Wages and Wedges in an Estimated Labor Search Model*

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Abstract

We estimate a search-based real business cycle economy using quantity data and a broad set of wage indicators, allowing the latent wage to follow a non-structural ARMA process. Under the estimated process, wages adjust immediately to most shocks and induce substantial variation in labor’s share of surplus. These results are not consistent with either a rigid real wage or Nash bargaining. The model fit is excellent, and smoothed realizations of the wage are consistent with empirical measures. According to the model, shocks to intertemporal preferences are the primary cause of inefficient fluctuations in the labor market and the driver of variation in labor’s share of surplus.

Keywords: Search and Matching, Wage Determination, DSGE, Bayesian Estimation

JEL Classification: E32, E24

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1 Introduction

At least since the criticism of efficient Nash bargaining by Shimer (2005), authors have debated the theoretical and empirical merits of a variety of alternative structural models of wage determination in labor search models. This paper steps back from committing to any particular model of wage determination, and instead asks what properties of a generic wage-determination process are needed to match the historical comovements of output, consumption, investment, unemployment, labor-market participation, and job creation in the US. To do this, we estimate a model which is structural in every way except for the determination of wages, which we treat as a latent ARMA process. By specifying a flexible process for the wage, our model nests a variety of possible structural models of wage determination. Compared to more-restricted models of wage determination, this added flexibility delivers an enormous improvement in the model’s fit of the historical series. According to the agnostically-estimated process, wages adjust quickly in response to most shocks and in a manner that causes labor’s share of match surplus to vary substantially over the business cycle. These features of the estimated wage process do not accord well with either Nash bargaining or the sluggish adjustment of the real wage, and provide a new set of prerequisites that should guide the continuing search for structural foundations of wage determination.

Our exercise begins with a standard real dynamic stochastic general equilibrium (DSGE) model, endowed with external habit formation, adjustment costs in investment and vacancy posting, variable capital utilization, and a search friction in the labor market. More or less complicated versions of this basic model underlie most of the recent literature on structural estimation of DSGE models. In contrast to most of this earlier literature, however, we choose to relax one structural restriction, that of committing to a particular wage-determination mechanism. As observed by Hall (2005) and Michaillat and Saez (2013), the process by which wages are determined in search models is logically and mathematically independent of the other constraints imposed by private optimization. It follows that, so long as they remain within a bargaining set, wages can coherently be treated as parametric.1 Our contribution is to treat the wage as a time-varying parameter endowed with a flexible ARMA specification that, given a rich-enough lag structure, is sufficient to describe the (linearized) dynamics of wages under virtually all assumptions the literature has made regarding wage determination.

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1See also Rogerson and Shimer (2011, p.659), who partly quote from Mortensen (1992, p.166), “...Unlike the Walrasian theory, there is no unique concept of equilibrium price inherent in the theory of markets with transactions costs.”
Given our flexible specification for wages, we proceed to estimate our model on quantity data and a set of five alternative wage series, which we assume are measured with error. We estimate the latent wage process jointly with the quantity data, and study the degree to which the model can reconcile the wage and quantity fluctuations. We find that it does quite well: unconditional implications for quantities are closer to the data than typically found in other estimated medium-scale DSGE models, and the implications for the latent wage are quite realistic. Since we have multiple wage measures, we then assess *ex post* which measures of wages correspond most closely to the model-implied wage, and find that a rather simple measure, total compensation-per-worker, does an excellent job of accounting for the aggregate fluctuations of quantities in the data.

As a basis for comparison, we also estimate equivalent models with Nash bargaining and non-adjustment of the wage following Hall (2005). We show that the model with the more general specification for wages substantially outperforms both alternatives in terms of overall fit of the data used in estimation: at many horizons it matches the data as well or better than a completely reduced-form VAR with standard lag-length. We also show that a single parameter, the vacancy adjustment cost, is crucial for the model to simultaneously fit quantity and wage data. Introducing this single adjustment cost parameter, which is analogous to the investment adjustment cost now ubiquitous in the DSGE literature, offers an improvement of model fit that is comparable to full relaxation of restrictions on the wage process. Moreover, our estimates suggest that, quantitatively, vacancy adjustment costs are a substitute for standard investment adjustment costs, which are estimated to be much smaller once we allow for non-zero vacancy adjustment costs. Since this vacancy adjustment cost parameter has important implications both for quantitative performance of the model as well as for its welfare implications, our results suggest further study of its micro foundations is warranted.

Having shown that the model performs quite well in terms of fit, we show that the estimated ARMA process governing the real wage exhibits distinctive features, which are precluded by at least two leading structural models of wage determination. First, we show that under the estimated wage process, the share of joint match surplus allocated to workers varies substantially over the business cycle. This type of variation is explicitly excluded by the constant-share Nash bargaining that is common in the literature. Second, we show that the estimated wage process is remarkably flexible in certain respects. In particular, wages respond quickly to permanent productivity shocks, reaching their long-run levels within
four quarters or less. Moreover, for temporary shocks including TFP, impact responses are maximal and fall over time.\textsuperscript{2} There is little in our estimation to suggest that the data call for hump-shaped responses to shocks. The evidence against the rigid wage hypothesis is not decisive, however. While we find no evidence of slow wage responses, wage responses to temporary shocks are generally small compared to a Nash bargaining benchmark. Thus, our estimated model leaves some role for incomplete adjustment to contribute to unemployment fluctuations.\textsuperscript{3}

Our estimated process for wages is tied to five wage indicator series, via a factor structure similar to that of Boivin and Giannoni (2006). These series are real compensation-per-worker, the real hourly wage, unit labor costs, the quality-adjusted wage series of Haefke et al. (2013), and the more recently introduced employment cost index. Since the correspondence of the model wage concept with the data is imperfect, and we have rather little reason to prefer any single measure, we let each series load differently on the underlying model wage and assume each rescaled series is measured with error. We tie the model closely to the wage data, however, by imposing relatively tight priors that the wage series do in fact contain substantial information regarding the wage in the model. Since these series have rather different properties, no single factor will be able to capture all series equally well and we find that the data prefer, by a substantial margin, the compensation-per-worker measure of wages. While it has a short history, and the level of it fluctuations are muted, the employment cost index also does a good job capturing the cyclical patterns of our estimated wage process.

Our estimation also has important implications for the sources of business cycles. Like other authors, we find a substantial role for technology shocks, both permanent and temporary. We also find an unusually large role for shocks to the efficiency of the labor matching mechanism, especially for the dynamics of labor market variables. These shocks drive 60 percent of unemployment fluctuations and an even greater share of the volatility of employment. According to the estimated model, shocks to matching efficiency were particularly important in driving dynamics in the period during and after the Great Recession. We also find a surprisingly large role for preference shocks, both to the discount factor and to labor-supply elasticity, in driving fluctuations in the rate of vacancy posting in the labor market.

\textsuperscript{2}It is worth acknowledging straightaway that our baseline assumption of AR(1) wage responses \textit{implies} this shape. This claim is based, instead, on our finding that increasing the order of the ARMA process for wages delivers minimal improvement in model fit.

\textsuperscript{3}Whether stickiness is real or nominal is not especially important, since nominal wage stickiness matters only to the extent it induces a sluggishness in real wages. Both types of frictions are potentially encompassed by our ARMA specification of the wage process.
These shocks contribute to vacancy volatility through a direct channel, because they affect
the incentives to engage in labor search given wages, and also because they are associated
in our estimation with small but very persistent changes in the level of wages. According to
our estimates, the latter channel is quantitatively far more important for understanding the
volatility of vacancies.

After establishing the model’s realistic positive implications, we then examine the effi-
ciency implications of the estimated wage process. To do this, we define two wedges in the
labor market, which correspond to efficiency in vacancy posting and in labor-force partic-
ipation. We show that both wedges are driven nearly two-thirds by the preference shock in
the economy. Much of this effect is driven by the small and persistent increase in wages we
find is associated with preference shocks. Wedges are driven to a lesser extent by matching
efficiency shocks and changes in labor supply elasticity, but nearly not at all by temporary
TFP shocks despite the fact that TFP drives a large fraction of output volatility in the
economy, and that the wage response to TFP changes is muted relative to Nash bargaining.
Additionally, we find that the same shocks that drive these two wedges also drive large fluc-
tuations in labor’s surplus share in the economy, which itself is both remarkably volatile and
strongly counter-cyclical.

This paper contributes primarily to the empirical literature building from the theory
of Diamond (1982), Mortensen (1982), and Pissarides (1985). Earlier contributors to this
of Nash bargaining, much of this literature has focused on alternative specifications for wage-
setting, and especially foundations for various types of wage stickiness, including Hall (2005),
Farmer and Hollenhorst (2006), Hall and Milgrom (2008), Gertler et al. (2008), and Kennan
(2010). Michaillat and Saez (2013) explicitly treat wages as parametric in order to focus on
the allocative role of market tightness. Our treatment here builds on this work, essentially
treating wages as *time-varying* parameters that may, or may not, respond to fundamental
shocks.

Methodologically, our approach belongs to a shorter literature that integrates non-structural
components within otherwise structural models. Structural models with non-structural
blocks have been estimated by, e.g., Sargent (1989), Altug (1989), Ireland (2004), and Boivin

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*Given the reduced-form nature of the wage process, we cannot credibly examine the consequences of
counterfactual policy changes. But this does not preclude a study of efficiency under the prevailing economic
structure, so long as it is stationary over the sample period and not expected to change by agents in the
economy.*
and Giannoni (2006). Also related is the literature initiated by McGrattan (1994), Hall (1996), and McGrattan et al. (1997), which uses calibrated dynamic models as measurement tools in order to infer the non-structural “wedges” driving the economy. Cheremukhin and Restrepo-Echavarria (2014) use this approach in a labor search model with (variable-share) Nash bargaining in order to decompose the sources of the RBC labor wedge and find an important role for shocks to matching efficiency. We contrast a few of our results with theirs in Section 4.

The remainder of the paper is organized as follows: Section 2 develops the model, Section 3 outlines our estimation strategy and Section 4 describes the results and their implications for modeling the real wage. Section 5 concludes.

2 The Model

The economy consists of a representative household and a representative firm who each trade in markets for consumption, labor, and capital. Consumption and capital markets are competitive, while transactions in labor markets are subject to search and matching frictions in the spirit of Mortensen and Pissarides (1994).

2.1 Households

The representative household consists of a continuum of ex-ante identical members each of whose unit time endowment can be allocated to working, searching for work, or leisure. The household derives utility at time $t$ from consumption and leisure (non-participation) according to the period utility function $u(c_t, f_t)$. Each period, the household dedicates a portion $s_t$ of its members to search for a match in the labor market. Searching members match with probability $p_t$, which the household takes as given. Moreover, newly-created matches become productive within-period, so that the total labor force participation of the representative household is given by

$$f_t = n_t + (1 - p_t) s_t,$$

5Consistent with the labor search literature incorporating a participation margin, we interpret non-participation in the labor force as leisure in the representative household’s optimization problem.

6We allow for an external habit in consumption as is common in the structural estimation of DSGE models. As aggregate consumption $C_{t-1}$ is taken as exogenous by the representative household, we suppress the dependence of $u(\cdot)$ on $C_{t-1}$.
where \( f_t \) denotes the measure of household members in the labor force, \( n_t \) denotes the measure of currently matched workers, and \( (1 - p_t) s_t \) gives the measure of household workers who search but fail to find a match in period \( t \). Each period, previously productive matches dissolve with exogenous probability \( \lambda \), so that the law of motion for matched workers faced by the household is

\[
  n_t = (1 - \lambda)n_{t-1} + p_t s_t.
\]

(2)

Figure 1 displays the timing of events in labor markets.

In addition to choosing its consumption and labor force participation, each period the household must also choose a level of capacity utilization for the current capital stock and a new level of investment subject to an investment adjustment cost as in Christiano et al. (2005). The law of motion for the stock of capital is given by

\[
  k_{t+1} = (1 - \delta(u_t))k_t + i_t,
\]

(3)

where \( \delta(u_t) \) is the depreciation rate of the capital stock, which is an increasing and convex function of capacity utilization, denoted \( u_t \).

The household budget constraint is given by

\[
  c_t + i_t Z_t \left( 1 + \Phi \left( \frac{i_t}{i_{t-1}} \right) \right) + \tau_t = R_t u_t k_t + W_t n_t + (1 - p_t) s_t \kappa_t + d_t.
\]

(4)

In equation 4, households take the rental rate of capital, the wage rate of labor, the relative price of investment in consumption units, and benefits paid to unemployed workers \( (R_t, W_t, Z_t \text{ and } \kappa_t \text{ respectively}) \), as given. They also receive \( d_t \), lump-sum dividends from firms to the household and pay \( \tau_t \), a lump-sum tax used to finance an exogenous stream of government expenditure and unemployment benefits paid to unemployed workers in economy. We assume that the benefit payed to unemployed workers is proportional to the real wage rate,

\[
  \kappa_t = \kappa W_t
\]

(5)

in order to maintain balanced growth.

Substituting the expression for \( f_t \) in equation (1) into the utility function, the represen-

\footnote{This timing convention is consistent with the evidence on labor market flows at quarterly frequency. See Davis et al. (2006).}
tative household’s problem may be expressed as

$$\max_{ct, s_t, i_t, u_t, n_t, k_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t b_t u(c_t, n_t + (1 - p_t)s_t)$$  \hspace{1cm} (6)$$

subject to (2), (3), (4) and (5), where \(b_t\) represents an exogenous shock to the discount factor, which previous DSGE estimations have often found to be an important shock. The period utility function in (6) also implicitly contains an intratemporal preference shock affecting labor supply, which we describe after specifying a functional form for utility.

The first-order conditions for investment, \(i_t\), and capital next period, \(k_{t+1}\), are given by

$$\mu^K_t = u_{c,t} Z_t \left[ 1 + \Phi \left( \frac{i_t}{i_{t-1}} \right) + \left( \frac{i_t}{i_{t-1}} \right) \Phi' \left( \frac{i_t}{i_{t-1}} \right) \right] - E_t \left\{ \beta \frac{b_{t+1}}{b_t} u_{c,t+1} Z_{t+1} \left( \frac{i_{t+1}}{i_t} \right)^2 \Phi' \left( \frac{i_{t+1}}{i_t} \right) \right\}$$  \hspace{1cm} (7)$$

$$\mu^K_t = E_t \left\{ \beta \frac{b_{t+1}}{b_t} \left[ (1 - \delta(u_{t+1})) \mu^K_{t+1} + u_{c,t+1} R_{t+1} u_{t+1} \right] \right\}$$  \hspace{1cm} (8)$$

where \(\mu^K_t\) denotes the Lagrange multiplier on the law of motion for capital (3). Equation (7) states that the utility gain from supplying an additional unit of capital must equal the marginal utility of current consumption, taking into consideration the costs associated with changing the level of investment. Equation (8) states that the utility gained from taking an additional unit of capital into the next period is the sum of its after-depreciation continuation value and the utility gained from consumption of future rental income. Observe that, absent investment adjustment costs and a utilization margin, (7) implies that \(\mu^K_t = u_{c,t} Z_t \forall t\), and so from (8) we recover the familiar RBC Euler equation. Optimality of capacity utilization requires

$$u_{c,t} R_t = \delta'(u_t) \mu^K_t.$$  \hspace{1cm} (9)$$

Finally, after some simplification, the household’s labor force participation condition may be expressed as

$$-\frac{u_{f,t}}{u_{c,t}} = p_t \left[ W_t + (1 - \lambda) E_t \left\{ \Omega_{t,t+1} \left( 1 - \frac{p_{t+1}}{p_t} \right) \left( -\frac{u_{f,t+1}}{u_{c,t+1}} - \kappa_{t+1} \right) \right\} \right] + (1 - p_t)\kappa_t.$$  \hspace{1cm} (10)$$

This condition states that the utility gained from keeping one more person out of the labor force (i.e. the marginal utility of leisure) is, at the optimum, equated to the utility gained from sending one more person to search for a job. The utility from joining the labor force
to search, in turn, depends on the utility derived from consuming out of the earned wage in the same period, plus the asset value of the job, both discounted according to the exogenous job-finding probability. Note that as the matching probability and separation rate tend to unity, equation (10) becomes the RBC labor supply condition.

2.2 Firms
The representative firm chooses labor, capital, and vacancy postings so as to maximize the present value of real dividends, discounted according to the consumer’s stochastic discount factor. The firm produces output with a production function of the form,

\[ y_t = A_t F(u_t k_t, X_t n_t), \]

where \( A_t \) captures exogenous, stationary shocks to total-factor productivity, while \( X_t \) is a non-stationary, labor-augmenting technology shock, with long-run growth rate \( \gamma_X \).

The law of motion of employed labor from the firm’s perspective is given by

\[ n_t = (1 - \lambda) n_{t-1} + q_t v_t \]

where \( v_t \) denotes vacancies posted in the labor market, while \( q_t \) denotes the probability of a vacancy posting returning a match. Firm profits are given by the expression

\[ d_t = A_t F(u_t k_t, X_t n_t) - W_t n_t - R_t u_t k_t - X_{t-1} Z^\alpha_{t-1} \left( a_0 + \xi \left( \frac{v_t}{v_{t-1}} \right) \right) v_t. \]

where \( a_0 \) governs the steady-state cost of posting a vacancy. We allow also for an adjustment cost in vacancy posting, the functional form of which we specify below. Notice that in the above, we have assumed that the cost of posting a vacancy grows with the stochastic trends in our model. This is necessary to ensure the economy has a balanced growth path.

The problem of the firm may therefore be expressed as

\[ \max_{v_t, n_t, k_t} E_0 \sum_{t=0}^{\infty} \beta^t b_t u_{t,t} d_t \]

subject to (12) and (13). The first order condition for capital is given by

\[ A_t F_{k,t} = u_t R_t. \]
The first-order condition for vacancies is given by,

$$\mu^N_t = \frac{X_{t-1}Z_{t-1}^{\alpha}}{q_t} \left[ a_0 + \xi \left( \frac{v_t}{v_{t-1}} \right) + \left( \frac{v_t}{v_{t-1}} \right) \xi' \left( \frac{v_t}{v_{t-1}} \right) - \gamma x t \gamma Z_{t-1}^{\alpha} E_t \Omega_{t,t+1} \left( \frac{v_{t+1}}{v_t} \right)^2 \xi' \left( \frac{v_{t+1}}{v_t} \right) \right]$$  \hspace{1cm} (16)$$

and the first-order condition for labor is given by

$$\mu^N_t = A_t F_{n,t} - W_t + (1 - \lambda) E_t \{ \Omega_{t,t+1} \mu^N_{t+1} \}$$  \hspace{1cm} (17)$$

where \( \Omega_{t,t+1} \equiv \beta \frac{b_{t+1}}{b_t} \frac{u_{e,t+1}}{u_{e,t}} \). Together, (16) and (17) constitute the vacancy posting condition, which states that the firm optimally posts vacancies until the marginal cost of so-doing is equalized with the marginal product of labor net of the wage bill, plus the continuation value of a match. Note that as the vacancy posting cost parameter \( a_0 \) tends to zero the conditions collapse to the familiar vacancy posting condition without adjustment costs. Moreover, as \( a_0 \) tends to zero and the separation rate tends to unity, the vacancy posting condition approaches the standard RBC labor demand condition.

2.3 Government

The government runs a balanced-budget, financing an exogenous stream of aggregate purchases \( G_t \) through lump-sum tax revenues \( \tau_t \) net of unemployment benefit transfers to households \( (1 - p_t) s_t \kappa_t \). The government’s resource constraint is thus given by

$$G_t = \tau_t - (1 - p_t) s_t \kappa_t.$$  \hspace{1cm} (18)$$

We follow Schmitt-Grohé and Uribe (2012) in assuming that government spending consists of a trend and stationary component

$$G_t = G_t^T G_t^S.$$  \hspace{1cm} (19)$$

The trend component gradually adjusts to restore the long-run share of government spending in the economy,

$$G_t^T = G_{t-1}^T \left( \frac{G_{t-1}^T}{g} \right)^{\phi_{g,y}} ,$$  \hspace{1cm} (20)$$

while transient changes in government expenditure follow an AR(1) process, specified below.
2.4 Wages

In order to close the model, we must make some assumption about how wages are determined in the economy. Rather than take a particular stand on the microeconomic foundations of wage determination in frictional markets, our baseline specification builds on the observation of Michaillat and Saez (2013) that prices can be treated as parameters in markets characterized by search and matching frictions. However, instead of taking the level of wages as fixed, as do both Hall (2005) and Michaillat and Saez (2013), we treat the statistical process by which wages evolve in the economy as parametric, and it is this process that we seek to estimate.

The starting point of our exercise, therefore, is a generic representation of equilibrium wages. Let \( w_t \) be the log of the wages appropriately detrended by dividing by the non-stationary model shocks. We then assume that

\[
\begin{align*}
\log w_t &= \beta^W(L) \epsilon_t. \\
\end{align*}
\]

Here, \( \beta^W(L) \) is an infinite-lag polynomial and \( \epsilon_t \) potentially includes all fundamental shocks related to technology and preferences. Equation (21) is generic, but we restrict this class in two ways. First, we assume that equilibrium wages are cointegrated with the stochastic trends in the economy. This assumption is necessary in order to ensure the economy has a balanced growth path, and is consistent with all price-setting mechanisms that we are aware of in the literature. Second, we need to assume that firms take the eventual wage a hired worker earns as given when making their vacancy posting decisions. With single-worker firms, or constant returns to scale production, there is very little loss of generality with this assumption.\(^8\) If we could estimate the infinity of coefficients implied in \( \beta^W(L) \), we would therefore directly nest the vast majority of wage determination mechanisms consisted in the literature. Of course, in practice, such an estimation is infeasible. In Section 3, we discuss the restrictions we place on equation (21) in order to implement our empirical strategy.

For comparison, we compare our baseline specification of wages with two alternatives. The first alternative we consider is the model of a “wage” norm, or fixed wage, as specified by Hall (2005). Since our model contains a trend, ensuring stationary requires that the real wage adjust, at some horizon, to permanent shocks. Hall (2005) shows that a deterministic

\(^8\)With multiple worker firms and decreasing returns, firms may internalize the effect on surplus, and therefore wages, created by hiring additional workers. In some contexts this leads to an in efficiency in which firms hire excess labor in order to suppress marginal products and therefore all wages.
and commonly known trend can easily be incorporated into the norm without changing the implications. The current case of a stochastic trend is not quite so straightforward, since the wage norm needs to adjust to surprise shocks at some horizon in order to stay within the bargaining set. Accordingly, we assume that wages adjust slowly to permanent shocks according to the equation

\[ W_{Hall}^t = c \left( X_t Z_t^{\alpha_1} \right)^\phi (W_{Hall}^{t-1})^{1-\phi}, \]  

(22)

where the constant \( c \) depends on the steady-state labor share of the economy and the parameter \( \phi \) captures the rate of adjustment, or error-correction, of the wage norm. In the Hall (2005) version of our economy, we assume \( \phi = 0.025 \), or an error-correction of 2.5 percent-per-quarter.

We also consider a version of the economy where the wage is determined by Nash bargaining (i.e. constant surplus split.) Following a standard derivation (reproduced in Appendix C) the Nash-bargained wage is given by

\[ W_{NB}^t = \eta_{1-\kappa} + \eta \kappa \left[ A_t F_{n,t} + (1-\lambda) E_t \{ \Omega_{t,t+1} P_{t+1} t+1 \mu_{t+1}^N \} \right]. \]  

(23)

This version of the model explicitly allows for a calibration of low worker bargaining power and high outside option which Hagedorn and Manovskii (2008) argue can address the challenge to Nash bargaining posed by Shimer (2005).

2.5 Wedges

The search and matching framework implies a strong link between macroeconomic efficiency and the wage-setting mechanism, as the famous result of Hosios (1990) emphasizes. In appendix D, we solve the social planner’s problem and use the resulting efficiency conditions to derive two wedges that define efficiency in the economy. The first wedge, which we call the vacancy posting wedge, describes the optimal tradeoff between the social cost of posting an additional vacancy and the benefits created by increasing the probability of a labor-market match forming. This first wedge is analogous to the static wedge of Arseneau and Chugh (2012), but the addition of vacancy adjustment costs adds a dynamic element to the condition. The second wedge, which we call the participation wedge, describes the optimal relationship between the cost of engaging in additional labor market search, and the social
benefits of a (potential) additional worker-firm match and is analogous to the dynamic wedge of Arseneau and Chugh (2012).

2.6 Equilibrium

The number of matches formed in the labor market is governed by a constant returns matching function \( m(V, S, \chi) \).\(^9\) \( S \) represents the aggregate mass of job-seekers, \( V \) represents the aggregate mass of vacancies posted by firms, and \( \chi \) allows for exogenous variation in the efficiency of the matching process.\(^10\) The probabilities of forming a match in the labor market, which both the consumer and firm take as given, are

\[
p_t = \frac{m(V_t, S_t, \chi_t)}{S_t} \\
q_t = \frac{m(V_t, S_t, \chi_t)}{V_t}.
\]

We assume the matching function is of the form specified by den Haan et al. (2000) so that matching probabilities are bounded, though we find that this choice is of no consequence to our estimation results.

Equilibrium is thus described by a set of allocations \( \{C_t, S_t, I_t, N_t, K_{t+1}, V_t\} \), a utilization rate \( u_t \) and a Lagrange multiplier \( \mu^K_t \) satisfying consumer optimality conditions (7), (8), (9) and (10), firm optimality conditions (15) and (17), the matching probability definitions in (24) and (25), the aggregate laws of motion for labor and capital

\[
N_t = (1 - \lambda)N_{t-1} + m(V_t, S_t, \chi_t)
\]

\[
K_{t+1} = (1 - \delta(u_t)K_t + I_t
\]

and the aggregate resource constraint

\[
C_t + I_tZ_t \left( 1 + \Phi \left( \frac{I_t}{I_{t-1}} \right) \right) + G_t = A_tF(u_tK_t, X_tN_t) - X_{t-1}Z^{\alpha}_{t-1} \left( a_0 + \xi \left( \frac{V_t}{I_{t-1}} \right) \right) V_t
\]

taking as given the exogenous processes for \( A_t, X_t, \chi_t, Z_t, b_t, \upsilon_t, G^T_t, \) and \( G^S_t \), as well as the parametric wage process described in equation (21).

\(^9\)Henceforth we denote by capitalized letters aggregate counterparts to household- and firm-specific variables.

\(^{10}\)See Andolfatto (1996) for an interpretation of matching efficiency shocks.
3 Empirical Strategy

3.1 Functional Forms

The representative household derives utility at time $t$ from the utility function

$$u(c_t, f_t) = \left[ (c_t - hC_{t-1}) \cdot \nu(f_t) \right]^{1-\sigma} - 1$$

(29)

where $h$ represents the degree of external habit formation with respect to last period’s aggregate consumption $C_{t-1}$, and

$$\nu(f_t) = \exp \left\{ \psi \left( \frac{(1 - f_t)^{1-\iota_t} - 1}{1 - \iota_t} \right) \right\}$$

(30)

where $\iota_t$ represents the inverse of the Frisch elasticity of labor force participation, which is allowed to vary stochastically.\(^\text{11}\)

Output is produced using capital and labor according to a standard Cobb-Douglas production function

$$F(u_t k_t, X_t n_t) = (u_t k_t)^\alpha (X_t n_t)^{1-\alpha}.$$ 

(31)

The investment adjustment cost is given by

$$\Phi \left( \frac{i_t}{i_{t-1}} \right) = \frac{\phi}{2} \left( \frac{i_t}{i_{t-1}} - \gamma x^{\frac{1}{\gamma Z}} \right)^2$$

(32)

so that in the steady state $\Phi(\gamma x^{\frac{1}{\gamma Z}}) = \Phi'(\gamma x^{\frac{1}{\gamma Z}}) = 0$, and $\Phi''(\gamma x^{\frac{1}{\gamma Z}}) = \phi > 0$.\(^\text{12}\)

Likewise, the vacancy adjustment cost is given by

$$\xi \left( \frac{v_t}{v_{t-1}} \right) = \frac{a_1}{2} \left( \frac{v_t}{v_{t-1}} - 1 \right)^2$$

(33)

so that in the steady state $\xi(1) = \xi'(1) = 0$, and $\xi''(1) = a_1$. The depreciation rate depends on utilization according to

$$\delta(u_t) = \delta_0 + \delta_1(u_t - \bar{u}) + \frac{\delta_2}{2}(u_t - \bar{u})^2$$

(34)

\(^\text{11}\)In the log-linearized model used in estimation, shocking $\iota$ is isomorphic to shocking $\psi$, which is used to calibrate the steady state.

\(^\text{12}\)Our investment adjustment cost function is a special case of the form specified in Christiano et al. (2005).
so that $\delta(\bar{u}) = \delta_0$, $\delta'(\bar{u}) = \delta_1 > 0$ and $\delta''(\bar{u}) = \delta_2 > 0$. Finally, as mentioned above, the matching technology follows the functional form suggested by den Haan et al. (2000):

$$m(V_t, S_t, \chi_t) = \chi_t \frac{S_t V_t}{(S_t^t + V_t^t)^{1/\epsilon}}. \quad (35)$$

There are seven fundamental exogenous processes hitting the economy:

$$\log(\gamma_{x,t}/\gamma_{x}) = \rho_x \log(\gamma_{x,t-1}/\gamma_{x}) + \epsilon_{x}^t \quad (36)$$
$$\log(\gamma_{z,t}/\gamma_{z}) = \rho_z \log(\gamma_{z,t-1}/\gamma_{z}) + \epsilon_{z}^t \quad (37)$$
$$\log(A_t) = \rho_g \log(A_{t-1}) + \epsilon_{a}^t \quad (38)$$
$$\log(\chi_t/\bar{\chi}) = \rho_x \log(\chi_{t-1}/\bar{\chi}) + \epsilon_{\chi}^t \quad (39)$$
$$\log(b_t) = \rho_b \log(b_{t-1}/\bar{\chi}) + \epsilon_{b}^t \quad (40)$$
$$\log(\iota_t/\bar{\iota}) = \rho_i \log(\iota_{t-1}/\bar{\iota}) + \epsilon_{i}^t \quad (41)$$
$$\log(G^S_t) = \rho_g \log(G^S_{t-1}) + \epsilon_{g}^t \quad (42)$$

In the above $\gamma_{x,t} \equiv X_t/X_{t-1}$ and $\gamma_{z,t} \equiv Z_t/Z_{t-1}$ describe the two stochastic trends of the economy. The preferences and technology described here are consistent with balanced growth. Since we will linearize the economy around a steady-state, we must first stationarize the first-order conditions of the economy. The details of these steps are given in Appendix A.

### 3.2 A Process for Wages

Since it is not feasible to estimate the infinity of coefficients of the MA representation of the wage in equation (21), we seek a more compact representation that spans, as much as possible, the same set of possible dynamics for wages. A large variety of representations is feasible, and the most appropriate one is not a priori obvious. A natural candidate representation for the stationary log-wage is

$$w_t = \phi_x x_t, \quad (43)$$

where $x_t$ represents the state vector, both exogenous and endogenous, of the economy. The advantage of this approach is that the endogenous states contain a summary of the full history of past shocks, and in a linear combination that is by construction relevant to economy. This approach also directly nests many wage setting mechanisms, such as Nash bargaining, that
do not themselves introduce additional states to the system.

This approach has drawbacks, however. First, it is hard to formulate priors on the parameter vector \( \phi \) and, in practice, the endogeneity of some elements \( x_t \) makes it especially difficult to constrain the Bayesian sampler (or any other search algorithm) to regions in which equilibrium both exists and is unique. The endogeneity of \( x_t \) also means that it is also quite difficult to characterize the restrictions implied by the representation in equation (43) on possible dynamics for wages. These challenges notwithstanding, we have explored representation (43) using a reliable global optimizer and found that it offers little or no advantages in terms of model fit relative to the representation we adopt below.

Notice that the above discussion of determinacy contrasts with the finding in other contexts that an exogenously specified price often leads to indeterminacy of equilibrium.\(^{13}\) Here, the observation of Michaillat and Saez (2013) that market tightness, rather than the wage, plays the market clearing role is crucial to pinning down a unique equilibrium. Our estimation procedure will discard parameter constellations with indeterminacy of equilibrium. If large regions of the parameter space are not easily transited by the sampling algorithm, this could potentially bias our results. While we do not have a theoretical result regarding uniqueness given this specification of prices, we are comforted by the strong fit of the model (suggesting that at least the quantitatively relevant part of the parameter space has not been excluded) and by the fact that the frequency of indeterminacy in the Bayesian simulations is similar across this and the structural Nash-bargaining formulation for the wage.

In order to maximize the transparency of the estimated parameters and minimize the complications cited above, we proceed to directly parameterize the impulse response of wages to each shock. Formally, we treat the (log of the) stationarized wage as the sum of seven independent components

\[
 w_t = \bar{w} + w_{x,t} + w_{z,t} + w_{a,t} + w_{\chi,t} + w_{b,t} + w_{\iota,t} + w_{g,t}. 
\]

(44)

The elements of this sum each evolve according to a univariate ARMA\((p,q)\) process, the precise nature of which depends on specification the of the underlying shock. For transitory shocks \( j \in \{a, \chi, b, \iota, g\} \), we assume a level-stationary process,

\[
 w_{j,t} = \Gamma_j(L)w_{j,t-1} + \Psi_j(L)\epsilon_{j,t}. 
\]

(45)

\(^{13}\)For a related discussion, see the literature on the Taylor principle, and also Giannoni and Woodford (2009).
For the non-stationary shocks, \( j \in \{x,z\} \), we assume a growth-stationary process

\[
\Delta w_{j,t} = \log(\gamma_j) + \Gamma_j(L)\Delta w_{j,t-1} + \phi_j (w_{j,t} - \gamma_{j,t}) + \Psi_j(L)\epsilon_{j,t},
\]

where the term multiplied by \( \phi_j \in (0,1] \) represents an error-correction term that ensures the level of the wage converges to a constant ratio with the other trending variables in the economy.

Given our interest in relaxing restrictive structural assumptions regarding wage determination, two comments on the modeling decisions embodied in (44) are in order. First, note that we have not included an error term in (44), and thus require that all of the variation in the real wage in our model be explained by our seven fundamental driving forces. This is not restrictive; re-estimating the exact same model but allowing for orthogonal variation in the real wage growth rate yields virtually no gains in terms of model fit. We view this result as an indication that the set of shocks specified above, acting both directly and through the real wage, are sufficient to explain the data.

We also estimate the model using alternative specifications of the wage. Rearrangement of equation (22) shows that the slow-adjustment specification is actually nested in our ARMA specification when \( \phi_x = \phi_j = \phi \) and the remaining ARMA parameters are set to zero. The Nash-wage specification is not explicitly nested in our ARMA process, and so requires specification of an additional parameter, \( \eta \), that is not present in either of our estimations. In order to ensure that our model comparison is not driven by prior choices on non-nested parameters, we use the least-restrictive priors that prove feasible for all parameters related to wage determination. See Appendix E.2 for details on the prior choices.

3.3 Data

We estimate the model using quarterly data from 1972Q1 to 2013QIV, with an additional six years of data used to initialize the Kalman filter. We classify the data used into two groups. The first set of observations, \( \hat{Y}_{1,t} \), consists of real quantity indicators for which we have a single measure. For these variables the link between the model and data is uncontroversial, and measurement errors are generally considered modest. Accordingly, we assume the data are a perfect measure of the corresponding model concept. The vector \( \hat{Y}_{1,t} \) consists of log-changes in real per-capita consumption, investment, government expenditure, the unemployment rate, and per-capita employment. The measurement equation linking
observables with model analogues is

\[
\hat{Y}_{1,t} = \begin{bmatrix}
\Delta \log c_t \\
\Delta \log i_t \\
\Delta \log g_t \\
\Delta \log ur_t \\
\Delta \log n_t
\end{bmatrix}.
\] (47)

Notice that we have not included GDP as an explicit observable; the resource constraint of the economy implies a relation between output, consumption, investment, government expenditure and vacancy postings (discussed below.) Including output therefore necessitates the addition of a measurement error term, since our model counterfactually assumes balanced trade, and we have found this additional data has no effect on the estimation. The covariance properties of these series are summarized in table 5.

To these “basic” data, we then append an additional set of variable \(Y_{2,t}\), for which the link to the model concepts, as well as the quality of measurement, is far less clear. In this vector, we include two measures of vacancy postings as well as five diverse measures of the aggregate wage. The first measure of vacancies comes from the BLS JOLTS survey, and is first available in 2001Q1. The second vacancy measure is given by the “Help-wanted” advertising index produced by the Conference Board through 2009. This series demonstrates a secular downward trend associated with the popularization of the internet and web-based job search, so we therefore drop all observations from 2000Q1 onward. Summary statistics for these series are included in table 7.\(^{14}\) Given the rather different inputs into these two series, we assume each represents a potentially rescaled version of the model concept \(\Delta v_t\) measured with error.

The choice of an appropriate empirical counterpart for the model wage is not obvious for at least two reasons. First, the failure of standard measures of the wage to account for unobserved worker heterogeneity can impart a countercyclical compositional bias upon real wages, an issue noted by Alan Stockman and subsequently studied by Solon et al. (1994). In recessions, unemployment should occur disproportionately among the low-skilled who earn low wages. Average measured wages thus rise during recessions due to the changing composition of the workforce, a fact that cannot be accounted for without comprehensive

\(^{14}\)The presence of missing observations means that our estimation occurs with an unbalanced panel. This creates no complication other than the appropriate modification of the Kalman filter.
individual-level data. Second, much recent literature has emphasized a distinction between the wage of new hires in the economy and the wage of ongoing matches. In many search and matching contexts it is the wage of new hires which plays the primary allocative role in the economy.\footnote{For some recent theory and empirical work on the issue see Pissarides (2009) and Kudlyak (2014), among others.}

For the reasons cited above, we incorporate five different real wage series in our estimation. The properties of these data are provided in table 8. The first three series are somewhat standard measures of compensation. The first is total real compensation-per-worker. Given the lack of an explicit hours margin in our model, and the growing importance of non-wage compensation in the US economy, we view this as conceptually the closest match to the real wage in our model economy. We add to this a measure of the real hourly wage, and real unit labor costs. These two measures are both somewhat more volatile than our preferred measure of the wage; the former is nonetheless highly correlated with the compensation measure while latter is less-so. To address concerns about composition bias, we include two additional series. The first is the Employment Cost Index compiled by the BLS, which seeks to follow wages holding constant the sectoral distribution of labor in the economy. The second is the quality adjusted wage series from Haefke et al. (2013), which seeks to adjust for individual-level characteristics to better account for the cyclical variation in the quality of workers. Neither series is available for the full sample period, and so each contributes to the unbalanced nature of the panel. As with the vacancy data, we assume each series represents a potentially rescaled version of the model concept $\Delta w_t$ measured with error.

In summary, the vector $\hat{Y}_{2,t}$ is related to the model according to the measurement equation

$$\hat{Y}_{2,t} = \begin{bmatrix} \gamma_{v,1} \Delta \log v_t \\ \gamma_{v,2} \Delta \log v_t \\ \gamma_{w,1} \Delta \log w_t \\ \gamma_{w,2} \Delta \log w_t \\ \gamma_{w,3} \Delta \log w_t \\ \gamma_{w,4} \Delta \log w_t \\ \gamma_{w,5} \Delta \log w_t \end{bmatrix} + \begin{bmatrix} \epsilon_{v1,t} \\ \epsilon_{v2,t} \\ \epsilon_{w1,t} \\ \epsilon_{w2,t} \\ \epsilon_{w3,t} \\ \epsilon_{w4,t} \\ \epsilon_{w5,t} \end{bmatrix}. \quad (48)$$

This approach to measurement is similar to the integration of DSGE and factor-based methods proposed by Boivin and Giannoni (2006). Further details on the data construction are
in Appendix E.2.

### 3.4 Identification

We log-linearize the model economy around the non-stochastic steady state, calibrate a subset of parameters to standard values, and estimate the remaining parameters via Bayesian methods. Cablibrated parameters are summarized in Table 1. We proceed to estimate the vector

$$\Theta_1 = \{h, \mu_{w/y}, \alpha, \phi, \xi, \bar{f}, \epsilon, p, \kappa, \phi_{g,y}\},$$

in addition to the parameters governing the fundamental exogenous processes and the ARMA wage process. Among the elements of $\Theta_1$, the compensation-to-output share $\mu_{w/y}$ is used to pin down the long-run wage in the economy, $\bar{f}$ pins down the scaling parameter of the disutility of labor $\psi$, and the long-run firm matching probability $q$ pins down the vacancy cost $a_0$, which is exactly neutral to dynamics in the economy. Model dynamics are driven by shocks to seven fundamental exogenous AR(1) processes: permanent total factor productivity ($\gamma_{X,t}$), the growth rate of the relative price of investment ($\gamma_{Z,t}$), temporary total factor productivity ($A_t$), matching efficiency ($\chi_t$), the discount factor ($b_t$), labor supply ($\iota_t$), and government spending ($g_t$).

In order to implement our procedure, we need to select the order the ARMA processes underlying wages in the economy. We experimented by adding subsequent AR and MA lags, but find that ARMA(1,0) provides very good fit without introducing an undue profusion of parameters.\(^{16}\) In principle, one could estimate the ARMA orders of each process within the Bayesian context following the strategy of Meyer-Gohde and Neuhoff (2014). Given the complication involved and the modest increase in fit that we find for models with higher order ARMAs, this step does not seemed warranted.

Given the large number of parameters required to parameterize the exogenous wage process, it is natural to wonder whether indeed the data contain sufficient information to identify the relevant parameters in the context of the model. We therefore test local identification using the method proposed by Iskrev (2010), which examines whether a set of model moments display linearly independent variation for a marginal adjustment of parameters. The model passes this test. We also check global identification using a numerical algorithm that

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\(^{16}\)Specifically, we treat all stationary shocks as affecting wages via an ARMA(1,0) process and treat growth shocks as affecting wages via ARMA(0,0) processes with an additional error-correction term that determines dynamics over time.
combines a genetic algorithm to discover a broad range of initial values with a hill-climbing procedure. Here we find some evidence of multiple local maxima with posterior density values that are relatively close to the global maximum, but whose dynamics are qualitatively different. For this reason, we impose priors that enforce two additional identification assumptions on the estimated parameters of the AR process. First, we impose the natural restriction that the impact response of the wage to an innovation in either temporary or permanent productivity is non-negative. Second, we impose the restriction that the first autoregressive coefficient of the polynomial $\Gamma_j(L)$ is non-negative for stationary shocks $j \in \{A, \chi, b, \iota, g\}$. This latter restriction prevents wage responses from exhibiting periodic “zig-zags”, a feature we believe is unlikely to be implied by structural models of wage determination. Importantly, these restrictions do not come close to binding at the global optimum of the posterior density, but rather help to prevent the MCMC algorithm from engaging in occasional diversions into regions of the state space that we believe are reasonably excluded on a priori grounds. See appendix E.2 for a detailed description of our choices for prior distributions.

4 Results

Table 2 reports estimates of deep parameters of our economy. The estimated values of these parameters are remarkably close to values found elsewhere in the literature. Particularly surprising, perhaps, is the finding that the long-run compensation share of output is around 69 percent, exactly as it appears in the data despite the fact that none of the data used in estimation bear directly on this value. These estimated parameter values imply a long-run share of vacancy posting in the economy 0.8 percent of GDP. While the correct empirical analogue is not clear, this is certainly within the range of plausible values. Finally notice that the vacancy adjustment parameter $\xi$ is estimated to be rather large. In fact, given the implied value of $a_0$ of around 0.3, in units of the vacancy good this implies a cost of adjusting vacancies that is about seven times larger than the investment adjustment cost, which itself is estimated to be in the low range of typical values. Table 2 also reports estimated autoregressive coefficients and standard deviations of innovations for the seven fundamental driving processes in our model. Our estimates of other parameters are broadly in line with those found in the DSGE estimation literature.\footnote{See, e.g., Christiano et al. (2005), Smets and Wouters (2007), Gertler et al. (2008).}

Table 3 reports the estimated parameters governing the evolution of the growth rate of
the real wage. Interpreting these values in isolation is somewhat difficult, but it is worth observing that only three shocks, $\epsilon_a, \epsilon_b, \epsilon_i$, have statically significant impact effects. The remaining growth shocks adjust over time due to their error-correcting terms, while the matching and government spending shocks have essentially no effect on wages.

Because we use a reduced-form model of wages, we need to check if the estimated model ever implies negative surplus for firms or households. In simulations, the estimated wage never violates the participation constraint of households (i.e. the wage never falls below the reservation wage). However, the participation constraint of firms is occasionally violated, as we will see presently in smoothed realizations. It it worth noting that, once linearized, even structural wage models will occasionally deliver wages outside the bargaining set. In principle, we could impose additional priors on the wage process that penalize parameter constellations that lead frequently to such violations. We have not pursued that avenue here, however.

4.1 Model Fit

The model does an exceptionally good job at matching moments from the data at business cycle frequencies. Tables 5 and 6 report the empirical and corresponding model-generated moments for the five primary quantity series used in estimation. Second moments, autocorrelations, and correlations with output growth all mirror the data very closely. One modest exception is that the model-implied volatility of unemployment is somewhat larger than that found in the data. Even for this variable, however, comovements and autocorrelations remain very close to the data. The covariogram in Figure 2 shows that the model fit is in fact very close at both short and medium horizons, with the data lying in between the Bayesian 95% confidence intervals at the vast majority of points. In fact, the alignment of the covariogram with the data between the 1-quarter and 8-quarter horizons is comparable to that which would be achieved with an unrestricted VAR of four lags; even more lags would be needed to replicate the model’s performance at matching longer-horizon covariances.

Much of the debate concerning the textbook search and matching model’s inability to generate adequate volatility in unemployment from productivity shocks focuses on the behavior of vacancies. Absent stochastic variation in the separation rate (which implies a counterfactually upward-sloping Beveridge curve), search and matching models require volatility in job creation in order to drive volatility in unemployment. Nash bargaining fails precisely because the wage is too elastic with respect to productivity; employers stand to gain little
from posting vacancies when increased productivity is matched by commensurate increases in real wages. As a result, models featuring Nash bargaining typically deliver inadequate volatility and procyclicality of the V-U ratio. Table 7 indicates that our model does not suffer from these issues, and instead matches the cross correlation of vacancies with the other quantity variables quite well.

Finally, figure 8 compares the model implied wage with the set of five empirical wage measures, while Table 4 provides estimates of the parameters of the measurement equation (48). Both the moments and the parameters of measurement strongly suggest that the compensation-per-worker measure of wages is the most consistent with observed quantities, given the model. The standard deviation, autocorrelation, and correlation with output are all quite close to the equivalent model-based wage. According to the measurement parameters, the hourly wage is also roughly consistent with the model in terms of scale (it is slightly too volatile) and has similar cyclical properties. Conversely, the Employment Cost Index, which has a relatively short history, seems somewhat too muted compared with the model. However, the estimate of its measurement error suggests it captures cyclical fluctuations in the wage rather well.

Figure 5 delves a bit deeper into the wage-measurement relationships by comparing the various measures of cumulative wage growth with the model-implied values over the same period. Once again, compensation-per-worker does remarkably well, with the measure and smoothed wages exhibiting a correlation of around 90 percent in both levels and growth rates. Meanwhile, the remaining wage measures seem to capture lower frequency movement of wages fairly well with the exception of unit labor costs which strays far from the smoothed wage during the first two-thirds of the sample.

4.2 Impulse Responses

Figure 3 documents impulse responses of the real wage, the two wedges, and labor’s share of match surplus. In all cases, wage responses to shocks under the estimated parameters are significantly different from those implied by Nash bargaining. All of the temporary shocks induce instantaneous changes in the real wage. However, with the exception of intertemporal preference shocks, which induce large responses, wage responses to temporary shocks are muted relative to the Nash case. It is also worth noting that wage responses to permanent shocks are markedly different from wage responses to temporary shocks. Permanent shocks do not induce instantaneous changes in the real wage, although in the case of permanent
TFP, the wage catches up to the trend in short order. Nothing in the estimation constrains this to be the case, the data simply demanded such a dichotomy between permanent and temporary shocks. Anticipating discussion of the variance decompositions, it should be noted here that shocks to permanent TFP are by far the dominant driver of variation in the real wage.

Turning now to Figure 4, it is quickly apparent that an agnostic specification of wages implies markedly different dynamics for vacancies and unemployment than the equal share Nash case. Large, hump-shaped dynamics of vacancies in response to shocks are consistent with the positive empirical autocorrelation in growth rates that others have documented. Interestingly, permanent increases in TFP induce large persistent contractions in vacancies and increases in unemployment after one period. Temporary TFP shocks induce instantaneous, small and short-lived increases in wages, thus driving increased vacancy posting and reduced unemployment. Shocks to intertemporal preferences induce huge, persistent swings in unemployment and vacancies, likely operating through their lasting effect on wages.

4.3 Variance Decomposition

Table 10 provides the unconditional variance decomposition for our estimated model. A significant fraction of variation in output, consumption, and investment growth is accounted for by temporary TFP shocks, as well the the permanent neutral and investment specific shocks. Somewhat surprising, however, is the important role that matching efficiency shocks play in determining output and investment, explaining roughly 30 percent of each. Matching efficiency shocks are by far the primary driver of fluctuations in the unemployment rate and overall employment, accounting for 62 and 77 percent of their variation respectively. Despite their importance elsewhere in the economy, and despite the small movement of the wage in response to TFP shocks, both temporary and permanent neutral productivity shocks play a decidedly small role in labor market dynamics.

Despite the importance of matching shocks for driving both employment and unemployment, it is the intertemporal preference shock that instigates the plurality (40 percent) of movements in vacancy posting. This seems to be caused almost entirely by the extremely persistent wage effect we estimate coming from this shock. The impulse response in panel (d) of Figure 4, shows that under Nash bargaining the wage response to this shock is of the opposite sign and the impact on vacancies is far smaller.

Importantly, over 70% of variation in the growth rate of the real wage is accounted for
by shocks to permanent TFP. Such shocks, however, have an otherwise negligible role in explaining variation in labor market quantities. The opposite is broadly true for matching efficiency shocks, which have effectively no role in causing wage fluctuations but alone explain the majority of movement in labor market quantities. Observing that shocks to the growth rate of the relative price of investment and temporary TFP together explain over half of the variation in output, consumption and investment, but less than 15 percent of employment and the unemployment rate, there appears to exist something of a dichotomy between the forces driving labor market dynamics, and those driving the rest of the economy.

4.3.1 The Great Recession

The model also delivers some interesting insights for the Great Recession period. Casual examination of the smoothed innovations to the fundamental shocks in figure 7 shows that, while the economy experienced some modestly bad realizations over the period, the overall size of the shocks required to match the Great Recession are not exceptional. Perhaps a bit more unusual from the perspective of the model, is the large number of consecutive negative realization for some variables over the recession period, most notably for matching efficiency.

To better understand the contribution of different shocks to the great recession, panel (a) of figure ?? plots unemployment realizations from the model in the counter-factual case that the realizations of matching efficiency shocks were zero starting from the first quarter of 2008. Eliminating these shocks would have cut peak unemployment during the period by about half. The second panel of figure ?? shows, furthermore, that even if no shocks had his the economy from 2008 onward, the model still would have predicted a rather substantial and sustained rise in unemployment, based on the state vector of the economy entering into the start of the recession period. This finding mirrors the results of Del Negro et al. (2014), who find that a new-Keynesian model with financial frictions could have forecast a substantial share of the Great Recession using only data through 2008.

4.4 Model Comparison

Having established that our baseline model fits the data very well, it is natural to ask whether either of the more structural assumptions about the wage that we consider can deliver a similarly good fit in an otherwise identical model. In order to quantify this difference in fit, we perform a Bayesian model comparison, computing marginal likelihoods for the three
models we wish to consider. Table 12 shows that, while the slow-adjusting wage model outperforms the model with Nash bargaining,\textsuperscript{18} both models are clearly dominated by the reduced-form specification for the wage, with a Bayes factor on the order of $exp(120)$.

The reasons for the failure of these two models are rather different, and in ways that corroborate previous observations about these two wage setting mechanisms. As demonstrated in table 13 and 14, the model estimated with Nash bargaining is unable to generate large enough variation in vacancy posting, while delivering an overly-volatile wage. Interestingly, the data do not seek a calibration that is consistent with the Hagedorn and Manovskii (2008) suggestion of low bargaining power combine with high replacement rate of benefits (median estimates are $\kappa = 0.46$ and $\eta = 0.41$.) Conversely, table 15 shows that the rigid-wage economy delivers counterfactually high volatility of unemployment, and to a less degree output growth.

Finally, table 12 also compares the model estimated with the non-structural wage process, but with the vacancy adjustment cost parameters $\xi$ set to zero. As the table indicates, this single parameter is crucial to matching the data, with a loss-of-fit from fixing that parameter that is fixed to zero of roughly one hundred log points. Interestingly, the importance of this particular parameter depends heavily on the presence of the wage series in the estimation; unconstrained by wage data, the model is able to “construct” an latent wage series that is largely consistent with the the comovements of aggregate quantities.

### 4.5 Rigidity and the Rate of Adjustment

We now turn to assessing the characteristics of the estimated wage process and what they can tell us about an empirically-relevant model of the real wage. While the impact responses of wages to the various shocks varies dramatically depending on the shock in question, the data do not appear to suggest that wages are intrinsically rigid independent of the source of underlying fluctuation. Wage responses to temporary TFP shocks are muted as one would expect if such shocks were the sole driver of business cycles, yet wages adjust to shocks to permanent TFP with haste after the first period of unresponsiveness. Accordingly, we believe the data suggest that wage adjustment can be quick but is often highly incomplete. Such properties do not accord well with either Nash bargaining or models embodying a

\textsuperscript{18}Because we assume that wages adjust to permanent shocks, albeit slowly, the rigid-wage model does in fact exhibit substantial variation in wages, variation that is consistent with the low-frequency movements in compensation-per-worker.
sluggish adjustment of the real wage. Indeed, our model dramatically outperforms both Nash bargaining and a Hall-style model augmented with shocks to trend growth in explaining the data.

The extent of wage rigidity is important. A large literature in macroeconomics has explored possible explanations for real rigidities as a means of generating realistic levels of unemployment during economic downturns. Examples include models of implicit contracts, insider-outsider wage bargaining, efficiency wages and social norms.\textsuperscript{19} It is perhaps theoretically plausible that such norms adjust quickly, and more fully, to permanent shocks relative to temporary changes, and such a story could potentially resolve the tension we find between the effects of temporary and permanent shocks.

### 4.6 Nash Bargaining

Much of the search and matching literature has focused on two special cases of our model: the Nash bargaining solution and the social planner’s solution.\textsuperscript{20} In the interest of comparing the dynamics of our estimated model with the dynamics of these two popular nested models, we derive the Nash bargaining solution associated with the framework described in Section 2 in Appendix C, and the social planner’s solution for the same framework in Appendix D. Figures 3-4 compare impulse responses of output, consumption, investment, wages, unemployment and vacancies to the six fundamental shocks for the estimated model and the Nash case.

As observed by Shimer (2005), the search and matching framework with Nash bargaining lacks an internal propagation mechanism because the wage is highly elastic with respect to labor productivity, thus dampening vacancy posting incentives for firms following shocks. In the context of a more complete search and matching model with capital and investment adjustment costs, productivity remains elevated for a longer period following shocks due to capital accumulation. Firms are nonetheless disinclined to exploit these productivity gains because any increases in vacancy posting quickly feed back into workers’ outside options, and thus wages. However, absent Nash bargaining, a protracted increase in productivity

\textsuperscript{19} Implicit contracts: Azariadis (1975); insider-outsider bargaining: Lindbeck and Snower (1986); efficiency wages: Yellen (1984); social norms: Akerlof (1980).

\textsuperscript{20}Hosios (1990) showed that, in the context of a standard search and matching model, the decentralized equilibrium coincides with that which would be chosen by a benevolent social planner when labor’s share of the match surplus is equal to the elasticity of the matching function with respect to searching workers. This condition does not hold precisely in our model because of the presence of an external habit in consumption. When we impose the Hosios condition and set the habit parameter to zero, the Nash and efficient outcomes coincide.
means that only a very limited measure of rigidity in real wages is required to generate large increases in the present discounted value of productivity net of the real wage. A small extent of rigidity, then, induces large fluctuations in vacancies relative to Nash bargaining. This intuition bears out in our impulse responses, which, despite varying considerably across shocks, appear to exhibit little intrinsic rigidity.

### 4.7 Efficiency and the Social Planner

Table 9 summarizes the dynamic consequences of our wage process for the vacancy posting and labor participation wedges in the economy. Both wedges are extremely volatile and persistent in the estimated economy, indicating substantial dynamic inefficiency. Table 11 shows that both wedges, and the labor share surplus, are substantially driven by intertemporal preference shocks, and to a lesser extent by matching efficiency and labor supply shocks. Surprisingly this pattern closely mirrors the variance decomposition of vacancy posting in the economy, suggesting a link. Indeed, the responses of these wedges to the intertemporal shock are also driven largely by the estimated wage responses to the shock; calibrating the wage response to this shock to be zero reduces the volatility of both wedges, and the shock’s contribution to vacancy variance, by an order of magnitude.

The pattern of wedge responses is also closely mirrored by the that of labor’s share of the match surplus, which also depends largely on the intertemporal preference shock. Efficiency and variations in labor’s share of the surplus are tightly linked, even when one moves away from the famous Hosios condition. Panels (b) and (c) of figure 3 shows impulse response of the wedges to the various shocks, while panel (d) shows the response of labors shares. All are shown alongside the analogous response in the model with Nash bargaining. The pattern of response shows a strong link between labor’s share of the surplus and the posting wedge. All wedges show starkly smaller responses to shocks under Nash bargaining, a difference that is especially noticeable for the two preference shocks.

Finally, figure 6 plots smoothed realizations for the labor share of surplus in the model economy. In addition to a modest downward trend over the sample period (according to the model this must be a temporary, if persistent change) the figure is notable because it shows that the labor share of surplus is strongly counter-cyclical. This result bears similarity to Cheremukhin and Restrepo-Echavarria (2014), who estimate a similarly counter-cyclical process in their model in which the Nash bargaining power varies exogenously over time. Our model corroborates the view that changes in labor’s share of surplus are important for
understanding labor-market fluctuations, and our model ties those changes closely to shifts in intertemporal preferences.

5 Conclusion

The search and matching framework has become a popular modeling device in mainstream macroeconomics. But no consensus has been reached on the nature of wage determination in such markets. We draw upon search theory for what we consider to be critical components for realistically representing market interactions—costly search, nontrivial job-finding rates, equilibrium unemployment—while dispensing with a priori wage determination mechanisms. Instead, we model wages as evolving according to an ARMA(1,0) process, thus approximately nesting all of the structural wage determination mechanisms consisted in the literature. We can do this because the search and matching framework per se requires nothing of wages except that they remain within the bargaining set. The flexibility of our approach allows us to study the empirical properties of the search and matching framework unadulterated by the influence of a particular theory of wages. The wage process that we estimate sheds light on the properties that a data-consistent theory of the wage must have.

Our model is able to match the data remarkably well. We match key business cycle frequency moments of output, consumption, investment, employment and the unemployment rate with a high level of accuracy relative to existing DSGE estimation literature. Moreover, our estimated model yields a implies an empirical realistic process for the real wage, one which is especially consistent with variations in the compensation-per-worker. Our estimated process for the wage suggests real wages need not be particularly rigid nor sluggish in order to generate empirically accurate cyclical fluctuations. This result stands in marked contrast with a large literature seeking amplification by way of nominal rigidities or other partial adjustment mechanisms. The success of our model indicates that, indeed, alternative structural model of wage determination could lead to qualitative improvement in the ability of search and matching models to match aggregate data.

While a variety of shocks affect aggregate output, consumption and investment in our economy, unemployment and labor force participation are driven in largely by shocks to matching efficiency. Despite the important of matching efficiency shocks, however, the most relevant shock for welfare is the intertemporal preference shock, which drives the majority of inefficient fluctuations in the labor market. These fluctuations which are closely tied to
variation in the labor share of surplus, which is both volatile and strongly counter-cyclical in the estimated economy.
Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Concept</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount Factor</td>
<td>0.992</td>
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<tr>
<td>$\iota$</td>
<td>Inverse Labor Supply Elasticity</td>
<td>1.500</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Risk Aversion</td>
<td>1.000</td>
</tr>
<tr>
<td>$u^n$</td>
<td>Steady-state U-rate (fixes $\lambda^N$)</td>
<td>0.062</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>Average Depreciation Rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>Curvature of Depreciation Rate</td>
<td>0.005</td>
</tr>
<tr>
<td>$g/y$</td>
<td>Government Share of GDP</td>
<td>0.205</td>
</tr>
<tr>
<td>$q^N$</td>
<td>Labor Matching Prob., firm side</td>
<td>0.500</td>
</tr>
<tr>
<td>$\gamma_x$</td>
<td>Long run tech growth</td>
<td></td>
</tr>
<tr>
<td>$\gamma_z$</td>
<td>Long run inv. spec. growth</td>
<td>0.996</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>Autocorrelation of permanent tech. growth</td>
<td>0.000</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Autocorrelation of permanent inv. spec. growth</td>
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Table 2: Posterior Estimates of Model Parameters, Part I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Concept</th>
<th>Median</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>Habit formation</td>
<td>0.811</td>
<td>0.720</td>
<td>0.862</td>
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<tr>
<td>$\mu_{w/y}$</td>
<td>Compensation Share of Output</td>
<td>0.689</td>
<td>0.646</td>
<td>0.730</td>
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<tr>
<td>$\alpha$</td>
<td>Labor elasticity of output</td>
<td>0.301</td>
<td>0.261</td>
<td>0.343</td>
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<tr>
<td>$\phi$</td>
<td>Investment adj. Cost</td>
<td>1.822</td>
<td>0.796</td>
<td>3.189</td>
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<tr>
<td>$\xi$</td>
<td>Vacancy adj. Cost</td>
<td>3.729</td>
<td>2.398</td>
<td>6.013</td>
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<tr>
<td>$f$</td>
<td>Steady-state labor-force (fixes $\psi$)</td>
<td>0.844</td>
<td>0.788</td>
<td>0.886</td>
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<tr>
<td>$\epsilon$</td>
<td>Elasticity of match function</td>
<td>0.356</td>
<td>0.111</td>
<td>0.703</td>
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<tr>
<td>$p^N$</td>
<td>Labor matching prob., consumer side</td>
<td>0.313</td>
<td>0.262</td>
<td>0.371</td>
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<td>$\kappa$</td>
<td>Replacement rate on unemployment benefits</td>
<td>0.041</td>
<td>0.014</td>
<td>0.098</td>
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<tr>
<td>$\phi_{g,y}$</td>
<td>Reversion to long-run government share</td>
<td>0.093</td>
<td>0.042</td>
<td>0.164</td>
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<tr>
<td>$\rho_a$</td>
<td>AR coeff on stationary TFP</td>
<td>0.627</td>
<td>0.523</td>
<td>0.715</td>
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<tr>
<td>$\rho_x$</td>
<td>AR coeff on matching efficiency</td>
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<td>0.752</td>
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<tr>
<td>$\rho_y$</td>
<td>AR coeff on demand</td>
<td>0.433</td>
<td>0.247</td>
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<tr>
<td>$\rho_i$</td>
<td>AR coeff on labor supply</td>
<td>0.994</td>
<td>0.988</td>
<td>0.998</td>
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<tr>
<td>$\rho_g$</td>
<td>AR coeff on government exp.</td>
<td>0.953</td>
<td>0.932</td>
<td>0.970</td>
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<tr>
<td>$\sigma^x$</td>
<td>Stddev of $\epsilon^x$</td>
<td>0.006</td>
<td>0.005</td>
<td>0.007</td>
</tr>
<tr>
<td>$\sigma^a$</td>
<td>Stddev of $\epsilon^a$</td>
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<td>0.006</td>
<td>0.011</td>
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<tr>
<td>$\sigma^z$</td>
<td>Stddev of $\epsilon^z$</td>
<td>0.016</td>
<td>0.009</td>
<td>0.020</td>
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<tr>
<td>$\sigma^\lambda$</td>
<td>Stddev of $\epsilon^\lambda$</td>
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<tr>
<td>$\sigma^b$</td>
<td>Stddev of $\epsilon^b$</td>
<td>0.016</td>
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<td>$\sigma^i$</td>
<td>Stddev of $\epsilon^i$</td>
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<td>0.008</td>
<td>0.011</td>
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<tr>
<td>$\sigma^g$</td>
<td>Stddev of $\epsilon^g$</td>
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<td>0.008</td>
<td>0.010</td>
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Table 3: Posterior Estimates of Model Parameters, Part II.

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</thead>
<tbody>
<tr>
<td>$\phi^x_0$</td>
<td>Wage impact - perm productivity</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>$\phi^z_0$</td>
<td>Wage impact - perm inv. spec.</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$\phi^a_0$</td>
<td>Wage impact - temp TFP</td>
<td>0.001</td>
<td>0.000</td>
<td>0.002</td>
</tr>
<tr>
<td>$\phi^\chi_0$</td>
<td>Wage impact - match efficiency</td>
<td>-0.000</td>
<td>-0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$\phi^b_0$</td>
<td>Wage impact - preference</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>$\phi^g_0$</td>
<td>Wage impact - labor supply</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>$\phi^\rho_0$</td>
<td>Wage impact - government</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>$\rho^{a}_0$</td>
<td>AR(1) - temp TFP</td>
<td>0.331</td>
<td>0.096</td>
<td>0.839</td>
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<tr>
<td>$\rho^{\chi}_0$</td>
<td>AR(1) - match efficiency</td>
<td>0.636</td>
<td>0.265</td>
<td>0.918</td>
</tr>
<tr>
<td>$\rho^{b}_0$</td>
<td>AR(1) - preference</td>
<td>0.985</td>
<td>0.966</td>
<td>0.996</td>
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<tr>
<td>$\rho^{g}_0$</td>
<td>AR(1) - labor supply</td>
<td>0.966</td>
<td>0.935</td>
<td>0.986</td>
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<tr>
<td>$\rho^{\rho}_0$</td>
<td>AR(1) - government</td>
<td>0.601</td>
<td>0.281</td>
<td>0.820</td>
</tr>
<tr>
<td>$\phi^{w,x}_0$</td>
<td>Error correction for perm productivity</td>
<td>0.892</td>
<td>0.695</td>
<td>0.975</td>
</tr>
<tr>
<td>$\phi^{w,z}_0$</td>
<td>Error correction for perm inv. spec.</td>
<td>0.100</td>
<td>0.057</td>
<td>0.150</td>
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Table 4: Posterior Estimates of Model Parameters, Part III.

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<tr>
<td>$\gamma^{w,1}$</td>
<td>Wage loading - compensation-per-worker</td>
<td>1.034</td>
<td>0.896</td>
<td>1.187</td>
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<tr>
<td>$\gamma^{w,2}$</td>
<td>Wage loading - hourly wage</td>
<td>1.364</td>
<td>1.207</td>
<td>1.537</td>
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<tr>
<td>$\gamma^{w,3}$</td>
<td>Wage loading - unit labor costs</td>
<td>1.120</td>
<td>0.916</td>
<td>1.342</td>
</tr>
<tr>
<td>$\gamma^{w,4}$</td>
<td>Wage loading - HSR quality adjusted wage</td>
<td>0.769</td>
<td>0.608</td>
<td>0.975</td>
</tr>
<tr>
<td>$\gamma^{w,5}$</td>
<td>Wage loading - employment cost index</td>
<td>0.396</td>
<td>0.341</td>
<td>0.460</td>
</tr>
<tr>
<td>$\gamma^{v,1}$</td>
<td>Vacancy loading - JOLTS survey</td>
<td>0.975</td>
<td>0.781</td>
<td>1.225</td>
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<tr>
<td>$\gamma^{v,2}$</td>
<td>Vacancy loading - Help-wanted</td>
<td>1.999</td>
<td>1.619</td>
<td>2.482</td>
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<td>$\sigma^{w,1}$</td>
<td>Measurement error - compensation-per-worker</td>
<td>0.003</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td>$\sigma^{w,2}$</td>
<td>Measurement error - hourly wage</td>
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<tr>
<td>$\sigma^{w,3}$</td>
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<td>$\sigma^{w,4}$</td>
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<td>0.010</td>
<td>0.009</td>
<td>0.012</td>
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<td>$\sigma^{w,5}$</td>
<td>Measurement error - employment cost index</td>
<td>0.003</td>
<td>0.003</td>
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<tr>
<td>$\sigma^{v,1}$</td>
<td>Measurement error - JOLTS survey</td>
<td>0.023</td>
<td>0.016</td>
<td>0.029</td>
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<tr>
<td>$\sigma^{v,2}$</td>
<td>Measurement error - Help-wanted</td>
<td>0.010</td>
<td>0.005</td>
<td>0.023</td>
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Table 5: Summary Moments - Data

<table>
<thead>
<tr>
<th></th>
<th>∆y</th>
<th>∆c</th>
<th>∆i</th>
<th>∆g</th>
<th>∆un</th>
<th>∆n</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ(X)</td>
<td>0.82</td>
<td>0.52</td>
<td>2.15</td>
<td>0.92</td>
<td>5.06</td>
<td>0.48</td>
</tr>
<tr>
<td>σ(X)/σ(∆Y)</td>
<td>1.00</td>
<td>0.64</td>
<td>2.61</td>
<td>1.12</td>
<td>6.15</td>
<td>0.58</td>
</tr>
<tr>
<td>ρ(Xₜ, Xₜ₋₁)</td>
<td>0.34</td>
<td>0.45</td>
<td>0.47</td>
<td>0.15</td>
<td>0.65</td>
<td>0.63</td>
</tr>
<tr>
<td>ρ(X, ∆Y)</td>
<td>1.00</td>
<td>0.62</td>
<td>0.75</td>
<td>0.22</td>
<td>-0.68</td>
<td>0.62</td>
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</table>

Table 6: Summary Moments - Estimated Model

<table>
<thead>
<tr>
<th></th>
<th>∆y</th>
<th>∆c</th>
<th>∆i</th>
<th>∆g</th>
<th>∆un</th>
<th>∆n</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ(X)</td>
<td>0.89</td>
<td>0.66</td>
<td>2.40</td>
<td>0.96</td>
<td>7.19</td>
<td>0.60</td>
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<tr>
<td>σ(X)/σ(∆Y)</td>
<td>1.00</td>
<td>0.75</td>
<td>2.70</td>
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<td>8.10</td>
<td>0.68</td>
</tr>
<tr>
<td>ρ(Xₜ, Xₜ₋₁)</td>
<td>0.54</td>
<td>0.40</td>
<td>0.57</td>
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<td>0.66</td>
<td>0.78</td>
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<tr>
<td>ρ(X, ∆Y)</td>
<td>1.00</td>
<td>0.82</td>
<td>0.93</td>
<td>0.14</td>
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</table>

Table 7: Moments - Vacancy-posting Measures

<table>
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<tr>
<th></th>
<th>Model</th>
<th>JOLTS</th>
<th>Help-Wanted</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ(X)</td>
<td>5.13</td>
<td>5.67</td>
<td>10.17</td>
</tr>
<tr>
<td>σ(X)/σ(∆Y)</td>
<td>5.78</td>
<td>6.90</td>
<td>12.38</td>
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<tr>
<td>ρ(Xₜ, Xₜ₋₁)</td>
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<td>0.65</td>
<td>0.68</td>
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<tr>
<td>ρ(X, ∆Y)</td>
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<td>0.41</td>
<td>0.78</td>
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Table 8: Moments - Wage Measures

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<tr>
<th></th>
<th>Model</th>
<th>Comp/Worker</th>
<th>Hrly Wage</th>
<th>ULC</th>
<th>HSR</th>
<th>ECI</th>
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<tbody>
<tr>
<td>σ(X)</td>
<td>0.60</td>
<td>0.60</td>
<td>0.77</td>
<td>0.98</td>
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<tr>
<td>σ(X)/σ(∆Y)</td>
<td>0.67</td>
<td>0.73</td>
<td>0.94</td>
<td>1.19</td>
<td>1.26</td>
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<tr>
<td>ρ(Xₜ, Xₜ₋₁)</td>
<td>0.16</td>
<td>0.01</td>
<td>-0.15</td>
<td>-0.27</td>
<td>-0.13</td>
<td>0.10</td>
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<tr>
<td>ρ(X, ∆Y)</td>
<td>0.31</td>
<td>0.33</td>
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<td>-0.48</td>
<td>0.02</td>
<td>0.18</td>
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</table>

Table 9: Moments - Wedges

<table>
<thead>
<tr>
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<th>Posting Wedge</th>
<th>Participation Wedge</th>
<th>Labor’s Share of Surplus</th>
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<tr>
<td>ρ(Xₜ, Xₜ₋₁)</td>
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<td>0.98</td>
<td>0.99</td>
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<tr>
<td>ρ(X, ∆Y)</td>
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Table 10: Variance Decomposition of Observables

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<th>$\sigma^i$</th>
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<tr>
<td>$\Delta y$</td>
<td>14.2</td>
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<td>25.0</td>
<td>4.6</td>
<td>3.3</td>
<td>1.6</td>
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<tr>
<td>$\Delta c$</td>
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<td>16.7</td>
<td>35.4</td>
<td>8.4</td>
<td>17.7</td>
<td>4.6</td>
<td>1.6</td>
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<tr>
<td>$\Delta i$</td>
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<td>26.6</td>
<td>24.8</td>
<td>32.6</td>
<td>2.1</td>
<td>2.1</td>
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<td>$\Delta g$</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<tr>
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<td>5.0</td>
<td>62.0</td>
<td>17.3</td>
<td>7.1</td>
<td>0.4</td>
</tr>
<tr>
<td>$\Delta n$</td>
<td>0.1</td>
<td>4.9</td>
<td>0.8</td>
<td>77.3</td>
<td>8.8</td>
<td>8.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Delta VN$</td>
<td>0.5</td>
<td>20.4</td>
<td>7.2</td>
<td>14.1</td>
<td>40.4</td>
<td>17.3</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Delta w$</td>
<td>72.9</td>
<td>7.3</td>
<td>5.3</td>
<td>0.0</td>
<td>7.3</td>
<td>5.6</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 11: Variance Decomposition of Wedges

<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma^x$</th>
<th>$\sigma^z$</th>
<th>$\sigma^a$</th>
<th>$\sigma^\chi$</th>
<th>$\sigma^b$</th>
<th>$\sigma^i$</th>
<th>$\sigma^g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Posting Wedge</td>
<td>0.7</td>
<td>8.8</td>
<td>0.7</td>
<td>7.9</td>
<td>67.8</td>
<td>13.4</td>
<td>0.7</td>
</tr>
<tr>
<td>Participation Wedge</td>
<td>0.9</td>
<td>8.3</td>
<td>0.4</td>
<td>10.4</td>
<td>65.9</td>
<td>13.3</td>
<td>0.7</td>
</tr>
<tr>
<td>Labor’s Share of Surplus</td>
<td>0.7</td>
<td>9.3</td>
<td>1.1</td>
<td>7.6</td>
<td>67.1</td>
<td>13.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 12: Model Comparison

<table>
<thead>
<tr>
<th>Model</th>
<th>$P(M)$</th>
<th>Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>5868.1</td>
<td>exp(0)</td>
</tr>
<tr>
<td>No Vac. Adj. Cost</td>
<td>5763.2</td>
<td>exp(105.0)</td>
</tr>
<tr>
<td>Fixed Wages</td>
<td>5744.8</td>
<td>exp(123.3)</td>
</tr>
<tr>
<td>Nash Bargaining</td>
<td>5652.2</td>
<td>exp(215.9)</td>
</tr>
</tbody>
</table>

Table 13: Moments - Vacancy-posting Measures for Nash-Bargaining economy

<table>
<thead>
<tr>
<th>Model</th>
<th>JOLTS</th>
<th>Help-Wanted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(X)$</td>
<td>0.98</td>
<td>5.67</td>
</tr>
<tr>
<td>$\sigma(X)/\sigma(\Delta Y)$</td>
<td>0.83</td>
<td>6.90</td>
</tr>
<tr>
<td>$\rho(X_t, X_{t-1})$</td>
<td>0.84</td>
<td>0.65</td>
</tr>
<tr>
<td>$\rho(X, \Delta Y)$</td>
<td>0.69</td>
<td>0.41</td>
</tr>
</tbody>
</table>

33
Table 14: Moments - Wage Measures for Nash Bargaining Economy

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Comp/Worker</th>
<th>Hrly Wage</th>
<th>ULC</th>
<th>HSR</th>
<th>ECI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(X)$</td>
<td>1.24</td>
<td>0.60</td>
<td>0.77</td>
<td>0.98</td>
<td>1.03</td>
<td>0.29</td>
</tr>
<tr>
<td>$\sigma(X)/\sigma(\Delta Y)$</td>
<td>1.05</td>
<td>0.73</td>
<td>0.94</td>
<td>1.19</td>
<td>1.26</td>
<td>0.35</td>
</tr>
<tr>
<td>$\rho(X_t, X_{t-1})$</td>
<td>0.41</td>
<td>0.01</td>
<td>-0.15</td>
<td>-0.27</td>
<td>-0.13</td>
<td>0.10</td>
</tr>
<tr>
<td>$\rho(X, \Delta Y)$</td>
<td>0.73</td>
<td>0.33</td>
<td>-0.02</td>
<td>-0.48</td>
<td>0.02</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 15: Summary Moments - Estimated Model with rigid wage

<table>
<thead>
<tr>
<th></th>
<th>$\Delta y$</th>
<th>$\Delta c$</th>
<th>$\Delta i$</th>
<th>$\Delta g$</th>
<th>$\Delta un$</th>
<th>$\Delta n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(X)$</td>
<td>1.37</td>
<td>0.63</td>
<td>2.93</td>
<td>0.91</td>
<td>9.48</td>
<td>0.63</td>
</tr>
<tr>
<td>$\sigma(X)/\sigma(\Delta Y)$</td>
<td>1.00</td>
<td>0.46</td>
<td>2.15</td>
<td>0.67</td>
<td>6.93</td>
<td>0.46</td>
</tr>
<tr>
<td>$\rho(X_t, X_{t-1})$</td>
<td>0.75</td>
<td>0.53</td>
<td>0.67</td>
<td>0.05</td>
<td>0.82</td>
<td>0.84</td>
</tr>
<tr>
<td>$\rho(X, \Delta Y)$</td>
<td>1.00</td>
<td>0.65</td>
<td>0.94</td>
<td>0.19</td>
<td>-0.87</td>
<td>0.86</td>
</tr>
</tbody>
</table>
Employment separation occurs ($p_{t-1}$ employees separate)

Aggregate state realized

Period $t-1$

Production (using $n_t$ employees and $k_t$ units of capital), goods markets and capital markets clear

Search and matching in labor market

Optimal labor-force participation decisions: $s_t$ individuals search for jobs

$nt = (1-\rho)nt-1 + m(st, vt)$

$yields$

$(1-\rho)nt-1$ individuals counted as employed, $st$ individuals counted as searching and unemployed

Period $t$

Firms post $v_t$ job vacancies

Period $t+1$

$n_t$

Figure 1: Timing of events in labor markets.
Figure 2: Correlogram comparing estimated model and actual data. Data moments largely fall within the 95 percent credible set of the estimated model.
Figure 3: Impulse responses of wages, wedges, and the labor share of surplus to each shock.
Figure 4: Quantity impulse responses to four different shocks.
Figure 5: Kalman-smoothed wages and the five wage indicator series.
Figure 6: Labor’s surplus share.
Figure 7: Smoothed fundamental innovations.
Figure 8: Counter-factual unemployment during the great recession. Figure shows log-unemployment in the economy for the counterfactual case where (a) $\epsilon^x_t = 0$ for 2008Q1 and on and (b) $\epsilon^j_t = 0$ for 2008Q1 and on, for all $j$ shocks. The vast majority of the unemployment during the great recession is explained by matching efficiency shocks.
Figure 9: Smoothed fundamental processes.
References


Appendices

A Stationary Representation

The model described in the body of the text is trend stationary with respect to labor-augmenting technological progress, $X_t$, and the relative price of investment, $Z_t$. Denoting by tildes the stationary counterparts to non-stationary variables, we can rewrite the model in terms of only stationary variables as follows:

\[
\tilde{Y}_t = A_t \left( u_t \tilde{K}_t \right)^\alpha (\gamma_{X,t} N_t)^{1-\alpha} \\
F_t = N_t + (1 - p_t) S_t \\
N_t = (1 - \lambda) N_{t-1} + M_t \\
\tilde{K}_{t+1} = \gamma_{X,t} \frac{N_t}{Z_t} \left[ (1 - \delta(u_t)) \tilde{K}_t + \tilde{I}_t \right] \\
\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t \left[ 1 + \Phi \left( \frac{\tilde{I}_t}{I_{t-1}} \gamma_{X,t-1} \gamma_{Z,t-1}^{\frac{1}{\alpha-1}} \right) \right] + \left[ a_0 + \xi \left( \frac{V_t}{V_{t-1}} \right) \right] V_t \\
\tilde{D}_t = \tilde{Y}_t - \tilde{W}_t N_t - \tilde{R}_t u_t K_t - \left[ a_0 + \xi \left( \frac{V_t}{V_{t-1}} \right) \right] V_t \\
\tilde{R}_t = \frac{\tilde{a}_t^{\lambda}}{\mu_t} \delta'(u_t) \\
\frac{\tilde{a}_t^K}{\mu_t} = E_t \left\{ \Omega_{t,t+1} \gamma_{Z,t} \left[ u_{t+1} \tilde{R}_{t+1} + (1 - \delta(u_{t+1})) \frac{\tilde{a}_t^K}{\mu_{t+1}} \right] \right\} \\
\frac{\tilde{a}_t^K}{\mu_t} = \gamma_{Z,t} \left[ 1 + \Phi \left( \frac{\tilde{I}_t}{I_{t-1}} \gamma_{X,t-1} \gamma_{Z,t-1}^{\frac{1}{\alpha-1}} \right) + \Phi' \left( \frac{\tilde{I}_t}{I_{t-1}} \gamma_{X,t-1} \gamma_{Z,t-1}^{\frac{1}{\alpha-1}} \right) \left( \frac{\tilde{I}_t}{I_{t-1}} \gamma_{X,t-1} \gamma_{Z,t-1}^{\frac{1}{\alpha-1}} \right) \right] \\
- E_t \left\{ \Omega_{t,t+1} \gamma_{Z,t} \left[ \gamma_{Z,t} \Phi' \left( \frac{\tilde{I}_{t+1}}{I_t} \gamma_{X,t} \gamma_{Z,t}^{\frac{1}{\alpha-1}} \right) \left( \frac{\tilde{I}_{t+1}}{I_t} \gamma_{X,t} \gamma_{Z,t}^{\frac{1}{\alpha-1}} \right)^2 \right] \right\} \\
\tilde{\mu}_t^N = \frac{1}{q_t} \left[ a_0 + \xi \left( \frac{V_t}{V_{t-1}} \right) + \left( \frac{V_t}{V_{t-1}} \right) \xi' \left( \frac{V_t}{V_{t-1}} \right) - \gamma_{X,t} \gamma_{Z,t}^{\frac{\alpha}{\alpha-1}} E_t \Omega_{t,t+1} \left( \frac{V_{t+1}}{V_t} \right)^2 \xi' \left( \frac{V_{t+1}}{V_t} \right) \right] \\
\tilde{\mu}_t^N = (1 - \alpha) A_t \left( \frac{u_t \tilde{K}_t}{\gamma_{X,t} N_t} \right)^\alpha \gamma_{X,t} - \tilde{W}_t + (1 - \lambda) E_t \left\{ \Omega_{t,t+1} \gamma_{X,t} \gamma_{Z,t}^{\frac{\alpha}{\alpha-1}} \tilde{\mu}_{t+1}^N \right\} \\
\tilde{R}_t = \alpha A_t \left( \frac{u_t \tilde{K}_t}{\gamma_{X,t} N_t} \right)^{\alpha-1} 
\]
\[-\frac{u_{f,t}}{u_{c,t}} = p_t \left( \tilde{W}_t + (1 - \lambda) E_t \left\{ \hat{\Omega}_{t+1} \left( \gamma_{X,t} \right) \frac{\alpha}{\alpha - 1} \right\} - \left( \frac{1 - p_{t+1}}{p_{t+1}} \right) \left( -\frac{u_{f,t+1}}{u_{c,t+1}} - \tilde{\kappa}_{t+1} \right) \right) \right\} + (1 - p_t)\tilde{\kappa}_t \]

(62)

where

\[
\Omega_{t,t+1} \equiv \beta \frac{b_{t+1}}{b_t} \frac{\mu_{t+1}}{\mu_t} \left( \gamma_{X,t} \right) \frac{\alpha}{\alpha - 1} \left( X_{t-1} Z_{t-1}^{\alpha-1} \right)^{\sigma} \]

(63)

\[
\bar{\mu}_t = \frac{\mu_t}{\left( X_{t-1} Z_{t-1}^{\alpha-1} \right)^{\sigma}} \]

(64)

\[
\bar{\mu}_K = \frac{\mu^K_t}{\left( X_{t-1} Z_{t-1}^{\alpha-1} \right)^{\sigma} Z_{t-1}} \]

(65)

\[
\bar{\mu}_N = \frac{\mu^N_t}{\left( X_{t-1} Z_{t-1}^{\alpha-1} \right)^{\sigma}} \]

(66)

and variables are detrended according to \( \tilde{\Delta}_t \equiv \frac{\Delta_t}{X_{t-1} Z_{t-1}^{\alpha-1}} \) for \( \Delta_t \in \{ Y_t, C_t, D_t, W_t \} \), \( \tilde{\Delta}_t \equiv \frac{\Delta_t}{X_{t-1} Z_{t-1}^{\alpha-1}} \) for \( \Delta_t \in \{ K_t, I_t \} \), and \( \tilde{R}_t \equiv \frac{R_t}{Z_t} \).
B Steady State and Calibration

We use the restrictions imposed by the deterministic steady-state of the model, together with long-run values for $\bar{p}$, $\bar{q}$, $\bar{N}$, $\bar{u}n$, $\bar{\phi}N$, $\bar{u}$ taken from the data, to analytically solve for values for all remaining endogenous variables, as well as $\chi$, $\lambda$, $\psi$, $a$, $\delta_1$ and $W$. In what follows, we make reference only to detrended variables, though we neglect tildes for ease of notation.

The first-order conditions for investment and utilization, evaluated at the deterministic steady state, imply

$$\mu^K/\mu = \gamma Z \quad (67)$$
$$R = (\mu^K/\mu)\delta_1. \quad (68)$$

Substituting these expressions into the household’s first-order condition for period-ahead capital and solving for $\delta_1$, we obtain

$$\delta_1 = \left[ (\Omega \gamma Z)^{-1} - 1 + \delta_0 \right] / \bar{u} \quad (69)$$

from which we recover

$$R = \gamma_z \delta_1. \quad (70)$$

From the firm’s first-order condition for capital, we obtain

$$K = \left[ \alpha A \bar{u}^{\alpha-1}(\gamma_X \bar{N})^{1-\alpha}r^{-1} \right]^{1/\alpha} \quad (71)$$

from which, in turn, we can solve for output

$$Y = A(uK)^{\alpha}(\gamma_X \bar{N})^{1-\alpha}. \quad (72)$$

The definition of the unemployment rate, together with its long-run value and that of total employment, can be used to solve for steady-state labor force participation

$$F = \frac{\bar{N}}{1 - \bar{u}n}. \quad (73)$$

The definition of the labor force, in turn, allows us to solve for the mass of searching workers,

$$S = \frac{F - \bar{N}}{1 - \bar{p}}. \quad (74)$$
The law of motion for employment (from the perspective of the household and the firm, respectively) yield

\[ \lambda = \frac{\bar{p}S}{\bar{N}} \]  
\[ V = \frac{\lambda \bar{N}}{q} \].

From the definition of the matching function, we obtain

\[ \theta = \frac{\bar{p}}{\bar{q}} \]

which in turn allows us to solve for \( \chi \),

\[ \chi = \bar{p} \theta^{1-\epsilon}. \]

Next, from the definition of labor's share, together with its steady state value \( \bar{\phi}^N \), we can solve for the real wage,

\[ W = \bar{\phi}^N \bar{N}^{-\alpha} A(\bar{u}K)^\alpha \frac{\gamma_1^{1-\alpha}}{\gamma_{X}^{1-\alpha}}. \]

With the steady state real wage in hand, we use the vacancy posting condition to solve for the vacancy posting cost

\[ a = \frac{\bar{q}}{1 - (1 - \lambda) \Omega \gamma_X \gamma_Z^{\alpha-1}} \left[ (1 - \alpha) A \left( \frac{\bar{u}K}{\bar{N} \gamma_X} \right)^\alpha - W \right] \]

where \( \Omega = \beta \left( \gamma_X \gamma_Z^{\alpha-1} \right)^{-\sigma} \). Finally, the law of motion for capital and the aggregate resource constraint imply, respectively,

\[ I = K \left[ \frac{1 - (1 - \delta_0) \gamma_X^{-1} \gamma_Z^{\alpha-1}}{\gamma_X^{-1} \gamma_Z^{\alpha-1}} \right] \]

and

\[ C = Y - G - a_0 V - \gamma_Z I. \]
Finally, making use of the labor force participation condition, we obtain

$$\psi = \frac{W \left[ \bar{p} + (1 - \bar{p}) \kappa - \kappa (1 - \lambda) \beta (1 - \bar{p}) \left( \gamma X \gamma \gamma Z^{-1} \right)^{1 - \sigma} \right]}{\left( 1 - (1 - \lambda) \beta (1 - \bar{p}) \left( \gamma X \gamma \gamma Z^{-1} \right)^{1 - \sigma} \right) \left( \left( 1 - F \right)^{-\gamma} C (1 - h (\gamma X \gamma \gamma Z^{-1})^{-1}) \right)} \cdot (83)$$
C Match Surplus and Nash Bargaining

In order to compare the dynamics of our estimated model with those implied by Nash Bargaining, we first use the expressions for household and firm match surpluses to derive the wage associated with Nash Bargaining in our model. This also yields expressions for household and firm match surplus shares, which we discuss in the body of the text.

The wage that results from Nash bargaining between firms and workers is given by

$$W_{t}^{NB} = [W_t - U_t]^{\eta}[J_t - V_t]^{1-\eta} \quad (84)$$

where $W_t$ denotes the value of a match for the household, $U_t$ denotes the value of unemployment for the household, $J_t$ denotes the value of a match for the firm, and $J_t$ denotes the value of a vacancy for the firm. Free-entry of firms implies that $V_t = 0$, and our specification of unemployment benefits, combined with the existence of a search margin for households, implies that $U_t = \kappa_t$. Thus, the standard Nash sharing rule reduces to

$$W_t - U_t = (\frac{\eta}{1-\eta}) J_t. \quad (85)$$

The household match surplus (in units of consumption) may be expressed as the sum of the wage payment earned in the period of the match (due our timing assumption) and the continuation value of the match, less the lump-sum transfer to the unemployed,

$$W_t - U_t = W_t - \kappa_t + (1 - \lambda) E_t \{ (1 - p_{t+1}) \Omega_{t,t+1} (W_{t+1} - U_{t+1}) \}. \quad (86)$$

The value of a match to the firm (again, in units of consumption) is given by current marginal product of the match net of the wage bill plus the continuation value,

$$J_t = A_t F_{n,t} - W_t + (1 - \lambda) E_t \{ \Omega_{t,t+1} J_{t+1} \}. \quad (87)$$

Note that the firm’s first-order conditions imply

$$J_t = \mu_t^N. \quad (88)$$

To solve for the wage associated with Nash bargaining, begin by substituting the expressions
for $W_t$ and $U_t$ into the Nash sharing rule,

$$(1 - \kappa)W^NB_t + (1 - \lambda)E_t \{(1 - p_{t+1})\Omega_{t,t+1}(W_{t+1} - U_{t+1})\} = \frac{\eta}{1 - \eta} J_t. \quad (89)$$

Iterating the sharing rule forward and substituting in for $W_{t+1} - U_{t+1},$

$$(1 - \kappa)W^NB_t + (1 - \lambda)E_t \left\{(1 - p_{t+1})\Omega_{t,t+1} \left(\frac{\eta}{1 - \eta} J_{t+1}\right)\right\} = \frac{\eta}{1 - \eta} J_t. \quad (90)$$

Replacing $J_t$ with the firm’s first-order condition for labor and then $J_{t+1}$ with the time $t + 1$ Lagrange multiplier,

$$(1 - \kappa)W^NB_t + (1 - \lambda)E_t \left\{(1 - p_{t+1})\Omega_{t,t+1} \left(\frac{\eta}{1 - \eta} \mu_{t+1}^N\right)\right\} = \frac{\eta}{1 - \eta} \left(A_tF_{n,t} - W^NB_t + (1 - \lambda)E_t \left\{\Omega_{t,t+1}\mu_{t+1}^N\right\}\right). \quad (91)$$

Solving for $W^NB_t,$ we obtain

$$W^NB_t = \frac{\eta}{1 - \kappa + \eta\kappa} \left[A_tF_{n,t} + (1 - \lambda)E_t \left\{\Omega_{t,t+1}\mu_{t+1}^N\right\}\right]. \quad (92)$$

From here, dividing by $X_{t-1}Z^\alpha_{t-1}$ yields the stationary representation,

$$\tilde{W}^NB_t = \frac{\eta}{1 - \kappa + \eta\kappa} \left[A_t\tilde{F}_{n,t} + (1 - \lambda)\gamma_{X,t}\gamma_{Z,t}^\alpha E_t \left\{\Omega_{t,t+1}\tilde{\mu}_{t+1}^N\right\}\right]. \quad (93)$$

Then, for a given wage determination mechanism, the standard expression for the household’s share of match surplus is given by

$$SS^H = \frac{W - U}{W - U + J - V} \quad (94)$$

and the expression for the firm’s share of match surplus is

$$SS^F = \frac{J - V}{W - U + J - V} = 1 - SS^H. \quad (95)$$
D Efficient Allocations

D.1 Non-stationary model

The social planner’s problem may be expressed as

$$\max_{C_t, V_t, F_t, N_t, I_t, K_{t+1}, u_t} E_0 \sum_{t=0}^{\infty} \beta^t b_t u(C_t, F_t)$$

subject to

$$A_t F(u_t K_t, X_t N_t) = C_t + I_t Z_t \left(1 + \Phi \left(\frac{I_t}{I_{t-1}}\right)\right) + X_{t-1} Z_{t-1}^{\alpha} \left(a_0 + \xi \left(\frac{V_t}{V_{t-1}}\right)\right) V_t$$

$$N_t = Z_t^{\alpha} \xi \left(\frac{V_t}{V_{t-1}}\right) V_t$$

where $\mu_t$, $\mu_t^N$, and $\mu_t^K$ are the respective Lagrange multipliers associated with the constraints of the planner’s problem. There are two distortions in the decentralized economy considered in the body of the paper. The first, search and matching frictions, arise as firms and households fail to internalize the aggregate implications of their search activities. The second, consumption externalities, arise because agents do not internalize the impact of their consumption decisions on aggregate consumption.\footnote{Note that the decentralized economy does not exhibit distortions associated with investment or vacancy adjustment costs because these costs are specific to the firm’s current and past decisions, and thus no externalities exist.} The conditions for optimality are given
where $\beta_{t,t+1} \equiv \frac{b_{t+1}}{b_t}$ and $m_S$ and $m_V$ are the partial derivatives of the matching function with respect to the mass of searching workers and vacancies, respectively. Note also that, in the case of the social planner,

\[
\mu_{C,t} = [(C_t - hC_{t-1}) v(F_t)]^{-\sigma} v(F_t) - h\beta E_t \left\{ [(C_{t+1} - hC_t) v(F_{t+1})]^{-\sigma} v(F_{t+1}) \right\}.
\]

The Hosios condition will therefore not, in general, be satisfied, due to the external habit in consumption internalized only by the social planner. In terms of stationary variables, the
planner’s optimality conditions may be expressed as

\[
\mu_t^N m_{V,t} = a_0 + \xi \left( \frac{V_t}{V_{t-1}} \right) + \left( \frac{V_t}{V_{t-1}} \right) \xi' \left( \frac{V_t}{V_{t-1}} \right) - \gamma_{X,t} \frac{\alpha}{\gamma_{Z,t}} E_t \left\{ \Omega_{t,t+1} \left( \frac{V_{t+1}}{V_t} \right)^2 \xi' \left( \frac{V_{t+1}}{V_t} \right) \right\} \tag{109}
\]

\[
\mu_t^N m_{S,t} = MRS_t \tag{110}
\]

\[
\mu_t^N = A_t \tilde{F}_{N,t} + (1 - \lambda) E_t \left\{ \Omega_{t,t+1} \gamma_{X,t} \frac{\alpha}{\gamma_{Z,t}} (1 - m_{S,t+1}) \mu_{t+1}^N \right\} \tag{111}
\]

\[
\mu_t^K = \gamma_{Z,t} \left[ 1 + \Phi \left( \frac{I_t}{I_{t-1}} \gamma_{X,t-1} \frac{1}{\gamma_{Z,t-1}} \right) + \Phi' \left( \frac{I_t}{I_{t-1}} \gamma_{X,t-1} \frac{1}{\gamma_{Z,t-1}} \right) \left( \frac{I_t}{I_{t-1}} \gamma_{X,t-1} \frac{1}{\gamma_{Z,t-1}} \right) \right] \tag{112}
\]

\[- E_t \left\{ \Omega_{t,t+1} \left[ \gamma_{Z,t+1} \Phi' \left( \frac{I_{t+1}}{I_t} \gamma_{X,t+1} \frac{1}{\gamma_{Z,t}} \right) \left( \frac{I_{t+1}}{I_t} \gamma_{X,t+1} \frac{1}{\gamma_{Z,t}} \right)^2 \right] \right\} \tag{113}
\]

\[
\mu_t^K = E_t \left\{ \gamma_{Z,t} \Omega_{t,t+1} \left[ A_{t+1} \tilde{F}_{K,t+1} + \mu_{t+1}^K (1 - \delta(u_{t+1})) \right] \right\} \tag{114}
\]

\[
\mu_t^K \delta'(u_t) = A_t \tilde{F}_{u,t} \tag{115}
\]

where we define

\[
\mu_t^N \equiv \frac{\mu_t^N}{\mu_t X_{t-1} Z_{t-1}^{\alpha-1}} \tag{116}
\]

\[
\mu_t^K \equiv \frac{\mu_t^K}{\mu_t Z_{t-1}} \tag{117}
\]

\[
MRS_t \equiv - \frac{u_{F,t}}{u_{C,t}} \left( X_{t-1} Z_{t-1}^{\alpha-1} \right)^{-1} \tag{118}
\]

\[
\tilde{F}_{u,t} \equiv \frac{F_{u,t}}{K_{u,t} Z_{t-1}} \tag{119}
\]

Equations (100) through (121) can be rearranged to deliver two separate conditions for optimality in the labor market, following Arseneau and Chugh (2012). We then define two efficiency wedges, \( \omega_{stat,t} \) and \( \omega_{dyn,t} \), as the ex-ante deviations from these optimality conditions.
In terms of the stationary model, the wedges are defined as

\[ MRS_t = \omega_{\text{posting},t} \frac{m_{S,t}}{m_{Y,t}} \left[ a_0 + \xi \left( \frac{V_t}{V_{t-1}} \right) + \left( \frac{V_t}{V_{t-1}} \right) \xi' \left( \frac{V_t}{V_{t-1}} \right) - \gamma_{X,t} \gamma_{Z,t} \right] E_t \left\{ \Omega_{t,t+1} \left( \frac{V_{t+1}}{V_t} \right)^2 \xi' \left( \frac{V_{t+1}}{V_t} \right) \right\} \]

(120)

\[ \frac{MRS_t}{m_{S,t}} = \omega_{\text{particip},t} A_t \tilde{F}_{N,t} + (1 - \lambda) E_t \left\{ \Omega_{t,t+1} \gamma_{X,t} \gamma_{Z,t} \left( 1 - m_{S,t+1} \right) \frac{MRS_{t+1}}{m_{S,t+1}} \right\} \] .

(121)

E Estimation Details

E.1 Data Construction

Nominal quantity variables are converted to their real per capita counterparts by dividing by the seasonally-adjusted chain-weighted GDP deflator and the civilian non-institutional population. Consumption is measured as the sum of personal consumption expenditures on non-durables and services, while investment is measured as the sum of personal consumption expenditure on durables and total private fixed investment. Unemployment is measured as the number of unemployed persons as a percentage of the civilian non-institutional labor force.

E.2 Estimation Details

The tables below describe the prior distributions used for the model estimation. The subsequent figures summarize the prior and posterior distributions for the same set of parameters.
Table 16: Prior Distributions - I

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Figure 10: Prior and posterior distributions of estimated parameters.
Figure 11: Prior and posterior distributions of estimated parameters.
Figure 12: Prior and posterior distributions of estimated parameters.