Effects of Driven Waves on Solar, Magnetospheric, Ionospheric, and Laboratory Plasma Dynamics

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Abstract

An essence of the driven wave formalism is discussed. This new method is suitable for wave processes in the inhomogeneous medium, and it reveals more complex nature of a initial waveform evolution than the standard WKB approach. Using this formalism, several very important processes, which have not been addressed before, were discovered.

In this talk, I will review results of our earlier studies about
1. how the driven waves are generated,
2. how they can be described analytically,
3. how driven waves can influence plasma particles, and
4. What are possible effects of the driven waves on the solar, magnetospheric, ionospheric and laboratory plasma dynamics.
Waves in Inhomogeneous Plasma

The method is **GENERAL** and can be applied to any kind of inhomogeneity in the background plasma.
Waves in Inhomogeneous Plasma

\[ A^{n\times n} \dot{e}^{n\times 1} + B^{n\times n} e^{n\times 1} = C^{n\times n} e^{n\times 1} \]

e – nx1 matrix representing linear disturbances
A – nxn matrix
B – nxn matrix-operator; derivatives operate on e
C – nxn matrix consists of linear derivatives of ambient plasma
MHD Waves In homogeneous Plasma

Solution = \[ \sum_{i=\text{Nat.Modes}} \sum_{\pm} A_{i0\pm} \exp(\pm i \omega_{i0\pm} t) + A_{i1\pm} \exp(\pm i \omega_{i1\pm} t) + A_{i2\pm} \exp(\pm i \omega_{i2\pm} t) + \ldots \]

\[ A_{in\pm} \propto S^n \]

\[ \omega_{in\pm} = \omega_{i0\pm} + \omega_{i1\pm} S^1 + \omega_{i2\pm} S^2 + O(S^3) \]
Driven Waves: Basic Idea

Inhomogeneties in the system

Alfven wave + Alfven wave with all natural freq.

Solution = \sum_{i=\{s,A,f\}} \sum_{i=\{0,1,2\}} A_{i0\pm} \exp(\pm i\omega_{i0\pm} t) + A_{i1\pm} \exp(\pm i\omega_{i1\pm} t) + A_{i2\pm} \exp(\pm i\omega_{i2\pm} t) + ... 

where \( A_{in\pm} \propto S^n \)

\( \omega_{in\pm} = \omega_{in0\pm} + \omega_{in1\pm} S^1 + \omega_{in2\pm} S^2 + S(3) \)

The method is **GENERAL** and can be applied to any kind of inhomogeneity in the background plasma.

**References:** Kaghashvili, 2007
Driven Waves: Basic Idea

References: Hollweg, Kaghashvili and Chandran, 2012
Applications
The Solar Puzzle

- Solar Structure:
  - Photosphere (visible light) $\approx 6000$K (from the blackbody spectrum)
  - Corona – a few $10^6$K (from the presence of highly ionized heavy ions)

- Puzzle: what heats the corona?

Reference: Priest, Solar Magnetohydrodynamics, 1982
What Heats the Corona?

Related questions:

- how is the energy reaching the photosphere channeled upwards and dissipated?
- what accelerates the solar wind?
- what produces the high-speed streams?
- what causes changes in the solar wind during the solar cycle?
- what triggers energetic events like CMEs, solar flares, etc.?
  
  (interaction of the solar wind and CMEs with Earth's magnetic field creates what we call Space Weather)

- what process(es) accelerates and produces super-thermal particles?
Some Key Advances in Our Understanding
(mainly related to Solar Physics)

• Photospheric motion + magnetism is the source

• Role of magnetic reconnection? Does reconnection launch waves?

• Waves are important throughout the solar atmosphere
  - high-frequency waves can resonantly interact with particles
  - wave pressure can play an important role
  - Alfvén wave energy may well be adequate to compensate energy losses
    (e.g. Hollweg 1973; Hollweg and Isenberg 2002; Cranmer, van Ballegooijen & Edgar 2007; Song and Vasyliūnas, 2011)

• Recent observations of waves in the solar atmosphere
  - low frequency spectrum (periods ~ minutes)
    (e.g. Tomczyk et al. 2007; De Pontieu et al. 2007; Jess et al. 2009; Tomczyk and McIntosh 2009; Gupta et al. 2010; McIntosh et al. 2011)

  (High frequency ion-resonant spectrum has not been observed without spacecraft)
Waves in the Solar Atmosphere

- MagnetoHydroDynamic (MHD) Waves
  - 3 restoring forces: gas pressure/magnetic pressure/magnetic tension
  - 2 characteristic speeds: sound speed/Alfvén speed
  - 3 different Modes: Alfvén wave, Fast and slow magnetosonic waves

- MHD waves are capable of transporting energy and momentum

- Slow magnetosonic waves
  - energy flux is low in low-β plasma
  - it cannot explain correlation between energetic structures and $B$
  - compressive; strongly damped

- Fast magnetosonic waves
  - most likely to be totally reflected (Hollweg 1978):
    \[ k_{\parallel}^2 = \frac{\omega_f^2}{v_a^2} - k_{\perp}^2 \]
  - compressive; strongly damped
Alfvén Wave

- Its existence was theoretically shown by Hannes Alfvén in 1942
  - incompressible; not readily damped
  - energy flux along $\mathbf{B}$ at $v_a$

- Omnipresent “super” mode
  - laboratory experiments:
    (e.g. Lundquist, 1949; Gekelman et al. 1997)
  - solar wind
    (e.g. Coleman 1966, 1967; Unti and Neugebauer, 1968; Belcher, Davis and Smith, 1969; Belcher and Davis, 1971)
  - geophysical plasma
    (e.g. Kletzing, 1994; Génot et al. 1999, 2004; Chaston et al. 2002; Lysak and Song, 2006)

- Inhomogeneities in the plasma can modify an initial waveform and couple into other modes
  (e.g. Melrose 1977; Hollweg and Lilliequist 1978; Wentzel 1989; Chagelishvili et al. 1996; Kaghashvili 1999)
Shear Flow Example

- **Problem:**
  How do Alfvén waves linearly evolve in a background velocity shear?

  - example profile: Couette flow

    \[
    \vec{V}_0 = (V_{0x}(y), 0, 0) \quad \vec{B}_0 = (B_{0x}, 0, 0) \quad S_y \equiv \frac{\partial V_{0x}(y)}{\partial y}
    \]

  - **Driven Waves:**

    \[
    \phi = \phi(S_y) \propto \sum \exp(\pm i\omega_A t) + \exp(\pm i\omega_S t) + \exp(\pm i\omega_F t)
    \]

  - **What are the driven waves?**

    - directly driven non-WKB waves due to the *linear interaction* of a single mode and the background inhomogeneity.
    - main source of energy is the flow inhomogeneity.
      
        (solar example: Alfvén wave recovers its original amplitude for a long time.)
    - they are not the usual normal modes

(References: Kaghashvili 2007; Hollweg and Kaghashvili 2012; Kaghashvili 2012)
SOLUTIONS:  

\( v_{\{y,z\}} = v_{\{y,z\}0}e^{-ik_xv_at} + \frac{k_{\{y,z\}}S_yv_y0}{2k^4v_a^2}\left\{ -i(k - k_x)^2e^{ikvt} - i(k + k_x)^2e^{-ikvt} + 2e^{-ikxv_at}\right\} \)

\( b_{\{y,z\}} = b_{\{y,z\}0}e^{-ik_xv_at} + \frac{k_xk_{\{y,z\}}S_yv_y0}{2kk^4v_a^2}\left\{ -i(k - k_x)^2e^{ikvt} + i(k + k_x)^2e^{-ikvt} - 2ke^{-ikxv_at}\right\} \)

\( v_x = -i\frac{S_yv_y0}{k_xv_a^2}e^{-ik_xv_at} \)

\( b_x = \frac{S_yv_y0}{2kk^2v_a^2}\left\{ i(k - k_x)^2e^{ikvt} - i(k + k_x)^2e^{-ikvt} + 4ik_xke^{-ikxv_at} \right\} \)

(Ref.: Hollweg and Kaghashvili, 2012; Kaghashvili, 2012)
Non-zero phase-averaged electric field:

\[ E_x = -\frac{1}{c} \{ \delta v_y \delta b_z - \delta v_z \delta b_y \} \equiv \langle E_x \rangle \]

\[ E_y = \frac{1}{c} V_0 \delta b_z - \frac{1}{c} B_0 \delta v_z - \frac{1}{c} \{ \delta v_z \delta b_x - \delta v_x \delta b_z \} \equiv \frac{1}{c} V_0 \delta b_z - \frac{1}{c} B_0 \delta v_z + \langle E_y \rangle \]

\[ E_z = -\frac{1}{c} V_0 \delta b_y + \frac{1}{c} B_0 \delta v_y - \frac{1}{c} \{ \delta v_y \delta b_x - \delta v_x \delta b_y \} \equiv -\frac{1}{c} V_0 \delta b_y + \frac{1}{c} B_0 \delta v_y + \langle E_z \rangle \]

An example, electric field parallel to \( B_0 \):

\[
\frac{c E_x}{B_0 v_a} = \frac{S_y v_{y0} (k_z v_{y0} - k_y v_{z0})}{2k k_{\perp} v_a^3} \times \cos(k v_a t - \vec{k} \cdot \vec{r})
\]

\[
\times \left\{ (k + k_x) \sin(k v_a t - \vec{k} \cdot \vec{r}) - (k - k_x) \sin(k v_a t + \vec{k} \cdot \vec{r}) + 2k \sin(\vec{k} \cdot \vec{r} - k x v_a t) \right\}
\]

(Reference: Kaghashvili, 2012)
Example Case: Solar Corona
(Cold-Plasma Limit)

• Initial Conditions:
  - \( k_x = 2\pi/(1.6 \times 10^5) \) \( km/s \) \( \quad k_y = k_z = 2\pi/(2 \times 10^4) \) \( km/s \)
  - \( |\delta v_{\perp 0}| = \sqrt{\delta v_y^2 + \delta v_z^2} = \sqrt{2}|\delta v_{y_0}| = 100 \) \( km/s \); \( \quad v_a = 800 \) \( km/s \) \( ; \quad B_0 = 5G \)
  - \( S_y \) calculation: 100 \( km/s \) flow speed change over \( 2 \times 10^4 \) \( km \): \( Sy = 0.005 \)
    (e.g. Kaghashvili et al. 2009; Hollweg and Kaghashvili 2012)

• Characteristics Periods:
  - \( T_a = 160 \) \( sec \) \( \quad and \quad T_s = 14 \) \( sec \)

• Electric Field Parallel to \( B_0 \) in \( mV/m \)
  \[
  E_x \approx 39.8 \times \cos\left(\frac{2\pi}{T_a} - \hat{k} \cdot \hat{r}\right) \times \left\{1.09 \times \sin\left(\frac{2\pi}{T_f} - \hat{k} \cdot \hat{r}\right) - 0.91 \times \sin\left(\frac{2\pi}{T_f} + \hat{k} \cdot \hat{r}\right) + 2 \times \sin\left(\hat{k} \cdot \hat{r} - \frac{2\pi}{T_a}\right)\right\}
  \]

• Dreicer electric field for electrons (run-away acceleration):
  - Cool Corona: \( 7 \times 10^{-2} \) \( mV/m \) \( \quad \) Hot Corona: \( 2 \times 10^{-1} \) \( mV/m \)

(Reference: Kaghashvili 2012)
Particle Dynamics in the DW Field

- Particle Parallel velocity
  \[
  \frac{dV_{\parallel}}{dt} = - \frac{q}{m} \left( \frac{B_0}{B} \right) E_x + \vec{V}_j \cdot \frac{d}{dt} \left( \vec{B} \right)
  \]

- Particle Kinetic Energy (related to the first adiabatic invariant)
  \[
  \frac{d}{dt} \left( \frac{m}{2} \left[ (V_{jx} - v_a - V_0)^2 + V_{jy}^2 + V_{jz}^2 \right] \right) = - S y m (V_{jx} - v_a - V_0) V_{jy} - \frac{q B_0}{c} V_a \left( V_{jy} \gamma_D - V_{jz} \gamma_D \right)
  \]

\[
\gamma_{D\{y,z\}} = b_{D\{y,z\}} + V_{D\{y,z\}} = \frac{k_{\{y,z\}} S y v y_0}{2 k \overline{k}_v^2 v_a^2} \left\{ (k - k_x) \sin(\vec{k} \cdot \vec{r} + k v_a t) + (k + k_x) \sin(\vec{k} \cdot \vec{r} - k v_a t) - 2 k \sin(\vec{k} \cdot \vec{r} - k x v_a t) \right\}
\]

- Acceleration in (ExB + Polarization Frame)
  \[
  \vec{a}_D = \frac{1}{B} \left( \vec{V}_E \times \frac{d (\vec{b})}{dt} \right) \frac{\vec{B}}{B}
  \]

(Reference: Kaghashvili 2012)
Driven Wave Generated Electric Field
(Other possible applications)

- In complex magnetic structures of the lower solar atmosphere where the flows are frequently detected (e.g. Winebarger et al. 2002; Del Zanna, 2008; Ofman and Wang, 2008; McIntosh et al. 2010)

- In the solar wind that exhibits regions of large cross-field velocity gradients. Such situations exist during the stream-stream interactions as well when the solar wind interacts with the Earth’s magnetosphere.

- Near Earth environment; waves present in the solar wind are able to penetrate into the magnetosphere-ionosphere system

- In the laboratory experiments where the plasma motion is localized by the magnetic field and complex sheared velocity field exists. The wall boundaries of the devices can introduce the velocity gradient in the plasma as well.

- Turbulence Generation

- Reconnection physics
- Electron-proton plasma,
- A homogeneous background magnetic field aligned flow,
- The x-axis is parallel to $B_0$,
- The background flow for both electrons and protons is along the magnetic field, where characterizes the shearing of the flow along the y-axis.
- The coordinate system is fixed relative to the mean flow, and it is in this frame where the background flow with the linear cross-field shear is given by

$$V_0 \equiv V_{0\{e,p\}} = \left(S_y y, 0, 0\right)$$
Kinetic Driven Waves

(relevant to: Magnetosphere, Solar flare, Lab. Plasma, etc.)

\[
\frac{d^2 v_y}{dt^2} + \frac{(k_x^2 + K_y^2) v_a^2}{1 + \lambda^2 k^2} v_y + \frac{K_y k_z v_a^2}{1 + \lambda^2 k^2} v_z + i \frac{k_x v_a}{\Omega_\mu} \frac{k^2 v_a^2}{1 + \lambda^2 k^2} b_z = f_{VY}
\]

\[
\frac{d^2 v_z}{dt^2} + \frac{K_y k_z v_a^2}{1 + \lambda^2 k^2} v_y + \frac{(k_x^2 + k_z^2) v_a^2}{1 + \lambda^2 k^2} v_z - i \frac{k_x v_a}{\Omega_\mu} \frac{k^2 v_a^2}{1 + \lambda^2 k^2} b_y = f_{VZ}
\]

\[
\frac{d^2 b_y}{dt^2} + \left[ \frac{(k_x^2 + K_y^2) v_a^2}{1 + \lambda^2 k^2} + \frac{k^2 v_a^2}{\Omega_\mu^2 (1 + \lambda^2 k^2)^2} \right] b_y + \frac{K_y k_z v_a^2}{1 + \lambda^2 k^2} b_z + i \frac{k_x v_a}{\Omega_\mu} \left[ \frac{K_y k_z v_a^2}{1 + \lambda^2 k^2} v_y + \frac{(k_x^2 + k_z^2) v_a^2}{1 + \lambda^2 k^2} v_z \right] = f_{BY}
\]

\[
\frac{d^2 b_z}{dt^2} + \frac{K_y k_z v_a^2}{1 + \lambda^2 k^2} b_y + \left[ \frac{(k_x^2 + k_z^2) v_a^2}{1 + \lambda^2 k^2} + \frac{k^2 v_a^2}{\Omega_\mu^2 (1 + \lambda^2 k^2)^2} \right] b_z - i \frac{k_x v_a}{\Omega_\mu} \left[ \frac{(k_x^2 + K_y^2) v_a^2}{1 + \lambda^2 k^2} v_y + \frac{K_y k_z v_a^2}{1 + \lambda^2 k^2} v_z \right] = f_{BZ}
\]

An example, forcing term:

\[
f_{VY} = -i S_y \left( \frac{2(1 + \lambda^2 k_z^2)}{1 + \lambda^2 k^2} K_y b_z + \frac{1 + 2\lambda^2 (1 + k_z^2)}{1 + \lambda^2 k^2} k_z b_z \right).
\]

(Reference: Kaghashvili, 1999, Kaghashvili, 2014)
Kinetic Driven Waves
(relevant to: Magnetosphere, Solar flare, etc.)

Figure 1. Modified Alfvén (dashed line) and fast magnetosonic wave (solid line) frequency plots. Three representative cases of the propagation angle between $k$ and $B_0$ are chosen to show the separation of the frequency curves. For a given driver Alfvén wave, vertical lines cross the frequency curves at the generated driven wave frequency values.

(Reference: Kaghashvili, 1999, 2014)
Driven Wave Effects on Turbulence Generation

see:

Hollweg, Kaghashvili and Chandran:
Velocity-shear-induced Mode Coupling in the Solar Atmosphere and Solar Wind: Implications for Plasma Heating and MHD Turbulence
Large Scale Modeling: Driven Wave Effects

\[
\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot (\bar{\rho} \bar{v}) = -\nabla \cdot \left( \langle \rho \, v \rangle \right)
\]

\[
\bar{\rho} \left( \frac{\partial \bar{v}}{\partial t} + \bar{v} \cdot \nabla \bar{v} \right) + \nabla \bar{P} + \frac{1}{4\pi} \left[ \nabla \times \bar{B} \right] \times \bar{B} = \langle V \rangle_{PE}
\]

\[
\frac{\partial \bar{B}}{\partial t} + \nabla \times \left( \bar{v} \times \bar{B} \right) = -\nabla \times \left( \langle v \times B \rangle \right)
\]

\[
\frac{\partial \bar{P}}{\partial t} + (\bar{v} \cdot \nabla)\bar{v} + \gamma \bar{P} (\nabla \cdot \bar{v}) = -\langle (v \cdot \nabla) v \rangle - \gamma \langle P (\nabla \cdot v) \rangle
\]

- Current Models: Only wave pressure exists for Alfvén waves,
- Considering the effects of inhomogeneities:
  All phase-averaged terms are contributing for driven waves.
Summary

• Driven waves exert the wave force on particles
  - it can be important for the solar wind dynamics

• Low-frequency spectrum can effectively interact with particles

• Electric fields generated by the driven waves can be important in the laboratory, terrestrial and astrophysical plasma processes where the inhomogeneous flows are present.

- Ready to be applied for many practical solar, magnetospheric, laboratory problems.