Recruiting, Preparing, and Retaining STEM Teachers for a Global Generation

Jacqueline Leonard, Andrea C. Burrows and Richard Kitchen (Eds.)

There is a critical need to prepare diverse teachers with expertise in science, technology, engineering, and mathematics (STEM) with the skills necessary to work effectively with underrepresented K-12 students. Three major goals of funded STEM programs are to attract and prepare students at all educational levels to pursue coursework in the STEM content areas, to prepare graduates to pursue careers in STEM fields, and to improve teacher education programs in the STEM content areas. Drawing upon these goals as the framework for *Recruiting, Preparing, and Retaining STEM Teachers for a Global Generation*, the 15 chapters contained herein highlight both the challenges and successes of recruiting, preparing, and sustaining novice teachers in the STEM content areas in high-need schools.

Recruiting, retaining and sustaining highly-qualified teachers with expertise in STEM content areas to work in hard-to-staff schools and geographic areas are necessary to equalize educational opportunities for rural and urban Title 1 students. High teacher turnover rates, in combination with teachers working out-of-field, leave many students without highly-qualified teachers in STEM fields. Most of the chapters in this volume were prepared by scholars who received NSF funding through Noyce and are engaged in addressing research questions related to these endeavours.

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CHAPTER 11

Exemplary Mathematics Teachers for High-Need Schools: A Two-Way Mentoring Model

Lillie R. Albert

Abstract

This study documents the experience of eight beginning teachers, eight experienced teachers, and six mathematicians participating in a professional learning community. The hallmark of the professional learning community is a two-way mentoring model, designed to incorporate content and pedagogical knowledge for teaching mathematics, whereby the beginning teachers have a mathematician and an experienced practicing teacher as mentors. Applying Vygotsky's concept of sociocultural historic theory, the mentor-mentee relationship is examined through the lens of intersubjectivity. Findings suggest that the development of intersubjectivity can move the mentoring process ahead, where this relationship is characterized by achieving a common understanding of mathematical activities and ideas.

Keywords

mentoring – professional learning community – subjective learning – intersubjectivity – collaboration – interactions – mathematician mentors

Introduction

The nation needs more outstanding mathematics teachers, which some suggest is critical to its competitiveness in an increasingly global economy (National Research Council, 2007, 2009; Stewart, 2012; U.S. Department of Education, 2016). This need is magnified in high-need school districts; at present, students in such districts are less likely to become part of our nation's future STEM workforce (National Research Council, 2009). With the reasoning-intensive *Common Core State Standards for Mathematics* (CCSS-M) just beginning to be implemented, it is critical to increase the nation's supply of highly-qualified

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teachers (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO]. The CCSS-M reflects essential mathematics knowledge and skills and an understanding of student learning. To successfully apply the CCSS-M in the classroom and to develop the human resources needed for mathematical leadership, our nation needs more teachers who are well prepared to teach the demanding, reasoning-intensive curriculum. These teachers will require mathematical knowledge and experience, which integrates subject matter knowledge with pedagogical content knowledge (Silver, 2003, Sowder, 2007; Supovitz & Turner, 2000). Therefore, we need effective professional learning communities for new and experienced teachers that will assist them in becoming highly-qualified instructional leaders and mentors (National Research Council, 2009).

The purpose of this study is to explore and document the experiences of eight beginning teachers and their mentors as they build relationships through a mentoring model, referred to here as the *Two-Way Mentoring Model*, in which the beginning teacher has an experienced practicing teacher and a mathematician as mentors for a period of five years. This study addresses how a two-mentoring model affects the building of mentor-mentee relationships, and specifically, how they achieve intersubjectivity. Intersubjectivity results from interaction whereby all the participants develop shared knowledge, meaning or understanding, which is mediated by a common goal and task (Albert, 2012; Eun, 2008; Wertsch, 1985). In this chapter, I present a brief discussion of the relevant literature about the role professional development plays in mentoring, shaping, and maintaining beginning teachers in the profession. Next, I provide a sketch of the theoretical framework, outlining conceptual and theoretical assumptions regarding the intersubjective nature of learning and understanding, while also considering how intersubjectivity could be achieved through collaborative conversations and interactions. Then, I present an overview of the methodology of the inquiry process, which includes a brief introduction to Wilber's Integral theoretical perspective (1995, 1998) as an analytic tool that foreshadows the theoretical framework and offers a graphical way to display major findings. The final two sections are comprised of a discussion of the findings with relevant conclusions and implications.

Mentoring in Professional Learning Communities

An important component that is necessary to assist beginning and experienced mathematics teachers in staying up-to-date in their field is their continued engagement in professional activities that include "strong content knowledge, practical pedagogical [preparation], and ongoing mentoring and education" (National Research Council, 2009, p. 134). The most compelling argument for providing teachers with experiences in which the focal point is mathematical knowledge for teaching and continuous mentoring is highlighted in research by Silver (2003), Sowder (2007), and Supovitz and Turner (2000). An important conclusion from this research is that effective professional development learning communities may influence teachers' understanding of content and subsequent pedagogical practices. Darling-Hammond (2009) and her colleagues summarized research in a report on the status of professional development in the United States and abroad. Their research concludes that "sustained and intensive professional development for teachers is related to student achievement gains. Effective professional development is intensive, ongoing, and connected to practice; [it] focuses on the teaching and learning of specific academic content" (p. 5). A key element is the need to involve mathematicians in professional and mentoring activities specifically designed for K-12 teachers (Conference Board of the Mathematical Sciences, 2012; Moyer-Packenham, Kitsantas, Bolyard, Huie, & Irby, 2009; Heaton & Lewis, 2011). This study is grounded in the assumption that when mentoring has a strong focus on pedagogy and content, it can benefit beginning teachers and their students.

Research on teacher mentoring emphasizes that it takes between three to five years for beginning teachers to develop effective teaching practices, which may lead to increased teacher retention and revitalization (Appleton, 2008; Feiman-Nemser, 2001; Feiman-Nemser & Carver, 2012; See, 2014; Simonsen, Luebeck, & Bice, 2009; Stanulis & Floden, 2009). To become competent teachers, beginning teachers need to engage in professional learning experiences with mentors who are well-prepared, and experienced teachers who effectively "combine the knowledge and skills of a competent classroom teacher with the knowledge and skills of a teacher of teaching" (Feiman-Nemser, 2001, p. 1036). Thus, from this research, we can make four major assumptions. First, mentors play an essential role in supporting beginning teachers' learning over time, which requires that mentors receive relevant preparation regarding how to be effective mentors to beginning teachers. Further, research suggests that mentor characteristics influence the mentoring experience of beginning teachers (Garza & Harter, 2016; Hansford, Tennent, & Ehrich, 2003). Yet, in some mentoring relationships, both the mentor and mentee unexpectedly develop productive relationships with each other and both personally and professionally benefit from the mentoring process (Ehrich, Hansford, & Tennent, 2004).

Second, if the mentoring experience is long-term, then beginning teachers tend to develop self-assurance and success that may lead to contentment with their career (Ingersoll, 2012; Ingersoll & May, 2012: Ingersoll & Strong, 2011).

Third, the mentoring program structure is fundamental to its success. It seems that outcomes evolving from mentoring and/or teacher induction programs are based on the quality of the program and not on the frequency of meetings or interactions (Richter, Kunter, Ludtke, Klusmann, Anders, & Baumert, 2013). Fourth, mentoring programs are favored by professional learning communities in both local and national school districts, because they may help beginning teachers continue in their professional career paths (Tschannen-Moran & Tschannen-Moran, 2011). A conclusion drawn from the research noted here is that although mentoring programs vary in objectives, processes, and structures, what appears to be a common thread for these programs is the implementation of a one-on-one mentor model or relationship in which an experienced educator supports a beginning teacher.

Achieving Intersubjectivity as a Framework for Two-Way Mentoring

An ultimate challenge in understanding mentoring relationships between experienced and beginning teachers and university mathematicians and beginning teachers is recognizing that neither the mentors nor mentees function in isolated subjective milieus; rather, what we find is that in the learning community, they function in intersubjective ways (Biesta, 1994; Esch & Tillema, 2015). Although individuals bring their unique sociocultural and mathematics backgrounds to the learning community, they do not function in isolation, but instead form intersubjective contexts (i.e., involving a social or an interactive process) by placing themselves within the lived experiences or learning communities of the other. The formation of intersubjective learning communities for mentors and their mentees is rooted in Vygotsky's (1978) construct of the *zone of proximal development* and referred to by Tharp and Gallimore (1994) as *teaching as assistance performance*.

An essential component to understanding Vygotsky's theoretical construction of the zone of proximal development (i.e., assistance performance or scaffolded learning) and its role in mentor-mentee dialogue and interaction is *intersubjectivity*. Vygotsky's (1978) zone of proximal development is the difference between what learners can accomplish independently and what they can accomplish with the assistance of more knowledgeable individuals. This theory helps to explain communicating processes that afford opportunities for mentor-mentee relationships to work with each other in pursuit of knowledge, skills, and ideas through social processes, such as conversations or interactions. These communicating processes create an opportunity through which a group of learners begin a task, activity, or conversation with different understandings but ultimately, through interaction, achieve a shared understanding or a "state of intersubjectivity" (Rommetveit, 1979, p. 94). Intersubjectivity (e.g., a shared meaning or understanding) results from this interaction, as the perspectives of all the participants intertwine, mingle, transform, and coalesce to develop shared meanings, which is mediated by a common goal and task. However, recent interpretations propose that developing "shared meanings" does not necessarily mean that the participants attain "identical conceptual structures," but rather, that "their conceptual structures are sufficiently compatible for successful reciprocal assimilation" (Steffe & Thompson, 2000, p. 193). In other words, communication allows participants to achieve what Rommetveit (1979), more appropriately, terms "states of partial intersubjectivity" in which their ideas and conceptions are well matched but may not be exactly the same. For example, a mentor and mentee may have a shared understanding that perseverance is a key element in helping students become more resourceful mathematical problem solvers. However, their conceptions of how to help students persevere with mathematics are not exactly the same. Explicitly, when students struggle with a challenging problem to the point of giving up, should modifying the task reduce the complexity of the problem? The differences between the mentor and mentee may occur in understanding how the task might be modified. Thus, they are in a state of partial intersubjectivity, suggested by Rommetveit (1979).

To exhibit intersubjectivity and to communicate effectively during joint activity, it is fundamental that the learners work toward the same goal (Berk & Winsler, 1995; Bruner, 1996). Yet, it is not essential for the parties to remain in a constant state of agreement or to reach a common end solution (Nathan, Eilam, & Kim, 2007). The current understanding of intersubjectivity, according to Nathan, Eilam and Kim (2007), is referred to as the *participatory view*, which emphasizes that both consensual agreement *and* disagreement are important in joint activity or discourse. This challenges the traditional view, which strictly judges agreement as advantaging and disagreement as disparaging. In fact, the role of disagreements in cognitive development has been established as a significant one. Mentoring is an uneven process in which both the mentor and mentee make immediate qualitative shifts. Therefore, intersubjectivity or joint awareness, in the service of joint activity, maintains the multiplicity and differences of the mentors and mentees. In the Vygotskian view, intellectual growth or understanding is advanced unevenly by "differentiation and contrast, not by reproduction of similarities" (Frawley, 1997, p. 90).

As a point of illustration, if a mentor and mentee aspire to reach a common understanding of an observed mathematics lesson, which might lead them to expressing different ideas about how the lesson progressed, what we might see is that both might be pursuing the lesson from individual lenses, e.g., teaching and observing, respectively. However, when they come together to engage in conversation about the observed lesson, their divergent points of views are challenged in respectful and productive ways, driving each other to present clearer and more articulate arguments. Thus, disagreements encourage critical discourse and guide the pair to consider more sophisticated ways of arguing their differing ideas; consequently, what may be evidenced is that intersubjectivity played an essential role in shaping the conversation between them, leading to a clearer understanding. This notion places Vygotsky's position of intersubjectivity at the "heart of learning and consciousness itself" (Nathan, et al., 2007 p. 524). For Vygotsky (1986), "We use consciousness to denote awareness of the activity of the mind–the consciousness of being conscious" (p. 170). Hence, we can see that being conscious of others' perspectives or ideas shape and contribute to the conversation, which may lead to intersubjectivity.

The importance of intersubjectivity in sociocultural contexts is well documented (Tharp & Gallimore, 1994; Matusov, 1996; Nathan et al., 2007). The participatory view of intersubjectivity focuses on "the coordination of individual participation in joint sociocultural activity" (Matusov 1996, p. 26). Social interactions that occur within the learning community provide the context for shared thinking. It is through social interactions that participants use "communicative tools" to negotiate meaning as they strive for a shared notion of the situation (Albert, 2000, 2002). Intersubjectivity therefore, becomes, "a condition for, or characteristic of, true human communication, implying for the interlocutors a reciprocal faith in a shared experiential world" (Smolka, DeGoes, & Pino, 1995, p. 169). Achieving intersubjectivity means mentors and their mentees must do more than just work together or allow one person to dominate the discourse. They must share power, "where inequality between partners resides only in their respective levels of understanding" (Driscoll, 1994, p. 236). Sharing the power or authority in joint conversation reduces the subjective difference between participants. New possibilities and opportunities are then opened up for mentors and their mentees, which may lead to a better understanding of the dialogue.

The study of intersubjectivity is becoming progressively important with the growing awareness of the social nature of human thought and development (Albert & McKee, 2001). A major purpose of this study is to examine how master teachers and mathematicians as a unit, working within a two-way mentoring model, scaffold and mentor beginning teachers while they are preservice teachers as well as during their first four years as beginning teachers. The Two-Way

Mentoring Model integrates pedagogy, content and pedagogical content knowledge, providing mentoring and scaffolding through these three domains.

The Larger Study Context

What is problematic with past and current mentoring relationships that are defined by one-on-one models is professional mismatch and non-productive relationships (Ehrich et al., 2004). For example, it is not uncommon for a beginning secondary mathematics teacher to be matched with an experienced teacher who is not a teacher of mathematics. This is frequently seen at the preservice level where experienced teachers not in the field of study of the student teachers are assigned to supervise them. The conjecture is that these experienced teachers are good instructional leaders and knowledgeable in effective pedagogical practices. This practice continues into the induction or mentoring phase, and it is guided by the notion that beginning teachers will receive the emotional and professional support they need as they enter the profession, resulting in teachers who are empowered and competent to teach in their field. Yet, as Ehrich et al. (2004) highlight, this result is not always the outcome that emerges, especially since many beginning mathematics teachers (close to 50% within the first five years) leave, or are in the process of leaving their position as practicing teachers. We believe that the lack of long-term support for beginning teachers, which includes an intensive focus on teaching mathematical content, contributes to this problem.

We acknowledge the need for teachers with strong mathematical content knowledge for teaching, a resilient understanding of mathematical pedagogy, and interest in and understanding of high-need schools. These characteristics may be developed through a strong mentoring model geared toward keeping beginning mathematics teachers current within their field as they continue to develop professionally. The major objective of this study is to document the experiences of eight beginning teachers, eight experienced teachers, and six mathematicians participating in a professional learning program. The hallmark of this program is the Two-Way Mentoring Model designed to incorporate content and pedagogical knowledge for teaching mathematics whereby the beginning teachers' mentors are a mathematician and an experienced practicing teacher.

The Two-Way Mentoring Model

The Two-Way Mentoring Model ("Model") is an innovative model where a beginning secondary mathematics teacher's mentors are an experienced practicing mathematics teacher and a mathematician (current or retired). In most cases, but not all, the mentees have the same set of mentors, starting during their preservice year while studying for their Master's degree in mathematics education, and continuing during their first four years of teaching. For consistency, the beginning teacher is referred to as a Teaching Fellow (TF), the experienced teacher as a Master Teacher (MT) and the mathematician as a Mathematician Mentor (MM). The mentors observe and conference with their mentees several times throughout the academic year. For MTs and MMs, this Model provides an opportunity to meet with the beginning teacher and other practicing teachers; to experience the range of challenges encountered by mathematics classroom teachers on-site; and to reconsider their own pedagogy in light of teacher preparation. For the TF, the Model offers additional expertise related to content and pedagogical content for teaching mathematics. Though the primary purpose is to offer support for teaching mathematics content and pedagogy, the Model acknowledges that content knowledge for teaching mathematics cannot be divorced from crucial issues, such as school culture and curriculum frameworks. Thus, during conversations and debriefings before and after an observation or during the mathematics practice seminars, the mentors offer support and give advice on curricular issues to support and guide their mentees during the critical stage of their professional development. Although TFs also receive advice from other individuals with whom they are working, such as administrators, department chairs, and other classroom teachers, their mentors are there to help them be cognizant of the big picture in regard to mathematics teaching and learning and to help them stay committed to the teaching profession. Mentors also benefit from the relationship by learning practical methods for mathematics pedagogy and gaining a better understanding of school cultures, teacher expectations, and the realities of teaching in high-need schools.

The Professional Development Program

The study is an extension of a much larger project that supports Teaching Fellows (beginning teachers) and Master Teachers (experienced teachers). It is a subset of a much larger study, currently, a six-year funded project by the NSF. Working together with two non-profits, the goal is to address the needs of highly-qualified mathematics teachers in high-need school districts. TFs and MTs engage in systematic professional development through a series of seminars that seamlessly interweave mathematics content, pedagogical content knowledge, and pedagogy. As the program advances, the MTs increase their leadership roles and responsibility for these professional learning activities and later extend them to their schools and districts. The program is based on prior successful models of mathematics teacher preparation programs that have a high teacher retention rate in urban districts (Luft, Bang, & Roehrig, 2007; Simonsen et al., 2009). It consists of a coherent program of professional development for teachers centered on mathematics, urban teaching support, and student learning through the involvement of teachers, mathematicians, and mathematics educators working as partners.

Methods and Procedures for the Present Study

A multicase study approach (Stake, 2006) is used to provide documentation of the mentoring experiences of eight beginning teachers and their mentors; the composition of a case involves a beginning mathematics teacher mentored by an experienced mathematics teacher and a mathematician. For the purposes and objectives of this study, case composition is referred to as two-way mentoring. Because the mentoring program is situated at eight different sites, and to explore as well as to understand the phenomenon of two-way mentoring, which is grounded in the theoretical perspective of intersubjectivity, the cases are considered as a collection. Limiting the number of cases to a collection allows for exploration of the mentoring experiences to a reasonable depth within the scope of time and resources that are available for this study. Stake (2006) suggests that in multicase study research, "the single case is of interest because it belongs to a particular collection of cases," representing a common feature, in which the "cases in the collection are somehow categorically bound together" (pp. 4–6). In this study, the professional learning program, in which two-way mentoring is embedded, bound the cases across spaces and sites.

Setting and Participants

The professional development context for this research uses a sociocultural lens to position mathematical content and practices as initially promoted by Vygotsky (1978, 1997), and later by the work of Davydov (1995, 1998), and then Goos (1999, 2005). Sociocultural practices, according to this research, need to engage participants in activities in which they acquire knowledge that also involve them in activities that further their intellectual development of content and pedagogy. Therefore, during the professional mathematics practice seminars, opportunities for social interaction were provided to benefit the participants' cognitive and social development (Albert, 2012).

The setting for this study is a professional learning community of beginning and experienced secondary mathematics teachers who serve urban, ethnically diverse, and low-income student populations within public schools located in the Northeast. In addition, mathematicians from the Arts and Science Mathematics Department at the university level are part of the learning community. The professional learning community included two spaces: the *mathematics practice seminars and colloquia*, which offer opportunities for engagement in pedagogical and content related activities, and the *classroom* where the teachers teach and the mathematicians and experienced teachers observe. These two spaces allow the researcher to focus on teachers' mathematics knowledge and their pedagogical knowledge of teaching mathematics. Sixteen teachers and six mathematicians participated in this study. Of the teachers and mathematicians who participated, the majority were white and female with mathematics teaching experience ranging from a few years (<4 years) to many years (>17 years); about one-third of participants have fewer than four years of experience, and those with more than 17 years of experience comprise a much smaller percentage of the total. All but one of the teacher participants have master's degrees in mathematics education, and the mathematicians have terminal degrees in mathematics and currently teach or are retired from university positions in mathematics. Table 11.1 presents a summary of the descriptive characteristics about the teachers and mathematicians who participated in this study.

TABLE 11.1 Participants' characteristics

	Gender		**Race**		**Degree**		**Teaching experience**	
Teaching	Male	25%	Asian	12.5%	Masters	100%	< 4 yrs.	100%
Fellows			Black	12.5%				
***n*=8**	Female	75%	White	75%				
Master	Male	50%	Latina	12.5%	Masters	100%	< 4 yrs.	0%
Teachers			Black	37.5%			4–10 yrs.	50%
***n*=8**	Female	50%	White	50%			11–15 yrs.	50%
Mathematicians	Male	33%	White	100%	Ph.D.	100%	5–10 yrs.	17%
***n*=6**							11–16 yrs.	17%
	Female	67%					>17 yrs.	67%

This group of participants, which includes eight beginning mathematics teachers, i.e., Teaching Fellow (TF); eight experienced mathematics teachers, i.e., Master Teacher Mentors (MF) and six mathematicians, i.e., Mathematician Mentor (MM) were selected to represent a range of mathematical content taught and student performance levels, such as low, regular, honors, and AP, consisting of subjects from Algebra I to AP Calculus. It is anticipated that their experiences in these different classroom settings could provide insights into how they dealt with pedagogical and content challenges and opportunities. Pseudonyms are used for all participants mentioned in this study.

Data Collection and Analysis Procedures

The mentoring process started while the TFs were enrolled in their secondary mathematics methods course. The secondary mathematics methods course integrated a range of techniques (e.g., modeling, hands-on tasks, graphic organizers) and teaching practices (e.g., collaborative groups, strategies for teaching content and language objectives, teaching scenarios) to help prospective mathematics teachers learn and understand how to make mathematics concepts clear to learners. In this course, prospective teachers examined strategies, techniques, and research-based ideas for teaching content, technology resources, assessment practices, and cultural and academic diversity, including the State Curriculum Frameworks and the CCSS-M. A major purpose of this study is to examine how master teachers and mathematicians as a unit, working within a two-way mentoring model, scaffold and mentor beginning teachers while they are preservice teachers, and during their first four years of teaching. The Two-Way Mentoring Model integrates pedagogy, content and pedagogical content knowledge, providing mentoring and scaffolding through these three domains.

In this study, the theoretical framework of achieving intersubjectivity guided data collection, as the aim is to understand this phenomenon and to make sense of participants' perceptions regarding mentoring. Using benchmark data from the larger study, documentation relies on three main data sources: (a) fieldnotes that focused on the interactions and conversations of participants for eight practice seminars each year over a three year period; (b) one 40-minute individual interview with all of the participants, two follow-up questions six months after the initial interview with the MTs and TFs, and two 45-minute focus group interviews—one focus group interview with MM and one with MTs and TFs; and (c) at least two observations (with protocols) of beginning teachers by the MT and MM mentors each year. A semi-structured observation protocol template was designed to capture content and pedagogical activities and interactions, regarding what the teacher and students do during the lesson. The observation protocol template also included a section to record specific and actionable comments that the MT and MM would share later with the TF, during the debriefing session.

Data Analysis

The semi-structured interviews provide the participants a way to offer insights about their experiences and understanding of the Two-Way Mentoring Model process. Observational field notes provide information about the emerging mentoring relationships and interaction between mentors and mentees during the practice seminars; whereas, the observations of the beginning teachers' instruction provide information about their teaching practices, including how they engage their students in learning and understanding content. Transcribed observational and interview data and audiotaped discussions were used to examine participants' perceptions and constructions of their understandings regarding the effectiveness of the Two-Way Mentoring Model as a framework for mentoring and scaffolding beginning teachers in their development and understanding of content and pedagogical content knowledge for teaching mathematics. In addition, all participants' interview and observation data, including fieldnotes in their contextual form, were reviewed and coded thematically. The theoretical framework of achieving intersubjectivity guided the coding process and findings are organized using quadrants based on Wilber's (1995, 1998) Integral Theory.

Integral, according to Wilber (1995), involves an all-inclusive process of holistic and balance perspectives. Wilber (1998) argued that all phenomena could be categorized according to four views or dimensions: interior, exterior,

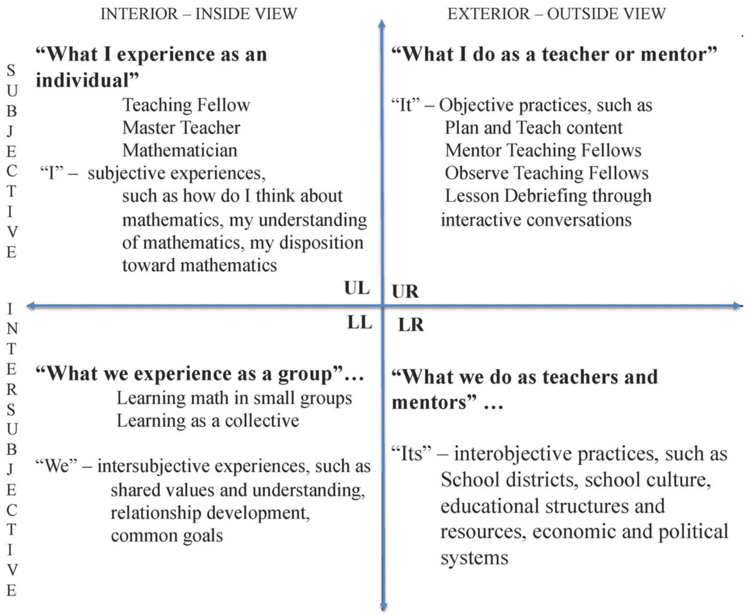

FIGURE 11.1 Four quadrants of two-way mentoring (adapted from Wilber, 1995, 1998)

individual, and collective. Figure 11.1 is an adaptation of Wilber's organizational structure to the Two-Way Mentoring Model, which is built on the ideas of achieving intersubjectivity. What is important to this structure is that each phenomenon is discernible in all four quadrants, and a modification or adjustment in any quadrant influences the other three quadrants (Wilbert, 1995). For example, an individual's internal experiences influence what the individual does as a classroom teacher, what is done as a group as well as what is done as teachers or mentors. Although there might be shared values and relationships within a group, external views, such as school culture and policies influence the group, incorporating the individual teacher's thinking and classroom practices. Examples explicit to the research are presented in the findings and discussion sections to show how Wilber's Integral Theory is a useful angle from which to understand the mentor-mentee relationship.

Findings

In this section, major findings are organized around themes and displayed through quadrants. First, themes were identified to characterize the mentor-mentee relationship of the participants. Second, as stated in the previous section, the use of quadrants applied here is grounded in Wilber's (1995, 1998) Integral Theory. Taken together, the goal here is to apply this approach to illuminate through themes and to reveal through quadrants how the participants engaged in professional activities that served as a catalyst for achieving intersubjectivity.

Intersubjective Learning: The Collective View

MFs came together to broaden and deepen mathematical and pedagogical content knowledge in the mathematical practice seminars to focus on content teaching and mentoring, which created collaborative learning contexts. In these contexts, TFs and MTs, occasionally joined by mathematicians, approached a problem-solving task with different understandings of the underlying concepts embedded problems. However, through the sharing of their knowledge about the task, they arrived at an intersubjective or a collective view regarding the task. Analysis from a Vygotskian perspective suggests that it is the social interactions and communications about the mathematics that may have led to a shared conception about the problem. These shared notions extended beyond the seminars into classrooms, schools, and districts. What the teachers learned in the practice seminars were in juxtaposition to the realities of what they are required to do at the school and district levels.

Although these realities might be viewed as disconnected experiences, they are all affected by the development and enhancement of the two-way mentoring process. For example, Figure 11.1 shows the four quadrants illustrating four primary perspectives through which the participants might experience mathematics teaching and learning as a mentor or mentee. To illustrate the four perspectives, representative examples from the data are presented.

The four perspectives are represented graphically considering four levels: the upper left, lower left, upper right and lower right, or the subjective, intersubjective, interior and exterior views, as illustrated in Figure 11.1. In the subjective or upper left quadrant, the participants are looking inward and considering or thinking about the teaching and learning of mathematics more from their own individual perspectives. For example, when a TF was asked to reflect on how she views receiving feedback from her mentoring teacher and mathematician, she replied, "I think [that] in person meetings are much more efficient than online feedback, I guess." Thus, this insight represents her perceptions after she had experienced receiving feedback electronically. Another illustration is a TF considering if he had any advice to give his mentors. He elaborates:

> I want to engage in conversations about content knowledge and I also have to accept that he is a mathematician and he hasn't been a [high school] teacher, he hasn't been in the classroom. So, I think I'm still doing a lot of the work there, figuring out how that relationship can work. So, I don't know if I necessarily have something to advise the mathematician about.

Thus, these examples represent subjective experiences and interior thoughts or memories of the individual TFs, the mentees. These thoughts and memories, as well as similar ones expressed by the other mentees, are their subjective states of mind operating from the inside, influenced or interfaced with the mentoring process, as the reality of their mentoring experiences is reduced to their individual mind. As individuals, they are the subjects, interpreting what they think of their mentors, through disjointed lived-experiences of them (Csordas, 2008).

In the lower left quadrant–the intersubjective view, learning mathematics consists of collective, interior experiences entailing interactions, shared values, meaning of concepts and relationships. In this quadrant, participants are required to listen and respond to each other to gain an understanding of the other, thus, entering into their lived-experiences. The excerpts below are instances when TFs and MTs shared a common goal or understanding about teaching the content during the practice seminar or a debriefing after an observation of a lesson to show when they achieve a state of intersubjectivity. A more detailed discussion of achieving intersubjectivity is discussed in the next subsection.

MT_1: As we both teach Algebra 2, we have had many discussions about the best way to teach content. At one seminar, we discussed common misconceptions that students had about how to graph a parabola. Together, we discussed how we both encounter the same issues with our respective students when teaching that material, and we talked about our preferred methods of teaching graphing quadratic functions.

MT_2: My mentee and I talked about ways to address getting students to have computational fluency with fractions, decimals, percent and their conversions. This was both in planning and debriefing a lesson on the topic. We discussed multiple ways of modeling these concepts through different representations and were very much on the same page, in terms of the content.

TF_1: My mentors observed me teach a lesson on factoring polynomials using area models. Factoring is always one of those tricky concepts for students; until they're able to create some kind of visual, they usually just think of factoring as a procedure. Joseph (master teacher) and I had a rich conversation about ways to engage students in factoring, and Newton (mathematician) had some interesting suggestions on extending with higher order polynomials.

As is clear from these instances, both TFs and MTs, within shared spaces and social interactions, are capable of co-constructing pedagogical content knowledge, modeling a common understanding about teaching mathematical content.

In the two external views–objective and interobjective quadrants–there are individual and collective views, represented as "What I do as a teacher or mentor." And "What we do as teachers and mentors," respectively. The upper right quadrant is the mentor and mentee's outside spaces or exterior things. It represents what each do as individuals in their classrooms, schools and districts. Below is a passage from notes of an observation by a mathematician mentor of his mentee in her second year of teaching. The mathematician noted:

> *I observed Marcela's sophomore CP (College Prep, i.e. not honors) Geometry class today from 8:05 to 9:20. There were 11 boys and 2 girls in the class. All but one student arrived on time, and one new student showed up halfway through class. A special education aide was present for the class period.*

> Marcela began by putting up the day's agenda. She then praised the students for their efforts on this week's [state] exams (although I don't think she knows how they did). Saying that she wanted information to improve her focus next year, she then asked students for their feedback on the exam. This led to a lively discussion, with students saying that it went well, but they had forgotten some formulas, etc. This was the first of many instances Marcela showed her respect for her students' experiences, and their answers were both boisterous and equally respectful. Marcela did not let students shout out answers, but a low level of trash talking was allowed. I think it is remarkable that Marcela and the students have worked out the balance between blowing off steam and keeping the class under control and moving through topics.

This passage represents the two external views, individual and collective. From an individual's view, it is an objective practice illustrating what the mathematics mentor does, he observes; yet, it also shows, as indicated in the lower right quadrant, interobjective practices–the influence of the collective world through structures or systems (e.g., the state high stake assessment system). Here, we see the relationship between participants' external views through objective and interobjective practices.

In sum, the process of mentoring is evidenced in all quadrants, as the four views do not exist in a vacuum. Interacting with others is an essential part of mentoring. Teachers do not learn in a vacuum; socialization or intersubjective interactions must precede the subjective aspects of understanding.

Intersubjective Learning Communities: Achieving Intersubjectivity

Without at least an opportunity for individuals to share and discuss their diverse viewpoints, which may or may not lead to achieving intersubjectivity, "we learn nothing, and do little to advance and refine our understanding and our means of communicating our understandings to others" (Nathan et al., 2007, p. 556). In Figure 11.2, it is evidenced that the mentor-mentee pairs bring diverse backgrounds, experiences, knowledge, and understandings about mathematics with them to the learning community, as noted in the four quadrants.

Presented in Figure 11.2 are instances when both the teacher and mathematician mentors shared a common understanding about teaching with their mentee. Although different perspectives are offered across the quadrants, intersubjectivity is achieved. For example, the MM's subjective experience about the reality of teaching mathematics at the high school level is his personal worldview, yet it is a view that is also shared by his mentee. The understanding of the MT and her mentee, regarding the role of classroom assessment, is not

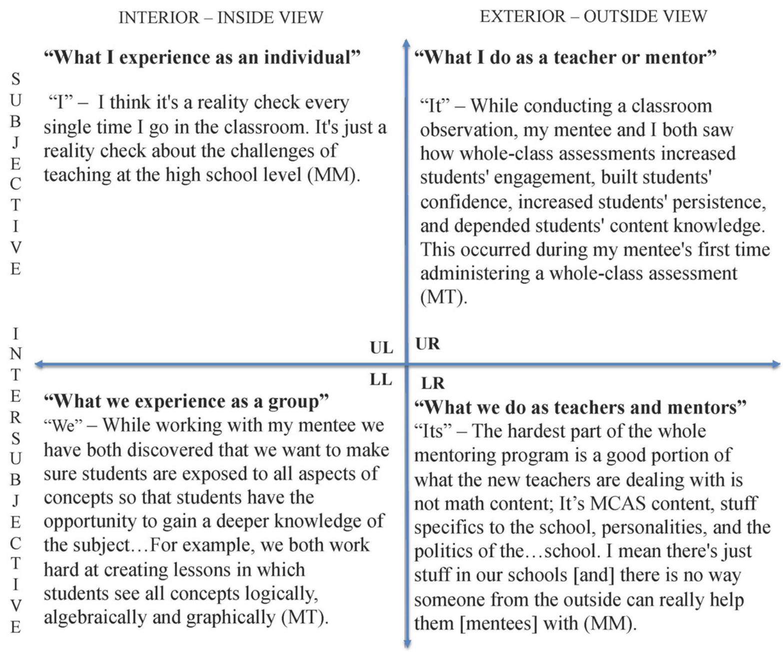

FIGURE 11.2 Mentoring across the four quadrants

only about what they do, but also it illustrates how external structures influence classroom teaching. At the intersubjective level, the collective interior-exterior, inside out, represents shared collective spaces for both the mentors and mentees. Thus, the frames of "I and We" are all inclusive intellectual and everyday knowledge embedded in the internal-external views, whether the experiences are subjective, objective, intersubjective or interobjective. We see how social and cultural influences affect mentor and mentees' learning and understanding.

The analysis of data showed that sharing the authority in intersubjective discourse lessens the subjective differences between the mentor and mentee. Again, quadrants are used to represent this finding. Figure 11.3 shows excerpts of responses, illustrating when the TFs are viewed as experts. We see comparable statements across the quadrants, regardless of view. Although not presented in the quadrants, the mathematicians also viewed TFs as experts in managing a challenging job, engaging their students, teaching mathematics at the high school level and having a developmental view of mathematics. The most common element that linked the mentor-mentee relationship was their love of the discipline.

In summary, using quadrant analysis based on Wilber's Integral Theory (1995, 1998), professional learning communities can be structured to foster the achievement of intersubjectivity. Also, from our examination of interrela-

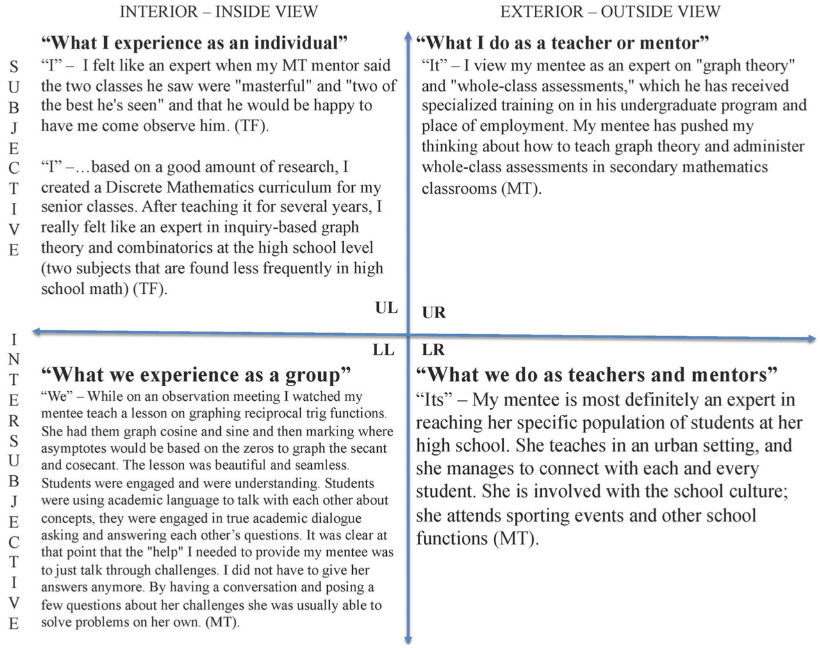

FIGURE 11.3 Beginning mathematics teacher (mentee) as expert

tionships, the "social interactions must be framed within an activity that has a clear purpose" (Shabani, 2016, p. 3), and the participants must share a common goal to reach a state of intersubjectivity. Therefore, there is formidable interweaving of individual mathematical learning and development and collective mathematical learning and development.

Advancing Intersubjectivity Unevenly: Differentiation and Contrast

What we saw in the previous section of the chapter are essential elements of achieving intersubjectivity. This section presents some flaws or problematic aspects of two-way mentoring. Surprisingly, the mentees showed little, if any, opposition towards having experienced practicing teachers and mathematicians as mentors. In fact, they believed that this type of mentor-mentee relationship is "super-useful, from a very thorough observation to the write-up," as stated by one of the TFs. However, the lack of frequent interactions, which were characterized as more classroom visits, lead to growing conflict between mentees and their mentors. Initially, it was the mentor teachers who voiced this concern, for example, stating,

MT_3: The time with my mentee has been great, but because we're in different schools, our professional relationship is limited. We need more time together. The lack of consistent interaction limits the effectiveness.

MT_4: I think that it's really hard because every time I go to see her teach, I have to take a day off work…that's fine but just being gone is an issue. And I think for that same reason like [mentee's name] never been able to come see me because she can't, she's you know in [mentee school's name] and I'm in [mentor school's name]. So, I think that the time we've been able to spend together has been really good, but I think it would be 1,000 times more beneficial if we were in the same school.

By the end of the TFs' third year in the program and second year as beginning teachers, they too expressed the lack of frequent classroom visits by their mentors, voicing, "I don't like just having them come in once a year and then giving feedback based on one lesson. I don't think it's super helpful. So, I think I was expecting more like every other month or every term or something that would have been better." This sentiment was consistent among the TFs. In contrast, another TF offered this commentary, "It is not clear what relationships should be developed vis-a-vis this mentoring program, so it's not clear I'm accessing all I can from my mentor." Instead of measuring mentoring grounded in frequency of classroom visits, she chose to focus more on relationship building. She continued highlighting that her relationship with her mathematician mentor was ambiguous because she did not know what to ask the mathematician and that at times it seemed that the mathematician was "out of touch with the education system at [her] school." It should be noted, however, that when discussing mentoring with the participants, relationship building emphasized the idea that mentoring relationships should advance organically and should not rely on forced structures. Consequently, due to a sense of disconnection, TFs were hesitant to reach out to their mentors. Since there were few structured mentor-mentee meetings, it was difficult to build organic relationships.

It was unfortunate that mentoring was viewed through a limited scope. Although mentors and mentees knew that the two-way mentoring model developed for this program would provide mentees with limited observation visits over the course of the program, mentoring relationships would be built on interactions during the practice seminars and the meetups following the colloquium. In individual and focus group interviews, most participants failed to recognize the elements of the program as ways to build mentoring relationships. Perhaps the program failed to make clear that mentoring goes beyond classroom observations to other types of face-to-face mediums, such as e-mail or Skype. Yet, ongoing suggestions for improving the program are encouraging more site visits, developing a platform to build relationships for all participants, such as a blog, a listserv, or a Facebook group. To date, there are a few mentor-mentee relationships developing via email; MTs are beginning to share resources with their mentees through Google Drive. Nevertheless, all of the participants agree that their attention to mathematics content and love of mathematics are what make this professional learning community and the mentoring relationship special.

Discussion

Achieving fundamental changes in teachers' content knowledge and instructional practices that influence student learning and performance require new approaches to professional development (Bay-Williams, Scott, & Hancock, 2007; Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Desimone, 2009; McLaughlin & Talbert, 2001). What we are learning from this study is that these new approaches entail more complex strategies that go beyond the *drive-by* or *one-shot* professional development in which an expert in the field provides a workshop on a particular topic or new educational software defined by the school administrator or the curriculum leader. Instead, they coalesce with a professional learning community that is teacher-driven and shared collectively by all participants (Albert, Terrell, & Macadino, 2014). Emerging from the analysis of data were approaches that included, but were not limited to, the teachers' commitment to understanding the mathematical content and standards they teach, such as *Common Core Standards*; the use of assessment data to ascertain relevant learning and pedagogical actions; professional activities that span over time; and adequate time for teachers and mathematicians to engage in professional activities. This was evidenced by the TFs' and MTs' reflections on instances when they shared a common goal or understanding about teaching the content.

Vygotsky (1978, 1997) theorized that when individuals participate in joint activities, the social situation transforms the cognitive development of the individual. Hence, the social construction of learning is grounded in participants' lived mathematics experiences and sociocultural histories, which leads to reinterpretations of mentees' and mentors' voices, unfolding within shared consciousness about the content (Albert, 2012; Bakhtin, 1984). In other words, the product that emerged should not just represent the co-construction (i.e., we) of mathematical knowledge and understanding, rather the very process undertaken should be considered. In this case, this element is evidenced as shown in Figure 11.2, when mentors show both internal and external perspectives that illustrate their understanding of the teaching challenges faced by their mentees. The Vygotskian view is that individuals achieve intersubjectivity by constructing a collective meaning of the activity or situation (Frawley, 1997).

To further understand how the mentor-mentee relationship achieves intersubjectivity, it is important to explore any potential power dimensions within the relationship. What we learned is that although the TFs often viewed their mentors as knowledgeable others, there were instances when the MTs and MMs viewed the TFs as experts (see Figure 11.3). In the theoretical framework, it is noted that to achieve intersubjectivity a successful mentor-mentee relationship must go beyond just working together or allowing the more knowledgeable individual to dominate the conversation or discussion. They must share power. According to Driscoll (1994), the only differences that exist between the mentor and mentee are in their individual levels of understanding. This element can limit not only the construction of mathematical teaching-learning experiences, but certainly the co-construction of them as well. This notion then points to the importance of considering not just subjective thinking of mentor-mentee pairs, but also how it led to intersubjective thinking about mathematics, for themselves and others in which their conversations and interactions are the primary tools used in the professional learning community. Frequently, collaborative interaction is suggested as a means of participating in lifelong professional learning and continuing education (Day, 1999; Duncombe & Armour, 2004). Vygotsky (1978, 1986) theorized that individuals and their social environments collaborate to shape learning and understanding. The findings presented in this study illustrate how the social nature of learning renders understanding regarding the ways in which learning unfolds as a socially mediated practice in learning communities (Albert, 2012; Albert, Terrell, & Macadino, 2014; Vygotsky, 1986).

Vygotsky (1978) argued that to understand and account for individual learning and development, one must consider the surrounding social environment of the individual. Vygotsky's epistemological argument suggests that learning is socially constructed and culturally shaped. Accordingly, mathematical learning involves the use of a variety of mental tools, such as language, that assist in the thinking process; these tools are culturally based and serve as signs that direct us to a deeper understanding of the social activity in which we are engaged (Vygotsky, 1981). Throughout this project, participants' conversations implicitly guided the mentor-mentee relationship. Central to the process of achieving intersubjectivity is trying to understand the perspectives of others. The findings mirror Rommetveit's (1979) notion of intersubjectivity in which common understandings are not always achieved in terms of a literal interpretation of the work but is achieved through negotiation of word meaning through conversations and interactions. A specific example is illustrated in the subjective and objective quadrants of Figure 11.3, showing both exterior and interior perspectives about teaching graph theory. For this study, it was through conversations where the negotiation of word meaning occurred; the participants were able to make sense of each other's perspectives about how to teach a specific mathematics concept of skills.

Conclusions and Implications

Developing intersubjectivity is a challenging, complex, and uneven process that involves various levels of communication. It requires time to maintain intersubjective relationships that include the expansion of trust and honesty. It should consider situations in which tensions have erupted beyond the normal underlying stress present in traditional one-on-one mentoring models (e.g., the interpretation of how to define the quantity and quality needed to build mentoring relationships). This conception was highlighted in the findings section in the discussion of the quality of mentoring. As a group, the participants did not believe that mentoring relationships would advance organically and should not rely on forced structures. They advocated for specific mentoring protocols to guide conversations and interactions.

Richter et al. (2013) posit that outcomes evolving from mentoring and/or teacher induction programs are based on the quality of the program and not essentially on its frequency. For the mentees, and some of their mentors, frequency regarding how often they interacted with their mentors at their school settings mattered. The finding that supports this conclusion was the lack of frequent interactions, which was characterized as more classroom visits and the lack of them, which led to growing conflict between mentees and their mentors. Thus, the frequency of interactions seemed to affect how mentoring relationships evolved and showed how advancing intersubjectivity can be an uneven process. However, I believe that the development of intersubjectivity can move the mentoring process ahead, where the relationship is characterized by achieving a common understanding of the mathematical activity, task, or idea. This process does not mean that individuals have to give up their own ideas, nor does it means they have to acquiesce. As the findings suggest, it allows for varying degrees of intersubjectivity.

Using Wilber's (1995, 1998) Integral Quadrant Theory as a vehicle for understanding the relevance of learning across views and perspectives, we can construct interpretations and conceptualizations around how to achieve intersubjectivity. The mentor-mentee pairs struggled with their respective professional roles because they had differing subjective views, perspectives, and interpretations. Thus, using Wilber's theory as an analytic way to portray findings can provide us with a tool to explore, challenge, and comprehend subjective and objective perspectives of teaching and learning mathematics. Based on this premise, we must be willing to explore emerging concepts about how to help the mentor-mentee pairs work toward achieving intersubjectivity and apply it to various mathematical teaching and learning environments.

In conclusion, I believe that the Two-Way Mentoring Model will change the existing mentoring model commonly used for induction of beginning teachers in two critical ways: (1) transitioning from the traditional one-on-one model to a team model of mentoring, and (2) shifting the mentors' assignment that characteristically focus on convenience or entitlement to smart selection of mentors that are mathematicians and experienced mathematics teachers. Based on research by Katzenbach and Smith (2003, 2005), this alternative model suggests that a team of successful individuals or teachers with various skills and expertise can provide greater support to any single teacher, as opposed to just a single mentor. Also, research on mentoring strongly suggests that content mentoring influences and supports beginning teachers' content knowledge and their pedagogical knowledge for teaching (Richter et al. 2013; Simonsen et al., 2009). For this study, findings suggest that participants agreed that the attention to mathematics content and the love of mathematics are what made the mentoring relationship special and helped them grow as professionals and, therefore, serves as a model for similar types of programs.

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