On the Edge of Doom:
A Model of Bank Recapitalization with Sovereign Default *

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Abstract

What is the best policy response to a financial crisis? At least ex post, theory and practice suggest to rebuild intermediaries’ balance sheets, generally with public funds. But what if the government faces a resource constraint herself? This paper addresses the optimal structure of bank recapitalization policy when sovereign debt is risky. I propose a model that combines a classic sovereign default model with private sector financial frictions, which generate fully endogenous and time-varying default costs. When the sovereign lacks commitment, I find that the impact of bank recapitalization on sovereign default risk follows a Laffer curve: Public capital infusions can decrease sovereign spreads when domestic banks are weak, even when transfers are fully financed by external borrowing. At the same time, if transfers are excessively large, recapitalization increases sovereign credit risk. Government bond holdings of domestic banks amplify the mechanism and can generate virtuous and vicious ("doom loop") cycles. This mechanism has implications for macroprudential regulation, optimal bank funding structure, and the workings of a banking union.

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1 Introduction

What is the optimal policy response to a financial crisis?

Not least since the Great Financial Crisis, much work has been done to model and study the impact of shocks to the financial system. While there is a wide range of models, the conclusion from many of them is that sufficient transfers of net worth to intermediaries can counteract frictions and erase much of the implicit costs. This insight was reflected in many of the crisis-era policy responses, such as TARP in the US and other government-led banking recapitalization programs in Europe. Recent empirical work on post-banking crisis recoveries (Homar and van Wijnbergen, 2016) also shows that recapitalization of the financial sector can drastically shorten the duration of post-crisis recessions.

In this context, seemingly little thought has been given to the source of recapitalization funds and the impact the related deterioration in a country’s fiscal balance might have on the economy. The European debt crisis of recent years provides a clear example why a broader model of the optimal policy response, one taking into account the other side of the balance sheet transfer, is needed: While many of the countries engulfed in the debt crisis were highly indebted a priori, Ireland and Spain were examples of a different type of crisis. With very low initial sovereign debt burdens, both countries suffered severe shocks to their borrowing rates in response to large-scale bailouts of their domestic banking systems. Following the textbook of financial crisis-fighting, the Irish banking guarantees were sweeping and large and initially succeeded in putting out the fire in the financial system. The subsequent flare-up of sovereign risk, however, was just as swift and severe, as the size of the fiscal commitment threatened to outweigh the government’s credible repayment capabilities.

With this motivation in mind, I propose a model of connected private and public credit risk, which allows to determine the optimal structure for bank recapitalization when both sectors are risky. The paper makes two main contributions: First, it develops a sovereign default model with endogenous default costs by incorporating a financial system and private international borrowing. Second, the model also allows for finding the optimal scale of bank recapitalization when sovereign default is possible. Extensions of the model show the pathways for mutual contagion between sovereign and banks through two different but connected sources of credit risk in each sector.

At its base, the model builds on the external financial accelerator in Gertler, Gilchrist and Natalucci (2007) and combines it with a classic sovereign default model of a sovereign without commitment. I then add two assumptions that make the model relevant to the question at hand: First, I allow the sovereign to borrow funds and transfer them to the private intermediary (recapitalization). Second, the intermediary’s funding options are determined by the sovereign’s standing in international credit markets.
In the model, banks are the sole owners of capital in the economy, while households provide fixed labor supply. Banks have the choice between raising funds from domestic households, which only require a fixed rate of return independent of the banks’ net worth (akin to deposit insurance), or from international lenders. Since borrowing internationally is risky because of observational frictions, external borrowing costs are determined by the banks leverage. If international borrowing costs are lower than domestic rates, the country will be a net debtor and benefit from capital inflows from abroad.

At the same time, the sovereign borrows from international lenders to fund required government spending and (potentially) transfers to the banks. Importantly, since the sovereign’s default decision determines the private sector's financing options, this generates an expanded Eaton Gersovitz (1981) -type punishment for sovereign default: Not only is the sovereign excluded from international financing markets if she fails to repay, but so are domestic banks. Domestic banks then have to rely solely on domestic financing.

This generates fully endogenous default costs for the sovereign, which vary with the state of the financial system. If domestic banks are well capitalized, then international financing will be cheap, and hence total capital (and total income) in the next period will be high if international financing access is preserved. If future income with international financing is sufficiently above total future income under domestic financing, the potential gain incentivizes the sovereign to repay her debts. However, if the financial system is weak, international borrowing costs are high, and the value of the foreign financing option will be low. Hence, the sovereign may choose to default even on small amounts of outstanding debt. Ex ante, international lenders can anticipate the risk of sovereign default based on economic risk as well as the initial state of bank balance sheets, and price bonds accordingly.

This mechanism highlights the potential benefits from recapitalization: Under some conditions, the model shows that transferring funds from the sovereign to the banking system increases the expected benefit from sovereign debt repayment. Hence, the sovereign can credibly lower her own risk of default (and increase welfare) even if recapitalization is funded with additional foreign borrowing. In this context, increasing banking system net worth can play the role of a commitment device.

However, bank recapitalization is a delicate endeavor. If the government already faces substantial outstanding debts, or if the recapitalization is too large, the increase in the required repayment can swamp the benefits from higher bank net worth. In combination, I can show that under some conditions, bank recapitalization follows a Laffer curve when taking sovereign default risk into account. When the government already faces a large debt burden and benefits from international financial integration are low, large-scale bank recapitalization can actually be destabilizing. At the same time, the model demonstrates how bank recapitalization can serve to outright reduce sovereign risk even when the amount of external sovereign debt increases.
In its most basic form, the model highlights the sensitivity of public creditworthiness to the state of the domestic banking system. But the model can be easily extended to incorporate a parallel sensitivity of bank balance sheets to sovereign risk, such as by allowing banks to hold domestic sovereign bonds. This generates a two-way risk transmission mechanism between public and private credit risk, and allows for virtuous and vicious ("doom") feedback loops: Lower (higher) sovereign credit risk lead to strengthened (weakened) bank balance sheets through a revaluation of domestic bond holdings, which further improves (worsens) the sovereign’s future repayment incentives. Similarly, stronger (weaker) bank balance sheets decrease (increase) sovereign credit risk, which further strengthens (weakens) bank net worth.

This paper builds on two different strands of the literature. First, the model suggests a new angle on default incentives and disincentives for the government of a net debtor economy. In thinking about the costs of sovereign default, Eaton and Gersovitz (1981) first proposed that the main cost of default to the sovereign comes from losing access to international capital markets. However, it is difficult to generate sufficiently high costs to the government through this channel to make default infrequent enough to match the data. In my model, by tying the domestic financial system’s financing options to the government’s default decision, I can generate higher endogenous and time-varying default costs, since market exclusion extends to the financial system and is hence more consequential. As a result, default is more likely when the financial system is weak, which makes the model consistent with the observation in Reinhart and Rogoff (2011), who show that sovereign default is often preceded by financial crises.

There has been some work on the interaction of sovereign default and the financial system, such as Genneaioli et al. (2014), Perez (2015), or Balloch (2016). In contrast to this paper, their emphasis is primarily on the "plumbing" of the domestic financial system. Sovereign default is costly because it destroys the safe asset that is used for intermediation purposes. These models broadly disregard international borrowing by the private sector and do not allow for discriminative default and government-led recapitalization (except Perez). Balke (2016), while predominantly interested in the labor market impact of sovereign default, relies on a similar mechanism.

In comparison, this paper directly models external borrowing by the private sector and relates it to sovereign default incentives. This step allows me to avoid a common, unintuitive feature of the models discussed above: If sovereign default is costly predominantly because of its effect on (domestic) intermediation, the government is more likely to default when bank balance sheets are strong. The opposite is true in the model at hand, which opens up the possibility to allow for recapitalization explicitly, and to study the synergies between private and public balance sheets.

Furthermore, because of the soft barrier between public and private balance sheets, there is now room to think about contagion from private to public credit risk. A comparable dynamic appears in Acharya et al.
where financial services are treated as a complimentary intermediary good. My work here allows for a more direct model of relative funding costs and hence allows for a more explicit trade-off between private and public credit spreads. By modeling the two different sources of credit risk, I am able to determine an optimal policy for recapitalization in the presence of endogenous sovereign default risk.

The second strand of the literature that this paper relates to is the study of bank recapitalization and bailouts. Here, a large body of work has focused on the moral hazard cost of bank bailouts, and how to structure resource transfers most efficiently in the face of agency frictions. One timely example of this work is Philippon and Schnabl (2013). Other research has focused on exploring the ways bank bailouts impact the financial system, for example as liquidity provision in Gorton and Huang (2004), all while taking the source of recapitalization funds as exogenous. Panageas (2010) discusses the optimal taxation approach to finance bank bailouts, however without including a feedback effect from banks to sovereign. Stavrakeva (2013) examines how fiscal capacity constraints affect the optimal level of risk-taking and regulation in domestic banking systems.

Farhi and Tirole (2016) provide another recent and important addition to the literature. Their paper is predominantly concerned with the banking system’s incentives when bailouts are possible, as well as with national and supra-national supervision. Their model features a strong feedback effect between sovereign and bank balance sheets but is less directly concerned with the optimal structure of bank recapitalizations ex post and their effect on the sovereign’s default incentives, the main focus of my work here.

Finally, Acharya et al. (2014), as discussed above, is closest to the discussion at hand since it explicitly models the feedback loop between sovereign and banks. There are some parallels to this in Leonello (2016), which focuses on the role of government guarantees for the banking sector. While guarantees work differently from explicit recapitalizations, and the model is based on roll-over risk, she also highlights some of the feedback effects I show in this paper.

The remainder of the paper is organized as follows: Section 2 describes the model and chapter 3 shows the main results. Chapter 4 extends the model to include the sovereign-banking nexus and explores the impact of alternative bank funding structures. Chapter 5 highlights the model’s implications for a banking union, and chapter 8 concludes.

2 Model

The three-period model features households, banks, firms, foreign investors and the government. Households provide a fixed amount of labor and solve a consumption-savings problem. Banks intermediate funds, either from domestic deposits or from international capital markets, to the firm, which hires inputs to production
in spot markets and produces the single consumption good. Importantly, while bank financing costs from domestic deposits are determined solely by the household’s time preferences, the cost of foreign borrowing depends on bank leverage. Finally, the government, which seeks to maximize household welfare, issues one-period bonds to international lenders in order to fund a fixed amount of government spending. It can issue additional debt to finance bank recapitalization, which lowers bank leverage, but may increase the risk of sovereign default, and hence the required interest rate on government bonds. Importantly, recapitalization is only possible at t=0, and risk only enters the model through uncertainty about production technology in period 1. Once capital and labor returns are realized, banks repay their creditors and the government subsequently decides whether to default or not, based on the possible payoff outcomes in the last period.

2.1 Setup

I now describe the time path of the model and the individual agents’ optimization problems:

- At t=0, the government issues debt to finance a required amount of government spending. It also observes the current level of net worth in the banking system and can inject additional funds (recapitalization). Both types of spending are funded by issuing one-period bonds to foreign investors. Foreign investors calculate the required risk spread on government bonds based on expectations about future default. Banks attract domestic deposits or borrow from foreign investors, and invest in capital for t=1, based on their expectation of future productivity $A_1$. Banks’ foreign borrowing costs are dependent on the bank’s external financing premium (leverage).

- At t=1, the productivity state is revealed and the firm hires capital and labor in the spot market to produce the single consumption good. Banks receive the market clearing rental rate on capital and repay their depositors or foreign lenders. Households receive their wage income, consume and/or save. The government decides whether to repay foreign borrowers, in which case households have to pay a lump-sum tax, or to default. If bondholders are paid back, banks can again borrow from abroad and invest in capital, this time while knowing returns at t=2 with certainty. In the case of default, banks lose access to foreign borrowing and can only access domestic deposits at a fixed interest rate.

- At t=2, the firm again hires capital and labor and produces according to current productivity. Banks receive capital income and pay back their borrowed funds or deposits with interest. Total resulting net worth is then transferred to the households. Households in turn consume their labor income, possible savings with interest, and the final net worth of the banks.
2.2 Households

Households are assumed to be risk-neutral consumers, discount future periods at the rate $\beta$, and inelastically provide a fixed amount of labor. For the three periods, households choose consumption to maximize:

$$E[c_1 + \beta c_2],$$  \hspace{1cm} (1)

subject to the budget constraints:

$$c_1 + d_1 = w_1 - b^* \cdot \mathbb{1}_{\{D=0\}},$$  \hspace{1cm} (2)

$$c_2 = R_2 d_1 + w_2 + n_2,$$  \hspace{1cm} (3)

Households earn wages $w_1$ and $w_2$, receive interest rate $R_2$ on deposits $d_1$, if they choose to save, and receive the leftover equity in the banks, $n_2$ in period two. In the no-default case, i.e. $D = 0$, the households are taxed lump-sum for the required repayment of outstanding foreign borrowing $b^*$.

2.3 Government

The government seeks to optimize household welfare in the economy, hence its optimal decision program has to satisfy:

$$V_0 = \max_{b^*} E[ \max_{D \in \{0,1\}} \{V_1, V_1^d\}],$$  \hspace{1cm} (4)

where $b^*$ denotes the amount of sovereign debt issued to international investors from $t=0$ to $t=1$, and $D$ captures the binary default decision in period $t=1$. The government only issues short-term debt at $t=0$, and makes the decision to repay at $t=1$. If the government does not default in period 1, then lump-sum taxes $T_1 = b^*$ are raised from households to pay for the maturing debt. Hence, the continuation values in period 1 are:

$$V_1(b^*, A_1) = c_1 + \beta c_2,$$

$$V_1^d(b^*, A_1) = c_1^d + \beta c_2^d,$$

where $c_t$ and $c_t^d$ denote consumption in the non-default and the default state respectively. $A_1 \in [A, \bar{A}]$ denotes the realization of the stochastic productivity level in the economy.

Since productivity is the only source of uncertainty, I can define a default and a repayment set given the outstanding levels of short-term debt, $b^*$, which make either decision optimal for the government:
\[ D(b^*) = \{ A_1 \epsilon A : V_1 < V_1^d \}, \]
\[ R(b^*) = \{ A_1 \epsilon A : V_1 \geq V_1^d \}. \]

Short-term debt is issued at \( t=0 \) in order to finance required government spending \( g \) and potentially also a recapitalization of the banking system, denoted \( rc \). This gives a set of budget constraints for the government:

\[ qb^* = \frac{1}{1 - \tau} rc + g, \quad (5) \]
\[ (b^* - T_1) \cdot I_{D=0} = 0, \quad (6) \]

where \( T_1 \) denotes tax revenue, and \( q \) denotes the price of the bond when issued. \( \tau \) describes the efficiency loss inherent in the bank recapitalization, i.e. the amount lost per unit of net worth transferred because of moral hazard or other inefficiencies inherent in the recapitalization process. \(^1\)

The government also provides an insurance scheme for domestic depositors. But since the required funds would similarly be raised by taxing households lump-sum, this has no effect in the model and I hence abstract from modeling it explicitly.

### 2.4 Foreign Lenders

Foreign lenders are assumed to be risk neutral and make similar consumption-savings decisions as domestic households, however with a foreign discount factor \( \beta^* \). This gives a required rate of return in the global savings markets:

\[ R_w = \frac{1}{\beta^*}. \quad (7) \]

I assume that global savings are provided perfectly elastically at this rate of return, and not at all at any lower rate. As a result of risk neutrality, the expected return on the sovereign bond thus needs to match the required rate of return in the global market. If the probability of sovereign default is non-zero, then the sovereign bond needs to be priced such that:

\[ \frac{1}{q} = \frac{1}{1 - p(b^*)} R_w, \quad (8) \]

where \( p(b^*) \) denotes the probability of default on the government bond, given the required repayment amount \( b^* \) (no partial default option is assumed). As a result, the bond price \( q \) takes values on the interval \( q \in [0, \frac{1}{\beta^*}] \).

\(^1\)Another reasonable assumption here would be to include a fixed cost on initiation of the capital injection. This would serve to make the model more realistic by creating larger "inaction zones" for the government (since bank recapitalization is a rare event) but since the model is primarily concerned with what happens around bank recapitalization, I will abstract from this here.
2.5 Firms

Firms competitively produce the single consumption good in the economy by hiring capital from banks and labor from households in spot markets in each period, after the state of technology $A$ is revealed. Firms optimize a simple Cobb-Douglas production function:

$$\max_{k,l} A k^\alpha l^{1-\alpha} - r_t^k k_t - w_t l_t$$

(9)

This optimization yields the conventional first order conditions for the return on capital and wages.

2.6 Banks

Banks are the sole owners of capital. Banks can lever their net worth $n_1$ by taking domestic deposits or by borrowing from foreign investors to invest in capital. Banks are risk-neutral and, given a starting position of net worth $n_0$, seek to maximize the amount of equity left in the last period

$$\mathbb{E}[n_2],$$

subject to the budget constraints:

$$n_0 = \bar{n} + rc$$

(11)

$$n_1 = R^k k_1 - R_1 d_0 - R^*_1 d^*_0,$$

(12)

$$n_2 = R^k k_2 - R_2 d_1 - R^*_2 d^*_1,$$

(13)

where $R^k$ denotes the gross return on capital, $R$ the interest rate on domestic deposits, $R^*$ the interest rate on foreign borrowing, and $d$ and $d^*$ domestic deposits and foreign borrowing respectively. Capital fully depreciates each period. As discussed above, $\bar{n}$ is the initial starting equity in the banks, and $rc$ captures the government recapitalization at $t = 0$.

As noted above, the interest costs banks face on the two different funding sources does not need to be the same. The key distinction here is that while domestic deposits are insured, foreign borrowing is risky. Because the government insures deposits for domestic households, the required rate on domestic deposits is time-invariant and pinned down by the household’s optimization problem’s FOC,

$$R = \frac{1}{\beta}.$$ 

(14)
Since the government does not insure foreign lenders and because of observational frictions, foreign borrowing costs are instead dynamically determined as in GGN, namely as combination of the world interest rate, and a leverage risk premium:

\[ R^*_t = \chi \left( \frac{k_t}{n_{t-1}} \right)^{1/\beta^*}. \] (15)

\( \chi \) is the standard expression for the external financing premium in the financial accelerator literature, a strictly increasing convex function of leverage. The premium depends on the amount of total capital held by the bank relative to the strength of its balance sheet. While I refer to the literature for the explicit microfoundations, intuitively one can think of the increasing costs of leverage as the result of costly state verification. When banks have less skin in the game, they may choose to do more opaque investments that require a tighter oversight by international lenders and hence require a higher rate of compensation. This spread is then combined with the required world interest rate, determined by foreign investors in the global savings market.

Since savings supply is perfectly elastic at the prevailing interest rates in the global market as well as by domestic savers (at least up to the limit of all available resources in the domestic economy), it is clear that unless the rates are exactly the same, banks will only use one source of funding. Hence, banks will rely on foreign borrowing rather than domestic deposits if:

\[ \frac{1}{\beta} > \chi \left( \frac{k_t}{n_{t-1}} \right)^{1/\beta^*}. \]

The main source of welfare losses from sovereign default in this model is domestic banks’ loss of access to international funding markets. As a result, for the model to deliver dynamics relevant for the questions at hand, it needs to be the case that the country is a net debtor, i.e. that the condition above holds in period 1.

Note while this needs to hold for \( n_0 \) as defined above, which includes the recapitalization, it does not necessarily need to hold in absence of it, i.e. for \( n_0 = \bar{n} \). It is possible for banks to face prohibitively high foreign funding costs and hence rely solely on domestic funding if the government decides not to recapitalize the system. In practice, this would mean that in absence of recapitalization, default on any amount of sovereign debt (i.e., any \( g \)) is certain, since there is no benefit at all to preserving international financial integration. However, recapitalization can shift the equilibrium outcome from certain to stochastic default under some conditions.

In short, bank recapitalization by the government serves to reduce the external financing premium banks

\[ \frac{1}{\beta} \geq \frac{1}{\beta^*} \] and external savings options are not attractive to domestic households.
face when capital levels are low. However, since the capital transfer comes with an increase in government debt, this might come with an increase in sovereign risk. The counteracting trajectories for the two sources of funding spreads suggests an interior solution for an optimal bank recapitalization.

2.7 Equilibrium

In equilibrium, the government maximizes welfare, households and foreign lenders maximize consumption utility, firms maximize profits, and banks maximize expected net worth. Markets for domestic deposits, foreign borrowing, sovereign bonds, labor and capital all clear.

Definition 1 Equilibrium in the economy is defined as a set of policy functions for consumption $c_1, c_2$, domestic deposits $d_1$, foreign borrowing $d_0^*, d_1^*$, government bond issuance $b^*$, capital returns $r_1^k, r_2^k$, wages $w_1, w_2$, repayment sets $R(b^*)$, default sets $D(b^*)$, and bond prices $q$ such that:

1. The plans for consumption and domestic deposits, $c_1, c_2$ and $d_1$, maximize households’ expected utility subject to their budget constraint, taking wages, interest rates, government policy, and banks’ behavior as given.

2. Banks maximize their expected net worth in period 2 by choosing capital, domestic deposits and foreign borrowing subject to budget constraints and financial frictions. Deposit and foreign borrowing markets clear.

3. Firms maximize profits by hiring the optimal amount of labor and capital, and labor and capital markets clear.

4. Government bond issuance $b^*$, the repayment and the default sets satisfy the government’s optimization problem, taking into account the impact of debt levels onto bond prices.

5. Bond prices $q$ reflect the government’s true default probabilities, and are consistent with the foreign lender’s maximization problem.

Since there is no uncertainty in period $t=2$, I can solve the model backwards. The lack of uncertainty allows for calculating the continuation values $V_1$ and $V_1^d$ for a given amount of short-term debt outstanding and realized technology level $A_1$. This yields a decision rule for when default is optimal. Assuming a probability distribution for $A_1$ allows to compute the probabilities of default, and then to back out the optimal $b^*$ in period 0, which is equivalent to finding the optimal amount of recapitalization for a given $g$ and $\bar{n}$. I describe the solution in the appendix.
3 Results

This section discusses the model’s equilibrium outcomes and what they imply about the relationship between sovereign risk, bank balance sheet health, and recapitalization. There are three main results: First, I show that sovereign default risk is lower when banks are well-capitalized ex ante. Second, as a result, bank recapitalization can serve to decrease sovereign default risk even when transfers are financed through additional external borrowing. Third, however, the model also demonstrates that when recapitalization transfers are excessively large, they become self-defeating and increase default risk. Taken together, the second and third result generate a 'Laffer curve' for bank recapitalization. Finally, the model can be shown to satisfy the standard criteria of sovereign default models, i.e. it features default in equilibrium, and the sovereign’s default incentives are increasing in the amount of external borrowing, all else equal.

In order for the model to generate interesting results, I first have to make a key assumption on the parameterization of the model. Since the model features two different financing sources for the domestic banking system, households need to discount the future more (and hence require a higher risk-free return) than international lenders. This ensures that access to international financial markets is valuable, at least in certain states.

Assumption 1. Domestic households are assumed to have a lower discount factor than international lenders:

$$\beta < \beta^*.$$  

I now discuss two basic properties of the model that simplify solving the equilibrium. First, for convenience, I split the no-default condition from above in period 1 value functions for the repayment and the default case:

$$v_1 = w_2(n_1) + R^k_2(n_1) \cdot n_1 - \frac{1}{\beta} b^*$$  \hspace{1cm} (16)

$$v^d_1 = w^d_2 + R^{k,d}_2 \cdot n_1,$$

so that $v_1$ and $v^d_1$ are truncations of the government’s respective welfare values $V_1$ and $V^d_1$. Wages $w$ and capital returns $R^k$ are dependent on bank net worth $n_1$ in the repayment outcome, but not in the default state. As these simplified value functions show, period 1 bank net worth has a larger effect on welfare in the repayment state (where wages and capital returns - both per unit and in total - depend on it) than in the default state, at least as long as $n_1$ lies in a reasonable range, which is formalized in the following:

Assumption 2. One additional unit of bank net worth at $t=1$ is assumed to be more valuable in the

\footnote{See the solution description in the appendix for more detail.}
repayment state than in autarky, i.e. $n_1$ is assumed to satisfy:

\[
\frac{\partial v_1}{\partial n_1} > \frac{\partial v_1^d}{\partial n_1}
\]

From above, recall that period 2 capital levels (and hence wages and total capital returns) are increasing concave functions of $n_1$, hence this is also true for $v_1$, keeping debt outstanding $b^*$ constant. At the same time, $v_1^d$ is linear in $n_1$. Under assumption 2, which states that $v_1$ increases faster in $n_1$ than $v_1^d$, this implies that the two value functions can cross at most once. In practice this means that there exists a unique default threshold $\tilde{n}_1$ at which the sovereign is indifferent between default and repayment, resulting in the first lemma.

**Lemma 1.** If default is optimal for some period 1 bank net worth $n_1$, then default is also optimal for all $\tilde{n}_1 < n_1$. Therefore, there exists a threshold $\tilde{n}_1$ below which the government always chooses to default, and above which it always repays.

Furthermore, $n_1$ is a function of initial bank net worth, $n_0$, and the realized productivity state $A_1$:

\[
n_1 = \alpha \left[ A_1 - \mathbb{E}(A_1) + \mathbb{E}(A_1) \frac{n_0}{K_1} \right] K_1^p,
\]

(18)

where $K_1$ is a function of $n_0$ and ex ante default probability $p$. Since banks gain more on their investments when productivity outcomes are high, $n_1$ is increasing in realized productivity $A_1$, holding $\mathbb{E}(A_1)$ constant. Hence,

\[
\frac{\partial n_1}{\partial A_1} > 0 \Rightarrow \frac{\partial v_1}{\partial A_1} > \frac{\partial v_1^d}{\partial A_1},
\]

i.e. assumption 2 also applies. As a result, lemma 1 can be extended to give lemma 2:

**Lemma 2.** If default is optimal for any $A_1 \in [A_1, \hat{A}_1]$, then there exists $\hat{A}_1$ s.t. $v_1(\hat{A}_1) = v_1^d(\hat{A}_1)$. Then, by assumption 2, $v_1(A_1) < v_1^d(\hat{A}_1)$ for all $A_1 < \hat{A}_1$, i.e. default is optimal for all productivity realizations below the default threshold $\hat{A}_1$.

Hence, the set of productivity realizations that trigger a default for a specific amount of sovereign debt outstanding is an interval. The probability of default can then be rewritten as

\[
p(b^*, n_0) = \int_{\hat{A}_1}^{\hat{A}_1} f(A) d(A),
\]

(19)

where $f(A)$ denotes the pdf of the stochastic productivity level, and $\hat{A}_1$ is the default threshold for a given $n_0$ and debt outstanding $b^*$, as described above.
3.1 Bank net worth

As lemma 1 shows, under reasonable assumptions about bank capitalization, higher ex post (i.e. t=1) net worth makes default less desirable for the sovereign. This suggests that ex ante, default risk should also be lower when banks' initial net worth is high. For this intuition to hold, it needs to be the case that an additional unit of initial net worth does not lead to a decrease in period 1 net worth for any realization of $A_1$.

This is trivial in good states, where higher net worth can support higher capital levels, and hence higher overall returns for the bank. Things are less obvious when realized productivity comes in below expectations, since banks suffer a loss on their invested capital, which grows with net worth. However, next period’s capital choice is a concave function of current period bank net worth, as returns to capital are decreasing in scale, given the fixed labor supply. Hence, leverage per unit of net worth actually decreases as bank equity increases. This means that losses per unit of net worth fall, so that higher initial net worth still leads to higher net worth in the next period relative to lower initial net worth, even in the lowest productivity state $A_1$. This logic formally holds for reasonable levels of $n_0$.

As a result, period 1 net worth $n_1$ is increasing in initial net worth $n_0$ for any realization of $A_1$. Then, lemma 2 applies, yielding proposition 1.

**Proposition 1.** For any fixed amount of required government spending $g$, default risk decreases with initial bank net worth $n_0$.

Why do better-capitalized banks lead to lower sovereign default risk? The answer lies in the role that bank net worth plays in the model. Keeping sovereign default probability fixed for the moment, a higher level of bank capitalization lowers international funding costs for banks and hence increases the amount of capital in the economy in the next period. As the amount of capital in autarky is fixed, this increases the welfare gain in the non-default state relative to the default state. Hence, bank capital takes on the role of a commitment device (which can be actively deployed, as shown in the next section). Well-capitalized banking systems promise higher relative welfare losses from default, and can therefore support higher levels of government borrowing.

This result is fundamental for the model and for motivating bank recapitalization even when the sovereign may already have large borrowing requirements. Importantly, it also sets this paper apart from most other sovereign default models with a financial sector, where the sovereign is actually more likely to default when banks have strong balance sheets.
3.2 Debt-funded bank recapitalization

Since stronger bank balance sheets reduce the risk of sovereign default, a logical next step is to think about government-led bank recapitalization. When a shock hits the financial system, can government intervention help in reducing private and public risk? This section explores how - and under what circumstances - recapitalization can reduce sovereign risk, even if it is funded by additional issuance of government debt.

At the beginning of the analysis of bank recapitalization in the model stands the key assumption from above: A unit of bank net worth is more valuable when the government repays its debts than when it defaults. Hence, since stronger balance sheets increase repayment incentives, bank net worth becomes a commitment device for the government, which illuminates the role that bank recapitalization plays in the model. However, when the transfer of resources from sovereign to private sector has to be funded by issuing more debt, this in turn increases the sovereign’s incentive to default. Will recapitalization hence be beneficial for sovereign risk or not? As visible in figure 1, the answer is ‘it depends.’ In fact, the interaction of default and repayment incentives generates a ‘Laffer curve’ for bank recapitalization, in which debt-financed transfers to the financial system can actually reduce sovereign default risk when banks are weakly capitalized. However, they can also worsen the sovereign risk profile when they become too large (or the financial system is already well-funded).

![Figure 1: Probability of repayment with debt-financed recapitalization](image)

In order to analyze the counteracting incentives of debt-funded recapitalization, I rewrite the govern-
ment’s truncated value functions from above such that

\[ v^*_1 = w_2(n_1) + R^k_2(n_1) \cdot n_1, \quad (20) \]

\[ v^b = \frac{1}{\beta} b^* = \frac{1}{\beta} \frac{1}{q} \left( g + \frac{1}{1-\tau} rc \right), \quad (21) \]

\[ v^d_1 = w^d_2 + R^{k,d}_2 \cdot n_1, \quad (22) \]

where \( w_2(n_1) \) and \( R^k_2(n_1) \) denote wages and capital returns in the repayment state, which are concave functions of bank net worth in period 1, \( n_1 \). \( v^b \) denotes the value of the required debt repayment, made up of required government spending \( g \) and recapitalization amount \( rc \), where \( q \) denotes the issuance price of the bonds, and \( \tau \) denotes the efficiency loss incurred in the resource transfer. \( v^d_1 \) captures the utility in the default case, as above.

From above, since \( v^*_1 - v^b = v_i \), we know that

\[ \frac{\partial v^*_1}{\partial n_0} > \frac{\partial v^d_1}{\partial n_0}. \]

Since \( n_0 = \bar{n} + rc \), this implies

\[ \frac{\partial v^*_1}{\partial rc} > \frac{\partial v^d_1}{\partial rc}. \]

Further, \( v^*_1 \) is a concave function of \( n_0 \) (and hence \( rc \)), since a fixed labor supply generates decreasing returns on capital. As default-state capital levels are fixed, this is not true for \( v^d_1 \), which is linearly increasing in \( n_1 \). This in turn means that the function \( \frac{\partial v^*_1}{\partial rc} - \frac{\partial v^d_1}{\partial rc} \) is decreasing in \( rc \), and concave. As the amount of recapitalization increases bank net worth, each additional unit produces a lower gain over the default scenario, which suggests that recapitalization becomes less useful as a commitment device when banks are already well-capitalized.

Finally, the important addition relative to the earlier section is that now the sovereign also faces a change in its outstanding debt:

\[ \frac{\partial v^b}{\partial rc} = \frac{\partial}{\partial rc} \frac{1}{\beta} \left[ \frac{1}{q} \left( g + \frac{1}{1-\tau} rc \right) \right] \]

\[ = \frac{1}{\beta} \left[ \frac{1}{q(1-\tau)} + \left( \frac{\partial q}{\partial rc} \right)^{-1} \left( g + \frac{1}{1-\tau} rc \right) \right]. \]

The debt burden of the sovereign responds to recapitalization in two ways. First, debt outstanding increases at the rate \( \frac{1}{q(1-\tau)} > 1 \) per unit of \( rc \), where \( q \) is the bond issuance price given current default risk. Secondly, if the recapitalization leads to a change in default risk, and hence in bond price \( q \), this leads to
a revaluation of total government debt, i.e. funds raised. As a result, when recapitalization leads to lower default risk, the face value of debt outstanding, $b^*$ can fall even as total funds raised in period 0, given by $g + \frac{1}{1-\tau} rc$, actually increase.

However, as long as there are no discontinuities in the distribution of productivity $A_1$ (such as in a discrete distribution), small changes in $rc$ should not lead to large changes in default risk, and therefore the first term should dominate the effect. This true at any point outside of the transition between certain and stochastic default that, depending on the assumed distribution of $A$ can be quite abrupt.

Taking it all together, $v^*_1 - v^d_1 - v^b$ is a concave function of $rc$. For changes in $rc$ small enough, i.e. such that overall changes the ex ante default probability are small enough so that the revaluation effects of the total debt stock are negligible,

$$\frac{\partial v^*_1}{\partial rc} > \frac{\partial v^d_1}{\partial rc} + \frac{\partial v^b}{\partial rc} \approx \frac{1}{\beta} + \frac{1}{\beta q(1-\tau)}.$$ 

But as $rc$ increases and the incremental returns to capital in the repayment state decrease, this relationship will switch such that

$$\frac{\partial v^*_1}{\partial rc} < \frac{\partial v^d_1}{\partial rc} + \frac{\partial v^b}{\partial rc},$$

as $v^*_1$ increases less rapidly than the linear increases in $v^d_1$ and $v^b$ combined. Figure 2 illustrates the behavior of the three functions graphically. This describes the origins of the recapitalization Laffer curve.

Figure 2: Behavior of truncated value functions
Lastly, depending on the probability distribution of $A_1$ and the amount of government debt outstanding, $g$, the effects of recapitalization on the value of the total debt outstanding can be large. This is especially true when a small amount of recapitalization shifts the equilibrium from certain to stochastic default. However, since the revaluation effect follows the same direction as the underlying welfare gain, this only serves to amplify the dynamics highlighted in the simplified value functions above: If an increase in the amount of recapitalization increases difference between utility in the repayment versus the default state, this leads to a decrease in the productivity threshold for default, $\tilde{A}_1$. As discussed above, this reduces the probability of default and hence lowers the required repayment $b^*$ in period 1, which further improves the attractiveness of the repayment state relative to the default state. The opposite is true when a change in the recapitalization worsens the benefit in the repayment state relative to default.

Hence, even when the distribution of $A_1$ is prone to large or even discrete shifts, which could generate large revaluation effects for small changes in recapitalization, this only provides amplifying, not countervailing force on the valuation of the two possible outcomes.

4 Extensions

4.1 Domestic government bond holdings

In the discussion of the links between sovereign default and the domestic financial system, an often-discussed component is the amount of sovereign debt that is domestically held. In fact, in other models that have used the financial system to generate endogenous costs of sovereign default, domestic bond holdings are essential. Default in those models only affects the banking system through their holdings of sovereign bonds, which can lead to balance sheet losses as well as take away a main instrument for financial intermediation.

In the European crisis, high concentration of domestic bonds on bank balance sheets was considered one of the main reasons for sovereign risk contagion. However, as I show in the following, integrating domestic bond holdings into the model provides some additional complexity, but mainly strengthens the underlying dynamics.

First, I assume that banks start off with an exogenously determined stock of long-term domestic government bonds, denoted $b$. I then assume that these bonds can be pledged in international borrowing (akin to a repo transaction) and hence function just like bank net worth - however with the slight modification that domestic bond holdings are 'mark-to-market,' i.e. that their pledge value will fluctuate with the price $q$ of bonds issued to foreigners. This means that for a given $n_0$ and $b$, banks are facing foreign borrowing costs such that:
\[ R_1^* = \chi \left( \frac{k_1}{n_0 + qb} \right)^{\frac{1}{\beta^*}}. \] (23)

This makes government default appear indiscriminate, i.e. the government cannot selectively default. This distinction is not particularly meaningful however, since in the default case, capital is funded exclusively through domestic deposits. Bank net worth is therefore irrelevant for future bank funding costs. Since bonds are also repaid by taxing domestic households (the final recipient of terminal bank net worth), welfare is unchanged if discriminate default is assumed. If the government does not default, bonds are repaid through the same lump-sum taxes that fund the repayment of foreign bonds.

The function above already shows the amplification that domestic bond holdings provide in the transmission of sovereign risk. Increasing sovereign default probability from lower bank net worth is further amplified by the commensurate decline in the mark-to-market value of bond holdings, leading to a further rise of foreign funding costs. This also opens the door to mutual risk migration between banks and sovereign. When the sovereign’s creditworthiness increases exogenously, this boosts bank balance sheets, which in turn increases the sovereign’s repayment incentives even further. Similarly, were bank balance sheets to exogenously deteriorate, sovereign repayment incentives would take a hit, which would drive down bank net worth down another notch through marking-to-market its sovereign bond portfolio.

Figure 3: Probability of debt repayment with debt-financed recapitalization and domestic bond holdings

In practice, this makes the incentives in bank recapitalization stronger, since it makes repayment more beneficial for the government: Since government bonds are of no value in autarky (since domestic households
are simply paying back themselves), they can be used to attract foreign financing at cheaper rates when access is preserved.

All this would suggest that large domestic bond holdings are actually a positive for the effectiveness of bank recapitalization. There are two important qualifications to this insight: First, just as domestic bond holdings can strengthen government repayment incentives through a ‘virtuous cycle,’ this can generate substantial costs when the government overextends herself with an excessively large recapitalization. This suggests that in economies with large domestically held debt stocks, even more caution may be warranted in not overstepping the appropriate recapitalization size.

Second, exogenous shocks, either to public or private balance sheets, become systemic through bond holdings, and their effect increases with the share of the bank balance sheet that is taken up by domestic bonds. Since this paper is predominantly concerned with the optimal sizing of recapitalization transfers after a shock has already occurred, and since bond holdings are hence assumed to be exogenous, I do not expand on this line of exploration here.

4.2 CoCo bond funding

The model can be extended further to analyze the impact of the recent innovation to the liabilities side of bank balance sheets. Following the bank bailouts of the Great Financial Crisis, European banking regulators have pushed banks to increase loss-absorbing capacity. A popular tool here has been the issuance of contingent convertible (CoCo) bonds. These bonds promise to provide banks with additional equity buffers in bad states without diluting current equity holders at the time of issuance. Most commonly, these hybrid securities are structured with a trigger level based on the ratio of equity capital to risk-weighted assets. Once this ratio falls below a pre-specified level, CoCo bonds automatically convert into common equity, or in some cases are even written down partially or completely. Furthermore, even before the trigger level is reached, coupon payments on CoCo bonds can be discontinued without the need to make up for foregone payments later.

This mechanism provides banks with an additional, automatic source of capital or borrowing relief when capital levels are low and equity capital raising is difficult, explaining its popularity with regulators. Research such as Avdjiev et al. (2015) has shown that CoCo issuance has indeed served to reduce banks’ credit risk. Interestingly, while corporate CoCo bonds had first been suggested in the early 1990s following the wave of junk bond issuance, the European debt crisis brought with it calls for CoCo sovereign bonds, such as in Brooke (2013), in order to reduce the risk of sovereign defaults.

However, the model at hand shows that integrating CoCos into the bank balance sheet alone already

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serves to reduce sovereign default risk, without having to extend CoCo funding to the sovereign directly. I add CoCo bonds to the model as follows: Banks’ international borrowing is assumed to consist of regular funding and a fixed percentage share of CoCo funding. CoCos are assumed to be structured conservatively to avoid creating large gains in bad productivity states. Instead of being written off or being converted into bank net worth, I assume instead that CoCo bondholders are only paid the gross return on capital instead of the pre-determined gross borrowing rate if – without this alteration – bank capital were to fall below a pre-determined trigger level.

Bank net worth with CoCo financing in period 1 is then

$$n_1^{\text{coco}} = n_1 + (R^*_1 - R^k_1)s^{\text{coco}}d^* \cdot \mathbb{I}_{n_1 < \bar{n}^{\text{coco}}}$$

(24)

i.e. the normally computed net worth plus a ‘rebate’ of the bank funding cost in excess of the actual earnings on capital, scaled by the share of CoCo financing, denoted $s^{\text{coco}}$, if the normally computed net worth would fall below the trigger level $\bar{n}^{\text{coco}}$.

This leads to a reshuffling of the distribution of possible bank net worth levels in period 1. In the case of technology outcomes being normally distributed, the altered distribution of $n_1$ now has a much thinner left tail, and a bunching just above the net worth trigger level (Figure 4).

This change in the distribution of possible bank net worth outcomes has direct implications for the probability of default. If the CoCo trigger level is set appropriately, CoCo financing can drastically reduce the chances that bad technology outcomes produce large enough losses on bank balance sheets to make
sovereign default optimal. This is especially noticeable when initial bank net worth is low relative to the trigger. In those circumstances, this also reduces the amount of optimal government recapitalization, while increasing the probability of repayment nonetheless. Since foreign lenders carry some of the risk of bad technology outcomes at some pre-determined capital level, higher recapitalization levels - which make it less likely that this level is hit - will reduce the value of this risk-sharing mechanism (Figure 5).

5 Policy implications

Clearly, the model presented here is highly stylized and, by construction, omits a number of factors that matter in the assessment of policy, especially as it pertains to bank recapitalization (such as moral hazard, or bank run risk during financial panics). Still, the model allows to make some conjectures about optimal policy and also provides a framework to think about current policy issues.

First, the main insight provided here is that, at least for a small open economy in a currency union, there exists an optimal amount for bank recapitalization, and under- as well as overshooting this amount can be very costly. Exceeding this level, as arguably was the case in Ireland during the Great Financial Crisis, can be especially dangerous even when government finances are generally in order, and can induce a 'doom loop' of mutual contagion between the banking system and the sovereign.
Figure 6: Banking sector borrowing rates

Second, the model allows for a simplified discussion of recent ECB policy and compare them to the effects a banking union might have. Beginning with the OMT announcement in 2012, the ECB has responded to the escalation of sovereign default risk in the Euro area with policy action directed at controlling sovereign borrowing rates across the Euro area. More recently, the ECB’s quantitative easing policies have been explicitly targeted at lowering sovereign rates, as policy makers have recognized that sovereign risk premia in the worst-hit economies have hampered the transmission of monetary policy.

Lower sovereign rates show up in the model at hand in two ways - once directly as one component of bank borrowing costs, and as a boost to balance sheets by revaluing current domestic sovereign bond holdings. This combination has led to a dramatic reduction in private funding rates in the most vulnerable Euro area countries since 2012, as visible in the bank credit spreads computed by Gilchrist and Mojon (2014) and shown in Figure 6.

But while the sovereign rates of Italy and Spain have continued to decline in tandem up until recently (figure 7), bank credit spreads in the two countries have begun to decouple in 2015\(^5\). In the framework of the model, this suggests that bank capitalization in Italy is lower than in Spain, even after the net worth gains from the revaluation of sovereign bond holdings are factored in. Italy’s banks do in fact have a substantial problem with nonperforming loans, and recapitalization has been proposed. However, there has been resistance to this move in other Euro area member states, and the model at hand can provide some

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\(^5\)The graph shows aggregate interest rates as calculated in Gilchrist-Mojon. Since this measure includes the full maturity structure of bank debt, this is not comparable directly in levels with the sovereign 10-year yields in figure 7.
indication why: Since the sovereign rate is practically capped by ECB policy, there are no market 'checks' on the scale of the fund transfer. Under the assumption that the ECB continues to cap sovereign rates, Italy would be incentivized to funnel more money into its banks than would be optimal under normal conditions, where overshooting the optimal amount of recapitalization would impose costs on the sovereign. Instead, these costs are now carried (to some extent) by the ECB, since its current holdings of Italian government bonds would have fundamentally lost value even as market prices stay constant.

While this "socialization" of losses has produced some complaints from other European countries, it is important to note that this is in essence how a Euro area banking union would work. A banking union would allow for bank recapitalization in an individual country without forcing it to raise additional funds individually. As such, the negative effects from bank recapitalization (and the danger of 'doom loops') would be avoided, potentially creating large cost savings for the countries involved. Finally, at least in this model, individual bank recapitalization when sovereign rates are capped, such as in the current Italian situation, arguably amounts to a banking union through the back door.

6 Conclusion

In this paper, I present a model of the interaction between the sovereign and the domestic banking system, where funding costs for the local economy and sovereign default incentives are closely intertwined. I show that weak private balance sheets in the banking system can be contagious for sovereign default risk, even
when the sovereign is relatively stable. The model incorporates banking system recapitalization, and points to the risk of excessively large transfers from public to private balance sheets - but also to the virtuous cycle appropriately-sized transfers can have. If done correctly, bank recapitalization may even pay for itself. In the sum, the model is able to match a number of different scenarios observed in the European sovereign debt crisis, where the close connection of sovereign and financial system loomed large.

I also analyze the role of domestic sovereign debt holdings, and find to amplify this mechanism. The model further show how using CoCo bonds in funding the banking system can serve in reducing sovereign default risk, and reduce the need for government recapitalization funds. Finally, the model also allows for a simple way to think about recent ECB policy targeting sovereign yields, as well as the implications these policies might have for a 'back door banking union.'
References


Appendix A  Solution

In order to determine when default is optimal for the government, I first need to determine the different consumption streams in the default and the non-default case. I operate from the assumption that in period 1, banks choose foreign borrowing rather than domestic deposits.

A.1 Utility under default

I work backwards, starting in period 2. In this period, households get to consume their wages, their deposits from the previous period plus interest, as well as the left-over equity in the banks. Since in this case, there is no foreign funding for the banks, households receive the gross returns on all capital in the economy:

$$c^d_2 = w^d_2 + R^d_k k^d_2.$$ 

Furthermore, since banks have to rely on domestic deposits and pay the required funding costs, I can solve for the capital level in period 2 given default by setting equal the required net rate of return on domestic deposits with the marginal return on capital:

$$R_2 = \frac{1}{\beta} = R^d_k = \alpha A_2 (k^d_2)^{\alpha - 1}.$$ 

This yields period 2 capital level:

$$K^d_2 = \left( \frac{1}{\alpha \beta A_2} \right)^{\frac{1}{\alpha}}.$$ 

Since technology level $A_2$ is certain, the return on capital is deterministic and identical with the return on deposits. Rewriting $c^d_2$ with the known capital stock $k^d_2$ gives:

$$c^d_2 = A_2 (K^d_2)^{\alpha}.$$ 

Final household consumption for period 2 is hence pinned down by the known technology level and parameters of the model. However, the required capital stocks have to be raised solely from domestic deposits and bank net worth in the previous period. Given wages and bank capital, this pins down household consumption in period 1:

$$c^d_1 = w_1 - (k^d_2 - n_1)$$

Under assumption 2, I can aggregate households’ total consumption stream in the non-default case, taking as given technology realization $A_1$, capital choice $k_1$, and bank initial net worth $n_0$, which includes the
original recapitalization amount. The total utility for the household given government default is described by:

\[ u(c_1^d, c_2^d) = w_1 - (k_2^d - n_1) + \beta(w_2^d + R_2^k k_2^d), \]

where both \( w_2^d \) and \( R_2^k \) are determined by \( k_2^d \), and hence are functions of model parameters.

A.2 Utility in absence of default

Again, I can solve backwards. Household consumption in period 2 in absence of default consists of wages and left-over bank equity. Domestic deposits from the previous periods are 0, since otherwise default would always be optimal. This yields:

\[ c_2 = w_2 + (R_2^k - R_2^*) k_2 + R_2^* n_1. \]

Since there is no uncertainty in period 2, capital returns and borrowing costs are the equal to each other, so the equation reduces to:

\[ c_2 = w_2 + R_2^* n_1, \]

where both wages as well as interest rate \( R_2^* \) are determined by the level of capital chosen by the banks. The bank sets equal the net cost of financing to the net marginal return on capital such that:

\[ R_2^* = \chi \left[ \frac{k_2}{n_1} \right] \frac{1}{\beta^*} = \alpha A_2^{k_2^{*} - 1}. \]

Therefore, I can solve explicitly for \( k_2 \) as a function of bank net worth \( n_1 \). As a result, household consumption at \( t=2 \) is also a deterministic function of \( n_1 \).

Moving on to period 1, in the case of no default and continued foreign borrowing, households consume everything that is not paid to the government as lump-sum tax to repay its short-term debt:

\[ c_1 = w_1 - b^* \]

In aggregate, the total streams of consumption utility for households add up to:

\[ u(c_1, c_2) = w_1 - b^* + \beta(w_2 + R_2^k n_1). \]

A.3 Default decision

As shown above, household utility in both the default and the non-default state are known with certainty in period 1, conditional on the amount of net worth in the banking system, \( n_1 \). This value only depends
on past variables and the realization of technology $A_1$, which is known before the government has to decide whether or not to default. This makes the government’s optimization problem in period 1 straightforward: It will pick the scenario under which household utility is highest. Hence, for the government not to default in period 1, the following inequality - taking $n_1$ as given - needs to hold:

$$u(c_1, c_2) \geq u(c_1^d, c_2^d).$$

Plugging in from above, this yields:

$$w_1 - b^* + \beta (w_2 + R^k_2 n_1) \geq w_1 - (k_2^d - n_1) + \beta (w_2^d + R^{k,d}_2 k_2^d)$$

This simplifies to:

$$\beta [(w_2 - w_2^d) + (R^k_2 - R^{k,d}_2) \cdot n_1] \geq b^* - (1 - \beta R^{k,d}_2)(k_2^d - n_1).$$

Since the return on capital in period 2 in default is necessarily the inverse of the household discount factor, this further simplifies to the no-default condition:

$$\beta [(w_2 - w_2^d) + (R^k_2 - R^{k,d}_2) \cdot n_1] \geq b^*,$$

i.e., the government defaults as long as the discounted difference in capital and labor returns in period 2 in the non-default state relative to the default state is not enough to compensate households for the repayment of foreign debt in period 1. Importantly, while wages increase with higher levels of capital that can be achieved through foreign borrowing, capital returns fall. Hence, the increase in labor income from higher capital levels does not only have to be enough to repay government debt, but also to make up for the decreased return on bank capital $n_1$. Further, the cost of capital in absence of default is again determined by the amount of bank capital, since it directly enters the financing cost. Finally, bank net worth is a function of technology outcome $A_1$. Therefore, under the assumption that external borrowing is optimal for banks at time 0, bank capital in period 1 is derived as follows:

$$n_1 = R^k_1 k_1 - R^d_1 d_0 = (R^k_1 - R^*_1)k_1 + R^*_1 n_0.$$  

As visible above, the banking system is carrying technology risk on its books: While the capital return $R^k_1$ is dependent on the realized technology state, $R^*_1$, the funding rate, is agreed-upon before the resolution of uncertainty, and is therefore based on $\mathbb{E}(A_1)$. In period 0, banks set equal their cost of financing to the expected return, such that:
\[ R_1^* = \chi \left( \frac{k_1}{n_0} \right) \frac{1}{\beta^*} = \mathbb{E}(R_1^k) = \alpha \mathbb{E}(A_1)k_1^{\alpha-1}. \]

This means that in a bad outcome, bank net worth is eroded, which lowers the value of having access to international capital markets going forward. To clarify, I can rewrite the above no-default condition as follows:

\[ \beta((1 - \alpha)A_2(k_2^2 - (k_2^d)^{\alpha}) + \alpha A_2(k_2^{\alpha-1} - (k_2^d)^{\alpha-1}) \cdot n_1) \geq b^*, \]

where \( k_2^d \) is defined as in section 4.1, and independent of period 1 values, but where \( k_2 \) is directly dependent on the value of \( n_1 \) (as in 4.2), and hence the realized technology state. Subsequently, for a given \( n_0 \), I can solve the condition to find the level of required debt repayment \( b^* \) for all possible values of \( A_1 \), at which the government is indifferent between defaulting and not defaulting.