Abstract

This paper provides a simple neoclassical framework for analyzing the recent financial crisis and the events that led up to it, emphasizing the role of liquidity premiums in investment decisions. I consider a money-in-the-utility-function model, in which “money” represents liquid claims that are either backed by the government or by private capital. Since developed economies have a comparative advantage in creating liquid claims, they export them to emerging economies. High liquidity premiums encourage investment for the purpose of creating capital-backed liquid claims. A shock to the private creation of liquidity results in the contraction of investment and a further rise in liquidity premiums.

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1 Introduction

In what sense can the recent financial crisis be represented as a contraction in the stock of money, and, in particular, a contraction in the liquidity properties of private money substitutes? Prominent work such as Lucas and Stokey [2011] and Gorton and Metrick [2012] document that traditional forms of money play only a minor role in transactions between large financial institutions, who hold minimal amounts of cash; instead, money for transactions is raised on a short-term basis through various private arrangements such as repurchase agreements (repo), often involving collateral such as treasuries or asset back securities. In 2008-9, many asset backed securities (and in particular, mortgage backed securities) lost much of their collateral value, which made it more difficult for financial institutions to raise liquidity. Within the financial system, this represented a contraction in stock of money.\footnote{Lucas and Stokey [2011] document a 30\% decline in repos held by primary dealers between 2008-2009, which is of comparable magnitude to the contraction in the stock of money during the Great Depression documented by Friedman and Schwartz [1963].}

While much has been written about the “bank run” dynamics of the crisis on the micro level and the short-run macroeconomic implications (see, for example, Uhlig [2010]), several important questions remain. This paper studies two of them. The first relates to the pre-crisis dynamics, and, in particular, the global environment that made the monetary system increasingly reliant on asset backed securities. The second relates to the long run dynamics associated with the contraction in the liquidity properties of asset backed securities, and their normative implications.

This paper provides a simple neoclassical account of recent events, that emphasizes the role of liquidity premiums in investment decisions. I adopt a money-in-the-utility-function approach (as in Sidrauski [1967] and Friedman [1969]), where “money” is interpreted as liquid claims held by financial institutions for the purpose of facilitating trading. The model features a global environment consisting of two countries: an emerging economy and a de-
veloped economy. There are two types of liquid claims: government-backed liquid claims (e.g., treasuries), which can be issued by both countries but are more abundant in the developed economy, and capital-backed liquid claims (e.g., mortgage backed securities), which are issued only by the developed economy.

The global integration of liquidity markets puts upward pressure on the liquidity premium, which fuels an investment boom in the developed economy. The private returns to capital in the developed economy increase with the higher liquidity premiums associated with capital-backed claims. The developed economy enjoys “international seigniorage rents” from exporting liquid claims to the emerging economy, which results in a permanent trade deficit. From the perspective of the emerging economy, these international “seigniorage” payments reduce permanent income and welfare. In the process of integration, the monetary system becomes increasingly reliant on capital-backed liquid claims.

The crisis is modeled as a shock to the liquidity properties of capital-backed claims. The recessionary dynamics that follow can be understood in neoclassical terms: lower expected liquidity benefits associated with capital lead to a contraction in investment. Furthermore, the expectation of permanently lower “seigniorage” revenues from issuing liquid claims implies lower permanent income, which induces agents in the developed economy to reduce consumption.

Importantly, while the shock implies a redistribution of wealth from developed to emerging economies, it is socially efficient in the sense that - with appropriate transfers - both economies can be made better off. The elimination of capital-backed liquid claims results in an efficient contraction in capital, and equates the marginal products of capital across countries. The inefficiency of private liquidity creation is a well-understood insight from the monetary tradition: for price-taking agents, there are incentives to spend real

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resources on “money printing machines” that print nominal bills, as those are valued in equilibrium. Socially, since money is neutral, such actions are inefficient.

In a quantitative interpretation of the model, I show that a permanent shock to private liquidity creation generates a contraction in the capital stock and an increase in liquidity premiums that are roughly comparable with the post-crisis experience of the US. The model also generates a modest contraction in consumption following the shock. However, the model cannot quantitatively account for the behavior of consumption during the crisis without allowing for additional feedback channels between liquidity supply and income. In terms of the pre-crisis dynamics, the quantitative interpretation of the model suggests that the integration of emerging markets into global liquidity markets can account for a large increase in liquidity premiums (of around 3%), and about a fifth of the growth of the US capital stock.

2 Related literature

It is useful to clarify how this paper is related to other models of the recent crisis. Broadly, theories attempting to explain the recent crisis fall into one of two categories: the “credit shock” view (e.g., Jermann and Quadrini [2012]) and the “demand shock” view (e.g., Mian and Sufi [2012] or Heathcote and Perri [2013]). The key difference between the two views is the implication for the investment wedge. According to the “credit shock” view, the tightening of a financial constraint creates an investment wedge that prevents agents from taking advantage of high-return investment opportunities. In contrast, according to the “demand shock” view, the contraction in investment is voluntary, as low demand lowers the expected return to capital.

This model falls somewhere in between. Mechanically, the shock is similar to a credit shock, or a fall in the underlying collateral value of physical capital. Furthermore, similar to the credit shock view, this model emphasizes
a malfunction in the financial system. However, the shock in this model hinders the financial system’s ability to create money-like assets, rather than its ability to extend loans. Similar to the demand shock view, the contraction in investment is voluntary (and even efficient): the incentives to accumulate physical capital are lower, as capital can no longer be used as collateral to back liquid claims.\(^3\)

Another prominent view of the crisis is the bubble-burst view (as in Martin and Ventura [2012]). The shock to private liquidity creation highlighted in this model shares the idea that a coordination failure among agents is at the core of the crisis: agents no longer expect capital-backed claims to provide liquidity services, so they cease to be liquid. However, the important difference between this model and models of rational bubbles is in the welfare implications. While rational bubbles are usually welfare-improving, “money creation” activities such as those studied here are typically welfare-reducing, due to the fundamental neutrality of money.

This paper is most closely related to others that emphasize the contraction in money-like assets, such as Lucas and Stokey [2011], Gorton [2010], Caballero and Farhi [2013], Guerrieri and Lorenzoni [2009] and Midrigan and Philippon [2011]. This literature mostly studies the mechanics of the shock and the short-run dynamics that follow; here, the emphasis is on long-run trends, and there are no short-run distortions to output associated with an unexpected, sharp contraction in the supply of money-like assets. In this sense, this paper is complementary to previous literature, as it studies the extent to which a shock to money-like assets can account for macroeconomic behavior around the crisis, without imposing any price stickiness or other non-neutralities. Indeed, the model grossly underestimates the response of output and consumption to the shock, leaving plenty of room for other potential frictions that are relevant in the short-run.

The focus on money substitutes is shared with Gorton and Metrick [2010],

\(^3\)This feature of the model is similar to Midrigan and Philippon [2011].
Krishnamurthy and Vissing-Jorgensen [2012] and Stein [2012]. Gorton and Metrick [2010] provide a detailed account of how the financial system was able to use securitized bonds as money; this account is a useful microfoundation for the reduced-form model of private money creation used in this paper. Stein [2012] presents a model that relates private money creation to excessive incidence of fire sales. Similar to the model used here, Stein [2012] and Krishnamurthy and Vissing-Jorgensen [2012] use a money-in-the-utility-function model, in which “money” represents liquidity services derived from liquid assets denominated in real terms; however, unlike the model used here, the liquid assets enter the utility function directly. In this model, agents do not derive utility from liquid assets directly but rather from the liquidity services that they produce, which depend crucially on their equilibrium price (in the same way that liquidity services from money depend on the nominal price level).

In terms of pre-crisis dynamics, the mechanisms emphasized in the model are closely related to the “asset shortage” view, summarized in Caballero [2006] and Caballero et al. [2008]. Subsequent work such as Caballero and Krishnamurthy [2009] and Maggiori [2008] observe that the surplus of emerging economies is composed primarily of treasuries and other safe assets, and relate the demand for assets to a demand for safety. In this paper, safety is de-emphasized, and the focus is on liquidity. In studying the composition of “global imbalances”, Gourinchas and Rey [2007] find that liquidity is an important property of US assets held by foreigners. Of course, the views are not mutually inconsistent as safety and liquidity often go together (see Krishnamurthy and Vissing-Jorgensen [2012] for the case of treasuries). However, the focus on liquidity is key for the welfare implications explored here.
3 Model

Consider a discrete time infinite horizon model, where time periods are indexed by $t = 0, 1, 2, \ldots$. There are two countries: a developed market (subscript $i = d$) and an emerging market (subscript $i = em$). In each economy, there is a measure $\rho_i$ of households with the following preferences defined over final-good consumption ($c_{i,t}$) and liquidity services ($m_{i,t}$):

$$U(\{c_{i,t}, m_{i,t}\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_{i,t}, m_{i,t})$$

where $u$ is a strictly increasing and concave function that is homogeneous of some degree $h$, and satisfies $\frac{\partial^2 u}{\partial c \partial m} \geq 0$. Section 3.1 describes the production and allocation process of the final good, and section 3.2 does the same for liquidity services. Section 3.3 defines the equilibrium concepts in this economy.

3.1 The final good

The final good is produced using capital ($k_{i,t}$), according to the production function:

$$y_{i,t} = A_i k_{i,t}^\alpha$$

where $\alpha \in (0, 1)$, and $A_d \geq A_{em} > 0$. Capital depreciates at a rate $\delta$, and capital accumulation follows:

$$k_{i,t+1} = (1 - \delta)k_{i,t} + i_{i,t}$$

where $i_{i,t}$ is investment.

Intertemporal trade. There is an international bond market for intertemporal trade in the final good. The bonds held by the representative household in country $i$ are denoted $b_{i,t}$. The world interest rate is denoted by $r_t$. The
bond market clearing condition is:

\[ \sum_{i=d,em} \rho_i b_{i,t} = 0 \]  

(4)

Bilateral transfers. There is a pre-determined sequence of bilateral transfers of the final good, \( T_{i,t} \). The purpose of introducing these transfers is methodological: it allows for the welfare comparison of equilibrium allocations net of redistribution effects. The bilateral transfers are, of course, in zero net supply (\( \sum_{i=d,em} \rho_i T_{i,t} = 0 \)).

3.2 Liquidity services

Agents derive liquidity services from holding money \( (M_{i,t}) \). Money does not impact the agent’s utility directly; rather, agents value money because others value it, a property which allows it to be used for transaction purposes. The extent to which a unit of money helps facilitate transactions naturally depends on the price at which agents are willing to exchange the final good for money. The value of liquidity services is then specified as a function of the purchasing power held by the agent:

\[ m_{i,t} = \frac{M_{i,t}}{p_{i,t}} \]  

(5)

where \( p \) is the price of the final good in terms of money.

For the time being, assume that each agent supplies \( M_{i,t}^s \) units of money. The agent’s problem is given by:

\[ \max_{c_{i,t},M_{i,t},k_{i,t+1},b_{i,t+1}} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}, \frac{M_{i,t}}{p_{i,t}}) \]  

(6)

s.t.

\[ p_{i,t}(c_{i,t} + k_{i,t+1} + b_{i,t+1}) + M_{i,t} = \]  

(7)
\[ p_{i,t}(A_i k_{i,t}^\alpha + (1 - \delta)k_{i,t} + (1 + r_t)b_{i,t} + T_{i,t}) + M^*_i \]

This framework departs slightly from the standard money-in-the-utility function model, in that the decision to hold money is intratemporal rather than intertemporal.\(^4\) However, the main departure of this model from the standard framework is in the specification of the money supply, and more broadly, in the interpretation of “money”.

**Money supply** \((M^*)\). In the standard framework, “money” is interpreted as nominal bills which are supplied by the central bank. Obviously, the appropriate specification of the money supply depends on the assets that the economy uses for transaction purposes. For example, in economies in which gold is used as a medium of exchange, the supply of money is determined by the supply of gold. The case of gold is instructive because, in addition to its role as a medium of exchange, gold has an independent consumption value, as it is valued for its beauty and can be used for making jewelry. In this case, it makes sense to think of gold as a “bundle” that includes some consumption goods and some money.

Here, I will assume that the supply of money is determined by the stock of liquid assets. Similar to the case of gold, liquid assets are valued both as claims on the final consumption good, and as a medium of exchange. As in the case of gold, it will be convenient to think of liquid assets as consisting of one (illiquid) claim on the final good, and one unit of money.

To elaborate on this idea, it is useful to think of a financial system, in which agents trade only in assets denominated in real terms. There are problems of “double coincidence of wants” in the financial market: each financial trader can supply assets with certain characteristics, but demands

\(^4\) In the standard model, money serves both as a medium of exchange and as a store of value (agents choose \(M_{i,t+1}\) at time \(t\)). In this case, the cost of holding money is related to the rate of return on money, which - if different from the rate of return on bonds - may distort the consumption of liquidity services (see Friedman [1969]). Here I chose to simplify this margin and assume that agents can choose their money holding within a period.
assets with other characteristics. Finding a trading partner that is willing to exchange one illiquid asset for another may be hard or impossible. In this context, certain assets emerge as mediums of exchange. For example, rather than trading illiquid asset $a$ for illiquid asset $b$, the trader can sell asset $a$ in exchange for treasury bills (or other highly liquid assets), and then use these assets to buy asset $b$ when the opportunity arises.⁵

Of course, in practice, such trades are often denominated in money: the trader sells asset $a$ in exchange for money, and then immediately uses that money to buy liquid assets. When the opportunity to buy asset $b$ arises, the agent sells the liquid assets for money and uses the money to buy asset $b$. As money is held for infinitesimal amounts of time, we can think of liquid assets as serving directly as the medium of exchange. Liquid assets are valued both because they promise to deliver units of the final good, but also - similar to standard money - because others value them, and they can thus be traded easily at no loss to facilitate the transaction process. Similarly, we can think of repo transactions in these terms: the trader can use liquid assets as collateral to raise money for buying asset $b$. After selling asset $a$, the trader can repay the loan and retake possession of the collateral. Liquid assets that can be used as collateral in the repo market - such as mortgage backed securities prior to the crisis - also play the role of money.

Formally, the supply of money is assumed to be proportional to the supply of liquid assets. Liquid assets are single-period IOU notes, issued at the beginning of the period and redeemed at the end of the period. There are two types of liquid assets: government-backed liquid assets (e.g., treasuries), and capital-backed liquid assets (e.g., mortgage backed securities).

The government can create liquid IOU notes against a fraction $\theta_i > 0$ of the economy’s output:

$$ l_{i,t}^g = \theta_i y_{i,t} \tag{8} $$

where $l_{i,t}^g$ denotes the stock of publicly-backed liquid IOU notes. These claims

⁵See Starr [2003] for a related model.
are distributed to households in a lump-sum fashion, and redeemed with lump-sum taxation.\footnote{\cite{7} It is possible to consider a richer framework in which the government chooses \( l_{i,t}^g \) subject to the constraint \( l_{i,t}^g \leq \theta_i y_{i,t} \). It will become evident that in this framework, \( l_{i,t}^g = \theta_i y_{i,t} \) is always weakly optimal from the global planner’s perspective: in the developed economy, this discourages the creation of privately-backed liquid claims to the extent possible, which is desirable since publicly backed claims provide the same service at a lower real cost. In emerging economies, this choice reduces the payment for imported liquidity services to the extent possible.} It will typically be assumed that \( \theta_d \geq \theta_{em} \).

In addition, private agents in the developed economy can issue liquid IOU notes against a fraction \( \gamma \geq 0 \) of their capital stock. The assumption is that the technology to transform capital into liquid claims is available only to developed economies. However, for notational purposes, it will be convenient to introduce the notation \( \gamma_d = \gamma \) and \( \gamma_{em} = 0 \).

To summarize, the supplies of liquid IOU notes at time \( t \) is given by:

\[
l_{d,t} = l_{d,t}^g + \gamma_d k_{d,t} = \theta_d y_{d,t} + \gamma k_{d,t} \tag{9}
\]

\[
l_{em,t} = l_{em,t}^g + \gamma_{em} k_{em,t} = \theta_{em} y_{em,t} \tag{10}
\]

Similar to the case of gold, each liquid IOU note can be thought of as a bundling of an illiquid claim on the final consumption good and a unit of the money-like asset. Thus, the supply of money-like assets is given by:

\[
M_{i,t}^s = l_{i,t} = l_{i,t}^g + \gamma_i k_{i,t} \tag{11}
\]

**The liquidity premium.** Note that \( p_{i,t} \) is not a “dollar” price; rather, it is the price of the final good in terms of the money-like asset. In this framework, the correct interpretation of \( p_{i,t} \) is the inverse of the liquidity premium. The liquidity premium can be calculated by comparing the price of a liquid IOU note to the price of an illiquid IOU note, that promises to deliver a unit of the final consumption good but does not provide any liquidity services. The additional liquidity benefits associated with the liquid IOU note are priced
at \( \frac{1}{p_{i,t}} \), the value of the “money” component of the liquid IOU note. Thus, the price of a liquid IOU note is \( 1 + \frac{1}{p_{i,t}} \) illiquid IOU notes. The liquidity premium is \( \frac{1}{p_{i,t}} \).

**Microfoundations.** Appendix A presents a model in which money (or liquid IOU notes) facilitates intra-period trade between agents who are subject to idiosyncratic demand shocks. Consumption takes place at the end of the period, but only a fraction of the agents have a positive marginal utility of consumption. Agents receive sequential information on their end-or-period demand, and trade based on that information. Agents who expect not to value consumption are willing to sell claims on the final good in exchange for the money-like asset, because they have confidence that, if new information arrives, they will be able to sell the asset at no loss. In this framework, money has a role in facilitating risk sharing across agents. I show that this model can be written in reduced form as the money in the utility function model presented here.

### 3.3 Equilibrium concepts

Each household takes \( r_t \) and \( p_{i,t} \) as given, and solves the following optimization problem:

\[
\max_{c_{i,t}, m_{i,t}} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}, m_{i,t})
\]

subject to \( k_{i,0}, r_0, b_{i,0}, T_{i,t}, \) equations 3, 5 and:

\[
\sum_{t=0}^{\infty} \frac{c_{i,t} + i_{i,t} + k_{i,t}}{\prod_{\tau=0}^{t}(1 + r_{\tau})} = \sum_{t=0}^{\infty} A_i k_{i,t}^{\alpha} + T_{i,t} + \frac{i_{i,t} + \gamma_i k_{i,t}}{p_{i,t}} + (1 + r_0) b_{i,0}
\]

In this specification, the only assets traded are the money-like assets and bonds; the within-period IOU notes are redundant in the household’s prob-
lem. Note that \( \frac{M^s_{i,t}}{p_{i,t}} = \frac{l^g_{i,t}}{p_{i,t}} + \gamma k_{i,t} \) is the market value of the agent’s supply of the money-like asset.

It will be useful to introduce two equilibrium concepts: a segmented equilibrium in which the market for liquid claims clears domestically, and an integrated equilibrium in which there is international trade in liquid claims (note that inter-temporal trade is allowed in both types of equilibria). Formally, the equilibria are defined as follows:

**Definition 1** 1. An equilibrium of the segmented economy is a sequence of interest rates \( r_t \), prices \( p_{i,t} \), final good consumption \( c_{i,t} \), output \( y_{i,t} \), investment \( i_{i,t} \), capital \( k_{i,t+1} \), bonds \( b_{i,t} \), holdings of the money-like asset \( M_{i,t} \), supplies of the money-like asset \( M^s_{i,t} \), publicly backed liquid claims \( l^g_{i,t} \), and liquidity services \( m_{i,t} \) that jointly satisfy equations 2-4, 8, 11, the households’ optimization problems, and the following domestic market clearing condition for liquid claims:

\[
M_{i,t} = M^s_{i,t} \tag{14}
\]

2. An equilibrium of the integrated economy is defined analogously, with the difference that the market clearing condition for liquid claims is given by:

\[
\sum_{i=d,em} \rho_i (M_{i,t} - M^s_{i,t}) = 0 \tag{15}
\]

Before presenting the theoretical results, two equilibrium conditions are worth emphasizing. The first is the household’s optimality condition with respect to \( k_{d,t+1} \):

\[
u_c(c_{d,t}, m_{d,t}) = \beta u_c(c_{d,t+1}, m_{d,t+1})(\alpha A_d k_d^{\alpha-1} + 1 - \delta + \frac{\gamma}{p_{d,t+1}}) \tag{16}
\]

\(^7\)To see this, note that a unit of the final good is equivalent to an illiquid IOU note (there is no discounting within a period), and a liquid IOU note is a bundle of money and an illiquid IOU note.
This condition departs from the standard Euler equation in the last term, \( \frac{\gamma}{p_{d,t+1}} \). This “investment wedge” reflects the component of investment returns that is due to the ability to issue capital-backed liquid claims. A higher liquidity premium increases the incentives to accumulate capital.

The second condition worth emphasizing is the household’s intratemporal optimality condition with respect to the choices of \( c_{d,t} \) and \( M_{d,t} \):

\[
u_c(c_{d,t}, m_{d,t}) = u_m(c_{d,t}, m_{d,t}) = u_m(c_{d,t}, \frac{M_{d,t}}{p_{d,t}})
\]  

This standard condition equates the marginal utility from consuming the final good with the marginal utility of consuming liquidity services (note that the relative price of \( c \) and \( m \) is always 1, since \( m = \frac{M}{p} \)). In this context, this standard optimality condition has strong implications for neutrality: regardless of the stock of money-like assets (\( M \)), the liquidity premium will adjust so that the above condition holds. In other words, the stock of liquid claims does not matter for the equilibrium consumption of liquidity services. The implication is that efforts taken by price-taking agents in order to increase \( M \) will, in general, prove socially wasteful, as they will be countered in equilibrium by a proportional increase in \( p \).

4 Theoretical results

This section presents two sets of results. The first compares the segmented equilibrium with the integrated equilibrium, and the second studies the long-run implications of a shock that sets \( \gamma = 0 \).

It is useful to begin with the following lemma:

\[\text{(17)}\]

\[\text{It is perhaps worth noting that this key implication presents an important departure from models that emphasize safety over liquidity. It is unlikely that the insurance services provided by safe assets diminish if their supply increases. However, the scarcity of liquid claims is central to their ability to provide liquidity services, since it determines their purchasing power in terms of the final good.}\]
Lemma 1 Consider an economy characterized by \( k_{i,0} > 0, T_{i,t} \) and \( b_{i,0} \), for \( i = d, em \). For \( \beta \to 1 \), both the segmented equilibrium and the integrated equilibrium converge to steady states, in which \( c_i, k_i, M_i, p_t, \) and \( m_i \) are constant.

The proof of the above lemma, together with other omitted proofs, is in the appendix. The existence of a steady state is less straightforward than in the standard neoclassical growth model, because a steady state solution for \( k_d \) is not guaranteed for any \( \beta \) (only for \( \beta \) sufficiently large).

The following set of results provides a useful comparison between the integrated and segmented equilibria:

Proposition 1 Consider an economy characterized by \( A_{i,0}, k_{i,0}, T_{i,t} r_0 \) and \( b_{i,0} \), for \( i = d, em \), as well as \( \theta_d \) and \( \gamma_d \). For \( \theta_{em} \) sufficiently small and \( \beta < 1 \) sufficiently large,

1. The steady state value of \( p_d \) is lower under the integrated equilibrium than under the segmented equilibrium.

2. The steady state value of \( k_d \) is higher under the integrated equilibrium than under the segmented equilibrium (and the steady state value of \( k_{em} \) is the same).

\[ \alpha A_d k_d^{\beta-1} + 1 - \delta + \frac{\gamma}{p_d} = \frac{1}{\beta} \tag{18} \]
\[ u_c(c_d, m_d) = u_m(c_d, m_d) \tag{19} \]

The steady state condition for capital implied by intertemporal maximization depends on the steady state value of the liquidity premium \( \frac{1}{p_d} \), which is pinned down by the household’s optimality condition with respect to liquid claims. In principle, these two equations may not have a solution: a higher \( k_d \) may lead to more consumption, and more consumption implies a higher liquidity premium (because of complementarity between \( c \) and \( m \)), which further increases the incentives to accumulate capital. The restriction \( \beta \to 1 \) guarantees that there is a solution because an increase in capital does not lead to a large increase in consumption (when \( \beta \to 1 \), the steady state value of \( k_d \) approaches its consumption maximizing level).
3. Welfare in em is lower under the integrated equilibrium.

4. With appropriate transfers, both economies can be made better off by choosing the segmented equilibrium over the integrated equilibrium.

These results indicate that a transition from a segmented equilibrium to an integrated equilibrium should be accompanied by several trends. From the perspective of the developed economy, integration is associated with an increase in the steady state liquidity premium \((\frac{1}{p_d})\). This has two effects: first, it increases the value of domestically-issued liquid claims, and increases steady state wealth (in terms of the final good). Second, the higher steady state liquidity premium implies a higher premium associated with capital-backed liquid claims; thus, the capital level in the developed economy converges to a higher steady state level.

The last two results of the proposition relate to the welfare properties of integration. Emerging economies are made worse off by integration; \(p_{em}\) increases, and thus the equilibrium value of \(em\)-issued liquid claims declines, resulting in lower wealth. The intuition for this welfare result is related to the notion of international seigniorage payments: liquid claims issued in \(d\) substitute for domestically issued liquid claims, and lead to “inflation” in their value. While in the segmented equilibrium, all rents from supplying liquidity services to the domestic economy are absorbed domestically, in the integrated equilibrium there are some rents absorbed by foreign liquidity suppliers.

Finally, the proposition states that, with appropriate transfers, both economies can be made better off by segmenting liquidity markets. This welfare result relates to the notion of wasteful creation of money substitutes. The increase in investment in \(d\) for the purpose of creating capital-backed liquid claims is analogous to investment in “money printing” machines, or “gold digging”: privately, since money-like assets are valued, increasing the supply of money-like assets is associated with rents. However, from a social
perspective, this merely reduces the equilibrium purchasing power of money-
like assets and does not produce additional liquidity services.

The second set of results studies the comparative statics of the model with
respect to $\gamma$. The value of $\gamma$ determines the degree to which the private capital
stock in the developed economy can be used to back liquid claims. Note that,
in this model, there is a strong coordination element associated with money-
like assets: since the liquidity services that they produce depend on the
equilibrium price $p$, they are, in principle, subject to multiple equilibria. In
particular, agents value money-like assets only because others value money-
like assets; if agents expect a subset of these assets to lose their liquidity
properties, they will cease to be money-like. In this sense, money-like assets
are similar to rational bubbles (or pure fiat money). A shock that sets $\gamma = 0$
can be thought of as a coordination shock, in which capital-backed claims
lose their liquidity properties.

**Proposition 2** Consider an economy characterized by $A_{i,0}$, $k_{i,0}$, $T_{i,t}$, $r_0$ and
$b_{i,0}$, for $i = d, em$, as well as $\theta_d$ and $\gamma_d$. For $\theta_{em}$ sufficiently small and $\beta < 1$
sufficiently large, both in the segmented equilibrium and in the integrated
equilibrium,

1. The steady state value of $p_d$ is increasing in $\gamma$.

2. The case $\gamma = 0$ minimizes the equilibrium value of $k_{d,t+1}$ for all $t$, as
well as the steady state value of $k_d$.

3. The case $\gamma = 0$ maximizes welfare: with appropriate transfers, both
economies can be made better off by imposing $\gamma = 0$.

The first part of the proposition states that $\frac{1}{p_d}$, the steady state liquidity
premium in the developed economy, is decreasing with $\gamma$. Intuitively, a higher
$\gamma$ means that liquid claims are more abundant, and hence their premium is
lower. This insight implies that a permanent reduction in $\gamma$ is associated
with a higher liquidity premium at the steady state.
The second part of the proposition establishes that $\gamma = 0$ minimizes investment in the developed economy. When $\gamma = 0$, the liquidity premium does not affect the private returns to capital, and the marginal product of capital is equated across countries. The elimination of the positive “investment wedge” in $d$ reduces its capital stock in all periods.\footnote{In general, investment may be non-monotone in $\gamma$: for large values of $\gamma$, liquid claims are highly abundant and the liquidity premium is low - thus, the ability to use capital to create privately backed liquid claims may not imply a large incentive to invest in additional capital.}

The final part of the proposition establishes that, with appropriate transfers, imposing $\gamma = 0$ can lead to a Pareto improvement. It is easy to verify that, when $\gamma = 0$, the equilibrium path corresponds to the standard neoclassical growth model in terms of consumption and investment. The intuition for this welfare result relates again to the inefficiency of private money creation: when $\gamma > 0$, there is a wedge between the private returns to investment and the social returns to investment, that results from the private incentives to create liquid claims. Imposing $\gamma = 0$ is efficient in the same way that a government monopoly on money printing is efficient: the government has an advantage in creating money-like assets because it internalizes their neutrality. In contrast, private agents have an incentive to spend real resources on creating money substitutes because of the private seigniorage rents that they absorb.\footnote{In general, since $\{k_{d,t+1}\}_{t=0}^\infty$ are not necessarily monotone in $\gamma$, neither is welfare. A change in $\gamma$ increases welfare only when it decreases investment in $d$.}

\section{Numerical illustration}

This section studies the model’s potential to quantitatively account for the pre-crisis investment boom and the post-crisis dynamics. In the model, the initial shock leading to an investment boom is the integration of liquidity markets. Of course, in practice, the integration of liquidity markets was not a shock but rather a gradual process. However, after the Asian crisis...
of 1998, many emerging economies have increased their treasury holdings dramatically.\footnote{See Beltran et al. [2013] for an in-depth analysis of the evolution of foreign treasury holdings since the mid 1990s.} I thus use 1990-1998 as a reference point for the segmented steady state, and date the initial “integration” shock at 1998.

The crisis is modeled as a shock to the liquidity value of privately-issued claims that sets $\gamma = 0$. To be roughly consistent with the onset of the crisis, I date this shock at 2007, though the liquidity shock was at its peak in 2008-9.

It is useful to compare the calibration results with their empirical counterparts. The empirical counterparts of $d$ and $em$ are chosen as the US and the BRIC economies (Brazil, Russia, India and China). To construct empirical counterparts for capital, I use data for fixed capital formation in constant prices from the world development indicators (WDI).\footnote{I construct aggregate capital levels using a depreciation rate of 0.1, and initial conditions corresponding to 10 times investment in the first year in which data is available, which is 1961 for the US and India, 1970 for Brazil, 1965 for China and 1990 for Russia. Since there are insufficient observations for Russia, I construct the BRIC aggregate as the sum of Brazil, China and India.} I compute the growth rates of the capital stock and de-trend the series by subtracting the average growth rate during 1990-1998.\footnote{The average growth rates are 6.3% for the Brazil, China and India aggregate and 3.4% for the US.}

To construct empirical counterparts for consumption, I use WDI data on consumption in constant prices. I compute growth rates for the US and for the average of BRIC countries, and de-trend the series by subtracting the average growth rates between 1991-1998.\footnote{Data for Russia is available only starting 1991.} To construct empirical counterparts for net exports as a percent of GDP, I use WDI data on imports, exports and GDP, all in constant prices.

While the construction of empirical counterparts for the real variables is fairly straightforward, the construction of empirical counterparts for the liquidity variables requires some thought. The literature on the measurement of liquidity premiums typically constructs relative liquidity premiums...
by comparing assets with similar risk but different liquidity properties (see, for example, Krishnamurthy and Vissing-Jorgensen [2012] for the case of treasuries). However, in this model, \( \frac{1}{p} \) is an *absolute* liquidity premium, that relates the price of a completely illiquid asset to the price of a completely liquid asset.

To construct an empirical counterpart for the liquidity premiums, I use the following procedure: (a) I compute an implied interest rate on illiquid assets based on the Euler equation relation \( \left( \frac{c_{t+1}}{c_t} = 1 + r_t \right) \), using realized consumption growth rates; (b) Then, I compute a real interest rate on liquid assets and compute a liquidity premium by subtracting the “liquid” interest rate from the “illiquid” interest rate. Since it is not obvious how to aggregate liquidity premiums, I compute the liquidity premiums for China only, instead of a BRIC aggregate. To compute the interest rate implied by the Euler equation relationship, I use WDI data on per-capita consumption (in constant prices) and a discount factor of \( \beta = 0.98 \). To compute the interest rate on liquid assets in the US, I use the real 1 year treasury rate.\(^{16}\) For China, I use the real deposit rate.\(^{17}\) The resulting series for the liquidity premium in the US is relatively smooth, and is reported at annual levels; for China, the series is very volatile, reflecting perhaps higher variability in consumption growth. To abstract from these fluctuations, I report averages for the periods 1986-1998,\(^{18}\) 1999-2006 and 2007-2011.

Unfortunately, there is no straightforward way to construct empirical counterparts for liquidity services \((m)\). Since the focus is on money substitutes used by the financial system, the standard money aggregates are

---

\(^{16}\)I use 1-Year Treasury Constant Maturity Rate (DGS1), Percent, Annual, Not Seasonally Adjusted from FRED (DGS1; annual frequency). I then subtract the 1 year inflation rate by using Consumer Price Index for All Urban Consumers: All Items (CPI-AUCSL), Percent Change from Year Ago, Annual, Seasonally Adjusted from FRED (CPI-AUCSL_PC1).

\(^{17}\)I use data on the deposit rate and the CPI from the WDI, and construct the real liquid rate as the deposit rate minus the inflation rate.

\(^{18}\)the series can be constructed starting 1986.
basically irrelevant. An ideal empirical counterpart would be an aggregate of all assets traded by financial firms, weighted by their liquidity premiums. While a construction of this aggregate is of independent importance, it is beyond the scope of this paper. The simulation results for liquidity services are presented without empirical counterparts.

Table 1: Calibration parameters

<table>
<thead>
<tr>
<th>Notation</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u(c, m)$</td>
<td>Utility</td>
<td>$-\ln\left(\frac{1}{c} + \frac{0.0025}{m}\right)$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.98</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in production</td>
<td>0.33</td>
</tr>
<tr>
<td>$A_d$</td>
<td>Productivity: developed economies</td>
<td>1</td>
</tr>
<tr>
<td>$A_{em}$</td>
<td>Productivity: emerging economies</td>
<td>0.4</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>Size of developed market</td>
<td>1</td>
</tr>
<tr>
<td>$\rho_{em}$</td>
<td>Size of emerging market</td>
<td>10</td>
</tr>
<tr>
<td>$\theta_d$</td>
<td>Public backing as a share of output: $d$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\theta_{em}$</td>
<td>Public backing as a share of output: $em$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Private liquidity as a share of capital: $d$</td>
<td>0.078</td>
</tr>
<tr>
<td>$\frac{b_d}{y_d}$</td>
<td>Segmented steady state NX (% GDP): $d$</td>
<td>-1.7%</td>
</tr>
<tr>
<td>$\frac{b_{em}}{y_{em}}$</td>
<td>Segmented steady state NX (% GDP): $em$</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

Table 1 summarizes the parameters chosen for the calibration. The parameters $\beta$, $\delta$, and $\alpha$, are assigned standard values. The choice of $A_d = 1$ is a convenient normalization. The productivity of the emerging market is calibrated based on Hsieh and Klenow [2009], who estimate manufacturing productivity in the US to be about 150% higher than in India and China.19

19 This figure also roughly matches the figure in Caselli [2013], who conducts a careful development accounting exercise and concludes that TFP in Latin America is about 50% of that of the US. While India, Russia and China may currently have lower TFP levels for the aggregate economy, these economies have also been experiencing fast TFP growth. Since the focus here is on long-run trends, the TFP gaps implied by the current gap in manufacturing and the TFP gap between the US and Latin America seem like a reasonable benchmarks.
The preferences over final good consumption and liquidity services follow the specification in Lucas [2000], with an intertemporal elasticity of substitution of 1. Lucas [2000] calibrates this functional form using the relationship between nominal interest rates and money demand in the US over the last century. Of course, the notion of money used in Lucas [2000] is different from that used in this paper. The use of this functional form is valid only under the assumption that currently used money-like assets substitute directly for money in its traditional usage.

Similarly, under the assumption that the government’s ability to create liquid claims is time invariant, I calibrate the values of \( \theta \) to match the historical averages of M3 over GDP. Prior to the surge in the use of money substitutes in the financial system, the government’s creation of liquidity was limited to the creation of M3. Historical values of M3 over GDP therefore provide some (imperfect) measure of \( \theta \).

I use the following procedure to calibrate \( \gamma \). According to Gorton et al. [2012], privately backed claims (including MBS, ABS and corporate bonds and loans) accounted for about 25% of the supply of liquid claims in the US in 2006. I choose \( \gamma \) to match this proportion in the integrated steady state.

I use the average trade balance in the US between 1990-1998 to calibrate the segmented steady state value of \( b_{s}\), and the market clearing condition to calibrate the corresponding level of \( b_{em} \).

\[^{20}\text{For the US, the average value of M3 as a percent of nominal GDP is about 70\% between 1961-1990. It is worth noting that } \frac{M3}{PY} \text{ in the US is essentially trend-less in this period, despite the fact that the US has experienced high GDP growth. This feature of the data is consistent with the assumption that } \theta \text{ is a time invariant parameter that does not change with GDP. Unfortunately, data for China is available only for the years 1977-1982. For these years, M3 was 30\% of nominal GDP on average. More data is available for India. Between 1977 and 1991, the average M3 over nominal GDP in India was 38\%. Pre-1990 data for Brazil and Russia is unavailable. Source: World Development Indicators.}\]

\[^{21}\text{Source: World Development Indicators.}\]

\[^{22}\text{Compared to the average trade surplus during 1990-1998, the steady state value of } b_{em} \text{ implied by the market clearing condition is lower for China and Russia, and higher for India and Brazil.}\]
5.1 The integration of liquidity markets

As a first quantitative exercise, I consider the implications of the global integration of liquidity markets, starting from the segmented steady state. I use Dynare to compute the deterministic convergence path.\(^23\)

The steady state values of the segmented and integrated equilibria, given the calibrated parameters in table 1, are summarized in the first two columns of table 2 (the third column presents steady state values for a permanent shock to the liquidity of capital-backed claims studied in the next section). The fourth column presents the ratio of integrated and segmented steady state variables. Figures 1 and 2 describe the transitional dynamics of the integrated equilibrium, starting from the segmented steady state.

Table 2: Steady state comparison

<table>
<thead>
<tr>
<th>Variable</th>
<th>Segmentation</th>
<th>Integration</th>
<th>Shock</th>
<th>Segmentation</th>
<th>Integration</th>
<th>Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{em} )</td>
<td>0.302</td>
<td>0.299</td>
<td>0.3</td>
<td>0.99</td>
<td>1.003</td>
<td></td>
</tr>
<tr>
<td>( c_d )</td>
<td>1.21</td>
<td>1.24</td>
<td>1.23</td>
<td>1.025</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>( m_{em} )</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
<td>0.99</td>
<td>1.003</td>
<td></td>
</tr>
<tr>
<td>( m_d )</td>
<td>0.06</td>
<td>0.062</td>
<td>0.061</td>
<td>1.025</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>( k_{em} )</td>
<td>1.147</td>
<td>1.147</td>
<td>1.147</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( k_d )</td>
<td>4.68</td>
<td>4.83</td>
<td>4.5</td>
<td>1.03</td>
<td>0.933</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{p_{em}} )</td>
<td>0.1</td>
<td>0.07</td>
<td>0.08</td>
<td>0.68</td>
<td>1.153</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{p_d} )</td>
<td>0.04</td>
<td>0.07</td>
<td>0.08</td>
<td>1.77</td>
<td>1.153</td>
<td></td>
</tr>
<tr>
<td>( \frac{\nu_x}{y}_{em} )</td>
<td>0.004</td>
<td>0.01</td>
<td>0.009</td>
<td>2.58</td>
<td>0.812</td>
<td></td>
</tr>
<tr>
<td>( \frac{\nu_x}{y}_d )</td>
<td>-0.01</td>
<td>-0.027</td>
<td>-0.022</td>
<td>2.56</td>
<td>0.83</td>
<td></td>
</tr>
</tbody>
</table>

From the perspective of the developed economy, the steady state liquidity premium (\( \frac{1}{p_d} \)) is about 3% higher; from the perspective of the emerging economy, the steady state liquidity premium is about 3% lower. Following the initial convergence, there is a slight decline in the liquidity premium, due

\(^{23}\)Throughout this analysis, I use the *simul* command to compute the deterministic convergence path.
Levels of $c$ and $k$ are normalized to match initial conditions.

Figure 1: The dynamics associated with the unanticipated integration of liquidity markets at $t = 1998$, starting from the segmented steady state: final good variables $(c, k, \frac{NX}{Y})$

Figure 2: The dynamics associated with the unanticipated integration of liquidity markets at $t = 1998$, starting from the segmented steady state: liquidity variables $(m, \frac{1}{p})$
to the accumulation of capital-backed liquid claims in the developed economy. The convergence in liquidity premiums is qualitatively consistent with observed trends (and the average liquidity premiums in China match the calibration remarkably well). However, liquidity premiums in the US seem to be somewhat lower than in China, reflecting perhaps incomplete integration.

Steady state consumption in the emerging economy is about 1% lower in the integrated steady state, while steady state consumption in the developed economy is about 2.5% higher. The increase in consumption in the developed economy reflects higher permanent income due to international liquidity rents. In the emerging economy, consumption drops, reflecting lower permanent income due to lower liquidity premiums associated with domestically issued liquid claims. Qualitatively, these trends are consistent with the data. However, quantitatively, the model under-predicts the consumption boom in the US. The predicted magnitudes for the consumption decline in BRIC are roughly consistent with the data.

Interestingly, consumption of liquidity services \( (m) \) moves proportionally to consumption; the standard intuition from the trade literature would suggest a “Pareto improving” trade, in which emerging economies trade the final good for more liquidity services. However, despite importing liquid claims, the emerging economy ends up consuming less liquidity services, while the developed economy ends up consuming more liquidity services. This counterintuitive result reflects the neutrality of liquid claims: importing additional liquid claims does not increase the value of liquidity services, because the scarcity value of liquid claims declines and with it their purchasing power in terms of the final good.

Upon integration, the developed economy starts accumulating capital, until it converges to its new steady state, which is about 3% higher. This increase corresponds to about a fifth of the growth in capital in the US.\(^{24}\) In

\(^{24}\)This is reasonable given other trends such as the IT revolution which can be thought of as capital-biased technological change. Within the framework of the model, additional factors contributing to the growth in capital may include financial innovation that increases
the model, the incentives to accumulate capital stem from higher liquidity premiums associated with capital-backed claims.

The steady state capital stock in the emerging economy remains unchanged. Initially, there is a modest amount of disinvestment in the emerging economy as a result of higher private returns in the developed economy. The model cannot account for the large observed growth in capital stocks in BRIC economies during the last decade. This may suggest that emerging economies are still far from their steady state capital levels, or that other structural changes are affecting investment.

Finally, the integration of liquidity markets is associated with an initial surge in trade balances. On impact, the high private returns to investment in the developed economy, together with higher permanent income due to international liquidity rents, induce agents in the developed economy to borrow. The trade surplus in the emerging economy reaches around 4% of GDP at its peak, and the trade deficit in the developed economy reaches roughly 8% of GDP at its peak, roughly consistent with its empirical counterpart.\(^{25}\) At the integrated steady state, the emerging economy imports liquid claims from the developed economy. This is reflected in higher steady state trade balances: the emerging economy increases its net exports from 0.4% of GDP to 1% of GDP, while the developed economy increases its net imports from 1% of GDP to 2.7% of GDP.

From a normative perspective, the integration of liquidity markets is associated with a decline in welfare for emerging economies that is equivalent to a permanent proportional reduction of \(c_{em}\) and \(m_{em}\) of about 1%. For the developed economy, integration improves welfare in an amount equivalent to a permanent proportional increase in \(c\) and \(m\) of around 2%. In line with Proposition 1, it is possible to establish that abstracting from distributional implications, both economies can be made better off by staying in the

\(^{25}\)The increase in the trade deficit in \(d\) predicted by the model are larger than those observed in the US, but they are also shorter-lived.
5.2 A permanent shock to private liquidity creation

The second quantitative exercise illustrates the effects of a permanent and unexpected shock to the liquidity properties of capital-backed claims. Formally, I consider an integrated equilibrium in which the value of $\gamma_d$ falls permanently to $\gamma_d = 0$.

Of course, the strong neutrality assumptions imply that the model has limited potential to account for the crisis itself: for example, a shock to $\gamma$ has no direct effect on output or even on liquidity services, as prices adjust instantaneously. This strong neutrality seems counter both to current experience and to past experiences, as sharp contractions in the money supply are often associated with contractions in output as well as long spells of unemployment. This model abstracts from any such potential “short run” effects, and is thus a conservative calibration of the effects of this shock.

The steady state variables associated with the shock are given by the third column of table 2, and figures 3 and 4 present the transitional dynamics associated with the shock, starting from the integrated steady state with $\gamma = 0.078$.

Since the possibility of creating capital-backed liquid claims no longer enters into the incentives to accumulate capital in the developed economy, its steady state capital stock drops by nearly 7%. This figure is broadly consistent the contraction in capital in the US between 2007-2011.

On impact, capital in the emerging economy increases somewhat (but the increase is small). This reflects consumption smoothing in the developed economy during its transition to a lower capital level. While quantitively understated, these dynamics may help account for part of the investment boom in emerging economies following the crisis.

The steady state liquidity premium increases from 7% to 8%, reflecting a contraction in the supply of liquid claims. This contraction results both from
Levels of $c$ and $k$ are normalized to match initial conditions.

Figure 3: The dynamics of the integrated equilibrium associated with an unanticipated and permanent shock to the liquidity of privately issued claims ($\gamma = 0$), at $t = 2007$, starting from the steady state of the integrated equilibrium with $\gamma = 0.078$: final good variables ($c$, $k$, $\frac{NX}{Y}$)
Figure 4: The dynamics of the integrated equilibrium associated with an unanticipated and permanent shock to the liquidity of privately issued claims ($\gamma = 0$), at $t = 2007$, starting from the steady state of the integrated equilibrium with $\gamma = 0.078$: liquidity variables ($m, \frac{1}{p}$)
the disappearance of capital-backed liquid claims, and from the depletion of the stock of publicly issued claims that follows the contraction in output in the developed economy. The 1% increase in liquidity premiums corresponds to about half of the increase in liquidity premiums in the US,\textsuperscript{26} and a third of the increase in liquidity premiums in China.

The higher liquidity premium has a positive wealth effect, as the value of publicly backed claims increases. For the emerging economy, this is the only effect, and steady state consumption increases by about 0.3% following the shock. For the developed economy, there is an additional effect: while the value of publicly backed liquid claims increases, the value of capital-backed liquid claims falls (to 0). The forgone seigniorage revenues and the lower equilibrium capital stock imply a net wealth loss, which is reflected in steady state consumption that is 1% lower. For both the emerging economy and the developed economy, the model qualitatively matches the consumption trends, but quantitatively understates them.

Finally, the shock is associated with a temporary reversal of trade balances, as the developed economy saves abroad in order to smooth the transition to a lower steady state capital stock. While the reversal of trade balances is at odds with the data, it is qualitatively consistent with the contraction in trade balances during the crisis. Furthermore, the long-run levels predicted by the model seem roughly consistent with their empirical counterparts.

In terms of welfare, the model generates an increase in welfare in the emerging economy that is equivalent to a permanent 0.5% increase in $c_{em}$ and $m_{em}$, and a decline in welfare in the developed economy that is equivalent to a permanent 1% decline in $c_d$ and $m_d$. The decline in welfare in the developed economy is a result of redistribution towards the emerging economy; by holding relative consumption levels constant, both economies can be made better off by the shock.

\textsuperscript{26}To gauge the increase in liquidity premiums in the US, I subtract the average premium during 1998-2007 from its peak at 2010. This results in about 2%.
Qualitatively, the dynamics of the model seem consistent with macroeconomic behavior following the crisis. The model can generate a surge in liquidity premiums, as well as a substantial contraction in capital in the developed economy. However, as expected, the model is less successful quantitatively in terms of consumption, as it assumes strong neutrality that abstracts away from any short-run implications of a contraction in the supply of liquid claims.

6 Concluding remarks

This paper illustrates that, when appropriately reinterpreted, a simple monetary model is useful for understanding the macroeconomic dynamics associated with the recent crisis. The model focuses on private money creation through investment, and is able to generate both a pre-crisis investment boom fueled by foreign demand for liquidity, and a large decline in investment following a shock to the liquidity properties of capital-backed claims.

Quantitatively, the model suggests that about a fifth of the increase in the US capital stock prior to the crisis was driven by foreign demand for liquid claims. The role of private money creation is potentially larger, as financial innovation and changes in regulation may have made it easier to create capital-backed money substitutes (for a detailed account of this view, see Gorton and Metrick [2010]).

The model also suggests that the contraction in investment in the US during the recent crisis may have been driven to a large extent by changes in liquidity benefits associated with capital. In the model, a shock to the liquidity properties of capital-backed claims generates a contraction in capital that is quantitatively consistent with observed magnitudes in the US during the crisis. However, the short-run dynamics of consumption remain largely unexplained by the model in its current form, suggesting a role for non-neutralities or other frictions associated with a sharp contraction in the stock
of money-like assets.

In terms of welfare, the model highlights both the social inefficiency associated with private money creation, and its potential distributional implications between emerging and developed economies. In terms of distributional implications, the model suggests that trade in liquid claims is not Pareto improving, as liquidity suppliers absorb seigniorage rents. Abstracting away from distributional implications, minimizing private money creation is socially optimal. Due to the neutrality of money, spending real resources on the creation of money substitutes is socially wasteful. From a global perspective, it is optimal to regulate the use of money substitutes. This may be difficult, as with high liquidity premiums on traditional liquid assets (such as treasuries), there are large rents associated with the creation of new liquid claims.

References


A model with intra-period trading

This appendix illustrates how the money-in-the-utility function model used in this paper can be micro-founded as a model in which agents’ utility depends only on consumption of the final good, and liquid claims are valued because they hold purchasing power that facilitates intra-period trading between agents. I show that this model can be written in reduced form as a discrete period money-in-the-utility-function model.

Consider the following within-period timing of the model. Timing within a period is continuous and is denoted \( \tau \in [0, 1] \), with \( \tau = 0 \) denoting the beginning of a period and \( \tau = 1 \) denoting the end of the period. Output is realized at \( \tau = 1 \), which is also when consumption takes place.

At \( \tau = 0 \), a market opens in which agents trade promises on output, and decide how many investment goods and how many consumption goods to produce and demand.\(^{27}\)

\(^{27}\)The same technology produces both investment and consumption goods; however, the decision whether the good will be used for investment or consumption must be made at \( \tau = 0 \).
There is both aggregate and idiosyncratic liquidity risk: at $\tau = 1$, only a fraction $\eta$ of consumers will value consumption, whereas a fraction $1 - \eta$ will not value consumption. The ex-ante distribution of $\eta$ is characterized by the probability density function $f(\eta)$. The agent’s expected consumption at time $\tau = 0$ is given by:

$$\int_0^1 f(\eta) \frac{1}{\eta} c_t(\eta, \chi = 1) d\eta$$

(20)

The indicator $\chi$ specifies whether the agent values consumption at $\tau = 1$; $c_t(\eta, \chi = 1)$ is the amount of consumption goods that the agent can expect to consume when he has positive demand and the aggregate demand state is $\eta$.

The aggregate state $\eta$ is revealed at $\tau = \frac{1}{2}$, at which point, agents can trade claims on consumption goods in exchange for money. A fraction $\eta$ of agents get a signal $\chi = 1$, and a fraction $1 - \eta$ get a signal $\chi = 0$. With probability $q$, the signals $\chi$ correspond to the final distribution of demand, and there is no further trading; with probability $1 - q$, there is a new independent draw of signals at $\tau = \frac{3}{4}$, and an additional round of trading. With probability $q$, the signals correspond to the final distribution of demand, and with probability $1 - q$, there is an independent draw at $\tau = \frac{3}{4} + \frac{1 - \frac{3}{2}}{2}$, and so on.

The problem from $\tau = \frac{1}{2}$ onwards can be written in recursive form. Denote by $V^n_{\chi=1}(c, M, \tilde{p})$ and $V^n_{\chi=0}(c, M, \tilde{p})$ the continuation values given $c$ claims on consumption, $M$ units of money, an aggregate demand state $\eta$, and signals $\chi = 1$ and $\chi = 0$ respectively, given that the spot price of the consumption good in terms of money is $\tilde{p}$. The continuation values are given by:

$$V^n_{\chi=1}(c, M, \tilde{p}) = \max_{c', \tilde{p}'} qc' + (1 - q)((1 - \eta)V^n_{\chi=0}(c', M', \tilde{p}') + \eta V^n_{\chi=1}(c', M', \tilde{p}'))$$

(21)
s.t. \( c', M' \geq 0 \) and:

\[
\tilde{p}c + M = \tilde{p}c' + M' \tag{22}
\]

And:

\[
V^\eta_{\chi=0}(c, M, \tilde{p}) = \max_{c', M'} (1 - q)((1 - \eta)V^\eta_{\chi=0}(c', M', \tilde{p}') + \eta V^\eta_{\chi=1}(c', M', \tilde{p}')) \tag{23}
\]

s.t. \( c', M' \geq 0 \) and:

\[
\tilde{p}c + M = \tilde{p}c' + M' \tag{24}
\]

Conjecturing an equilibrium in which \( \tilde{p} = p_\eta \) is constant given \( \eta \), it is possible to obtain the following solution:

\[
V^\eta_{\chi=1}(c, M, p_\eta) = (1 - (1 - \eta)(1 - q))(c + \frac{M}{p_\eta}) \tag{25}
\]

and:

\[
V^\eta_{\chi=0}(c, M, p_\eta) = (1 - q)\eta(c + \frac{M}{p_\eta}) \tag{26}
\]

In this case, substituting in the budget constraint, \( V^\eta_{\chi=1} \) solves:

\[
V^\eta_{\chi=1}(c, M, p_\eta) = \max_{M'} q(c + \frac{M - M'}{p_\eta}) + (1 - q)(\eta(1 - (1 - \eta)(1 - q)) + (1 - \eta)(1 - q)\eta(c + \frac{M}{p_\eta})
\]

The optimization results in \( M' = 0 \) and \( c' = c + \frac{M}{p_\eta} \). Similarly, it is possible to show that agents with \( \chi = 0 \) are indifferent with respect to \( c' \) and \( M' \). In equilibrium, claims on consumption are held only by \( \chi = 1 \) agents, who hold no money. At the beginning of each trading period (after \( \tau = \frac{1}{2} \)), \( \eta \) agents hold claims on consumption; each agent holds \( \frac{c}{\eta} \) claims. Of these agents, \( (1 - \eta) \) are willing to sell their claims for money, so the supply of consumption goods is \( \eta\frac{c}{\eta}(1 - \eta) \). Using similar considerations, the nominal
demand for goods is \( (1 - \eta) \frac{M}{(1 - \eta)} \eta \). The market clearing price \( p_\eta \) must satisfy:

\[
\eta M = (1 - \eta) p_\eta c \Rightarrow p_\eta = \frac{\eta M}{(1 - \eta)c}
\]

(28)

At \( \tau = 0 \), agents solve:

\[
V(k, M^s, b, r, T) = \max_{k', c, M} u(\int_0^1 f(\eta) \frac{1}{\eta} (c + \frac{M}{p_\eta}) d\eta) + V(k', (M^s)', b', r', T')
\]

(29)

s.t.

\[
p(c + k' + b') + M = p(Ak^\alpha + (1 - \delta)k + (1 + r)b + T) + M^s
\]

(30)

The first order conditions with respect to \( c \) and \( M \) yield:

\[
\frac{1}{p} \int_0^1 f(\eta) \frac{1}{\eta} d\eta = \int_0^1 f(\eta) \frac{1}{\eta} \frac{1}{p_\eta} d\eta = \frac{M}{c} \int_0^1 f(\eta) \frac{1}{\eta} \frac{1}{1 - \eta} d\eta
\]

(31)

Thus, both \( p_\eta \) and \( p \) are proportional to \( \frac{M}{c} \). In reduced form, we can write:

\[
u(\int_0^1 f(\eta) \frac{1}{\eta} (c + \frac{M}{p_\eta}) d\eta) = u(c, \frac{M}{p_\eta}) = u(c, \frac{M}{p})
\]

(32)

\section*{B Proofs}

\textbf{Existence of steady state.} The first order conditions of the households’ problems are:

\[
u_c(c_{i,t}, m_{i,t}) = u_m(c_{i,t}, m_{i,t})
\]

(33)

\[
u_c(c_{i,t}, m_{i,t}) = \beta u_c(c_{i,t+1}, m_{i,t+1})(1 + r_{t+1})
\]

(34)

\[1 + r_{i,t+1} = \alpha A_i k_{i,t+1}^{\alpha-1} + 1 - \delta + \frac{\gamma_i}{p_{i,t+1}}
\]

(35)
At the steady state, these equations can be written as:

\[ u_c(c_i, m_i) = u_m(c_i, m_i) \]  \hspace{1cm} (36)

\[ u_c(c_i, m_i) = \beta u_c(c_i, m_i)(1 + r) \Rightarrow 1 + r = \frac{1}{\beta} \]  \hspace{1cm} (37)

\[ 1 + r = \frac{1}{\beta} = \alpha A_i k_i^{\alpha - 1} + 1 - \delta + \frac{\gamma_i}{p_i} \]  \hspace{1cm} (38)

Note that the last condition pins down \( k_{em} \) since \( \gamma_{em} = 0 \). Since \( r \) and \( k_{em} \) have unique steady state values, there are 7 steady state values to solve for: \( c_d, c_{em}, M_d, M_{em}, p_d, p_{em} \) and \( k_d \) (\( m_i \) is then determined by \( m = \frac{M}{p} \)).

Note that equation 34 implies that:

\[ \frac{u_c(c_{d,t+1}, m_{d,t+1})}{u_c(c_{d,t}, m_{d,t})} = \frac{u_c(c_{em,t+1}, m_{em,t+1})}{u_c(c_{em,t}, m_{em,t})} \Rightarrow \frac{u_c(c_{d,t}, m_{d,t})}{u_c(c_{em,t}, m_{em,t})} = \Psi \]  \hspace{1cm} (39)

Where \( \Psi \) is such that the intertemporal budget constraint is satisfied for both countries. With this condition, 6 additional steady state conditions are required. In the segmented equilibrium, these are given by equation 36 for \( i = d, em \), equation 38, the goods market clearing condition and the domestic money market clearing conditions. In the integrated equilibrium, instead of two domestic money market clearing conditions there is one global money market clearing condition, but in addition there is a restriction \( p_d = p_{em} \).

Using this, the household’s optimality conditions can be derived from solving the following “pseudo-planner’s” problem:

\[ \max \sum_{t=0}^{\infty} \beta^t (u(c_{d,t}, m_{d,t}) + \Psi u(c_{em}, m_{em})) \]  \hspace{1cm} (40)

s.t. \[ \sum_{i=d,em} \rho_i (c_{i,t} + i_{i,t} - y_{i,t} + m_{i,t}) = \sum_{i=d,em} \rho_i \frac{M_{i,t}^g + \gamma_i k_{i,t}}{p_{i,t}} \]  \hspace{1cm} (41)

Note that the first order conditions of this problem correspond to the house-
holds’ optimality conditions. Using homogeneity, the first order conditions are satisfied when \( \frac{c_{em}}{c_d} = \frac{m_{em}}{m_d} = \lambda \), and \( \lambda^k \Psi = 1 \). The problem can be rewritten as:

\[
\max \sum_{t=0}^{\infty} \beta^t u(c_t, m_t) \tag{42}
\]

s.t.

\[
\sum_{i=d,em} \rho_i(i_t - y_{i,t}) + c_t + m_t = \sum_{i=d,em} \rho_i \frac{M_{i,t}^y + \gamma_i k_{i,t}}{p_{i,t}} \tag{43}
\]

where \( c_t = \sum_{i=d,em} \rho_i c_i \) and \( m_t = \sum_{i=d,em} \rho_i m_i \). At the steady state, \( c = \sum_{i=d,em} \rho_i(y_i - i_i) = \sum_{i=d,em} \rho_i(A_i k_i^\alpha - \delta k_i) \), and \( m = \sum_{i=d,em} \rho_i \frac{\theta_i A_i k_i^\alpha + \gamma_i k_i}{p_i} \).

The steady state values of \( c \) and \( m \) are therefore functions of \( k_d \) and \( p_d \) (and \( k_{em} \), which has a unique steady state value). Thus, the steady state values of \( k_d \) and \( p_d \) are jointly determined by the two equations:

\[
u_c(x) - u_m(x) = 0 \tag{44}
\]

(at the point \( x = (\sum_{i=d,em} \rho_i(A_i k_i^\alpha - \delta k_i), \sum_{i=d,em} \rho_i \frac{\theta_i A_i k_i^\alpha + \gamma_i k_i}{p_i}) \)) And:

\[
\alpha A_d k_d^{\alpha-1} + 1 - \delta + \frac{\gamma}{p_d} - \frac{1}{\beta} = 0 \tag{45}
\]

It is easy to see that the second condition implies a decreasing relationship between \( p_d \) and \( k_d \): when \( k_d \) is higher, the marginal product of capital is lower and a higher value of \( \frac{1}{p_d} \) is necessary for the equality to hold - thus, \( p_d \) is lower. In what follows, I show that when \( \beta \to 1 \), the first condition implies an increasing relationship between \( k_d \) and \( p_d \) in the relevant region for the steady state.

To establish this increasing relationship, I take the following steps. First, I show that \( u_c(x) - u_m(x) \) is decreasing in \( p_d \). Then, I show that \( u_c(x) - u_m(x) \) is increasing in \( k_d \) (for the relevant region of \( k_d \)) - thus, for a higher value of \( k_d \) the intersection with 0 requires a higher value of \( p_d \).

To establish that \( u_c(x) - u_m(x) \) is decreasing in \( p_d \), note that the derivative
with respect to $p_d$ yields the following conditions, for the integrated and the segmented equilibria respectively:

$$
\left( \frac{\partial^2 u}{\partial m^2} - \frac{\partial^2 u}{\partial m \partial c} \right) \frac{1}{p_d^2} \sum_{i=d,em} \rho_i(\theta_1 A_i k_i^\alpha + \gamma_i k_i) < 0
$$

(46)

and, for the segmented equilibrium (where $p_d$ may be different from $p_{em}$):

$$
\left( \frac{\partial^2 u}{\partial m^2} - \frac{\partial^2 u}{\partial m \partial c} \right) \frac{1}{p_d^2} \rho_d(\theta_d A_d k_d^\alpha + \gamma k_d) < 0
$$

(47)

To show that $u_c - u_m$ is increasing in $k_d$, note that the derivative $\frac{\partial (u_c - u_m)}{\partial k_d}$ is given by:

$$
\rho_d((\alpha A_d k_d^{\alpha - 1} - \delta)(\frac{\partial^2 u}{\partial c^2} - \frac{\partial^2 u}{\partial m \partial c}) + \frac{1}{p_d} (\alpha \theta_d A_d k_d^{\alpha - 1} + \gamma)(\frac{\partial^2 u}{\partial m \partial c} - \frac{\partial^2 u}{\partial m^2}))
$$

(48)

The second term of this expression is always positive (since $\frac{\partial^2 u}{\partial m \partial c} \geq 0$ and $\frac{\partial^2 u}{\partial m^2} < 0$). However, the first term may be positive or negative, depending on the sign on $\alpha A_d k_d^{\alpha - 1} - \delta$. The condition $\beta \to 1$ guarantees that this expression is either negative or close to 0. To see this, note that the Euler condition for capital implies:

$$
\alpha A_d k_d^{\alpha - 1} - \delta = \frac{1}{\beta} - 1 - \frac{\gamma}{p_d} < \frac{1}{\beta} - 1 \to_{\beta \to 1} 0
$$

(49)

It follows that for $\beta$ sufficiently close to 1, the expression in equation 48 is positive. Thus, the steady state values of $k_d$ and $p_d$ are uniquely determined by the intersection of the increasing relationship given by equation 44 and the decreasing relationship given by equation 45 (the intersection exists because the Euler condition takes all values in $(0, \infty)$).

**Steady state $k_d$ is higher under integration, and $p_d$ is lower.** Denote segmented steady state values with superscript $s$, and integrated steady state
values with superscript $i$. Furthermore, denote:

$$x^s(p_d, k_d) = \left( \sum_{i=d,em} \rho_i (A_i k_i^\alpha - \delta k_i), \sum_{i=d,em} \rho_i \frac{\theta_i A_i k_i^\alpha + \gamma_i k_i}{p_i} \right)$$  \hspace{1cm} (50)$$

where $p_{em}$ and $k_{em}$ are given by their segmented steady state values, and:

$$x^i(p_d, k_d) = \left( \sum_{i=d,em} \rho_i (A_i k_i^\alpha - \delta k_i), \sum_{i=d,em} \rho_i \frac{\theta_i A_i k_i^\alpha + \gamma_i k_i}{p_d} \right)$$  \hspace{1cm} (51)$$

where $k_{em}$ is given by its steady state value (which is the same in both the segmented and integrated steady state).

Note that for $\theta_{em}$ sufficiently small, the segmented equilibrium features $p^*_d > p^s_{em}$: to see this, note that $c^*_d$ and $c^s_{em}$ are independent from $\theta_{em}$ - thus, $m^s_{em}$ is independent from $\theta_{em}$, and $p^s_{em} = \frac{M^s_{em} \theta_{em} k^\alpha_{em}}{m^s_{em}} \rightarrow \theta_{em} \rightarrow 0$. Thus, at the point $(p^s_d, k^s_d)$ (using equation 47),

$$u_c(x^s(p^s_d, k^s_d)) - u_m(x^s(p^s_d, k^s_d)) = 0 > u_c(x^i(p^s_d, k^s_d)) - u_m(x^i(p^s_d, k^s_d))$$  \hspace{1cm} (52)$$

Using equation 46 that establishes that $u_c(x^i) - u_m(x^i)$ is decreasing in $p_d$ (for a given $k_d$), it follows that the solution $p^i_d(k^s_d)$ that is defined by:

$$u_c(x^i(p^i_d(k^s_d), k^s_d)) - u_m(x^i(p^i_d(k^s_d), k^s_d)) = 0$$  \hspace{1cm} (53)$$

satisfies $p^i_d(k^s_d) < p^s_d$. Since the decreasing relationship implied by the Euler condition ($MPK_d + \frac{s}{p_d} = \text{const}$) is the same in both the segmented and integrated equilibria, the intersection between the $u_c = u_m$ curve and the Euler condition in the $(p_d, k_d)$ space features $p^i_d < p^s_d$ and $k^i_d > k^s_d$.

43
Welfare for \( em \) is lower under the integrated equilibrium. First, note that at the steady state \( p_{em}^* < p_{d}^i = p_d^i \). To see this, note that:

\[
\sum_{i=d,em} \rho_i \frac{\theta_i A_i(k_i^s)^\alpha + \gamma_i k_i^s}{p_i^s} = \frac{1}{p_d^i(k_d^s)} \sum_{i=d,em} \rho_i (\theta_i A_i(k_i^s)^\alpha + \gamma_i k_i^s) \tag{54}
\]

(where \( p_d^i(k_d^s) \) is defined by equation 53). It follows that \( p_{em}^* < p_d^i(k_d^s) < p_d^i \). Thus,

\[
p_{em}^* < p_d^i(k_d^s) < p_d^i = p_{em}^i \tag{55}
\]

For \( \beta \to 1 \), this implies lower permanent income and a tighter budget constraint for \( em \):

\[
\sum_{t=0}^{\infty} A_{em} k_{em,t}^\alpha + (1 - \delta) k_{em,t} - k_{em,t+1} + \frac{\theta_{em} A_{em} k_{em,t}^\alpha}{p_{em,t}} \prod_{t'=0}^t (1 + r_{t'}) \tag{56}
\]

For \( \beta \) sufficiently close to 1, this sum is well approximated by its steady state value:

\[
\frac{\beta}{1 - \beta} (A_{em} k_{em}^\alpha - \delta k_{em} + \frac{\theta_{em} A_{em} k_{em}^\alpha}{p_{em}})
\]

which is decreasing with \( p_{em} \). Thus, in the integrated equilibrium, \( em \) faces a tighter budget constraint than in the segmented equilibrium, and thus equilibrium welfare in \( em \) is lower.

The integrated equilibrium is Pareto inferior (with appropriate transfers). First, write the household’s problem and the social welfare function in recursive forms. The pseudo-representative household’s problem can be written as:

\[
V^h(k_i, M_i^s, p_i | i = d, em) = \max_{c,m,k_i'} u(c, m) + \beta V^h(k_i', M_i'^s, p_i' | i = d, em) \tag{57}
\]
s.t.

\[ c + m = \sum_{i=d,em} p_i (A_i k_i^a + (1 - \delta) k_i - k_i' + \frac{\theta_i A_i k_i^a + \gamma_i k_i}{p_i}) \]  \tag{58}

The household’s optimization with respect to \( k_d' \) yields:

\[ u_c(c, m) = \beta (\frac{\partial V^h}{\partial k_d'} + \frac{\partial V^h}{\partial M_d'} \frac{\partial M_d'}{\partial k_d'}) = \beta (\frac{\partial V^h}{\partial k_d'} + \frac{\gamma}{p_d'}) \]  \tag{59}

**Lemma 2** The equilibrium values of \( c, m, k'_d, k'_e \) are invariant to \( M_d \) and \( M_e \).

To see this, note that the condition \( u_c = u_m \) pins down \( m \) as a function of \( c \); by the goods market clearing condition, \( c \) and hence \( m \) are pinned down as a function of \( k \) and \( k' \); the value of \( k' \) is determined by \( k \), given the Euler condition:

\[ u_c(c(k, k'), m(k, k')) = \beta (\frac{\partial V^h}{\partial k_d'} + \frac{\gamma}{p_d'}) \]  \tag{60}

Thus, \( k', k, m, \) and \( c \) do not depend on \( M \). Therefore, equilibrium welfare can be written as:

\[ V(k_d, k_e) = V^h(k_d, k_e, M_d, M_e, p_d, p_e) \]  \tag{61}

Where \( \frac{\partial V}{\partial k_d} = \frac{\partial V^h}{\partial k_d} \).

To establish that equation 60 has a unique solution, I prove the following lemma:

**Lemma 3** The price \( p_d \) is increasing in \( k_d \).

To see this, consider a marginal increase in \( k_d \) of \( \epsilon \). Note that, using homogeneity and the condition \( u_c = u_m \),

\[ \frac{\partial \ln c}{\partial k_d} = \frac{\partial \ln m}{\partial k_d} \]  \tag{62}

45
Denote aggregate income by:

\[ I = \sum_{i=d, em} \rho_i(A_i k_i^\alpha + (1 - \delta)k_i) \]  

Write:

\[ \frac{\partial \ln c}{\partial k_d} = \frac{\partial \ln c}{\partial \ln I} \frac{\partial \ln I}{\partial k_d} = \frac{\partial \ln c}{\partial \ln I} \frac{\rho_d(\alpha A_d k_d^{\alpha-1} + 1 - \delta)}{\sum_{i=d, em} \rho_i(A_i k_i^\alpha + (1 - \delta)k_i)} \]  

Since \( \frac{\partial \ln c}{\partial \ln I} \leq 1 \) (using standard considerations),

\[ \frac{\partial \ln c}{\partial k_d} < \frac{\rho_d(\alpha A_d k_d^{\alpha-1} + 1 - \delta)}{\sum_{i=d, em} \rho_i(A_i k_i^\alpha + (1 - \delta)k_i)} \]  

In the integrated equilibrium,

\[ \frac{\partial \ln m}{\partial k_d} = \frac{\partial \ln(\sum_{i=d, em} \rho_i(\theta_i A_i k_i^\alpha + \gamma_i k_i))}{\partial k_d} - \frac{\partial \ln \rho}{\partial k_d} \]  

Thus,

\[ \frac{\partial \ln \rho}{\partial k_d} \geq \rho_d(\frac{\alpha \theta_d A_d k_d^{\alpha-1} + \gamma}{\sum_{d, em} \rho_i(\theta_i A_i k_i^\alpha + \gamma_i k_i)} - \frac{\alpha A_d k_d^{\alpha-1} + 1 - \delta}{\sum_{i=d, em} \rho_i(A_i k_i^\alpha + (1 - \delta)k_i)}) \]  

(The subtraction uses \( \frac{\partial \ln m}{\partial k} = \frac{\partial \ln c}{\partial k} \)). It can be verified that this expression is weakly positive:

\[ \frac{\alpha \theta_d A_d k_d^{\alpha-1} + \gamma}{\sum_{d, em} \rho_i(\theta_i A_i k_i^\alpha + \gamma_i k_i)} \geq \frac{\alpha A_d k_d^{\alpha-1}}{\sum_{i=d, em} \rho_i(A_i k_i^\alpha + (1 - \delta)k_i)} \]  

The second inequality follows from \( \theta_d \geq \theta_{em} \), and the third inequality follows from \( A_d \geq A_{em} \). Thus, \( \rho \) is increasing in \( k_d \) in the integrated equilibrium. In
the segmented equilibrium, the argument is the same except that:

\[
\frac{\partial \ln p_d}{\partial k_d} = \frac{\partial \ln M_d}{\partial k_d} - \frac{\partial \ln c_d}{\partial k_d} \geq \frac{\partial \ln M_d}{\partial k_d} - \frac{\partial \ln c}{\partial k_d} \geq \frac{\partial \ln M}{\partial k_d} - \frac{\partial \ln c}{\partial k_d} \tag{69}
\]

This means that equation 60 has a unique solution, since the RHS is decreasing in \(k_d\) and the LHS is increasing in \(k_d\).

The solution for \(k'_d\) is higher under the integrated equilibrium: let \((k^s_d)'\) denote the segmented equilibrium value of \(k'_d\). For \(\theta_{em}\) sufficiently small, \((p^s_d)' > (p^s_{em})'\). Thus, the integrated equilibrium satisfies \((p^i_d)' < (p^s_d)'\), and thus \((k^i_d)' > (k^s_d)'\).

Assume that \(\theta_{em}\) is such that these inequalities hold for all \(k^s_{d,t}, k^s_{em,t}\) along the segmented equilibrium path. Using standard arguments, it is possible to show that \(k^i_d\) is increasing in \(k_d\), and thus \(k^i_{d,t+1} > k^s_{d,t+1}\) for all \(i\).

It is left to show that a decrease in \(k'_d\) is welfare improving. To see this, note that in equilibrium, a marginal increase in \(k'_d\) leads to the following change in welfare:

\[
-u_c + \beta \frac{\partial V}{\partial k'_d} = -u_c + \beta \frac{\partial V^h}{\partial k'_d} < -u_c + \beta \left( \frac{\partial V^h}{\partial k'_d} + \frac{\gamma}{p'_d(k'_d)} \right) = 0 \tag{70}
\]

Thus welfare is higher if \(k'_d\) is lower in the next period. It has been established that segmentation at \(t = 1\) improves welfare. The standard recursive argument shows that welfare is improved if markets are segmented in every period.

**Steady state** \(p_d\) is increasing with \(\gamma\). A higher \(\gamma\) is an upward shift of equation 45 in the \((p, k)\) space. What happens for the relationship in equation 44? For a given \(k\) and \(p\), a higher \(\gamma\) implies a higher value of \(m\); thus, \(u_c - u_m\) is larger. Thus, \(p(k, \gamma)\) is increasing in \(\gamma\).
$k_{d,t+1}$ is minimized for every $t$ when $\gamma = 0$. Note that, for a given $k_d$, $k'_d$ is lowest when $\gamma = 0$ (see equation 60). Since $k'_d$ increases with $k_d$ (using standard arguments), it follows through a recursive argument that $k_{d,t+1}$ is lower for all $t$ when $\gamma = 0$.

With appropriate transfers, welfare is maximized with $\gamma = 0$. To see this, note that by equation 70, holding $V^h$ constant, welfare of the pseudo-representative household is highest when $\gamma = 0$. The recursive argument establishes that welfare is maximizes when $\gamma = 0$ for every period.