Diagnostic Assessments in Algebra and Geometry

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The Role of Assessment

Assess Student Understanding

Instruction

Modify Instruction based on Current State of Student Knowledge
Cognitive Diagnostic Assessment

- Achievement Tests
  - Measure student ability on a unidimensional scale
  - Provide correct/incorrect results

- Error Analyses
  - Identify procedural errors
  - Instruction can focus on correct algorithms and processes

- Diagnostic Assessment
  - Explore cognitive processes underlying responses- misconceptions or under-developed reasoning
  - Provide a method for teachers to explore why students struggle with concepts or get items incorrect
  - Instruction can focus on improving conceptual understanding
A Word about Our Use of the Word Misconception…

- *Misconception* is used to represent:
  - Flawed preconceptions
  - Underdeveloped reasoning
  - Traditional misconceptions
  - Any systematic source of difficulty that students have in their reasoning.

- Targeted instruction can be used to help students refine, reorganize, or build upon their knowledge to overcome misconceptions.
1. Test Items
   - Each item has multiple response options:
     - Correct response
     - Distracter responses
     - Misconception responses
   - Students who select 40% or more of the misconception responses were classified as having the misconception.
   - 10-12 items for each misconception
   - Each test/item is scored two ways: ability and misconception

2. Performance Reports
   - Ability Report
   - Diagnostic Misconception Report

3. Instructional Resources
Development of Diagnostic Assessments

1. Develop items
   - DAA- Closed-Response
   - DGA- Open-Response Items

2. Pilot items and select a subset of the best-performing items

3. Develop instructional resources

4. Validity Testing
   - Content-Related- Do the items represent the domain?
   - Construct-Related- Do the items measure what we think?
     - Transference tasks and interview teaching with cognitive probes
   - Criterion-Related- Do the items give consistent information?
     - Classroom assessments, teacher predictions, standardized test items
   - Consequential-Related- What are the consequences of the results?

5. Efficacy test of the complete intervention
   - Assessment
   - Diagnostic Reports
   - Instructional Resources
DAA- Concept of a Variable

Misconception

- Students fail to understand the role of letters in equations
- Students interpret letters in equations as labels referring to concrete objects
- Students assign a concrete numerical value to a letter intended to represent a variable

M is a positive whole number. How many possible values can 10m have?

A. 5
B. 10
C. 20
D. Infinitely many

D is correct response
B is misconception response which represents assigning the number 10 to the letter representing a variable
Students associate $=$ with a command to perform an operation instead of expressing a symmetric transitive relation between the expressions on the left and right side.

Students have difficulty with the idea that adding or subtracting the same amount from both sides of an equation maintains equality.

$8 + 4 = [\ ] + 5$

A. 6
B. 7
C. 10
D. 12

B is correct response
D is misconception response, which represents adding $8+4$
Students have difficulty in interpreting the graph of a real world situation

Students confuse slope and height

Which provider has the lowest per-month cost?

A. Simple.com

B. Call.com

C. At less than 5 hours per month, Simple.com has the lower per-month cost. At more than 5 hours per month, Call.com has the lower per-month cost.

D. At less than 5 hours per month, Call.com has the lower per-month cost. At more than 5 hours per month, Simple.com has the lower per-month cost.
DGA- Shape Properties Misconception

- Students have difficulties when reasoning with Concept Images without Concept Definitions
- Students can identify prototypical examples of shapes
- Students might consider orientation and proportion as characteristic of the shape

Misconceivers may say no to rectangle, square, rhombus, or rotated parallelograms.
Students have difficulties when not transforming every point in a plane.

Most problematic with distant points of rotation or distant lines of reflection.

Kelly is working on a problem with two steps.
Step 1: Translate (slide) line segment YZ two units to the right.
Step 2: Rotate (turn) the line segment 180 degrees counterclockwise around point K.
Watch the movie that shows what Kelly did.

What was one mistake that Kelly made in her transformation?

- **A** She did not translate the correct number of units in step 1.
- **B** She translated and rotated in Step 2 instead of just rotating.
- **C** She did not rotate 180 degrees in Step 2.
- **D** She actually didn't make any mistakes in her transformations.

B is correct response.

D is the misconception response where students slide a figure to a distant point of rotation.
Students have difficulties in mentally structuring space and connecting structured space to measurement formulas.

A student was using isosceles right triangle units to measure the area of a rectangle. He placed 6 tiles along one side and 4 tiles along the other side like this:

What is the area of the rectangle in triangle units?

- A. 10 units
- B. 20 units
- C. 24 units
- D. 48 units
- E. 96 units
- F. None of the above

D is correct response. C and E are misconception responses that plug in numbers without connecting formulas to structured space.
Question 17

Here are three shapes:

How many of these shapes are parallelograms?

A. None
B. One
C. Two
D. Three
Performance Reports - Overview

Graphing Misconception Report

Teacher: Sample Teacher
Class: 1st period

> View the Item by Item Results for the Graphing Misconception.

On average, your students' responses were:

- 45% correct
- 30% indicate a misconception

These students have a Probable Graphing Misconception:

- Jessica Yue: chose the misconception option for 8 out of 10 items
- Tom Hoffmann: chose the misconception option for 5 out of 10 items
- Wei Tao: chose the misconception option for 5 out of 10 items

These students have a Possible Graphing Misconception:

- Kevon Tucker-Seeley: chose the misconception option for 4 out of 10 items
- Octavio Suarez Munist: chose the misconception option for 4 out of 10 items

These students got less than 65% of the questions correct (Struggling w/ items):

The Graphing Misconception

In algebra, students learn that a graph is a representation for a function. Students learn to translate between graphs, equations, and table of values.

But just as the translation between equations and word problems is more difficult, students sometimes find interpreting the graph of a real world situation more difficult. Students may forget the algebraic relationships they have learned and resort to graphical misconceptions. The most common graphing misconceptions are treating a graph as a picture and slope-height confusion.

An example of interpreting a graph as a picture might be a problem asking a student to draw a speed vs. time graph for a biker riding over a hill. Students with the misconception would draw the hill, and ignore that speed is asked for. Students do not look at the graph as showing speed as a function of time, but think of it more literally.
## 1st Period Item Summary: Graphing Misconception

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C = correct  
M = incorrect AND this answer may indicate a misconception  
X = incorrect due to other error
Instructional Resources- Interpretive Summary of Misconception

**Equality Symbol Misconception**

As algebra teachers, we all know how frustrating it can be to teach a particular concept and to have a percentage of our students not get it. We try different approaches and activities but to no avail. These students just do not seem to grasp the concept. Often, we blame the students for not trying hard enough. Worse yet, others blame us for not teaching students well enough.

Students often learn the equality symbol misconception when they begin learning mathematics. Rather than understanding that the equal sign indicates equivalence between the expressions on the left side and the right side of an equation, students interpret the equal sign as meaning “do something” or the sign before the answer. This problem is exacerbated by many adults solving problems in the following way:

\[ 5 \times 4 + 3 = ? \]
\[ 5 \times 4 = 20 + 3 = 23 \]

Students may also have difficulty understanding statements like \( 7 = 3 + 4 \) or \( 5 = 5 \), since these do not involve a problem on the left and an answer on the right.

Falkner presented the following problem to 6th grade classes:

\[ 8 + 4 = [ ] + 5 \]

All 145 students gave the answer of 12 or 17. It can be assumed that students got 12 since \( 8 + 4 = 12 \). The 17 may be from those who continued the problem: \( 12 + 5 = 17 \).

Students with this misconception may also have difficulty with the idea that adding or subtracting the same amount from both sides of an equation maintains equality. Kieran gives this example:

Solve for \( x \): \( 2x + 3 = 5 + x \)

\[ 2x + 3 - 3 = 5 + x \]
\[ 2x = 5 + x - x - 3 \]
\[ 2x - x = 5 - 3 \]
\[ x = 2 \]

The answer is correct, but several steps of the solution contain incorrect equations.
Square units are usually used to measure area but other shapes can be used as a unit also. Let's use isosceles right triangles to measure the area of this rectangle.

A student counts 6 isosceles right triangles along one side and 3 isosceles right triangles along another side. What is the area of the rectangle in triangle units?

18 triangle units

How do you know?

I know because to find the area of an object, you multiply length times width. So six triangles times three triangles equals eighteen triangles. This is the area.

Student 1: (reads from paper) "Will your method allow you to transform any parallelogram into a rectangle? Explain." So, get a marker. If there was like a small parallelogram, if we cut it, do you think we could make a rectangle with it?

Student 2: If we cut it like that (pointing and tracing onto paper) wouldn't it?

Student 1: Oh, we could use the same strategy.

Student 2: Try cutting another parallelogram.

Student 1: (draws and cuts a parallelogram that is a smaller, mathematically similar version of the one they had worked on) This is what we did before right?

Other two students: mm hmm...

Student 1: Here's a square (pointing to a rectangle within the parallelogram) and then... (lines up cut out shape onto paper), yeah, you could do it with any parallelogram.

Student 2: But the point is, that was the same one as that (pointing at shapes) and we have to try making different ones to see if it will get into a rectangle.

Student 1: But isn't this shape, wait, I don't get what you're saying. Isn't this a parallelogram? This shape (pointing to paper)

Student 2: Yeah it is, but there are more parallelograms we could try to make go into a rectangle.

Student 1: What other ones?

(Student 2 doesn't answer and looks at her paper)

Student 1: I think like, that this is just the shape... You know how there's square, and rectangle and then, there's a triangle. I think that is just a parallelogram. I think that how you said before that, if you cut the two right angles on the sides then you can make a rectangle.

Student 2: So the answer would be yes right?
Instructional Resources - Lesson Plans

GRAPHING

Lesson 1

Objective
Students will:
- differentiate between a picture and a graph.
- identify a graph that represents a certain situation.

Algebra Misconception Addressed by This Lesson
Students who are unclear on graph interpretation may consider a graph a picture of the event rather than a pictoral representation of data about the event.

Rationale
In this lesson, students will view a computer applet (online tool) showing a picture of a real-world event (water flowing out of a tube). As animation shows how the picture changes, a graph is drawn to represent the event. Using this applet, students will observe that changing the parameters in the problem situation changes the graph, but not the picture. Since one of the graphing misconceptions algebra students have involves misinterpreting a graph as a picture, this activity sets the stage for a discussion of the differences between a graph and a picture of an event.

Materials
You will need:
- Paper and pencils
- Activity sheets
- Computer with Internet access

Definitions
Graph:
- a way to display data; examples include a line graph or a scatter plot.
Line graph:
- a data display that shows points connected by line segments;
- often used to show changes over time
- x-axis:
- the horizontal number line in a coordinate graph
- y-axis:
- the vertical number line in a coordinate graph
Small-Group or Individual Instruction

Activity 1

1. On a computer, navigate to the following website:
   http://illuminations.nctm.org/ActivityDetail.aspx?ID=16
   This applet will show an animation of water flowing out of a tube. As the water flows out, the applet represents this event with both a picture and a graph.

2. Have students gather around the computer screen (or look on their own computers, or use overhead). Show students that the tube on the left contains water. Ask students to imagine that this tube has a hole in the bottom that is 0.5 unit in diameter. Ask students what will happen to the tube as water flows through the hole in its bottom. Students should indicate that the water level in the tube will drop vertically. Then run the applet by hitting “Start.”
DAA- Pilot Efficacy Study

- **Validity Testing**
  - Results: classification consistency ranged from .73 to .87

- **Four-group cluster-randomized controlled trial**
  - Full intervention
    - Test, Ability Reports, Diagnostic Reports, Instructional Resources
  - Control
    - Test, Ability Reports
  - Partial intervention
    - Test, Ability Reports, Diagnostic Reports
  - Partial intervention
    - Test, Ability Reports, Instructional Resources

- **Pre-test, Intervention (three weeks), Post-test**
  - 10-12 items per misconception test

- **Sample**
  - $n = 44$ teachers, 905 students
  - Grades 6-12
## DAA- Characteristics of Pre- and Post-Tests

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DAA- Pilot Study Results

- Prevalence of Misconceptions
  - Concept of a Variable- 14%
  - Equality- 11%
  - Graphing- Slope– 12%
- Effect sizes were calculated for standardized comparisons of post-test means
- Ability
  - Full intervention had higher ability than three other groups (.76sd Partial- Reps, .91sd Partial- Instrc, .81sd Ctrl)
  - Full Intervention and Partial- Instrc had higher ability than Partial- Reps and Ctrl (.36sd)
- Misconception
  - Full Intervention had lower misconception scores than three other groups (.63sd Partial- Reps, .80sd Partial- Instrc, .72sd Ctrl)
  - Full Intervention and Partial-Instrc had lower misconception scores than Partial-Reps and Ctrl (.30sd)
## Conclusions

- Diagnostic assessments identify underlying cognitive difficulties in algebra and geometry.
- Diagnostic assessment results, combined with targeted instructional resources, can help refine and improve student understandings and misconceptions.
- Diagnostic assessments should be designed to be easily integrated into classroom instruction.