Can the emergence of new and more productive intangible technologies hurt capital reallocation and reduce aggregate output? We develop a model of an economy in which firms use tangible and intangible capital. Firms' only source of external finance is the collateral value of tangible assets, so a shift towards new technologies with a higher reliance on intangible capital decreases corporate borrowing and interest rates. The decrease in interest rates increases the price of intangible assets and decreases the rate at which non-investing firms accumulate savings, reducing the ability of credit constrained expanding firms to purchase capital. An increase in the share of intangible capital in production analogous to the one observed in the US can jointly replicate several key trends: i) corporate net savings increased; ii) household net leverage increased; iii) the real interest rate fell; iv) asset prices increased; and v) output progressively declined relative to its previous trend. Finally, a decline in interest rates caused by an increase in aggregate household savings has significantly different implications for reallocation and aggregate output in an economy with a low share of intangibles when compared to an economy with a high share of intangibles.

Keywords: Intangible Capital, Borrowing Constraints, Capital Reallocation, Secular Stagnation

JEL Classification:
1 Introduction

Real interest rates have decreased in the last decades, while economic growth has fallen short of previous trends, developments that have been linked to a process of ‘secular stagnation’ (Summers (2015), Eichengreen (2015)). At the same time, the developed world has experienced a technological change towards a stronger importance of information technology and knowledge, human and organizational capital, which has gradually reduced the reliance on physical capital (Corrado and Hulten (2010a)), and which has been linked to the significant decrease in corporate net borrowing (Falato et al (2013), Döttling and Perotti (2015)).

This paper argues that the rise in importance of intangible capital as a factor of production is key to explain both the declining real interest rates and the sluggish growth. Aggregate productivity depends on an efficient reallocation of resources from declining or exiting firms to new entrants or expanding firms. The rise of intangible capital implies a growing importance of the reallocation of intangible assets such as patents, brand equity, and human and organizational capital. These assets cannot be collateralized and their acquisition has to be financed mostly using retained earnings. An increase in the share of intangible capital in production thus decreases corporate borrowing and in equilibrium depresses interest rates. The decrease in interest rates, in turn, increases the price of these intangible assets, and reduces the ability of credit constrained expanding firms to purchase them. Lower interest rates also decrease the rate at which non-investing firms can accumulate savings to finance future expansions.

This alternative explanation of secular stagnation is consistent with crucial stylized facts about recent trends in industrialized economies, such as declining interest rates, below-potential growth, and large increases in net corporate savings and asset prices over GDP, and has potentially important policy implications.

We formalize this intuition by developing a stylized model of an economy in which a productive sector uses a technology with tangible capital, intangible capital and labor as complementary factors in the production of consumption goods. Moreover, we follow Kiyotaki and Moore (2012) in assuming that this sector is populated by a continuum of firms that can only invest occasionally. Firms suffer from financing constraints that prevent them from issuing equity, or from borrowing any amount in excess of the collateral value of their holdings of tangible capital. They have finite lives, and this prevents them from accumulating enough savings to overcome
their financial constraints. In equilibrium, they save as much as possible in non-investing periods, and invest all of their accumulated net savings plus their borrowings in investing periods. Both tangible and intangible capital are assumed not to depreciate and are in fixed supply. Any residual capital not absorbed by the firm sector is used by an unproductive alternative sector, which as a result of being the marginal buyer of capital also prices it. Therefore, aggregate productivity in this economy depends on the ability of growing productive firms to absorb the assets liquidated by the exiting firms. The consumer sector is modelled as overlapping generations of three-period lived credit constrained households, as in Eggertsson and Mehrotra (2014), a device that enables us to obtain an equilibrium interest rate in the steady state which is not necessarily equal to the household rate of time preference.

Our first exercise consists in simulating the effects of an increase in firms’ reliance on intangible capital in their production technology. We follow Falato et al. (2013) and capture this trend by increasing the production function parameter that determines the intangible to tangible capital ratio, so that such ratio increases from the value observed at the end of the 1970s, to the value observed at the beginning of the 2000s. Since we assume that intangible capital is more productive in equilibrium, this experiment can be interpreted as a shortcut for an endogenous process of adoption of more productive technologies. The increase in the relative importance of intangible capital in the production function induces productive firms to shift from tangible capital to intangible capital. Since the latter is not collateralizable, the firm’s borrowing capacity declines, and the interest rate must fall to stimulate a reduction in households saving and clear the bond market. The decline in the interest rate influences reallocation and aggregate output through multiple channels. On the one hand, a lower interest rate enables investing firms to borrow more, both because borrowing costs are lower and because a low interest rate increases the value of tangible assets, which firms use as collateral. This collateral value channel has a positive effect on reallocation and aggregate output. On the other hand, a lower interest rate also increases the price of intangible assets, thus reducing the amount of capital firms can purchase with given savings and borrowing capacity. This capital purchase price channel has a negative effect on reallocation. Furthermore, lower rates also affect firms without an investment opportunity. They have an additional positive effect on reallocation by helping firms that borrowed recently to invest to repay their debt (debt overhang channel).
However, firms that have not been investing for a while are net savers, and lower rates slow
down the rate at which they accumulate earnings to finance future investment opportunities.
This *savings channel* has a negative effect on reallocation. While the intangible ratio is small,
the collateral value channel and the debt overhang channel dominate, generating a moderate
increase in aggregate output following a decrease in the interest rate, thanks to the higher pro-
ductivity of intangible capital. However, as the share of intangible assets keeps increasing and
the interest rate declines further, the other two negative channels become stronger, hampering
reallocation. As a consequence, despite the higher productivity of intangible assets, aggregate
output is 6% lower in the equilibrium with a share of intangible capital equal to 50% (as in the
post-2000 period) than in the equilibrium with a share of intangible capital equal to 20% (as in
the pre-1980 period).

As mentioned above, we interpret this comparative static exercise as the developments in
the US economy following the rise in the share of intangible capital in the last 40 years. In
this respect, this simple model is remarkably consistent with a series of well documented trends
during this period: i) corporate savings increased as a fraction of GDP; ii) household leverage
increased as a fraction of GDP; iii) the real interest rate fell; iv) asset prices relative to aggregate
output increased; and v) output progressively declined relative to its previous trend. While the
importance of the rise in intangible capital for stylized facts i)-iii) has been already shown by
Falato et al (2013) and Dottling and Perotti (2015), this paper is, to the best of our knowledge,
the first to show that they are potentially very important in explaining the secular stagnation
fact v).

In order to further understand the relation between interest rates, reallocation of intangible
assets, and aggregate output, we perform another simulation exercise where a decline in the
interest rate is caused by a deleveraging process in the household sector, as in Eggertsson
and Mehrotra (2015). This exercise is relevant because recent empirical studies demonstrate
that demand side factors such as demographic forces, higher inequality within countries, and a
preference shift towards higher saving by emerging market governments, have been responsible
for a large increase in savings and the declining trend in real interest rates over the last three
decades (Rachel and Smith, 2015). We first study this deleveraging process holding fixed the
share of intangible capital at the pre-1980 level of 20%. In this case, the *collateral value channel*
dominates and the deleveraging process actually improves reallocation and raises aggregate output by 0.5%. However, when we repeat the same exercise holding fixed the share of intangible capital at the post-2000 level of 50%, we find that the two negative channels, the capital purchase price channel and savings channel, are much more important, and result in a 5% decline in aggregate output.

2 Related Literature

The secular stagnation hypothesis as an explanation of recent economic trends has been proposed, amongst others, by Summers (2015) and Eichengreen (2015). One prominent example of a formalization of these ideas is Eggertsson and Mehrotra (2014), who show how a persistent tightening of the debt limit facing households can reduce the equilibrium real interest rate and, in the presence of a zero lower bound and sticky prices, generate permanent reductions in output.\(^1\) Common to most of these accounts of secular stagnation is that the excess savings arise from the household or the foreign sector, but not from a decrease in the demand for those savings from the corporate sector. For example, Summers (2015) and Eichengreen (2015) mention factors such as population aging and a rise in savings of developing economies. An exception is Thwaites (2015), who explains the decrease in interest rates as a result of the decrease in the relative price of investment goods. In our model, a realistically calibrated increase in the use of intangible capital can achieve a substantial decrease in interest rates.

The rising use of intangible capital has been documented by Corrado and Hulten (2010a), and its relation to the decrease in corporate borrowing and the rise in corporate cash holdings has been shown empirically by Bates et al. (2009). Falato et al. (2013) and Döttling and Perotti (2015) introduce models that describe how the rise in intangibles can lower the equilibrium interest rate by decreasing firms’ net borrowing. Giglio and Severo (2012) link the decrease in interest rates caused by the rise of intangibles to the appearance of asset price bubbles. Our contribution to this literature is to describe a mechanism through which the rise in intangibles can have a negative impact on aggregate reallocation and growth.

Finally, our paper is related to the literature on the determinants of aggregate investment. A broad class of investment models predict that lower interest rates reduce the user cost of capital.

\(^1\)Other recent theoretical papers with alternative explanations of secular stagnation are Bachetta et al (2015) and Benigno and Fornaro (2015).
and stimulate investment. However, a large body of empirical research finds very little evidence of this negative relation (e.g. see, Caballero (1999), and Schaller (2007)). More recently, Kothari, Lewellen and Warner (2015), using a multivariate regression framework that includes as additional determinants of investment corporate profits, stock market returns, credit spreads and GDP growth, find a positive relation between lagged risk free interest rates and aggregate investment up to 2 quarters into the future. In our general equilibrium model, aggregate capital and interest rates are both endogenous and they may correlate positively or negatively with each other depending on the relative importance of tangible and intangible factors of production, with a positive relation which prevails, because of the rise in intangibles, during the post-1980 period.

3 Model

We introduce an infinite-horizon, discrete-time economy populated by firms, who use labor and tangible and intangible capital to produce consumption goods, and households, who provide labor to the firms and own them. We describe the firm sector and firms’ optimization problem in Section 3.1, and discuss the households’ problem and optimal choices in Section 3.2.

3.1 Firms

We consider an economy with two types of firms, productive and unproductive. The unproductive sector is not essential for the analysis, and is introduced because it allows for a simple characterization of the equilibrium prices of tangible and intangible capital and simplifies the analysis considerably.

3.1.1 The Productive Sector

The productive sector is composed of a continuum of firms of mass 1.

Technology and financing opportunities

They produce a final good using a constant returns to scale production function which is Cobb-Douglas in labor and capital. The firms use two different types of complementary capital, tangible and intangible. For simplicity, we assume that both tangible and intangible capital do not depreciate and are in fixed supply, with aggregate stocks represented respectively by $K^T$
and $K^I$, and that they are perfect complements. The production function takes the form:

$$y_t^p = g(\mu) z_t n_t^{(1-\alpha)} \left[ \min \left( \frac{k_{T,t}}{1-\mu}, \frac{k_{I,t}}{\mu} \right) \right]^\alpha$$

(1)

where $0 < \alpha \leq 1$, $0 < \mu < 1$. The terms $k_{T,t}$ and $k_{I,t}$ represent tangible and intangible capital installed in period $t-1$ that produce output in period $t$. Finally, $z_t$ is a productivity parameter, $n_t$ is labor, and the productivity term $g(\mu)$ is a monotonic and increasing function of the intangible share parameter $\mu$.

The budget constraint for productive firms is given by the following dividend equation:

$$d_t = y_t^p + (1 + r_t) a_{f,t} - a_{f,t+1} - q_{T,t} \left( k_{T,t+1} - k_{T,t} \right) - q_{I,t} \left( k_{I,t+1} - k_{I,t} \right) - w_t n_t.$$

(2)

where $r_t$ is the interest rate, $q_T$ and $q_I$ are the prices of tangible and intangible capital, respectively, and $w_t$ is the wage. $a_{f,t} > 0$ indicates that the firm is a net saver, and $a_{f,t} < 0$ indicates that the firm is a net borrower.

Productive firms are subject to frictions in their access to external finance. They are unable to issue equity, which means that dividends are subject to a non-negativity constraint:

$$d_t \geq 0.$$  

(3)

They can issue one-period riskless debt, subject to the constraint that they can only pledge a fraction $\theta$ of tangible capital as collateral. This translates into the following borrowing constraint:

$$a_{f,t+1} \geq -\frac{\theta q_{T,t+1} k_{T,t+1}}{1 + r_{t+1}}. $$

(4)

We conjecture, and check later, that in equilibrium firms are credit constrained and choose not to pay dividends. Imposing that $d_t = 0$ in budget constraint (2), and substituting for $a_{f,t+1}$ in (4), we can express the borrowing constraint as:

$$\left( q_{T,t} - \frac{\theta q_{T,t+1}}{1 + r_{t+1}} \right) k_{T,t+1} + q_{I,t} k_{I,t+1} \leq y_t^p - w_t n_t + (1 + r_t) a_{f,t} + q_{T,t} k_{T,t} + q_{I,t} k_{I,t}.$$

(5)

From the Leontief structure of the production function it follows that $k_{T,t} = \frac{1-\mu}{\mu} k_{I,t}$. Therefore, from now onwards, we use this result to express all equations as a function of intangible capital only, and (5) becomes:
Solving for $k_{I,t+1}$, we obtain:

$$k_{I,t+1} \leq \frac{y_t^p - w_t n_t + (1 + r_t)a_{f,t} + \left( q_{T,t} \frac{1 - \mu}{\mu} + q_{I,t} \right) k_{I,t}}{q_{T,t} \frac{1 - \mu}{\mu} + q_{I,t} - \theta q_{T,t+1} \frac{1 - \mu}{\mu}}$$ (6)

The right hand side of equation (6) is the maximum feasible investment in intangible capital for a firm. The numerator of equation (6) is the total wealth available to invest. The denominator captures the downpayment necessary to purchase one unit of $k_{I,t+1}$ and $\frac{1 - \mu}{\mu}$ units of $k_{T,t+1}$. The term $q_{T,t} \frac{1 - \mu}{\mu} + q_{I,t}$ represents the total cost necessary to purchase these amounts of both types of capital, and the term $\theta q_{T,t+1} \frac{1 - \mu}{\mu}$ is the amount that can be financed by borrowing.

**Timing, value function, and optimality conditions**

At the beginning of each period, both types of capital are predetermined and in their optimal ratio $k_{T,t} = \frac{1 - \mu}{\mu} k_{I,t}$, and therefore the production function can be written as:

$$y_t^p = g(\mu) z_t n_t \left( \frac{k_{I,t}}{\mu} \right)^{1 - \alpha}.$$ (7)

Given the wage $w_t$ and its predetermined capital $k_{I,t}$, a firm will choose the profit maximizing level of labor, which determines the optimal capital labor ratio:

$$\frac{k_{I,t}}{n_t} = \mu \left[ \frac{w_t}{(1 - \alpha) g(\mu) z_t} \right]^{\frac{1}{\alpha}}.$$ (8)

After producing, the firm’s technology becomes useless with probability $\psi$. In this case, the firm liquidates all its capital, and pays out as dividends all of its savings, including the liquidation value of capital, and exits.

Firms cannot invest every period. More specifically, they can only invest in a given period with probability $\eta$. This assumption, in addition to capturing the realistic feature that firms’ investment is lumpy (Caballero (1999)), is meant to allow firms to have the opportunity to accumulate significant amounts of liquid savings, in line with the empirical evidence.

Let $\lambda_t$ and $\mu_t$ be the Lagrange multipliers of constraints (3) and (6), respectively. We define the value functions conditional on investing and not investing, respectively $V^+(k_{I}, a_{f,t})$ and
$V^-(k_I, a_{f,t})$, as follows:

$$V_t^+(k_{I,t}+1, a_{f,t}+1) = \max_{a_{f,t+1}, k_{I,t+1}} \left[ \left(1 + \lambda_t\right)d_t + \mu_t \left[ y_t^p - w_t n_t + \left(1 + r_t\right)a_{f,t} + \left(q_{T,t} - \frac{1}{\mu} + q_{I,t}\right)k_{I,t} - \frac{1}{\mu} + q_{I,t} - q_{T,t} \frac{1}{1 + \tau_{t+1}} \frac{1}{1 + \mu} \right] k_{I,t+1} q_{I,t} \right]$$

$$+ \frac{1}{1 + r_{t+1}} \left(1 - \psi\right)V_{t+1}(k_{I,t+1}, a_{f,t+1}) + \psi d_{t+1}^{exit},$$

(9)

and

$$V_t^-(k_{I,t}+1, a_{f,t}+1) = \max_{a_{f,t+1}} \left[ \left(1 + \lambda_t\right)d_t + \mu_t \left[ y_t^p - w_t n_t + \left(1 + r_t\right)a_{f,t} + \left(q_{T,t} - \frac{1}{\mu} + q_{I,t}\right)k_{I,t} - \frac{1}{\mu} + q_{I,t} - q_{T,t} \frac{1}{1 + \tau_{t+1}} \frac{1}{1 + \mu} \right] k_{I,t+1} q_{I,t} \right]$$

$$+ \frac{1}{1 + r_{t+1}} \left(1 - \psi\right)V_{t+1}(k_{I,t+1}, a_{f,t+1}) + \psi d_{t+1}^{exit},$$

(10)

where $d_{t+1}^{exit}$ is the dividend in case of liquidation and exit from activity:

$$d_{t+1}^{exit} = y_{t+1}^p + \left(1 + r_{t+1}\right)a_{f,t} + q_{T,t} \frac{1}{\mu} k_{I,t} q_{I,t} - w_t,$$

(11)

and $V_{t+1}(k_{I,t+1}, a_{f,t+1})$ is the value function conditional on continuation but before the investment shock is realized:

$$V_{t+1}(k_{I,t+1}, a_{f,t+1}) = \eta V^+(k_{I,t+1}, a_{f,t+1}) + (1 - \eta) V^-(k_{I,t+1}, a_{f,t+1})$$

(12)

The firm solves (9) or (10), subject to (2), (3) and (6).

We claim that in equilibrium the marginal return on capital for firms in the productive sector is always higher than its user cost:

$$\frac{\partial y_{t+1}^p}{\partial k_{I,t+1}} = \alpha g(\mu) z_{t+1}^{\alpha (1-\alpha)} \left(\frac{k_{I,t+1}^{\alpha - 1}}{\mu}\right) \left(q_{T,t} - \frac{1}{\mu} + q_{I,t}\right) - \left(q_{T,t}^{\frac{1}{\mu}} + q_{I,t}^{\frac{1}{\mu}}\right) \frac{1}{1 + \tau_{t+1}},$$

(13)

and, therefore, that the borrowing constraint (6) is always binding. If this is the case, then the optimal intangible capital is given by:

$$k_{I,t+1} = \frac{y_{t+1}^p - w_t n_t + \left(1 + r_t\right)a_{f,t} + \left(q_{T,t} - \frac{1}{\mu} + q_{I,t}\right)k_{I,t} - \frac{1}{\mu} + q_{I,t} - q_{T,t} \frac{1}{1 + \tau_{t+1}} \frac{1}{1 + \mu} \right] k_{I,t+1} q_{I,t}}{\left(q_{T,t} - \frac{\theta q_{T,t+1}}{1 + \tau_{t+1}} \frac{1}{\mu} + q_{I,t}\right)}.$$

(14)

Moreover condition (13) also implies that a firm that cannot invest will not sell any of its capital and for those firms $k_{I,t+1} = k_{I,t}$.

Regarding the dividend and cash accumulation policy, the first order condition for cash
holdings $a_{f,t+1}$ is:

$$(1 + \lambda_t) = (1 - \psi) \left[ \eta (1 + \lambda_{t+1}^+ + \mu_t) + (1 - \eta) (1 + \lambda_{t+1}^- + \mu_t) \right] + \psi. \quad (15)$$

Substituting (15) recursively forward, it is clear that if the firm expects $\mu_t$ to be positive now or in the future, then $\lambda_t > 0$, and the firm will always retain all earnings and $d_t = 0$. It is important to note that this is so because there is no cost of holding cash. The general formula for cash holdings for investing and non-investing firms is obtained by substituting $d_t = 0$ in (2):

$$a_{f,t+1} = y_t^p + (1 + r_t) a_{f,t} + \left( q_{T,t} \frac{1 - \mu}{\mu} + q_{I,t} \right) k_{I,t} - \left( q_{T,t+1} \frac{1 - \mu}{\mu} + q_{I,t+1} \right) k_{I,t+1}, \quad (16)$$

and simplifies to:

$$(a_{f,t+1} \mid \text{not invest}) = y_t^p + (1 + r_t) a_{f,t} - w_t n_t \quad (17)$$

for non-investing firms. Investing firms in equilibrium borrow as much as possible, and:

$$(a_{f,t+1} \mid \text{invest}) = -q_{T,t+1} \frac{1 - \mu}{\mu} k_{I,t+1} < 0. \quad (18)$$

Equations (17) and (18) determine the wealth dynamics of firms. A firm that invested in period $t - 1$ but is not investing in period $t$ has debt equal to $-a_{f,t} = q_{T,t} \frac{1 - \mu}{\mu} k_{I,t}$. It uses current profits $y_t^p - w_t n_t$ to pay the interest rate on debt $-r_t a_{f,t}$ and to reduce the debt itself. As long as the firm is not investing, the debt $-a_{f,t}$ decreases until the firm becomes a net saver and has $a_{f,t} > 0$. At this point, wealth accumulation is driven both by profits $y_t^p - w_t n_t$ and by interest on savings $r_t a_{f,t}$, until the firm has an investment opportunity and its accumulated wealth $(1 + r_t) a_{f,t}$ is used to purchase capital (see equation (14)). This discussion clarifies that a lower interest rate $r_t$ helps the non-investing firm to repay existing debt, but it slows down the accumulation of savings after the firm has repaid the debt. Later we will refer to these two effects as the \textit{debt hangover channel} and the \textit{savings channel}.

### 3.1.2 The Unproductive Sector

There is a mass one of identical firms in the unproductive sector. They have production functions which are linear in tangible and intangible capital, taking the form:

$$y^u_t = z_t^{u,I} k_{I,t}^u + z_t^{u,T} k_{T,t}^u.$$
This sector is assumed to be able to finance intangible capital with equity from the household sector and to pay out all profits as dividends to households every period. Their budget constraint is

\[ d_t^u = y_t^u - q_{I,t} \left( k_{I,t+1}^u - k_{I,t}^u \right) - q_{T,t} \left( k_{T,t+1}^u - k_{T,t}^u \right) \]  \hspace{1cm} (19)

Given its linear technology, and provided that its return on capital is lower than in the productive sector, the unproductive sector is willing to absorb all the capital not demanded by the productive sector, at a price equal to its marginal return on capital.

### 3.1.3 Aggregation and Pricing of Assets

Since all productive firms produce at the optimal capital labor ratio determined by equation (8), and the production function is constant returns to scale, we can aggregate it across firms and substitute aggregate labor supply \( N = 1 \) to obtain:

\[ Y_t^p = g(\mu) z_t \left( \frac{K_{I,t}}{\mu} \right)^\alpha. \]  \hspace{1cm} (20)

The wage is determined in competitive markets by the marginal return of labor:

\[ w_t = (1 - \alpha) z_t \frac{K_{I,t}}{\mu}, \]  \hspace{1cm} (21)

Aggregate capital is determined as follows. A fraction \((1 - \psi)\) of productive firms continues activity and a fraction \(\eta\) of those has an investment opportunity. They have a fraction \((1 - \psi)\) \(\eta\) of total assets in the productive firm sector, and use it to buy capital following equation (14). A fraction \(\psi\) of productive firms exits, and is replaced by an equal number of firms with an initial endowment of \(W_0\) and no capital. Therefore:

\[ K_{I,t+1} = (1 - \psi) \left[ \frac{Y_t^p - w_t + (1 + r_t) A_{f,t} + \left( q_{T,t} - \frac{q_{T,t+1}}{1+r_{t+1}} \right) \frac{1-\mu}{\mu} + q_{I,t} K_{I,t}} {q_{T,t} - \frac{q_{T,t+1}}{1+r_{t+1}} \frac{1-\mu}{\mu} + q_{I,t}} \right] + (1 - \eta) K_{I,t} \]

\[ + \psi \eta \frac{W_0}{q_{T,t} - \frac{q_{T,t+1}}{1+r_{t+1}} \frac{1-\mu}{\mu} + q_{I,t}}. \]  \hspace{1cm} (22)

We define \(W_t\) as total wealth at the beginning of period \(t\):

\[ W_t \equiv Y_t^p - w_t + (1 + r_t) A_{f,t} + \left( q_{T,t} \frac{1-\mu}{\mu} + q_{I,t} \right) K_{I,t} \]  \hspace{1cm} (23)
Rearranging (22), we get:

\[ K_{I,t+1}^* = \eta K_{I,t+1}^{IV} + (1 - \psi) (1 - \eta) K_{I,t}, \tag{24} \]

where

\[ K_{I,t+1}^{IV} = \frac{(1 - \psi) W_t + \psi W_0}{(q_{T,t} \cdot \frac{q_{T,t+1}}{1 + r_{t+1}}) \frac{1 - \mu}{\mu} + q_{I,t}} \tag{25} \]

is total intangible capital in the hands of investing agents at the end of period \( t \), expressed in aggregate terms. Aggregate tangible capital of the productive sector is equal to:

\[ K_{T,t+1}^* = \frac{1 - \mu}{\mu} K_{I,t+1}^* \tag{26} \]

Furthermore we can aggregate firms in the unproductive sector and obtain:

\[ Y_t^u = z_t^{u,I} \left( K_t^I - K_{I,t} \right) + z_t^{u,T} \left( K_t^T - K_{T,t} \right). \]

The marginal return of capital in the productive sector is as follows. In order obtain a marginal increase \( \frac{\partial Y_t^P}{\partial K_{I,t}} = \alpha \bar{g} \left( \mu \right) z_t \left( \frac{K_{I,t}}{\mu} \right)^{\alpha - 1} \), the productive sector purchases one unit of intangible capital and \( \frac{1 - \mu}{\mu} \) units of tangible capital. The return of this investment in the unproductive sector is \( z_t^{u,I} + \frac{1 - \mu}{\mu} z_t^{u,T} \). The equilibrium described above requires that the productive sector have the highest return on capital:

\[ \alpha \bar{g} \left( \mu \right) z_t \left( \frac{K_{I,t+1}^*}{\mu} \right)^{\alpha - 1} > z_t^{u,I} + \frac{1 - \mu}{\mu} z_t^{u,T} \tag{27} \]

If condition 27 is satisfied, then it follows immediately that the prices of capital are:

\[ q_{I,t} = z_t^{u,I} + \frac{q_{I,t+1}}{1 + r_{t+1} + \xi_{t+1}} \tag{28} \]

and

\[ q_{T,t} = z_t^{u,T} + \frac{q_{T,t+1}}{1 + r_{t+1} + \xi_{t+1}} \tag{29} \]

where the parameter \( \xi_{t+1} \) is a positive wedge that increases the required return on assets above the risk free rate \( r_{t+1} \). We interpret it as a short cut to a risk premium factor. By substituting
(28) and (29) into (27), it follows that:

\[
\alpha g(\mu) 2\left(\frac{K_{I,t+1}^*}{\mu}\right)^{a-1} > q_{I,t} - \frac{q_{I,t+1}}{1 + r_{t+j} + \xi_{t+1}} + \frac{1 - \mu}{\mu} \left( q_{T,t} - \frac{q_{T,t+1}}{1 + r_{t+j} + \xi_{t+1}} \right),
\]

which implies that the claim (13) is correct.

To compute aggregate financial assets of the productive sector \(A_{f,t+1}\), we take into account that, among the fraction \(1 - \psi\) of continuing firms, a fraction \(1 - \eta\) simply accumulates savings, while a fraction \(\eta\) borrows up to the maximum to invest. Among the fraction \(\psi\) of new firms, a fraction \(\eta\) borrows up to the maximum, while the rest save their initial endowment \(W_0\):

\[
A_{f,t+1} = (1 - \psi) [(1 - \eta) (Y_t^p + (1 + r_t)A_{f,t} - w_t)] + \psi (1 - \eta) W_0
\]

\[
-\eta \theta \frac{q_{T,t+1}}{1 + r_{t+1}} \frac{1 - \mu}{\mu} K_{I,t+1}^{INV}.
\]

At the aggregate level total investment \(q_{T,t} \frac{1 - \mu}{\mu} + q_{I,t}\) \((K_{I,t+1} - (1 - \psi) K_{I,t})\) is also equal to total resources available to invest:

\[
\left(q_{T,t} \frac{1 - \mu}{\mu} + q_{I,t}\right) (K_{I,t+1} - (1 - \psi) K_{I,t}) = [(1 - \psi) \eta (Y_{t}^p - w_t + (1 + r_t)A_{f,t}) + \psi W_0]
\]

\[
+ \eta \theta \frac{q_{T,t+1}}{1 + r_{t+1}} \frac{1 - \mu}{\mu} K_{I,t+1}^{INV}.
\]

Substituting (32) into (31) we obtain:

\[
A_{f,t+1} = (1 - \psi) (Y_t^p - w_t + (1 + r_t)A_{f,t}) + \psi W_0 - \left(q_{T,t} \frac{1 - \mu}{\mu} + q_{I,t}\right) (K_{I,t+1} - (1 - \psi) K_{I,t}),
\]

Finally, total dividends paid out by exiting productive firms to households are equal to:

\[
D_t^p = \psi \left[Y_t^p - w_t + (1 + r_t)A_{f,t} + \left(q_{T,t} \frac{1 - \mu}{\mu} + q_{I,t}\right) K_{I,t}\right] - \psi W_0,
\]

and the dividends paid by the unproductive sectors are:

\[
D_t^u = Y_t^u - q_{I,t} \left[\left(\overline{K}_t - K_{I,t+1}\right) - \left(\overline{K}_t - K_{I,t}\right)\right] - q_{T,t} \left[\left(\overline{K}_t - K_{T,t+1}\right) - \left(\overline{K}_t - K_{T,t}\right)\right],
\]

so that total aggregate dividends are:

\[
D_t = D_t^p + D_t^u.
\]
### 3.2 Households

We follow Eggertsson and Mehrotra (2014) in considering a simple overlapping generations model where households live for three periods. They are born in period 1 (young), they become middle aged in period 2 (middle age), and retire in period 3 (old). Only the middle and old generations receive dividend income from the firm sector, and the middle generation also supplies labor to the productive firms in return for a wage. The young borrow from the middle-aged households, which in turn will save for retirement when old, when they spend all of their remaining income and assets.

A household born in time $t$ maximizes the following objective function

$$
\max_{c_t^y, c_{t+1}^m, c_{t+2}^o} E_t \left\{ \log (c_t^y) + \beta \log (c_{t+1}^m) + \beta^2 \log (c_{t+2}^o) \right\},
$$

subject to the borrowing constraints

$$
\begin{align*}
    c_t^y &= b_{t+1}^y, \\
    c_{t+1}^m &= \gamma d_{t+1} + w_{t+1} - (1 + r_{t+1})b_{t+1}^y + b_{t+2}^m, \\
    c_{t+2}^o &= (1 - \gamma)d_{t+2} - (1 + r_{t+2})b_{t+2}^m
\end{align*}
$$

where $d_t$ are aggregate dividends from the productive sector, and a fraction $\gamma$ goes to the middle age households, the remaining $1 - \gamma$ to the old households. $w_t$ is the wage, and $b_t$ is borrowing.

There is a limit on the amount of debt the young can borrow:

$$
(1 + r_{t+1})b_{t+1}^y \leq \bar{b}. 
$$

We assume that the borrowing constraint (39) is binding, so that borrowing by the young is

$$
b_{t+1}^y = \frac{\bar{b}}{1 + r_{t+1}},
$$

and $c_t^y = b_t^y$. The middle aged are at an interior solution:

$$
\frac{1}{c_t^m} = \beta \frac{1 + r_{t+1}}{c_{t+1}^o}
$$

Combining (41), (38), (37) and (40), and solving for $c_m$, we get:
\[ c_t^m = \frac{(1 - \gamma) d_{t+1} + (1 + r_{t+1}) (\gamma d_t + w_t - \bar{b})}{(1 + \beta) (1 + r_{t+1})}, \]  

(42)

and substituting this result back into (37) and rearranging, results in:

\[ b_{t+1}^m = \frac{(1 - \gamma) d_{t+1} - \beta (1 + r_{t+1}) (\gamma d_t + w_t - \bar{b})}{(1 + \beta) (1 + r_{t+1})} \]  

(43)

The old consume their wealth:

\[ c_t^o = \frac{[(1 + \beta) (1 + r_t) - 1] (1 - \gamma) d_t + \beta (1 + r_t) (\gamma d_{t-1} + w_{t-1} - \bar{b})}{(1 + \beta) (1 + r_t)} \]  

(44)

Finally, total borrowing \( B_t \) and consumption \( C_t \) are given by:

\[ B_{t+1} = b_{t+1}^g + b_{t+1}^m \]  

(45)

\[ C_t = c_t^g + c_t^m + c_t^o \]  

(46)

### 4 Steady State Equilibrium

We consider a steady state equilibrium and we drop the subscript \( t \). Using (40) and (43), we can compute aggregate household borrowing as:

\[ B = \frac{[(1 + \beta) + \beta (1 + r)] \bar{b} + [(1 - \gamma) - \beta (1 + r) \gamma] d - \beta (1 + r) w}{(1 + \beta) (1 + r)} \]  

(47)

Aggregate firm sector cash holdings in the steady state are can be obtained by combining (33), (20) and (21) to obtain:

\[ A_f = \frac{(1 - \psi) \left( g(\mu) z \left( \frac{K_f}{\mu} \right)^\alpha - (1 - \alpha) z_t \left( \frac{K_h}{\mu} \right) \right) + \psi W_0 - \left( q_T \frac{1-\mu}{\mu} + q_I \right) \psi K_I}{[1 - (1 - \psi) (1 + r)]} \]  

(48)

Aggregate borrowing is equal to aggregate savings, or

\[ A_f = B, \]  

(49)

and by Walras’ Law, the aggregate resource constraint is satisfied. In order to determine optimal capital, equation (24) in the steady state is equal to:

\[ K_I = \frac{(1 - \psi) W + \psi W_0}{\left[ q_T \left( 1 - \frac{\theta}{1+r} \right) \frac{1-\mu}{\mu} + q_I \right] [1 - (1 - \psi) (1 - \eta)]} \]  

(50)
where $W$ is defined using equation (23) in steady state:

$$W \equiv g(\mu) z \left( \frac{K_I}{\mu} \right)^{\alpha} - (1 - \alpha) z_t \left( \frac{K_I}{\mu} \right) + (1 + r) A_f + \left( q_T \frac{1 - \mu}{\mu} + q_I \right) K_I$$

(51)

Finally, the prices of capital are determined by recursively iterating forward equations (28) and (29):

$$q_I = \frac{z^{u,I}}{r + \xi}$$

(52)

$$q_T = \frac{z^{u,T}}{r + \xi}$$

(53)

The steady state values of $W$, $A_f$, $B$, $K_I$, $q_I$, $q_T$, and $r$ are jointly determined by equations (47), (48), (49), (50), (51), (52), and (53).

### 4.1 Characterization of the Equilibrium

To illustrate the main properties of the model, we now develop some important features of the equilibrium that can be characterized analytically. The analysis clarifies the main mechanisms through which interest rates interact with the degree of reliance on intangible capital to affect the allocation of capital and aggregate output.

#### 4.1.1 Partial Equilibrium Analysis of Firm Investment Policies

In this section, we analyze firms’ investment policies to provide a greater understanding of the **collateral value channel**, the **capital purchase price channel**, the **debt overhang channel**, and the **savings channel**, the four channels through which interest rate variations interact with firm financial constraints to affect capital reallocation. To provide clearer results, we take the interest rate $r$ and capital prices $q_T$ and $q_I$ as given, assume $\alpha = 1$, so that the production function is linear in capital, and assume that a risk spread $\xi = 0$. Combining equations (48), (50), and (51), we obtain the following expression for the total amount of steady state reallocation of intangible capital $\psi K_I$:

$$\psi K_I = \frac{\eta(1 - \psi) \left[ g(\mu) z \left( \frac{K_I}{\mu} \right) + (1 + r) A_f \right]}{\left( q_T \frac{1 - \mu}{\mu} + q_I \right) - \left( q_T \frac{\theta}{1 + \theta} \frac{1 - \mu}{\mu} \right) \left[ 1 + \eta \frac{(1 - \psi)}{\psi} \right]},$$

(54)

where

$$A_f = \eta A_f + (1 - \eta) A_S,$$

(55)
and

\[
A_I = \left( 1 - \psi \right) g(\mu) z \left( \frac{K_I}{\mu} \right) + \psi W_0 + A_f (1 - \psi) (1 + r) \left[ - \left( q_T \frac{1 - \mu}{\mu} + q_I \right) \right] \psi K_I, \quad (56)
\]

\[
A_S = \left( 1 - \psi \right) g(\mu) z \left( \frac{K_I}{\mu} \right) + \psi W_0 + A_f (1 - \psi) (1 + r). \quad (57)
\]

We can identify the four channels clearly in (54). The debt overhang and savings channels are captured by the term \((1 + r)A_f\) in the numerator. If the productive firm sector is a net saver, then \(A_f > 0\), and increases in the interest rate \(r\) increase the speed of accumulation of savings and help capital reallocation. This is the savings channel. Net savings are composed of the net savings \(A_I\) of the \(\eta\) fraction of firms that where investors in the previous period, and net savings \(A_S\) of the fraction \((1 - \eta)\) that were not. Conversely, if \(A_f < 0\), increases in the interest rate \(r\) lower the speed of accumulation of savings and hurt capital reallocation. This is the debt overhang channel. An inspection of expressions (54), (56), and (57) shows that the strength of both channels is negatively related to the probability of firm exit \(\psi\), and disappear when \(\psi = 1\). The intuition for this is that when \(\psi = 1\) firms only live for one period and are unable to save or to carry over debt from previous periods.

The capital purchase price channel is captured by the term \(q_T \frac{1 - \mu}{\mu} + q_I\) in the denominator. Lower interest rates that increase the price of tangible and intangible assets reduce the amount of capital firms can purchase for a given amount of net worth and borrowing capacity. The collateral value channel, by which a lower interest rate enables investing firms to borrow and invest more, is captured by the second term in the denominator. It works by increasing the value of tangible capital used as collateral, indirectly because of the increase in \(q_T\), and directly because of the decrease in \(r\) which increases the present value of the collateral pledged next period. The amount of tangible capital available to be used as collateral is equal to the amount bought, plus and additional \(\eta \frac{(1-\psi)}{\psi}\) units per unit bought, which captures existing holdings of tangible capital that can be used as collateral.

To isolate and gain a further understanding of the capital purchase price and collateral value channels, we consider the case in which \(\psi = 1\) and \(\eta = 1\). This is a situation in which firms only live for one period, and all firms can invest, so the debt overhang and the savings channels are mute. A closed form solution for \(K_I\) can be obtained, which is:
The collateral value channel and the capital purchase price channel operate in opposite directions, so to analyze which dominates we study the sensitivity of $K_I$ to variations in $r$ using expression (58):

$$\frac{\partial K_I}{\partial r} = \frac{W_0 \left( - \left( \frac{\partial q_T}{\partial r} \frac{1-\mu}{\mu} + \frac{\partial q_I}{\partial r} \right) + \frac{\partial q_T}{\partial r} \frac{\theta}{1+r} \frac{1-\mu}{\mu} + q_T \frac{\partial \left( \frac{q_T}{1+r} \right)}{\partial r} \frac{1-\mu}{\mu} \right)}{\left( q_T \frac{1-\mu}{\mu} + q_I \right)^2},$$

where the sign of this derivative is given by:

$$\text{sign} \left[ - \left( \frac{\partial q_T}{\partial r} \frac{1-\mu}{\mu} + \frac{\partial q_I}{\partial r} \right) + \frac{\partial q_T}{\partial r} \frac{\theta}{1+r} \frac{1-\mu}{\mu} + q_T \frac{\partial \left( \frac{q_T}{1+r} \right)}{\partial r} \frac{1-\mu}{\mu} \right] \leq 0. \quad (59)$$

The first term in brackets captures the capital price channel, and is positive, given that $\partial q_T/\partial r$ and $\partial q_I/\partial r$ are both negative. The second and third terms capture the collateral value channel, and are both negative. The term inside the brackets in (59) increases in $\mu$, meaning that the effect of higher interest rates is more likely to be expansionary when the share of intangibles is high. As $\mu$ approaches 1, the collateral value channel disappears, and the capital price channel remains. A similar effect would obtain if we decrease the ability to collateralize tangible capital, captured by $\theta$.

### 4.1.2 Characterization of the Equilibrium

To provide a deeper understanding of how the features of the equilibrium of the economy described in Section 3 change as a result of a transition from an economy reliant on tangible capital to one in which intangible capital acquires a larger importance, we represent the equilibrium in the credit market in Figure 1. In the graph, upward sloping savings curve captures net savings of the household sector and net savings of the non-investing firm sector. Higher interest rates induce households to save more, while the savings of the non-investing firms are not sensitive to interest rates, as they simply save as much as possible. The demand for capital by the investing firms is equal to the amount borrowed by them plus (minus) the savings (debt) they carry over from the previous period. This curve can be upward or downward sloping depending on the relevance of intangible capital in the production function.
The left panel in the figure represents a "tangibles" economy with a low $\mu$. In such an economy, an increase in aggregate savings has the standard effect of lowering interest rates and increasing capital purchases from expanding firms. The collateral value channel and the debt overhang channels dominate. As a result, a larger share of the capital stock is in the hands of the productive sector, and output increases. The right panel considers the case of an "intangibles" economy with a high $\mu$. The demand for capital curve is upward sloping when an economy has a high reliance on intangible capital due to the strength of the capital price and savings channel. As interest rates rise, firms demand more capital because they have larger savings and because the price of capital is lower. In this case, an outwards shift in the savings schedule generates a decrease in equilibrium capital purchases, because the decrease in interest rates it generates hurts reallocation of capital towards productive firms.
Table 1: Benchmark Calibration - Parameter Choices

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>Capital (tangible + intangible) share</td>
<td>$\alpha$</td>
<td>0.6</td>
</tr>
<tr>
<td>Intangible share of total capital</td>
<td>$\mu$</td>
<td>0.2</td>
</tr>
<tr>
<td>Relative productivity of intangible capital</td>
<td>$z^{u,I}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Borrowing limit</td>
<td>$\bar{b}$</td>
<td>13.16</td>
</tr>
<tr>
<td>Productivity parameter</td>
<td>$z$</td>
<td>40</td>
</tr>
<tr>
<td>Collateral value of tangible capital</td>
<td>$\theta$</td>
<td>0.9</td>
</tr>
<tr>
<td>Probability of an investment opportunity</td>
<td>$\eta$</td>
<td>0.06</td>
</tr>
<tr>
<td>Aggregate capital stock</td>
<td>$K$</td>
<td>12</td>
</tr>
<tr>
<td>Exit probability of productive firms</td>
<td>$\psi$</td>
<td>0.05</td>
</tr>
<tr>
<td>Endowment of new firms</td>
<td>$W_0$</td>
<td>2.5</td>
</tr>
<tr>
<td>Risk premium</td>
<td>$\xi$</td>
<td>0.06</td>
</tr>
<tr>
<td>Share of dividends to middle aged households</td>
<td>$\gamma$</td>
<td>1</td>
</tr>
</tbody>
</table>

5 Calibration

For the purpose of evaluating the qualitative and quantitative importance of the four channels explained above for the real economy, we calibrate the model on US data. Our benchmark calibration, illustrated in table 1, is meant to capture the US economy during the period immediately preceding 1980, with a small share of intangible capital and high real interest rates. In this respect we follow Falato et al. (2013) in setting $\mu = 0.2$, so that the share of intangible capital over total capital is 20%, the parameter $\bar{b}$ in order to obtain a real interest rate $r = 6\%$, and the elasticity of output with respect to capital $\alpha$ equal to 0.6. The remaining parameters are chosen as follows. The pledgeability parameter $\theta$ is equal to 0.9, implying that the firms can borrow up 90% of the value of their tangible capital. The probability of having an investment opportunity $\eta$ is set equal to 6%. This value is consistent with the empirical evidence from the lumpy investment literature. The probability to exit $\psi$ is equal to 0.05, which implies an average firm age of 20 years. The initial endowment of newborn firms $W_0$ is equal to 2.5, which corresponds to 2% of average firm output. The productivity parameter $z$ is equal to 40, and it matches a value of the ratio of marginal productivity of capital in the productive sector relative to the unproductive sector (which in this model corresponds to Tobin’s Q) equal to 2. The risk premium parameter $\xi$ is set equal to 6%. The value of the parameter $z^{u,T}$ is normalized to 1, while $z^{u,I}$ is set equal to 0.8, which is equivalent to assuming that in the productive sector
intangible capital is 20% more productive, per unit cost, than tangible capital. Finally, the share of dividends that go to middle aged households $\gamma$ is set to 1.

6 The Rise in Intangible Capital

This section studies the effects of an increase in firms’ reliance on intangible capital in their production technology. Recent decades have seen a gradual process of technological change towards a larger importance of information technology and knowledge, human and organizational capital, which have progressively reduced the relative use of physical capital (Corrado and Hulten (2010a), Falato et al (2013), Döttling and Perotti (2015)).

We follow Falato et al (2013) and capture this trend by an increase in the parameter that determines the intangible capital ratio, from a value of $\mu = 0.2$, which delivers a ratio of intangible to tangible assets consistent with the value observed in the 1970s, to a value of $\mu = 0.5$, representing the early 2000’s, while keeping all the other parameters constant at the benchmark level. In this exercise, the rise of intangible capital is driven by an exogenous change in $\mu$. However, since we assume that intangible capital is more productive in equilibrium, it can be interpreted as a shortcut for an endogenous process of adoption of more productive technologies.

In figure 2 we show the response of key endogenous variables to changes in $\mu$. The increase in the relative importance of intangible capital in the production function induces productive firms to shift from tangible capital to intangible capital. Since the latter is not collateralizable, the firm’s borrowing capacity for a given amount of combined tangible and intangible investment declines, and the interest rate must fall to stimulate a reduction in households saving and clear the bond market. The decline in the interest rate influences aggregate capital and output through the channels illustrated in section 4.1. A lower interest rate enables investing firms to borrow more, and helps those non-investing firms that still need to repay their debt contracted during their last investment episode. These collateral price and debt overhang channels have positive effects on $K_I$ and $K_T$. However, the capital purchase price channel and the savings channel act in the opposite direction, by increasing the cost of capital and reducing the saving rate of the other non-investing firms. While $\mu$ is small, the collateral value and debt overhang channels are strong, because the tangible share of total capital is still large, and compensates
Figure 2: Equilibrium response of aggregate variables to an increase in the intangible capital ratio parameter $\mu$ from 0.2 to 0.5 in the benchmark model.
the other two opposite effects. Output increases moderately with $\mu$, reflecting the higher productivity per unit cost of intangible capital. However, as $\mu$ keeps increasing and $r$ declines further, the other two effects become stronger, both because asset prices $q_I$ and $q_T$ increase more than proportionally as $r$ falls, and because tangible capital becomes less important. As a result, the overall stock of productive inputs declines, and output is 6% lower in the equilibrium with $\mu = 0.5$ than in the equilibrium when $\mu = 0.2$. The drop in output is large because the endogenous decline in the interest rate is also very large, from 6.0% to -2.6%.

As mentioned above, we interpret the comparative static exercise in figure 2 as the developments in the US economy following the rise in the share of intangible capital in the last 40 years. In this respect, this simple model is remarkably consistent with a series of well documented trends during this period: i) corporate savings increased as a fraction of GDP; ii) household leverage increased as a fraction of GDP; iii) the real interest rate fell; iv) asset prices relative to aggregate output increased; and v) output progressively declined relative to its previous trend, a phenomenon referred to sometimes as secular stagnation.

While the importance of the rise in intangible capital for stylized facts i)-iii) has been already shown by Falato et al (2013) and Dottling and Perotti (2015), this paper is, to the best of our knowledge, the first to show that they are potentially very important in explaining the secular stagnation hypothesis.

To better understand the mechanism that drives the effects of the rise in intangibles, figure 3 compares the reaction of the economy described before, in which the interest rate is endogenous, with an economy with a fixed interest rate, which could be interpreted as a small open economy exposed to a world interest rate that it cannot affect. The constant interest rate implies that capital prices are also fixed, and mutes the savings, collateral value, debt hangover and capital purchase price channels. As a consequence, since intangible capital is more productive because it is relatively cheaper, output increases by 1%. Corporate borrowing decreases substantially due to the lower availability of tangible capital to be used as collateral by firms. The price of both types of capital is constant because the interest rate does not vary, and this precludes any effects on investment driven by variation in the collateral value of tangible capital or the purchase price of intangible capital.
Figure 3: Equilibrium response of aggregate variables to an increase in the intangible capital ratio parameter $\mu$ from 0.2 to 0.5 in the benchmark model and in a model in which the interest rate is kept constant at the equilibrium level when $\mu = 0.2$. 
Figure 4: Equilibrium response of aggregate variables to a decrease in young households’ borrowing capacity. Calibration with a pre-1980 level of intangible capital (20% of total capital).
7 Household Deleveraging and Intangible Capital Intensity

In this section, we consider a comparative static exercise in which the household sector progressively increases its savings, which puts downward pressure on the equilibrium interest rate. This exercise is relevant because recent empirical studies demonstrate that demand side factors such as demographic forces, higher inequality within countries, and a preference shift towards higher saving by emerging market governments, have been responsible for a large increase in savings and the declining trend in real interest rates over the last three decades (Rachel and Smith, 2015). We implement the increase in aggregate household savings by progressively reducing the parameter $b$, which captures young households’ borrowing capacity. We interpret it as a shortcut for all the different factors mentioned above.

We compare the impact of this progressive increase in household savings in two different economies. The first is the benchmark pre-1980 calibration, in which intangible capital accounts only for 20% of aggregate capital. The second is an alternative calibration with high usage of intangible capital, which matches the same empirical observables as the benchmark calibration, except for the share of intangible capital in the economy, which is set to the post-2000 value of 50% of total capital ($\mu = 0.5$). The only other difference between the two calibrations is that in order to match Tobin’s $Q$, we recalibrate the parameter $z$. In figure 4 we present the equilibrium values of selected aggregate variables in response to a decrease in $b$ under the benchmark calibration. The reduction in $b$ lowers the borrowing of young households, and increases the savings of the combined household sector. As a consequence, the interest rate drops from its initial value of 6% to around 1%, to generate an increase in corporate borrowing that clears the debt market. Firms are able to borrow more because of the collateral value channel. However, despite the large increase in borrowing, capital increases only by 2% in the steady state, because of the counteracting savings and capital purchase price channels. Aggregate output increases by 0.5%.

Figure 5 illustrates the results of same exercise under the alternative post-2000 calibration with a share of intangible capital equal to 50%. To make the two exercises comparable, we apply in both cases the same reduction in $b$ as a percentage of initial aggregate output $Y$. Figure 5 shows that also in this economy with a higher reliance on intangible capital, the adjustment in the bond market requires a fall in the interest rate, which allows firms to increase their...
Figure 5: Equilibrium response of aggregate variables to a decrease in young households’ borrowing capacity. Calibration with a post-2005 level of intangible capital (50% of total capital).
borrowing capacity. Since firms have relatively less collateralizable capital, the interest rate has to fall relatively more than in an economy with a higher share of tangible capital (figure 4) to clear the bond market. The most striking difference in this economy is that now output falls by as much as 5%, while the same deleveraging process caused an increase of 0.5% in the calibration with low intangible capital. The reason is that, in this economy, tangible capital is less important, and the negative effects of the savings and capital purchase price channels dominate the other two channels.

8 Conclusion

This paper suggests that the changes in firms’ financing behavior brought about by technological evolution might help explain the subpar growth and low interest rates associated with secular stagnation. Furthermore, it suggests that they might interact with the other forces behind secular stagnation to amplify their negative effects.

Our insights could also be extended to develop interesting policy implications. On one hand, the mechanisms described in this paper, operating mostly through the endogenous reaction of interest rates, suggest that the rise in intangibles might have important implications for monetary policy. On the other hand, the negative externality in households’ and firms’ saving decisions might introduce a role for a fiscal policy that discourages such saving.
References


