This is the first course of the Ph.D. econometrics sequence at Boston College. Its primary objective is to provide a statistical foundation and its application to econometric models (especially regression models) for further coursework in econometrics at the Ph.D. level. Linear algebra and calculus are pre-requested. Topics that are related to econometrics are emphasized.

• **Instructor:** Zhijie Xiao

• **Textbooks:**


• The material covered in this course can be found in a number of textbooks (at different levels). e.g.:

  – Knight, Keith, Mathematical Statistics.
• Requirements:

There will be 8-10 problem sets. Late assignments will not be accepted. You are allowed to work together in groups, but it is necessary that you attempt each problem yourself and turn in individual answers. The Boston College policy on academic integrity is outlined at:

http://www.bc.edu/bc_org/avp/enmgt/stserv/acd/univ.html#integrity.

There will be 2 exams. The grade will be determined by homework (20%), and the exams (80%).

If you are a student with a documented disability seeking reasonable accommodations in this course, please contact Kathy Duggan, (617) 552-8093, dugganka@bc.edu, at the Connors Family Learning Center regarding learning disabilities and ADHD, or Paulette Durrett, (617) 552-3470, paulette.durrett@bc.edu, in the Disability Services Office regarding all other types of disabilities, including temporary disabilities. Advance notice and appropriate documentation are required for accommodations.
Course Outline

   
   (a) Introduction to Probability, Conditional Probability, Limit of Events: DeGroot and Schervish, Ch1 and 2., Casella and Berger, Ch1.
   
   (b) Random Variables and Distributions, Bayes Theorem and It's Applications: DeGroot and Schervish, Ch3., Casella and Berger, Ch1, 2 and 4.
   
   (c) Moments and Expectation, Martingales: DeGroot and Schervish, Ch 4., Casella and Berger, Ch 2.
   
   (d) Special Distributions: DeGroot and Schervish, Ch 5., Casella and Berger, Ch3.
   
   (e) Some Important Results in Probability, Inequalities and bounds for probability: DeGroot and Schervish, Ch 4.6, 4.8., Casella and Berger, Ch3 and 4.
   
   (f) Introduction to Asymptotics, Convergence of Random Variables, Delta Method, Law of Large Numbers and Central Limit Theorem: DeGroot and Schervish, Ch 4.8, 5.7., Casella and Berger, Ch5.

2. Part II: Statistical Inference and Linear Regression
   
   (a) Introduction to Statistics: Decision Procedures. DeGroot and Schervish, Ch 6, Casella and Berger, Ch6.
   
   (b) Estimation: Method of Moments, GMM, MLE. DeGroot and Schervish, Ch 6, 7.7., Casella and Berger, Ch7.
   
   (c) Properties of Estimators: Moments and distributions of conventional estimators. DeGroot and Schervish, Ch 7., Casella and Berger, Ch10.
   
   
   (e) An Introduction To Linear Model: Linear Regression, Gauss-Markov Theorem. Basic Properties of OLS. Basic Inference Procedure in Linear Models. DeGroot and Schervish, Ch 10., Casella and Berger, Ch10, 12.
A Detailed List of Topics

PART I: Probability

1 Introduction

Background knowledge: Uncertainty, Probability, and Statistics
Introduction to Econometrics

2 Set Theory and The Definition of Probability

Experiment, Set and Elements, Event and Outcomes
Collection of Sets: Set of Sets, Field and \( \sigma \)-field, Subfield and sub \( \sigma \)-field
Definition of Probability
Basic Properties of Probability Measure

3 Probability and Conditional Probability

Limits of Events (Limit Sets)
The Borel-Cantelli Lemma
Updating Probabilities, Conditional Probability
Independent Events
Conditional independence
Law of Total Probability
Bayes’s Theorem
Prior and Posterior Probabilities

4 Random Variable

Definition of Random Variable
Distribution Function of a Random Variable
Continuous Random Variable and Density
Discrete Random Variable

5 Multivariate RVs and Conditional Probability Distribution

Bivariate Joint Probability Distribution
Conditional Probability and Independence
Bayes Theorem and It’s Applications
Multivariate Distributions

6 Functions of Random Variables

Distribution of a Monotone Transformation of a r.v.
The distribution of a function of a r.v.
Function of r.v.s
Examples: Linear transformation; The Maximum and Minimum Values; Sums of random variables

7 Expectations and Conditional Expectations

Mean/Mathematical Expectation/Expected Value
Moments
Quantiles
Bivariate Moments, Correlation, Conditional Expectation
Filtration
Martingale, submartingale, supermartingale
Doob (-Meijer) decomposition
Martingale Difference Sequences
The Martingale Convergence Theorem

8 Generating Functions

Generating function
Probability Generating Function
Moment Generating function.
Characteristic function

9 Some Special Distributions

• Discrete Distributions

Hypergeometric Distribution
Bernoulli Distribution
Binomial Distribution
Multinomial Distribution
Poisson Distribution
Continuous Distributions

Uniform Distribution
Beta$(a, b)$ distribution
Gamma$(a, b)$ Distribution
Exponential$(\theta)$
Distribution Derived from Normal

10 Some Useful Inequalities and Limiting Results

• Bounds for Probabilities: Markov Inequality and Chebyshev’s Inequality
  Bernstain inequality
  Hoeffding’s inequality.
  Jensen’s Inequality

• Covariance Inequalities
  Cauchy-Schwarz inequality:
  Holder’s inequality
  Minkowski’s Inequality
  Lyapunov’s inequality:
  Marcinkiewicz–Zygmund inequality
  Yokoyama, R. (1980) inequality for mixing sequences
  Rosenthal inequality

• Convergence of Random Variables
  Modes of Convergence
  Convergence of Functions of Random Variables:
  Some important limiting results.
  Law of Large Numbers
  Central Limiting Theorems
  Order of magnitude
  Law of Iterated logarithm (LIL)
  The Delta Method
PART II: Statistics

11 Introduction

Data and Model

- Parameters and Identification
  
  Parametric, Nonparametric, Semiparametric

- The decision theoretic framework: the information we want to draw

Estimation, Hypothesis testing, Ranking, Prediction

- Comparison of Decision Theory Framework

Loss Function, The Risk Function
Evaluating Decision Process

12 Estimation and Testing Based on Sample Mean and Variance

Basic Concepts
  
  Point Estimation and Interval estimation: Bias, Variance, and MSE, Unbiasness and Consistency
  
  Sufficiency and "Reduction of the Data", Factorization Theorem
  
  Minimum Sufficient Statistic
  
  Properties of Sample Mean and Sample Variance Under Normality
  
  Distribution of the estimators without assuming normality - asymptotic distributions via CLT and LLN
  
  Confidence Intervals

13 Estimation: GMM and MLE

- Method of Moment and Generalized Method of Moment

- MLE

Large sample result

- Minimum Variance unbiased estimator (MVU) and Rao-Blackwell Theorem
Lehmann-Scheffé Theorem

- Maximum Likelihood Estimation in General Form

One-Step Estimator
Fisher Information and Cramer-Rao inequality

- Properties of MLE
Consistency
Asymptotic Normality:
Asymptotic Efficiency:
Invariance

14 Hypothesis Tests

- Hypothesis, Null hypothesis
Test
Simple hypothesis
Composite hypothesis
alternative hypothesis

- Testing a Hypothesis
Type I and Type II Error, Level of significance, Power of the test
Testing and confidence interval

- Optimal Tests
Most Powerful Test and the Neyman-Pearson Lemma
Uniformly Most Powerful Test, Monotone likelihood ratio
UMP Unbiased Tests

- Likelihood Ratio tests

15 Introduction to Regression

Using Regression to Study the Relationship among variables
Bivariate Linear Regression
Classical Assumptions for Linear Models
Properties of the OLSE
Inference about the OLSE
GAUSS-MARKOV THEOREM
16 Multiple Regression

Multiple OLS Regression in Matrix form, GAUSS-MARKOV THEOREM
Measurement of Goodness of Fit
Testing Significance of the Regression
Multicollinearity
Heteroskedasticity/Heteroscedasticity
Autocorrelation