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Does the Preference Reversal Phenomenon Necessarily Contradict the Independence Axiom?

By Uzi Segal*

One of the most puzzling paradoxes in decision theory is the preference reversal phenomenon. This phenomenon seems to contradict the transitivity axiom, for a long time, one of the cornerstones of utility theory. This paradox is established whenever a decision maker prefers lottery X to lottery Y, but is willing to put a lower selling price on X than on Y. Such experiments, first reported by Harold Lindman, 1971; and Sarah Lichtenstein and Paul Slovic, 1971, were repeated by David Grether and Charles Plott, 1979; Werner Pommerehne et al., 1982; and Robert Reilly, 1982. Although the later researchers improved the mechanism through which the selling price emerges, they all found systematic reversals.

A new approach to this problem was developed by Charles Holt, 1986, and Edi Karni and Zvi Safra, 1987. From two different starting points, these authors showed that the preference reversal phenomenon does not necessarily prove a violation of the transitivity axiom, as it may contradict the independence axiom. This implies that people do not maximize expected utility, but in that case the preference reversal phenomenon becomes just another evidence against expected utility theory, but not against the transitivity axiom.

These results have their own disadvantages. Although the independence axiom is not as fundamental as transitivity, it is nevertheless very appealing on normative grounds. Of course, the preference reversal phenomenon necessarily contradicts at least one of the assumptions of expected utility theory, but empirical evidence shows that despite its normative appeal, violations of the reduction of compound lotteries axiom may happen (see, for example, Joshua Ronen, 1971, and Doug Snowball and Clif Brown, 1979). In this paper I therefore suggest a decision mechanism for the preference reversals lotteries which is transitive and satisfies the independence axiom, but not the reduction axiom.

I. The Preference Reversal Phenomenon

Consider the two lotteries $P = (-1, 1/36; 4, 35/36)$ and $S = (-1.5, 25/36; 16, 11/36)$.\(^1\) $P$ is sometimes called the $P$ bet (or lottery) and $S$ is called the $S$ lottery. After the decision maker expresses his preferences between these two lotteries, he is asked to announce the prices for which he is willing to sell $P$ and $S$. This is done by using the Gordon Becker, M. DeGroot, and J. Marschak (1964) mechanism. After the decision maker declares his selling price of $P$, call it $r_p$, one number out of the set (0.00, ..., 9.99) is selected at random (the offer price). If it exceeds $r_p$, the decision maker receives the offer price. Otherwise, he participates in the lottery $P$. Similarly, the selling price of $S$, $r_s$, is found. Denote these new compound lotteries by $X_P$ and $X_S$, respectively. This mechanism seems to force the decision maker to announce his true certainty equivalent\(^2\) of $P$ and $S$. Indeed, if $r_p > CE(P)$, he may have to participate in $P$ even though he would rather sell it, while $r_p < CE(P)$ may force him to sell $P$ against his will. If $r_p = CE(P)$ and $r_s = CE(S)$, then

*Department of Economics, University of Toronto, Toronto, Canada M5S 1A1. I wrote this paper while visiting the Economics Department at UCLA. I am grateful to Edi Karni, Mark Machina, Zvi Safra, and an anonymous referee for their comments.

\(^1\)The lottery $(x_1, p_1; \ldots; x_n, p_n)$ yields $x_i$ dollars with probability $p_i$, $i = 1, \ldots, n$.

\(^2\)The certainty equivalent of a lottery $A$ is given implicitly by $(CE(A), 1) \sim A$. 

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by the definition of the certainty equivalent and by the transitivity axiom, \( P \succeq S^3 \) iff \( r_p \succeq r_S \). However, most people prefer \( P \) to \( S \), but announce \( r_p \), which is less than \( r_S \).

Before presenting other explanations for this phenomenon, one should define precisely the independence and reduction axioms. By the later one, the decision maker is indifferent between a compound lottery, that is, a lottery at which the prizes are by themselves tickets for other (standard) lotteries, and its actuarially equivalent one-stage lottery. By the independence axiom, the simple lottery \( X \) is preferred to the simple lottery \( Y \), iff the compound lottery \( A \), yielding with probability \( p \) a ticket for \( X \) and with probability \( 1-p \) a ticket for another lottery \( Z \), is preferred to \( B \), which is the same as \( A \) but with \( Y \) instead of \( X \). These two axioms imply together that the decision maker prefers the simple lottery \( (x_1, p_1; \ldots; x_n, p_n) \) to \( (y_1, q_1; \ldots; y_m, q_m) \), iff for every \( 0 < p \leq 1 \) he prefers the simple lottery \( (x_1, pp_1; \ldots; x_n, pp_n; z_1, (1-p)r_1; \ldots; z_t, (1-p)r_t) \) to \( (y_1, pq_1; \ldots; y_m, pq_m; z_1, (1-p)r_1; \ldots; z_t, (1-p)r_t) \). Most recent writers refer (at least implicitly) to this last axiom as the independence axiom. However, in this paper the independence preference axiom is to be understood in its two-stage lotteries context.

Two recent articles showed that the preference reversal phenomenon, although inconsistent with expected utility theory, could alternatively be attributed to a violation of the independence axiom and not the transitivity axiom. Holt (1986) pointed out that the experiment itself might have some influence on the announced selling prices. Indeed, to save money, the subjects participated only in one lottery. The decision maker first decided whether he prefers \( P \) or \( S \) (say, \( P > S \)), and announced \( r_p \) and \( r_S \). Then, one out of the lotteries \( P \), \( X_p \), and \( X_S \) was selected at random, and the decision maker played it. Formally, he had to choose between \( A = (P, 1/3; X_p, 1/3; X_S, 1/3) \) and \( B = (S, 1/3; X_p, 1/3; X_S, 1/3) \). By the inde-

\( P \succeq S \) iff \( P \) is weakly preferred to \( S \). \( P > S \) iff \( P \succeq S \), but not \( S \succeq P \), and \( P - S \) iff \( P \succeq S \) and \( S \succeq P \).
II. Preference Reversals and the Reduction Axiom

Consider again the lottery $X_S$. In this lottery the decision maker announces a number $r_S$. Then he is offered a random price from the set $(0.00, \ldots, 9.99)$. If it exceeds $r_S$ he receives this amount, otherwise he plays the lottery $S$. Karni and Safra claimed that this lottery gives with probability $0.1(r_S + 0.01)$ the lottery $S$, and with probability $1/1000$ each of the prizes $r_S + 0.01, \ldots, 9.99$. However, other possible interpretations lead to different results. Let $\langle \alpha, \beta \rangle$ be the uniform distribution on the segment $[\alpha, \beta]$. It may be that the decision maker considers $X_S$ as the following two-stage lottery. With probability $0.1(r_S + 0.01)$ it gives a ticket to the lottery $S$. On the other hand, if the offer price exceeds $r_S$, the probability of which is $0.1(9.99 - r_S)$, then he participates in the lottery $\langle r_S + 0.01, 9.99 \rangle$ which is the uniform distribution over the segment $[r_S + 0.01, 9.99]$. In other words, $X_S = (S, 0.1(r_S + 0.01); \langle r_S + 0.01, 9.99 \rangle, 0.1(9.99 - r_S))$. By using the independence axiom and the certainty equivalent mechanism one obtains that

1. $X_S \sim (CE(S), 0.1(r_S + 0.01))$;
2. $CE(\langle r_S + 0.01, 9.99 \rangle, 0.1(9.99 - r_S))$.

Similarly,

1. $X_P \sim (CE(P), 0.1(r_P + 0.01))$;
2. $CE(\langle r_P + 0.01, 9.99 \rangle, 0.1(9.99 - r_P))$.

It may now happen that $P > S$, but $r_S > r_P$. This is demonstrated by the following example.

**Example**: Let $A$ be a lottery. Define $F_A(x) = \Pr(A \leq x)$, and let $A^0 = \{(x, p) \in \mathbb{R} \times [0,1]: F_p(x) \leq p \}$. Let $L = \{ A: F_A(-2) = 0 \}$. $L$ is the set of all lotteries with loss floor of $-2$. For every $A, B \in L$, let $A \succeq B$ iff $V(A) \geq V(B)$, where

\[ V(A) = \mu(A^0 \cap [-2, \infty) \times [0,1]) + 1.44 \cdot 10^8 \mu\left( A^0 \cap \left[ \frac{555}{144}, \frac{556}{144} \right] \times \left[ \frac{1}{1000}, 1 \right] \right) \mu\left( A^0 \cap \left[ \frac{936}{144}, \frac{937}{144} \right] \times \left[ \frac{1380}{3600}, \frac{1381}{3600} \right] \right). \]

($\mu$ denotes the Lebesgue’s measure.) It is easy to verify that $CE(P) = 278/72$ and $CE(S) = 277/72$, hence $P > S$. Similarly, for $3.03 \leq x \leq 4.32$,

\[ CE(\langle x, 9.99 \rangle) = \frac{x + 9.99}{2} - \frac{5}{2592}. \]

From (1), (2), and (3) we now obtain Table 1. It follows that $r_P = 3.82$, $r_S = 3.85$, and $r_P < r_S$.

**Remark 1**: The preference relation in this example is transitive and continuous, satisfies the first-order stochastic dominance axiom for preferences\(^4\) and the independence axiom, but not the reduction of compound lotteries axiom.

\(^4\) First-order stochastic dominance axiom for preferences: If, for every $x$, $F_A(x) \leq F_B(x)$, then $A \succeq B$. 

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**Table 1 — Values of $X_P$ and $X_S$**

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<thead>
<tr>
<th>$r_P$</th>
<th>$V(X_P)$</th>
<th>$r_S$</th>
<th>$V(X_S)$</th>
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Remark 2: $V$ is not a measure of $A^0$. It can be proved that if $V$ is a measure, then this analysis will not solve the preference reversal phenomenon.

III. Concluding Remarks

Holt (1986) and Karni and Safra (1987) claimed that the preference reversal phenomenon does not necessarily contradict the transitivity axiom, as it may indicate a violation of the independence axiom. In this paper I showed that this phenomenon may suggest a departure from the reduction of compound lotteries axiom, and not from the independence axiom. What is missing in this approach, as well as in the above-mentioned two works, is a general framework that explains the preference reversal phenomenon together with other paradoxes like the Allais paradox and the common ratio effect. First steps in this direction were taken by Holt (1986).

REFERENCES


