotes fiscal inflation, i.e., the amount of inflation that is tolerated by the central bank due to unfunded fiscal shocks. With respect to fiscal inflation, monetary policy is passive: the central bank reacts less than one-to-one, $\phi^F \leq 1$. Instead, the central bank is active in stabilizing inflation in deviations from fiscal inflation: $\phi^M > 1$.

Linearizing the fiscal rule in equation (7) we obtain:

$$\hat{\tau}_t = \gamma^M \left(\hat{b}_{t-1} - \hat{b}_{t-1}^F\right) + \gamma^F \hat{b}_{t-1}^F + \epsilon_t^M + \epsilon_t^F. \tag{9}$$

Given that $\gamma^F < \beta^{-1} - 1$, the expression above makes clear that the fiscal adjustments are not large enough to cover the entirety of the fiscal burden.

Linearizing the monetary rule, we obtain:

$$\hat{r}_t = \phi^M \left(\hat{\pi}_t - \hat{\pi}_t^F\right) + \phi^F \hat{\pi}_t^F. \tag{10}$$

8

If we further assume $\phi^F = 0$, we obtain a Taylor rule that is isomorphic to a rule with a time-varying target: $\hat{r}_t = \phi^M \left(\hat{\pi}_t - \hat{\pi}_t^F \right)$. Crucially, the time-varying target $\hat{\pi}_t^F$ is not an additional shock, but it is instead tightly related to the amount of inflation tolerated by the central bank to stabilize a portion of the overall fiscal burden, leading to a fiscal theory of trend inflation.

Appendix A exploits the linearity of the model to prove that the components of debt and inflation in deviations from their corresponding targets, $\hat{b}_t^M = \hat{b}_t - \hat{b}_t^F$ and $\hat{\pi}_t^M = \hat{\pi}_t - \hat{\pi}_t^F$, are *exactly* the amounts of debt and inflation that would arise if the Monetary-led policy mix were always in place and only funded shocks occurred. We can then interpret $\hat{b}_t^M$ as the amount of funded debt that the fiscal authority is committed to stabilize through fiscal adjustments. Analogously, $\hat{\pi}_t^M$ corresponds to movements in inflation originating from shocks that the central bank does not accommodate and that are instead responsibility of the fiscal authority. We use the superscript M to emphasize that the Monetary-led policy mix applies with respect to these variables.

Using the fact that in the linearized model the total amount of debt is the sum of two components, funded and unfunded debt, $\hat{b}_t = \hat{b}_t^M + \hat{b}_t^F$, we can rewrite the fiscal rule as:
$$\hat{\tau}_t = \gamma^M \hat{b}_{t-1}^M + \gamma^F \hat{b}_{t-1}^F + \epsilon_t^M + \epsilon_t^F.$$
Similarly, exploiting the fact that in the linearized model $\hat{\pi}_t = \hat{\pi}_t^M + \hat{\pi}_t^F$, the monetary rule can be re-written as:
$$\hat{r}_t = \phi^M \hat{\pi}_t^M + \phi^F \hat{\pi}_t^F.$$

Thus, the linearized model allows two equivalent ways to interpret the policy rules. First, the policy rules can be interpreted as describing a situation in which policymakers react to time-varying targets that are driven by the need of stabilizing the amount of unfunded debt. Alternatively, the policy rules can be interpreted as shock specific rules in which policymakers react differently to the different components of the endogenous target variables depending on the shocks that generate the fluctuations.

Substituting the monetary rule (10) into the Fisherian equation (1) yields the monetary block of the model with partially unfunded debt:
$$\mathbb{E}_t \hat{\pi}_{t+1} = \phi^M \left(\hat{\pi}_t - \hat{\pi}_t^F \right) + \phi^F \hat{\pi}_t^F. \tag{11}$$
Plugging the policy rules in the law of motion of debt (2), yields the fiscal block:
$$\hat{b}_t = \left(\beta^{-1} - \gamma^M \right) \hat{b}_{t-1} + \gamma^M \hat{b}_{t-1}^F - b \left(\beta^{-1} - \phi^M \right) \hat{\pi}_t - b \left(\phi^M - \phi^F \right) \hat{\pi}_t^F - \epsilon_t^M - \epsilon_t^F, \tag{12}$$
where to simplify the exposition, and without loss of generality, we have assumed that the fiscal authority completely disregards the amount of unfunded debt: $\gamma^F = 0$.

To close the model, we need to characterize the dynamics of fiscal inflation, $\hat{\pi}_t^F$, and of the
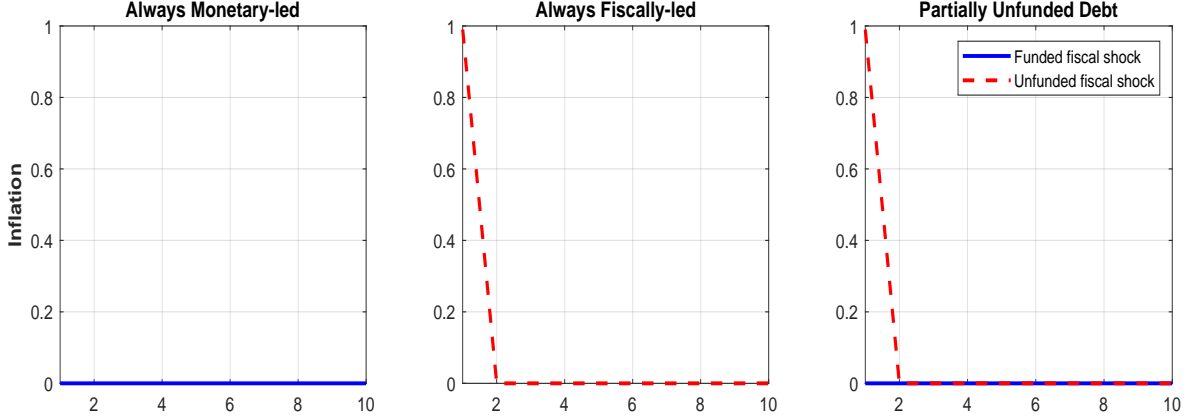
9

Figure 1: Impulse response of inflation to a fiscal shock. The discount factor β is set to 0.99 and the steady-state value of debt b to 1. In the model with partially unfunded debt, the monetary policy parameters are $\phi^M = 2.0$ and $\phi^F = 0$ and the fiscal policy parameters are $\gamma^M = 0.2$ and $\gamma^F = 0$. The Always Monetary-led model is parameterized as follows: $\phi = \phi^M$ and $\gamma = \gamma^M$. The Always Fiscally-led model is parameterized as follows: $\phi = \phi^F$ and $\gamma = \gamma^F$. Fiscal shocks are i.i.d. with unit variance.

associated amount of unfunded debt, $\hat{b}_t^F$. To do so, we construct a shadow economy in which the Fiscally-led policy mix is always in place and only the shocks to unfunded spending ϵ_t^F occur. The shadow economy keeps track of fiscal inflation and the amount of unfunded debt.

The monetary and fiscal blocks for the shadow economy are then:

$$\mathbb{E}_t \hat{\pi}_{t+1}^F = \phi^F \hat{\pi}_t^F, \tag{13}$$

$$\hat{b}_t^F = \beta^{-1} \hat{b}_{t-1}^F - b(\beta^{-1} - \phi^F)\hat{\pi}_t^F - \epsilon_t^F. \tag{14}$$

Note that the monetary and fiscal blocks for the shadow economy are isomorphic to those in equations (5) and (6) once the parameter restrictions for the Fiscally-led policy mix are imposed and the only fiscal shocks are the funded ones.

The set of equations (11), (12), (13), and (14) describe the model with partially unfunded debt. Since there are two non-predetermined variables ($\hat{\pi}_t$ and $\hat{\pi}_t^F$) and two non-stationary eigenvalues associated with equations (11) and (14), the model satisfies the Blanchard and Khan conditions and is thereby determinate—there exists a unique stable Rational Expectations equilibrium.

The third panel of Figure 1 presents the impulse responses in the model with partially unfunded debt. As before, we assume that the monetary authority fully accommodates the increase in fiscal inflation: $\phi^F = 0$. In response to a funded spending shock (solid blue line), the economy with partially unfunded debt behaves exactly as in the left panel, where policymakers always follow the Monetary-led policy mix, and inflation is unaffected by the shock. In response to an unfunded spending shock, inflation increases. The economy with partially unfunded debt behaves exactly as in the middle panel, where policymakers always follow the Fiscally-led policy mix. The policy rules in the model with partially unfunded

debt are shock-specific and policymakers respond differently depending on the nature of the fiscal shocks. Thus, the properties of the Monetary-led and Fiscally-led policy mix coexist in the model with partially unfunded debt.

At a more intuitive level, our modeling formalism connects to a central idea of the fiscal theory of price level, namely that debt stability can be achieved with a mix of fiscal adjustments and movements in inflation that are accommodated by the central bank. With respect to this idea, the shadow economy serves the purpose of measuring the evolution of the share of debt that is expected to be covered with fiscal adjustments, while the remaining, unfunded share is expected to be covered via movements in inflation. In this sense, the partition between funded and unfunded spending is akin to an accounting exercise, under the typical assumption of perfect and symmetric information of both agents and policymakers.

The focus of this paper is on the effects of unfunded fiscal shocks. However, this new class of models can be used to study other forms of heterogeneity in policy responses. Furthermore, the results presented here extend to more complex models in which fiscal policy is distortionary and more shocks are present. In our empirical analysis, we consider a state-of-the-art TANK model with a rich set of shocks. With respect to the typical business cycle shocks, the economy behaves as in other TANK models studied in the literature. However, unfunded fiscal shocks lead to the dynamics typical of a Fiscally-led regime.

Finally, it is worth mentioning that we could also have solved the model by constructing a different (Monetary-led) shadow economy in which all public debt is funded, the central bank always follows the Taylor principle, but only the funded fiscal shocks occur. This duality in solving models with shock specific rules stems from the linearity of the model. Linearity implies that the two shadow economies are indeed additive sub-economies of the actual economy. This means that the sum of the inflation rates and the sum of debts in the two parallel economies are equal to their counterparts in the actual economy, as shown in Appendix A.

2.2 Unfunded fiscal shocks and nominal rigidities

We now introduce a production side to the model presented in the previous section. We will use this model to investigate the role of price rigidity in the propagation of funded and unfunded transfer shocks to inflation, output, debt, and the real interest rate.

Specifically, we modify the setup presented above by assuming the period utility function $U(C_t, N_t) = lnC_t + \phi ln(1 - N_t)$, where N_t represents hours worked. Households receive real wage income $W_t N_t$ in exchange for supplying labor services to the firms, and the production function is $Y_t = N_t^{1-\alpha}$. All other assumptions are the same as described in the previous
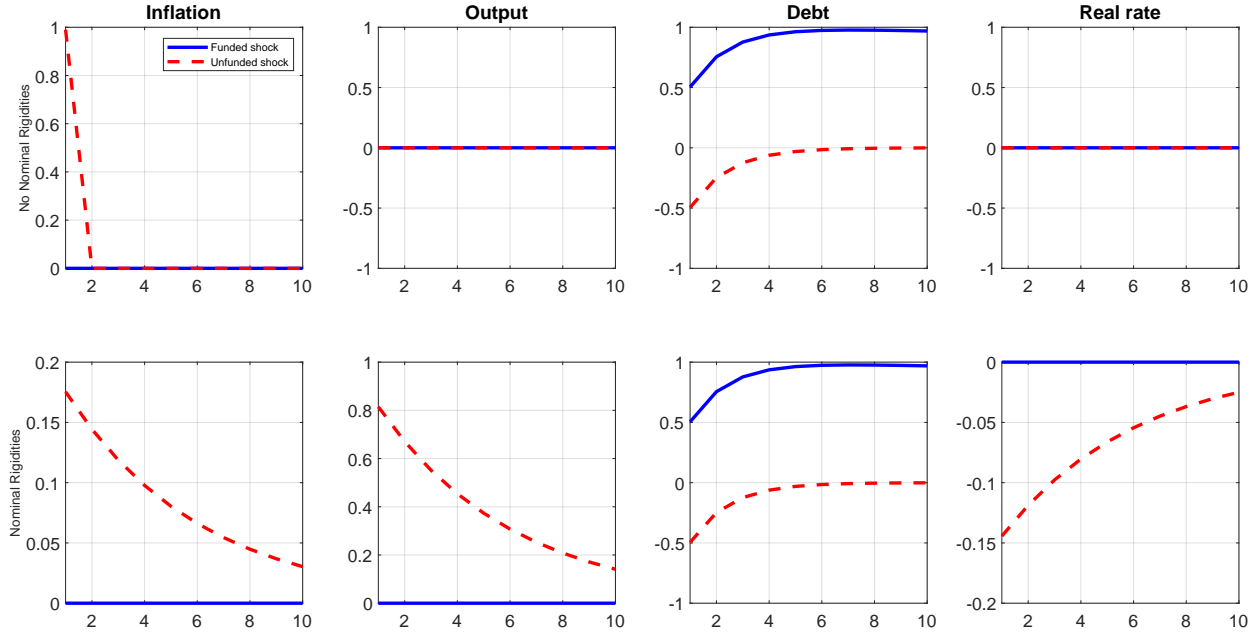
Figure 2: **Propagation of funded and unfunded fiscal shocks in the stylized model**. Impulse response functions of inflation, real output, government debt, and the real interest rate to funded (blue solid line) and unfunded shocks (red dashed line) to primary surpluses. The upper panels show the propagation in the model with perfectly flexible prices. The lower panels show the propagation in the New Keynesian model.

section, including the specification of the fiscal rules in eq.(7). In the case of flexible prices, which provides the benchmark for a classical economy, we assume perfect competition in both goods and labor markets. We calibrate this stylized model consistently with the parameters of the large model that we estimate in Section 4. Both the linearized equations for this standard flexible-price economy and the parameter values assigned in the calibration are reported in Appendix B.

The graphs in the first row of Figure 2 illustrate the impulse responses to funded and unfunded fiscal shocks (the blue and red lines, respectively) in the case of flexible prices. Two points emerge from these graphs. First, expansionary fiscal shocks lead to an increase in inflation only if they are unfunded; that is, only if they are not backed by future adjustments in taxes. Second, regardless of whether an expansionary fiscal shock is funded or unfunded, the real economy is perfectly insulated from fiscal shocks. Like the typical model with flexible prices, the real economy is not affected by movements in inflation. Also note that the debt-to-output ratio falls in the aftermath of an unfunded fiscal shock because the increase in inflation raises nominal output (but not real output). Thus, introducing a production economy with flexible prices does not change the behavior of inflation with respect to the Fisherian model. This should not come as a surprise, as in this class of models, real activity and real interest rates are exogenous with respect to inflation.

We now make a single change to this model by introducing nominal rigidities. Specifically,

we assume that the good market is monopolistically competitive and price-setting firms face quadratic price adjustment costs a la Rotemberg. These changes make the linearized model similar to the types of stylized New Keynesian models studied in the textbooks of Galí (2008) and Woodford (2003). The linearized equations for this model are reported in appendix B.[4]

In the lower plots of Figure 2, we show the propagation of funded (blue solid lines) and unfunded shocks (the red dashed line) based on the model with nominal rigidities. Comparing these responses to the ones shown in the first row highlights the effects of nominal rigidities on the propagation of these two fiscal shocks. Noticeably, introducing nominal rigidities does not affect the response to funded shocks. These shocks are still irrelevant for output and inflation. However, nominal rigidities deeply affect the response to unfunded fiscal shocks. Unlike the model with fully flexible prices, in the aftermath of an unfunded fiscal shock, the change in inflation is spread out over time, leading to a fiscal theory of trend inflation. Furthermore, now unfunded fiscal shocks affect the path of real interest rates and hence output. Given the very stylized nature of this model, it is remarkable that inflation rises so persistently after an unfunded fiscal shock.[5]

It is important to emphasize that the lower panels of Figure 2 reveal some key empirical cross-correlations, which will be crucial to identify unfunded fiscal shocks in the large-scale model that we estimate in Section 4. Namely, unfunded shocks increase inflation while decreasing both the real interest rate and the ratio of government debt to output; funded shocks also lead to an increase in inflation, but the responses of both the real rate and debt are instead non-negative.

3   A Quantitative Model

In this section, we build and estimate a state-of-the-art Two Agents New Keynesian (TANK) model with a rich fiscal block and partially unfunded debt. The model features all the ingredients that have been proven successful in matching US business cycle dynamics, including a large set of business cycle shocks. With respect to these shocks, monetary policy satisfies the Taylor principle and the fiscal authority is in control of debt stabilization. Thus,

---

[4]The Appendix shows that the model boils down to the simplest three-equations New Keynesian model, plus a fiscal rule for the primary surplus and a law of motion for debt.

[5]We obtained our results setting the response to fiscal inflation in the Taylor rule to zero. Allowing for a positive response to fiscal inflation would generate persistent movements in inflation even absent nominal rigidities, but there would not be effects on real activity. Furthermore, Cochrane (2001) shows that in a frictionless model in which debt has a maturity structure, fiscal shocks can generate persistent movements in the *price level*. The results in Cochrane (2001) are for the price level, while we focus on inflation, and are based on a model in which nominal interest rates are not controlled by the central bank, unlike in the models discussed in our paper. In our quantitative analysis, we also allow for a maturity structure, but in our setting nominal rigidities play a dominant role, as illustrated in Section 6.

in this respect, the model behaves as its counterparts extensively studied in the literature (see among many others, Christiano et al. (2005) and Leeper et al. (2017)). However, the model also features unfunded fiscal shocks. These shocks to transfers are not backed by future fiscal adjustments, implying that a share of the overall government debt is unfunded. The central bank accommodates the increase in inflation necessary to stabilize the unfunded amount of debt. As a result, these shocks allow the model to generate persistent movements in inflation and a decline in real interest rates, leading to a fiscal theory of trend inflation. In what follows, we outline the model in detail.

3.1 The economy

The economy is populated by a unit measure of households, of which a fraction μ are hand-to-mouth consumers. The remaining fraction, $1 - \mu$, are savers and we indicate them with an S superscript. The presence of hand-to-mouth households, together with distortionary taxation, breaks Ricardian equivalence and makes funded transfers relevant for a fraction of the population even under a Monetary-led policy mix.

Savers A household of optimizing saving agents, indexed by j, derives utility from the consumption of a composite good, $C_t^{*S}(j)$, which comprises private consumption $C_t^S(j)$ and government consumption G_t such that $C_t^{*S}(j) = C_t^S(j) + \alpha_G G_t$. The parameter α_G governs the substitutability between private and government consumption. When negative, the goods are complements; when positive, they are substitutes. External habits in consumption imply that utility is derived relative to the previous period value of aggregate savers' consumption of the composite good θC_{t-1}^{*S}, where $\theta \in [0, 1]$ is the habit parameter. Saver households also derive disutility from the supply of differentiated labor services from all its members, indexed by l, $L_t^S(j) = \int_0^1 L_t^S(j, l) \, dl$. The period utility function is given by $\mathcal{U}_t^S(j) = u_t^b \left(\ln \left(C_t^{*S}(j) - \theta C_{t-1}^{*S} \right) - L_t^S(j)^{1+\xi} / (1 + \xi) \right)$, where u_t^b is a discount factor shock and ξ is the Frisch elasticity of labor supply.

Households accumulate wealth in the form of physical capital $\bar{K}_t^S$. The stock of capital depreciates at rate δ and accrues with investment I_t^S, net of adjustment costs. The law of motion for physical capital is: $\bar{K}_t^S(j) = (1 - \delta) \bar{K}_{t-1}^S(j) + u_t^i \left[1 - s \left(I_t^S(j) / I_{t-1}^S(j) \right) \right] I_t^S(j)$, where u_t^i is a shock to the marginal efficiency of investment and s denotes an investment adjustment cost function that satisfies the properties $s(e^\gamma) = s'(e^\gamma) = 0$ and $s''(e^\gamma) \equiv s > 0$, where γ is the steady-state growth rate of the economy.

Households derive income from renting effective capital $K_t^S(j)$ to the intermediate firms. Effective capital is related to physical capital according to $K_t^S(j) = \nu_t(j) \bar{K}_{t-1}^S(j)$, where $\nu_t(j)$ is the capital utilization rate. The cost of utilizing one unit of physical capital is given

by the function $\Psi(\nu_t(j))$. Given the steady-state utilization rate $\nu(j) = 1$, the function Ψ satisfies the following properties: $\Psi(1) = 0$, and $\frac{\Psi''(1)}{\Psi'(1)} = \frac{\psi}{1-\psi}$, where $\psi \in [0, 1)$. We further denote the gross rental rate of capital as R_t^k and the tax rate on capital rental income as τ_t^K.

The household can also save by purchasing two types of zero-coupon bonds which differ in their maturity. One-period bonds promising a nominal payoff $B_{s,t}$ at time $t + 1$ can be purchased at the present discounted value $R_{b,t}^{-1} B_{s,t}$, where the gross nominal interest rate $R_{b,t}$ is related to the interest rate set by the Central Bank R_t through the equation $R_{b,t} = u_t^{rp} R_t$, and the wedge u_t^{rp} can be interpreted as a risk premium shock. Long-term government bond B_t with a maturity decaying at a constant rate $\rho \in [0, 1]$ and duration $(1 - \beta\rho)^{-1}$, can be purchased at price P_t^B.

Each period, the household receives after-tax nominal labor income, after-tax revenues from renting capital to the firms, lump-sum transfers from the government Z_t^S and dividends from the firms D_t. These resources can be spent to consume and to invest in physical capital and bonds. The nominal budget constraint for the saver household is:

$$P_t \left(1 - \tau_t^C\right) C_t^S(j) + P_t I_t^S(j) + P_t^B B_t(j) + R_{b,t}^{-1} B_{s,t} \tag{15}$$
$$= \left(1 + \rho P_t^B\right) B_{t-1}(j) + B_{s,t-1}(j) + \left(1 - \tau_t^L\right) \int_0^1 W_t(l) L_t^S(j,l)\, dl$$
$$+ \left(1 - \tau_t^K\right) R_t^k \nu_t(j) \bar{K}_{t-1}^S(j) - \psi(\nu_t) \bar{K}_{t-1}^S(j) + P_t Z_t^S(j) + D_t(j),$$

where $W_t(l)$ denotes the wage rate that applies to all household members, and τ_t^C and τ_t^L denote the tax rates on consumption and labor income, respectively. The household maximizes lifetime discounted utility $\sum_{t=0}^{\infty} \beta^t \mathcal{U}_t^S(j)$ subject to the sequence of budget constraints in equation (15).

Hand-to-mouth households Every period, hand-to-mouth households consume all of their disposable, after-tax income, which comprises revenues from labor supply and government transfers. The hand-to-mouth households supply differentiated labor services, and set their wage to be equal to the average wage that is optimally chosen by the savers, as described below. Using the superscript N to indicate the non-saving, hand-to-mouth households, their budget constraint can be written as follows:

$$\left(1 + \tau_t^C\right) P_t C_t^N(j) = \left(1 - \tau_t^L\right) \int_0^1 W_t(l) L_t^N(j,l)\, dl + P_t Z_t^N(j),$$

where it is assumed that both savers and non-savers face the same tax rates on consumption and labor income.

Final good producers A perfectly competitive sector of final good firms produces the homogeneous good Y_t at time t by combining a unit measure of intermediate differentiated

inputs using the technology $Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\eta_t^p+u_t^{NKPC}}} di \right)^{1+\eta_t^p+u_t^{NKPC}}$, where η_t^p denotes exogenous i.i.d. changes in the elasticity of substitution among good varieties. In the linearized version of the model, these shocks shift the New Keynesian Phillips curve (NKPC) and are often dubbed cost-push shocks. The variable u_t^{NKPC} is also a cost-push shock but it is assumed to follow a near-unit-root process. This highly persistent cost-push shock is meant to capture external forces such as the effects of international trade on firms' costs, advancements in firms' technologies to control production costs (e.g., the introduction of just-in-time inventory management method) etc. All these long-run factors can affect the cost of producing goods and inflation in the long run. Profit maximization yields the demand function for intermediate goods $Y_t(i) = Y_t \left(P_t(i)/P_t \right)^{-\left(1+\eta_t^p+u_t^{NKPC}\right)/\left(\eta_t^p+u_t^{NKPC}\right)}$, where $P_t(i)$ is the price of the differentiated good i and P_t is the price of the final good.

Intermediate good producers Intermediate firms produce using the technology $Y_t(i) = K_t(i)^\alpha (A_t L_t(i))^{1-\alpha} - A_t\Omega$, where Ω is a fixed cost of production that grows with the rate of labor-augmenting technological progress A_t and $\alpha \in [0,1]$ a parameter. The technological progress A_t follows an exogenous process that is stationary in the growth rate. Specifically, we assume that $u_t^a = (1-\rho_a)\gamma + \rho_a u_{t-1}^a + \varepsilon_t^a$, where $u_t^a = \ln(A_t/A_{t-1})$ and γ is a drift parameter capturing the logarithm of the rate of technology growth in steady state. Intermediate firms rent capital and labor in perfectly competitive factor markets. L_t is a bundle of all the differentiated labor services supplied in the economy, which are aggregated into a homogeneous input by a labor agency, as described below. The nominal rental rate of capital is denoted by R_t^K and the wage rate by W_t. Cost minimization implies that all firms incur the same nominal marginal cost $MC_t = (1-\alpha)^{\alpha-1}\alpha^{-\alpha} \left(R_t^k\right)^\alpha W_t^{1-\alpha} A_t^{-1+\alpha}$.

When setting prices, intermediate producers face frictions à la Calvo, i.e., at time t a firm i can optimally reset its price with probability ω^p. Otherwise it adjusts the price with partial indexation to the previous period inflation rate according to the rule $P_t(i) = (\Pi_{t-1})^{\chi_p}(\Pi)^{1-\chi_p}P_{t-1}(i)$, where $\chi_p \in [0,1]$ is a parameter, $\Pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$ and Π denotes the aggregate rate of inflation at steady state.

Intermediate producers that are allowed to reset their price maximize the expected discounted stream of nominal profits:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega_p)^s \frac{\Lambda_{t+s}^S}{\Lambda_t^S} \left[\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\chi^p}\Pi^{1-\chi^p} \right) P_t(i) Y_{t+s}(i) - MC_{t+s}Y_{t+s}(i) \right]$$

subject to the demand function of the final good sector, where Λ^S denotes the marginal utility of the savers.

Wages We assume that both savers and hand-to-mouth households are monopoly suppliers of a unit measure of differentiated labor service, indexed by l. Each period, a saver household gets an opportunity to optimally readjust the wage rate that applies to all of its workers, $W_t(l)$, with probability ω_w. If the wage cannot be reoptimized, it will be increased at the geometric average of the steady-state rate of inflation Π and of last period inflation Π_{t-1}, according to the rule $W_t(l) = W_{t-1}(l)(\Pi_{t-1}e^\gamma)^{\chi_w}(\Pi e^\gamma)^{1-\chi_w}$, where $\chi_w \in [0,1]$ captures the degree of nominal wage indexation.

All households, including both savers and non-savers, sell their labor service to a representative, competitive agency that transforms it into an aggregate labor input, according to the technology $L_t = \left(\int_0^1 L_t(l)^{\frac{1}{1+\eta_t^w}} dl\right)^{1+\eta_t^w}$, where η_t^w is an exogenous wage mark-up shock. The agency rents labor type $L_t(l)$ at price $W_t(l)$ and sells a homogeneous labor input to the intermediate producers at price W_t. The static profit maximization problem yields the demand function $L_t(l) = L_t(W_t(l)/W_t)^{-(1+\eta_t^w)/\eta_t^w}$.

Monetary and Fiscal Policy Assuming that one-period government bonds are in zero net supply, the government nominal budget constraint can be written as:

$$P_t^B B_t + \tau_t^K R_t^k K_t + \tau_t^L W_t L_t + \tau_t^C P_t C_t = \left(1 + \rho P_t^B\right) B_{t-1} + P_t G_t + P_t Z_t, \qquad (16)$$

where $C_t = \mu C_t^N + (1-\mu) C_t^S$ denotes aggregate consumption and $Z_t = \int_0^1 Z_t(j)\,dj = Z_t^S = Z_t^N$, using the assumption that every household receives the same amount of transfers regardless of whether it is hand-to-mouth or saver. The budget constraint in equation (16) implies that the fiscal authority finances government expenditures, transfers, and the rollover of expiring long-term debt by raising taxes on consumption, labor and capital, and by issuing new long-term debt obligations.

We rescale the variables entering the fiscal rules by defining $g_t = G_t/A_t$ and $z_t = Z_t/A_t$. For each variable x_t, $\widehat{x}_t$ denotes the percentage deviation from its own steady state. Let $s_{b,t} = \frac{P_t^B B_t}{P_t Y_t}$ be the debt-to-GDP ratio. As in the model presented in Section 2, the debt-to-GDP ratio in deviations from the steady state, $\hat{s}_{b,t}$, is the sum of two components, funded $\hat{s}_{b,t}^M$ and unfunded $\hat{s}_{b,t}^F$ debt. As before, we use superscripts M and F to emphasize that the Monetary-led policy mix applies to funded debt, while the Fiscally-led policy mix applies to unfunded debt. For the shocks, we use the superscripts only to label the two types of transfer shocks, while we assume that all other shocks only affect the funded portion of debt.

The fiscal authority adjusts government spending $\hat{g}_t$, transfers $\hat{z}_t$, and tax rates on capital

income, labor income, and consumption $\hat{\tau}^J$, $J \in \{K, L, C\}$ as follows:

$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1 - \rho_G) \; \gamma_G \hat{s}^M_{b,t-1} + \zeta_{g,t}, \tag{17}$$

$$\hat{z}_t = \rho_Z \hat{z}_{t-1} - (1 - \rho_Z) \; \gamma_Z \hat{s}^M_{b,t-1} + \phi_{zy} \hat{y}_t + \zeta^M_{z,t} + \zeta^F_{z,t}, \tag{18}$$

$$\hat{\tau}^J_t = \rho_J \hat{\tau}^J_{t-1} + (1 - \rho_J) \; \gamma_J \hat{s}^M_{b,t-1} \tag{19}$$

where $\hat{s}^M_{b,t-1} = \hat{s}_{b,t-1} - \hat{s}^F_{b,t-1}$ denotes the portion of the debt-to-GDP ratio that the fiscal authority is committed to stabilize with fiscal adjustments. This commitment is captured by the values for the reaction parameters γ_G, γ_Z, and $\gamma_J > 0$ that are large enough to guarantee that the portion $\hat{s}^M_{b,t-1}$ of debt remains on a stable path. The fiscal authority does not make fiscal adjustments in response to the remaining, unfunded, portion of debt $\hat{s}^F_{b,t-1}$. The variables $\zeta^M_{z,t}$ and $\zeta^F_{z,t}$ denote shocks to transfers that are respectively funded and unfunded; the realizations of the unfunded shock, cumulated over time, determine the evolution of the shares of debt that is unfunded. The amount of funded debt is instead determined by the funded fiscal shocks and the business cycle shocks. The shocks $\zeta_{g,t}$, $\zeta^M_{z,t}$ and $\zeta^F_{z,t}$ follow AR(1) Gaussian stochastic processes.

The central bank is fully committed to move the short-term interest rate $\hat{R}_t$ in response to the movements of inflation originating from the typical business cycle shocks and the funded fiscal shocks, while it fully accommodates the movements in inflation necessary to stabilize the unfunded portion of debt. As explained in Section 2, this shock specific monetary policy rule can be captured by a standard Taylor rule in which the central bank reacts to deviations of inflation from the level of inflation needed to stabilize the unfunded share of debt. We call this level of inflation tolerated by the central bank, fiscal trend inflation, $\hat{\pi}^F_t$. It follows that the linearized monetary policy rule with an effective lower bound constraint (ELB) can be written as:

$$\hat{R}_t = \max \left[-\ln \underline{R}, \rho_r \hat{R}_{t-1} + (1 - \rho_r) \left[\phi_\pi \left(\hat{\pi}_t - \hat{\pi}^F_t \right) + \phi_y \hat{y}_t \right] + u^m_t \right]. \tag{20}$$

where u^m_t is a monetary policy shock.

The parameter $\phi_\pi > 1$ implies that the Taylor principle is satisfied and monetary policy is active when responding to deviations of inflation, $\hat{\pi}_t$, from fiscal trend inflation, $\hat{\pi}^F_t$. Indeed, the variable $\hat{\pi}^F_t$ measures the increase in trend inflation, relative to the central bank long-term target (and steady-state rate), that the central bank accommodates so as to stabilize the share of unfunded debt $\hat{s}^F_{b,t-1}$. The policy mix characterized by equations (17) - (27) therefore implies that monetary policy is active in response to deviations of inflation from fiscal trend inflation and passive (no response) with respect to the inflation needed to stabilize the share of unfunded debt in deviations from its long-term target. Concurrently, fiscal policy is passive with respect to its commitment in stabilizing the share of funded government debt $\hat{s}^M_{b,t-1}$, and active (no response) with respect to the unfunded share of debt. Thus, a monetary-led

policy mix with respect to the typical business cycle shocks coexists with a fiscally-led policy mix with respect to the unfunded fiscal shocks.

The way fiscal trend inflation $\hat{\pi}_t^F$ enters the Taylor rule is similar to a time-varying target or an inflation drift that are typically added to estimated medium-scale DSGE models to explain trend inflation in the data (e.g., Smets and Wouters 2007). However, while the inflation drift in these other models evolves exogenously according to a close-to-random-walk process, the fiscal trend inflation in our model varies in response to the need to stabilize the share of unfunded debt, which is endogenous. Therefore, changes in trend inflation $\hat{\pi}_t^F$ are made credible by the coordination between monetary and fiscal authorities regarding the stabilization of the existing public debt.

3.2 Solving the Model

The unit-root process followed by the labor-augmenting technology A_t implies that some variables are non-stationary. Hence, we first detrend the non-stationary variables and then we log-linearize the model equations around the steady-state equilibrium (transfers and primary surplus are linearized).[6]

To solve the model, we need to track the evolution of the shares of funded and unfunded debt and fiscal trend inflation. To do so, we construct a shadow economy that keeps track of the funded portion of debt and the associated evolution of the endogenous variables. This economy is characterized by the same set of equations as the actual economy except for the monetary and fiscal rules. Specifically, the rules in the shadow economy differ from those in the actual economy insofar as: i) the shocks to unfunded debt are shut down, implying that the entire debt in the shadow economy is funded; ii) the central bank always responds to inflation deviations from its fixed target (i.e., steady-state inflation). Denoting by the superscript M any variable that belongs to the parallel economy, the shadow rules for the monetary and fiscal authorities read as follows:

$$\hat{g}_t^M = \rho_G \hat{g}_{t-1}^M - (1 - \rho_G) \ \gamma_G \hat{s}_{b,t-1}^M + \zeta_{g,t}, \tag{21}$$

$$\hat{z}_t^M = \rho_Z \hat{z}_{t-1}^M - (1 - \rho_Z) \ \gamma_Z \hat{s}_{b,t-1}^M + \phi_{zy} \hat{y}_t^M + \zeta_{z,t}^F, \tag{22}$$

$$\hat{\tau}_t^{J,M} = \rho_J \hat{\tau}_{t-1}^{J,M} + (1 - \rho_J) \ \gamma_J \hat{s}_{b,t-1}^M \tag{23}$$

$$\hat{R}_t^M = \max \left[-\ln \underline{R}, \rho_r \hat{R}_{t-1}^M + (1 - \rho_r) \left[\phi_\pi \hat{\pi}_t^M + \phi_y \hat{y}_t^M \right] + u_t^m \right], \tag{24}$$

where any difference with respect to the actual economy can be grasped by comparing equations (22) and (24) with (18) and (27), respectively. By construction, the shadow economy represents a counterfactual economy with no unfunded debt. Consequently, it

---

[6]The list of the log-linearized equations of the model is reported in Appendix C.

allows us to track the evolution of the amount of funded debt that the fiscal authority is committed to stabilize with fiscal adjustments, i.e. the share of debt in the actual economy that is stabilized by a Monetary-led policy mix.

As explained earlier, in the actual economy the monetary authority responds to deviations of inflation, $\hat{\pi}_t$, from fiscal trend inflation, the level of inflation needed to stabilize the share of unfunded debt, $\hat{\pi}_t^F$. As shown for the stylized model in Section 2, this level of inflation is precisely the difference between the inflation rate in the actual economy and the inflation rate in the shadow economy; that is, $\hat{\pi}_t^F = \hat{\pi}_t - \hat{\pi}_t^M$. By plugging $\hat{\pi}_t^M = \hat{\pi}_t - \hat{\pi}_t^F$ into the Monetary policy rule of the actual economy – equation (27) – we obtain

$$\hat{R}_t = \max\left[-\ln\underline{R}, \rho_r\hat{R}_{t-1} + (1-\rho_r)\left[\phi_\pi\hat{\pi}_t^M + \phi_y\hat{y}_t\right] + u_t^m\right]. \tag{25}$$

This rule implies that the monetary authority conducts active monetary policy only with respect to the inflation rate in the shadow economy.

With the level of debt entering the fiscal rules in the actual economy equal to the stock of debt in the shadow economy and the monetary rule of the actual economy expressed as a function of the inflation rate in the shadow economy, we can combine the policy functions characterizing the actual and the shadow economy to get a system of linear Rational Expectations equations. The model with partially unfunded debt can then be solved with standard solution algorithms and evaluated with a structural estimation.

4   Inference

The model is estimated using Bayesian techniques. The posterior distribution is obtained combining the priors for the model parameters with the model's likelihood function. The likelihood is evaluated with the Kalman filter. Section 4.1, introduces the data set used for estimation and presents the estimation strategy. The prior and posterior distributions for the model parameters are discussed in Subsection 4.2 and Subsection 4.3, respectively.

4.1   Data

The data set we use for estimation comprises ten variables for the U.S. economy observed at quarterly frequency over the period 1960:Q1 to 2020:Q4: real per-capita GDP growth; real per-capita consumption growth; real per-capita investment growth; a measure of the hours gap; the effective federal funds rate; the growth of average weekly earnings; price inflation based on the GDP deflator; the growth of real government transfers; the growth of government consumption and investment; the government debt-to-GDP ratio. Appendix D shows how these series are constructed.

To account for the federal funds rate being stuck at the effective lower bound from 2008:Q1 through 2015:Q3, we estimate the model over two subsamples: from 1960:Q1 to 2007:Q4 and then from 2008:Q1 to 2020:Q4. When estimating the model on the latter subsample, we add to the dataset the expectations for the federal funds rate one- through ten-quarters ahead, based on overnight index swaps.[7] Formally modeling the lower bound for the interest rate would raise substantially the computational challenge of our empirical exercise because it would introduce a non-linearity in the model, which requires using non-linear Monte Carlo filters to evaluate the likelihood (Fernandez-Villaverde and Rubio-Ramirez 2007). We adopt a simpler approach, following Campbell et al. (2012), who use data on market-based future federal funds rates to estimate the model after the fourth quarter of 2008. Agents' expectations about the future interest rates are informed by the market forecasts, which enforce the effective lower bound in the model. Therefore, agents in the model are not surprised about not seeing negative interest rates in every period during the Great Recession and the Pandemic Recession.

4.2 Priors

To elicit the prior distributions for the model parameters, we follow the approach proposed by Del Negro and Schorfheide (2008). Some parameter values are fixed in estimation or implied by steady-state restrictions. We fix the discount factor β to the value of 0.99, so that the steady-state real interest rate is broadly consistent with its sample average. The quarterly rate of capital depreciation, δ^K, is set to target an investment rate of 2.5%. The parameters governing the steady-state markups on wages and prices cannot be separately identified in estimation, so we set them to 0.14, following Leeper et al. (2017). The elasticity of output to capital in the production function α is set to the standard value of 0.33. The parameter s_{gc}, capturing the ratio of government expenditures to GDP, is set to 0.11 following Leeper et al. (2017). Finally, the steady-state tax rates on labor, capital and consumption, denoted by the parameters τ_L, τ_K, and τ_C, are set to the values of 0.186, 0.218 and 0.023, respectively, also based on Leeper et al. (2017). The consumption tax rate τ^C is assumed to be constant, so the parameters γ^C and ρ_C are set to zero.

The right panels in Tables 1 and 2 report the priors for the structural parameters and for the exogenous processes, respectively. The priors for both macroeconomic and fiscal variables are generally quite diffuse. We note that the decay rate of the maturity of long-term bonds, ρ, is set to 0.9593 to match an average duration of six years, as estimated by the Congressional Budget Office (2020). We follow Kaplan et al. (2014) and center the share of

---

[7]We construct series of the market-expected federal funds rate in the same way as Campbell et al. (2017). See Appendix D for further details.

Prior and Posterior Distributions for the Structural Parameters

Param	Description	Posterior Distribution				Prior Distribution		
		Mode	Median	5%	95%	Type	Mean	Std
s_b	Debt to GDP annualized	2.1703	2.1834	2.0147	2.3497	N	1.82	0.10
$100\ln\mu$	Steady state growth rate	0.4000	0.4001	0.3255	0.4925	N	0.50	0.05
$100\ln\Pi$	Steady state inflation	0.5402	0.5195	0.4267	0.6104	N	0.50	0.05
ξ	Inverse Frisch elasticity	1.9704	1.9167	1.7493	2.1217	N	2.00	0.25
μ	Share of hand-to-mouth	0.0771	0.0778	0.0652	0.0925	N	0.11	0.01
ω_w	Wage Calvo param	0.8041	0.8063	0.7861	0.8243	B	0.50	0.10
ω_p	Price Calvo param	0.8663	0.8666	0.8375	0.8897	B	0.50	0.10
ψ	Capital utilization cost	0.6596	0.6572	0.5755	0.7502	B	0.50	0.10
s	Investment adjust. cost	5.7144	5.5214	5.0185	5.9213	N	6.00	0.50
χ_w	Wage infl. indexation	0.0372	0.0437	0.0164	0.0923	B	0.50	0.20
χ_p	Price infl. indexation	0.3117	0.2782	0.1279	0.4101	B	0.50	0.20
θ	Habits in consumption	0.9106	0.9091	0.8985	0.9187	B	0.50	0.20
α^G	Subs. private/gov. cons.	-0.0455	-0.0396	-0.1832	0.0838	N	0.00	0.10
ϕ_y	Interest response to GDP	0.0012	0.0019	0.0001	0.0074	N	0.25	0.10
ϕ_π	Interest response to infl.	2.0577	2.0963	1.9462	2.2525	N	2.00	0.10
ϕ_{zy}	Transfer response to GDP	0.0715	0.0439	0.0198	0.0719	G	0.10	0.05
γ_G	Gov. cons. response to debt	0.3800	0.3463	0.2218	0.4279	N	0.25	0.10
γ_K	Capital tax response to debt	0.0043	0.0064	0.0003	0.0335	N	0.25	0.10
γ_L	Labor tax response to debt	0.0163	0.0133	0.0009	0.0461	N	0.25	0.10
γ_Z	Transfers response to debt	0.0017	0.0063	0.0003	0.0249	N	0.25	0.10
ρ_r	AR coeff. interest rate	0.7250	0.7223	0.6650	0.7746	B	0.50	0.10
ρ_G	AR coeff. gov. cons.	0.9637	0.9627	0.9340	0.9803	B	0.50	0.10

Table 1: Posterior modes, medians, 90% posterior credible sets, and prior moments for the structural parameters. The letters in the column with the heading "Prior Type" indicate the prior density function: N, G, and B stand for Normal, Gamma, and Beta, respectively.

hand-to-mouth households μ to 0.11, to match the share of poor hand-to-mouth consumers.

We note that the priors for the autocorrelation coefficients of both the funded and unfunded transfer shocks are tightly centered around a very persistent mean to reflect the econometrician's beliefs that changes in funded and unfunded transfers are very persistent. Conversely, cyclical increases in government transfers are expected to be backed by the increase in tax revenues and the decline in spending ensuing the next economic recovery.

We also set the prior on the autocorrelation coefficient of the NKPC expectation shock so as to provide the model with a competing mechanism to explain trend inflation. Thus, the model allows, but does not require, trend inflation to be generated by the emission of unfunded debt. The autocorrelation coefficients of the fiscal rules (ρ_Z, ρ_K, ρ_L) are set to 0.5 because they are only weakly identified in the estimation. The prior for the standard deviation of the shocks is the same except for that of the unfunded transfer shocks and the near-permanent cost-push shock, for which we pick a tighter prior. The priors for these two

Priors and Posteriors for the Exogenous Processes

Param	Description	Posterior Distribution				Prior Distribution		
		Mode	Median	5%	95%	Type	Mean	Std
ρ_{eG}	AR coeff. gov. cons.	0.2868	0.3045	0.1506	0.3782	B	0.500	0.100
ρ_{eZ}^{F}	AR coeff. funded trans.	0.9954	0.9953	0.9933	0.9968	B	0.995	0.001
ρ_{eZ}^{U}	AR coeff. unfunded trans	0.9958	0.9956	0.9937	0.9971	B	0.995	0.001
ρ_a	AR coeff. technology	0.2987	0.2803	0.1711	0.3610	B	0.500	0.100
ρ_b	AR coeff. preference	0.8237	0.8237	0.7774	0.8609	B	0.500	0.100
ρ_m	AR coeff. mon. policy	0.2407	0.2573	0.1692	0.3105	B	0.500	0.100
ρ_i	AR coeff. investment	0.9205	0.9206	0.8990	0.9395	B	0.500	0.100
ρ_{rp}	AR coeff. risk premium	0.9085	0.9062	0.8880	0.9220	B	0.500	0.100
$\rho_{\mu^{NKPC}}$	AR coeff. pers. cost push	0.9965	0.9966	0.9951	0.9977	B	0.995	0.001
σ_G	St.dev. gov. cons.	1.9046	1.9306	1.7416	2.1419	IG	0.500	0.200
σ_Z^{F}	St.dev. funded transfers	2.9635	2.8922	2.6631	3.0924	IG	0.500	0.200
σ_Z^{U}	St.dev. unfunded transfers	0.5166	0.5500	0.4194	0.7319	IG	0.100	0.050
σ_a	St.dev. technology	1.2113	1.1989	1.0895	1.3349	IG	0.500	0.200
σ_b	St.dev. preference	4.9850	4.9782	4.9214	4.9986	IG	0.250	0.200
σ_m	St.dev. mon. policy	0.2375	0.2406	0.2154	0.2691	IG	0.500	0.200
σ_i	St.dev. investment	0.5192	0.5318	0.4734	0.5955	IG	0.500	0.200
σ_w	St.dev. wage markup	0.3487	0.3512	0.3156	0.3912	IG	0.500	0.200
σ_p	St.dev. trans. cost push	0.1625	0.1640	0.1427	0.1877	IG	0.500	0.200
σ_{rp}	St.dev. risk premium	0.3914	0.3990	0.3441	0.4586	IG	0.500	0.200
$\sigma_{\mu^{NKPC}}$	St.dev. pers. cost push	1.3255	1.3763	1.2106	1.6382	IG	0.100	0.050
σ_{GDP}^{m}	Measur. error GDP	0.4330	0.4352	0.3947	0.4831	IG	0.500	0.200
σ_{by}^{m}	Measur. error Debt/GDP	0.3160	0.3032	0.2221	0.4217	IG	0.500	0.200

Table 2: Posterior modes, medians, 90% posterior credible sets, and prior moments for the structural parameters. The letters in the column with the heading "Prior Type" indicate the prior density function: N, G, and B stand for Normal, Gamma, and Beta, respectively.

shocks are chosen to be the same in that these two shocks are (ex-ante) expected to provide the model with two competing explanations of the dynamics of trend inflation in the data. It can be shown that loosening those priors would not affect our main results.

4.3 Posterior Distributions

The left panels of Tables 1 and 2 report the posterior distributions for the structural parameters and the exogenous processes, respectively, obtained over the sample period 1960Q1 to 2007Q4. The estimates obtained over the second subsample, 2008Q1 to 2020Q4, are reported in Appendix E. We note that the parameters governing the response of the tax instruments to debt, i.e., γ_L, γ_K and γ_C are positive but quantitatively small. The stabilization of the share of funded debt is therefore ensured by the relatively higher estimate of the parameter γ_G, which implies that debt stabilization is mostly achieved by changing government purchases rather than taxes. We estimate a relatively high degree of price and wage rigidities,

consistent with the findings in Leeper et al. (2017), implying a relatively flat Phillips curve. Our estimate of the habit parameter lies towards the upper end of the multitude of estimates reported in the literature, but are smaller than those obtained by Leeper et al. (2017). The output coefficient in the Taylor rule is close to zero, and smaller than typically obtained in the estimation of similar models, suggesting that changes in the share of transfers that are funded cannibalize the interest rate response to output.

5 Results

In this section, we use the estimated TANK model with partially fiscally unfunded debt to analyze the role of fiscal trend inflation with respect to the historical evolution of inflation and to understand what role we should reasonably expect monetary and fiscal interactions will play in a large-debt environment. In Section 5.1, we show how shocks to funded and unfunded transfers propagate to the macroeconomy in the estimated model. The results in this section will help us to understand what drives the identification of the share of unfunded transfers in the data. In Section 5.2, we use the estimated model to investigate the historical dynamics of the unfunded share of transfers. In Section 5.3, we show that shocks to unfunded transfers explain the bulk of the dynamics of U.S. inflation in the postwar period. In Section 5.4 and in Section 5.5, we study the model predictions for inflation after the Pandemic Recession, which brought about a sizable increase in the U.S. fiscal imbalance.

5.1 Identification of Unfunded Transfer Shocks

In this subsection, we study how unfunded transfer shocks, $\zeta_{z,t}^F$, funded transfer shocks, $\zeta_{z,t}^M$, and shocks to long-run cost-push shocks, u_t^{NKPC}, propagate through the economy. This analysis sheds light on how the three shocks are identified in the estimation. Figure 3 shows that the three shocks give rise to very different impulse responses for key macroeconomic variables, i.e. the inflation rate, the real interest rate, and the debt-to-GDP ratio.

The propagation of a funded transfer shock (dashed black line) produces only a modest impact on the macroeconomy, as the expansionary impulse of current transfers is offset by the expectations of higher taxes and/or a decrease in government spending in the future; qualitatively, inflation rises, following the positive stimulus to aggregate demand and real marginal costs. Concurrently, the debt-to-GDP ratio increases to fund the rise in transfers.

Unfunded transfer shocks, conversely, have a quantitatively strong expansionary effect on the macroeconomy (solid blue line). In sharp contrast with the propagation of funded transfer shocks, unfunded transfers lead to a fall in the real interest rate, as the fiscal and
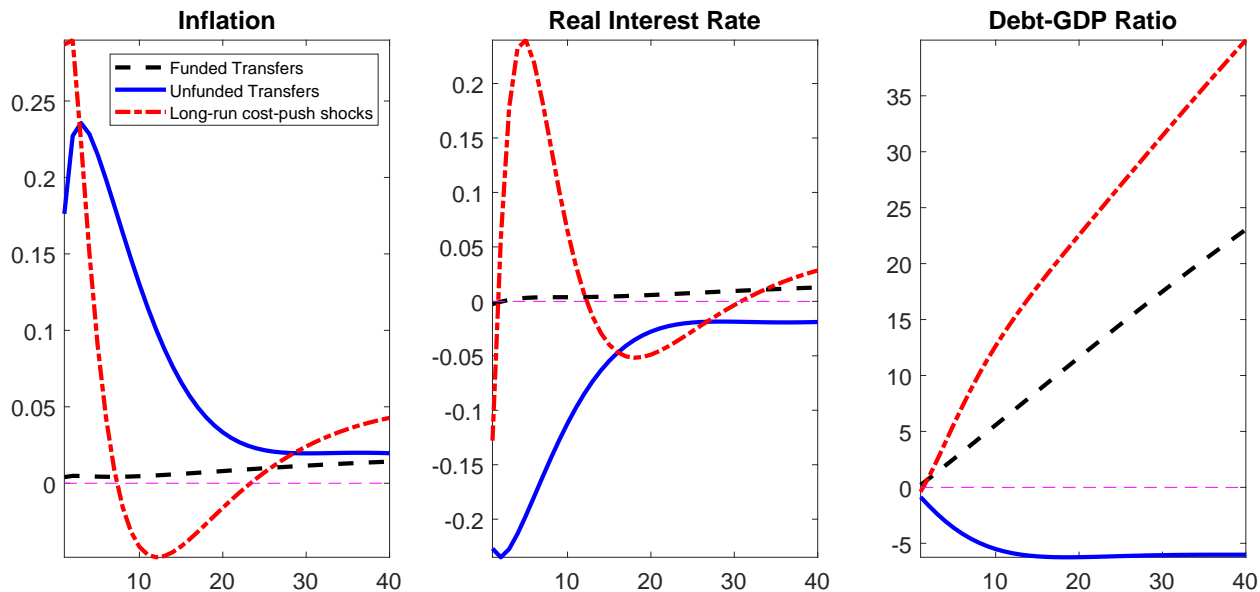
Figure 3: Impulse Response Function of Inflation (left), Real Interest Rate (middle), and Debt-To-GDP Ratio (right) to a one-standard deviation shock to the share of funded transfers (black dashed line), to the share of unfunded transfers (blue solid line), and to the persistent non-policy shifter of the Phillips curve (red dotted-dashed line). Units: percentage deviations from steady-state values. The magnitude of the initial shocks is set to be equal to one-standard deviation estimated in the second sample (2008Q1-2020Q4).

the monetary authorities coordinate to let inflation rise to stabilize the increase in transfers. This coordinated policies increase both inflation and inflation expectations. Note that 5 years after the shock, inflation remains above its long-run value. The lower real interest rate also stimulates aggregate production. Lower financing costs and higher GDP determine a fall in the debt-to-GDP ratio, despite the increase in spending. The opposite responses of the debt-to-GDP ratio produced by funded vs. unfunded transfers, together with the large effects on the macroeconomy of unfunded shocks that are absent in response to funded shocks, allow us to separately identify these two transfers shocks, $\zeta_{z,t}^M$ and $\zeta_{z,t}^F$ in the estimation.

A shock to firms' long-run cost-push shocks (dot-dashed red line) produces a temporary but short-lived rise in inflation. The real interest rate falls, but only on impact, and then rises persistently as the central bank reacts to the inflationary pressure. The rise in real rates and the associated contraction in aggregate production lead to an increase in the debt-to-GDP ratio. The opposite responses of the real interest rate and of the debt-to-GDP ratio following a shock to unfunded transfer shocks and a shock to long-run inflation expectations, provide identification for the two shocks, $\zeta_{z,t}^F$ and u_t^{NKPC}. The response of the debt-to-GDP ratio to all the three shocks is very persistent because of the highly inertial process, which is behind the debt-to-GDP ratio in structural models and is governed (up to the first approximation) by the inverse of agents' discount factor (β^{-1}).[8]

---

[8]It takes roughly 600 quarters for the debt-to-GDP ratio to converge to steady state after an unfunded transfers shocks and roughly twice as long to converge following a shock to the funded transfer shocks and to
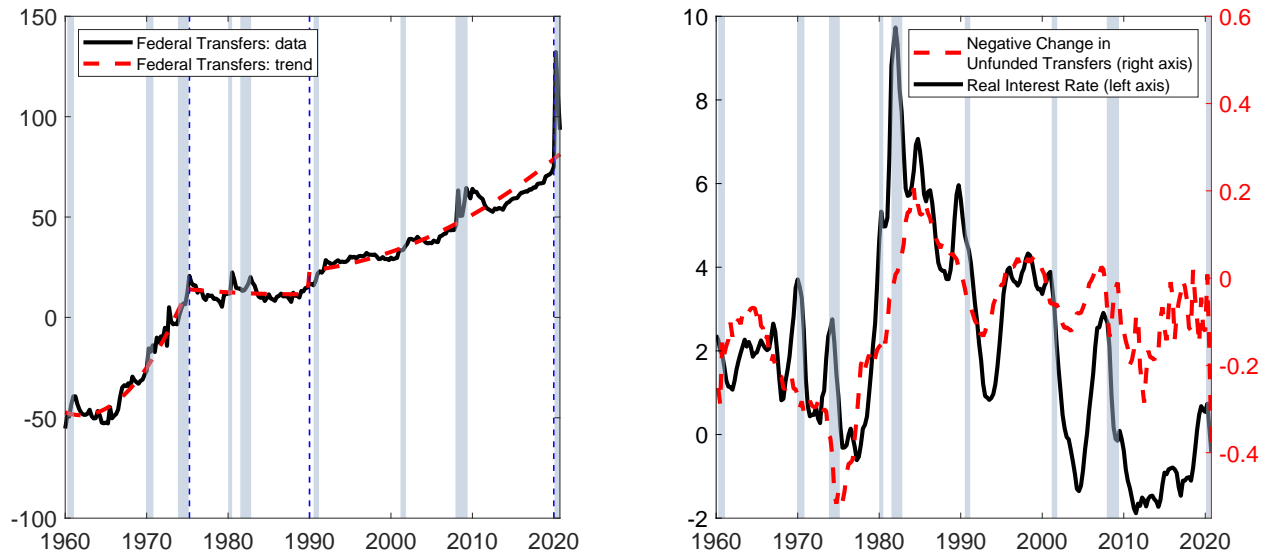
Figure 4: U.S. Real Federal Transfer Payments and Real Interest Rate. Left graph: Federal transfers in the data and its time trend measured by a quadratic polynomial fitted on each of the four time periods marked by the vertical blue bars. Right plot: The negative of the change in the share of unfunded transfers (black line) and the real interest rate (the red dashed line). The former is computed by taking the 3-year moving average of the changes in the share of unfunded transfers predicted by the model (smoothed estimates). The latter is computed by taking the one-year moving average in the real rate of interest predicted by the model (smoothed estimates).

5.2   Funded and Unfunded Transfers

Before using the model to infer the historical behavior of the share of funded and unfunded federal transfers, it is useful to take a look at how total per-capita real federal transfers have evolved over time. The left panel of Figure 4 plots the evolution of real U.S. government transfers from 1960Q1 to 2020Q4 (black line), which are observed in the structural estimation. The red dashed line in the figure corresponds to the time trend measured by a quadratic polynomial fitted on each of the four time periods marked by the vertical blue bars in the figure. The first period, spanning the 1960s and going up to the mid 1970s, was characterized by a sharp increase in real government transfers. These transfers reflect policy initiatives, initiated by President Johnson, aimed at reducing poverty levels. These initiatives were part of the Great Society program, which aspired to reduce racial injustice and crime and to improve the environment.

After President Johnson ended his second term in 1969, the level of transfers continued to increase during the Nixon's presidency (1969-1974). This is consistent with the fact that many of the welfare programs introduced in the 1960s shifted the long-term path of spending. But in the subsequent period, which starts from the mid 1970s and ends around 1990, the

---

the long-run cost-push shocks. The fact that the debt-to-GDP is still away from the steady state while the other variables have already adjusted is due to the high value for the inverse of the deterministic discount factor (β^{-1}). However, debt is on a stable path in virtue of the adjustments of the other variables that can move faster.

growth in transfers came to a halt, and their level remained broadly unchanged. Next, after 1990, the level of transfers started to increase again; the rate of growth has been rather stable throughout this third period, although smaller than the one observed between the mid 1960s and the mid 1970s. Finally, the very last period, which captures the pandemic recession, has witnessed a large jump in the level of transfers, whose magnitude exceeds by far any increase observed over the estimation sample.

What portion of these changes can be attributed to funded and unfunded transfers? The right plot of Figure 4 shows the *negative* of the changes in the share of unfunded transfers based on the model estimates and its relation with the real interest rate. This plot illustrates how the structural estimation is able to attribute the observed changes in total transfers to the two components, funded and unfunded. The figure highlights that the real interest rate declines when the share of estimated unfunded transfers increases, and vice versa. This result is consistent with the impulse response functions shown in Figure 3, where the real interest rate responds negatively to increases in the share of unfunded transfers. Changing the share of unfunded transfers requires monetary and fiscal coordination. Specifically, monetary policy has to accommodate the movements in inflation resulting from this share of transfers, leading to fluctuations in the real interest rate.

Figure 5 presents the decomposition of the evolution of transfers into the shares that are estimated to be funded ($\zeta_{z,t}^{M}$) and unfunded ($\zeta_{z,t}^{F}$), as depicted by the white and black bars, respectively. The four panels of this figure refer to the same four periods discussed above and highlighted in Figure 4. The red line in the right panel of Figure 5 plots the sum of the the fundend and unfunded components ($\zeta_{z,t}^{M}+\zeta_{z,t}^{F}$).

Changes in the share of unfunded transfers capture revisions in expectations about the monetary and fiscal commitment to use fiscal instruments to repay the persistent flow of total transfers. For instance, total transfers may fall while the share of unfunded transfers rises (e.g., in 2020q4). In the estimation, the changes in the share of unfunded transfers are chiefly informed by the joint dynamics of inflation, real interest rate, and debt-to-GDP ratio, as shown in the previous section (Figure 3). Historical events like an exceptionally large recession, the creation of large welfare programs, the appointment of a new Chairman can be linked to the estimated movements in the share of unfunded transfers.

The panel in the top left corner shows that the increase in transfers occurred between the mid 1960s and the mid 1970s was partially funded, and partially unfunded. Specifically, the rise in the share of unfunded transfers over this period is substantial (black bars). In the following period, which ranges between the mid 1970s and 1990, the share of unfunded transfers exhibits an hump-shaped pattern, but by the end of this subsample, it ends up again hovering around the same value already reached by the mid-1970s.
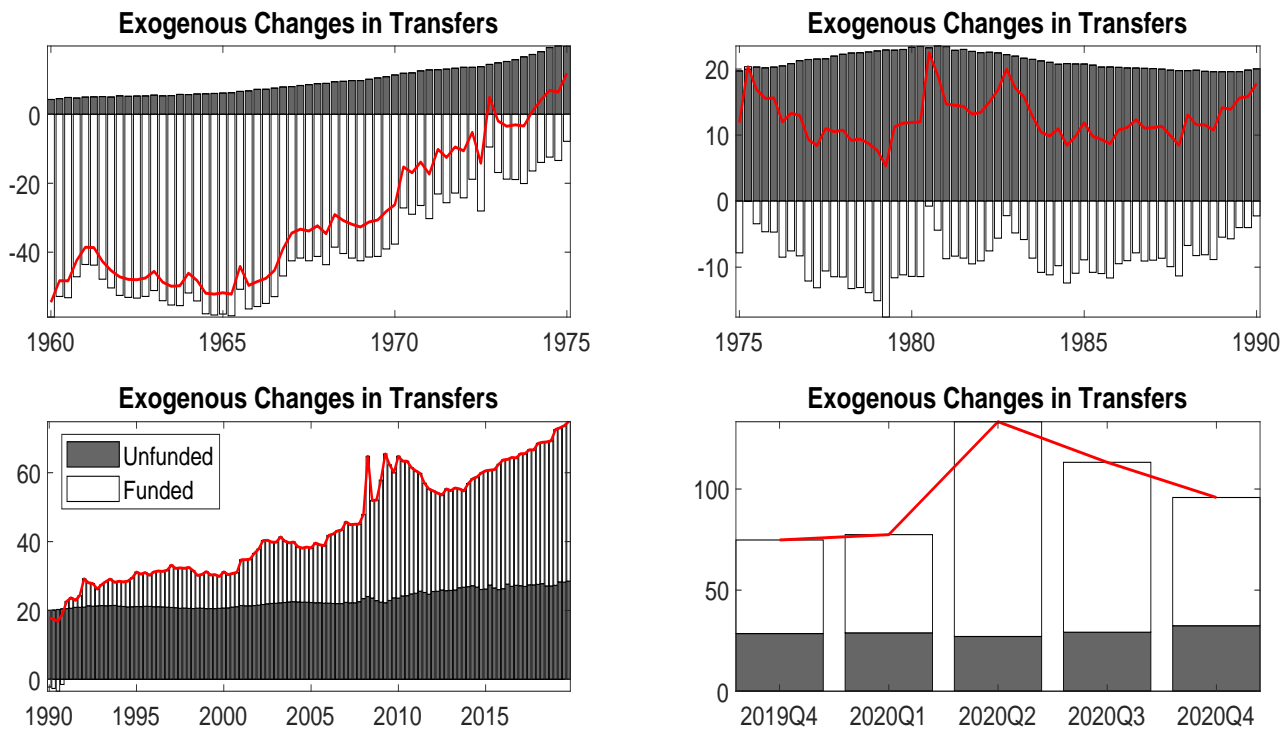
Figure 5: Estimated decomposition of exogenous changes in transfers (red solid line) $((\zeta_{z,t}^F + \zeta_{z,t}^M))$ into their Funded and Unfunded components ($\zeta_{z,t}^F$ and $\zeta_{z,t}^M$). These components isolate the cumulative effects of the historical realizations of funded or unfunded shocks on transfers in percentage points. These realizations are estimated using the smoother Kalman filter. Parameters are set at their posterior mode. Units: percentage deviations from steady-state values.

As we will discuss in more detail below, the initial acceleration in the late 1960s and the subsequent slowdown in the second half of the 1970s play an important role in accounting for the rise and fall in inflation. The sharp rise in the real interest rate in the first half of the 1980s –primarily due to Volcker's aggressive monetary tightening – explains why a smaller fraction of transfers are interpreted as fiscally unfunded (the black bars). As we will see, this change in expectations implies a sharp reduction in the inflation rate that the central bank is expected to tolerate ($\hat{\pi}_t^F$). Concurrently, the overall level of transfers (the red line) exhibits quite an erratic behavior, which mainly affects the funded share of transfers (the white bars). These movements are mostly due to a quite volatile economy, with two large recessions in the late 1970s and early 1980s.

The bottom left panel illustrates that the steady rise in transfers observed after 1990 was partially unfunded; the black bars rise steadily, albeit sluggishly, over this period. In the post-Millennial period, the model recovers a more rapid increase in the share of unfunded debt observed in the 2010s in light of a very accommodating monetary policy that engendered a decade-long negative real interest rate (Bianchi et al. 2016). As shown in the right panel of Figure 4, this pattern corresponds to a decline in the real interest rate, which is interpreted by the model as a sign that the central bank is willing to tolerate a higher amount of inflation.

28

Finally, the last panel of Figure 5 shows that the federal government increased total transfers sharply in the second quarter of 2020 in an attempt to combat the severe consequences of the pandemic crisis. However, the share of unfunded transfers slightly fell in that quarter and only increased in the third and fourth quarter of 2020. Interestingly, the increase in the expected share of unfunded transfers happened concomitantly with the introduction of new monetary framework in the last days of the third quarter of 2020 (August 27). The new framework contemplates the possibility for the Federal Reserve to let inflation overshoot its two-percent target after the Pandemic recession. This new monetary policy strategy is reflected in the change of the expected path of the future federal funds rate, which we observe in the estimation using overnight index swaps.

5.3   Drivers of Inflation and GDP growth

We now turn our attention to the analysis of the relation between unfunded transfer shocks and the the historical dynamics of inflation and GDP growth. Figure 6 provides a historical shock decomposition of inflation (red line). The black bars in the figure illustrate the level of inflation originating from unfunded transfer shocks, which we labelled fiscal trend inflation. The gray and white bars highlight the role of the other policy shocks and the non-policy shocks, respectively.[9]

The key result that emerges from Figure 6 is that fiscal trend inflation accounts for the bulk of inflation dynamics. The rise in trend inflation over the mid-1960s and up to the mid-1970s was accounted for, almost entirely, by the inflationary effects of the rise in unfunded transfers that took place in that period, as illustrated in Figure 5. The inflationary effects of this rise in the level of transfers started to wither away during the late 1970s, while non-policy shocks were pushing up on the rate of inflation. Even though the share of unfunded transfers rose in the second half of the 1970s (upper right panel of Figure 5), the pace of this increase was too slow to sustain the high level of fiscal trend inflation caused by the large expansion in the share of unfunded transfers of the first half of the 1970s (upper left panel of Figure 5). As a result, fiscal trend inflation fell steadily in the second half of the 1970s, even if it remained elevated at the end of the 1970s, when unfunded transfers still explain about half of observed inflation.

The fall in inflation accelerated in the first half of the 1980s – mostly driven by a fall in fiscal trend inflation (the black bar), the amount of inflation tolerated by the central bank to stabilize the unfunded transfers. In the first five years of the 1980s, fiscal trend inflation declined by 3%, moving from 3.8% to 0.8% in deviations from the long-term inflation

---

[9]Other policy shocks include the monetary policy shock, funded transfer shocks, and the other fiscal shocks. Non-policy shocks include all other shocks.
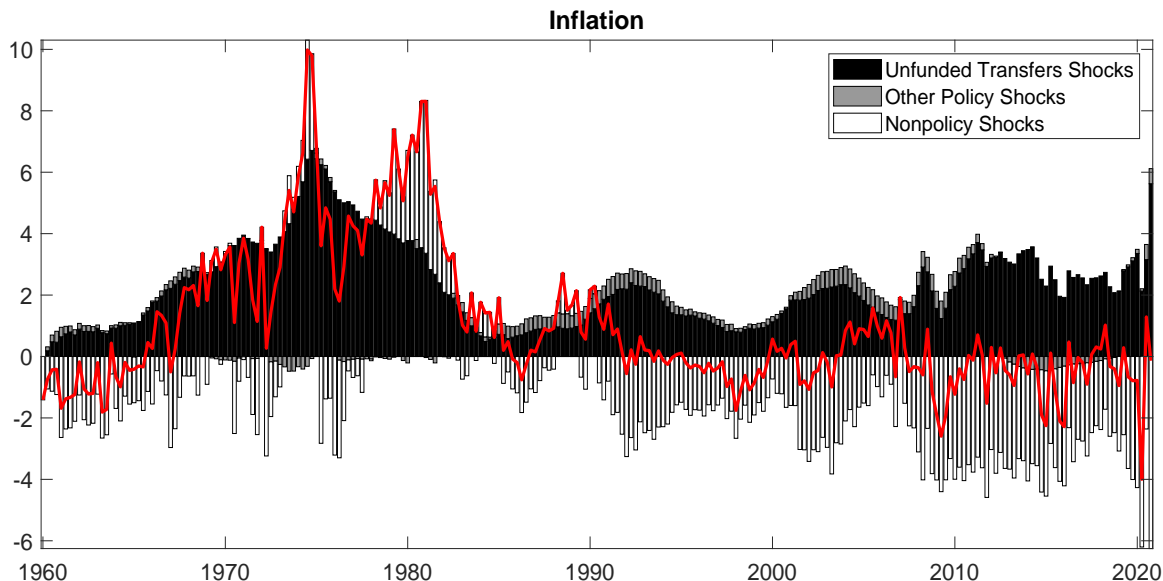
Figure 6: Drivers of Inflation. The red line is the inflation rate in the data in deviations from model's steady-state inflation. The bars represent the cumulative effects of unfunded transfers shocks (black bars), other policy shocks (gray bars), and nonpolicy shocks (white bars) on inflation (in deviations from steady state). Other policy shocks include shocks to funded transfers and surprise and anticipated monetary policy shocks. Shocks are estimated using the Kalman smoother and setting the model parameters at their posterior mode.

target. The sharp increase in the real interest rate due to the aggressive monetary tightening conducted by the Federal Reserve Chairman Volcker led to a large fall in fiscal trend inflation $\hat{\pi}_t^F$, in the first five years of the 1980s. Thus, as in Sims and Zha (2006), we document that a policy change occurred before the appointment of Fed Chairman Volcker in August 1979. However, we also find that the early 1980s led to an acceleration in the change of the policy environment, with the election of President Reagan who arguably provided the political backing for the actions of the Federal Reserve.[10] These results are also consistent with the evidence provided by Hazell et al. (2020). They argue that the Phillips curve has always been flat in the US and that the primary force behind the Volcker disinflation was a change in long-term inflation expectations triggered by the policy change, rather than high unemployment working through a steep Phillips curve.

Importantly, from about 1990 until the most recent years, fiscal inflation generated by the steady rise in the level of transfers contributes persistently to reflate the economy, offsetting the deflationary bias set off by non-policy shocks (the white bars consistently lying in the

---

[10]Our results suggest an interesting interpretation for the finding of the seminal paper by Clarida et al. (2000) that the systematic response of the Federal Reserve to inflation was weaker in the 1970s than in the following decade, a result confirmed by subsequent work by Lubik and Schorfheide (2004), Fernandez-Villaverde and Rubio-Ramirez (2007b), and Bianchi (2013) in estimated DSGE models. In light of our results, the estimated coefficient of a standard Taylor rule could be interpreted as a weighted average of two different coefficients whose weights change depending on the type of shocks that determine movements in inflation.
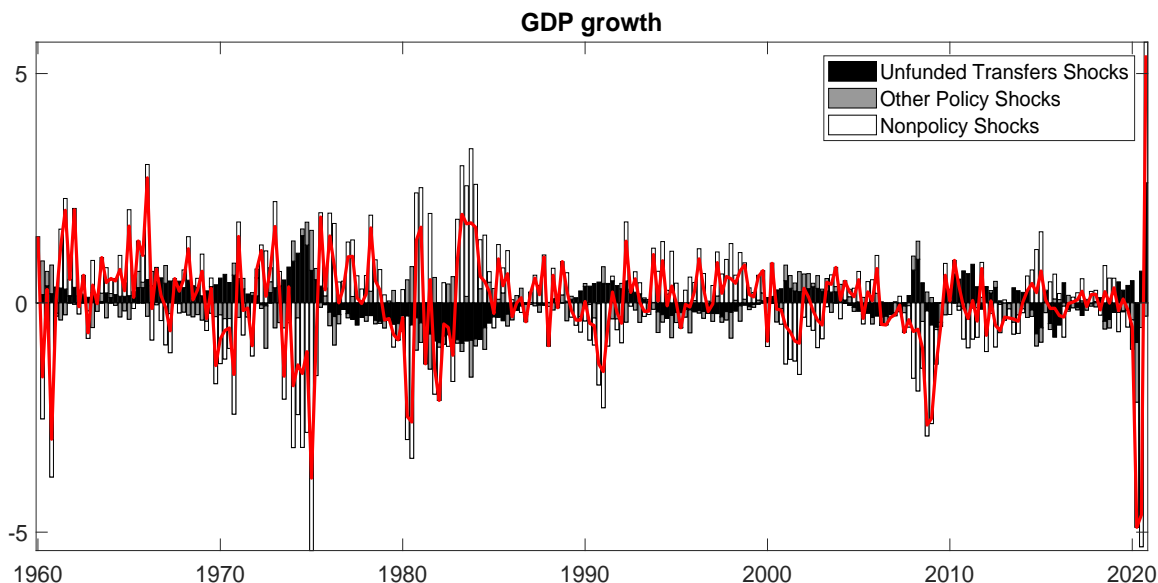
30

GDP growth

Figure 7: Drivers of GDP Growth. The red line is the annualized quarter-over-quarter growth rate of real percapita GDP in deviations from model's steady-state inflation. The bars represent the cumulative effects of unfunded transfers shocks (black bars), other policy shocks (gray bars), and nonpolicy shocks (white bars) on GDP growth (in deviations from steady state). Other policy shocks include shocks to funded transfers and surprise and anticipated monetary policy shocks. Shocks are estimated using the Kalman smoother and setting the model parameters at their posterior mode.

negative territory). The deflationary effects of these non-policy forces are persistent and keep dragging inflation down for a long period of time.

Figure 7 provides a similar historical shock decomposition for the series of GDP growth. A key take away from this figure is that unfunded transfers shocks have played an important role in driving real activity, beyond their effects on inflation. Unfunded transfers are estimated to play an important role in counteracting the productivity slowdown of the late 1960s and mid-1970s, providing a positive contribution to growth. At the same time, their effects reverted in the 1980s, contributing negatively to growth.

Over the years of the Great Moderation (1984-2008), the contribution of unfunded transfers to fluctuations in growth becomes more modest, consistent with a more sluggish evolution. We see an important resurgence of their role during the 2008-2009 recession and even more so in correspondence of the pandemic recession. The model attributes a large portion of the rebound in economy activity to a robust increase in the amount of unfunded transfers implemented in response to the pandemic.

Interestingly, the change in the amount of unfunded transfers and the associated contribution to the rebound of the economy do not coincide with the increase in fiscal transfers, but rather with the announcement of the change of the policy strategy followed by the Federal Reserve. Thus, it is the coordination between monetary and fiscal authorities that triggers the large rebound of the economy. The increase in funded transfers alone has limited effi-
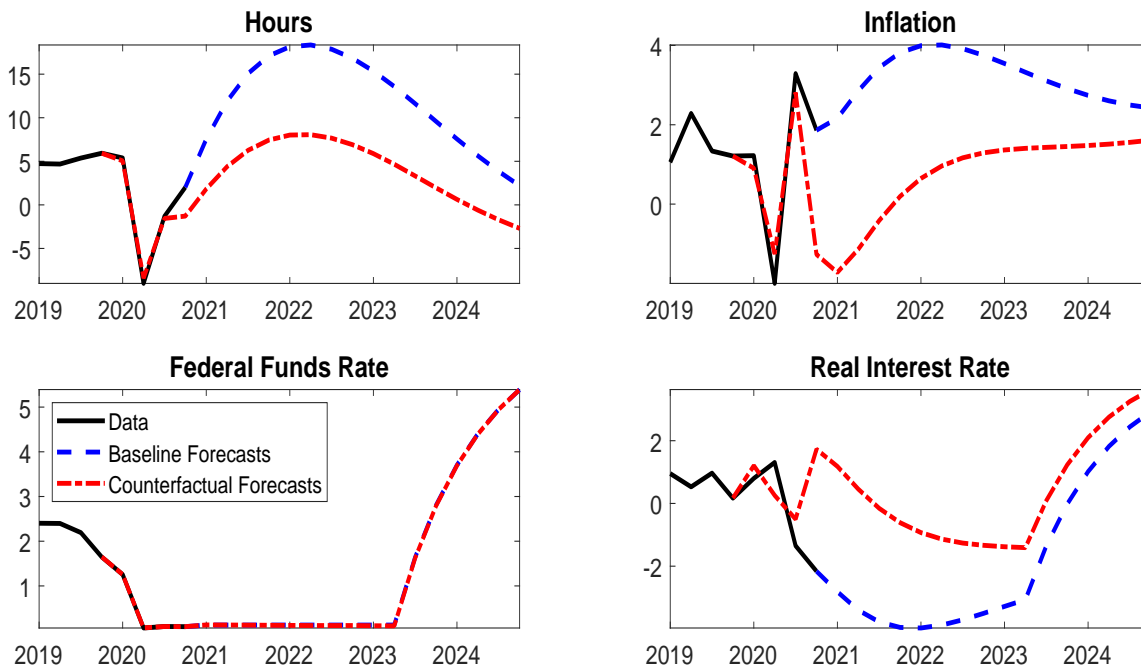
Figure 8: The Role of Unfunded Debt in The Past Recession. Model's forecast on hours worked, inflation, the Federal Funds rate, and the real interest rate, conditional on using filtered data up to 2020Q4 (dashed blue line). The counterfactual scenario is based on the assumption that all transfer shocks estimated during the Pandemic Recession (i.e., 2020Q1 through 2020Q4) are funded (dot-dashed red line). Shocks are estimated using the Kalman smoother. Model parameters are set at their posterior mode.

cacy because it also generates an expectation of large tax increases in the future. This result holds despite the fact that we allow for hand-to-mouth consumers that immediately spend the transfers that they receive. Instead, an increase in unfunded transfers leads to a reflation of the economy, a decline in real interest rates, and an increase in real activity.

5.4 Unfunded Fiscal Shocks during the Pandemic

In this section, we analyze the importance of monetary and fiscal policy coordination in the context of the pandemic recession. To do so, we isolate the role played by the expected share of unfunded fiscal transfers in contrasting deflationary pressure and providing a boost to real economic activity. Specifically, Figure 8 shows the model out-of-sample forecasts of some key macroeconomic aggregates, e.g., hours, inflation, the Federal Funds rate, and the real interest rate, conditional on using filtered data up to 2020Q4 (dashed blue line). To isolate the contribution of the recent unfunded transfer shocks, we contrast this baseline forecast with a counterfactual scenario in which we assume that all transfer shocks estimated during the Pandemic Recession (i.e., 2020Q1 through 2020Q4) are funded (dot-dashed red line).

A first key takeaway from the figure, is that the rise in inflation necessary to wear away

32

the share of 2020 transfers that are unfunded is predicted to be moderate and relatively persistent. As noted earlier, this overshoot of inflation over the central bank's 2% target allows the central bank to regain space for future monetary policy in an environment of elevated ELB risk.

If all transfer shocks that occurred in 2020 had been funded, the rate of inflation would have temporarily fallen in negative territory, persistently undershooting the long-run target of 2% throughout the forecast horizon. This deflationary bias is the consequence of the long-run drag on inflation played by a number of persistent non-policy shocks (including the almost-unit root shifter of the price Phillips curve, u_t^{NKPC}) on inflation at the end of the sample. The highly persistent effect of these shocks on inflation is denoted by the white bars in Figure 6. The increase in the share of unfunded transfers in the fourth quarter of 2020 gives rise to inflationary pressure that dominates these long-lasting deflationary forces allowing the central bank to overshoot its long-term 2% target and reflate the economy.

The fall in real interest rates generated by the rise in inflation expectations provides persistent stimulus to the economy, boosting total hours up to 18% above their steady-state value. Furthermore, since part of the transfers paid by the government to help the economy weather the pandemic recession are expected to be unfunded, agents anticipate that future fiscal adjustments will be lighter in the baseline scenario, also contributing to the economic recovery that follows the pandemic recession.

The increase in inflation required to provide this stimulus is tolerated by the central bank, as it can be seen by noting that the central bank does not anticipate the lift-off of the interest rate. Indeed, the forecasts on the path of the federal funds rate shown in the bottom left plot of Figure 8 is no different in the baseline scenario and in the counterfactual scenario. This behavior of the nominal interest rate reflects the coordinated action of the monetary and fiscal authorities, aimed at stabilizing a fraction of the 2020 fiscal stimulus with inflation. From this point of view, the interaction of monetary and fiscal policies plays a key role in implementing the asymmetric policy strategy outlined by the Federal Reserve following its 2020 policy review (Bianchi et al. (2019)). The increase in the share of unfunded transfers makes the asymmetric strategy not only credible, but also necessary.[11]

5.5   ARPA Fiscal Stimulus and Post Pandemic Inflation

Since the model is estimated using data up to 2020Q4, the baseline forecast in Figure 8 accounts for the fiscal stimulus introduced by President Trump's administration, but not for the subsequent stimulus of President Biden, i.e., the American Rescue Plan Act (ARPA).

---

[11] This finding echoes the remarks by Sims (2016) at the 2016 Jackson Hole symposium: "[...] interest rate policy, tax policy, and expenditure policy, [...] jointly determine the price level."

This additional $1.9 trillion stimulus has led influential commentators and scholars to express concern about price stability. (e.g., Blanchard, 2021, and Summers, 2021)

To investigate the macroeconomic implications of the ARPA stimulus, Figure 9 compares the baseline forecast illustrated in Figure 8 (the blue dashed line in both figures), with a revised forecast that accounts for the transfers observed in 2021Q1 (the red dot-dashed line). We emphasize that the dataset used to produce this revised forecast differs from the dataset used to produce the baseline forecast only to the extent that it adds information on the transfers implemented in 2021Q1. In other words, we do not include observations for the other series of the model. ARPA stimulus mostly rested on providing additional transfers to households, which received a new stimulus check in their mailbox the last weeks of March 2021. Hence the difference between the two projections isolates the contribution of the ARPA stimulus predicted by our model, everything else being equal. Importantly, the increase in transfers observed in 2021Q1 is attributed by the filter to funded and unfunded shocks according to their historical pattern.[12] The upper bound of the pink area is obtained by increasing the estimated variance of the unfunded fiscal shock, σ_Z^U, by a factor of two and by decreasing the estimated variance of the funded fiscal shocks, σ_Z^F, by the same factor. In this way, the forecasts attributes a larger share of the observed increase in transfers due to the ARPA stimulus to the unfunded fiscal shocks. The lower bound is obtained by increasing the estimated standard deviation of the funded transfers shocks and by decreasing the estimated standard deviation of the unfunded transfers shocks by the same factor.

Figure 9 shows that the ARPA stimulus produces a further increase in inflation, up from close to 4% to about 5% at its peak. Our conditional forecast can replicate very well the inflation data for 2021 (the four star points in Figure 9), data points that are not used in the filtering exercise. After completing a larger overshoot, inflation retrenches to the central bank target in 2025. Because of the larger increase in inflation, the real interest rate falls further, providing an even bigger boost to the economy in 2021 and over the following years, as shown by the forecasts on hours (upper left panel). This is despite an anticipated lift-off of the nominal interest rate, relative to the baseline case. This anticipated lift-off occurs because part of the increase in transfers due to the fiscal stimulus is fiscally funded and the central bank does not accommodate the (small) increase in inflation attributed to funded fiscal shocks (note that in the exercise we keep the anticipated MP shocks fixed).

In light of these results, we conclude that the post-pandemic macroeconomic situation is largely in line with the historical experience for the United States, where unfunded spending has played an important role in accounting for inflation dynamics. In fact, the post-pandemic

---

[12]The decomposition of the transfers in 2021Q1 into the funded and unfunded components is shown in Figure 14 in Appendix F.
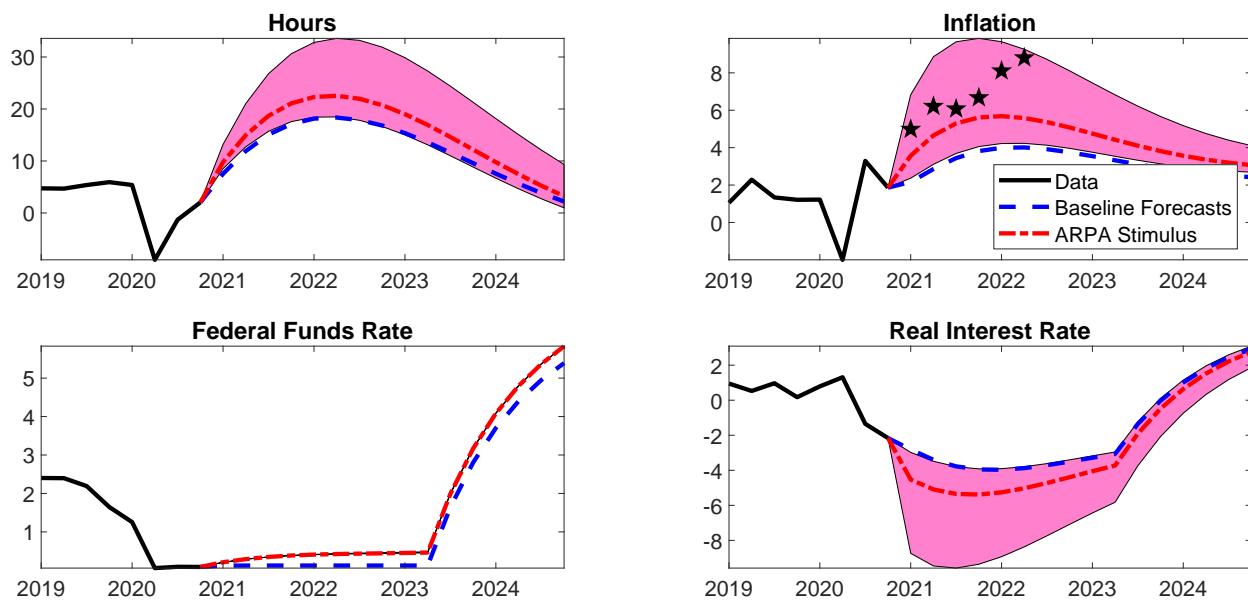
Figure 9: The Macroeconomic Effects of the ARPA Stimulus. Model's forecast on hours worked, inflation, the Federal Funds rate, and the real interest rate, conditional on using filtered data up to 2020Q4 (dashed blue line). The counterfactual scenario is conditional on filtered data up to 2020Q4 and on the federal transfers payment in the first quarter of 2021, which is when most of the ARPA stimulus checks were sent out (dot-dashed red line). Shocks are estimated using the Kalman smoother. The upper bound pf the pink area is obtained by increasing the standard deviation of the unfunded transfers shocks and by decreasing the standard deviation of the funded transfers shocks by a factor of 2. Analogously, the lower bound is obtained by increasing the standard deviation of the funded transfers shocks and by decreasing the standard deviation of the unfunded transfers shocks by a factor of 2. Model parameters are set at their posterior mode.

fiscal interventions might have been necessary to help the recovery and move away from a low interest rate environment that limits the actions of the Federal Reserve. However, as 2022, spending is at an historical maximum. This implies that even small changes in the way debt will be stabilized can lead to large swings in inflation. An implication of these findings is that a clear path for both monetary and fiscal policy is necessary to regain control of inflation in a post-pandemic world.

6 Robustness checks and external validation

In this section, we conduct a series of additional exercises. We fist trim down the model to the textbook NK model of Section 2 to highlight that what matters for the propagation of unfunded shocks in our quantitative model is the presence of nominal rigidities. We then discuss how fiscal trend inflation differs from an exogenous time-varying inflation target. Finally, We provide external validation for our results based on a VAR.
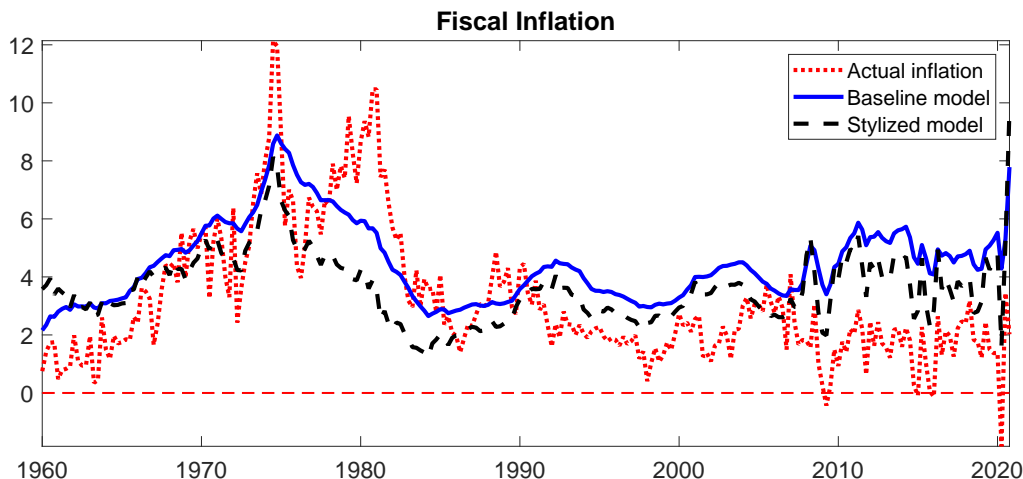
Figure 10: **Fiscal Inflation.** The red dotted line shows the GDP deflator in the data. The blue solid line shows the inflation rate simulated from the baseline New Keynesian DSGE model studied in the paper using unfunded transfer shocks. The dashed black line shows the inflation rate simulated from the stylized New Keynesian model using the same sequence of estimated unfunded transfer shocks. Relative to Figure 5 of the paper we add the sample mean of inflation to the simulated series of fiscal inflation. Figure 5 instead shows fiscal inflation in deviation from the steady state.

6.1 Trend inflation in a textbook model

The reason for estimating the large-scale model of Section 3, rather than a bare-bone New Keynesian model, is that the former features multiple competing mechanisms that can explain trend inflation. Whether our fiscal mechanism plays a dominant role is therefore an empirical question. Indeed, in Figure 6, we see that from the early 1990s, non-policy factors have persistently contributed to keeping inflation down. The model predicts, however, that fiscal inflation has played an important role in dampening these deflationary pressures due to non-policy factors (primarily persistent cost-push shocks and favorable technology shocks). Estimating a simple New Keynesian model, such as the one presented in Section 2.2, would not be appropriate to address the quantitative importance of our mechanism, as it rules out by construction alternative ways to account for movements in inflation. Thus, the additional features of the baseline model mostly serve the purpose to render the propagation of familiar business-cycle shocks consistent with standard general-equilibrium models and with decades of empirical work based on VAR analysis and to avoid ruling out a priory other sources of inflation fluctuations.

However, a drawback of estimating large models, is that it is often hard to assess to what extent the results are driven by all the intricacies of the auxiliary assumptions. The purpose of this section is to show that the fiscal mechanism proposed in this paper is successful in explaining trend inflation even when it is studied using a bare-bone New Keynesian model.

To make this point, we take the stylized model with nominal rigidities of Section 2.2, and calibrate its parameters consistently with the estimated parameters of the large New

36

Keynesian model. It goes without saying that many of the parameters estimated in the large model do not have a counterpart in the bare-bone New Keynesian model, so we do not use these parameters. We then simulate the stylized model using only the shocks to unfunded transfers as estimated using the large New Keynesian model. These are the shocks that are used to quantify fiscal inflation in Figure 6. The simulated series of inflation for the simple model is shown in Figure 10 (the black dashed line) along with the U.S. GDP deflator in the data (the red dotted line). For comparison, we also report fiscal inflation based on the large model (the blue solid line). The important takeaway from this picture is that the inflation simulated from the stylized model explains trend inflation quite successfully.

In addition, if we compare the black dashed line with the blue solid line, which corresponds to fiscal inflation implied by the estimated large New Keynesian model (i.e., the black bars in Figure 5 of the paper), we observe that the simple New Keynesian model endowed with the fiscal mechanism proposed in our paper generates a series of fiscal inflation that is remarkably similar to what the large New Keynesian model predicts. This is despite the fact that the shocks are not re-estimated to account for the differences between the two models. This indicates that what makes the large model successful in accounting for the movements in trend inflation is the presence of nominal rigidities, and not the ancillary frictions that are popular in the New Keynesian literature.

In order to illustrate this result from a different angle, in Appendix G we conduct one more exercise. We estimate a version of the stylized New Keynesian model consisting of the canonical three equations plus a fiscal block, and equipped with a number of shocks that are required to fit the behavior of GDP, inflation, the Federal Fund Rate, transfers, taxes, debt and government expenditures. The key result is that unfunded transfer shocks explain the trend of actual inflation remarkably well, even in the stylized model.

To sum up, we conclude that the many bells and whistles of the estimated New Keynesian model do not play a key role in accounting for our results. Even when a bare-bone New Keynesian model is used in place of the large New Keynesian model, we obtain very similar results. The fiscal theory of trend inflation arises because of the interaction between the fiscal mechanism and nominal rigidities, in line with what explained in Section 2.2.

6.2 Time-varying exogenous target

In Section 2, we discussed that unfunded fiscal shocks generate low frequency movements in inflation that are accommodated by the central bank. When the response to these movements in inflation is restricted to zero, like in our baseline analysis, the central bank's Taylor rule can be rewritten in a way that is isomorphic to having a time-varying inflation target. However, while the Taylor rule appears as such, the movements in inflation have a fiscal origin and need
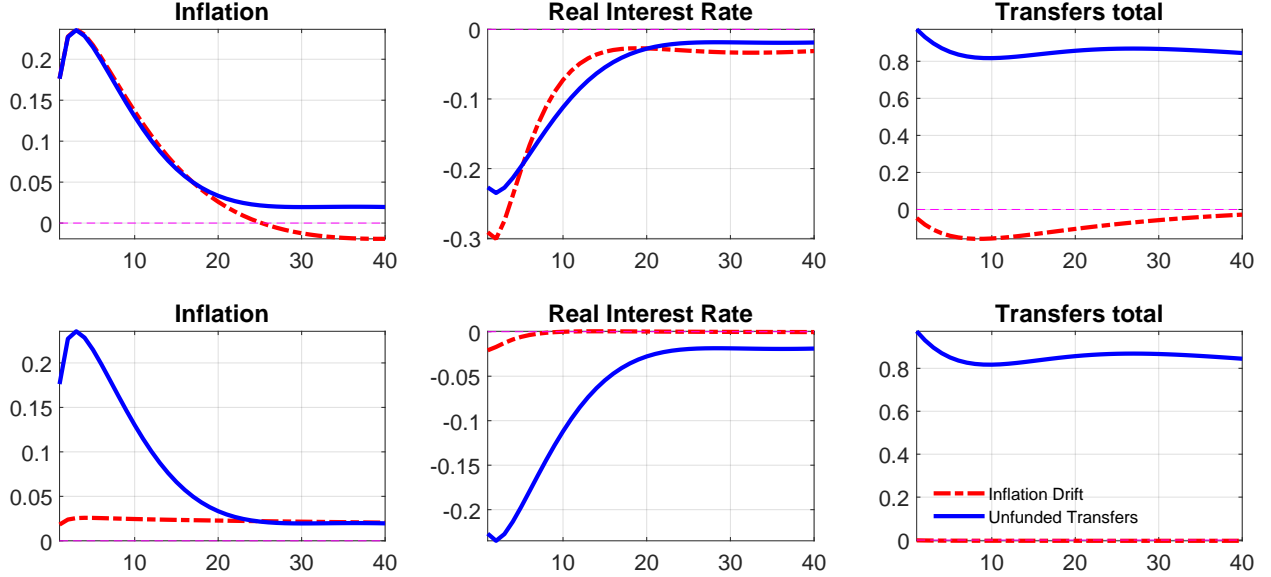
Figure 11: **Propagation of unfunded fiscal shocks vs. exogenous inflation target in the estimated model**. In the first row, the impulse responses of a shock to the exogenous inflation target are obtained by calibrating standard deviation and autocorrelation of that shock to match the short-term impulse response of inflation to an estimated shock to unfunded transfers. In the second row, the impulse responses of a shock to the exogenous inflation target are obtained by calibrating standard deviation and autocorrelation of the shock to match the long-term response (e.g, in the last four quarters) that is generated by an estimated shock to unfunded transfers.

to obey specific cross equation restrictions, leading to our fiscal theory of trend inflation. In this subsection, we further elaborate on this point by showing that our estimated measure of fiscal inflation is *not* observationally equivalent to having an exogenous time-varying inflation target. This is because fiscal inflation and an exogenous inflation target generate different commovement of the variables of interest.

To illustrate this point, we compare the impulse responses to an unfunded fiscal shock to the impulse responses to a shock to the exogenous inflation target. In this second scenario, we modify our baseline model to have an exogenous target that evolves according to an AR(1) process. Specifically, ignoring the effective lower bound constraint, the the Taylor rule becomes:

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) \left[\phi_\pi \left(\hat{\pi}_t - \hat{\pi}_t^{ET} \right) + \phi_y \hat{y}_t \right] + u_t^m. \tag{26}$$

where the exogenous target follows the process:

$$\hat{\pi}_t^{ET} = \rho_{ET} \hat{\pi}_{t-1}^{ET} + u_t^{ET}. \tag{27}$$

Figure 11 presents the results based on two alternative ways to calibrate the exogenous time-varying inflation target. In the first row, we choose the persistence and volatility of the exogenous inflation target in a way to replicate the short-term response of inflation to an unfunded fiscal shock. In the second row, we choose the persistence and size of the exogenous inflation target shock to match the long-term response of inflation (10 years).

A clear trade-off arises when trying to replicate the dynamics of fiscal inflation with an exogenous inflation target shock. If the properties of the shocks are chosen to match the short-term response of inflation (first row), then the model is not able to account for the persistent increase in inflation above the long-term inflation target. Inflation becomes negative after 25 quarters in response to the exogenous target shock, while it remains persistently positive under the unfunded fiscal shock. Furthermore, there is a key difference between the two impulse responses in the behaviour of transfers. In response to an exogenous target shock, transfers go down as the economy is booming. Instead, in the case of fiscal inflation, the rise in inflation and real activity is triggered by the persistent increase in transfers.

If the properties of the exogenous target shock are chosen to match the long-term response of inflation, then the real interest rate and transfers barely move and inflation does not feature the large increase at the time of the shock.[13] This second point is particularly important because it is what allows our model to account for the observed dynamics of inflation following the pandemic. More broadly, this second case shows that fiscal inflation can at the same time generate large and persistent movements in inflation, while the time-varying exogenous target necessarily fails in one of the two dimensions because it does not obey the cross-equation restrictions that identify the unfunded fiscal shocks.

Finally, it is important to underscore that the direction of causality is different. In the case of fiscal trend inflation, the fiscal shock triggers inflationary pressure that the central bank accommodates. In the case of a shock to the exogenous inflation target, the central bank cuts rates to achieve a higher target. To make sense of the post-pandemic inflation, with interest rates already stuck at the zero lower bound, the first mechanism appears much more plausible. The ability of the model to account for the post-pandemic rise of inflation out of sample is an important validation for the mechanism proposed in this paper.

6.3 Model-free evidence of fiscal trend inflation

In this section, we estimate a VAR model using the same observable variables used to estimate the empirical DSGE model (Section 4.1) over the first sample period (1960:Q1 to 2007:Q4).[14] The objective of this section is to provide some "model-free" evidence to support the general mechanism studied in this paper.

Unfunded fiscal shocks are identified by imposing minimal parametric restrictions con-

---

[13]Transfers still respond negatively to the shock in the lower panel of Figure 11.

[14]We do not estimate the model in the subsequent period to avoid issues with the federal funds rate being mired at zero for several years. Since the trend component of inflation is very volatile in the first sample period, our exercise is pertinent. Following Sims and Zha (1998), we adopt a unit root prior for the parameters of the VAR with a pre-sample of four quarters. The prior hyperparameters are chosen so as to maximize the marginal likelihood.
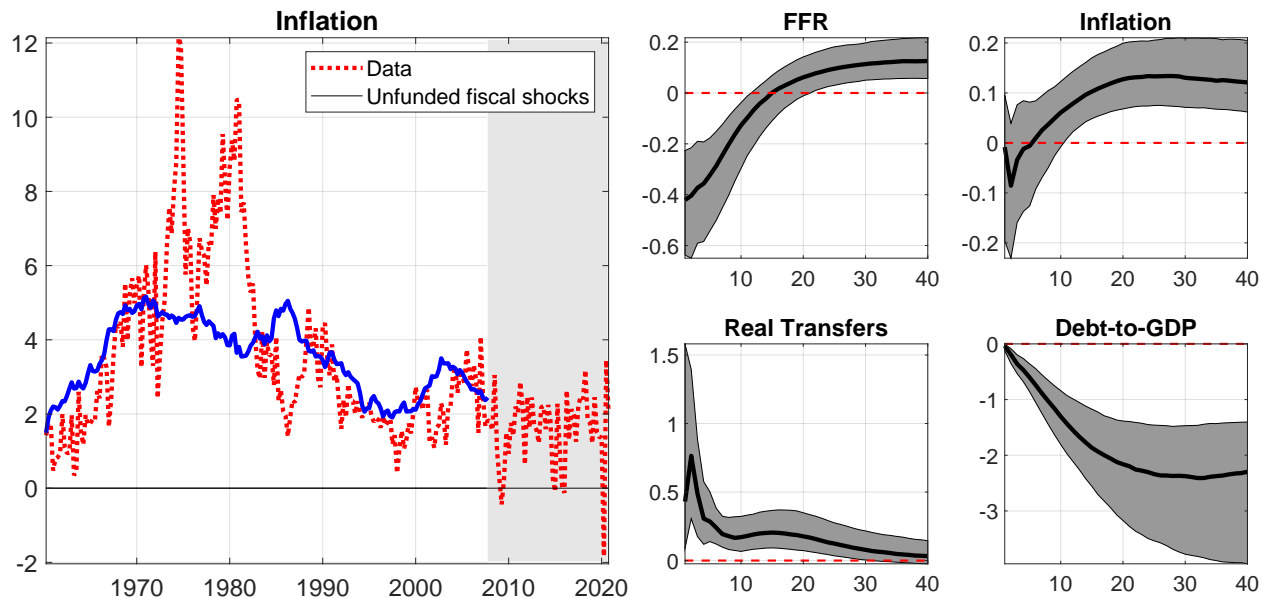
Figure 12: **Propagation of unfunded fiscal shocks in a model-free setting**. Unfunded fiscal shocks are identified by imposing that these shocks raise real transfer growth for two years while the monetary authority refrains from responding by increasing the federal funds rate (FFR) in that period. The debt-to-GDP ratio is expected to fall for five years after the shock. The first figure on the left compares inflation in the data (the red dotted line) with inflation explained by the identified unfunded fiscal shocks in the first sample 1960:Q1-2007:Q4 (the blue solid line). The figures on the right show the response of the FFR, inflation, real transfers growth, and debt-to-GDP ratio to the identified unfunded fiscal shock. The black lines denote the posterior median of these responses and the gray bands their 60 percent posterior interval. The horizontal axes report the quarters after the shock. Inflation and the response of inflation, FFR, and real transfer growth are expressed in percentage points of annualized rates.

cerning the responses of the monetary and fiscal authorities to inflation and the debt-to-GDP ratio. Specifically, we assume that an unfunded fiscal shock raises real transfer growth for two years while the monetary authority refrains from responding by raising the FFR over the same period. The debt-to-GDP ratio is expected to fall for five years after the shock. It should be noted that all business cycle shocks can also affect real government transfers. For instance, a recessionary shock is likely to raise transfers. However, such shock would also increase the debt-to-GDP ratio, which would violate one of our identifying restrictions. An exogenous change to the central bank's inflation target would lower the debt-to-GDP ratio and the FFR. However, as shown in Figure 11, this shock to the central bank's target would also lower transfers, which is at odd with our identifying restrictions. Finally, an expansionary shock to funded transfers would increase the debt-to-GDP ratio and, most likely, the federal funds rate. Both responses are ruled out by our identifying assumptions.[15]

---

[15]One might be concerned about the consistency of the VAR with the estimated structural model since we do not explicitly model the shadow economy in the VAR model. However, the shadow economy is just an accounting exercise that is justified by the assumptions that (i) agents are rational and have full information and (ii) the monetary and fiscal authority always coordinate their policies consistently with fiscal trend inflation $\hat{\pi}_t^F$. We do not make these assumptions in the VAR model since we want to be agnostic as to whether agents are perfectly informed and how policymakers coordinate monetary and fiscal policies.

The VAR is not subject to the theoretical cross-equation restrictions characterizing the micro-founded models considered so far. Thus, this exercise provides further corroborating evidence that our main findings are not driven by auxiliary assumptions embedded in the baseline model. The plot on the left of Figure 12 compares inflation in the data (the red dotted line) with inflation explained by the unfunded fiscal shocks identified in the VAR model (the blue solid line). Consistently with the predictions of our estimated structural model shown in Subsection 5.3, the identified unfunded transfer shocks play an important role in explaining the run-up and the subsequent slow down in trend inflation in the first three decades of the sample. Since we use only minimal restrictions to identify the unfunded fiscal shocks, one should not expect that the historical contribution of this shock to inflation mimics exactly that of the unfunded fiscal shocks in the structural model. For instance, shocks to unfunded transfers may not increase total government transfers. This is the case when a larger funded transfer shock lowers total transfers. To the extent that funded and unfunded transfers shocks are orthogonal, this is not an issue for our sign-restriction identification strategy. However, in a short sample, the estimated shocks may violate the orthogonality condition and, when this is the case, our identification procedure may discard some genuine shocks to unfunded transfers. That said, it is reassuring that unfunded shocks identified with minimal theoretical restrictions play an important role in explaining trend inflation analogously to the unfunded transfers shocks estimated in our microfounded model.

The four figures on the right panel show the response of the FFR, inflation, real transfers growth, and debt-to-GDP ratio to the identified unfunded fiscal shock. Inflation is the unrestricted variable and responds positively and very persistently to the unfunded fiscal shocks identified in the VAR. This pattern is consistent with our fiscal theory of trend inflation. The persistent response of inflation to the identified unfunded transfer shock is consistent with the important contribution of this shock to explaining movements in trend inflation, as shown in the historical decomposition of inflation in the left panel. Our identifying restrictions do not impose that the monetary authority does not respond to fiscal inflation $\hat{\pi}_t^F$, as we do in the microfounded models considered so far. For the fiscal theory to hold true, what is needed is that monetary policy is passive with respect to changes in inflation resulting from unfunded transfer shocks. This condition turns out to be satisfied in the VAR model even though it is not imposed. See the upper plots in the right panel of Figure 12.[16]

---

[16]We also consider a VAR where (i) we replace the series of the nominal FFR with the real FFR, (ii we add the series of the ratio of the primary deficit to debt, and (iii) we add the four-quarter-ahead GDP deflator inflation expectations from the *Survey of Professional Forecasters*. We identify the shock by assuming that the unfunded transfer shock increases government transfers and the primary deficit to debt ratio and lowers the real FFR, the debt-to-GDP ratio for the next 5 years. We show that inflation expectations respond positively and very persistently to this shock. The corresponding figure is available upon request.

7   Conclusions

In this paper, we have developed a novel class of models in which unfunded fiscal shocks coexist with typical business cycle shocks to propose a fiscal theory of trend inflation. Our empirical results show that fiscal trend inflation plays a major role in explaining movements in inflation. As external validation, we use the model to study the post-pandemic events. We show that the current situation is not necessarily different from the past. The model predicts out-of-sample an increase in trend inflation above the 2% inflation target as a result of the partially unfunded fiscal interventions implemented in response to the pandemic. Absent further unfunded fiscal shocks, inflation is expected to slowly revert to its long-term target.

References

Aiyagari, S. R. and M. Gertler (1985). The backing of government bonds and monetarism. *Journal of Monetary Economics 16*(1), 19–44.

Barro, R. (2022). Understanding Recent US Inflation. Technical report, project syndicate.

Barro, R. J. (1974, Nov.-Dec.). Are Government Bonds Net Wealth? *Journal of Political Economy 82*(6), 1095–1117.

Bassetto, M. (2002). A game-theoretic view of the fiscal theory of the price level. *Econometrica 70*(6), 2167–2195.

Bassetto, M. and T. J. Sargent (2020). Shotgun wedding: Fiscal and monetary policy. *Annual Review of Economics 12*(1), 659–690.

Benhabib, J., S. Schmitt-Grohe, and M. Uribe (2002). Avoiding Liquidity Traps. *Journal of Political Economy 110*(3), 535–563.

Bianchi, F. (2013). Regime switches, agents' beliefs, and post-world war ii us macroeconomic dynamics. *The Review of Economic Studies*, rds032.

Bianchi, F. and C. Ilut (2017). Monetary/Fiscal Policy Mix and Agents' Beliefs. *Review of Economic Dynamics 26*, 113–139.

Bianchi, F., M. Lettau, and S. C. Ludvigson (2021). Monetary policy and asset valuation. *Journal of Finance, forthcoming*.

Bianchi, F. and L. Melosi (2017). Escaping the Great Recession. *American Economic Review 107*(4), 1030 – 58.

Bianchi, F. and L. Melosi (2019). The dire effects of the lack of monetary and fiscal coordination. *Journal of Monetary Economics 104*, 1–22.

Bianchi, F. and L. Melosi (2022, 08). Inflation as a Fiscal Limit. Working paper series, Federal Reserve Bank of Chicago.

Bianchi, F., L. Melosi, and M. Rottner (2019). Hitting the Elusive Inflation Target. NBER working paper.

Blanchard, O. (2021). In defense of concerns over the $1.9 trillion relief plan. PIIE opinion, 18 February 2021.

Campbell, J. R., C. L. Evans, J. D. Fisher, and A. Justiniano (2012). Macroeconomic Effects of Federal Reserve Forward Guidance. *Brookings Papers on Economic Activity 43*(1 (Spring), 1–80.

Campbell, J. R., J. D. M. Fisher, A. Justiniano, and L. Melosi (2017). Forward guidance and macroeconomic outcomes since the financial crisis. *NBER Macroeconomics Annual 31*(1), 283–357.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal Rigidities and the

Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy 113*(1), 1–45.

Clarida, R., J. Galí, and M. Gertler (2000). Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. *Quarterly Journal of Economics 115*, 147–180.

Cochrane, J. H. (1998). A Frictionless Model of U.S. Inflation. In B. S. Bernanke and J. J. Rotemberg (Eds.), *NBER Macroeconomics Annual 1998*, pp. 323–384. Cambridge, MA: MIT Press.

Cochrane, J. H. (2001). Long Term Debt and Optimal Policy in the Fiscal Theory of the Price Level. *Econometrica 69*, 69–116.

Cochrane, J. H. (2022). A fiscal theory of monetary policy with partially-repaid long-term debt. *Review of Economic Dynamics 45*, 1–21.

Congressional Budget Office (2020). Federal debt: a primer. (CBO Publication No. 56165). Washington, DC: Congressional Budget Office.

Del Negro, M. and F. Schorfheide (2008). Forming Priors for DSGE Models (And How it Affects the Assessment of Nominal Rigidities). *Journal of Monetary Economics 55*(7), 1191–1208.

Fernandez-Villaverde, J. and J. F. Rubio-Ramirez (2007a). Estimating Macroeconomic Models: A Likelihood Approach. *Review of Economic Studies 74*(4), 1059–1087.

Fernandez-Villaverde, J. and J. F. Rubio-Ramirez (2007b). How Structural are Structural Parameters? *2007 NBER Macroeconomics Annual*.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.

Hall, G. J. and T. J. Sargent (2011). Interest Rate Risk and Other Determinants of Post-WWII U.S. Government Debt/GDP Dynamics. *American Economic Journal: Macroeconomics 3*(3), 192–214.

Hazell, J., J. Herreno, E. Nakamura, and J. Steinsson (2020). The slope of the phillips curve: evidence from us states. Technical report, National Bureau of Economic Research.

Kaplan, G., G. Violante, and J. Weidner (2014). The Wealthy Hand-to-Mouth. *Brookings Papers on Economic Activity 44*(1), 77–138.

Leeper, E. M. (1991). Equilibria Under Active and Passive Monetary and Fiscal Policies. *Journal of Monetary Economics 27*, 129–147.

Leeper, E. M., N. Traum, and T. B. Walker (2017, August). Clearing Up the Fiscal Multiplier Morass. *American Economic Review 107*(8), 2409–2454.

Leeper, E. M. and X. Zhou (2013). Inflation's role in optimal monetary-fiscal policy. *NBER Working Paper*.

Lubik, T. and F. Schorfheide (2004). Testing for Indeterminacy: An Application to U.S.

Monetary Policy. *American Economic Review 94*(1), 190–217.

Reis, R. (2016). QE in the future: the central bank's balance sheet in a financial crisis. Discussion Papers 1620, Centre for Macroeconomics (CFM).

Sargent, T. and N. Wallace (1981). Some Unpleasant Monetarist Arithmetic. *Federal Reserve Bank of Minneapolis Quarterly Review Fall*, 1–17.

Schmitt-Grohe, S. and M. Uribe (2000). Price level determinacy and monetary policy under a balanced-budget requirement. *Journal of Monetary Economics 45*(1), 211–246.

Sims, C. A. (1994). A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy. *Economic Theory 4*, 381–399.

Sims, C. A. (2016). Fiscal policy, monetary policy and central bank independence. In *Kansas City Fed Jackson Hole Conference*.

Sims, C. A. and T. Zha (1998). Bayesian Methods for Dynamic Multivariate Models. *International Economic Review 39*(4), 949–968.

Sims, C. A. and T. Zha (2006). Were There Regime Switches in US Monetary Policy? *American Economic Review 91*(1), 54–81.

Smets, F. and R. Wouters (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *The American Economic Review 97*(3), 586–606.

Summers, L. H. (2021). Opinion: The inflation risk is real. Washington Post, 24 May 2021.

Woodford, M. (1994). Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy. *Economic Theory 4*, 345–389.

Woodford, M. (1995). Price Level Determinacy without Control of a Monetary Aggregate. *Carnegie-Rochester Series of Public Policy 43*, 1–46.

Woodford, M. (2001). Fiscal Requirements of Price Stability. *Journal of Money, Credit, and Banking 33*, 669–728.

Woodford, M. (2003). *Interest and prices: Foundations of a theory of monetary policy*. Princeton, New Jersey: Princeton University Press.

A   Solving Economies with Partially Unfunded Debt

To prove that the system of equations (11), (12), (13), (14), are the correct policy functions of the model with partially unfunded debt, we have to show that the two following claims are true. First, the difference between the overall stock of debt and its unfunded share is funded, that is, $\hat{b}_t - \hat{b}_t^F = \hat{b}_t^M$. Second, the inflation rate the central bank strives to stabilize with active monetary policy in the actual economy is precisely the actual rate of inflation net of the inflation needed to stabilize the unfunded debt (i.e., $\pi_t^M = \pi_t - \pi_t^F$).

   Both claims can be proved by constructing yet another parallel economy to pin down (i) the share of inflation the monetary authority has to control with active monetary policy, π_t^M, and (ii) the share of funded debt, $\hat{b}_t^M$, which is the share of debt the fiscal authority is responsible to repay by raising future surpluses. This parallel economy is as follows:

$$\mathbb{E}_t \hat{\pi}_{t+1}^M = \hat{r}_t^M, \tag{28}$$

$$\hat{b}_t^M = \beta^{-1}\hat{b}_{t-1}^M - \hat{\tau}_t^M - b\beta^{-1}\pi_t^M + b_{\hat{r}_t^M}, \tag{29}$$

$$\hat{r}_t^M = \phi^M \hat{\pi}_t^M, \tag{30}$$

$$\hat{\tau}_t^M = \gamma^M \hat{b}_t^M + \epsilon_t^M. \tag{31}$$

In this parallel economy, all fiscal shocks are funded, ϵ_t^M, and the policy mix is monetary led ($\phi_M > 1$ and $\gamma > \beta^{-1} - 1$). The monetary and fiscal blocks are obtained as done for the other economies we studied in the main text.

$$\mathbb{E}_t \hat{\pi}_{t+1}^M = \phi^M \hat{\pi}_t^M, \tag{32}$$

$$\hat{b}_t^M = \left(\beta^{-1} - \gamma^M\right)\hat{b}_{t-1}^M - b(\beta^{-1} - \phi^M)\hat{\pi}_t^M - \epsilon_t^M. \tag{33}$$

To prove the first claim, we need to show that $\hat{\pi}_t^M = \hat{\pi}_t - \hat{\pi}_t^F$. If the equation is rolled one period forward and we apply the expectation operator on both sides, we obtain

$$\mathbb{E}_t \hat{\pi}_{t+1}^M = \mathbb{E}_t \hat{\pi}_{t+1} - \mathbb{E}_t \hat{\pi}_{t+1}^F. \tag{34}$$

   The right-hand side can be pinned down using the monetary block of the actual economy (11) and that in the parallel economy in the main text – equation (13). After making these substitutions we obtain:

$$\mathbb{E}_t \hat{\pi}_{t+1}^M = \phi^M \left(\hat{\pi}_t - \hat{\pi}_t^F\right). \tag{35}$$

Substituting the monetary block of the parallel economy in equation (32) to replace the expectations on the left hand side proves the second claim.

   Let us now turn to the second claim. This claim requires us to show that $\hat{b}_t^M = \hat{b}_t - \hat{b}_t^F$. Substituting the fiscal rule in equation (9) (with $\gamma_F = 0$) and the monetary rule in equation
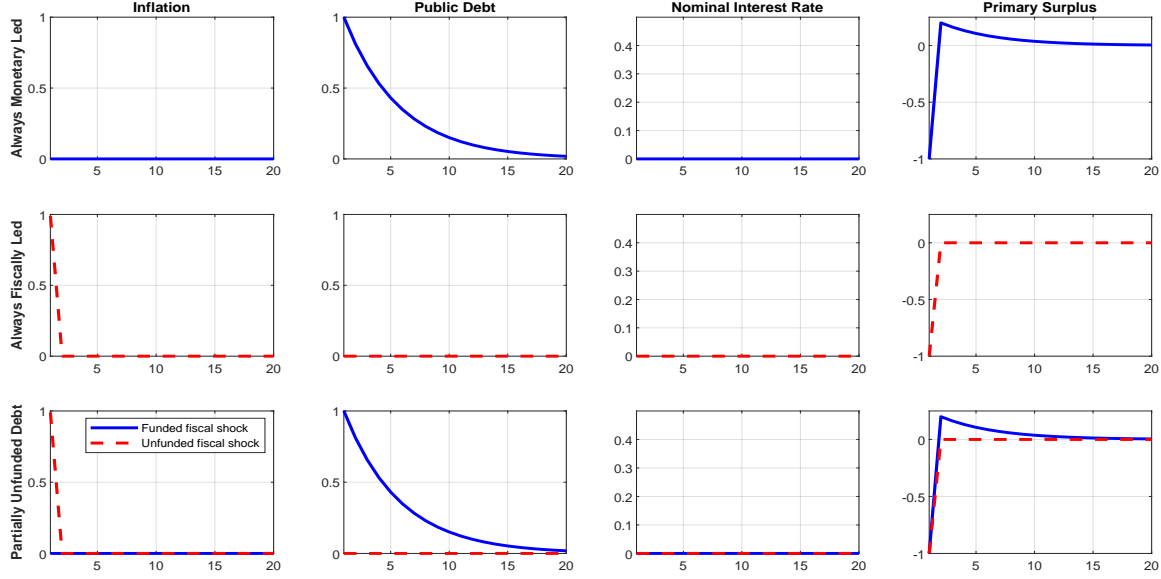
1

Figure 13: Response of Model Variables to a Fiscal Shock.

(10) into the law of motion of debt in equation (2), yields:[17]

$$\hat{b}_t = \left(\beta^{-1} - \gamma^M\right)\hat{b}_{t-1} + \gamma^M \hat{b}_{t-1}^F - b\left(\beta^{-1} - \phi^M\right)\hat{\pi}_t - b\left(\phi^M - \phi^F\right)\hat{\pi}_t^F - \epsilon_t^M - \epsilon_t^F. \tag{36}$$

We plug the second claim, $\hat{\pi}_t = \hat{\pi}_t^M + \hat{\pi}_t^F$, into the above equation to obtain

$$\hat{b}_t = \left(\beta^{-1} - \gamma^M\right)\hat{b}_{t-1} + \gamma^M \hat{b}_{t-1}^F - b\left(\beta^{-1} - \phi^M\right)\hat{\pi}_t^M - b\left(\beta^{-1} - \phi^F\right)\hat{\pi}_t^F - \epsilon_t^M - \epsilon_t^F. \tag{37}$$

We now subtract $\hat{b}_t^F$ from both sides of this equation and use equation (14) that defines the share of unfunded debt in equilibrium to get

$$\hat{b}_t - \hat{b}_t^F = \left(\beta^{-1} - \gamma^M\right)\left(\hat{b}_{t-1} - \hat{b}_{t-1}^F\right) - b\left(\beta^{-1} - \phi^M\right)\hat{\pi}_t^M - \epsilon_t^M. \tag{38}$$

We then subtract $\hat{b}_t^M$ from both sides of this equation and use its definition from the fiscal block of this parallel economy – equation (33) to get

$$\hat{b}_t - \hat{b}_t^F - \hat{b}_t^M = \left(\beta^{-1} - \gamma^M\right)\left(\hat{b}_{t-1} - \hat{b}_{t-1}^F - \hat{b}_{t-1}^M\right). \tag{39}$$

The only way to satisfy the equation above is when the first claim $\hat{b}_t^M = \hat{b}_t - \hat{b}_t^F$ is satisfied in every period, which is precisely the first claim.

Finally, Figure 13 shows the complete plot of the response of all the variables of the toy model to funded and unfunded fiscal shocks.

---

[17]The fiscal rule could be equivalently expressed as $\tau_t/\tau = \left(b_{t-1}/b_{t-1}^M\right)^{\gamma^F}\left(b_{t-1}^M/b\right)^{\gamma^M}e^{\epsilon_t^M + \epsilon_t^F}$.

2

B Production Economies: Linearized Equations

B.1 Flexible-price model

Euler equation

$$E_t \hat{y}_{t+1} = \hat{y}_t + \hat{r}_t \tag{40}$$

Labor supply

$$\frac{n}{1-n}\hat{n}_t = \hat{y}_t + \widehat{w}_t^r \tag{41}$$

Labor demand

$$\hat{w}_t^r = -\alpha \hat{n}_t \tag{42}$$

Production function

$$\hat{y}_t = (1-\alpha)\hat{n}_t \tag{43}$$

Equations (40) to (43) denote an autonomous system that solves for the real block of the economy, $\hat{y}_t$, $\hat{n}_t$, $\hat{w}_t^r$, $\hat{r}_t$. However, inflation is not determined. We introduce the behavior of the monetary and fiscal authorithies to pin down the path of inflation.

Real rate definition

$$\hat{r}_t = \hat{R}_t - E_t \hat{\pi}_{t+1} \tag{44}$$

Taylor rule

$$\hat{R}_t = \phi_\pi^M \hat{\pi}_t + \phi_\pi^F (\hat{\pi}_t - \hat{\pi}_t^*) \tag{45}$$

Evolution of debt

$$b\hat{b}_t = -\tau \hat{\tau}_t + \beta^{-1} b \left(\hat{y}_{t-1} - \hat{y}_t + \hat{R}_{t-1} - \hat{\pi}_t + \hat{b}_{t-1} \right) \tag{46}$$

Fiscal rule

$$\hat{\tau}_t = \delta^M \hat{b}_{t-1}^* + \delta^F (\hat{b}_{t-1} - \hat{b}_{t-1}^*) + \varepsilon_t^F + \varepsilon_t^U \tag{47}$$

The system of equations above is supplemented with a block of equations that characterize the shadow economy. This block consists in an additional set of equations (40) to (47), where any variable that refers to the actual economy x_t is replaced by the same variable in the shadow economy x_t^*.

B.2 New Keynesian model

Euler equation

$$E_t \hat{y}_{t+1} = \hat{y}_t + \hat{r}_t \tag{48}$$

3

Labor supply

$$\frac{n}{1-n}\hat{n}_t = \hat{y}_t + \widehat{w}_t^r \tag{49}$$

New Keynesian Phillips Curve

$$\hat{\pi}_t = \kappa \hat{w}_t^r + \beta E_t \hat{\pi}_{t+1} \tag{50}$$

Production function

$$\hat{y}_t = (1-\alpha)\hat{n}_t \tag{51}$$

Real rate definition

$$\hat{r}_t = \hat{R}_t - E_t \hat{\pi}_{t+1} \tag{52}$$

Taylor rule

$$\hat{R}_t = \phi_\pi^M \hat{\pi}_t + \phi_\pi^F (\hat{\pi}_t - \hat{\pi}_t^*) \tag{53}$$

Evolution of debt

$$b\hat{b}_t = -\tau \hat{\tau}_t + \beta^{-1}b \left(\hat{y}_{t-1} - \hat{y}_t + \hat{R}_{t-1} - \hat{\pi}_t + \hat{b}_{t-1} \right) \tag{54}$$

Fiscal rule

$$\hat{\tau}_t = \delta^M \hat{b}_{t-1}^* + \delta^F (\hat{b}_{t-1} - \hat{b}_{t-1}^*) + \varepsilon_t^F + \varepsilon_t^U \tag{55}$$

The block of equations that characterize the shadow economy consists in an additional set of equations (48) to (55), where any variable that refers to the actual economy x_t is replaced by the same variable in the shadow economy x_t^*.

Plugging eq.(52) into (48), and combining eq.(49) and (51) to substitute for $\hat{w}_t$ into eq.(50), it is possible to express the New-Keynesian block (eq.48) to (53)) as three equations:

$$E_t \hat{y}_{t+1} = \hat{y}_t + \hat{R}_t - E_t \hat{\pi}_{t+1}, \tag{56}$$

$$\hat{\pi}_t = \tilde{\kappa} \hat{y}_t + \beta E_t \hat{\pi}_{t+1}, \tag{57}$$

where $\tilde{\kappa} = \left(\frac{\eta}{1-\alpha} - 1 \right) \kappa$ and $\eta = \frac{n}{1-n}$, and

$$\hat{R}_t = \phi_\pi^M \hat{\pi}_t + \phi_\pi^F (\hat{\pi}_t - \hat{\pi}_t^*) + \phi_y \hat{y}_t. \tag{58}$$

B.3   Calibration

The parameter values of the model are calibrated to the estimated model and reported in Table 3. See the posterior mode of Table 1 and Table 2 in the paper.

Parameter	Value	Description
Preferences, technology and price frictions		
β	0.99	Discount factor
α	0.33	Elasticity production fn
N	0.4	Average hours worked
Π	1	Gross steady-state inflation
b	2.17	debt to output ratio
κ	0.02	Slope Phillips curve
Monetary and fiscal authorities		
ϕ_π^M	2	Taylor rule response to regular inflation (π^*)
δ_b^M	1.5	Debt response with surplus (b^*)
ϕ_π^F	0	Taylor rule response to unfunded inflation ($\pi - \pi^*$)
δ_b^F	0	Debt response with surplus ($b - b^*$)
Shocks		
ρ_t	0.5	Autocorrel. fiscal shock
STD_τ	1	St. dev. fiscal shock

Table 3: Calibrated parameter values in the simple model with price rigidities

C   The Log-Linearized Model

This model features a trend in the state of labor-augmenting technological progress. In order to make the model stationary, we define the following variables: $y_t = \frac{Y_y}{A_t}$, $c_t^{*S} = \frac{C_t^{*S}}{A_t}$, $c_t^{S} = \frac{C_t^S}{A_t}$, $c_t^N = \frac{C_t^N}{A_t}$, $k_t = \frac{K_t}{A_t}$, $g_t = \frac{G_t}{A_t}$, $z_t = \frac{Z_t}{A_t}$, $b_t = \frac{P_t^B B_t}{P_t A_t}$, $w_t = \frac{W_t}{P_t A_t}$, and $\lambda_t^S = \Lambda_t^S A_t$. We list below the equations of the log-linear model, starting with those that characterize the actual-economy block.

Production function:
$$\hat{y}_t = \frac{y + \Omega}{y} \left[\alpha \hat{k}_t + (1 - \alpha) \hat{L}_t \right]. \tag{59}$$

Capital-labor ratio:
$$\hat{r}_t^K - \hat{w}_t = \hat{L}_t - \hat{k}_t. \tag{60}$$

Marginal cost:
$$\widehat{mc_t} = \alpha \hat{r}_t^k + (1 - \alpha) \hat{w}_t. \tag{61}$$

Phillips curve:
$$\hat{\pi}_t = \frac{\beta}{1 + \chi_p \beta} \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\chi_p}{1 + \chi_p \beta} \hat{\pi}_{t-1} + \kappa_p \widehat{mc_t} + \kappa_p \hat{\eta}_t^p + \kappa_p \hat{u}_t^{NKPC}, \tag{62}$$
where $\kappa_p = \left[(1 - \beta \omega_p)(1 - \omega_p) \right] / \left[\omega_p (1 + \beta \chi_p) \right]$.

Saver household's FOC for consumption:

$$\hat{\lambda}_t^S = \hat{u}_t^b - \frac{\theta}{e^\gamma - \theta}\hat{u}_t^a - \frac{e^\gamma}{e^\gamma - \theta}c_t^{*S} + \frac{\theta}{e^\gamma - \theta}c_{t-1}^{*S} - \frac{\tau^C}{1 + \tau^C}\hat{\tau}_t^C, \tag{63}$$

where $\hat{u}_t^a = u_t^a - \gamma$.

Public/private consumption in utility:

$$\hat{c}_t^* = \frac{c^S}{c^S + \alpha_G g}\hat{c}_t^S + \frac{\alpha_G g}{c^S + \alpha_G g}\hat{g}_t. \tag{64}$$

Euler equation:

$$\hat{\lambda}_t^S = \hat{R}_t + \mathbb{E}_t\hat{\lambda}_{t+1}^S - \mathbb{E}_t\hat{\pi}_{t+1} - \mathbb{E}_t\hat{u}_{t+1}^a + \hat{u}_t^{rp}. \tag{65}$$

Maturity structure of debt:

$$\hat{R}_t + \hat{P}_t^B = \frac{\rho}{R}\mathbb{E}_t\hat{P}_{t+1}^B + \hat{u}_t^{rp}. \tag{66}$$

Saver household's FOC for capacity utilization:

$$r_t^k - \frac{\tau^K}{1 - \tau^K}\hat{\tau}_t^K = \frac{\psi}{1 - \psi}\hat{\nu}_t. \tag{67}$$

Saver household's FOC for capital:

$$\hat{q}_t = \mathbb{E}_t\hat{\pi}_{t+1} - \hat{R}_t + \beta e^{-\gamma}\left(1 - \tau^K\right)r^k\mathbb{E}_t\hat{r}_{t+1}^k - \beta e^{-\gamma}\tau^K r^k\mathbb{E}_t\hat{\tau}_{t+1}^K + \beta e^{-\gamma}\left(1 - \delta\right)\mathbb{E}_t\hat{q}_{t+1} - \hat{u}_t^{rp}. \tag{68}$$

Saver household's FOC for investment:

$$\hat{\imath}_t + \frac{1}{1 + \beta}\hat{u}_t^a - \frac{1}{(1 + \beta)\,se^{2\gamma}}\hat{q}_t - \hat{u}_t^i - \frac{\beta}{1 + \beta}\mathbb{E}_t\hat{\imath}_{t+1} - \frac{\beta}{1 + \beta}\mathbb{E}_t\hat{u}_{t+1}^a = \frac{1}{1 + \beta}\hat{\imath}_{t-1}. \tag{69}$$

Effective capital:

$$\hat{k}_t = \hat{\nu}_t + \widehat{\overline{k}}_{t-1} - \hat{u}_t^a. \tag{70}$$

Law of motion for capital:

$$\widehat{\overline{k}}_t = (1 - \delta)\,e^{-\gamma}\left(\widehat{\overline{k}}_{t-1} - \hat{u}_t^a\right) + \left[1 - (1 - \delta)\,e^{-\gamma}\right]\left[(1 + \beta)\,se^{2\gamma} + \hat{\imath}_t\right]. \tag{71}$$

Hand-to-mouth household's budget constraint:

$$\tau^C c^N\hat{\tau}_t^C + \left(1 + \tau^C\right)c^N\hat{c}_t^N = \left(1 - \tau^L\right)wL\left(\hat{w}_t + \hat{L}_t\right) - \tau^L wL\hat{\tau}_t^L + z\hat{z}_t. \tag{72}$$

Wage equation:

$$\hat{w}_t = \frac{1}{1 + \beta}\hat{w}_{t-1} + \frac{\beta}{1 + \beta}\mathbb{E}_t\hat{w}_{t+1} - \kappa_w\left[\hat{w}_t - \xi\hat{L}_t + \hat{\lambda}_t^S - \frac{\tau^L}{1 - \tau^L}\hat{\tau}_t^L\right]$$
$$+ \frac{\chi^w}{1 + \beta}\hat{\pi}_{t-1} - \frac{1 + \beta\chi^w}{1 + \beta}\hat{\pi}_t + \frac{\beta}{1 + \beta}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\chi}{1 + \beta}\hat{u}_{t-1}^a - \frac{1 + \beta\chi - \rho_a\beta}{1 + \beta}\hat{u}_t^a + \kappa_w\hat{\eta}_t^w \tag{73}$$

where $\kappa_w \equiv \left[(1 - \beta\omega_w)(1 - \omega_w)\right]/\left[\omega_w(1 + \beta)\left(1 + \frac{(1+\eta^w)\xi}{\eta^w}\right)\right]$.

Aggregate households' consumption

$$c\hat{c}_t = c^S\left(1 - \mu\right)\hat{c}_t^S + c^N\mu\hat{c}_t^N. \tag{74}$$

Aggregate resource constraint:

$$y\hat{y}_t = c\hat{c}_t + i\hat{u}_t + g\hat{g}_t + \psi'(1)k\hat{\nu}_t. \tag{75}$$

Government budget constraint:

$$\frac{b}{y}\hat{b}_t + \tau^K r^k \frac{k}{y}\left[\hat{\tau}_t^K + \hat{r}_t^k + \hat{k}_t\right] + \tau^L w\frac{L}{y}\left[\hat{\tau}_t^L + \hat{w}_t + \hat{L}_t\right] + \tau^C \frac{c}{y}\left(\hat{\tau}_t^C + \hat{c}_t\right)$$

$$= \frac{1}{\beta}\frac{b}{y}\left[\hat{b}_{t-1} - \hat{\pi}_t - \hat{P}_{t-1}^B - \hat{u}_t^a\right] + \frac{b}{y}\frac{\rho}{e^\gamma}\hat{P}_t^B + \frac{g}{y}\hat{g}_t + \frac{z}{y}\hat{z}_t. \tag{76}$$

Fiscal Rules

$$\hat{g}_t = \rho_G\hat{g}_{t-1} - (1-\rho_G)\gamma_G\hat{s}_{b,t-1}^M + \zeta_{g,t} \tag{77}$$

$$\hat{z}_t = \phi_{zy}\hat{y}_t + \rho_Z\hat{z}_{t-1} - (1-\rho_Z)\gamma_Z\hat{s}_{b,t-1}^M + \zeta_{z,t}^M + \zeta_{z,t}^F \tag{78}$$

$$\hat{\tau}_t^L = \rho_L\hat{\tau}_{t-1}^L + (1-\rho_L)\gamma_L\hat{s}_{b,t-1}^M + \zeta_{\tau_L,t} \tag{79}$$

$$\hat{\tau}_t^K = \rho_K\hat{\tau}_{t-1}^K + (1-\rho_K)\gamma_K\hat{b}_{b,t-1}^M + \zeta_{\tau_K,t} \tag{80}$$

Monetary Rule:

$$\hat{R}_t = \max\left(-\ln R, \rho_r\hat{R}_{t-1} + (1-\rho_r)\left[\phi_\pi\hat{\pi}_t^M + \phi_y\hat{y}_t\right] + \epsilon_{R,t}\right) \tag{81}$$

The variables with the superscript M in equations (77) to (81) above belong to the shadow economy. In turn, the block of equations that characterize the shadow economy consists in an additional set of equations (59) to (76), where any variable that refers to the actual economy x_t is replaced by the same variable in the shadow economy x_t^M, plus the rule for the monetary authority

$$\hat{R}_t^M = \max\left(-\ln R, \rho_r\hat{R}_{t-1}^M + (1-\rho_r)\left[\phi_\pi\hat{\pi}_t^M + \phi_y\hat{y}_t^M\right] + \epsilon_{R,t}\right) \tag{82}$$

and the rules for the fiscal authority,

$$\hat{g}_t^M = \rho_G\hat{g}_{t-1}^M - (1-\rho_G)\gamma_G\tilde{b}_{t-1}^M + \zeta_{g,t} \tag{83}$$

$$\hat{z}_t^M = \phi_{zy}\hat{y}_t^M + \rho_Z\hat{z}_{t-1}^M - (1-\rho_Z)\gamma_Z\tilde{b}_{t-1}^M + \zeta_{z,t}^M \tag{84}$$

$$\hat{\tau}_t^{LM} = \rho_L\hat{\tau}_{t-1}^{LM} + (1-\rho_L)\gamma_L\tilde{b}_{t-1}^M + \zeta_{\tau_L,t} \tag{85}$$

$$\hat{\tau}_t^{KM} = \rho_K\hat{\tau}_{t-1}^{KM} + (1-\rho_K)\gamma_K\tilde{b}_{t-1}^M + \zeta_{\tau_K,t}. \tag{86}$$

D   The Dataset

Real GDP growth is computed as the growth rate of nominal GDP (GDP), divided by the GDP deflator (JGDP). Real consumption growth is the growth rate of the sum of personal consumption expenditures in non durable goods (PCND) and services (PCESV), divided by their price indexes (DNDGRG3M086SBEA and DSERRG3M086SBEA, respectively). Real

investment growth is the growth rate of the sum of gross private domestic investment (GP-DICTPI) and personal consumption expenditures in durable goods (PCDG), divided by the respective price deflators (GPDICTPI and DDURRG3M086SBEA), and scaled by the 16+ US civilian population (CNP16OV). We construct a measure of hours per capita by dividing total hours worked (PRS85006023) by population (CNP16OV). We then construct a measure of the hours gap by taking the difference of hours per capita from its trend, which is computed as a fourth degree polynomial. We compute a measure of hourly wages dividing wage compensation (A576RC1) by average weekly hours in the nonfarm business sector (PRS85006023). Based on this series, we create a nominal wage index, which we divide by an index of the GDP deflator (based on JGDP) and take growth rates. The debt to GDP ratio is constructed dividing the nominal market value of gross federal debt (MVGFD027MNFRBDAL) by nominal GDP (GDP). The growth of government consumption and investment expenditures is computed as follows. We add nominal federal government consumption expenditures (A957RC1Q027SBEA) to nominal gross government investment (A787RC1Q027SBEA), divide by the implicit price deflator (A822RD3Q086SBEA) and by an index of the U.S. population, with base 2012Q3 (CNP16OV) and finally take growth rates. The growth of real government transfers is computed as follows. We add government social benefits (B087RC1Q027SBEA) to other current transfer payments, which include grants-in-aid to state and local governments (FGSL), create an index with base 2012Q3, divide by an index of the U.S. population (CNP16OV) and an index of the GDP deflator (GDPDEF) with the same base year and finally take growth rates. Finally, inflation is computed as the rate of growth of the GDP deflator (JGDP) and the interest rate is given the Effective Federal Funds Rate (FEDFUNDS).

E Second Sample Estimates

		Posterior	Prior Distribution		
Priors and Posteriors Distributions: Second sample					
Param	Description	Mode	Type	Mean	Std
σ_G	St.dev. gov. cons.	3.2021	IG	0.500	0.200
σ_Z^F	St.dev. funded transfers	4.9982	IG	0.500	0.200
σ_Z^U	St.dev. unfunded transfers	1.0214	IG	0.100	0.050
σ_a	St.dev. technology	3.7944	IG	0.500	0.200
σ_b	St.dev. preference	4.9975	IG	0.250	0.200
σ_m	St.dev. mon. policy	0.1242	IG	0.500	0.200
σ_i	St.dev. investment	2.5281	IG	0.500	0.200
σ_w	St.dev. wage markup	0.6567	IG	0.500	0.200
σ_p	St.dev. price markup	0.1630	IG	0.500	0.200
σ_{rp}	St.dev. risk premium	2.8727	IG	0.500	0.200
$\sigma_{\pi^{NKPC}}$	St.dev. infl. drift	4.9939	IG	0.100	0.050
σ_{GDP}^m	Measur. error GDP	1.7952	IG	0.500	0.200
σ_{by}^m	Measur. error Debt/GDP	4.9963	IG	0.500	0.200

Table 4: Posterior modes, medians, 90% posterior credible sets, and prior moments for the structural parameters. The letters in the column with the heading "Prior Type" indicate the prior density function: N, G, and B stand for Normal, Gamma, and Beta, respectively.

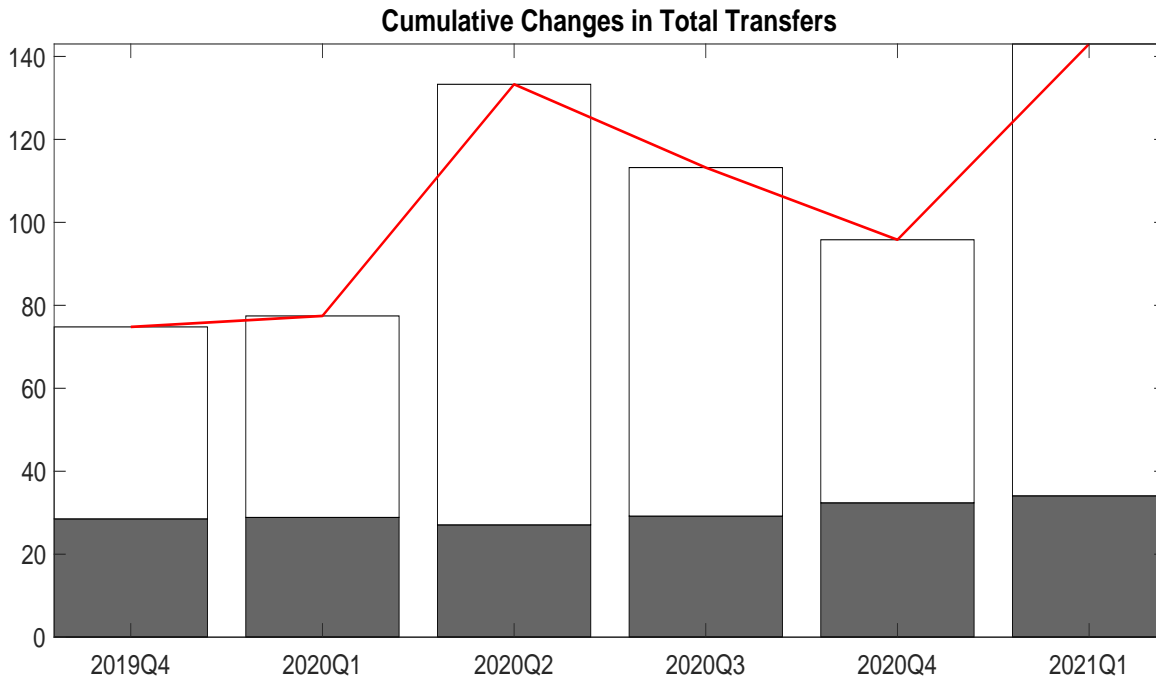# F ARPA Stimulus Decomposition: Funded vs. Unfunded Transfers

Figure 14: stimated Decomposition of U.S. Government Transfers (red solid line) into their Funded and Unfunded Cumulative Components. These components isolate the cumulative effects of the historical realizations of funded or unfunded shocks on transfers in percentage points. These realizations are estimated using the smoother Kalman filter. Parameters are set at their posterior mode.

G Estimation of the Small-Scale Model

We estimate a version of the simple New Keynesian model presented in Section (3) and augmented to include shocks to the growth rate of TFP, preferences, markups, taxes, government spending, funded and unfunded shock to long-term transfers, a shock to short term transfers a monetary shock. We include the following observable variables in estimation: real GDP growth, inflation as measured by the GDP deflator, the Federal Funds rate, the debt-to-GDP ratio, government purchases, and the growth rate of taxes and transfers. All fiscal variables are expressed as a fraction of GDP. For the construction of these series and their sources we refer to Section (D). We follow the same two-sample approach to estimation explained in Section (4.1). Specifically, over the post-2008 period we add the data for the path of the Federal funds rates and a full array of forward guidance shocks.

The model is a stripped down version of the model in Bianchi and Melosi (2022), under the restrictions that there are no different regimes, no habits, and no long term bonds. We refer to that paper for further details. This model consists of a standard three-equation New Keynesian block and a fiscal block that allows for unfunded fiscal transfer shocks. The system of linearized equations is reported below, using the same notation of the medium-scale model in Section (C).

IS:

$$\hat{y}_t = -\left[\hat{R}_t - E_t\hat{\pi}_{t+1} - (1 - \rho_b)\,\hat{u}_t^b\right] + \rho_a a_t + E_t\hat{y}_{t+1} + (1 - \rho_g)\tilde{g}_t, \tag{87}$$

where $ln(A_t/A_{t-1}) = \gamma + a_t$, $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$.

Phillips curve:

$$\hat{\pi}_t = \kappa\left[\left(1 + \frac{\alpha}{1 - \alpha}\right)\hat{y}_t - \tilde{g}_t\right] + \beta E_t\hat{\pi}_{t+1} + \tilde{\eta}_t^p, \tag{88}$$

where we have used the rescaled markup $\tilde{\eta}_t^p = \kappa\left(\frac{v}{1-v}\right)\tilde{v}_t$ and v denotes the elasticity of substitution.

Taylor rule:

$$\hat{R}_t = \max\left\{-\ln R, \rho_r\hat{R}_{t-1} + (1 - \rho_r)\left[\phi_\pi\hat{\pi}_t^M + \phi_y\left(\hat{y}_t - \hat{y}_t^*\right)\right] + \varepsilon_{R,t}\right\}, \tag{89}$$

where the output target $\hat{y}_t^*$ is given by

$$\frac{\alpha}{1 - \alpha}\hat{y}_t^* = \tilde{g}_t$$

Government budget constraint

$$\tilde{s}_{b,t} = \beta^{-1}\tilde{s}_{b,t-1} + s_b\beta^{-1}\left(\hat{R}_t - \hat{y}_t + \hat{y}_{t-1} - \hat{u}_t^a - \hat{\pi}_t\right) - \tilde{\tau}_t + \tilde{z}_t + g^{-1}\tilde{g}_t, \tag{90}$$

Government spending:

$$\tilde{g}_t = \rho_g\tilde{g}_{t-1} + \zeta_t^g \tag{91}$$

Tax rule:

$$\tilde{\tau}_t = \rho_\tau\tilde{\tau}_{t-1} + (1 - \rho_\tau)\left[\gamma_\tau\tilde{s}_{b,t-1}^M + \gamma_{e^*}\left(\tilde{z}_t^* + g^{-1}\tilde{g}_t\right) + \gamma_{\tau,y}\left(\hat{y}_t - \hat{y}_t^*\right)\right] + \zeta_t^\tau \tag{92}$$

Transfers:

$$\tilde{z}_t - \tilde{z}_t^* = \rho_z\left(\tilde{z}_{t-1} - \tilde{z}_t^*\right) + (1 - \rho_z)\gamma_{z,y}\left(\hat{y}_t - \hat{y}_t^*\right) + \zeta_{z,t}^M \tag{93}$$

Long-term component of transfers

$$\tilde{z}_t^* = \rho_z^*\tilde{z}_{t-1}^* + \zeta_{z,t}^{*M} + \zeta_{z,t}^{*F}. \tag{94}$$

The variables with superscripts M in equations (89) and (92) to (94) above belong to the shadow economy. In turn, the block of equations characterizing the shadow economy consists in an additional set of equations (87) to (93), where any variable that refers to the actual economy x_t is replaced by the same variable in the shadow economy x_t^M, plus the rule
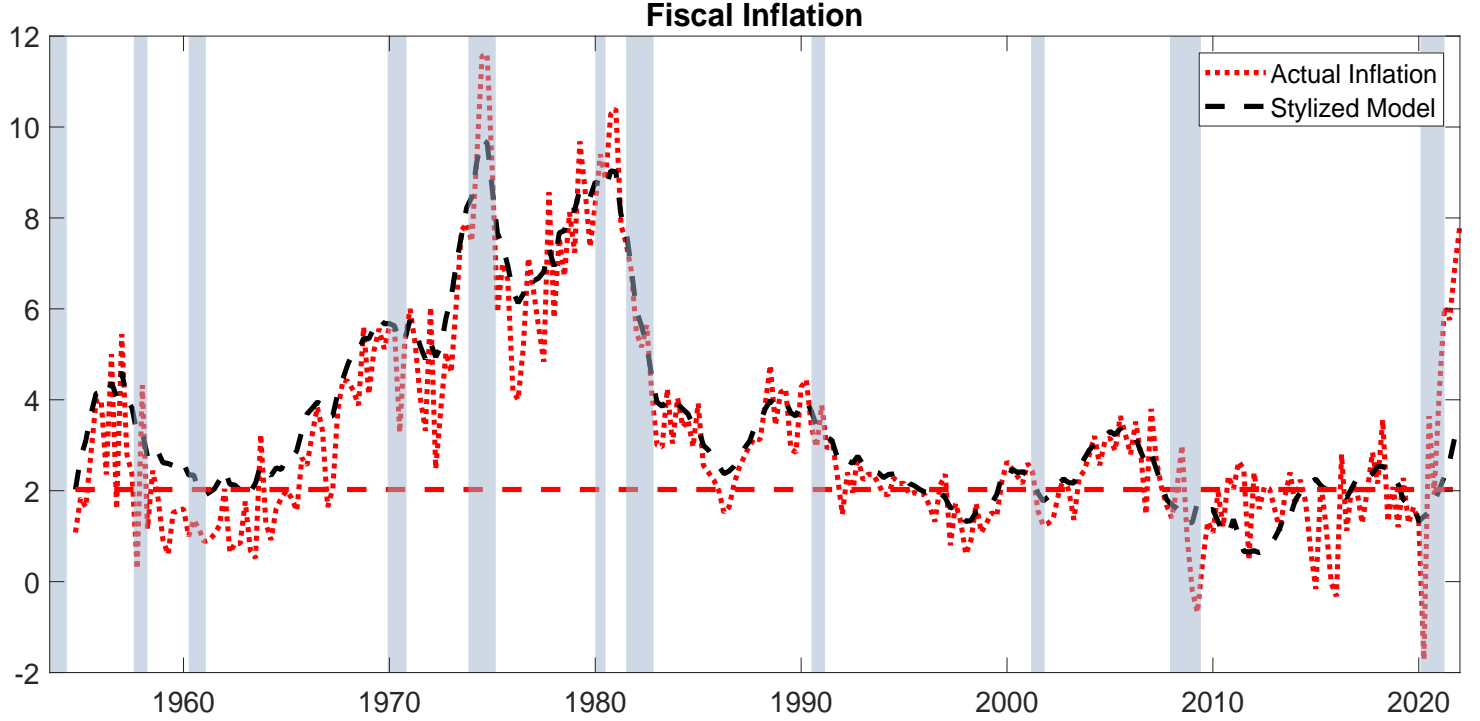
Figure 15: The black dashed line is the inflation rate in the data. The blue solid line is the predicted rate of inflation conditional on unfunded fiscal transfer shocks.

for the long component of transfers:

$$\tilde{z}_t^{*M} = \rho_z^* \tilde{z}_{t-1}^{*M} + \zeta_{z,t}^{*M}. \tag{95}$$

Prior and posterior distributions related to the estimation of this small-scale NK model are reported in Tables 5 and 6.

The main output of the estimation is reported in Figure 15, which shows the dynamics of inflation (black dashed line) together with the rate of inflation predicted by fiscally unfunded transfer shocks (solid blue line). The result illustrate that the unfunded shocks explain trend inflation remarkably well. Hence, we conclude that the result that unfunded fiscal shocks explain trend inflation is not due to the many auxiliary assumption that characterize the medium-scale model of Section (3).

Priors and Posteriors for the Structural Parameters

Param	Descr.	Posterior Mode	Type	Mean	Std
κ	Slope Phillips Curve	0.0113	G	0.300	0.15
π	Steady state inflation	0.5066	N	0.500	0.05
γ	Steady state growth	0.4221	N	0.400	0.05
b	Steady state debt/GDP	2.1386	N	2.400	0.05
g	Steady state gov. spending	1.0922	N	1.080	0.05
τ	Steady state tax rate	0.1616	N	0.175	0.05
$\gamma_{z,y}$	Elasticity of transf. to output	-0.0197	N	-0.400	0.20
ϕ_y	Interest response to y	1.4128	N	0.400	0.20
ϕ_π	Interest response to π	1.8924	N	2.500	0.30
ρ_r	AR1 coeff Taylor rule	0.7963	B	0.500	0.20
γ_{e*}	Spending elasticity of taxes	0.0043	B	0.500	0.20
$\gamma_{\tau,y}$	Debt elasticity of taxes	0.0163	G	0.070	0.02
ρ_Z	AR1 coeff short term transfers	0.5007	B	0.200	0.05
ρ_τ	AR1 coeff taxes	0.4977	B	0.500	0.20

Table 5: Posterior modes and prior moments for the structural parameters. The letters in the column with the heading "Prior Type" indicate the prior density function: N, G, and B stand for Normal, Gamma, and Beta, respectively.

Priors and Posteriors for the Exogenous Processes

Param	Description	Posterior Mode	Type	Mean	Std
ρ_g	AR coeff. gov. cons	0.9787	B	0.500	0.100
ρ_Z^*	AR coeff. long transfers	0.9951	B	0.995	0.001
ρ_a	AR coeff. technology	0.5765	B	0.500	0.100
ρ_b	AR coeff. preference	0.8735	B	0.500	0.100
ρ_μ	AR coeff. markup	0.2082	B	0.500	0.200
σ_m	AR coeff. mon. policy	0.2052	IG	0.500	0.200
σ_g	St dev gov cons.	0.0064	IG	0.500	0.200
σ_a	St dev technology	0.4247	IG	0.100	0.050
σ_τ	St dev tax	0.0072	IG	0.500	0.200
σ_b	St dev preference	2.3997	IG	0.250	0.200
σ_z	St dev short transfers	0.0071	IG	0.500	0.200
σ_μ	St dev markup	0.1662	IG	0.500	0.200
σ_z^{*F}	St dev long funded trans.	0.0163	IG	0.500	0.200
σ_z^{*U}	St dev long unfunded trans.	0.0158	IG	0.100	0.050

Table 6: Posterior modes and prior moments for the exogenous processes. The letters in the column with the heading "Prior Type" indicate the prior density function: N, G, and B stand for Normal, Gamma, and Beta, respectively.