Targeting Constant Money Growth at the Zero Lower Bound

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Abstract: Unconventional policy actions, including quantitative easing and forward guidance, taken by the Federal Reserve during and since the financial crisis and Great Recession of 2007-2009, have been widely interpreted as attempts to influence long-term interest rates after the federal funds rate hit its zero lower bound. Alternatively, similar actions could have been directed at stabilizing the growth rate of a monetary aggregate, so as to maintain a more consistent level of policy accommodation in the face of severe disruptions to the financial sector and the economy at large. This paper bridges the gap between these two views, by developing a structural vector autoregression that uses information contained in both interest rates and a Divisia monetary aggregate to infer the stance of Federal Reserve policy and to gauge its effects on aggregate output and prices. Counterfactual simulations from the SVAR suggest that targeting money growth at the zero lower bound would not only have been feasible, but would also have supported a stronger and more rapid economic recovery since 2010.

Keywords: Constant money growth rate rules, Divisia monetary aggregates, Quantitative easing, Structural vector autoregressions, Zero lower bound.


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Targeting Constant Money Growth at the Zero Lower Bound

The financial crisis and Great Recession of 2007-2009 seemingly required major changes in Federal Reserve operating procedures. Pre-crisis, the Fed conducted monetary policy by targeting the interest rate on overnight, interbank loans: the federal funds rate. When the Fed wished to tighten monetary policy, it raised its target for the federal funds rate; conversely, when it wished to ease, it lowered its funds rate target. After the target was reduced to a range of 0 to 0.25 percent in December 2008, however, the zero lower bound on the federal funds rate forced the Fed to look for other ways of providing additional monetary stimulus to help output and employment recover and prevent inflation from falling further.

Bernanke (2012) describes two sets of tools that the Fed adopted, under his Chairmanship, to continue pursuing these goals. Multiple waves of large-scale asset purchases of US Treasury bonds and US government agency mortgage-backed securities, known more popularly as “quantitative easing” or “QE,” aimed to put direct downward pressure on long-term interest rates. Meanwhile, “forward guidance,” in the form of official policy statements promising to keep short-term interest rates low for an extended period of time, even as the economy began to recover, were directed at reducing long-term rates further, working through expectational channels on the yield curve. Thus, from this perspective, the focus of Federal Reserve policy appears to have remained squarely on interest rates, but the Fed’s interventions in bond markets grew enormously in size and scope while its communication strategy became vastly more ambitious and complex.1

Viewed from a different angle, however, there may be less to distinguish between monetary policy before and after the crisis. This is because, in more normal times, when the Fed raised its federal funds rate target, it still had to bring about its desired increase in the equilibrium funds rate by conducting open market operations that drained reserves from the

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1 Bernanke, Reinhart, and Sack (2004) discuss alternative monetary policy strategies once the zero lower bound constraint becomes binding. Written before the Great Recession, these options were examined because, under sustained low inflation, the zero lower bound was becoming something more than a theoretical curiosity. One of the suggested strategies was to stimulate economic activity by increasing the size and altering the composition of the Fed’s balance sheet in a manner that would raise asset values and reduce yields. The authors, however, do not consider the potential effects of changes in the quantity of money.
banking system. This decrease in reserves worked through textbook channels to slow the growth rate of money and thereby dampened economic activity generally and slowed inflation specifically. Likewise, when the Fed lowered its target for the funds rate, it engineered the desired fall through open market operations that increased the supply of reserves, which caused broad money growth to accelerate with more rapid economic activity and inflation to follow. From this viewpoint, large-scale asset purchases might have worked mainly to increase the quantity of reserves supplied to the banking system leading, all else equal, to faster money growth, higher output, and more stable inflation. Forward guidance might have worked, as well, to convince the public that open market operations designed to stimulate broad money growth would continue even after the economy began to recover; working again through expectational channels, this more persistent increase in money growth might have contributed to a stronger economy right away.

All else was not equal, however. At the same time it was conducting large-scale asset purchases, the Federal Reserve began paying interest on reserves. Ireland (2014) shows that the ability to pay interest on reserves gives the central bank a second instrument of monetary policy: one that works to shift the demand curve, rather than the supply curve, in the market for reserves. In particular, at any given level of the federal funds rate, movement from an initial equilibrium in which interest is not paid on reserves to a new equilibrium in which interest is paid on reserves at a rate that is very close to the federal funds rate target itself triggers a potentially large rightward shift in the demand curve for reserves, which the central bank can then accommodate with an equally large increase in the supply of reserves without generating additional broad money growth or inflation. Indeed, with the adjusted monetary base increasing from $872 billion in August 2008 to a peak value $4,097 billion in August 2014 and excess reserves representing 84% of that increase, it appears that quantitative easing, largely or perhaps even completely, simply accommodated the increased demand for
reserves generated by the Fed’s new interest on reserves policy, circumstances that do not represent a genuine “easing” of overall monetary conditions.\(^2\)

At the same time this transition to a new equilibrium with interest on reserves was underway, flight-to-quality dynamics set off by the financial crisis itself were surely shifting the demand curve for reserves even farther to the right. This would occur because, as discussed by Anderson, Bordo, and Duca (2016), the same flight-to-quality dynamics that increased banks’ demand for reserves during the financial crisis also increased the public’s demand for the safe and liquid assets included in the M2 monetary aggregate. These shifts would have worked against the expansionary effects that the Fed’s large-scale asset purchases might have otherwise had with the implication that even multiple rounds of QE and the attendant large increases in reserves supply might have simply worked to accommodate this shift in demand without bringing about additional growth in broad money.

To illustrate the disconnect between what has been labeled as “quantitative easing” and money growth, the three panels on the left-hand side of figure 1 plot year-over-year growth rates in the Divisia M1, M2, and MZM monetary aggregates we use throughout this study. The assets included in Divisia M1 and M2 are the same as those in the Federal Reserve’s official, simple-sum M1 and M2 measures of money. Divisia MZM excludes the small time deposit component of M2 but adds institutional money market mutual fund balances to obtain the “nonterm M3” aggregate originally proposed by Motley (1988) and relabeled MZM – “money, zero maturity” – by Poole (1991). All three measures of Divisia money are complied, however, using the economic aggregation techniques first outlined by Barnett (1980) and reviewed more recently in Barnett (2012). They correct, as well, for the distortions in the Federal Reserve’s official monetary aggregates induced by the proliferation of deposit sweep programs, described by Cynamon, Dutkowsky, and Jones (2006), that before the financial crisis allowed banks to reduce their holdings of required reserves without changing the public’s perception of the amount of funds held on deposit at those banks. Barnett, Liu, Mattson, and van den Noort

\(^2\) As of April 2016, excess reserves still represented 60 percent of the adjusted monetary base.
describe the construction of these Divisia monetary aggregates in full detail; the series themselves are available through the Center for Financial Stability’s website.

The shaded portions of each panel on the left-hand side of figure 1 identify periods during which the Fed was conducting large-scale asset purchases in order to highlight the “consistently inconsistent” effects those programs had on broad money growth. Measured by any of the three aggregates, money growth rose then fell during QE1, accelerated throughout QE2, and drifted lower during much of QE3. The three panels on the right-hand side of figure 1, meanwhile, show that the downward trend in the velocity of the Fed’s official, simple-sum M2 measure studied by Anderson, Bordo, and Duca (2016) also appears in the income velocity of all three Divisia aggregates. This downward trend in velocity continues, even after short-term interest rates reached their zero lower bound in 2008, a pattern consistent with the same flight-to-quality portfolio shifts. The implication of this first set of graphs is that Fed policy succeeded only partially in supporting the monetary system against the severe disruptions set off by the financial crisis of 2007 and the sharp downturn in aggregate economic activity that followed. The money supply did not fall during the Great Recession as Friedman and Schwartz (1963) show that it did during the Great Depression. On the other hand, multiple waves of QE have not consistently generated the growth in broad monetary aggregates that, given the decline in velocity, would have been necessary to stabilize nominal income and spending.

Zooming out to give a broader view, figure 2 plots longer time series for year-over-year growth in all three Divisia aggregates, now shading in periods corresponding to recessions as identified by the National Bureau of Economic Research. Noticeable declines in money growth pre-date every US recession since 1968 and, more generally, fluctuations in money growth appear distinctly pro-cyclical; procyclical money growth, of course, has been viewed as a by-product of implementing monetary policy in a manner that uses an interest rate as its intermediate target. Tables 1 and 2, meanwhile, report correlations between real GDP, the

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3 See Currie (1934) for a much earlier analysis that emphasizes many of the same points as Friedman and Schwartz (1963).

GDP deflator, and lagged values of each measure of money after the logarithms of all series are passed through the filter developed by Baxter and King (1999) to isolate fluctuations occurring at business cycle frequencies corresponding to periods between 8 and 32 quarters. These tables show that, in fact, modest correlations between money, output, and prices seen over the entire sample of quarterly data running from 1967:1 through 2016:1 become much stronger when recomputed with data from 2000:1 through 2016:1, that is, the most recent period dominated by the financial crisis, Great Recession, and slow recovery that followed. The lags at which peak correlations between money, output, and prices can be found lengthen, as well, moving from the full sample to the most recent period, following broader trends documented and discussed by Belongia and Ireland (2015a, 2015b, 2016).

Taylor (2009) argues that the Federal Reserve might have conducted monetary policy in more systematic fashion by switching to a version of Friedman’s (1960) k-percent rule for constant money growth after the federal funds rate hit its zero lower bound in 2008. Our own preliminary examination of the data, meanwhile, indicates that the recent episode of zero nominal interest rates was marked by surprisingly wide fluctuations in broad money growth that also turned out to be highly correlated with subsequent movements in aggregate output and prices. This begs the question of whether the Federal Reserve could have dealt with economic stabilization more successfully if it had targeted a constant rate of money growth since 2008. And, if so, would the US economy’s recovery from the Great Recession been stronger or more rapid?

In this paper, we provide answers to these questions by modifying the structural vector autoregression (SVAR) developed previously in Belongia and Ireland (2015a, 2015b), as needed to fit the new issues raised by the most recent period spanning 2000:1 and 2016:1. We capture, in particular, the effects that the Fed’s large scale asset purchases and forward guidance have had on the economy, working through traditional interest rate channels, by estimating the model using Wu and Xia’s (2016) measure of the “shadow” federal funds rate in place of the federal funds rate itself. This measure is derived from a nonlinear model that accounts for the zero lower bound on the actual funds rate, but follows Black (1995) by using
information in the term structure of interest rates to deduce the shadow rate – which may be negative – consistent with the behavior of yields on longer-term bonds. We also follow our own previous work by imposing enough structure on both money supply and money demand relationships to use information in the Divisia M1 monetary aggregate to help identify monetary policy shocks and estimate their effects on the economy. Thus, in particular, our SVAR brings data on both interest rates and the money stock to bear in gauging the stance and consequences of Federal Reserve policy before, during, and since the financial crisis of 2007.

We use our model to consider a range of counterfactual scenarios in which the Federal Reserve succeeds in maintaining a constant rate of Divisia M1 growth while its funds rate target is up against the zero lower bound.5 Reassuringly, the shadow funds rate does not have to fall as far below zero along paths implied by these counterfactuals as it did historically, suggesting that these alternative paths for money would have been feasible in practice. Additionally, our simulations suggest that by switching to a constant money growth rule, the Fed would have provided more consistent support for the economic recovery and expansion, generated more rapid output growth and, to some extent, faster inflation as well. We interpret these results as evidence that the Fed could have successfully directed its efforts towards stabilizing money growth while the funds rate remained at its zero lower bound and would have generated more favorable macroeconomic outcomes by doing so. Finally, we conclude by discussing the broader implications of our results, both for the conduct of monetary policy during a cyclical downturn marked by zero nominal interest rates and the design of models for monetary policy analysis and evaluation.

5 Our analysis is in the spirit of McCallum (1990), who examined whether a rule for growth in the monetary base could have prevented the Great Depression, and Bordo, Choudhri, and Schwartz (1995), who simulated the potential effects on output and prices during the Depression under two variants of Friedman’s k-percent rule. Although these studies show effects on nominal income or prices and output separately that are of different magnitudes, it is interesting to note that either monetary rule suggests economic performance would have substantially better than that depicted by the actual data.
Interest Rates, Money, and Monetary Policy in a Structural VAR

The structural vector autoregression developed in Belongia and Ireland (2015a, 2015b) describes the behavior of six variables: the GDP deflator \( P_t \), real GDP \( Y_t \), the federal funds rate \( R_t \), the Divisia M1 monetary aggregate \( M_t \), the Divisia M1 user cost aggregate \( U_t \), and the Commodity Research Bureau/Bureau of Labor Statistics (CRB/BLS) spot commodity price index \( C_t \). Here, as noted above, we replace the actual federal funds rate with Wu and Xia’s (2016) shadow federal funds rate, to more fully capture the effects that monetary policy has had, working through the traditional interest rate channel; of special interest is the period from 2009:1 through 2015:4 when the Federal Reserve’s funds rate target was up against its zero lower bound. Our preference for Divisia M1 over M2 and MZM as a measure of money reflects the shorter lags at which its peak correlations with output and prices appear in tables 1 and 2, especially since the brevity of our sample period limits the number of lags we can include in the VAR. All variables enter the VAR in logarithms, except for the shadow funds rate and the Divisia M1 user cost, which enter as decimals, e.g., \( R_t = 0.05 \) or \( R_t = -0.01 \) for a shadow funds rate equal to +5 or -1 percent.

Figure 3 plots the six quarterly series used to estimate the model. The data for real GDP and the GDP deflator are drawn from the Federal Reserve Bank of St. Louis’ FRED database; those for the Divisia M1 quantity and user cost aggregates are from the Center for Financial Stability’s website. The series for the shadow federal funds rate comes from Jing Cynthia Wu’s webpage at the University of Chicago’s Booth School of Business, and historical data for the CRB/BLS commodity price index are compiled from various editions of the CRB’s Commodity Price Yearbook and the Barchart.com website. The sample of data runs from 2000:1 through 2016:1, so as to focus on both the lead-up to and aftermath of the financial

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6 The user cost variable is the price dual to the economic quantity aggregate, a result that identifies a proper own-price of monetary services and avoids an arbitrary choice of an interest rate to serve as its proxy. See, e.g., Barnett (1978, 2012) and Belongia (2006).
crisis of 2007 and the Great Recession that followed, while also providing enough observations
to estimate the parameters of the SVAR with a reasonable degree of precision.

Collecting the variables into the 6 x 1 vector

\[ X_t = [P_t \ Y_t \ R_t \ M_t \ U_t \ C_t]', \]  \hspace{1cm} (1)

the structural model can be written as

\[ AX_t = \mu + \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \Sigma \varepsilon_t, \]  \hspace{1cm} (2)

where \( A \) is a 6x6 matrix of impact coefficients with ones along the diagonal, \( \mu \) is a 6x1 vector
of constant terms, \( \Phi_1 \) and \( \Phi_2 \) are 6x6 matrices of autoregressive coefficients, \( \Sigma \) is a 6x6
matrix with standard deviations of the structural disturbances along the diagonal and zeros
elsewhere, \( \varepsilon_t \) is a 6x1 vector of serially and mutually structural shocks, normally distributed
with zero means and

\[ E\varepsilon_t \varepsilon_t' = I_6, \]  \hspace{1cm} (3)

and \( I_6 \) is the 6x6 identity matrix. As noted above, the short sample of data used to estimate
the model dictates our choice to place two lags of \( X_t \) on the right-hand side of (2). Multiplying
(2) by \( A^{-1} \) leads to the reduced form

\[ X_t = \nu + \Gamma_1 X_{t-1} + \Gamma_2 X_{t-2} + x_t, \]  \hspace{1cm} (4)

where \( \nu = A^{-1} \mu \), \( \Gamma_j = A^{-1} \Phi_j \) for \( j = 1,2 \), and the 6x1 vector of zero mean disturbances

\( x_t = A^{-1} \Sigma \varepsilon_t \) is such that

\[ E x_t x_t' = \Omega, \]  \hspace{1cm} (5)

with

\[ A^{-1} \Sigma \Sigma' (A^{-1})' = \Omega \]  \hspace{1cm} (6)

Since the reduced-form covariance matrix \( \Omega \) has only 21 distinct elements, at least 15
restrictions must be imposed on the 36 elements of \( A \) and \( \Sigma \) that have not been normalized to
equal zero or one in order to identify the structural disturbances based on information in the data. A popular approach to solving this identification problem follows Sims (1980) by imposing a lower triangular structure on $A$. With the variables ordered as shown in (1), this identification scheme is based partly on the assumption that monetary policy shocks, measured by the third element $\varepsilon_{mp}^i$ in the vector of structural disturbances $\varepsilon_i$, affect the aggregate price level and output with a one-period lag. Leeper and Roush (2006) note, however, that when a monetary aggregate also appears in the list of variables used to estimate the model, as is the case here, a triangular scheme that orders interest rates behind prices and output but ahead of money also reflects assumptions that distinguish money supply from money demand. In particular, when $A$ is lower triangular, the third equation of (2) can be written as

$$a_{31}P_t + a_{32}Y_t + R_t = \sigma_{33}\varepsilon_{mp}^i,$$

where $a_{ij}$ and $\sigma_{ij}$ denote the coefficients from row $i$ and column $j$ of the matrices $A$ and $\Sigma$ and where reference to the constant and autoregressive terms from the VAR is suppressed to focus on the contemporaneous relationships between the observable variables and the identified structural policy disturbance. With the timing assumptions that prevent $P_t$ and $Y_t$ from responding immediately to the monetary policy shock in mind, (7) can be interpreted as a monetary policy rule taking the same general form as Taylor’s (1993), describing how the Federal Reserve sets its target for the funds rate with reference to the current period’s values of aggregate prices and output. Under this interpretation, the money supply then adjusts elastically so as to satisfy the fourth equation from (2), which can be written as a flexibly-specified money demand relationship

$$a_{41}P_t + a_{42}Y_t + a_{43}R_t + M_t = \sigma_{44}\varepsilon_{md}^i,$$

linking the quantity of money demanded to the price level, aggregate output as a scale variable, and the short-term interest rate as an opportunity cost variable. Thus, while the lagged terms that appear implicitly in (7) and (8) and more explicitly in (2) do allow for flexible dynamic
interactions between lags of interest rates and money, the view of monetary policy reflected in this triangular identification strategy resembles closely the one taken by the canonical New Keynesian model, as depicted in textbook presentations such as Gali’s (2015): the Federal Reserve is described as targeting the funds rate based on output and inflation, leaving the money stock to expand or contract as needed to fully accommodate changes in money demand.

Belongia and Ireland (2015a, 2015b) take an alternative approach to identifying the structural shocks in (2) and (3) based on the reduced form described by (3) and (4), imposing additional structure on the money demand relationship in order to allow for a finite elasticity of money supply and, by extension, a richer set of interactions between the federal funds rate and the Divisia M1 money stock in shaping the effects of monetary policy disturbances. This alternative model, in its most general form, parameterizes the matrix of impact coefficients in (2) as

$$A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & 1 & 0 & 0 & 0 & 0 \\
a_{31} & a_{32} & 1 & a_{34} & 0 & 0 \\
-1 & -1 & 0 & 1 & a_{45} & 0 \\
-a_{54} & 0 & a_{53} & a_{54} & 1 & 0 \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & 1
\end{bmatrix}.$$  \(9\)

The first two rows of the matrix \(A\) shown in (9) impose the same timing restrictions used in the fully recursive, triangular model: that the aggregate price level and output respond to monetary policy (and other) shocks with a one-period lag. In defense of this timing assumption, note from tables 1 and 2 that for our sample period running from 2000:1 through 2016:1, the contemporaneous correlations between the cyclical components of money and output and money and the price level are consistently negative, a reduced-form relationship more easily explained if monetary policy responds immediately to output and inflation than if output and inflation respond immediately to monetary policy.

The third row of (9) describes a policy rule more general than (7), taking the form
\[ a_{31}P_t + a_{32}Y_t + R_t + a_{34}M_t = \sigma_{35} \epsilon_{t+1}^{\text{mp}}, \]  

(10)

again suppressing the constant and autoregressive terms from (2) to focus on the contemporaneous relationships between the observables and the structural shock. Following Ireland (2001), (10) can be interpreted as a generalized Taylor (1993) rule that includes the money stock together with the aggregate price level and output in the list of variables that Federal Reserve policymakers refer to when setting their target for the funds rate. Following Leeper and Roush (2003), (10) can also be interpreted as a monetary policy rule that features a finite elasticity of money supply, in contrast to the implicit assumption of infinite money supply elasticity reflected in (7), the original Taylor (1993) rule, and the standard New Keynesian model. Our preferred interpretation, based on reduced-form connections between money, output, and prices shown in figures 1 and 2 and tables 1 and 2, however, is that (10) captures simultaneous movements in the federal funds rate and the money stock, both of which are important in transmitting the effects for monetary policy shocks through the economy.

Belongia and Ireland (2015a) test the general specification in (10) against the more constrained alternative originally proposed by Sims (1986) and used more recently by Leeper and Roush (2003) and Sims and Zha (2006) in which \( a_{31} = a_{32} = 0 \), so that the price level and output are excluded from the monetary policy rule. Using data from 1967:1 through 2007:4, we found that imposing these constraints did not lead to statistically significant deterioration in the model’s fit and therefore adopted the simpler model as our benchmark in both Belongia and Ireland (2015a) and Belongia and Ireland (2015b). As described below, however, with data from 2000:1 through 2016:1, these same constraints are rejected quite decisively; hence, we use the more flexible model here. Keating, Kelly, Smith, and Valcarcel (2014) and Arias, Caldara, and Rubio-Ramirez (2016) are two other recent papers that experiment, successfully, in bringing information in both interest rates and money aggregates to bear in identifying monetary policy shocks and estimating their effects on the economy.

Row four of (9) draws on economic theory to parameterize the money demand relationship more tightly as
\[ M_t - P_t - Y_t + a_{45} U_t = \sigma_{44} \epsilon_t^{md}, \tag{11} \]

Relative to (8) from the triangular model, (11) imposes a unitary price elasticity, so that the demand for money is described explicitly as a demand for real cash balances. Here, again in contrast to our previous work in Belongia and Ireland (2015a, 2015b), we also impose a unitary income elasticity of money demand; though not essential for identification, this constraint sharpens our other parameter estimates, is not rejected by the data, and is consistent with theories of money demand that predict a stable relationship between monetary velocity and an opportunity or user cost variable. Finally, compared to (8), (11) replaces the federal funds rate with the Divisia M1 user cost index, which measures the “price” of monetary services in theoretically coherent way; interest rates, by contrast, are linked to the price of bonds as money substitutes. Thus, drawing on the logic behind identification in more traditional simultaneous equation systems, (10) and (11) work to disentangle shocks to money supply from those to money demand, first, by using quantity-theoretic restrictions that associate “money supply” with nominal cash balances and “money demand” with real cash balances expressed relative to income. These shocks also are distinguished by including the shadow federal funds rate as a variable that the Fed cares about in the money supply rule and the Divisia user cost of money as a variable that private depositors care about in the money demand equation.

Row five of (9) provides the equation

\[ a_{53} R_t + a_{54} (M_t - P_t) + U_t = \sigma_{55} \epsilon_t^{mu} \tag{12} \]

describing how the private banking system, together with the Federal Reserve, create the liquid assets in the Divisia M1 monetary aggregate. Belongia and Ireland (2014) and Ireland (2014) incorporate this “monetary system” into a dynamic, stochastic, general equilibrium model in which bank deposits and currency substitute imperfectly for one another in providing monetary services, showing in particular how changes in the federal funds rate get passed along to consumers in the form of a higher user cost of a Divisia monetary aggregate. Equation (12) adds flexibility to the simpler relationship implied by the DSGE models, allowing the
quantity of real monetary services created to affect the user cost as well, as it would if banks also pass rising costs along to consumers as they expand their scale of operation. Finally, row six of (9) allows commodity prices to respond instantaneously to all other shocks that hit the economy.

Hamilton (1994, Ch.11) and Lutkepohl (2006, Ch.9) outline methods for estimating structural VARs like ours via maximum likelihood. These references show that even with the non-recursive restrictions imposed on $A$ in (9), fully efficient estimates of the reduced-form constant and autoregressive coefficients in (4) can be obtained by applying ordinary least squares separately to each equation; estimates of the free parameters in $A$ and $\Sigma$ can then be obtained by maximizing a concentrated log-likelihood function. Since (9) imposes 18 restrictions on the off-diagonal elements of $A$ whereas only 15 restrictions are needed for identification, the statistical adequacy of the over-identified structural model can be tested by comparing its maximized log-likelihood function to that of the exactly-identified triangular model. We do this below, before using the estimated model to assess the role that Federal Reserve policy played during and since the Great Recession and to trace out the implications of various counterfactuals.

Estimates and Counterfactuals

Table 3 reports estimates of key parameters from the matrices $A$ and $\Sigma$ of impact coefficients and structural shock volatilities, focusing on the monetary policy and money demand relationships (7) and (8) from the triangular model and the monetary policy, money demand, and monetary system equations (10)-(12) from our preferred, non-recursive structural model. Despite the three overidentifying restrictions imposed on the matrix $A$ in (9), the two specifications fit the data equally well: the maximized log-likelihoods of 1403.52 for the non-recursive model and 1403.97 for the triangular alternative imply a p-value of 0.83 for the likelihood ratio test statistic for the null hypothesis that the three additional constraints are
satisfied. Standard errors for the individual coefficients, also shown in the table, are computed using the formulas from Proposition 9.5 in Lutkepohl (2006, Ch.9, p.373).

For the triangular model, the estimated coefficients on the GDP deflator and real GDP in the monetary policy equation are consistent with the interpretation of (7) as a variant of the Taylor (1993) rule for the federal funds rate. In the estimated money demand relationship (8), the coefficient on real GDP turns out to be negative and statistically significant. An inverse relationship between nominal money and output, however, is what one would expect to see in a money supply relationship as opposed to a money demand function. Thus, this finding suggests that excluding money from the monetary policy rule (7) – that is, assuming an infinitely elastic money supply schedule – forces the money demand relationship (8) to account for dynamics associated with both money supply and money demand. Although the triangular model is exactly identified and therefore “fits” the data as well or better than any other, we take this as evidence that it fails to discriminate adequately between monetary policy and money demand and work, from this point forward, exclusively with the non-recursive specification described by (9)-(12).

The estimated parameters from our preferred model all have the appropriate signs. Although the coefficients on the aggregate price level and output in the monetary policy rule (10) are individually insignificant when compared to the size of their standard errors, a chi-squared test for their joint significance has a p-value of 0.0003, decisively rejecting the null hypothesis that both are equal to zero. The Divisia M1 money stock enters with a large, though imprecisely estimated, positive coefficient that, when combined with the positive coefficient on the aggregate price level, captures a strong and sensible tendency for the shadow funds rate to fall in response to building deflationary pressures. Likewise, the incorporation of money into the expanded policy rule works to magnify the estimated positive coefficient on aggregate output, suggesting that (10) also does a better job than (7) in recognizing the Federal Reserve’s efforts to stabilize the real economy during and after the Great Recession. The Divisia user cost variable enters the estimated money demand equation (11) with the expected, negative
sign, and both the shadow federal funds rate and the real money stock significantly influence the user cost via the estimated parameters from the monetary system equation (12).

The solid lines in figure 4 plot the impulse responses of the shadow federal funds rate, Divisia M1, real GDP, and the GDP deflator to a one-standard-deviation contractionary monetary policy shock, identified using the non-recursive restrictions imposed through (9). The dashed lines, meanwhile, provide plus-and-minus one standard error bands around the estimated responses, computed as suggested by Hamilton (1994, Ch.11, pp.336-337) by treating each impulse response as a vector-valued function of the estimated VAR parameters and using the numerical derivatives of that function to convert standard errors for the parameters into standard errors for the impulse responses. The monetary policy shock lifts the shadow funds rate by 25 basis points over the first four quarters; the shadow rate remains higher for another year before falling back below its initial level in response to the lower levels of output and prices that also follow the unanticipated tightening. Leeper and Zha (2003) point out that an impulse response with these properties captures the same short-run liquidity effect and longer-run expected inflation effect that Friedman (1968) and Cagan (1972) associate with monetary policy actions that decrease the money supply. In fact, as figure 4 also shows, the identified policy shock has large and persistent contractionary effects on Divisia M1. Real GDP responds to the disturbance with a lag, moving lower with effects that build over a period of several years. Finally, the impulse response for the GDP deflator exhibits a short-run “price puzzle,” rising slightly for several quarters after the shock before falling more persistently later on. Overall, however, the effects of the monetary policy shock on the aggregate price level are small and imprecisely estimated. The absence of strong effects of monetary policy on inflation is, in fact, a feature that runs consistently through all of our results.

Table 4 decomposes the forecast error variances in real GDP and the GDP deflator, reporting percentages due to each of the three structural disturbances identified by (10)-(12): shocks to monetary policy, money demand, and the monetary system. Of these three shocks, only monetary policy plays a large role in accounting for output fluctuations. Although the large standard errors, computed for these variance decompositions in the same way as for the
impulse responses described above, highlight the considerable degree of uncertainty surrounding these results, the estimated model attributes about 20 percent of the forecast errors in real GDP over horizons of four to five years to the effects of monetary policy shocks. By contrast, monetary policy shocks explain almost none of the volatility in the GDP deflator.

The top left-hand panel of figure 1 shows that Divisia M1 grew at an average annual rate of about 9 percent between 2008:1 and 2016:1. With stable velocity, of course, 9 percent money growth would have translated directly into 9 percent annual growth in nominal spending and resulted in much higher rates of output growth and/or inflation than those seen historically. The sharp decline in velocity seen in the top right-hand side panel of the same figure, however, implies that even more rapid monetary expansion was needed to fully stabilize the economy. In addition, because money growth itself exhibited wide fluctuations about its mean, monetary policy may have amplified, rather than ameliorated, fluctuations in output and prices during and since the financial crisis. All of these observations suggest that the lesson drawn from US monetary history by Friedman and Schwartz (1963) and Brunner and Meltzer (1968) continues to have relevance today: By interpreting low nominal interest rates as a sign of monetary ease and neglecting a sign of monetary tightness revealed by slow money growth, it is possible to understand how the Federal Reserve contributed to both the length and severity of the Great Depression and why the Fed behaved as it did during the Great Recession.

These observations prompt us to use our estimated model to answer two sets of questions. First, what would the trajectories of key macroeconomic variables have looked like in the absence of monetary policy disturbances? Did these identified shocks work to stabilize, or destabilize, output and inflation? Second, what would have happened if the Federal Reserve had acted more deliberately to stabilize the rate of money growth around an average at, or possibly even above, the historical mean of 9 percent? Would switching to Milton Friedman’s (1960) k-percent rule for constant money growth once the zero lower bound for the funds rate had been reached – an option specifically mentioned and advocated by Taylor (2009) – have
worked to promote a more rapid, or at least a more stable, recovery and expansion following the Great Recession?

To begin answering these questions, the top left-hand panel of figure 5 plots the realized, historical path for the monetary policy shock $\varepsilon_{t}^{mp}$ implied by (10). As noted above, uneven growth in the Divisia M1 aggregate in the face of declining velocity is an initial sign that, despite the massive increase in base money during and after the financial crisis, monetary policy on balance remained far from being uniformly or excessively easy during this period. The path for the policy shocks confirms this: none of the realizations of $\varepsilon_{t}^{mp}$ from 2007:1 through 2011:2 exceeds two standard deviations, and an extended string of expansionary (negative) values does not appear until 2013:3 through 2014:2, in the second half of the QE3 period.

The left-hand column of figure 6 shows what happens, according to the estimated model, when the realized shocks from 2008:1 forward are “turned off” in order to simulate the paths of output, prices, money, and interest rates in their absence. The graphs in the top three rows plot percentage-point differences between the simulated counterfactual and actual levels of real GDP, the GDP deflator, and the Divisia M1 aggregate; the graph in the bottom row compares the counterfactual path for the shadow federal funds rate traced out by the solid line with the actual path for the shadow funds rate traced out by the dashed line. The top two panels in the left-hand column, therefore, indicate that real GDP and the deflator would have been higher – real GDP by more than 0.75 percentage points – without the historical series of policy shocks. Only since 2015 has the extra monetary stimulus applied during QE3 produced more rapid output and money growth than was experienced historically; without monetary policy shocks, the aggregate price level would still have been slightly higher than it was at the beginning of 2016.

The remaining panels of figure 5 show the hypothetical series of monetary policy shocks from 2008:1 forward that, when fed through the estimated model, keep Divisia M1 growing along a constant path even as all of the other shocks take on their historical values. The first
simulation fixes the constant annual rate of money growth at 9 percent – almost exactly the same as the historical average in the data. The two additional counterfactuals raise the constant annual growth rate of money to 9.5 percent and then to 10 percent. All three additional graphs reveal that, to have kept the broad money supply growing at a constant rate, the Federal Reserve would have had to apply more monetary stimulus earlier on: in the context of the VAR, this stimulus takes the form of much larger (in absolute value) negative values for $\varepsilon_i^{mp}$ during the first two quarters of 2008 and again from 2009:2 through 2010:4. An extended run of expansionary shocks also becomes necessary to support continuing stable money growth towards the end of the sample period. Maintaining constant 10 percent money growth, in fact, requires $\varepsilon_i^{mp}$ to be negative from 2011:4 through 2016:1. Although in only one of these quarters, 2015:2, does the shock exceed two standard deviations, the serial correlation evident in the bottom, right-hand panel of figure 5 makes us stop here, without considering even faster rates of monetary expansion – as discussed by Leeper and Zha (2003), doing so would surely raise concerns, related to the Lucas (1976) critique, that our VAR with constant coefficients could not credibly address.

The second through fourth columns of figure 6 illustrate the effects that these constant money growth paths would have had on the US economy. The panels in the bottom row reveal that the shadow funds rate falls more quickly under the constant monetary growth counterfactuals than it did historically. Reassuringly, however, the counterfactual paths for the shadow funds rate never decline as far below zero as the actual historical path eventually does. Even in the case of 10 percent annual money growth, for instance, the bottom, rightmost graph in figure 6 shows that the shadow funds rate hits its low of −2.15 percent in the third quarter of 2010, whereas the actual shadow funds rate falls to −2.92 percent – more than 75 basis points lower – in the second quarter of 2014. In addition, all three of the counterfactual paths for the shadow funds rate lie above the actual shadow funds rate for most of the period since 2011:3. We take this as evidence that the Fed could have used some combination of its
existing policy tools – quantitative easing and forward guidance – to generate outcomes with more stable paths for money growth.

In the top two rows of figure 6, the positive effects on real GDP and the GDP deflator become larger and more persistent, of course, as the constant rate of money growth increases from 9 to 10 percent. With constant 9 percent Divisia M1 growth, aggregate output and prices in 2016:1 would be at the same levels observed historically, but those levels would have been approached more quickly: the graphs show, for example, that real GDP would have been 1.25 percentage points higher by 2011:4 under the constant money growth counterfactual than what was observed in reality. The model predicts that with constant 10 percent money growth, real GDP would have been almost 2 percentage points higher by the end of 2011 and would have remained higher all the way through the end of the sample in 2016:1.

Table 5 looks at these same results in a different way, comparing the average annual growth rates in real GDP, the GDP deflator, and the Divisia M1 monetary aggregate historically and under the four counterfactuals: without monetary policy shocks and with the three rates of constant money growth. Because of the severity of the Great Recession and the estimated lags in the effects of monetary policy, differences across actual and counterfactual histories appear small when compared, in the top panel, over the entire period from 2008:1 through 2016:1. Focusing on the period of economic recovery from 2010:1 forward, however, the alternative policy of stabilizing money growth at a 10 percent annual rate increases the average annual rate of real GDP growth from 2.03 to 2.25 percent. Looking at the results from this angle helps us avoid the temptation to overstate their implications. Even under these counterfactual scenarios, the US economy would still have experienced the full force of the Great Recession, and much of the disappointing slow recovery that followed. Nevertheless, the results show that by switching to a constant money growth policy in 2008, as soon as the severity of the downturn had become apparent, the Fed could have provided additional monetary stimulus that would have made the recovery both stronger and faster.

Finally, the results from figure 6 and table 5 show the same, very modest effects of monetary policy shocks on the aggregate price level seen previously in the impulse responses
and variance decompositions. The estimated model predicts, for example, that even with 10 percent money growth the annual inflation rate would only have been 5 basis points higher from 2008:1 through 2016:1, leading to an aggregate price level that is just 40 basis points higher overall. Reynard (2007) argues that the lags between monetary policy actions and their effects on inflation have become too long to be fully captured by conventional VAR models such as ours. Indeed, table 2 shows that these lags extend out to three or four years when measured by peak correlations between Divisia money and the GDP deflator. But while a persistent shortfall in inflation below the Federal Reserve’s two percent target may have been an inevitable consequence of the financial crisis and Great Recession, even our estimated SVAR suggests that the Fed could at least have moved inflation somewhat closer to target by generating faster growth in the aggregate quantity of money.

Conclusions and Implications

Could – and should – the Federal Reserve have attempted to stabilize the growth rate of a monetary aggregate once its federal funds rate target reached the zero lower bound in 2008? Our analysis suggests that the answer to both questions is “yes.” Counterfactual simulations conducted with a structural vector autoregression generate constant money growth paths that require smaller movements for Wu and Xia’s (2016) shadow federal finds rate than those observed historically. Moreover, these constant money growth paths lead to a stronger and faster recovery in real GDP and slightly higher rates of GDP price inflation, findings that point to their desirability.

Targeting constant money growth at the zero lower bound would likely have other advantages, too, which our SVAR does not even attempt to capture. As suggested by Taylor (2009), for example, targeting money growth would allow the Federal Reserve to continue conducting monetary policy in a systematic, rule-like fashion even after the zero lower bound makes the prescriptions of more conventional interest rate rules difficult or impossible to follow. A policy of targeting constant money growth also would acknowledge the key point made by Friedman and Schwartz (1963), Brunner and Meltzer (1968), and Friedman (1968)
that, especially during sharp cyclical downturns when deflationary expectations threaten to take hold, very low nominal interest rates no longer serve as a sign that monetary policy is fully accommodative; in these environments, slow rates of money growth or even outright monetary contraction will show that monetary policy is too restrictive instead.\textsuperscript{7} Thus, by working harder to stabilize broad money growth, the central bank can continue to emphasize its commitment to stabilizing inflation and the real economy even under the most extraordinary circumstances.

In fact, our empirical findings illustrate a key role for changes in the quantity of money as well as interest rates in describing Federal Reserve policy and its effects on the US economy over the period before, during, and since the financial crisis and Great Recession of 2007-2009.\textsuperscript{8} These findings, therefore, raise basic questions about the adequacy of state-of-the-art New Keynesian models, which capture only those effects working through traditional interest rate channels. At the same time, however, our empirical model itself has difficulty connecting historical monetary policy shocks and counterfactual monetary policy interventions to significant movements in the aggregate price level or inflation. These results highlight that understanding the full range of effects that monetary policy has on the economy remains nearly as elusive to us today as it did to Friedman (1968) nearly half a century ago. Perhaps, for this reason too, a constant money growth rate rule that attempts to remove monetary policy itself as a source of macroeconomic instability remains an attractive benchmark against which to judge more activist alternatives that try, but fail, to do more.

\textsuperscript{7} This also illustrates the difference between a “liquidity trap” and a “credit deadlock” as discussed by Laidler (2004) and Sandilands (2010). While the former has been central to most discussions of the efficacy of monetary policy during the Great Recession, the latter concept, which suggests an important role for expanding the quantity of money, has received virtually no attention. An exception is Belongia and Ireland (2016).

\textsuperscript{8} Results in Romer (1992) indicate, similarly, that recovery from the Great Depression can be attributed to rapid growth in the money supply rather than fiscal measures or any self-correcting mechanism.
References


Table 1. Correlations Between the Cyclical Components of Real GDP and Lagged Divisia Money

A. Sample Period: 1967:1 – 2016:1

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B. Sample Period: 2000:1 – 2016:1

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Note: Each entry shows the correlation between the cyclical component of real GDP in quarter $t$ and the cyclical component of the indicated Divisia monetary aggregate in quarter $t-k$. 
Table 2. Correlations Between the Cyclical Components of the GDP Deflator and Lagged Divisia Money

A. Sample Period: 1967:1 – 2016:1

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*Note:* Each entry shows the correlation between the cyclical component of the GDP deflator in quarter $t$ and the cyclical component of the indicated Divisia monetary aggregate in quarter $t-k$. 
Table 3. Estimated Impact Coefficients from Vector Autoregressions

| A. Triangular Model | Monetary Policy | \( R = 0.31P + 0.19Y \) | \( \sigma = 0.0029 \) |
| | | (0.23) (0.07) |
| | Money Demand | \( M = 0.22P - 0.59Y - 0.67R \) | \( \sigma = 0.0070 \) |
| | | (0.56) (0.19) (0.31) |
| | \( L^* = 1403.97 \) |

| B. Non-recursive Model | Monetary Policy | \( R = 0.29P + 0.91Y + 0.99M \) | \( \sigma = 0.0084 \) |
| | | (0.66) (0.59) (0.75) |
| | Money Demand | \( M - P - Y = -6.61U \) | \( \sigma = 0.0502 \) |
| | | (3.92) |
| | Monetary System | \( U = 6.82R + 1.48(M - P) \) | \( \sigma = 0.0185 \) |
| | | (2.15) (0.64) |
| | \( L^* = 1403.52 \) |

Notes: The table reports estimates of the coefficients shown in equations (7) and (8) for the triangular model and (10)-(12) for the non-recursive model. \( \sigma \) denotes the standard deviation of the structural disturbance associated with each equation; \( L^* \) denotes the maximized value of the log-likelihood function. Standard errors of the estimated parameters are in parentheses.
| Quarters Ahead | Real GDP | | | GDP Deflator | | |
| | Monetary Policy | Money Demand | Monetary System | Monetary Policy | Money Demand | Monetary System |
| 2 | 0.0 | 0.5 | 0.6 | 0.8 | 0.0 | 0.0 |
| | (0.1) | (1.1) | (1.2) | (1.6) | (0.1) | (0.3) |
| 4 | 0.3 | 0.6 | 0.5 | 0.6 | 0.2 | 0.0 |
| | (1.1) | (2.0) | (1.3) | (1.7) | (1.1) | (0.3) |
| 8 | 3.7 | 0.6 | 1.0 | 0.3 | 0.1 | 1.4 |
| | (5.8) | (2.8) | (2.6) | (0.7) | (1.1) | (3.9) |
| 12 | 10.7 | 1.2 | 2.0 | 0.7 | 0.1 | 3.4 |
| | (11.2) | (4.5) | (5.7) | (2.5) | (0.8) | (7.7) |
| 16 | 17.9 | 1.5 | 2.8 | 1.2 | 0.1 | 4.8 |
| | (15.8) | (5.3) | (7.6) | (4.1) | (0.8) | (10.2) |
| 20 | 21.6 | 1.4 | 3.3 | 1.5 | 0.1 | 5.6 |
| | (18.3) | (5.0) | (8.3) | (5.6) | (0.9) | (11.7) |

*Notes: Each entry indicates the percentage of the forecast error variance in real GDP or the GDP deflator due to the indicated shock, identified using the non-recursive strategy described in the text. Standard errors are in parentheses.*
Table 5. Average Annual Growth Rates: Historical and Counterfactual

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<td>Without Monetary Policy Shocks</td>
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<td>With Constant 9 Percent Money Growth</td>
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<td>With Constant 9.5 Percent Money Growth</td>
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<td>With Constant 10 Percent Money Growth</td>
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<th>GDP Deflator</th>
<th>Divisia M1</th>
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<td><strong>B. 2010:1 – 2016:1</strong></td>
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*Note:* Each entry reports the annualized growth rate of indicated variable over the indicated period historically or under one of the counterfactuals without monetary policy shocks or with constant money growth.
Figure 1. Divisia Money Growth and Velocity. Panels on the left show year-over-year growth rates of the Divisia monetary aggregates, with periods of quantitative easing shaded. Panels on the right show the income velocities of the Divisia monetary aggregates.
Figure 2. Divisia Money Growth. Panels show the year-over-year growth rates of the Divisia monetary aggregates, with NBER recession dates shaded.
Figure 3. Data Used To Estimate the Vector Autoregressions.
Figure 4. Impulse Response Functions. Panels show the percentage-point response of the indicated variables to a one-standard-deviation monetary policy shock (solid blue line) together with plus-and-minus one standard errors bands (dashed red lines).
Figure 5. Monetary Policy Shocks: Historical and Counterfactual. Panels show the historical path for the identified monetary policy shock, together with counterfactual paths required to support constant rates of money growth.
Figure 6. Counterfactual Simulations. Panels in the top three rows show the percentage-point difference of real GDP (Y), the GDP deflator (P), and the Divisia M1 monetary aggregate (M) under the indicated counterfactual compared to the historical path of the same variable. Panels in the bottom row compare the counterfactual (solid blue lines) and historical (dashed red lines) trajectories for the shadow federal funds rate.