Investment Shocks, Uninsurable Unemployment Risk, and Macroeconomic Comovement *

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Abstract

Standard (complete markets) macroeconomic models rely on total factor productivity (TFP) shocks to explain the observed comovement between consumption and investment. However, recently advanced two major explanations for the collapse of both consumption and investment during the Great Recession are based on non-TFP shocks: one, shocks to contraction in firm investment; the other, shocks to household deleveraging. Neither sources are convincing in the perspective of standard models, in which both fail to generate the observed comovement between consumption and investment. This paper shows that an incomplete markets New Keynesian model, in which uninsurable unemployment risk is counter-cyclical, can generate comovement in response to investment shocks. A simulation of the US economy during the Great Recession indicates that the incomplete markets model can account for 70 percent of the total decline in aggregate consumption between the peak of expansion and the trough of the recession.

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1 Introduction

A defining feature of the business cycle is comovement between investment and consumption. Therefore, the ability to generate aggregate comovement is a natural test for macroeconomic models. However, while shifts in productivity can deliver the comovement in most macroeconomic models, other types of aggregate shocks do not (Barro and King (1984)). Achieving the comovement in response to non-productivity shocks is of particular importance to our efforts to understand the nature of the Great Recession during which both consumption and investment plunged more than ever since the Great Depression. There are two main explanations for the sources of Great Recession that have been explored: one, events that are specific to investment; the other, shocks that affect household balance sheet. On the one hand, Bloom (2009) illustrates that uncertainty is responsible for the decline in the economy by causing firms to temporarily pause their investment activities. Gilchrist and Zakrajšek (2012) provide evidence that spike in credit spreads and in particular, excess bond premia, had considerable predictive power for recent economic slump. On the other hand, Mian, Rao, and Sufi (2013) argue that household deleveraging in 2007 and 2008 reduced consumption, leading to poor sales in businesses and a collapse of the financial institutions.

But neither views can explain the observed comovement between consumption and investment in standard representative agent macroeconomic model pointing modifications in the existing model are necessary. For example, negative shock to investment predicts short-run rise or, at most, flat movements in consumption.\(^1\) On the other hand, any shock that raises household net savings reduces consumption but results in a rise in investment cushioning its impact on aggregate demand.\(^2\)

Even with New Keynesian ingredients that make aggregate demand channel stronger, it is surprisingly difficult to achieve comovement of consumption and investment. In particular, while studies based on estimated representative agent New Keynesian models with multiple shocks are conclusive that shocks to investment are the most important drivers of output, investment, and employment fluctuations in US economy, any changes investment induce opposite movement in consumption. In this paper, I show that when markets are incomplete and idiosyncratic risk is counter-cyclical, New Keynesian model can generate comovement in response to investment shocks and thus a large part of the drop in consumption during the Great Recession is explained.

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1Justiniano, Primiceri, and Tambalotti (2010, 2011) show that shocks to marginal efficiency of investment account for between 7 and 8 percent of the variance of consumption at business cycle frequencies. Christiano, Motto, and Rostagno (2014) show that, under the environment of financial friction à la Bernanke, Gertler, and Gilchrist (1999), unexpected shocks to cross-sectional idiosyncratic uncertainty explain 3 percent of the movements in consumption. Similarly, Carlstrom, Fuerst, Ortiz, and Paustian (2014) present the result that shocks to net worth, the most important driver of output fluctuations in their environment, lead to the response of consumption that is opposite to that of investment.

2In standard representative agent models, a shock to discount factor (preference shock) is often represented as a household deleveraging shock in reduced form. A preference shock induces negative comovement of consumption and investment. Even in borrower-saver type models that incorporate debt and collateral constraints as in Justiniano, Primiceri, and Tambalotti (2015), a reduction in debt leads to a mild fall in consumption but a rise in investment.
by these shocks. A key contribution is to create a recession that jointly captures a fall in both consumption and investment and a desire to save more – main features of the recent recession.

To see why comovement problem arises in standard New Keynesian models, consider a recession originated from a shock that leads to a collapse of investment. Phillips curve implies that inflation falls on impact in response to a fall in current and expected future real activities. Monetary policy rule dictates nominal interest rate has to fall. As price level falls immediately, inflation is expected to rise to restore its long-run equilibrium and therefore real interest falls. The resulting intertemporal substitution offsets income effects associated with a fall in current and expected future labor income and hence consumption rises in short-run.\(^3\)

However, is it a realistic prediction of the standard model in which households increase consumption even their labor income is expected to fall? First, it is inconsistent with empirical evidence from structural VARs presented by Gilchrist and Zakrajšek (2011, 2012). They show that a one standard deviation increase in the financial bond premium causes reductions in consumption, investment, employment, and output in short-run. Second, an emerging body of empirical studies document that some households are borrowing constrained and are not fully insured against the risk of unemployment. In particular, workers suffer substantial losses in both earnings and consumption levels during unemployment.\(^4\) Accordingly, investment slump associated with a rise in unemployment risk would increase households’ desire to hold a buffer stock of savings reducing their current consumption rather than taking full advantage of the opportunity for intertemporal substitution.

To investigate the quantitative impact of investment shocks on aggregate consumption through a change in unemployment risk, I consider a general equilibrium model in which some fraction of households face uninsurable unemployment risk and borrowing constraints. I assume that these households experience consumption losses of 21 percent upon job loss consistent with the micro-economic evidence from Chodorow-Reich and Karabarbounis (2015). The model has two additional key ingredients. First, search frictions in labor market are added so that unemployment risk faced by households varies endogenously through firms optimal choices for vacancies. Second, the model includes New Keynesian features. Inclusion of these features such as imperfect competition and price rigidities has been shown to improve the performance of macroeconomic

\(^3\)Sticky wages and habit formation are the elements that could ease the comovement problem but they are not definitive solutions.

\(^4\)Using data from the first four waves of the Health and Retirement Study (HRS), Stephens (2004) finds that annual food consumption falls by roughly 16 percent upon being displaced. Similarly, using the 1999-2009 biannual waves of the Panel Study of Income Dynamics (PSID), Saporta-Eksten (2014) finds that job loss leads to a drop in expenditure on non-durables and services of 17 percent, of which, about half occurs before a job loss and the other half occurs around job loss. Chodorow-Reich and Karabarbounis (2015), using the Consumer Expenditure Survey (CE), report 21 percent decline in expenditure on non-durables and services upon unemployment of one year. Exploiting data from the Continuing Survey of Food Intake of Individuals (CSFII), Aguiar and Hurst (2005) find unemployed experience a decline in total food expenditure of 19 percent. Kolsrud, Landais, Nilsson, and Spinnewijn (2015) use Swedish data to document that annual consumption expenditures drop on average by 27 percent for those who are unemployed for longer than 20 weeks.
models and make investment shocks account for most of the output fluctuations (Justiniano et al. (2010)). Moreover, Ravn and Sterk (2013) show that sticky prices and real wage rigidity are key elements for the market incompleteness to matter in business cycles.  

I then use the model to construct the series for marginal efficiency of investment shocks so that investment series implied by the model coincides with those in the data during the Great Recession periods. With the series of estimated shocks, I simulate the aggregate consumption responses over the same periods. My findings are as follows. First, the model can account for 70 percent of the total decline in aggregate consumption between the NBER peak and the second quarter of 2009. Second, the decline in aggregate consumption is mostly driven by unemployed households who reduce consumption for precautionary reasons at all levels of cash-on-hand.

The debate on the sources and relative importance that caused the recent recession still remains. Was reduced consumption caused by household deleveraging or depressed firm investment due to a rise in uncertainties or difficulties in obtaining external funds? My results indicate the latter as the more appealing cause of the recession suggesting that policies aimed to stimulate firm investment such as injecting capital to financial intermediaries that are specialized in business lending may be effective in boosting both consumption and investment.

Relationship with the literature

My work is related to burgeoning literature that integrates market incompleteness and nominal rigidities. In the literature, the papers that incorporate endogenous uninsurable unemployment risk as mine are Ravn and Sterk (2013), Gornemann, Kuester, and Nakajima (2014), Challe et al. (2015), and den Haan, Rendahl, and Riegler (2015).

Ravn and Sterk (2013) are the first who study the interaction of precautionary savings, aggregate demand and uninsurable unemployment risk and show that imperfect insurance amplifies labor market quantities to an exogenous shock to the job separation rate. Similarly, Challe et al. (2015) allow a variety of structural shocks and take the model with imperfect insurance to the data and quantifies the extent to which precautionary savings effect amplifies the aggregate quantities. Both papers exploit wealth distribution of finite support that implies only employed households are able to save and unemployment households consume in a hand-to-mouth fashion. den Haan, Rendahl, and Riegler (2015) study an environment in which combination of incomplete markets and sticky nominal wages magnifies output in response to productivity shocks. In contrast to first two papers, by allowing a large dimensional cross-sectional wealth distribution, I can analyze decision of households with different levels of wealth and different employment states. In my

5Krusell and Smith (1998) and Krusell, Mukoyama, and Sahin (2010) find that imperfect insurance does not help in generating more volatile business cycles under flexible prices.
model, precautionary savings motive of unemployed households is crucial in driving aggregate consumption. Moreover, instead of focusing on the possibility of generating more volatile business cycles as above three papers, I focus on the possibility of having consumption that moves in the same direction as investment in response to investment disturbances. Gornemann, Kuester, and Nakajima (2014) focus on the distributional consequences of monetary policy whereas the current paper is concerned with the aggregate consequences of investment shocks.

Several other papers suggest solutions to the comovement problem associated with investment shocks. Khan and Tsoukalas (2011) argue that the comovement problem is mitigated if the cost of capital utilization is specified in terms of increased depreciation of capital proposed by Greenwood, Hercowitz, and Huffman (1988) alternative to the one proposed by Christiano, Eichenbaum, and Evans (2005). Gilchrist and Zakrajšek (2011) show that, with preferences that imply consumption-hours complementarity and eliminate the wealth effect on labor supply proposed by Greenwood, Hercowitz, and Huffman (1988), financial shocks lead to a positive co-movement between consumption and investment. Eusepi and Preston (2015) argue that if agents’ preferences display a reasonable degree of non-separability between consumption and hours and hours are mainly driven by extensive margin then consumption moves together with investment after marginal efficiency of investment shocks. The employed households consume more than unemployed households in compensation for disutility of work but households are assumed to be perfectly insured from unemployment risk in their economy. Therefore, the comovement of aggregate consumption and investment is achieved due to the compositional effect arising from the variation in numbers of employed. Instead, I provide complementary channel that highlights the role of imperfect insurance. In my model economy, household reduces consumption due to an unfavorable income prospect even its employment state is unchanged.

The work by Bayer et al. (2015) is close to mine in the use of models with market incompleteness and uninsurable idiosyncratic uncertainty to study macroeconomic comovements. Their goal is to generate a fall in physical investment when consumption demand is reduced due to an exogenous rise in income uncertainty. However, the goal of this paper is to achieve a fall in consumption when negative investment-specific shock hits.

The paper proceeds as follows. Section 2 presents the incomplete markets model augmented with search frictions and New Keynesian features. Section 4 compares the aggregate consumption dynamics in the incomplete markets model relative to the standard complete markets setup. Section 5 illustrates the results based on alternative calibration and different monetary policy rule. Section 6 concludes.
2 Model

The extent to which the aggregate consumption in the incomplete markets model differs relative to that in the complete markets setup depends on the population of households that face uninsurable unemployment risk and borrowing constraints. These households will trade off the benefits of intertemporal substitution and the costs associated with having lower buffer stock savings, when real interest rate falls in recessions. To analyze these effects I depart from representative household framework by assuming that the economy is populated by two groups of households similar to McKay and Reis (2016). The first group is relatively more patient and fully insured from unemployment risk, and owns the capital and firms. The second group are more impatient and imperfectly insured from unemployment risk, and face borrowing constraints (henceforth, uninsured households). Having heterogeneous discount factor among households helps to generate wealth distribution of rich holding most of the wealth as in US (Krusell and Smith (1998)). The comparison between incomplete markets model and the complete markets model can be easily made by changing only one parameter, a fraction of insured households, maintaining the steady state aggregate quantities and prices.

Throughout this paper, variables with subscript \( i \) are household specific, variables with a superscript \( L \) or \( H \) are group specific. The variables without a subscript or superscript are variables that are identical across households such as aggregate quantities or prices.

2.1 Uninsured Households

Decision problem

There is a measure \( 1 - \Omega \in [0, 1] \) of continuum of uninsured households indexed by \( i \in [0, 1 - \Omega] \). An uninsured household is either employed or unemployed. Upon employment, he supplies labor inelastically and receive after-tax real wage. Upon unemployment, he receives unemployment insurance with replacement rate \( b^u \), assumed to be taxable. A household working at the beginning of the period may lose a job within the period with probability \( \rho_x \). However, I assume he may find a job right away upon separation with probability \( f \) within the period. Therefore, the event that each employed household falls into unemployment pool at the end of the period occurs with probability \( (1 - \rho_x) f \). I refer to this rate as job-loss rate.

Uninsured households cannot purchase unemployment insurance contracts and do not own shares in the firms or own the capital stock. They can only self-insure through trading riskless and liquid bonds but cannot take short position. The budget constraint of uninsured household \( i \) at period \( t \) is given by

\[
\begin{align*}
  c_{i,t}^L + b_{i,t+1}^L &= (1 - \tau_t)w_t e_{i,t} + (1 - \tau_t)b^u w_t(1 - e_{i,t}) + \frac{R_{t-1} b_{i,t}}{\Pi_t},
\end{align*}
\]

(2.1)
together with borrowing constraint, \( b^L_{i,t+1} \geq 0 \), where \( c^L_{i,t} \) denotes consumption of uninsured household \( i \), \( b^L_{i,t+1} \) is the quantities of bond purchased in the beginning period \( t \) in terms of consumption good at period \( t \), \( \Pi_t \) denotes the gross inflation rate, \( R_{t-1} \) is the gross nominal interest rate paid on bonds purchased in period \( t-1 \). \( w_t \) is real labor income and \( b^w_t \) refers to unemployment insurance. \( \tau_t \) denotes tax rate on real wage and unemployment insurance and \( e_{i,t} \) refers to an indicator for employment status: \( e_{i,t} = 1 \) for employed and \( e_{i,t} = 0 \) for unemployed. Let \( V(b^L_t, 1; S) \) be the value of being employed given its bond position and the aggregate state vector \( S \), which will be described when we define equilibrium. The decision problem faced by an employed household is

\[
V(b^L_i, 1 : S) = \max_{c^L_i, b^L_i'} \left\{ \left[ \frac{c^L_{11} - \sigma}{1 - \sigma} \right] + \beta^L \mathbb{E} \left( 1 - \rho_x (1 - f(S)) \right) V(b^L_i', 1; S') \right. \\
\left. + \beta^L \mathbb{E} \rho_x (1 - f(S)) V(b^L_i', 0; S') \right\} \tag{2.2}
\]

subject to borrowing constraint, budget constraint (2.1), and law of motion for aggregate states which I specify below. \( \beta^L \) is the discount factor for uninsured household, \( \sigma \) refers to risk aversion and \( \mathbb{E} \) is the conditional expectations operator over aggregate uncertainties. Similarly, the problem of an unemployed household with the same bond level is given by

\[
V(b^L_i, 0; S) = \max_{c^L_i, b^L_i'} \left\{ \left[ \frac{c^L_{11} - \sigma}{1 - \sigma} \right] + \beta^L \mathbb{E} f(S) V(b^L_i', 1; S') + \beta^L \mathbb{E} (1 - f(S)) V(b^L_i', 0; S') \right\} \tag{2.3}
\]

subject to borrowing constraint, budget constraint (2.1), and law of motion for aggregate states.

**Law of motion for distribution**

The distribution of wealth across uninsured households in the next period is determined in current period by the households’ savings rules. These rules are governed by distribution of wealth in beginning of the current period and current realization of aggregate uncertainties. The distribution of employment states across uninsured households in the next period is determined by job loss rate and job-finding rate that households face in current period. These rates are governed by firm’s optimal policies that are determined by distribution of wealth in beginning of the current period and current realization of aggregate uncertainties. Therefore, distribution of uninsured households over wealth and employment states in next period is a function of aggregate states in current period and is characterized as follows:

\[
\Gamma'(B, e') = \sum_{e' \in \{0,1\}} \text{Pr}(e'|e; S) \int \mathbf{1}\{g(b^L, e; S) \in B\} d\Gamma(b^L, e), \tag{2.4}
\]
where \( \Gamma(\cdot) \) is the CDF of the distribution of uninsured households in the beginning of the current period, \( g(\cdot) \) refers to savings rule, \( B \) is a subset of the space of bond holdings and \( \Pr(e'|e;S) \) denotes the transition rate from employment state \( e \) to state \( e' \) which varies endogenously.

### 2.2 Insured Households

There is a measure \( \Omega \) of insured households. This group is fully insured from idiosyncratic unemployment risk due to a following structure. Each insured household can be thought of a large family that consists of a unit measure of continuum of uninsured households. Therefore, what matters for the consumption of insured household is the pooled income of uninsured households which is governed by aggregate risk as individual unemployment risk is shared across the members. The fraction of uninsured households that are employed (unemployed) within the family is assumed to be equal to aggregate employment (unemployment) level and they receive identical after-tax real wage (unemployment insurance). Therefore, labor income of a family is total sum of after-tax real wage received from employed members and after-tax unemployment insurance from unemployed members.

Insured households have the same utility function as uninsured households, but they are more patient. They enjoy significant wealth by owning bonds and physical capital. Their consumption and investment in physical capital are financed by five sources: trading bonds with uninsured households and government, revenue from renting capital to intermediate-goods firms, labor income, and dividend earned from intermediate-goods firms. Unlike uninsured households, they are not borrowing constrained and, thus, their Euler equation holds with equality. The \( t \)-period budget constraint looks

\[
c_t^H + b_{t+1}^H + i_t^H = (1 - \tau_t) (w_t n_t + UI_t(1 - n_t)) + \frac{R_{t-1}}{1 + \tau_t} b_t^H + r_k^H k_t^H + d_t, \tag{2.5}
\]

where \( c_t^H, b_{t+1}^H \) and \( i_t^H \) are consumption, holdings of bond, and investment by insured households in period \( t \), respectively. \( r_k^H \) represents real rental rate of capital and \( k_t^H \) is capital stock. \( d_t \) denotes dividend from owning intermediate-goods firms. The physical capital is accumulated via following technology,

\[
k_{t+1}^H = (1 - \delta) k_t^H + \mu_t \left( 1 - S \left( \frac{i_t^H}{i_{t-1}^H} \right) \right) i_t^H, \tag{2.6}
\]

where \( \delta \) is depreciation rate. As in Christiano, Eichenbaum, and Evans (2005), investment in physical capital is subject to adjustment cost captured by quadratic function \( S(\cdot) \). In steady state, \( S = S' = 0 \) and \( S'' > 0 \). \( \mu_t \) is marginal efficiency of investment which governs the efficiency at which investment good are transformed into physical capital that is used for production in the
next period.\footnote{Another interpretation to disturbances to capital production is an investment-specific technology shocks that affects the transformation of consumption into investment goods as in Greenwood, Hercowitz, and Krusell (1997). Justiniano, Primiceri, and Tambalotti (2011) nest both stories and show disturbances that affect the transformation of investment goods into the future capital input is the most important driver of US output fluctuations, whereas investment-specific technology contributes little.}

This shock follows the stochastic process

\[
\log(\mu_t) = \rho^\mu \log(\mu_{t-1}) + \sigma^\mu \varepsilon_t^\mu \\
\text{with} \quad \varepsilon_t^\mu \overset{iid}{\sim} \mathcal{N}(0, 1),
\]  

(2.7)

where $\rho^\mu$ and $\sigma^\mu$ denote persistence and the standard deviation of the shock, respectively.

Justiniano, Primiceri, and Tambalotti (2011) show that movements in marginal efficiency of investment is highly correlated with the fluctuation of credit spreads which measure the vulnerability of financial system. Moreover, shocks to marginal efficiency of investment leads to qualitatively similar responses to financial shocks or uncertainty shocks in the model that incorporates balance-sheet channel of the firms. For instance, Christiano, Motto, and Rostagno (2014) and Carlstrom, Fuerst, Ortiz, and Paustian (2014) show that shocks to the volatility of idiosyncratic productivity of entrepreneur or net worth in the financial accelerator model of Bernanke, Gertler, and Gilchrist (1999) predicts too little volatility of consumption relative to that of investment. Therefore, instead of explicitly specifying financial shocks or uncertainty shocks, I interpret a decline in marginal efficiency of investment as a reduction of credit supply to firms caused by a weakened balance-sheet of financial intermediary or a rise in uncertainties.\footnote{Gilchrist, Schoenle, Sim, and Zakrajšek (2014) argue that the impact of uncertainty on investment occurs primarily through changes in credit spreads rather than through the traditional wait-and-see effect}

2.3 Matching

Each firm posts multiple of identical jobs. Vacant jobs and unemployed households are randomly matched according to the aggregate matching function, $m(u_{a,t}, v_t) = \psi(u_{a,t})z(v_t)^{1-z}$, where $m(u_{a,t}, v_t)$ is the number of matches in period $t$, when there are $u_{a,t}$ job seekers and $v_t$ vacancies. $\psi$ is the matching efficiency and $z$ represents elasticity of matches with respect to job seekers. Recall that I assumed employed households are able to search for the jobs right away, once they are separated. Therefore, job seekers consist of unemployed households in previous period and households that were on job in previous period but are separated in this period. The number of job seekers in period $t$, in turn, is given by

\[
u_{a,t} = u_{t-1} + \rho_x n_{t-1}.
\]  

(2.8)

Given the matching function, probability of a vacant job to be filled and probability of a job
seeker to be employed equal
\[ \lambda_t = m(1/\theta, 1) = \psi \theta_t^{-z} \] (2.9)
and
\[ f_t = m(1, \theta) = \psi \theta^{1-z}, \] (2.10)
respectively, where, \( \theta = v_t / u_{a,t} \) denotes the labor market tightness. The number of unemployed in period \( t \) equals the number of job seekers who failed to find a job and is given by
\[ u_t = (1 - f_t)(u_{t-1} + \rho x n_{t-1}). \] (2.11)

2.4 Production Sector

Final-goods firms A continuum of identical and competitive firms combine differentiated intermediate goods and produce final good according to Dixit-Stiglitz aggregator,
\[ y_t = \left( \int y_{j,t}^{1-1/\varepsilon} dj \right)^{1/(1-1/\varepsilon)}, \] (2.12)
where \( y_{j,t} \) is the amount of intermediate good \( j \) used as inputs and \( \varepsilon \) is the elasticity of substitution between any pair of intermediate goods. The final-good firm’s problem is to minimize the expenditures on intermediate goods taking the prices as given subject to production function (2.12). Its optimal choices imply the demand function for intermediate good \( j \),
\[ y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\varepsilon} y_{t}, \] (2.13)
where \( P_{j,t} \) is the price of intermediate good \( j \) in period \( t \). \( P_t \) denotes the aggregate price index which is given as,
\[ P_t = \left( \int P_{j,t}^{1-\varepsilon} dj \right)^{1/(1-\varepsilon)}, \] (2.14)
obtained from the zero-profit condition for final-good firm.

Intermediate-goods firms There is a unit continuum of intermediate-goods monopolistic producers. Intermediate-good firm \( j \) produces differentiated good \( j \) according to a production function,
\[ y_{j,t} = k_{j,t}^{\alpha} n_{j,t}^{1-\alpha} - \xi, \] (2.15)
where \( k_{j,t} \) denotes the capital used and \( n_{j,t} \) is the stock of employers used. \( \alpha \) is the elasticity of production with respect to capital and \( \xi \) refers to the fixed cost measured in units of final good. In every period, it posts vacancies, \( v_{j,t} \), which are filled with probability \( \lambda_t \). Therefore, the evolution of employees in firm \( j \) is given as

\[
n_{j,t} = (1 - \rho_x)n_{j,t-1} + \lambda_t v_{j,t}, \tag{2.16}
\]

In addition, it faces price-setting frictions which is modeled as quadratic costs of price adjustment following Rotemberg (1982). A firm \( j \) maximizes present-discounted stream of profits,

\[
\max_{p_{j,t},n_{j,t},v_{j,t},k_{j,t}} \mathbb{E}_t \sum_{s=0}^\infty \Lambda_{t+s} \left[ \left( \frac{P_{j,t+s}}{P_{t+s}} \right) y_{j,t+s} - w_{t+s}n_{j,t+s} - r^{k}_{t+s}k_{j,t+s} - \kappa v_{j,t+s} \right] - \frac{\phi}{2} \left( \frac{P_{j,t+s}}{P_{j,t+s-1}} - 1 \right)^2 y_{t+s} \tag{2.17}
\]

subject to (2.13), (2.15) and (2.16). Costs for the firm are forgone resources from searching new employees and setting prices, wage bill paid to all employees, and the rental of capital. \( \kappa \) is the costs associated with posting a vacancy. \( \Lambda_{t+t+s} \) is the the stochastic discount factor of insured households who are the owners of the intermediate-goods firms.

As in Gornemann, Kuester, and Nakajima (2014), I assume that real wage is set according to the rule,

\[
\ln(w_t) - \ln(w) = \phi_y (\ln(y_t) - \ln(y)), \tag{2.18}
\]

where \( \phi_y \in [0,1] \) is the elasticity of real wage with respect to output and controls the extent to which real wages vary with business cycle. I adopt the exogenous wage rule instead of having wage determined by solving Nash bargaining problem for the following reason. In the model of household heterogeneity in wealth, wages become non-trivial function of individual wealth under Nash bargaining. In the procedure of obtaining stationary competitive equilibrium, the first step of the solution method used in this paper, one needs to solve for the fixed wage function which is characterized by many discrete points in addition to one-dimensional fixed point, which is the only requirement for solving Bewley-Hugget-Aiyagari type model. Even after going through such computational burden, the literature indicates that very little wage dispersion is created by wealth inequality.\(^9\) As the bargaining theory admits that any wage within the bargaining set could be an outcome of the bargain, in Appendix, I verify that the series of wages predicted by the model lies within the bargaining set.

\(^9\)Krusell, Mukoyama, and Sahin (2010) show that mean-min ratio – the ratio of the mean wage to the minimum wage – implied by Nash bargaining is very close to 1.
2.5 Government

A stock of government debt outstanding, \( b^g \), is assumed to be invariant in all periods. The government raises tax revenue and issues bonds to finance interest payments on debt and unemployment insurances. Then the government budget constraint is

\[
\tau_t (w_t n_t + b^u w_t u_t) + b^g = \frac{R_{t-1}}{\Pi_t} b^g + b^u w_t u_t.
\] (2.19)

The left-hand side is the revenue and the right-hand side is the expenditure. From this budget constraint, we get an expression for the tax rate, which depends on the employment rate, real wage, real interest rate, and government debt. That is,

\[
\tau_t = \frac{(\frac{R_{t-1}}{\Pi_t} - 1)b^g}{w_t (n_t + b^u (1 - n_t))} + \frac{b^u (1 - n_t)}{(n_t + b^u (1 - n_t))}.
\] (2.20)

During expansions of aggregate demand, inflation rate and real wage rise. This means that there is a reduction in the costs of debt and a rise in tax base. Moreover, a rise in employment and a reduction in unemployment increases revenue and reduces outlays. All these forces lead to an increase in budget surplus which implies a reduction in the tax rate according to the fiscal rule.

Monetary policy in the baseline follows a simple Taylor rule,

\[
\ln \left( \frac{R_t}{R} \right) = \alpha_{\pi} \ln \left( \frac{\Pi_t}{\Pi} \right),
\] (2.21)

where \( \alpha_{\pi} \) measures the extent to which interest rate responds to a deviation of inflation rate from its target. \( R \) and \( \Pi \) are steady state gross nominal interest rate and gross inflation rate, respectively. I omit output gap term because with incomplete markets, it is no longer clear what a constrained-welfare natural level of output is. The term that captures interest rate smoothing is considered in section 5 and its aggregate implication is studied.

2.6 Market Clearing and Equilibrium

There are four markets operating in the model – bonds, labor, capital and final goods. The bond market clears when

\[
(1 - \Omega) \int g_t(b^L, e; S) d\Gamma_t(b^L, e) + \Omega b^H_t = b^g,
\] (2.22)
where the terms on the left-hand side are bond holdings by uninsured and insured households. The markets for labor, capital, and final goods clear if

\[ n_t = \int_0^1 n_{j,t}dj \]  
\[ \Omega k_t = \int_0^1 k_{j,t}dj \]  

and

\[ c_t + i_t = y_t - \frac{\phi}{2} (\Pi_t - 1)^2 y_t - \kappa v_t \]  

holds, where \( c_t = (1 - \Omega) \int c^L_i di + \Omega c^H_i \) and \( i_t = \Omega i^H_t \).

A symmetric equilibrium in this economy is a sequence of aggregate quantities, \( \{c_t, i_t, y_t, k_t, d_t, n_t, u_t, u_d, t, f_t, \lambda_t, v_t, \theta_t, \mu_t\}_{t=0}^{\infty} \) prices, \( \{w_t, r_t, \Pi_t\}_{t=0}^{\infty} \) uninsured household decision rules, \( \{g_t(b^L, e; S)\}_{t=0}^{\infty} \) insured household variables, \( \{c^H_t, i^H_t, k^H_t\}_{t=0}^{\infty} \) the distribution of uninsured households over bond wealth and employment states, \( \{\Gamma_t(b^L, e)\}_{t=0}^{\infty} \) and policy instruments, \( \{R_t, \tau_t\}_{t=0}^{\infty} \) such that

1. the uninsured household decision rules maximize (2.2) and (2.3) subject to (2.1),
2. insured households maximize the same felicity function as uninsured subject to (??),
3. the distribution of households over bond wealth and employment states evolves according to law of motion (2.4),
4. final-goods firm’s decisions are (2.13) and (2.14),
5. intermediate-goods firms maximize (2.17) subject to (2.13), (2.15), and (2.16) given factor prices,
6. dividends received by insured households result from the optimal decision of the intermediate-goods firms,
7. real wages are consistent with (2.18) and real rental rate of capital and inflation respect the optimal decision of the intermediate-goods,
8. the stock of capital, the stock of job seekers, job filling rate, job finding rate and the stock of unemployed vary consistently with (2.6), (2.8), (2.9), (2.10) and (2.11),
9. the process of marginal efficiency is (2.7),
10. the government adjust taxes subject to (2.19) and monetary policy follows (2.21),
11. markets that operate in the economy clear, (2.22) - (2.25),

where aggregate state vector \( S \) is given by

\[ S = \{\Gamma(b^L, e), b^H, i^H, k^H, \mu\}. \]

Note that employment stock before the labor market transitions can be computed from the cross-
sectional distribution of wealth, that is \( n_{t-1} = \int_{b_L} d\Gamma_t(b^L, 1) \).

## 3 Calibration

The model period is one quarter. As my simulation focuses on the periods of the Great Recession, I fix the starting point (steady state) of the simulation at 2007Q3, one quarter before the peak of expansion defined by the NBER. Because, the goal of this paper is how limited insurance faced by households shape the aggregate consumption dynamics, the parameters that affect the extent to which the households are insured should be carefully chosen. To do so, I first fix the parameters that determine the steady state values of aggregate quantities and prices and then choose the values of parameters that determine the extent to which uninsured households are insured.

The capital depreciation rate, \( \delta \), is fixed to 0.015 implying a 6 percent annual depreciation of physical capital. Elasticity of production with respect to capital, \( \alpha \), is set to 0.33. I fix the discount factor of insured households to match the real return on bonds to 3 percent, in line with the average real Federal funds rate from 1984Q1 to 2008Q3. The unemployment rate in the third quarter of 2007 is 4.7 percent. For the choice of elasticity of substitution between intermediate-goods, \( \epsilon \), I target the steady state markup of 1.2. The fixed cost, \( \xi \), is set so that steady state profits of monopolistic competitive firms are zero.

The steady state employment transition is governed by job finding rate, \( f \), and separation rate, \( \rho \). Following Shimer (2005), I first compute the monthly job-finding rate using unemployment and short-term unemployment data constructed and seasonally adjusted by the BLS from the Current Population Survey (CPS). I average the resulting series over each quarter and, then, convert them into quarterly terms. The resulting job finding rate in 2007Q3 is 0.71. Using equation (2.11), I then compute the job separation rate. The matching function elasticity to job seekers, \( z \), is chosen to be 0.5, a standard value. For the matching efficiency, \( \psi \), I exploit the relation between vacancy filling rate and job finding rate using equation (2.10) and (2.9) and match the quarterly vacancy filling rate of 0.71, computed by den Haan, Ramey, and Watson (2000). Expected costs of posting a unit vacancy, \( \kappa/\lambda \), is calibrated to match 4.5 percent of quarterly wages following Hagedorn and Manovskii (2008a), whose calculation is based on the time spent for hiring one worker. The value of steady state real wage is obtained from the optimal vacancy posting condition under free-entry. I set the share of the insured households, \( \Omega \), to 0.2. Kaplan, Moll, and Violante (2015) report that the liquid wealth share of top 10 percent by liquid wealth is equal to 86 percent of total liquid wealth and the share of top 20 percent is 97 percent using data from the 2004 Survey of Consumer Finances. Therefore, I assume that top 20 percent hold enough liquid wealth to fully insure idiosyncratic risks. \(^{10}\)

Because the key assumption in this paper is the inability to insure against the unemployment

\(^{10}\)My calibration implies the liquid wealth share held by top 20 percent is 94 percent in steady state
spell, it is important to determine how worse the households are when unemployed. Following Chodorow-Reich and Karabarbounis (2015), I assume that consumption level of households that are unemployed is 21 percent below its level of employed households on average. There are four parameters that contribute to the consumption difference between the employment states. First, the coefficient of risk aversion, $\sigma$, determines how much households can bear the fluctuations in individual income. Second, the supply of liquid assets influences after-tax income of the uninsured households and thus their probability of hitting a borrowing constraint. In section 5, I show how aggregate supply of liquid assets plays a role in aggregate consumption volatility. Third, replacement rate, $b^u$, influences consumption who are unemployed through a change in unemployment benefit. Fourth, a shift in discount factor of uninsured households shifts the relative importance of current consumption to future consumption and, hence, determines the current amount of savings. In particular, the lower the discount factor, the more likely households are near the borrowing constraint and thus exposed to a large consumption fluctuations.

The coefficient of risk aversion is set equal to 2. I set the supply of government bonds, $b^g$, to match the average ratio of aggregate liquid assets to GDP. I use the same definition of liquid assets as Guerrieri and Lorenzoni (2015) and are calculated from aggregate household balance sheets reported in the Flow of Funds Accounts over the period from 1984Q1 to 2012Q4. The replacement is chosen to be 0.4, following Shimer (2005). I fix three parameters and, then, adjust the discount factor of uninsured households to match the consumption differential.

I now discuss the parameters that affect the aggregate dynamics. Ravn and Sterk (2013) show that the business cycle dynamics in incomplete markets model are substantially different from those of complete markets model under an environment with sticky prices and real wage rigidity. Accordingly, I choose the parameters related to the price rigidity as follows. For the parameter that governs the costs of adjusting prices, I exploit the equivalence of the coefficient of marginal cost in the linearized Phillips curve implied by Rotemberg model and the one derived from the Calvo model. I, then, find $\phi_p$ that corresponds to the price adjustment frequency of 4 quarters. With regard to the elasticity of real wage with respect to output, $\phi_y$, I adopt the value computed by Hagedorn and Manovskii (2008b). In fact, in Hagedorn and Manovskii (2008a), labor productivity is assumed to be a cyclical indicator, so the real wage rigidity is measured by the wage elasticity with respect to labor productivity. In their working paper (Hagedorn and Manovskii (2008b)), they compute the elasticity of real wages with respect to output using data predicted from the calibrated model and find that the value, 0.25, is equal to that computed from the data. Because there is no shock to productivity and business cycle is assumed to be driven by investment shock in this paper, I use the output as a cyclical indicator.

The sensitivity of the nominal interest rate with respect to inflation in the Taylor rule, $\alpha_{11}$, is set at 1.5, which is in the standard range of values used in the literature. The target level for gross in-

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11 Flow of Funds Table B.100 Lines 10 (deposits), 17 (treasury securities), 18 (agency and GSE securities), 19 (municipal securities), 20 (corporate and foreign bonds), 25 (corporate equities) and 26 (mutual fund shares).
flation, $\Pi$, is set at 1. The investment adjustment cost parameter, $S''$, is set equal to 2.85, to capture a hump-shaped response of investment in response to investment shocks, consistent with SVAR based evidence from Gilchrist and Zakrajšek (2012). The persistence of the investment shocks, $\rho^\mu$ is assumed to be 0.72. The value of latter two parameters are taken from Justiniano, Primiceri, and Tambalotti (2010). The standard deviation of the shock is chosen so that the volatility of investment implied by the model matches that computed from investment data that ranges from 1984Q1 to 2012Q2. I report standard deviation of shock for both incomplete markets model and complete markets model.

4 Business Cycle Analysis

4.1 Solution Method

The model laid out in this paper requires a method that solves the incomplete market models with aggregate uncertainty. The well-known challenge involved in solving these models is that aggregate quantities and prices depends not only on aggregate shocks but also on distribution of wealth which is of infinite-dimensional object (Krusell and Smith (1998)). I use the method developed by Reiter (2009), because this method can easily handle a large number of aggregate state variables with rich elements in aggregate level. For example, McKay and Reis (2016) show that the incomplete markets model can be incorporated with many features that found to be important in monetary business cycle models.

The application of Reiter (2009) method to the model in this paper can be summarized as follows. First, the distribution of wealth is approximated with a histogram that has large number of bins. The mass of households in each bin becomes a state variable of the model, so the infinite-dimensional object is approximated with many but finite number of state variables. Second, the household decision rule is discretized with finite number of knot points which are interpolated with linear splines. Using a block that describes labor market development, optimal policies of production side, and fiscal rule, I obtain the stationary competitive equilibrium using standard algorithm that is used to solve Bewley-Huggett-Aiyagari models in which there are idiosyncratic but no aggregate uncertainties. The model is then linearized around the steady state and the solution is computed using method for solving linear rational expectation systems (Sims (2002)).

The resulting solution preserves the nonlinear relationship between uninsured household decisions and individual state variables, so that consumption function exhibits a kink at cash-on-hand where borrowing constraint starts to bind. However, the uninsured household decisions are linear in aggregate states. More details on the solution method is described in Appendix. In addition, to check the accuracy of the solution, Euler equation errors are reported.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Target (Source)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>δ</strong></td>
<td>Capital depreciation rate</td>
<td>0.015</td>
<td>6% annual depreciation rate</td>
</tr>
<tr>
<td><strong>α</strong></td>
<td>Production wrt capital</td>
<td>0.33</td>
<td>Standard</td>
</tr>
<tr>
<td><strong>β^H</strong></td>
<td>Discount factor of insured household</td>
<td>0.993</td>
<td>3% annual real interest rate</td>
</tr>
<tr>
<td><strong>ε</strong></td>
<td>Elasticity of substitution b/w goods</td>
<td>6</td>
<td>Markup of 1.2</td>
</tr>
<tr>
<td><strong>ξ</strong></td>
<td>Fixed costs</td>
<td>0.55</td>
<td>Zero-profit condition</td>
</tr>
<tr>
<td><strong>ρ_x</strong></td>
<td>Job separation rate</td>
<td>0.12</td>
<td>Equation (2.11)</td>
</tr>
<tr>
<td><strong>z</strong></td>
<td>Matching function elasticity</td>
<td>0.5</td>
<td>Standard</td>
</tr>
<tr>
<td><strong>ψ</strong></td>
<td>Matching efficiency</td>
<td>0.71</td>
<td>Job filling rate of 0.71</td>
</tr>
<tr>
<td><strong>κ</strong></td>
<td>Cost of posting vacancy</td>
<td>0.061</td>
<td>4.5% of quarterly wages</td>
</tr>
<tr>
<td><strong>Ω</strong></td>
<td>Share of insured households</td>
<td>0.2</td>
<td>McKay and Reis (2016)</td>
</tr>
</tbody>
</table>

**Parameters associated with imperfect insurance**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Target (Source)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>σ</strong></td>
<td>Risk aversion</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td><strong>b^S</strong></td>
<td>Supply of liquid assets</td>
<td>1.74 x annual GDP</td>
<td>Liquid assets/GDP of 1.74</td>
</tr>
<tr>
<td><strong>b^U</strong></td>
<td>Replacement rate</td>
<td>0.4</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td><strong>β^L</strong></td>
<td>Discount factor of uninsured household</td>
<td>0.983</td>
<td>Consumption differences of 21%</td>
</tr>
</tbody>
</table>

**Parameters associated with aggregates dynamics**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Target (Source)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ϕ_p</strong></td>
<td>Price stickiness</td>
<td>58.7</td>
<td>Adjustment freq. of 4 quarters</td>
</tr>
<tr>
<td><strong>ϕ_y</strong></td>
<td>Wage elasticity wrt output</td>
<td>0.25</td>
<td>Hagedorn and Manovskii (2008b)</td>
</tr>
<tr>
<td><strong>α_π</strong></td>
<td>Interest rate rule on inflation</td>
<td>1.5</td>
<td>Standard</td>
</tr>
<tr>
<td><strong>S''</strong></td>
<td>Investment adjustment cost</td>
<td>2.85</td>
<td>Justiniano et al. (2010)</td>
</tr>
<tr>
<td><strong>ρ^k</strong></td>
<td>Persistence of MEI</td>
<td>0.72</td>
<td>Justiniano et al. (2010)</td>
</tr>
<tr>
<td><strong>σ^I</strong></td>
<td>Std. of MEI shock (IM)</td>
<td>0.038</td>
<td>Std of of investment 4.85</td>
</tr>
<tr>
<td><strong>σ^l</strong></td>
<td>Std. of MEI shock (CM)</td>
<td>0.057</td>
<td>Std of of investment 4.85</td>
</tr>
</tbody>
</table>

Notes: IM stands for 'Incomplete Market' and CM stands for 'Complete Market'.

17
4.2 Results

Table 2: Business cycle statistics: Data vs Model

<table>
<thead>
<tr>
<th>Variable (x)</th>
<th>Data</th>
<th>Model (IM)</th>
<th>Model (CM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>std(x)</td>
<td>1.85</td>
<td>1.33</td>
<td>0.95</td>
</tr>
<tr>
<td>Output (y)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>std(x)/std(y)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>corr(x,y)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>std(x)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption (c)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>std(x)/std(y)</td>
<td>0.79</td>
<td>0.38</td>
<td>0.52</td>
</tr>
<tr>
<td>corr(x,y)</td>
<td>0.85</td>
<td>0.93</td>
<td>-0.08</td>
</tr>
<tr>
<td>std(x)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment (i)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>std(x)/std(y)</td>
<td>4.85</td>
<td>3.65</td>
<td>5.10</td>
</tr>
<tr>
<td>corr(x,y)</td>
<td>0.98</td>
<td>0.99</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Notes: The table compares moments of the data and from 10,000 simulations of the model. Standard deviations are scaled by 100. The moments are taken from log of the data and then detrended using HP-filter with smoothing parameter 1600. The first row for each variable reports the standard deviation of the variable and the its second row denotes its standard deviation relative to the standard deviation of output. The third row show the contemporaneous correlation with output. The third column reports results from the incomplete markets model and the fourth column show results from the complete markets model.

I now assess how uninsurable unemployment risk alters the dynamics of aggregate consumption in response to marginal efficiency of investment shocks. To do so, I compare the baseline model, the incomplete markets model, against the complete markets model. The complete markets economy is obtained from the baseline model by adjusting the fraction of insured households, $\Omega$, to 1.

Business cycle statistics

Table 2 displays the business cycle facts and assesses how well the incomplete markets model and complete markets models are able to capture these. All data are quarterly and seasonally adjusted. Both the data and the model-generated data are taken log and detrended using HP filter with smoothing parameter of 1600. The source of the data is the St. Louis Fed’s FRED II database and the period ranges from 1984Q1 to 2012Q4. Consumption corresponds to personal consumption expenditures on non-durables and services, while investment is the sum of personal consumption expenditures on durables and gross private domestic investment. Then the real series are constructed by dividing the nominal series by working age population, Aged 15-64, and GDP deflator. For the measure of output, I take the sum of consumption and investment.

The standard deviation of investment shocks is adjusted for each model so that model-implied
investment series display exactly the same volatility as the data. The standard deviation of con-
sumption is 37 percent larger in the complete markets model than in the baseline incomplete mar-
kets model. However, the standard deviation of output is 29 percent less in the complete markets
model than in the incomplete markets counterpart, while the investment volatility is the same in
both models. This reflects that consumption in the complete markets model and consumption in
the incomplete markets model are not moving in same directions. The different pattern of con-
sumption movements between the two models is clearly revealed in the correlations of output and
consumption. The correlation suggests that consumption is strongly procyclical in the incomplete
markets model, whereas it is slightly countercyclical, at most acyclical, in the complete markets
counterpart. Therefore, the procyclical pattern of consumption is what makes the output in the
incomplete markets model more volatile relative to the one in complete markets model. However,
the baseline model still under-predicts the output volatility observed in the data.

Impulse responses

Figure 1 displays the impulse responses to a contractionary investment shock. To highlight the di-
mensions in which the the baseline model differs from the complete markets benchmark, impulse
responses are plotted under the same size of shocks, the standard deviation of shocks from the
incomplete markets model.

Note that consumption declines upon impact under the incomplete market model (solid line),
displaying comovement with output and investment. By contrast, under the complete markets
benchmark, consumption is positive for initial several periods and then gradually falls in medium
and long run. There are two competing forces that deliver the short-run response of consumption.
First, a contraction in investment is associated with lower household income as there are more
households who become unemployed. To see how employment is affected, consider intermediate
good firm’s decision over creation of vacancies. A firm’s optimal level of vacancies are determined
by the following equation,

\[ \frac{K}{\lambda_t} = \sum_{s=0}^{\infty} \Lambda_{t,t+s} (1 - \rho_x)s E_t [mc_{t+s}MP_{n,t+s} - \omega_{t+s}] , \quad (4.1) \]

where \(mc_t\) and \(MP_{n,t}\) are real marginal cost and marginal products of labor in period \(t\). The left
hand-side is, again, the expected costs of posting a unit vacancy, and right-hand side is present-
discounted value of current and expected future profits given by a filled job. A decline in current
investment implies a lower level of expected future capital stock and thus a lower level of ex-
pected future marginal products of labor. As long as real wages are moderately procyclical, the
profits associated with the filled job become less attractive, inducing less postings of jobs. An
increased number of unemployed pushes down the aggregate consumption. Second, as the mon-
eyary policy authority lowers the interest rate to stabilize the economy, households engage in in-
tertemporal substitution spending more in current period and less in future periods. If the latter is strong enough to offset the former income effects, aggregate consumption in current period remains stable or could, even, rise. This situation arises in the complete markets benchmark where households are perfectly insured from idiosyncratic risks by trading state-contingent financial securities and thus able to run down their savings without any fear of having lower buffer stock of assets.\footnote{Although I do not explicitly model state-contingent securities in insured households budget constraint, we could think that these households have sufficient amount liquid wealth and hence are close to full insurance.}

However, under the incomplete markets model, some fraction of households are borrowing constrained and face uninsurable unemployment risk that increases as the recession develops. These households anticipate that the probability of meeting the event tree of long sequences of low income (unemployment state) will rise. For them, increasing expenditures would require depletion of savings, which is costly as it makes them more painful during the spell of unemployment. Because the spell of unemployment is expected to be longer, they expect the costs of running down the liquid wealth will increase, giving up the gains from intertemporal substitution. As the costs keep growing, the uninsured households accumulate precautionary wealth by reducing current consumption.

The response of investment is relatively more muted under the incomplete markets model. This is because the increase in precautionary savings to hedge against the rise in expected duration of unemployment reduces the costs of purchasing capital goods, the real interest rate.

The responses of vacancies, unemployment and job filling rate are relatively more amplified under the incomplete markets model. As the fall in investment is followed by a less demand for hires, increased difficulties in finding jobs leads to precautionary savings, which lowers aggregate demand, reduces hire more, and so causes more unemployment fear. The feedback loop between consumption and unemployment fear is the source of the amplification. Relatively lower demand for hires under incomplete market model reduces the price of labor by more and thus the real marginal cost. As the response of inflation is determined by the current and future expected path of real marginal cost, inflation falls by more. Nominal interest rate drops by more according to monetary policy rule. Lastly, real wages falls by more as output responds more strongly under the incomplete markets model.

**Consumption of uninsured households**

I have discussed that a fall in aggregate consumption in short-run to negative investment shocks is due to the dominance of uninsured household’s incentive to hold precautionary savings over his incentive to engage in intertemporal substitution. Recall that there are two types of uninsured households: employed and unemployed. One might think that a decline in aggregate consumption could be driven by higher number of unemployed who has higher marginal propensity to
consume (MPC) than employed on average rather than precautionary savings effect. I, thereby, show that precautionary savings effect is much stronger for unemployed households, suggesting that the result is not governed by pure compositional effect. To do so, it is useful to know how decision rules of each type change to a surprise change in investment.

The left panel of Figure 2 plots the steady state consumption policy rules for each employment state evaluated at different levels of cash-on-hand. Cash-on-hand represents the resources available for consumption and savings in given period, which is the right-hand side of budget constraint (2.1). The maximum level of cash-on-hand in the figure is obtained from the level of bond at which mass of uninsured households is close to zero premultiplied by steady state real real interest rate plus the steady state after-tax labor income of employed. The consumption rules for both types display strong nonlinearity around the cash-on-hand at which the curve is kinked. The uninsured households who hold cash-on-hand at which optimal consumption overlaps with 45 degree-line consume all the current available resources by hitting a borrowing constraint. The cut-off value of cash-on-hand above which savings are positive is higher for employed households, as they are relatively more likely to become employed in next period than unemployed and thus have less incentive to leave resources for future consumption.\(^\text{13}\)

The right two panel in Figure 2 illustrates how consumption policies deviate from its steady state policies in response to unit standard deviation increase in investment shock. The vertical axis of the two graphs represents the deviations of consumption from its steady state values and are scaled by 100. First, note that the difference in the response of consumption rules between the types is stark. Particularly, for employed households, there exists a level of cash-on-hand that divides the region in which consumption falls from the region where consumption rises. Although the job-loss rate has increased due to a reduction in hires, employed are still more likely to be employed than unemployed and thus are expected to receive relatively more income in the future. As a consequence, those who are even below the the cut-off cash-on-hand reduces consumption only by small amount. Employed households whose cash-on-hand position is above the cut-off level are insured enough to find that running down the buffer stock of savings is less costly and raise their consumption in response to a fall in real interest rate. By contrast, unemployed households significantly reduce consumption level at almost all cash-on-hand positions.

The stark difference in response of consumption policies between employed and unemployed is confirmed by responses of average consumption by type. Figure 3 portrays impulse responses of average consumption for each type to unit standard deviation fall in marginal efficiency of investment. In the case of unemployed, the consumption falls by significant amount for initial several periods and then quickly reverts back to long-run equilibrium. This is because the expectation of extended unemployment spell boosts the precautionary savings which make unemployed employees

\(^{13}\)The steady state transition rate from employed to employed is 0.965 and the rate from unemployed to employed is 0.71. Moreover, it is trivial to show \(1 - \rho_c (1 - f) > f\) as long as \(\rho_c < 1\), implying the former is greater than the latter even along the business cycles.
consume less than employed today but consume more in the future when their level of cash-on-hand is relatively higher. In the case of employed, in contrast, the consumption response is much smaller to start out with, and tends to maintain below the steady state equilibrium for long time as they enjoy more from intertemporal substitution than unemployed.

The Great Recession

Impulse responses in figure 1 illustrate that while consumption rises initially, it eventually declines in the complete markets model. Hence, whether or not the complete markets model cannot explain a drop in consumption during recessions is not clear from impulse responses alone. In order to highlight the meaning of differences in short-run consumption movements between the two model economies, I use the models to simulate aggregate consumption during the Great Recession periods. Assuming that the starting point of the economy is 2007Q3, I simulate each economy using the sequences of shocks recovered by the following procedure. I assume investment is only driven by shocks to marginal efficiency of investment and choose those so that model-implied investment series exactly equals to its detrended data counterpart, from 2007Q3 to 2012Q4. This requires to find 22 unknowns that match 22 targets.

The top panel of figure 4 depicts the paths for consumption implied by the two models and its data starting in 2007Q3, which are indexed to equal zero in 2007Q4, the peak of the expansion as defined by the NBER. In the data, consumption falls by 3.4 percent by 2009Q2 while the incomplete markets model predicts a 2.4 percent decline. However, in the complete markets model, consumption starts to decline in 2009Q1 and reaches its peak in 2011Q4. Its initial rise followed by its delayed decline mirrors the impulse responses of the complete markets model.

A notable observation is a slower recovery of aggregate consumption predicted by incomplete markets model relative to its data since mid 2010. This is because of the existence of fully insured households that accounts for 20 percent of the whole population. The consumption path of these households mimics the persistent decline of aggregate consumption path predicted by the complete markets and thus partially undoes the reversion of uninsured households’ consumption to its long run-trend. Were the incomplete markets economy populated by only uninsured households, aggregate consumption would have recovered faster.

The bottom two panels in figure 4 plot the paths for output and employment implied by the two models and their data starting in 2007Q3, which are indexed to equal zero in 2007Q4. Output is defined as the sum of consumption and investment and the incomplete markets model does relatively better job in predicting its magnitude reflecting consumption response generated by the model. As expected, a reduction in job-finding rate contributes to a fall in employment in the two models and its response is more amplified in the incomplete markets model due to the presence of feedback loop between unemployment risk and consumption demand.  

---

14 One reason that employment response in the model economies is more volatile than the data is that the adjustment
I now assess the contribution of unemployed households to aggregate consumption. The circed solid line in figure 5 represents the counterfactual aggregate consumption in response to the recovered series of shocks when consumption of uninsured and unemployed households is replaced by that of uninsured and employed households.\textsuperscript{15} Thus, the difference between the circed solid line and the real line captures the contribution of uninsured and unemployed households on aggregate consumption for given sequences of shocks, prices, and unemployment risk. If uninsured and unemployed households consumed as much as uninsured but employed households, the decline of aggregate consumption by 2009Q2 would be less than 1 percent. Of course, once we consider general equilibrium effects, the contribution of uninsured and unemployed households would be greater. Prices and unemployment risk are also affected by decisions made by these households and hence it is very difficult to isolate their true contribution in general equilibrium perspective.

Overall, the endogenous variation in unemployment risk appears to have contributed substantially to the decline in aggregate consumption between the start and the peak of the Great Recession mainly through the precautionary savings incentive of unemployed households and their increased number.

5 Extensions and Robustness Analysis

In this section, I extend the analysis above under alternative calibrations. First, I examine how monetary policy rule that considers interest rate smoothing changes aggregate consumption dynamics of the incomplete markets model differently from its complete market counterpart. Second, I explore how the supply of liquid assets matter for aggregate consumption fluctuations.

5.1 Interest Rate Inertia

Often, Taylor rule allows current interest rate to respond to last period’s interest rate in order to capture the Federal Reserves tendency to smooth changes in interest rates. Its inclusion has been proven to make the monetary policy rule characterize the behavior of Fed funds rate better (Clarida, Galí, and Gertler (2000)).

To investigate its aggregate implication, I modify monetary policy rule to: \[ \ln \left( \frac{R_t}{R_{t-1}} \right) = \rho_R \ln \left( \frac{R_{t-1}}{R_{t-2}} \right) + (1 - \rho_R) \phi_{\Pi_t} \], where \( \rho_R \) captures the degree of interest rate inertia and is set equal to 0.8, which is within the range of estimates in the literature. The upper two panels in figure 6 compare the

\textsuperscript{15}That is, the solid line portrays the path of \( \Omega c_{Ht}^{bl} + (1 - \Omega) n \int c_t^L (b^L, 1) d\Gamma_t (b^L | 1) + (1 - n) \int c_t^L (b^L, 0) d\Gamma_t (b^L | 0) \), whereas the circed solid line portrays the path of \( \Omega c_{Ht}^{bl} + (1 - \Omega) \int c_t^L (b^L, 1) d\Gamma_t (b^L | 1) \), where \( c_t(b^L, e) \) is the consumption of household with bond level of \( b^L \) and employment state \( e \in \{ 0, 1 \} \).
impulse responses for aggregate consumption to unit standard deviation of negative investment shock under different values of interest rate smoothing parameter with the left panel corresponding to the complete markets model and the right one to the incomplete markets model. In the complete markets model, smoothness of interest rate helps to smooth out the path of consumption. Since nominal interest rates do not adjust much to poor economic activity and expected inflation is relatively stable variable due to sticky prices, real interest rates fall relatively less. Accordingly, households are less willing to substitute intertemporally. In contrast, under the incomplete markets environment, smoothness of real interest forced by monetary policy rule makes purchases of capital more costly than in the regime in which real interest rate adjusts more, so the resulting fall in investment reduces postings of jobs more. Consequently, uninsured households face more uncertain job prospects and hence devote themselves more to accumulation of precautionary wealth.

For completeness, I plot the simulation results for the Great Recession periods when monetary policy rule allows for interest rate inertia, depicted in the bottom panel of figure 6. Under the incomplete markets environment, the decline of consumption at the peak of the recession is almost three times larger than the one portrayed in figure 4. However, under the complete markets environment, path for consumption appears flatter relative to the one depicted in figure 4, consistent with the impulse responses shown in the top-left panel of figure 6. The result that persistent monetary policy rule induces larger unemployment risk and thus amplifies the response of aggregate consumption is in line with the result discussed by Ravn and Sterk (2013) who show that less aggressive response of policy rate to deviations of inflation from its target produces large amplification of unemployment.

5.2 Supply of Liquid Assets

The quantity of liquid assets available in the incomplete markets economy governs the sensitivity of aggregate consumption with respect to aggregate shocks because of the way the tax system is introduced in the model. Consider a permanent increase in supply of bonds. Equation (2.20) tells that an increase in supply of bonds would require government to raise the tax rate to pay the increased interest associated with higher debt. Because uninsured households receive less after-tax income, they expect to hit a borrowing constraint soon, as their stock of savings run out more quickly. In response, they choose to accumulate more savings by reducing consumption permanently. As they are further away from borrowing constraints in new steady state, their consumption becomes less responsive to aggregate shocks.\footnote{If the tax revenue fully consists of lump-sum tax on the insured households, amount of asset supply does not matter for aggregate dynamics.}

The literature provides two different definitions of liquid assets. In the baseline calibration, the average ratio of assets to GDP is calculated based on the definition of liquid assets by Guerrieri and
Lorenzoni (2015). Alternatively, Kaplan, Moll, and Violante (2015) exclude corporate equities from liquid assets, because nearly 3/4 of total equity is either indirectly held in the form of retirement accounts or private businesses. Based on their definition, my computed average ratio of assets to GDP using the data from the Flow of Funds is 0.9.\textsuperscript{17} I label the model economy with liquid assets defined by Guerrieri and Lorenzoni (2015) as \textit{high asset economy} and the one with liquid assets defined by Kaplan, Moll, and Violante (2015) as \textit{low asset economy}.

The upper two panels in figure \ref{fig:impulse_responses} compare the impulse responses for aggregate consumption to unit standard deviation of negative investment shock under different quantities of liquid assets with the left panel corresponding to the complete markets model and the right one to the incomplete markets model. Under the incomplete markets, reduction in the supply of bond makes uninsured households hold less wealth and, accordingly, aggregate consumption becomes more volatile. By contrast, in the complete markets model, the supply of bond does not matter in shaping the response of aggregate consumption. As the changes in liquid wealth is completely offset by the changes in tax payment, insured households whose consumption profile depends on lifetime labor and asset income do not make any adjustments.

For completeness, the simulation results for the Great Recession periods under the high asset environment is presented in the bottom panel of figure \ref{fig:impulse_responses}. Aggregate consumption of the incomplete markets model responds stronger at all periods compared to its low asset economy counterpart, portrayed in figure \ref{fig:consumption_response}.

6 Conclusion

The unprecedented and unexpected fall in house prices led to a collapse of prices of various financial assets, causing severe tightening of credit conditions for both businesses and households. This led to the debate on whether the contraction in real activities originated from collapse in household consumption or disruption of firm investment. However, neither explanations are fully appealing in the perspective of standard macroeconomic models, in which both disturbances fail to generate a plunge in consumption and investment. In this paper, I present a heterogeneous agent New Keynesian model featuring imperfect insurances consistent with microeconomic evidence and show that investment disturbances not only lead to a fall in investment but also a substantial decline in aggregate consumption. This contrast with a prediction of complete markets New Keynesian model in which consumption rises for first several periods and, thereafter, declines slowly. My result suggests that events that hit investment may be a coherent explanation for the sources of the Great Recession.

\textsuperscript{17}Kaplan, Moll, and Violante (2015) construct liquid assets by combining data from Flow of Funds (FoF) and Survey of Consumer Finances (SCF). In particular, they choose to use SCF measures for the three main categories of liquid assets – deposits, government and corporate bonds. For the rest of the categories, they use data from FoF. However, to be consistent with the data source for the baseline calibration, I, instead, choose to use FoF measures for all assets.
Of course, the result does not imply that a shock to household is irrelevant to aggregate consumption movements during the recession. Recently, motivated by an unprecedented leveraging cycle since 2000, there has been a rising interest in modeling the link between the Great Recession and household debt. In the incomplete markets environment as mine, Guerrieri and Lorenzoni (2015) and Huo and Ríos-Rull (2013, 2015) study a recession that starts with reduction in the household borrowing limit, delivering a decline of consumption demand. My work provides alternative explanation that highlights the recession that starts with reduction in firm investment and ends with reduction in consumption. Nesting the two competing stories in richer incomplete markets model and investigating which story matters more for the response of aggregate consumption during the Great Recession seems fruitful research area.
Figure 1: Impact of an Investment Shock on Macroeconomic Variables

Notes: The solid lines (IM) depict the impulse response functions of the Incomplete markets model, while the dashed lines (CM) are those of the complete markets model. The size of investment shock is set to the standard deviation obtained from the incomplete markets model.
Notes: The left panel depicts steady state optimal consumption rules for uninsured households. The right panel portrays the deviation from the steady state consumption rules in response to one standard deviation investment shock and are scaled by 100. The size of the shock is set to the standard deviation obtained from the incomplete markets model.
Figure 3: Impact of an Investment Shock on Uninsured Consumption

Notes: The solid lines depict the impulse response functions for average consumption of unemployed, while the solid lines with circles are those for average consumption of employed. The size of investment shock is set to the standard deviation obtained from the incomplete markets model.
Figure 4: Aggregate Dynamics during the Great Recession

Notes: The three panels of the figure depict the behavior of selected macroeconomic variables in the periods of 2007Q3-2012Q4 implied by the models and the data. The dashed line (IM) corresponds to the incomplete markets model, while the line with ‘x’ (CM) corresponds to the complete markets model. Consumption, employment, and output are taken log and then detrended using HP-filter with smoothing parameter 1600. Employment is the log of employment-population ratio. All series are indexed to equal zero in 2007Q4.
Notes: The figure depicts the behavior of aggregate consumption in the periods of 2007Q3-2012Q4 implied by the models and the data. The dashed line (IM) corresponds to the incomplete markets model, while the line with 'x' (CM) corresponds to the complete markets model. Circled solid line denotes counterfactual response when unemployed is replaced by employed. Consumption is taken log and then detrended using HP-filter with smoothing parameter 1600. All series are indexed to equal zero in 2007Q4.
Figure 6: Consumption Dynamics (Different Monetary Policy Rules)

Notes: The upper two panels in the figure portray the impulse response functions for aggregate consumption under different monetary policy rules. The bottom panel depict the behavior of aggregate consumption in the periods of 2007Q3-2012Q4 implied by the models ($\rho_R = 0.8$) and the data. The dashed line (IM) corresponds to the incomplete markets model, while the line with 'x' (CM) corresponds to the complete markets model. Consumption is taken log and then detrended using HP-filter with smoothing parameter 1600. All series are indexed to equal zero in 2007Q4.
Figure 7: Consumption Dynamics (Different Asset Supplies)

Notes: The upper two panels in the figure portray the impulse response functions for aggregate consumption under different supplies of assets. The bottom panel depict the behavior of aggregate consumption in the periods of 2007Q3-2012Q4 implied by the models (low asset economy) and the data. The dashed line (IM) corresponds to the incomplete markets model, while the line with ‘x’ (CM) corresponds to the complete markets model. Consumption is taken log and then detrended using HP-filter with smoothing parameter 1600. All series are indexed to equal zero in 2007Q4.
References


Appendices

A Aggregate Equations

Here I describe the aggregate equations that result from the optimization of insured households and firms and those related to labor market environment but not presented in the main text.

**Insured household’s problem** The insured household chooses \( \{c^H_t, b^H_{t+1}, i^H_t, k^H_{t+1}\} \) to maximize the same utility as uninsured subject to equations (2.5) and (2.6). Setting up the Lagrangian, with \( \Lambda_t \) and \( \Lambda_t q_t \) as the Lagrangian multipliers on constraints (2.5) and (2.6), respectively, and then rearranging the resulting optimality conditions using relation \( i_t = \Omega^H_t \), we obtain

\[
1 = \mathbb{E}_t \beta^H \frac{R_t}{\Pi_{t+1}} \left( \frac{c^H_{t+1}}{c^H_t} \right)^{-\sigma}, \quad \text{(A.1)}
\]

\[
1 = q_t [1 - S \left( \frac{i_t}{i_{t-1}} \right) - \frac{i_t}{i_{t-1}} S' \left( \frac{i_t}{i_{t-1}} \right)] + \mathbb{E}_t \beta^H \left( \frac{c^H_{t+1}}{c^H_t} \right)^{-\sigma} q_{t+1} \left( \frac{i_{t+1}}{i_t} \right)^2 S' \left( \frac{i_{t+1}}{i_t} \right), \quad \text{(A.2)}
\]

and

\[
q_t = \mathbb{E}_t \beta^H \left( \frac{c^H_{t+1}}{c^H_t} \right)^{-\sigma} [r^k_{t+1} + q_{t+1} (1 - \delta)]. \quad \text{(A.3)}
\]

**Intermediate-goods firm** A firm j’s problem is (2.17) subject to (2.13), (2.15), and (2.16). Resulting optimality conditions are

\[
r^k_t = a k_t^{\alpha-1} n_t^{1-a} m c_t, \quad \text{(A.4)}
\]

\[
\frac{\kappa}{\lambda_t} = (1 - \alpha) k_t^\alpha n_t^{1-a} m c_t - w_t + \mathbb{E}_t \beta^H \left( \frac{c^H_{t+1}}{c^H_t} \right)^{-\sigma} (1 - \rho_x) \frac{\kappa}{\lambda_{t+1}}, \quad \text{(A.5)}
\]

and

\[
1 - \gamma + \gamma m c_t = \phi_p (\Pi_t - 1) \Pi_t - \phi_p \mathbb{E}_t \left[ \beta^H \left( \frac{c^H_{t+1}}{c^H_t} \right)^{-\sigma} (\Pi_{t+1} - 1) \Pi_{t+1} \frac{y_{t+1}}{y_t} \right]. \quad \text{(A.6)}
\]

Note that subscript \( j \) is omitted, because intermediate-goods firms face identical factor prices, aggregate matching function, pricing frictions, and production technology, their decisions are the same.

**Labor market environment** Substituting \( v \) out in equation (2.16) using (2.9), we obtain,

\[
n_t = (1 - \rho_x) n_{t-1} + \psi v_t \theta^{-z}, \quad \text{(A.7)}
\]
where labor market tightness is

$$\theta_t = \frac{v_t}{u_{a,t}}.$$  

(A.8)

Definition of unemployment is

$$u_t = 1 - n_t,$$  

(A.9)

B Computational Method

Here I describe the procedure used to solve for an equilibrium path of the heterogeneous agent model with aggregate shocks considered in Section 4.

B.1 Solving for the household’s decision rules without aggregate shocks

For each type of uninsured household characterized by an employment state, I solve for the level of cash-on-hand, $\chi_1$, at which the household starts to consume all of his available resources and therefore is just on the threshold of being borrowing constrained. Taking the first grid point equal to $\chi_1$, I create additional 99 grid points on cash-on-hand, $\chi_2, ..., \chi_{100}$, 100 grids in sum. Constructing the grid in this fashion allows for a more accurate solution because we do not interpolate across the kink in the policy rule where borrowing constraint stops binding. Because there is generally a fair amount of curvature in the savings policies, especially for values of cash-on-hand near the borrowing constraint, the grid points are unevenly spaced with more points near the borrowing constraint. Between the grid points, I interpolate household savings rule with linear splines. I then solve for the household’s savings policies using Broyden (1965) method by imposing that the Euler equation holds with equality at the grid points. The resulting solution consists of $\chi_1$ and values of savings policy evaluated at $\chi_2, ..., \chi_{100}$. In total, the household policy rule for savings is parameterized by 200 variables, 100 points for each employment state.

B.2 Finding the stationary equilibrium

This part of the algorithm is similar to the one described in Aiyagari (1994). I assume the consumption differential between employed and unemployed is 21 percent and search for the value of uninsured households’ discount factor, $\beta^L$, for which this is an equilibrium. When simulating the stationary distribution of wealth, I use non-stochastic simulation as described by Young (2010). Given such $\beta^L$, I compute total bond holdings and consumption of uninsured households. I then use standard techniques from the analysis of representative agent models to find the rest of the aggregate variables. Once I obtain aggregate consumption and total asset supply, consumption and
bond holdings of insured household are obtained by subtracting those of uninsured households from aggregate variables.

B.3 Solving for aggregate dynamics

Household Decision Rules The 200 variables that summarize the household savings policy depend on the aggregate state. As the aggregate state changes, I require that these variables satisfy the Euler equation on the grids described above, which yields nonlinear 200 restrictions.

Evolution of the wealth distribution The non-stochastic simulation algorithm tracks the distribution of wealth using a histogram. The mass in each of these bins is considered to be a variable. I create evenly spaced 250 bins between 0 and and maximum level of bond, $\bar{b}$, for each type of uninsured household. $\bar{b}$ is chosen such that, in the steady state distribution, the mass in $\bar{b}$ bin is very close to zero. In total there are 499 variables that characterize the distribution because one variable is redundant considering that distribution must sum to one. For a given set of household savings rules, transition probabilities between employment states, and prices we can formulate a linear equation system of size 499 that describe transition dynamics of the wealth distribution. Therefore, we have 499 variables and 499 linear restrictions.

Aggregate Equations In addition to the equations that represent the solution of the uninsured households problem and the distribution of wealth across these households, we have equations related to the optimal decision of insured households: (A.1), (A.2), (A.3), (2.6), those of firms: (A.4), (A.5), (A.6), (2.15), (2.18), labor market environment, (A.7), (2.8), (2.11), (A.8), (A.9), (2.9) government polices: (2.20), (2.21), market clearing: (2.25) and process of marginal efficiency: (2.7).

I introduce a set of auxiliary variables that carry extra lag of variables: $i_t^{lag} = i_{t-1}$, $R_t^{lag} = R_{t-1}$, $k_t^{lag} = k_{t-1}$. I use these equations with $c_t = (1 - \Omega) \int c_i^H di + \Omega c_t^H$ to solve for $\mu_t$, $k_t$, $y_t$, $u_t$, $u_{u,t}$, $n_t$, $\theta_t$, $\lambda_t$, $f_t$, $w_t$, $r_t^k$, $q_t$, $\Pi_t$, $mc_t$, $R_t$, $\tau_t$, $c_t$, $i_t$, $v_t$, $i_t^{lag}$, $k_t^{lag}$, $R_t^{lag}$ and $c_t^H$.

Linearization and Solution At this stage, we have a large system of 722 restrictions, some of which are nonlinear, that 722 variables must satisfy. Following Reiter (2009), the system is linearized around the stationary equilibrium using automatic differentiation and then solved using Sims (2002) method for linear rational expectations model.

B.4 Accuracy check: Euler equation errors

To be written
C Wage in the Bargaining Set

Here, I verify whether the real wages predicted by the model lies in the bargaining set. To do so, I check that households and intermediate-goods firms all extract a positive surplus from the match during the Great Recession period, given the sequences of investment shocks. This states wage should be determined in a way that hiring is profitable to intermediate-goods firms and, at the same time, an unemployed should feel that compensation is attractive enough to accept a job offer. Household’s surplus from the match is always positive since real wage, $w_t$, exceeds unemployment benefits, $b_t w_t$ and the probability of remaining employed, $1 - \rho_x (1 - f_t)$ exceeds the probability of finding a job, $f_t$. Note that there is neither disutility from working nor home production. The intermediate-good firm’s surplus is

$$J_t = (1 - \alpha) k_t^\alpha n_t^{-a}mc_t - w_t + \mathbb{E}_t \beta^H \left( \frac{c_t^H}{c_t^{H+1}} \right)^{-\sigma} (1 - \rho_x) J_{t+1}, \quad (C.1)$$

assuming that value of posting a vacancy is converged to zero from perfect competition. Figure A.1 reports the intermediate-good firm’s surplus during the financial crisis given the estimated sequences of shocks used in plotting figure 4 in the main text.